An introduction to the Pollution Routing Problem and its behaviour

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Glossary

I Set containing the identifier for each point

$\bar{I} := (i, j)$ Set containing the identifier for each arc

S Set containing the sequence of stops

$J := (j_s)_{s \in S}$ A family of values that represents each point on a given path

$\bar{J} := (i_r, j_s)_{s,r \in S}$ A family of values that represents each arc on a given path

$\mathscr{I}$ Specifies the variant of the Excel implementation

$\mathscr{N}$ A set which contains all points/nodes in a given scenario, with $\acute{n} \in mathscrN$ representing a point.

$\mathscr{A}$ A set which contains all arc in a given scenario, with $\acute{a} \in mathscrA$ representing an arc.

$\mathscr{C}$ Represents the parameter group *Costs*

$\mathscr{P}$ Represents the parameter group *Product*

$\mathscr{V}$ Represents the parameter group *Vehicle*

$\mathscr{W}$ Represents the parameter group *World*

$X := (x_i)_{i \in I}$ Family of values that contains the x-coordinate for each point

$Y := (y_i)_{i \in I}$ Family of values that contains the y-coordinate for each point

$Q := (q_i)_{i \in I}$ Family of values that contains the demand of each point

$\widetilde{AB} := (\widetilde{ab_i})_{i \in I}$ Family of tuples that contains the lower and upper bound of the time window at each point

$A := (a_i)_{i \in I}$ Family of values that contains the lower bound at each point

$B := (b_i)_{i \in I}$ Family of values that contains the upper bound at each point

$\acute{T} := (\acute{t_i})_{i \in I}$ Family of values that contains the serving time required at each point

$V := (v_{ij})_{i,j \in I}$ Family of values that contains the speed travelled on each arc

$\widetilde{LU} := (\widetilde{lu_{ij}})_{i,j \in I}$ Family of tuples that contains the lower and upper speed limit on each arc

$L := (l_{ij})_{i,j \in I}$ Family of values that contains the lower speed limit on each arc

$U := (u_{ij})_{i,j \in I}$ Family of values that contains the upper speed limit on each arc

$D := (d_{ij})_{i,j \in I}$ Family of values that contains the distance of each arc

$\Theta := (\theta_{ij})_{i,j \in I}$ Family of values that contains the road angle on each arc

$F := (f_{ij})_{i,j \in I}$ Family of values that contains the weight of the vehicle at on each arc

$T := (t_{ij})_{i,j \in I}$ Family of values that contains the time required for travelling an arc

$P := (p_{ij})_{i,j \in I}$ Family of values that contains the total energy consumed on each arc

$Pw := (pw_{ij})_{i,j \in I}$ Family of values that contains the curb weight based energy consumption on each arc

$Pf := (pf_{ij})_{i,j \in I}$ Family of values that contains the cargo weight based energy consumption on each arc

$Pv := (pv_{ij})_{i,j \in I}$ Family of values that contains the travelling speed based energy consumption on each arc

$\hat{\alpha} := (\alpha_{ij})_{i,j \in I}$ Family of values that contains the arc coefficient of each arc

β Vehicle coefficient

$C := (c_{ij})_{i,j \in I}$ Family of values that contains the total cost that occurred on each arc

$Cd := (cd_{ij})_{i,j \in I}$ Family of values that contains the driver cost that occurred on each arc

$Cf := (cf_{ij})_{i,j \in I}$ Family of values that contains the fuel cost that occurred on each arc

$Ce := (ce_{ij})_{i,j \in I}$ Family of values that contains the emission cost that occurred on each arc

$T_{0j} := (t_{0j})_{j \in I}$ Family of values that contains the travelling time from the origin to a point j

$\acute{p}_d$ Wage of the driver

$\acute{p}_f$ Fuel price

$\acute{p_e}$ Emission cost

$\rho_{product}$ Density of the product

$V_{product}$ Volume of the product

K_v Capacity of the vehicle (Volume)

K_w Capacity of the vehicle (Weight)

K Capacity of the vehicle

M_{max} Maximum weight of the vehicle

V_{max} Volume of the trailer

w Curb weight of the vehicle

A Surface area of the vehicle

η Fuel efficiency of the vehicle

ω_F Energy content in a litre of fuel

ω_{GHG} Carbon content in a litre of fuel

$\acute{c_d}$ Drag coefficient

a Acceleration of the vehicle

ρ_{air} Density of the air

$\acute{c_r}$ Rolling resistance coefficient

g Gravity

r Radius

Chapter 1

Introduction

This paper is concerned with building a basic understanding of the Pollution Routing Problem (PRP) and its behaviour. This is accomplished by investigating the topic on several levels of detail. As depicted in Figure 1.1, the most general level

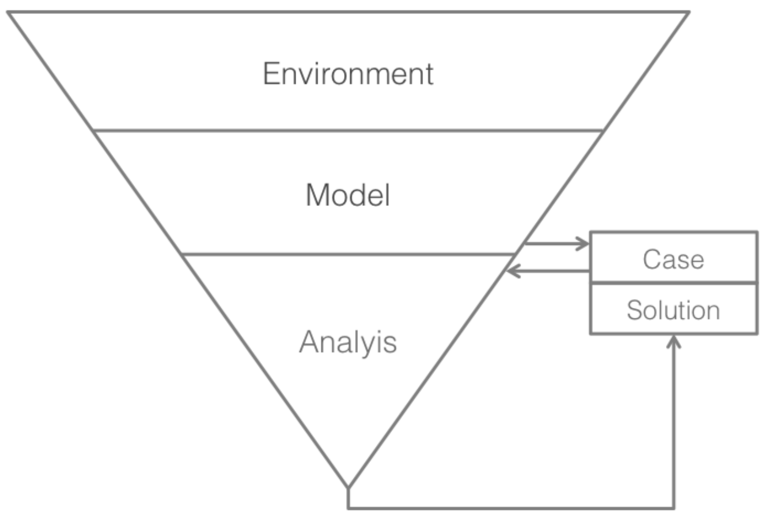


Figure 1.1: Introduction - Structural Overview

investigates the environment in which the PRP is placed. That is, before any further investigations investigations into the behaviour of the model are made, the context in which the PRP exists has to be introduced. The intent behind this is to paint a general picture, which should aid with the mental placement of the content introduced at later stages. Chapter 2, which is constructed as a literature review provides this general overview. Since, the PRP is part of the Green Vehicle Routing Problems (GVRP), which is a group of Vehicle Routing Problem (VRP) variants that include environmental concerns in conventional VRP variants, Chapter 2 will not only investigate the motivation behind introducing environmental factors into the VRP, but also introduce the different varieties of VRP. Having introduced the environment in which the PRP is placed, Chapter 2 continues, by investigating essential components of the PRP. That is, it briefly introduces various solving algorithms and models for estimating the green house gas (GHG) output of a vehicle. This includes the most common parameter used for estimating fuel consumption, as well as the three main types of fuel consumption models. Lastly, based on the insight obtained throughout this chapter a method for categorising and identifying VRPs is suggested. By discussing the above mentioned topics the following questions should be answered.

- Why is it important to introduce environmental factors into the VRP?
- What is a GVRP, how is it structured and what is its relationship to the VRP?
- How can a GVRP be categorised?

While the first stage is concerned with providing an introduction into and overview of the topic of the PRP, by researching the various sources of literature, the second level has a more narrow view onto the topic, as it only discusses one version of the PRP. That is, Chapter 3 introduces a version of the PRP presented by Bektaş and Laporte (2011), as well as suggests a possible method for implementing such a model into Excel. This is done by conveying the structure of the model, by introducing the required parameter and by means of providing a step by step solution for implementing said model into Excel. Due to the descriptive nature of that chapter, no research questions are answered. In order to properly investigate the behaviour of the model it has to be executed. However, to do so, input values are required. Therefore, a case study is created, which is introduced in Chapter 4. This case not only provides a set of parameter, against which the model is executed, but also packages these values in a narrative. That is, the case consists out of background information used for setting up the narrative, as well as a set of problems, which serve as foundation for the last level, i.e. the Analysis. Finally, the last level provides the most detailed view onto the PRP, which by extension indicates a rather narrow perspective on the topic. Generally speaking, the one version of the PRP introduced in Chapter 3 is combined with the scenarios given in Chapter 4 to create the analysis found in Chapter 5. This chapter does not only provide a more detailed solution for the case, but also investigates a variety of different aspects related to the behaviour of the PRP. That is, the first section tries to identify whether objective load or objective distance performs closer to the objective that minimises emissions. The second section investigates whether it is possible to approximate the behaviour of the fuel consumption model, given a minimal amount of information. Additionally, it also discusses a possibility for comparing two similar objective functions. The third part of Chapter 5 investigates methods for selecting an appropriate vehicle and how the choice of the vehicle impacts the output. The forth section of this chapter compares behaviour of the objective cost against the behaviour of the objective emissions. Moreover, it also tries to determine, how the implementation of variable speed influences the model and more importantly, when is it sensible to actually implement variable speed. Finally, the last section tries to investigate the restrictive capabilities of time windows with and without variable speed. Given the topics introduced above, Chapter 5 should answer the research questions provided below.

- By comparing the objectives distance and load, is it possible to identify which objective produces results that are closer to the results obtained by optimising for GHG emissions?
- Is it possible to mimic the behaviour of the fuel consumption model, given a minimal amount of input?
- How can the most suitable vehicle be chosen and how does the choice of vehicle impact the output of the model?
- What are the differences between the objective cost and the objective emission?
- In which cases is it necessary to incorporate variable speed into the model and how does it effect the output of the model?
- How does the introduction of time window constraints impact the output of the model?

Lastly, the results discussed in Chapter 5, are compressed into a brief summary, which only provides the most important arguments required for solving the case. This summary is structured in form of a teaching note and can be found in the second part of Chapter 4. Apart from presenting a possible solution it also briefly summarises the case and its objectives.

To summarise, Chapter 2 serves as an introduction into the topic, which places the PRP into context. Secondly, Chapter 3 introduces a version of the PRP presented by Bektaş and Laporte (2011), as well as provides a guideline for implementing said model into Excel. Thirdly, Chapter 4 constructs a case, which servers as a foundation for analysing the behaviour of the model. Additionally, a brief teaching note is provided. Thereby opening up the possibility for utilising the case for educational purposes. Lastly, Chapter 5 expands upon the solution provided by the teaching note, by investigating the underlying behaviour of the model in detail.

Chapter 2

Literature Review

The following chapter tries to serve as an introduction into the topic of the PRP. However, as it is intended to provide a general overview over said topic, it mainly revolves around the GVRP, which is the group of VRP variants that contains the PRP. This introduction is accomplished by investigating the questions

- Why is it important to introduce environmental factors into the VRP?
- What is a GVRP, how is it structured and what is its relationship to the VRP?
- How can a GVRP be categorised?

The first Section 2.1, tries to answer the first question by briefly introducing the VRP. Furthermore, this section also investigates the negative impact of freight transport on the environment as well as the predicted effects of climate change. Section 2.2 is concerned with the environment in which the GVRP and especially the PRP is embedded. In this case environment refers to how the GVRP is connected to the VRP, how are environmental factors introduced into the VRP and how are these kind of problems solved. Lastly, Section 2.3 tries to build upon the knowledge obtained in Section 2.2 by trying to create an easy method for distinguishing a GVRP based on its elements.

2.1 Introduction

Before, any variant of GVRP can be investigated further, the original idea behind the VRP shall be introduced. This is done in the first part of this section. Furthermore, the second part of this section investigates the motivation behind introducing environmental concepts into the VRP. To do so some of the negative effects associated with freight transport are introduced. Additionally, potential environmental consequences resulting from the accumulation of GHG emissions in the atmosphere are also explored, since this development can to some extend be related to freight transport.

2.1.1 The Vehicle Routing Problem

The Vehicle Routing Problem or short VRP is part of a greater group of routing problems, which include the Shortest Path Problem, the Chinese Postman Problem, the Rural Postman Problem, the Dial-a-ride Service Tour Problem, the Arc Routing Problem and the Travelling Salesmen Problem. The VRP was introduced in Dantzig and Ramser (1959) under the name *The Truck Dispatching Problem*. It took five years until Clarke and Wright (1964) coined the term VRP. This problem builds upon the idea of the Travelling Salesmen Problem or short TSP, which is concerned with finding the shortest route required in order to visit each city of a given set of cities. The VRP, although building on the core idea of the TSP, distinguishes itself by being more general. That is, it can be expanded by imposing additional restrictions such as having a path, which includes the point of origin twice, i.e. a round trip, or having to dispatch products at each point. In order to understand more complicated versions of the VRP one first has to grasp the underlying concept of the VRP, this is done by examining the original version proposed in Dantzig and Ramser (1959). It is important to express that the notation found in the example below is adapted to be consistent with the notation in the rest of this paper:

There exists a family of points $\mathscr{N} := (n_i)_{i \in I}$, whereby $\forall i \in I \setminus \{o\} : n_i :=$ *station point* and $n_o :=$ *terminal point*. For convenience $I^X := I \setminus \{o\}$. The points are connected by arcs which can be expressed as family $\mathscr{A} := (a_{ij})_{i,j \in I}$. Each point and each arc has a certain set of attributes. For example, $\forall i, j \in I : a_{ij}$ there is an element in $D := (d_{ij})_{i,j \in I}$, which specifies the length of a_{ij}. Moreover, $\forall i \in I : n_i$ there is an element in $Q := (q_i)_{i \in I}$, which describes its demand. If the total demand $\sum_{i \in I} q_i$ surpasses the capacity of the truck C, i.e. $\sum_{i \in I} q_i > C$ the truck has to return to the terminal point, which concludes a sub-route m. However, it is vital that the demand q_i must never exceed the capacity of the truck C, i.e. $\forall i \in I : C \geq qi$. The family $X = (x_{ij})_{i,j \in I}$ indicates the pairing of two points. For example, $\exists! i, j \in I : x_{ij} = x_{ji} = 1$ means that point n_i and point n_j are connected by a path, whereas $x_{ij} = x_{ji} = 0$ indicates the opposite case. These x values have to be assigned in a way that every point is connected to at most one other point,$\forall i \in I^X \exists! j \in I^X : x_{ij} = x_{ji} = 1$ which means each station point can only be visited once. The exception in this case would be the terminal point, as it has to be connected with two points in order to ensure a round trip, i.e. $\exists! o \in I : \exists j \in I : x_{oj} = x_{jo} = 1$. Additionally, these relations have to be defined in order to minimise the objective, e.g. distance, which can be calculated as seen in the following equation (Eksioglu et al., 2009, Dantzig and Ramser, 1959, Laporte, 2007)

$$D = \sum_{i,j \in I} d_{ij} \cdot x_{ij}.$$

Even if the VRP originated from the TSP, the latter can be viewed as merely a specification of the former. That is, the TSP is a VRP in which the total demand $\sum_{i \in I} q_i = \infty$ and the amount of sub-routes $m = 1$. Moreover, one generalisation of the original VRP is the classical VRP, which allows to attribute any desired metric to a specific arc. This opens the model up to other objectives than distance, such as cost or even emissions (Laporte, 2007). Having the VRP introduced the next step is to investigate the utility of combining the VRP with environmental metrics.

2.1.2 Environmental Impact of Freight Transport

The VRP is closely related to freight transport, thus before it is possible to determine the motivation behind including environmental concerns into the VRP, the environmental impact of freight transport has to be investigated. Firstly, freight transport is correlated with economic growth. Additionally, freight transport is a major source of pollution. This means the amount of freight traffic correlates with economic growth and with the total output of greenhouse gas emissions (GHG emissions). Hence, with an increase in Gross Domestic Product (GDP) it is likely that the total amount of freight transport will also increase, thus leading to a rise in GHG emissions. This assumption is further strengthened by the observation that the world experienced a decline in GHG emissions, due to the financial crisis, which can partly be attributed to a reduction of freight transport during that time. In 2010 the transport sector accounted for 23% of the global fossil fuel consumption and was responsible for about 15% of the total greenhouse gas emissions. However, the road based freight transport alone emitted around 30% to 40% of the emissions caused by the freight transport industry. Unfortunately, it is predicted that this share will rise above 50% by 2050. This is especially alarming, since the total amount of freight transport is expected to increase by over 230% during the same time (OECD/ITF, 2010, 2015, Demir et al., 2014). Furthermore, with an increase in road based freight transport, one can assume that other externalities such as accidents, other variants of pollution, noise, resource consumption and land deterioration will become more prominent as well. This assumption only holds if major technological changes are neglected. In order to understand the scope and impact of the externalities a brief overview is provided. Cullinane and Edwards (2010) served as a foundation for the quick overview over the negative consequences of freight transport.

Accidents: Apart from imposing additional economic costs on the society as a whole, this by-product of road traffic poses a direct threat to human lives and their well-being. The momentum of a vehicle, the state of the traffic, the condition of the road and the weather are all factors that directly influence the likelihood of an accident to occur, as well as its severity. In commercial freight transport another highly significant cause of accidents is the lack of sleep. With estimations attributing 15% to 25% of the accidents in commercial freight transport to the driver fatigue (Cullinane and Edwards, 2010, Goel and Vidal, 2013).

Emissions: Many components found in the exhaust of a vehicle are the product of an incomplete combustion process. According to Cullinane and Edwards (2010) these components impact the environment of three different scopes. Firstly, the local level. Hereby the area of concern are the health risks associated with the pollution produced during the combustion process. Some major areas of impact on human health are damage of the respiratory system and the cardiovascular system. Apart from that their carcinogenic qualities pose an additional threat to human well-being. The effects measured on a regional level include acid rain or photochemical smog. Apart from the fact that the latter one is suspected to cause respiratory problems, the regional level tends to be concerned with the negative impacts of emissions on the regional environment. Fortunately, acid rain, which interferes with the growth of flora and fauna, is in decline, since low-sulphur fuels have been introduced. The last area of impact resides on the global scale and is concerned with climate change. Hereby the above mentioned greenhouse gas emissions, which are measured in CO2 equivalents, are responsible for an increase in global temperatures (Cullinane and Edwards, 2010).

Noise: The propulsion noise of the engine, the road contact noise and the aerodynamic noise all contribute to the problem of noise pollution. This noise pollution negatively influences human health. May it be by impacting the quality of sleep, the cardiovascular system, mental health or the hearing capabilities of humans. It is estimated that in the year 2000 around 44% of the European population was exposed to potential dangerous levels of noise, with the young, elderly and poor being disproportionally effected by these consequences. It is estimated that the economic cost of these adverse effects on health amounted to costs equal to 0.4% of the GDP. However, not only humans suffer the adverse effects of noise pollution. Since, it also impairs the behaviour and the development of animal communities, especially if they rely mainly on acoustic communication, e.g. birds (Francis et al., 2009, Den Boer and Schroten, 2007, Cullinane and Edwards, 2010).

Resource consumption and land use: During the life span of a vehicle a variety of different resources are consumed. The same is true for the infrastructure required to properly operate said vehicles, e.g. roads. Ranging from construction until its discharge, materials have to be extracted, processed, recycled and disposed. Moreover, a more frequent use accelerates the deterioration process and thus increases the total materials and energy consumed. For example, an increase in traffic decreases the lifetime of the pavement, thereby advancing the need for repairs. Since a pavement of poor quality increases the rolling resistance of a vehicle, the fuel consumption will also rise. Thereby other negative externalities such as emissions or noise are reinforced (Kanari et al., 2003, Newbery, 1988, Wang et al., 2012, Yang et al., 2010).

Even though, the focus of this paper is the reduction of green house gas emissions by utilising the capabilities of the VRP, it seems reasonable to assume that many of these negative externalities described above, can be alleviated by employing similar models. Some of these ideas are presented below.

- Suboptimal scheduling can contribute to driver fatigue. Therefore, one of the approaches for reducing accidents could be the use of Vehicle Routing Problems with Time Windows or short VRPTW (see Section 2.2) to overcome poor scheduling(Goel and Vidal, 2013).

- Emissions can be reduced by either minimising the energy consumed or by minimising the produced pollution. This application of the VRP is the main focus of this paper (Lin et al., 2014).
- No studies concerning a solution for reducing noise emissions by using the VRP were found. However, it could be possible to reduce the traffic noise by, using a VRPTW to prevent the vehicle from entering densely populated areas during certain times. Additionally, it could be possible to include the noise pollution into the cost function of the model. However, it has to be mentioned that these thoughts disregard any economic feasibility.
- Reducing the resource consumption could also be mitigated by VRP which minimises the required energy. Since a reduction of consumed energy would lead to the decrease in the consumption of fossil fuels. Thus, saving additional consumption required for acquiring said energy source in the first place (Lin et al., 2014). Additionally, one could assume that a VRP that intends to reduce the total distance travelled, could lead to a reduction in road and vehicle deterioration.

As stated above the main focus of this paper lies on the issue of reducing greenhouse gas emissions. Therefore, the Green VRP or short GVRP, which among other applications can be used for reducing the GHG emissions, is a cornerstone of this paper. Its relevance becomes apparent, if one considers that climate change is becoming an increasingly prominent problem with serious repercussions. Hence, the GVRP could be used to reduce the greenhouse gas output of the freight transport industry and thus aid in the process of reducing the total global greenhouse gas output. Thereby, slowing down the warming process and delaying its negative effects. The negative effects of an increase in average global temperatures could manifest in an increase in the frequency and severity of extreme weather patterns, such as heat waves, droughts, storms and flooding. Furthermore, it is predicted that the worlds oceans will not only become more acidic, which heavily impacts marine life, but will also increase in volume, i.e. an increase in sea levels. Such a development could lead by the end of the century to massive habitat loss on land and sea. Combined these effects may also result in a reduction of agricultural crop yields and subsequently to unprecedented immigration (Schellnhuber et al., 2013). Since these consequences impact society as a whole, it is to assume that the transportation sector will be influenced by some of these upcoming problems. For example, the increase in sea levels, might render many coastal roads useless due to increase in flooding. A more general problem arises when considering the impacts of bad weather, such as snow or rain, on traffic. May it be a higher likelihood of accidents or an increase in congestion, the transportation sector will have to adapt to the changes to come (Koetse and Rietveld, 2009). Therefore, may it be the reduction of GHG emissions or any other kind of negative effects, in order to positively impact the future development of society a more thoughtful use of resources may help alleviating some of the negative consequences. One option that could aid in addressing these issues is the VRP, which will be investigated in depth in Section 2.2.

2.2 The Green VRP and its Components

This section offers a broad overview over the topic of Green VRP at each level.

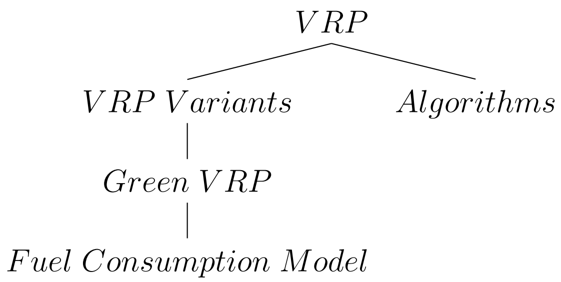


Figure 2.1: Literature Review - VRP Environment

At first the left part of the tree is investigated, Subsection 2.2.1 introduces a variety of different VRP variants. The Green VRP which is a subset of all VRP variants, is excluded from this overview. This is due to the fact that the Green VRP is the focus of this paper and is therefore covered in greater detail in Subsection 2.2.2. As some Green VRP variants, e.g. Pollution Routing Problem, require an accurate assessment of the energy consumption, fuel consumption models are an integral part of these types of problems. Hence, Subsection 2.2.3 covers the major types of fuel consumption models as well as the most common parameter required by these models. Lastly, in order to fully cover the environment within which the Green VRP is placed, the most common algorithms required for solving a VRP are briefly introduced. This is done in Subsection 2.2.4.

2.2.1 Vehicle Routing Problem Variants

After its introduction in 1959 the amount of VRP literature increased exponentially, which to some extent reflects the importance of the vehicle routing problem. However, this development can partially be attributed to the advances in algorithms, as well as computational power. Both of which opened up new possibilities for addressing the VRP, thus explaining why the diversity and the amount of research around this topic increased during the 1990, which led to a plethora of more complex VRP variants, such as dynamic VRP or fuzzy VRP (Eksioglu et al., 2009). Some of the important variants of VRP are presented below. To ensure that only important VRP variations are selected, the VRP has to be mentioned in at least two of the four VRP surveys Lin et al. (2014), Caceres-Cruz et al. (2014), Eksioglu et al. (2009) and Marinakis and Migdalas (2007). The selected variants are then briefly described and then sorted based on the year of origin. In case that two survey paper credit a different paper as the original paper, the earlier one presented is checked. If the variant is mentioned in said paper, the variant is dated based on the year in which the paper was published. If it was possible to obtain the original paper and if its information was useful, the description is based on said paper. Otherwise the descriptions are based on the summary provided in the survey paper or is drawn from other papers entirely.

1959 | Capacitated-VRP (CVRP): The CVRP is the original form of the VRP and contains a set of station points and one terminal point. The aim is to fulfil the demand of these station points in a manner that it minimises the costs, e.g. distance (Lin et al., 2014, Dantzig and Ramser, 1959). See Section 2.1

1964 | Heterogeneous Fleet-VRP (HVRP): In its core the HVRP acknowledges the fact that a normal fleet does not consist out of identical vehicles. That means, in the HVRP one can include conditions that enable the modelling of a variety of different vehicles at the same time. This problem was originally introduced in Clarke and Wright (1964). However, further variants of this idea, such as the Fleet size and Mix VRP emerged in later years (Lin et al., 2014, Clarke and Wright, 1964).

1967 | Pickup and Delivery Problem (PDP): A problem is a PDP, if the vehicle has not only to deliver goods to the station points, but simultaneously also has to pick up goods at the respective point. Moreover, it is often the case that these goods have to be transmitted to a different station point. This implies two specific points are directly connected by a delivery request. Additionally, the vehicle has to depart and return to the terminal point. The PDP is a generalisation of the VRP. Thereby making the VRP a problem in which the terminal point is either the destination or the origin of all goods (Marinakis and Migdalas, 2007, Savelsbergh and Sol, 1995).

1969 | Multi-depot-VRP (MDVRP): The MDVRP includes several terminal points, e.g. several warehouses. In this problem one route can start at one terminal point and can conclude at an entirely different terminal point (Marinakis and Migdalas, 2007, Tillman, 1969).

1969 | Stochastic-VRP (SVRP): A SVRP is a VRP where at least one parameter is subjected to uncertainty. Such parameters are typically the demand, the travelling time or even the set of station points. Thus these problems encompass not only integer programming, but also include stochastic concepts. Hence, the solving such problems becomes far more difficult. This explains why the rapid advancements in computational power, led to a rise in its popularity (Gendreau et al., 1996, Eksioglu et al., 2009).

1974 | Periodic-VRP (PVRP): The PVRP extends the VRP by adding an additional time horizon in which the solution should be optimal. That is, given a certain time frame, which is segmented into smaller instances. For each instance the route has to be recalculated, with the additional restriction that the set of station points differs between each instance of the time frame. It is the essence of the PVRP to find the optimal solution for the complete time frame, by defining the rout of each instance. For example, a truck collecting waste, only has to visit a house once a week and the PVRP intends to choose the daily route in order to minimise the costs per week (Beltrami and Bodin, 1974, Lin et al., 2014).

1977 | VRP with Time Windows (VRPTW): The VRPTW enables one to impose time windows onto the model, thereby restricting the space of possible solutions. These time windows can be applied to either the time at which a vehicle has to reach a certain station point or the time during which a vehicle can be used. The modern VRPTW contains either Hard Time Windows, i.e. being precisely within the specified time frame, or Soft Time Windows, which allow some kind of violation.The latter are often enforced by using a cost function (Lin et al., 2014, Marinakis and Migdalas, 2007).

1980 | Dynamic-VRP (DVRP): A classical VRP can be seen as a closed system, i.e. all the informations required for solving the problem are already known. By comparison, the DVRP receives its information over time and has to recalibrate the route accordingly. This information can be anything, e.g. a vehicle breaks down, a path is not possible or as it was in the original example, a new station point appears or disappears (Lin et al., 2014, Psaraftis, 1980).

1989 | Split-delivery-VRP (SDVRP): This variant enables one to split the demand of one station point between multiple vehicles. For example, one vehicle could satisfy 25% of the demand, while a second one caters the rest. Thereby additional flexibility is introduced, which can significantly improve the outcome (Dror and Trudeau, 1989).

1995 | Fuzzy-VRP (FVRP): Fuzzy logic enables the program to make decisions based on a gradient function rather than a merely True or False statement. The need for such becomes especially apparent when dealing with time windows. For example, a customer may prefer to be visited by the vehicle at a certain time, while also tolerating a certain time frame. The closer the visiting time is in relation to the upper and lower bounds of said frame, the lower the satisfaction of the customer will be. Fuzzy logic allows to consider such an ambiguity. While originally introduced for time windows, fuzzy logic can be applied for other elements of the VRP (Lin et al., 2014, Cheng et al., 1995).

2000 | Open-VRP (OVRP): The OVRP can easily be distinguished from the VRP, as its route closes not at the terminal point, but at a station point. That is, the vehicle does not return to its point of origin. However, if a vehicle has to return to the terminal point, it has to return on a path with station points. An example would be the postal service. Since in this line of work after a vehicle is finished distributing its cargo, it can either leave the service or return to the terminal point while at the same time collecting mail (Sariklis and Powell, 2000).

An overview over the research that created the list above can be found in Table 2.1. It cites the original paper of every VRP variant. Additionally, it tracks and informs about the accessibility of said papers. Furthermore, it depicts whether the VRP was mentioned in a survey paper.

Variant	Paper	Lin et al. (2014)	Caceres-Cruz et al. (2014)	Eksioglu et al. (2009)	Marinakis and Migdalas (2007)	Access
CVRP	Dantzig and Ramser (1959)	X	X	X	X	Yes
HVRP	Clarke and Wright (1964)	X	X	X	X	Yes
PDP	Wilson and Weissberg (1967)	X	X	-	X	No
MDVRP	Tillman (1969)	X	X	-	X	Yes
SVRP	Tillman (1969)	X	X	X	X	Yes
PVRP	Beltrami and Bodin (1974)	X	X	-	X	Yes
VRPTW	Russell (1977)	X	X	X	X	Yes
DVRP	Psaraftis (1980)	X	-	X	X	Yes
SDVRP	Dror and Trudeau (1989)	X	X	-	-	Yes
FVRP	Cheng et al. (1995)	X	-	X	-	Yes
OVRP	Sariklis and Powell (2000)	X	-	-	X	Yes

Table 2.1: Literature Review - VRP Variants: Analysis

In the case of the SVRP the paper Eksioglu et al. (2009) and Lin et al. (2014) disagree on the origin of said variant. The latter refers to the paper Tillman (1969) as the one that introduces the SVRP, while the former cites Cook and Russell (1978). However, since Tillman (1969) already includes a probabilistic demand it seems reasonable to date the problem at 1969. This controversy applies in a similar fashion to the DVRP, since the same survey paper cites two different sources. Unfortunately, the paper proposed by Lin et al. (2014), namely Speidel (1976), was not accessible. However, it referenced a second paper, which is still earlier than any other reference. Therefore, due to a lack of certainty, the DVRP had to be attributed to Psaraftis (1980). For the VRPTW the same controversy occurred, it was resolved by checking the paper referenced with the earliest publishing date. Thus confirming that Cheng et al. (1995) was indeed the paper of origin. It is important to reiterate that any claim of origin is solely based on the methodology introduced above.

The variants listed above are not exclusive, thus it is likely that most VRP models incorporate several of these variants in order to create more realistic results. A combination of several VRP variants can be called a Rich-VRP and due to increases in computational power and algorithmic efficiency a shift toward such models can be observed (Caceres-Cruz et al., 2014). Overall having introduced a variety of different VRP variants, none of which directly addresses environmental concerns. The one particular VRP variant is missing is the Green VRP (GVRP). It was introduced in 2006 and tries to reduce externalities that negatively impact the environment, e.g. greenhouse gas emissions, energy consumption or waste production is missing in this overview. The reason behind its intentional exclusion from this overview is that it is already discussed in detail in Subsection 2.2.2 (Caceres-Cruz et al., 2014, Lin et al., 2014).

2.2.2 The Green Vehicle Routing Problem

The Green VRP or GVRP is a VRP variant which incorporates environmental factors into the optimisation process. Such an approach becomes especially important considering the increasing severity of environmental problems. In general a GVRP can be defined as a VRP that minimises or aids the process of minimising any kind of externalities that negatively impact the environment. Prominent examples are VRP with objectives that minimise the required energy, the fuel consumption or the pollution. However, this definition also includes the collection of waste, the routing of vehicles with alternative fuel sources or end-of life goods collection. As a result of this variety, one can further distinguish three groups of GVRP. This categorisation was introduced by Lin et al. (2014) and includes the Green-VRP (G-VRP), the Pollution Routing Problem (PRP) and the VRP in reverse logistics (VRPRL) (Caceres-Cruz et al., 2014, Lin et al., 2014).

VRP in reverse logistics (VRPRL) The first GVRP was a VRPRL and emerged in 2006. Reverse logistics is an integral part of the process of reusing and recycling materials, which subsequently should, if done correctly increase the environmental efficiency of the respective endeavour. Hence, a company operating under the principles of reverse logistics has to be concerned with regular material distribution and with reverse distribution in order to ensure the back flow of material required for recycling purposes. Thereby, not only is it relevant to properly manage the back flow of the materials, but also to minimise the required materials in the first step of distribution (Carter and Ellram, 1998). To do so a broad variety VRPs can be used to aid and optimise these distribution processes. Some examples of such VRP are Selective Pick-ups and Pricing, Waste Collection, End-of-live Goods Collection and Simultaneous Distribution and Collection. The Waste Collection VRP models the back flow of waste to several waste deposit sites and can therefore be associated with PVRP or MDVRP. Fairly similar is the End-of-live Goods Collection, which models the back flow of used goods to the manufacture. Selective Pick-ups and Pricing integrates the choice to visit a station point or not. Which can be done by using a modified PDP that incorporates prices. The above mentioned VRPRL variants are only concerned with segments of the whole reverse logistics concept. The Simultaneous Distribution and Collection VRP differs. It enables one to model the entire flow in a reverse logistic supply chain (Lin et al., 2014). However, the focus of this paper lies especially on the other two GVRP categories G-VRP and PRP, thus the VRPRL is not investigated further.

Green-VRP (G-VRP) The second GVRP to be mentioned is the G-VRP, which is mainly a VRP that minimises energy consumption. The G-VRP was first introduced in 2007 by Kara et al. (2007). The model presented in this paper was a modified CVRP, that minimises based on the load of a vehicle and the distance travelled. These factors where then used to approximate the energy required to complete the entire trip. Additionally, the G-VRP is also concerned with routing vehicles that utilise alternative fuels, e.g. electrical vehicle which require the incorporation of factors such as recharging times, battery degradation, partial recharging and recharging facilities (Felipe et al., 2014, Lin et al., 2014).

Pollution Routing Problem (PRP) Its objective is to minimise the amount of pollution produced during a trip. Therefore, implying that a widespread application of this variant could aid in reducing the total GHG output of the freight transport industry. Furthermore, it might be possible that this concept could also be applied to other forms of pollution, as introduced in Section 2.1. However, the focus of this paper remains on the former, i.e. the reduction of GHG emissions. The first instance of such a PRP can be found in Palmer (2007). Pollution Routing Problems are especially relevant in this paper since the case study presented in Chapter 4 requires the implementation of the PRP presented in Bektaş and Laporte (2011) (Felipe et al., 2014, Lin et al., 2014).

Both G-VRP and PRP rely to some extend on fuel consumption models. This is because both of these categories, or at least the variants that include energy consumption into their modelling process, require a fuel consumption model in order to assess the required energy or the amount of pollution emitted. Unfortunately, the average fuel consumption provided by the manufacturer is not an appropriate basis for modelling of such kind. Therefore, the following section is concerned with providing a brief overview over fuel consumption model variants, as well as over the main factors that influence the fuel consumption (Demir et al., 2014).

2.2.3 Fuel Consumption Models

At the heart of the PRP and other GVRP variants lies the fuel consumption model. Hence, this section explores the different parameter that can influence fuel consumption as well as the different types of fuel consumption models. The subsequent review of fuel consumption models is heavily influenced by the paper Demir et al. (2014), thus it is recommended, if a more detailed summary of these models is desired. Firstly, any model requires input, thus the most common input factors required by fuel consumption models are summarised in Table 2.2.3. These factors can be classified into vehicle related, environment related, traffic related, driver related and operation related. A brief overview of the specific components is seen in Figure 2.2.

Table 2.2.3 summarises the top 15 factors, ranked based on the frequency of inclusion in the fuel consumption models presented in Demir et al. (2014). The "Other" factors found in the category vehicle and environment are not included in the ranking. Additionally, it is addressed how the respective factor affects the fuel consumption. The column "Influences" visualises the direction of influence a factor has on fuel consumption, if said factor increases, by using the following notation

- If it correlates negatively , i.e. factor X increases and fuel consumption decrease: $\downarrow$
- If it correlates positively, i.e. factor X increases and fuel consumption increases: $\uparrow$
- If depends on the circumstance, i.e. factor X increases and depending on the circumstance fuel consumption decrease or fuel consumption increases: $\updownarrow$
- If the nature of the factor does not allow scaling e.g. a nominal scale: $-$

To expand upon the information given in the column "Influence" the column "Explanation", provides a short description of the connection between fuel consumption and the respective factor.

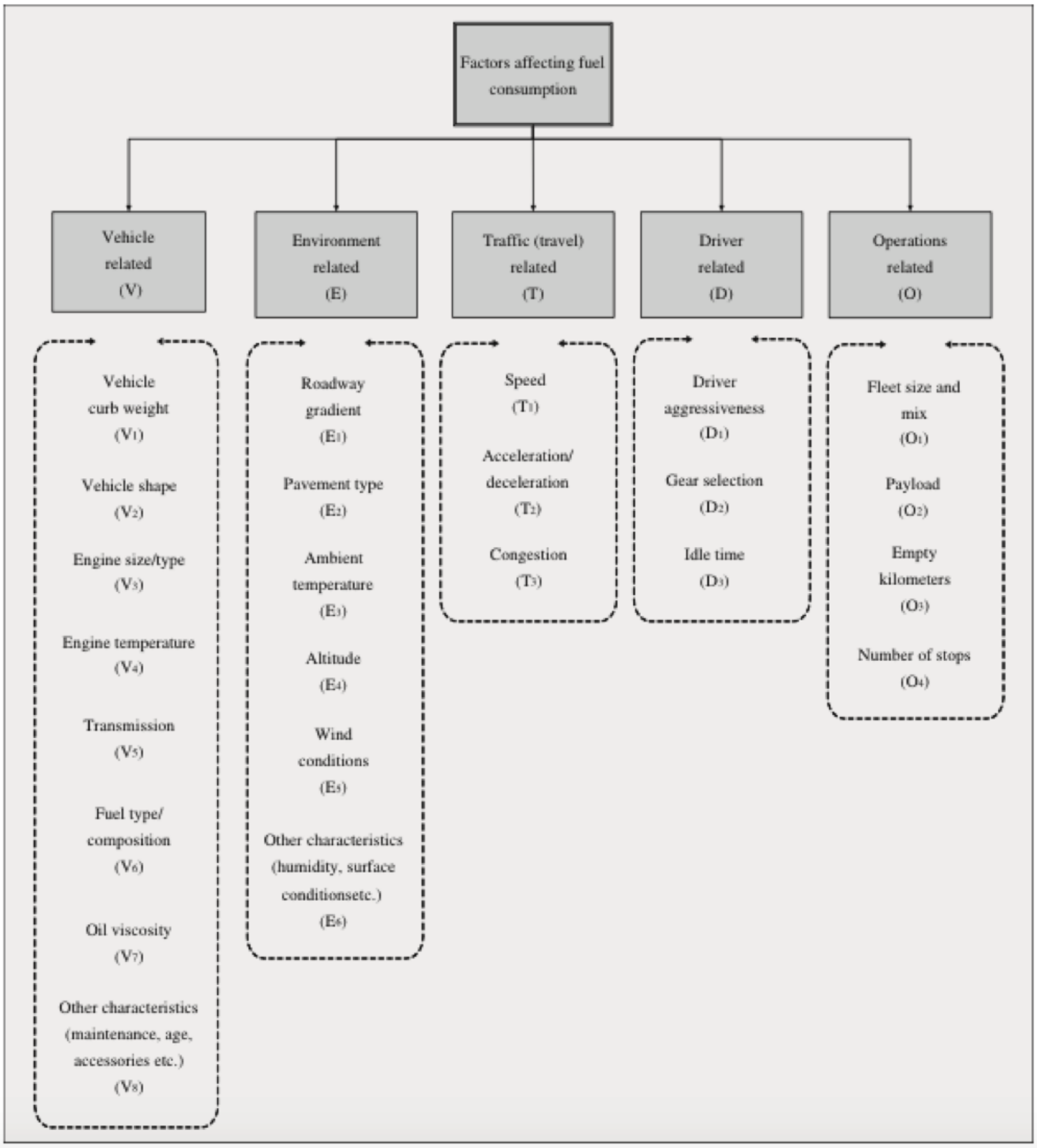


Figure 2.2: Literature Review - Fuel Consumption Factors - Overview (Demir et al., 2014)

Frequency	Category	Name	Influence	Explanation
100%	Vehicle	Vehicle Curb Weight	↑	All else equal a heavier vehicle requires more energy than a lighter one (Demir et al., 2014).

Frequency	Category	Name	Influence	Explanation
100%	Operation	Fleet Size and Mix	–	The fleet size and mix is influenced by the decision of which and how many vehicles are used. For example, if one selects two smaller or one bigger vehicle for the trip. Even if smaller vehicles require less fuel, a bigger vehicle can sometimes need less fuel than two smaller ones. Hence, an increase in fuel consumption depends on the decisions made (Demir et al., 2014).
100%	Operation	Payload	↑	All else equal a heavier vehicle requires more energy than a lighter one. Therefore, the fuel consumption increases (Demir et al., 2014).
92%	Traffic	Speed	↑	The factor speed is viewed as the one with the most significant impact on the vehicles fuel economy. Since with an increase in speed opposing forces such as air resistance, inertia, rolling resistance increase as well (Demir et al., 2014).
88%	Environment	Road Gradient	↕	The energy required to overcome the same distance on a steeper road is higher when travelling upwards than on a flatter road. However, the opposite is true if the vehicle travels the road downwards (Demir et al., 2014).

Frequency	Category	Name	Influence	Explanation
76%	Vehicle	Fuel Type	–	For example diesel engines are more efficient that petrol ones, thus total fuel consumption is lower. This can be explained that on the one hand the type of fuel dictates what type of engine is required and on the other that different kinds of fuels contain a different amount of energy per unit, i.e. energy density (Cullinane and Edwards, 2010, McKendry, 2002). Therefore, one could assume that if the energy density of a fuel is high, less of it will be required for the same amount of work. This assumption neglects the differences between engines.
68%	Vehicle	Vehicle Shape	↑	The shape of the vehicle dictates its drag coefficient. A higher drag coefficient increases drag and thereby the energy required to propel the vehicle. However, regardless of the shape if the frontal area increases the total energy consumption rises as well (Hall, 2015b, Bektaş and Laporte, 2011).
64%	Environment	Pavement Type	↑	There is a direct relationship between the roughness of the pavement and the rolling resistance coefficient. The higher the rolling resistant coefficient the more energy will be required. Therefore, as the roughness of the pavement increases so does the fuel consumption of the vehicle (Du Plessis et al., 1990, Bektaş and Laporte, 2011).

Frequency	Category	Name	Influence	Explanation
52%	Environment	Altitude	↑	At a higher altitude the atmospheric pressure declines, this leads to a decrease in air density. However this is compensated by the fact that a higher altitude leads to a decline in oxygen in the air. A lack of oxygen negatively influences the combustion process, thus leading to an increase in fuel consumption (Alwakiel, 2011).
52%	Traffic	Acceleration	↑	Given the same time frame a vehicle in acceleration requires more energy than one at a constant speed. Therefore, if the intensity of acceleration increases the fuel consumption rises as well (Ahn et al., 2002, Alwakiel, 2011).
40%	Vehicle	Transmission	–	The transmission type of the vehicle also influences its fuel consumption. For example, a vehicle with a manual transmission system will be out performed by a vehicle with an automatic one, since the automatic one can change gears faster and is able to select the optimal gear ratio, thus achieving a better overall transition (Carbone et al., 2001).
36%	Vehicle	Engine Type	–	As mentioned by the factor *Fuel Type*, diesel engines are more efficient than petrol ones, thus total fuel consumption is lower. Similarly, electrical engines have even better efficiency than ones based on combustion (Cullinane and Edwards, 2010, Helms et al., 2010).
36%	Vehicle	Engine Temperature	?	No clear information regarding the impact of Engine Temperature on fuel consumption was found.
32%	Driver	Idle Time	↑	For every unit of time a vehicle is idle it consumes energy. Therefore, the longer the idle time, the higher the fuel consumption (Akcelik and Besley, 2003).

Table 2.2: Literature Review - Fuel Consumption Factors

The accuracy of a fuel consumption model varies based on how many and which factors it incorporates. Therefore, the trade-off every fuel consumption model is confronted with is the one between accuracy and simplicity. When trying to provide a precise estimate, it is only possible to do so in a narrow scope, e.g. one vehicle, as on a greater scale this becomes infeasible in both informations requirements and computational time. Therefore, general estimates are required, if one intends to model a traffic network. According to Demir et al. (2014) these models can be categorised into factor models, macroscopic models and microscopic models (Zegeye et al., 2013, Demir et al., 2014).

Factor models These models operate on the highest level, as it only provides rough estimates by relating an emission factor to a specific type of vehicle and a specific driving behaviour. These factors are defined by repeated controlled measurements. These measurements are then averaged over all measured driving cycles (Demir et al., 2014).

Macroscopic models These models employ, if compared to factor models, already a more complex composition of factors. They do not describe the dynamics of individual vehicles, but are based on average aggregated network parameter. That is, it is suited for modelling groups of vehicles instead. These kind of models are often used to estimate network wide emissions, which then can be used for emission inventories or in the planning of green supply chains (Zegeye et al., 2013, Demir et al., 2014).

Microscopic models These models are also called instantaneous emission models, which are required if one intends to precisely estimate the fuel consumption, by modelling individual vehicles and their behaviour, i.e. acceleration, deceleration or idle time. Therefore, the amount of variables and the amount of computational time required increases proportionally with the amount of different vehicles (Zegeye et al., 2013, Demir et al., 2014).

These fuel consumption models can then be used to assess how much energy is consumed by a vehicle during its trip. Hence, these kind of models provide a solid foundation for developing objective functions of VRP that include environmental factors, e.g. PRP. However, the last components missing from providing an overview of the environment in which GVRP are placed, are the algorithms used for solving such problems. This missing element is introduced in the last subsection, i.e. Subsection 2.2.4.

2.2.4 Algorithms

This section is concerned with giving a basic non-technical overview over the methods designed for solving the VRP. This is especially important, since finding a solution to a VRP is highly difficult and becomes even harder as the number of station points increases. Therefore, solving the VRP for an optimal solution above a certain threshold of points, is highly impractical. Hence, problems with larger sets rely on heuristics, which are faster and more versatile, but might only provide a good solution and not an optimal solution (Laporte, 2007). There are several approaches for solving the VRP and its variants, which can roughly be divided into three subsections, namely Exact Algorithms, Heuristics and Meta-Heuristics.

Exact algorithms These algorithms can solve a VRP for its optimal value. That means, it can guarantee that the value returned is indeed the best option available. However, since a VRP is a NP-hard Problem (NP := non-deterministic polynomial time) the solving time increases disproportionally for every additional staring point. Therefore, these algorithms can only be applied on small problem sets. Prominent elements in this category are Integer Linear Programming, Dynamic Programming and Direct Tree Search Methods. The branch-and-cut algorithm, which is part of the integer linear programming based algorithm family is currently the most promising approach for solving VRP with exact algorithms.

Heuristics In comparison to exact algorithms the classical heuristic algorithms only returns a good solution. Many classical heuristics used in VRPs are based on a descend mechanism. That is, they investigate the neighbourhood of the current solution for a better value, if such a value is found it becomes the next solution. This process continues until there is no better value to be found. However, this does not imply that this value has to be the optimal solution. These classical heuristics can further be divided into constructive heuristics, which incrementally build a solution based on predefined rules, and improvement heuristics, which start at a feasible solution and improve based on the initial solution. This initial solution is in the most cases provided by a constructive heuristic. The most common algorithm variants belonging to the constructive branch are the savings algorithms, sweep algorithms, petal algorithms and the Fisher and Jaikumar two phase algorithms. Whereas improvement algorithms can be further separated into intra-route heuristics and inter-route heuristics.

Meta-Heuristics One variant of meta-heuristics, called local-search heuristics, also employ the same descend methodology as most classical heuristics, however, it is embedded into sophisticated neighbourhood search structures, which allows a more rapid reduction of possible solutions and a variation of neighbourhood structures. This category includes heuristics such as the tabu search, simulated annealing, greedy randomising adaptive search procedure, variable neighbourhood search and the large neighbourhood search. Another type of meta-heuristics are population-search heuristics, which are also called genetic algorithms. Herby a population of solutions is created. Out of this population solutions are selected, which then employ the role of parents. Then two solutions are crossed over to generate offspring solutions. This process allows enough freedom for slight mutations to occur. This offspring is then selected based on its fitness, thus leading to the replacement of the worst elements of the previous generation. Other population-search mechanism such as adaptive memory procedure exist. However, they work fairly similar to the genetic algorithms. The final category employs learning mechanisms, which can be separated into neural networks and ant algorithms. While neural networks are yet to be successfully implemented in a VRP, ant algorithms are already utilised. Algorithms of this kind mimic the behaviour of ants, which use pheromone trails to search and identify lucrative food sources (Laporte, 2007, Cordeau et al., 2005, Oliveira and Carravilla-FEUP, 2009, Lin et al., 2014).

Having now introduced the environment in which GVRP are placed the next step is to develop an easy measure for identifying the different VRP variants, the different Green VRP and the different fuel consumption models from each other. Such a method is developed in Section 2.3. However, even though this subsection introduced several types of different algorithms for solving VRP, they will not be included in Section 2.3 as the current Subsection was only included to provide a more complete picture of the GVRP environment.

2.3 Classification Methodology

The environment surrounding the GVRP that was introduced in Section 2.2, consists out of a variety of different VRP, GVRP and fuel consumption models. Therefore to provide a more concrete method for identifying these components, this section will attempt to create a rough methodology for distinguishing between these components. Similar to Section 2.2 this section will at first investigate the differences between the VRP variants. This can be found in Subsection 2.3.1. From there it tries to separate G-VRP and PRP in Subsection 2.3.2. The last GVRP variant is, as mentioned in Section 2.2 excluded, since the focus resides on the other two variants. Lastly, the methodology for categorising fuel consumption model variants is coined.

2.3.1 Classification of Vehicle Routing Problem Variants

In order to better identify the different variations of VRP presented in Subsection 2.2.1 the following classification scheme is crated. This is done by building on the definitions given in Section 2.2.1 to analyse the respective optimisation programs for patters that can provide a more concrete method for identifying the different variants. This process follows certain steps. (notation for Table 2.3 is included):

1. Based on the definitions given in Section 2.2.1 a question is formed. This question tries to capture the essence of the respective variant.
2. Based on the survey paper Lin et al. (2014) its oldest and its youngest reference of this variant is selected. This time difference is chosen to ensure that only characteristics that are fundamentally engrained into the respective variant are selected.
3. If one of these papers is not available (*NA*) or not useful (*NU*), e.g. no optimisation program or no common ground was found, the next paper that is closest to the specific time frame is chosen (*n+1*). However, if no such thing is found, other survey papers are used to obtain the respective reference.
    a) Papers referenced in Lin et al. (2014) have the marker $A(n)$
    b) Papers referenced in Marinakis and Migdalas (2007) have the marker

$B(n)$

c) Papers referenced in Eksioglu et al. (2009) have the marker $C(n)$

d) Other sources have the marker X

4. based on the formulated questions the identifying characteristics are selected. These characteristics are chosen to be as simple as possible, while at the same time complex enough to still have identifying properties. Thereby, ensuring that the characteristic identified enables one to easily categorise a variant with a relatively high degree of confidence.
5. The characteristics identified are compared. If both papers do not inhibit the same characteristic, the previous step is repeated. Otherwise the result is adapted and included in the Table 2.3.1.

Table 2.3 contains the sources for the content found in Table 2.3.1. Additionally, it contains information about the paper, on which the content of Table 2.3.1 is constructed. For example, "Access Information" $B(4) : NA$ means that the fourth option found in the paper Marinakis and Migdalas (2007) was selected, because the other three paper with an earlier publishing date where not accessible and that paper Lin et al. (2014) did not provide a better reference.

Variant	Earliest Paper	Access Information	Latest Paper	Access information
CVRP	Dantzig and Ramser (1959)	$A(1)$	Clarke and Wright (1964)	$A(1)$
HVRP	Clarke and Wright (1964)	$A(1)$	Choi and Tcha (2007)	$B(1)$
PDP	Thangiah et al. (1996)	$B(4) : NA$	Parragh et al. (2007)	$A(1)$
MDVRP	Tillman (1969)	$A(1)$	Dondo and Cerdá (2007)	$A(2) : NU$
SVRP	Tillman (1969)	$A(1)$	Mendoza et al. (2010)	$A(1)$
PVRP	Christofides and Beasley (1984)	$B(1) : NU$	Pirkwieser and Raidl (2009)	$A(1)$
VRPTW	Solomon (1987)	$A(2) : NU$	Qureshi et al. (2010)	$A(2) : NU$
DVRP	Powell (1986)	$C(1) : NU$	Hong (2012)	$A(2) : NU$
SDVRP	Dror and Trudeau (1990)	$X : NU$	Archetti et al. (2011)	$A(2) : NU$
FVRP	Cheng et al. (1995)	$A(1)$	Xu et al. (2011)	$A(3) : NA$
OVRP	Brandão (2004)	$A(2) : NU$	Li et al. (2012)	$A(1)$

Table 2.3: Literature Review - VRP Variants: Methodology

The results produced by following this methodology can be found in Table 2.3.1. It contains the VRP categorisation scheme. Herby, both the "Questions" and the "Identifying Characteristics" can be used. Additionally, it is vital to mention that the notation of the following table (Table 2.3.1) is not harmonised with the notation of the paper. Furthermore, it is important to understand that the elements found in Table 2.3.1 in column "Identifying Characteristics" are not a distillation of all unique aspects of the respective variant, since it is only necessary to identify one fundamental attribute to categorise the VRP at hand. Unfortunately, due to the underlying method is not possible to ensure that all VRP can be categorised with this categorisation scheme.

Variant	Question	Identifying Characteristics
CVRP	Is the trip limited by the total capacity of the vehicle?	$C \gg \sum_i q_i$
HVRP	Is the routing of multiple and different vehicles possible?	There has to be a set of vehicles $X = \{1, .., x\}$ or a set of vehicle types $T = \{1, .., t\}$
PDP	Does the model include pickup and delivery points?	There has to be a set of delivery points $D = \{1, .., d\}$ and a set of pickup points $P = \{1, .., p\}$.
MDVRP	Does the model include more than one terminal point?	There has to be a set of terminal points $T = \{1, .., t\}$
SVRP	At least one input parameter has to have a probabilistic nature?	The objective is only an expected objective e.g. objective cost $E[C]$
PVRP	Are multiple periods considered?	There has to be a set of periods $T = \{1, .., t\}$. Additionally a set of feasible combinations of visits $C = \{1, .., c\}$ should exist.
VRPTW	Is any object, i.e. point, vehicle and others, restricted by a time window?	There has to be a set of values that defined the upper and lower bounds of the time window $[a_i, b_i]$
DVRP	Is it possible that the input information changes over time?	There has to be a planing horizon and sets of elements attributed to one planing horizon.

Variant	Question	Identifying Characteristics
SDVRP	Is it possible to only partially satisfy the demand of the station point?	There has to be a variable to keep track of the demand of the customer. For example, $y_{(iv)}$ being the quantity delivered by a vehicle or during a delivery pattern.
FVRP	Is there a fuzzy membership function that is used for decision making?	There has to be a fuzzy membership function integrated into the optimisation program. For example such a function can found in Cheng et al. (1995) $\mu_i(t_i) = \begin{cases} 0 & t_i < e_i \\ \frac{t_i - e_i}{u_i - e_i} & e_i \leq t_i \leq u_i \\ \frac{l_i - t_i}{l_i - u_i} & u_i \leq t_i \leq l_i \\ 0 & t_i < l_i \end{cases}$ Such a function requires a fuzzy number, which in this case is defined by e_i, u_i, l_i. This particular fuzzy number gives an ideal time, an upper and a lower boundary. This function can then be implemented into the optimisation program as so $\mu_i(t_i) > 0$.
OVRP	Is the terminal point visited only once?	A distinction between the terminal point and the station points has to be made, i.e. there have to be separate sets on nodes. For example,$V = \{0, .., v\}$ is the total set of points, while $C = \{1, .., c\}$ is the set of station points.

Table 2.4: Literature Review - VRP Variants: Categorisation Scheme

The variant SDVRP was problematic to resolve, since the two paper cited in Lin et al. (2014) that were published in the 20th century, did not provide the necessary insight into the optimisation problem. Therefore, the paper was selected based on the criteria of publishing date and author by using Google Scolar. That is, Google Scholar was used to find the earliest feasible paper published by the author of the original reference in Lin et al. (2014) was chosen. In the case of the FVRP an additional paper was used for the enquiry, since both paper selected were fairly similar. This paper, namely Cao and Lai (2010), was the latest one mentioned in Lin et al. (2014). Being now able to distinguish various VRP variants the next step is to find a method for identifying the difference between G-VRP and PRP.

2.3.2 Classification of Green Vehicle Routing Problems

Unfortunately, the difference between G-VRP and PRP is somewhat fuzzy. Therefore, in order to better understand said differences, the papers presented in Lin et al. (2014), are summarised and classified by their defining characteristic. As the definition provided, stated that G-VRP are concerned with minimising energy or with the routing of alternative fuel vehicles, its defining characteristic is an optimisation function that minimises either energy, fuel or fuel cost. However, this does not necessarily include the G-VRP concerned with the routeing of alternative fuel vehicles. Therefore an additional defining characteristic has to be coined, which is that the model has to include some sort of constraint that can be related to fuel levels, fuel consumption or recharging requirement. The characteristic of a PRP is an optimisation function that minimises emissions or the costs of emissions. Table 2.5 provides an overview over which paper includes which categorisation. However, multi selection is possible that means if an optimisation function includes two or more objective, it is included into the table. The paper Schneider et al. (2014) was not obtainable and is therefore not included in the analysis.

As seen in Table 2.5 the distinction between G-VRP and the PRP are not as clear as desired. Since if one ignores the fuel cost objective from Bektaş and Laporte (2011) and Demir et al. (2012), around 29% of the all PRP have characteristics from the G-VRP. This would not be particularly problematic, because the concept of Rich VRP states that it is possible that one model can include characteristics from a variety of different VRP variants. However, these 29% have no PRP characteristics. Therefore, the utility of distinguishing between G-VRP and PRP has to be questioned. Furthermore, this argument is supported by the fact that energy consumption is directly related to fuel consumption, which also directly relates to greenhouse gas emissions (Demir et al., 2014, Caceres-Cruz et al., 2014). Therefore, it is suggested that these types of VRP are regrouped under a name such as

Variant	Paper	Energy	Fuel	Fuel Cost	Recharging	Emissions	Emissions Cost
G-VRP	Erdoğan and Miller-Hooks (2012)	-	-	-	X	-	-
G-VRP	Kara et al. (2007)	X	-	-	-	-	X
G-VRP	Kuo (2010)	-	X	-	-	-	-
G-VRP	Xiao et al. (2012)	-	-	X	-	-	-
PRP	Bauer et al. (2010)	X	-	-	-	-	-
PRP	Faulin et al. (2011)	-	-	-	-	-	X
PRP	Bektaş and Laporte (2011)	-	-	X	-	-	X
PRP	Demir et al. (2012)	-	-	X	-	-	X
PRP	Fagerholt et al. (2010)	-	X	-	-	-	-
PRP	Palmer (2007)	-	-	-	-	X	-
PRP	Ubeda et al. (2011)	-	-	-	-	X	-

Table 2.5: Literature Review - GVRP Classification: Analysis

Energy Routing Problem (ERP). However, as this is only a suggestion, the original terms are used in this paper. In conclusion, however, even if it is somewhat fuzzy the characteristics summarised above, may be used for categorisation purposes. For most of these models it is certain that the energy consumption of the vehicle has to be assessed. As already established this is done be means of fuel consumption models. Thus Subsection 2.3.3 will investigate possible characteristics for identifying the variations of these models.

2.3.3 Classification of Fuel Consumption Models

In order to categorise fuel consumption models, the relationship between the categories of fuel consumption models and the occurrence of specific input factors, is key for classifying papers based on the fuel consumption models is use. This section focuses on the distinction between microscopic and macroscopic models. The first categorisation variable can be obtained by analysing the distribution of factors between the macroscopic and microscopic models. Table 2.6 shows in percent how many factors of each category the average microscopic and the average macroscopic fuel consumption model includes. This table is based on the data found in the overview in Demir et al. (2014) see Figure 2.3. In Demir et al. (2014) every fuel consumption model is contrasted, based on the factors used to calculate the fuel consumption. Based on this the amount of factors for each model is then used to calculate the average amount of factors for each fuel consumption model category, i.e. microscopic and macroscopic models. These averages are then related to the total number of factors per category. Additionally, for better comparison the difference between microscopic and macroscopic models is calculated.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	E_1	E_2	E_3	E_4	E_5	T_1	T_2	D_1	D_3	O_1	O_2	O_3	O_4
Macroscopic models																				
MEET	✓			✓		✓		✓		✓		✓	✓			✓	✓	✓	✓	
NTM	✓					✓		✓	✓								✓	✓	✓	
COPERT	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓				✓	✓		
ECOTRANSIT	✓	✓				✓		✓	✓			✓	✓	✓	✓		✓	✓	✓	✓
NAEI	✓			✓		✓		✓		✓			✓		✓	✓	✓	✓	✓	✓
MOBILE	✓				✓	✓		✓		✓			✓				✓	✓		
MOVES	✓				✓	✓		✓		✓			✓			✓	✓	✓		
HBEFA	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓		
GREET	✓			✓		✓							✓				✓	✓		
LEM	✓					✓		✓					✓				✓	✓		✓
VERSIT-LD	✓	✓			✓	✓		✓	✓				✓	✓			✓	✓		
IVE	✓	✓				✓		✓					✓		✓		✓	✓		
EMFAC	✓	✓				✓		✓	✓	✓	✓	✓	✓			✓	✓	✓		
Microscopic models																				
IFCM	✓	✓						✓	✓	✓	✓	✓	✓	✓		✓	✓	✓		
F-MEFCM	✓	✓						✓	✓	✓	✓	✓	✓	✓		✓	✓	✓		
RSFCM	✓	✓						✓	✓	✓	✓	✓	✓			✓	✓	✓		
ASFCM	✓	✓						✓	✓	✓	✓						✓	✓		
VSP	✓	✓	✓			✓		✓	✓	✓	✓	✓	✓	✓			✓	✓		
EMIT	✓	✓						✓	✓	✓	✓	✓	✓	✓			✓	✓		
VT-Micro	✓							✓					✓	✓			✓	✓		
OFCM	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓	✓		✓	✓	✓		
PERE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
CPFM	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
PHEM	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		
CMEM	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓		

Figure 2.3: Literature Review - Fuel consumption model: Factor Overview (Demir et al., 2014)

Model Type	Vehicle	Environment	Traffic	Driver	Operation	Total
Microscopic	56.25%	62.5%	58.33%	11.11%	50%	51%
Macroscopic	51%	33.33%	38.5%	20.5%	55.77	42%
Difference	5.3%	29.17%	19.88%	−9.4%	−5.77%	9.4%

Table 2.6: Literature Review - Fuel Consumption Models: Factor comparison

Based on these values two deductions can be made. Firstly the total difference between these two model types only amounts to 9.4%, which is not convincing enough to conclude that microscopic models consider more factors. Secondly, it can be deduced that microscopic models tend to focus on factors such as environment and traffic. Hence, the conclusion that the microscopic models simply have a different focus than macroscopic ones, is to some extend supported. Therefore, it seams reasonable to coin the following weak identification criteria.

If a fuel consumption model contains a higher amount of factors that belong to the category Environment and Traffic, it tends to be a microscopic model.

The next possible characteristic to investigate is that if the quality of the input differs between these two model variants. Firstly, the quality of input is in this case defined by its purity and the level of detail. For example, when the factors included into the fuel consumption equation are calculated based on precise physical attributes rather than selected based on certain classification schemes, its inputs are of higher quality. Fortunately, Demir et al. (2014) point out that the level of precision in microscopic models requires more of said high quality inputs. These factors are concerned with vehicle shape such as frontal surface area, road conditions such as road angel or rolling resistance. By looking at the core equations concerned with fuel calculation, it is tested if microscopic models include a higher quantity of such factors. The mentioned factors that serve as a basis for comparing the fuel consumption models can be found in Table 2.7

Model	Distance	Speed	Mass	Time	Acceleration	Road Angle	Gravity	Air density	Surface Area	Drag	Rolling Resistance
Macro											
MEET	X	X									
NTM	X										
COPERT	X	X									
ECOTRANSIT	X		X								
NAEI	X	X									
Micro											
IFCM		X	X	X	X	X	X				
F-MEFCM	X		X	X		X	X				
RSFCM		X	X	X		X	X		X		
ASFCM		X	X								
VSP		X	X	X	X	X	X	X	X	X	X
EMIT		X	X	X	X	X	X			X	X
VT-MICRO		X			X	X	X				
OFCM		X	X	X			X				
PERE		X	X	X	X	X	X	X	X	X	X
CPFM		X	X	X	X		X	X	X	X	X
PHEM		X	X	X	X	X	X	X	X	X	X
CMEM		X	X	X	X	X	X	X	X	X	X

Table 2.7: Literature Review - Fuel Consumption Models: Input comparison

As seen in Table 2.7 the microscopic fuel consumption models tend to require a specific kind of inputs, which describe the environment (road angle, gravity , rolling resistance, air density), vehicle (surface drag area) and behaviour (acceleration) in greater detail. Hence, another characteristic can be formulated:

If a fuel consumption model contains parameter that describe the environment, the vehicle and the traffic in greater detail, it tends to be a microscopic model.

Unfortunately, both of these two characteristics are alone not enough to distinguish between microscopic and macroscopic models, but still can be used to aid the categorisation decision. To conclude, microscopic models are more likely to incorporate parameter describing the environment and the traffic. At the same time, however, factors concerning the vehicle, the environment and the traffic are integrated in greater detail.

Chapter 3

The Model and its Implementation

This chapter introduces one variant of the GVRP mentioned above, namely the PRP presented in the paper Bektaş and Laporte (2011). Hence, if no other citation is made, most of the information found in this chapter has its roots in Bektaş and Laporte (2011). Anyhow, the intent behind this chapter, is to create the necessary understanding of the model in order to solve the case presented in Chapter 4. Furthermore, this chapter also introduces the notation used in Chapter 5. That is, in Section 3.1 the general idea behind the model, as well as its structure is introduced. Moreover, this section also defines all the parameter used in the model. The following section, i.e. Section 3.2, contains a step by step guide for implementing the model into Excel. Above that it also introduces the notation required by Chapter 5, thus serving contentiously as a reference point for the following chapters.

3.1 Introduction of the model

As mentioned earlier the model presented in this section is based on the paper Bektaş and Laporte (2011). However, while the basic elements of this model remain true to its origin, the actual implementation of the optimisation program differs greatly, since in contrast to this version, the version presented in Bektaş and Laporte (2011) is a linear integer problem. The following subsection introduces the underlying idea and structure of the model as well as introduces and explains the parameter that are required and produced by the model.

3.1.1 The general idea

This subsection intends to convey a basic structure of the model, upon which the rest of this paper builds. The beauty of this model is that it consists of several layer, with each layers serving as a foundation for the subsequent ones. This is true for its core structure, as well as for any other restrictions or calculations. Hence, the best way to generate a basic understanding of this model is to specify what the model wants to achieve as well as introduce its components.
Beginning with the goal of this model. Its aim is it to find, given a specified set of stops, an optimal sequence of paths in order to return the optimal trip. The previous sentence already contains four highly important terms to be understood, namely stops, optimal, paths and trip. At first the term optimal shall be examined. What is optimal? In this case it refers to finding a solution to the VRP, which provides the most desirable outcome. The values on the basis of which the desired outcome is calculated are called objectives. This model is designed to minimise objectives such as the distance travelled by the vehicle (Od), the total load a vehicle has to carry (Of), the emissions a vehicle produces (Oe) and costs that were required for completing the task (Oc). To reiterate, the task that has to be completed is the delivery of goods to customers. In this case customers can be understood as stops, which are separated into station points and terminal points. While station points can be viewed as customers or points that receive goods, the terminal point is the warehouse or the point of origin. That is, it is the point from which vehicles depart and return after they completed their deliveries. These create a family of points $\mathscr{N} := (\acute{n}_i)_{i \in I}$ with I being a set which containing the indices of the points. Furthermore, $\forall i \in I \setminus \{o\} : n_i := \textit{station point}$ and $\exists! o \in I : n_o := \textit{terminal point}$. For convenience $I^X := I \setminus \{o\}$. These points create the fundamental building blocks of this model. Every point is granted a position in space, may it be a two or three dimensional. Indicating that its position can be defined by merely using $x_i \in X$ and $y_i \in Y$ values, with

$$\begin{aligned} X &:= \{(x_i)_{i \in I} | \forall i \in I : x_i = \textit{Position of } n_i \textit{ in dimension one}\} \\ Y &:= \{(y_i)_{i \in I} | \forall i \in I : y_i = \textit{Position of } n_i \textit{ in dimension two}\}. \end{aligned}$$

However, it is possible to view a point in three dimensions, if this is the case one additional value, representing the third dimension, has to be added to precisely specify its position. That is $z_i \in Z$ with

$$Z := \{(z_i)_{i \in I} | \forall i \in I : z_i = \textit{Position of } n_i \textit{ in dimension three}\}.$$

Most of these nodes resemble customers, thus each point has its own attributes such as its demand $Q := (q_i)_{i \in I^X}$ or a serving time window $\widetilde{AB} := (\widetilde{ab_i})_{i \in I}$. It has to be added that $\forall i \in I : \widetilde{ab_i} = (a_i, b_i) = b_i - a_i$ with $a_i \in A := \{(a_i)_{i \in I} | \forall i \in I : a_i = \textit{lower bound of time window for Point } n_i\}$ and $b_i \in B := \{(b_i)_{i \in I} | \forall i \in I : b_i = \textit{upper bound of time window for Point } n_i\}$. These attributes can later on be used for restricting the optimisation program. An example for one set of points $\mathscr{N}$ can be seen in Figure 3.1. While the points can be referenced by numerical indices, in this example, the customers n_i are referenced with A to D and the and the warehouse n_o, which resembles the start and endpoint of a trip is called *Origin*. Therefore, the set of Indices is equal to $I := \{Origin, A, B, C, D\}$.

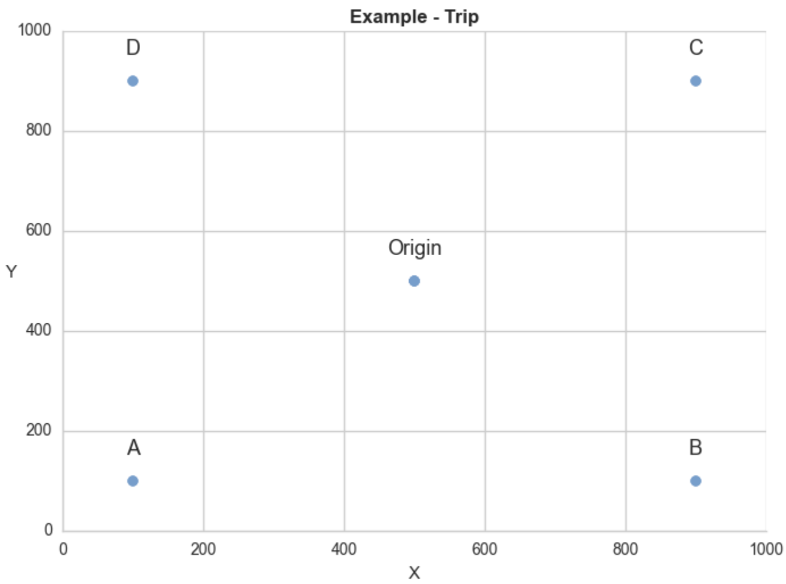


Figure 3.1: Model - Example: Nodes

The next level is a path or arc, which is defined by two points. That is, $\mathscr{A} := (a_{ij})_{i,j \in I}$. For easier reference let $\bar{I} = I \times I$ with $\bar{I} \ni i := (i, j) | i, j \in I$ and thus $\mathscr{A} := (a_i)_{i \in \bar{I}}$. Similar to the real world an arc a_i has also attributes specific to it, which are the length $D := (d_i)_{i \in \bar{I}}$, the road angle or slope $\Theta := (\theta_i)_{i \in \bar{I}}$, the vehicles travelling speed $V := (V_i)_{i \in \bar{I}}$ and the vehicles lower travelling speed constraint $L := (l_i)_{i \in \bar{I}}$, as well as its upper travelling speed constraint $U := (u_i)_{i \in \bar{I}}$. In this model, every node is connected with every other node by one arc, as seen in Figure 3.2.

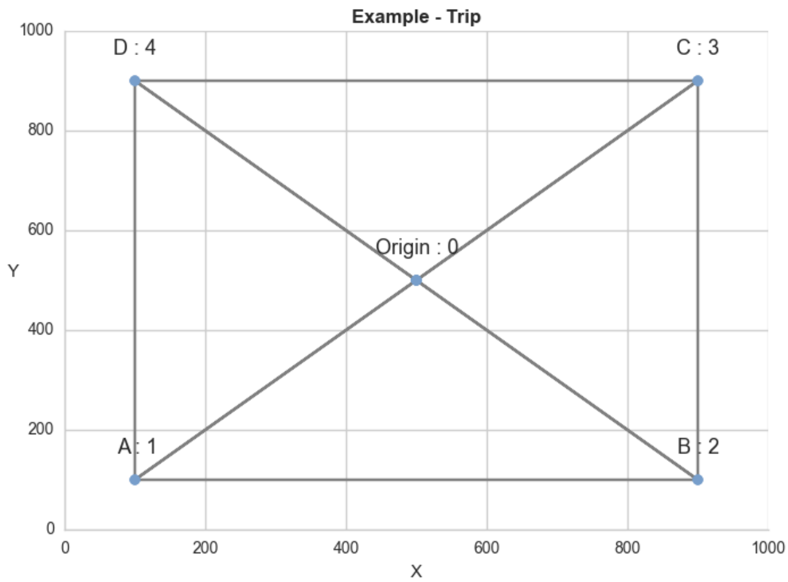


Figure 3.2: Model - Example: Arcs

The third foundational component is the trip. It is simply an accumulation of arcs and by extension of points, which adheres to specific criteria. These vary based on the implementation of this model. However, there are some criteria that are generally applicable, the first one being that a trip has to contain every point. Furthermore, a trip has to start and end at the point of origin, while every other point must only be visited once. This implies that each path in the trip has to be unique. Generally speaking it is upon the optimisation algorithm to find the total set of arcs that satisfies the given objective best, while still adhering the general criteria and specific restrictions. This means the optimisation algorithm in its simplest form is a function

$$\omega : S \to I, (s) \mapsto \omega(s) := i_s \in J \subset S \times I \tag{3.1}$$

that creates a family $J := (i_s)_{s \in S}$ in $S \times I$, with $S \subseteq \mathbb{N} \wedge \sharp S = \sharp I + 1 \wedge \forall s \in S \forall r \in S \setminus \{s\} : s \neq r$. Thereby, apart from the terminal point n_o each point, or to be more precise each index of a point, is paired with exactly one unique natural number, i.e. $\forall i, j \in I^X \forall s, r \in S : \omega(s) = \omega(r) \Rightarrow i_s = j_r \Rightarrow s = r \wedge i = j$ and $\exists! m, n \in S \forall s \in$

$S : m \leq s \wedge n \geq s$ with $\exists o \in I : m \neq n \wedge \omega(m) = \omega(n) \Rightarrow o_m = o_n$. Similar to the notation for an arc let $\bar{J} = J \times J$ with $\bar{J} \ni j_{sr} := (i_s, j_r) = (\omega(s), \omega(r)) | i_s, j_r \in J$ and $\forall s \in S \exists r \in S : s < r \wedge \forall r \in S \exists t \in S : r \leq t$, thus $\mathscr{A} := (a_j)_{j \in \bar{J}}$ is the set of arcs representing the trip. One possible example of such a trip can be seen in Figure 3.3.

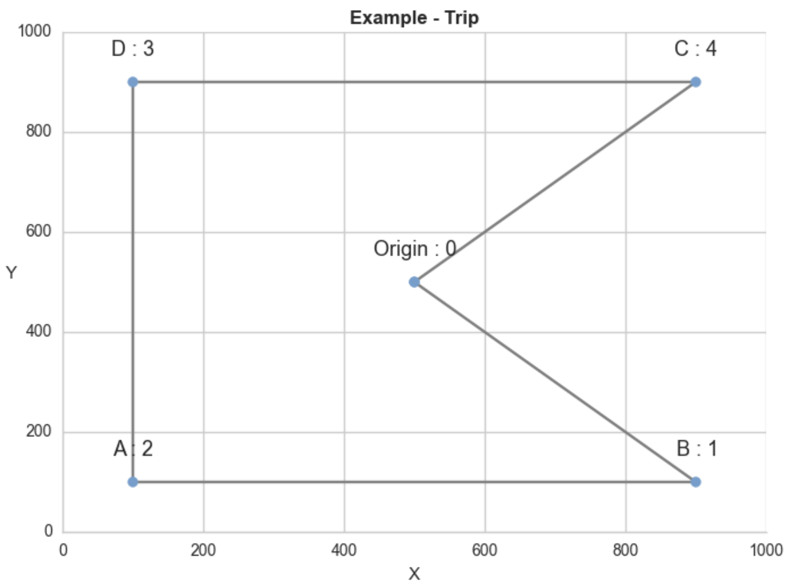


Figure 3.3: Model - Example: Trip

As mentioned above, the incrementing structure of this model mostly holds true for the objectives as well. Moreover, each objective has also an underlying assumption, which is integral for understanding, why it is presented as a valid objective in the first place. The most relevant objectives presented in this paper have already been named. However, further elaboration follows.
Firstly, the objective that lies at the very foundation of this model is the objective distance or Od. By selecting said objective, one usually intends to find the shortest sequence of arcs, that fulfil the requirements of a trip. This means:

$$min\ D := \sum_{j \in \bar{J}} d_j \tag{3.2}$$

Thereby, it is ensured that only the distance of the arc $a_j \in \mathscr{A}$ with $j \in J$ connecting the points $\mathscr{N}$ based on the sequence defined by $\omega(S)$ are part of the sum. The idea behind this objective is that for every meter a vehicle travels it consumes energy. This leads to the assumption that if the total distance travelled is reduced, the energy consumed will also decrease. Secondly, the objective load or Of. This objective is the only one that does not directly build upon the objective distance. Therefore, it can also be considered as one of the foundational objectives of this model. However, it can be argued that the objective load indirectly includes some measure of distance, namely the arc. Because, the intention behind it, is the minimisation of load carried by the vehicle during the whole trip. That means the sum of the load carried $F = (f_j)_{j \in \bar{J}}$ on each arc of the trip has to be minimised, i.e.

$$min\ F := \sum_{j \in \bar{J}} f_j \tag{3.3}$$

For example, if there is the choice between point n_i with a demand of q_i and a point n_j with a demand of q_j and $q_i < q_j$ is true, than it is in the best interest of the algorithm to choose q_j first. Expressed in a more formal manner $\forall i, j \in I^X \exists! s, r \in S : q_i < q_j \Rightarrow r < s$ with $\omega(r) = j$ and $\omega(s) = i$. The assumption behind this is that in order to move a heavy object more energy is required than for a lighter object. Hence, if the weight of the vehicle decreases earlier in the trip, the total amount of weight carried reduces. The two objectives mentioned above are fairly straightforward. However, the other two objectives require a more sophisticated set of parameters. Both, the objective emissions and the objective cost require an approximation of the vehicles energy consumption. Therefore, before any other objective is described properly, the fuel consumption model has to be introduced.

The fuel consumption model lies at the heart of the PRP. The fuel consumption model used in Bektaş and Laporte (2011) for solving the PRP is based on the Comprehensive Modal Emission Model or short CMEM. The model is based on a collection of data obtained by measuring tailpipe emissions and can be classified as a microscopic fuel consumption model, as indicated by a broad use of different environmental factors and the requirement for detailed input parameters. In order to accurately estimate the amount of fuel required, this model demands detailed parameters such as engine friction factor or the engine speed for the calculation (Demir et al., 2014). Such level of detail is not always necessary, thus further implementations rely on a simplified version of this model. That is, rather than considering engine friction factor, the engine speed, drive train efficiency, additional energy consumption, engine displacement, the total tractive power demands and a measure for efficiency for specific engine types, a short cut has been chosen. This short cut simply considers the total tractive power demand and the engine efficiency and the amount of energy present in one litre of fuel. The same approximation is used by Bektaş and Laporte (2011). However, this simplified version of this model, only provides a correct approximation of the energy consumption above a speed of $40[km/h]$. In summary this model can be formally described by the equation:

$$P_i = d_i \cdot w \cdot \alpha_i + d_i \cdot f_i \cdot \alpha_i + d_i \cdot v_i^2 \cdot \beta = Pw_i + Pf_i + Pv_i \quad i \in \bar{I} \tag{3.4}$$

This model consists of three parts. The first part is concerned with estimating the energy required for transferring the empty mass of the vehicle w over a certain distance d_{ij}. The second part models the energy required to carry the load of the vehicle f_{ij} over the distance d_{ij}. Naturally, simply multiplying the weight and the distance together is not sufficient enough to calculate the energy required. In order to do so one has to integrate the environment and the behaviour of the vehicle into the model. This is done by α_{ij}. It includes factors such as acceleration a, gravity g, rolling resistance Cr and road angel θ_{ij}. This means

$$\alpha_i := a + g \cdot sin(\theta_i) + g \cdot Cr \cdot sin(\theta_i) \quad i \in \bar{I}. \tag{3.5}$$

Thirdly, the speed of the vehicle has to be considered as well. This is accomplished by part three of this model. It approximates the fuel consumption of a certain vehicle travelling over a certain distance d_{ij} at a certain speed v_{ij} . Hereby, parameters such as the surface area of the vehicle A, its drag coefficient Cd and the air resistance ρ are required. These are combined into the parameter β. Formally, expressed

$$\beta = 0,5 \cdot Cd \cdot A \cdot \rho. \tag{3.6}$$

The weight and speed based components are then summed together, thus the total energy consumption is obtained. As already stated above this value can, based on the efficiency of the vehicle, directly be converted into the total fuel consumption, based on which total GHG output can be calculated. This is possible because there is a direct relationship between the fuel consumed and the emissions produced. Therefore, one only has to multiply the fuel consumption, with the relevant factor, i.e. ω_{GHG}. Since the aim of the next objective is to minimise the energy required $P_{ij_{[kWh]}}$ or the emissions produced $P_{ij_{[kg]}}$. It builds upon the distance objective, as well as the load objective, and combines them. Additionally, this objective also considers the speed of a vehicle as a relevant factor. Hence, a variety of factors are merged into one parameter, which is done by the fuel consumption model. Thereby, a more wholesome optimisation approach is possible. The formal definition below is used to minimise the emissions produced.

$$min\ P := \sum_{j \in \bar{J}} p_j \tag{3.7}$$

The last objective would be the objective cost. This seems at first fairly straight forward, as it only tries to minimise the cost $C_{ij_{Total}}$of one trip. However, this objective includes the highest number of different factors. Since the costs include the fuel costs, the emission costs and the wage of the driver. Admittedly, the fuel and the emission costs require the insights obtained from the objective emissions. However, this objective builds upon that and adds the hourly wage of the driver. Thereby increasing the complexity further. Similar to the previous objective, the formal definition of this objective looks as follows:

$$min\ C := \sum_{j \in \bar{J}} c_j \tag{3.8}$$

As soon as the objectives are defined one has to impose restrictions onto the model. These restrictions or constraints ensure that a sensible output is provided as the restrict the set of possible solutions. However, this subsection is primarily concerned with conveying a basic picture of the PRP. Therefore, may it be setting a fixed travelling speed or by imposing time windows onto the model. The constraints are covered in Section 3.2. In conclusion, the version of the PRP covered in this paper consists of three basic elements. Firstly, the atoms of the model, i.e. points, which represent customers or warehouses. Secondly, the arcs, which is defined by two points. Lastly, a trip is a collection of arcs, that was selected based on certain criteria in order to optimise a certain objective. The most commonly used objectives in this paper are load, distance, emissions and cost. The last layer of the model, i.e. the constraints can then be used for adjusting the optimisation process by restricting the set of possible solutions. This basic description of the model introduced a variety of different parameter. However, such a brief and incomplete introduction is not sufficient. Hence, these parameter are defined properly in the following section, i.e. Subsection 3.1.2.

3.1.2 Parameter

The model presented above requires a certain set of input values as well as produces a variety of output parameter. This section addresses these values. Firstly, the input parameter are introduced. For the sake of simplicity they are divided into five different groups. The first one is the *Trip*, which includes attributes that relate to a specific point or a specific arc. The second one is the parameter group *World*, this category includes general physical information. Thirdly, parameter representing vehicle specific attributes are introduced. The forth parameter group is required in order to calculate different costs, thus it is named *Costs*. Lastly, the parameter group *Product* includes certain attributes of the carried product. These groups are created to provide a better overview for the excel implementation. After all input parameter are defined, the output parameter will be introduced. Output in this case is any parameter that is the result of a calculation introduced in Section 3.2.
Firstly, parameter group *Trip*. As stated above all input variables in this category, are either arc or point specific. Therefore, it can be divided into a subgroup containing point specific and a subgroup containing arc specific parameter. The set of points consists of $\mathscr{N} := (\acute{n}_i)_{i\in I}$, with each point being a set of parameter on its own, thus $\forall \acute{n} \in \mathscr{N} : \acute{n} := \{x, y, q, \widetilde{ab}, \acute{t}\}$. Therefore, implicitly $\mathscr{N} := \{X, Y, Q, \widetilde{AB}, \acute{T}\}$.

$X := (x_i)_{i\in I}, Y := (y_i)_{i\in I}[m]\ or\ [°]$: The model used for calculating the emissions, requires the distance between each stop as an input factor. However, it is far more practical to reference each stop by its absolute geographical position rather than having to enter the distance between each stop individually. Therefore, this implementation of the model requires the coordinates of each stop as an input. This can be accomplished by using the Universal Transverse Mercator coordinate system (UTM-System) or by using the World Geodetic System. The first one is a system, that splits the earth into 60 zones and projects them onto a two-dimensional plane. While the second one describes the position of a point by using longitude and latitude, i.e. spherical coordinates. When using the first system the parameter are measured in $[m]$, while with the second the are given in [°] (Wikipedia, 2016h,g).

$Q := (q_i)_{i \in I}, [units]$: In order to calculate the weight of the vehicle at each arc, it is vital to know the demand of every customer, i.e. at each point. Even though, the demand can directly be measured in $[kg]$, in this case the demand will be measured in $[units]$. This makes it easier to incorporate other factors such as the size of the product.

$\widetilde{AB} := (\widetilde{ab_i})_{i \in I} = (b_i)_{i \in I} - (a_i)_{i \in I}, [s]$: Is the notation for the time window. A time window is the time frame during which the vehicle has to visit the customer i.e. point. It is defined by its lower bound $(a_i)_{i \in I}$ and the upper bound $(b_i)_{i \in I}$. In the calculation the time window is measured in $[s]$

$\acute{T} := (\acute{t_i})_{i \in I}, [s]$: This is the notation for the serving time of a customer. Serving time can include elements such as the time required to discharge the load or the time required to fulfil the necessary paperwork. It is measured in $[s]$

The next step is to define the subgroup that contains the attributes of individual arcs. That is, $\mathscr{A} := (\acute{a}_i)_{i \in \bar{I}}$, with each arc having a set of parameters on its own, thus $\forall \acute{a} \in \mathscr{A} : \acute{a} := \{v, \theta, \widetilde{lu}\}$, which by extension means $\mathscr{A} := \{V, \Theta, \widetilde{LU}\}$

$V := (v_i)_{i \in \bar{I}}, [m/s]$: The speed, at which a vehicle travels on one arc, is one of the possible decision variables in this model. Hence, in some implementations it has to be set before the optimisation process can be started. In other the optimal values are calculated by the optimisation algorithm. Nevertheless, in both cases the speed is measured in $[m/s]$.

$\Theta := (\theta_i)_{i \in \bar{I}}, [°]$: Represents the road angle of a trip between two stops, which indicates how steep a certain path is. Hence, it depends on the difference in altitude between stop i and stop j. It is measured in [°] i.e. degrees.

$\widetilde{LU} := (\widetilde{lu_i})_{i \in \bar{I}} = (u_i)_{i \in \bar{I}} - (l_i)_{i \in I}, [m/s]$: This parameter defines the speed at which a vehicle is allowed to travel at a certain arc. In order to do so two bounds are required. In this case is the lowest speed is defined by $(u_i)_{i \in \bar{I}}$, while the highest speed is set by $(l_i)_{i \in \bar{I}}$. All of the values used are expressed in $[m/s]$.

The next input category includes parameter that specify the physical attributes of the environment, this it is called parameter group *World* and is defined as $\mathscr{W} := \{c'_r, \rho_{air}, g, r_s\}$.

$\rho_{air}, [kg/m^3]$**:** This is a factor used to indicate the density of a liquid or gas. In this case it is the density of the air, through which the vehicle has to move. Said density depends on temperature and pressure. Fortunately, there are standard values available that aid with estimating the appropriate density. However, it is possible to calculate the air density by using

$$\rho_{air} = \frac{p}{R_{air} \cdot T}$$

In this case p is absolute pressure in $[Pa]$, R_{air} being the gas constant for air measured in $[J/(kg \cdot K)]$ and T being the temperature in $[K]$. Therefore, ρ_{air} is measured in $[kg/m^3]$ (Wikipedia, 2016a).

$\acute{c_r}$**:** Is the rolling resistance coefficient, which expresses the force required to propel a vehicle of a certain weight, forward. Similar to the drag coefficient it is also reverse engineered from the measured rolling resistance in a controlled environment (Committee for the National Tire Efficiency Study, 2006, Wikipedia, 2015d).

$g, [m/s^2]$**:** Defines the gravitational pull experienced by a certain object. That is, it expresses the acceleration an object experiences towards the earth. It is possible to assign one standard gravitational acceleration, because of the massive differences in mass between the earth and any other object on it. This standard value amounts to $9.81 m/s^2$ (Wikipedia, 2015c, Hall, 2015c).

Having defined the environment the next step is to describe the vehicle in use, which is accomplished by the parameter group $Vehicle$, which is defined as $\mathscr{V} := \{K_v, K_w, w, A, \eta, \acute{c_d}, \omega_{GHG}, \omega_F\}$.

$K := min(K_v, K_w), [units]$**:** Represents the total amount of product units a vehicle can carry. Hereby, two limiting factors can be identified. Firstly, the maximum amount of units a vehicle can carry by weight, i.e. K_w with $K_w = \dfrac{M_{max} - w}{\rho_{product}}$. This is limited by the maximum legal weight and the curb weight of the vehicle [1]. Secondly, the amount of units a vehicle can carry can also be restricted by

[1] M_{max} being the Gross Vehicle Weight Rating or short GVWR, which is a combination of the curb weight and the maximum legal amount of load a vehicle is allowed to carry. Hence, it can be used in order to classify the vehicle. In Austria these classes are N1, N2 and N3. While N1 includes all vehicles with a GVWR below 3500 kg, N2 vehicles have to be between 3500 kg and 12000 kg and a N3 vehicle would be any vehicle above 12000 kg(Akcelik and Besley, 2003,

its storage volume, i.e. K_v with $K_v = \dfrac{V_{max}}{V_{product}}$. Both of which differer between vehicles. That is, K is equal to the lower value and is measured in $[Units]$.

$w, [kg]$: This value is the empty mass of the vehicle, which is also called the curb weight. In the context empty mass refers to the weight of the vehicle and its fluids, such as fuel or engine oil, but without cargo.

$A, [m^2]$: This is the notation for the frontal surface area of the vehicle. That is, it can easily be calculated based on the dimensions of the vehicle. The surface area is a vital for approximating the drag of an object (Hall, 2015b).

$a, [m/s^2]$: This represents the acceleration of the vehicle. It can be used for describing different traffic situations, e.g. congestion (Demir et al., 2014).

η: Is the fuel efficiency of a vehicle. To be more specific it is the thermal efficiency of the vehicle, which describes the completeness of its combustion. That is, it determines how much of the energy stored in the fuel source, can be utilised by the engine. While the average fuel efficiency of a gasoline engine amounts to 20%, some diesel engines can utilise from 32% to 39% of the fuel source (Bektaş and Laporte, 2011, Wikipedia, 2015b).

$\acute{c}_d$: The drag coefficient can be used to calculate the drag of an object under a variety of different circumstances. In order to assess the drag coefficient of an object its drag has to be measured in a controlled environment, e.g. a wind tunnel. This means the drag coefficient has to be reversed engineered by measuring the object in question. This implies that the drag coefficient is more or less unique to an object. However, for the purpose of this case an approximation is sufficient (Akcelik and Besley, 2003, Hall, 2015a).

$\omega_{GHG}[kg/l]$: This parameter describes the carbon content of the fuel. Naturally, the value varies between different fuel types, e.g. diesel and gasoline. However, using this value allows the approximation of CO2 output per litre of fuel. Hence, it is measured in $[kg/l]$ (Facts, 2005).

$\omega_F[l/kWh]$: This parameter describes the energy content in one litre of fuel. Similar to the ω_{GHG}, its value varies between different fuel types, e.g. diesel and gasoline. Nevertheless, it enables the expression of the energy consumption in litre of a certain fuel type. Therefore, it is measured in $[l/kWh]$ (Center, 2013).

Wikipedia, 2015a, BMF, 2015).

The next set is the parameter set *Costs*. It can be used to compare values with different units of measurement, by means of pricing them. This means one can, if the prices are selected appropriately, introduce externalities such as GHG output into the economic argumentation. Anyhow, it is defined as $\mathscr{C} := \{\acute{p_d}, \acute{p_f}, \acute{p_e}\}$.

$\acute{p_d}, [Euro/h]$**:** This factor represents the wage of the driver. Hence, since it translates travelling time into Euro it is measured in $[Euro/h]$.

$\acute{p_f}, [Euro/l]$**:** This factor translates the fuel consumption measured in litre into Euro. The value of which may depend on the fuel type, but it is always measured in $[Euro/l]$.

$\acute{p_e}, [Euro/t]$**:** This factor translates the GHG output which is measured in $[t]$, into Euro. Its unit of measurement is $[Euro/t]$.

Finally, the last parameter group classified as input, is the parameter group *Product*. It only includes two entries. This is due to the fact that apart from the weight and size of the product no additional information is required. However, it is conceivable that given HVRP and a combination with different products, additional information might be necessary. Thus, indicating that the information requirement for a product might vary between VRP variants. In this case however, this group is defined as $\mathscr{P} := \{\rho_{Product}, V_{Product}\}$

$\rho_{product}, [kg/Unit]$**:** This is merely a factor that specifies the weight of a product in $[kg]$. Hence, it expresses the density of the product in $[kg/Unit]$.

$V_{product}, [m^3/Unit]$ This is merely a factor that specifies the size of a product, thus it is measure in $[m^3/Unit]$.

Having now every, important input parameter defined the next step is to investigate the parameter that are "created" by executing the model at hand. Hence, the remainder of this subsection is concerned with clarifying their notation and their function.

$D := (d_i)_{i \in \bar{I}}, [m], (\mathscr{A})$**:** This represents the length of one arc, i.e. the distance between two points. Theoretically it would be possible to use the actual length a vehicle has to travel. However, for the sake of simplicity, this model uses the linear distance between two points. The distance is naturally measured in $[m]$.

$F := (f_i)_{i \in \bar{I}}, [kg], (\mathscr{A})$**:** It represents the load a vehicle has to carry across one arc. However, this is not expressed in $[Units]$, but rather in $[kg]$.

$T := (t_i)_{i \in \bar{I}, (\mathscr{A})}, [s]$**:** This is simply the time required by the vehicle to travel across one arc, i.e. from one point to another. That means the service time is not included. This parameter is measured in $[s]$.

$T_{oj} := (t_{oi})_{i \in I}, [s]$**:** This represents the time required by a vehicle to reach a point within a trip. That means, it is the accumulation of every time consuming factor from the moment the vehicle leaves the terminal point to the station point i.

$\hat{\alpha} := (\alpha_i)_{i \in \bar{I}}, (\mathscr{A})$**:** It describes the condition of the arc and is thus an arc specific factor. This factor is required by the fuel consumption model and is for the most part comprised out of parameter, which describe the environment.

β**:** This is the second parameter required by the fuel consumption model. It is concerned with combining vehicle specific attributes and is closely related to the energy loss caused by drag.

$Px := (px_i)_{i \in \bar{I}}[x], (\mathscr{A})$**:** The output P is the direct result of the fuel consumption model and is in its original form measured in $[J]$. This indicates that it represents the energy required to get from one point to another. Across the model this parameter can be measured in $[kWh]$, which is simply another measurement unit for energy, in $[l]$, which expresses the energy requirement in litres of fuel and lastly in $[kg]$, which represents the GHG output of the vehicle. If necessary these different instances of P are differentiated in the implementation by incorporation the units of measurement into the notation, e.g. $Px_i[J] = Px_{i_{[J]}}$. Furthermore, there are four different variation of Px_i, which are described below.

$Pw := (pw_i)_{i \in \bar{I}}, [x], (\mathscr{A})$**:** This is the notation for the energy required to carry the curb weight w of the vehicle from one point to another, while disregarding the speed of the vehicle.

$Pf := (pf_i)_{i \in \bar{I}}, [x], (\mathscr{A})$**:** It is the energy required to carry the load measured in weight f_{ij} from one point to another. Similar to Pw the speed of the vehicle is also not considered.

$Pv := (pv_i)_{i \in \bar{I}}, [x], (\mathscr{A})$**:** This represents the energy required by a vehicle to travel a certain distance at a certain speed.

$P := (p_i)_{i\in\bar{I}}, [x], (\mathscr{A})$**:** It is the sum of the different energy consumption components, thus it represents the total energy required to travel form one point to another.

$Cd := (cd_i)_{i\in\bar{I}}, [Euro], (\mathscr{A})$**:** This represents the costs that occur due to the fact that one has to employ a driver. To be more specific, the expense of paying a driver is in relationship to the time it takes to travel form one point to another, i.e. Tij.

$Cf := (cf_i)_{i\in\bar{I}}, [Euro], (\mathscr{A})$**:** This simply is the notation for the costs that are caused by the fuel consumption of the vehicle.

$Ce := (ce_i)_{i\in\bar{I}}, [Euro], (\mathscr{A})$**:** These costs are also related to the energy consumption, since it represents the amount ought to be paid by producing a certain amount of emission.

$C := (c_i)_{i\in\bar{I}}, [Euro], (\mathscr{A})$**:** This represent the total cost that can be attributed to a single path. That is, it contains Cd, Cf and Ce.

This section has provided a general overview over the idea behind the model and its structure as well as introduced a variety of different parameter and their notation. With the necessary components defined Section 3.2 will introduce the implementation of this model into Excel.

3.2 Guideline

The previous section conveyed the general idea behind the model and introduced the necessary parameter. The last step is to provide a guideline, which describes the process of actually implementing the model into Excel. Therefore, the first part will briefly introduce the notation used to describe calculations in the Excel Implementation. The second part of the section will provide a step by step guide to do so. Additionally, the calculations are presented in a pure mathematical notation, as well as in the notation presented in the first part of this section. The last part of this section introduces the different solver constellations as well as their notation, both of which are used in Chapter 4 and Chapter 5.

Firstly, in order to harmonise the following guideline for implementing the PRP with the Excel Workbook some notation has to be introduced. $\langle\rangle$ - braces indicate that everything containing said braces, references elements in the excel workbook.

Furthermore, $\|$ is used to separate the calculation from the range of cells affected, while : is an indicator for the range. In some cases $\ulcorner\urcorner$-braces are used to specify the source worksheet of the reference, with | separating the worksheet name and the range. This is, however, only used if necessary. If there is \$ in front of an index or part of an index this particular part is classified as static. That is this reference is not influenced by the selected solution range, while the non-static or variable cells have to be interpreted in conjunction with the selected solution range. For example, $\langle\ulcorner WS1|\$M\$85 * D31 * I31\urcorner\|\ulcorner WS1|J31 : J37\urcorner\rangle$ indicates, apart from the fact that the calculation takes place entirely in $WS1$, that the cell $M85$ is multiplied with the cell $D31$ and the cell $I31$, while the result is displayed in the cell $J31$. Moreover, since this calculation is applied to the range $J31 : J37$ every element within is filled with the result of a similar calculation, as seen below.

$$\begin{aligned}
\langle\ulcorner WS1|\$M\$85 * D31 * I31\urcorner \quad &\| \quad \ulcorner WS1|J31 : J37\urcorner\rangle \\
\Leftrightarrow \langle\$M\$85 * D31 * I31 \quad &\| \quad J31 : J37\rangle \Rightarrow \\
\langle J31 \quad &= \quad \$M\$85 * D31 * I31\rangle \\
\langle J31 \quad &= \quad \$M\$85 * D31 * I31\rangle \\
\langle J32 \quad &= \quad \$M\$85 * D32 * I32\rangle \\
\langle J33 \quad &= \quad \$M\$85 * D33 * I33\rangle \\
\langle J34 \quad &= \quad \$M\$85 * D34 * I34\rangle \\
\langle J35 \quad &= \quad \$M\$85 * D35 * I35\rangle \\
\langle J36 \quad &= \quad \$M\$85 * D36 * I36\rangle \\
\langle J37 \quad &= \quad \$M\$85 * D37 * I37\rangle
\end{aligned}$$

With the notation defined, the language used in the guideline below can be understood. Therefore, it is now possible to present the various steps required for building the PRP in Excel. These steps are enumerated in order to ensure easy referencing.

1. **Positioning the Input:** Since the input parameters of the model range from single factors to two dimensional arrays, the input has to be arranged appropriately. For single parameter the actual positioning is irrelevant, however, for better understanding it is useful to separate these values based on the groups introduced in Subsection 3.1.2. Firstly, a parameter that takes on a

different value for each point, can be depicted as an one-dimensional array. In this array every value belongs to a different point. For example, given index set I with $\sharp I = 4$, the family of values $(a_i)_{i \in I}$ can be stored in the range $\langle B2 : B5 \rangle$. Secondly, parameter that are assigned to an arc take on individual values based on the arc in question. Hence, they are defined by two point indices, i.e. $\forall \bar{i} \in \bar{I} : \bar{i} := (i, j) \in I \times I$. Thus it becomes apparent that a range resembling a two-dimensional array is required. Thereby it becomes possible to depict information that is unique to every arc. For example, the family of values $(a_{\bar{i}})_{\bar{i} \in \bar{I}}$ has to be stored in $\langle B2 : E5 \rangle$. After this illustration of how values are and can be structured in the Excel implementation, the positions of the input values can be defined.

- The values of the category vehicle are located at the range $\langle N5 : P12 \rangle$.
- The values of the category cost are located at the range $\langle N16 : P18 \rangle$.
- The values of the category product are located at the range $\langle N22 : P23 \rangle$.
- The values of the category world are located at the range $\langle N27 : P31 \rangle$.
- The point specific values are located at the range $\langle D5 : K12 \rangle$.
- The arc specific values L, U ,V and Θ are located at the range $\langle D15 : K22 \rangle$, $\langle D25 : K32 \rangle$, $\langle D35 : K42 \rangle$ and $\langle D45 : K52 \rangle$.

In order improve the structure of the excel implementation, these input values are linked to the worksheets at which further calculation is done. That is, on each worksheet the cells used for further calculation are:

- The values of the category vehicle are linked with the range $\langle N102 : P109 \rangle$.
- The values of the category cost are linked with the range $\langle N114 : P116 \rangle$.
- The values of the category product are linked with the range $\langle N120 : P121 \rangle$.
- The values of the category world are linked with the range $\langle N125 : N129 \rangle$.
- The point specific values are linked with the range $\langle D102 : K109 \rangle$.
- The arc specific values L, U ,V and Θ are linked with the range $\langle D112 : K119 \rangle$, $\langle D122 : K129 \rangle$, $\langle D132 : K139 \rangle$ and $\langle D142 : K149 \rangle$.

2. **Prepare path selection:** The foundation of this implementation is the range of cells which contains the sequence of points visited, i.e. J. The number of

cells required is therefore equal to the number of station points plus two, since the terminal point has to be visited twice. Therefore, the space attributed to the index numbers of station points is $\langle D54 : D59 \rangle$, thus the cells that contain the index of the terminal point are $\langle D53 \rangle$ and $\langle D60 \rangle$. The range $\langle D54 : D59 \rangle$ will be set aside, as they are later part of the decision cells for the solver, i.e. they are filled with decision variables. Moreover, these cells are vital they are used to reference any point or arc specific value. For the sake of simplicity most results of future calculations are arranged beside $\langle D54 : D59 \rangle$, thus creating some kind of "solution space".

3. **Calculating the Distance:** In order to obtain the length of every arc one has to calculate the distance between every point. This can be done by either using the Pythagoras theorem or by using the Great Circle equation. If one chooses to calculate the distance with the Pythagoras theorem one is refereed to the approximations found in Wikipedia (2016c). Since these implementations become increasingly inaccurate when the distance between points is greater or when the points are closer to the geographical poles (Wikipedia, 2016c). Therefore, the Great Circle equation is chosen for further implementations, with which it is possible to better approximate the curvature of the planet. This is done by calculating the shorter segment of a great circle, since every distance between two points on the planet is part of a circle with the radius r. This distance measured in spherical coordinates is then translated into a distance between Cartesian coordinates, i.e. d. In this case this coordinates x and y are based on the World Geodetic System (WolframMathWorld, 2016). Given these coordinates one can calculate the distance between every point by

$$
\begin{aligned}
&\forall i, j \in \bar{I} : \\
&\qquad d_{i,j} \quad := \quad r \cdot \arccos(\sin(y_i) \cdot \sin(y_j) + \cos(y_i) \cdot \cos(y_j) \cdot \cos(x_i - x_j))
\end{aligned}
$$

which translates into

$$\begin{aligned}
\langle \quad & ARCCOS(SIN(INDEX(\$F\$103:\$G\$109;\$C82+1;2)*(PI()/180)) \\
* \quad & SIN(INDEX(\$F\$103:\$G\$109;D\$81+1;2)*(PI()/180)) \\
+ \quad & COS(INDEX(\$F\$103:\$G\$109;\$C82+1;2)*(PI()/180)) \\
* \quad & COS(INDEX(\$F\$103:\$G\$109;D\$81+1;2)*(PI()/180)) \\
* \quad & COS(ABS(INDEX(\$F\$103:\$G\$109;\$C82+1;1)*(PI()/180) \\
- \quad & INDEX(\$F\$103:\$G\$109;D\$81+1;1)*(PI()/180)))) \\
* \quad & \$N\$100*1000 \| D82:J88 \quad \rangle
\end{aligned}$$

in the Excel implementation. The extension $\frac{\pi}{180}$ has to be added in order to convert the radiant used in Excel into degrees. Additionally, since for future calculation the unit of distance has to be $[m]$ the radius of the planet has to be entered in the same unit.

4. **Indexing and summing the distance:** The set D, which was obtained through Step 3, contains the distance between every arc. This type of implementation, however, requires only the distance between points, if said arc is part of the trip J. Therefore, one has to select the required values based on the sequence specified in Step 2. This is done by using the index function

$$\langle INDEX(\$D\$82:\$J\$88;D53+1;D54+1) \| E54:E60 \rangle.$$

It uses the value of the previous row as the index for point j_s and the current row for indexing point j_r. Additionally, the total distance

$$D := \sum_{j \in \bar{J}} d_j$$

has to be calculated. At this point it has to be mentioned that in order to prevent an even more nested notation, generally speaking X has two meanings. Firstly, it represents the family $(x_i)_{i \in \bar{I}}$ as well as the sum over this family. In this case X can be interpreted as D. Anyhow, in the excel implementation this equates to

$$\langle SUMME(E54:E60) \| E61 \rangle.$$

5. **Indexing and summing the demand:** Similar to Step 4 the demand has to be mapped into the solution space. However, this time only the index found in the current row is required, while the second dimension remains 1 resulting in

$$\langle INDEX(\$H\$103 : \$H\$109; D54 + 1; 1) \| F54 : F60\rangle.$$

Similarly the sum of the demand is

$$\sum_{j \in J} q_j$$

, which is the same as

$$\langle SUMME(F54 : F60) \| F61\rangle.$$

6. **Calculating the load in weight:** For further calculations the load has to translated into weight. Additionally, given the nature of the VRP the total amount of load will decrease at each station point. Hence, the demand of point $q_{i_s} = q_{\omega(s)}$ has to be multiplied by the weight per unit $\rho_{product}$ and then subtracted from the load carried on the arc before, i.e. $f_{\omega(\acute{r})\omega(s)}$, which formally is given $\acute{r} < s < r$

$$\begin{aligned} \forall j \in \bar{J} : f_{j_{sr}} = f_{\omega(s)\omega(r)} &= f_{\omega(0)\omega(1)} - \sum_{i=1}^{n} q_{\omega(i)} \cdot \rho_{product} \quad n := \{1, ..., s\} \subset S \\ &\Leftrightarrow \quad f_{\omega(s)\omega(r)} = f_{\omega(\acute{r})\omega(s)} - q_{\omega(s)} \cdot \rho_{product} \end{aligned}$$

This equates to

$$\langle G54 - F54 * \$O\$120 \| G55 : G60\rangle.$$

However, the only exception is the arc from Stop 0 to Stop 1. In this model $f_{\bar{j}_{01}}$ has to amount to the entire demand of the trip. Therefore, it is simply

$$f_{\omega(0)\omega(1)} = \sum_{j \in J} q_j \cdot \rho_{product}$$

which depicted in Excel equates to

$$\langle F61 * O120 \| G54\rangle.$$

Having the weight calculated for every arc, the next step is to generate its sum which is formally

$$F := \sum_{j \in \bar{J}} f_j$$

and in the Excel implementation

$$\langle SUMME(G54 : G60) \| G61 \rangle.$$

7. **Indexing the Speed:** This step is virtually the same as 4. The only difference is that this time the values are selected from the range that contains the vehicles travelling speed on every arc and that the speed has to be converted from $[km/h]$ to $[m/s]$, i.e. $\forall j \in \bar{J} : v_j \cdot \frac{1}{3,6}$. This results in

$$\langle INDEX(\$E\$133 : \$K\$139; D53 + 1; D54 + 1)/3,6 \| G61 \rangle$$

in the Excel implementation.

8. **Calculate the travel time:** The time a vehicle requires to travel from one point to another can be calculated by dividing the arcs distance with the vehicles travelling speed on the same arc. Formally this means

$$\forall j \in \bar{J} : t_j = \frac{d_j}{v_j},$$

which in Excel is

$$\langle E54/H54 \| I54 : I60 \rangle.$$

The time required for the complete trip is

$$T := \sum_{j \in \bar{J}} t_j$$

which is the same as

$$\langle SUMME(I54 : I60) \| I61 \rangle$$

in the Excel implementation.

9. **Calculate alpha:** One of the factors required for estimating the fuel con-

sumption. This factor is a combination of several environmental factors, such as gravity g, acceleration a, road angel Θ and rolling resistance $\acute{c_r}$. It is defined as

$$\forall i \in \bar{I}: \ \alpha_i = a + g \cdot sin(\theta_i) + g \cdot \acute{c_r} \cdot sin(\theta_i),$$

which in Excel is

$$\langle \$0\$129 + \$O\$127 * SIN(E143) + \$O\$127 * \$O\$125 * COS(E143) \| M82 : S88 \rangle.$$

10. **Index alpha:** This step is virtually the same as Step 4. The only difference is that this time the values are selected from the range that contains α, which in Excel is

$$\langle INDEX(\$M\$82 : \$S\$88; \$D53 + 1; \$D54 + 1) \| J64 : J60 \rangle.$$

11. **Calculate beta:** The other factor required by the fuel consumption model is β. This factor is specific to the vehicle and requires input such as the drag coefficient $\acute{c_d}$, the surface area of the vehicle A and the air density ρ_{air}. It is defined as

$$\beta = 0.5 \cdot \acute{c_d} \cdot A \cdot \rho_{air},$$

which translated into Excel equates to

$$\langle 0, 5 * O126 * O105 * O107 \| L79 \rangle.$$

12. **Calculate the energy required to move the empty vehicle:** The first part of the fuel consumption model is concerned with defining the base energy consumption of an empty vehicle. This is done by simply multiplying the factors distance D, curb weight w and the environmental factor α. Thus it is defied as

$$\forall j \in \bar{J} : pw_j := d_j \cdot \alpha_j \cdot w,$$

which in Excel is

$$\langle \$N\$103 * E54 * J54 \| K54 : K60 \rangle.$$

Therefore, the total energy over the whole trip is

$$Pw := \sum_{j \in \bar{J}} pw_j$$

and in Excel

$$\langle SUMME(K54 : K60) \| K61 \rangle.$$

13. **Calculate and sum the energy required to move the load:** The second part of the fuel consumption model calculates the energy required to move the load of the vehicle. Similar to Step 12 this is done by multiplying the distance D with the weight of the load carried F and the environmental factor α. Described mathematically this means

$$\forall j \in \bar{J} : pf_j := d_j \cdot \alpha_j \cdot f_j,$$

which in Excel is

$$\langle G54 * E54 * J54 \| L54 : L60 \rangle.$$

Thus the sum is

$$Pf := \sum_{j \in \bar{J}} pf_j,$$

which is represented in Excel as

$$\langle SUMME(L54 : L60) \| L61 \rangle.$$

14. **Calculate and sum the energy required to move at a certain speed:** The third part of the fuel consumption model provides an estimation on how the fuel consumption changes based on a certain speed. Therefore, it requires input factors such as distance D, the vehicle specific factor β and of course the speed of the vehicle V. Yet again, all these factors are multiplied, however, in contrast to Step 13 and Step 12 the factor V is squared in beforehand. Hence, it is defined as

$$\forall j \in \bar{J} : pv_j := d_j \cdot \beta \cdot v_j^2,$$

which in Excel equates to

$$\langle H54 \wedge 2 * \$L\$79 * E54 \| M54 : M60 \rangle.$$

This values are then summed up to

$$Pv := \sum_{j \in \bar{J}} pv_j,$$

which in Excel is

$$\langle SUMME(M54 : M60) \| M61 \rangle.$$

15. **Calculate total energy required:** In this step all values generated during the Steps 12, 13 and 14 are simply added. That is

$$\forall j \in \bar{J} : p_j := pw_j + pf_j + pv_j,$$

which in Excel equates to

$$\langle SUMME(K54 : M54) \| N54 : N60 \rangle.$$

Thus now the total energy consumed during the trip can be calculated by

$$P = \sum_{j \in \bar{J}} p_j,$$

which in Excel is

$$\langle SUMME(N54 : N60) \| N61 \rangle.$$

16. **Energy unit translation:** At this state the total energy consumption is measured in unit $[J]$. However, further calculations require it to be given in $[kWh]$. Therefore, one simply has to multiply the energy required with the translation factor, which dictates that $1[J] = 2.77\overline{7} \cdot 10^{-7}[kWh]$ (Wikipedia, 2016e). For easier calculations $PX := \{Pw, Pf, Pv, P\}$ the set of all energy consumption variants. Thus one defines

$$\forall px \in PX \forall j \in \bar{J} : px_{[kWh]_j} := px_{[J]_j} \cdot 2.77\overline{7} \cdot 10^{-7}.$$

In Excel this is

$$\langle K54 * \$C\$95 \| E67 : H74 \rangle.$$

17. **Translate the Energy into fuel consumption:** The next step is to translate the energy measured in $[kWh]$ into the fuel consumption in $[l]$. This is

done by dividing the required energy with the energy content of one litre of fuel ω and with the efficiency of the vehicle η. That is

$$\forall px \in PX \forall j \in \bar{J} : px_{[l]_j} := px_{[kWh]_j} \cdot \frac{\omega_F}{\eta},$$

which is in Excel

$$\langle E67 * \$O\$109/\$O\$106 \| I67 : L74 \rangle.$$

18. **Translate the fuel consumption into GHG emissions:** The last conversion to be done is to translate the fuel consumption in $[l]$ into GHG emissions measured in $[kg]$. This is achieved by multiplying the consumed fuel with amount of CO2 in a litre of fuel, i.e. ω_{GHG}. Hence, formally defined this equates to

$$\forall px \in PX \forall j \in \bar{J} : px_{[kg]_j} := px_{[l]_j} \cdot \omega_{GHG},$$

which is in Excel

$$\langle I67 * \$0\$108 \| M67 : P74 \rangle.$$

19. **Calculate the driver costs:** One part of the total costs originate from the wage of the driver, which if combined with the travel time amounts to the cost of the driver. Hence, it is defined

$$\forall j \in \bar{J} : cd_j := \acute{p}_d \cdot t_j,$$

which is in Excel

$$\langle I54/3600 * \$O\$114 \| O54 : O60 \rangle.$$

The factor $\frac{1}{3600}$ is necessary, because the wage is normally given in $[Euros/h]$. Furthermore the sum of this parameter is

$$Cd := \sum_{j \in \bar{J}} cd_j,$$

which is in Excel

$$\langle SUMME(O54 : O60) \| O61 \rangle.$$

As a final remark one could argue that the serving time $\acute{t}_j$ for $j \in J$ should also be considered, however, to simplify the calculation this is neglected.

20. **Calculate the fuel costs:** The second component is the fuel costs, which can simply be calculated by multiplying the fuel consumption with the fuel price. Thus,

$$\forall j \in \bar{J} : cf_j := \acute{p_f} \cdot p_j,$$

which translated into

$$\langle L67 * \$O\$115 \| P54 : P60 \rangle$$

in the Excel implementation. The sum in this case would be

$$Cf := \sum_{j \in \bar{J}} cf_j,$$

which is in Excel

$$\langle SUMME(P54 : P60) \| P61 \rangle.$$

21. **Calculate the cost of emissions:** The last component of the total costs is the emission costs, which is defined by multiplying the produced emissions with the estimated price of CO2. In mathematical terms this is

$$\forall j \in \bar{J} : ce_j := \acute{p_e} \cdot p_j,$$

while in the Excel implementation this is defined as

$$\langle O67 * \$O\$116/1000 \| Q54 : Q60 \rangle$$

The sum in this case would be

$$Ce := \sum_{j \in \bar{J}} ce_j,$$

which is in Excel

$$\langle SUMME(Q54 : Q60) \| Q61 \rangle.$$

Additionally, it has to be said that since the price is normally given in $[Euro/t]$ it first has to be divided by 1000 to get the price of one $[kg]$ of CO2.

22. **Calculate the total cost:** The total cost of each arc can be calculated by

simply adding all previously generated cost related values together. Formally this is

$$\forall j \in \bar{J} : c_j := cd_j + cf_j + ce_j,$$

which in Excel equates to

$$\langle SUMME(O54 : Q54) \| R54 : R60 \rangle.$$

In order to obtain the total cost of the entire trip one simply has to

$$C := \sum_{j \in \bar{J}} c_j,$$

which in Excel is

$$\langle SUMME(R54 : R60) \| R61 \rangle.$$

23. **Index servicing time:** Similar to Step 5 the servicing time has to be mapped onto the solution space. To do so the function

$$\langle INDEX(K\$103 : K\$109; \$D54 + 1; 1) * 3600 \| V54 : V60 \rangle$$

is defined. The factor 3600 ensures that the servicing time, which is given in hour is translated into seconds, in order to ensure a clean calculation.

24. **Index lower bound of the time window:** This step is virtually the same as Step 23. That is the lower bound of the time window is indexed into the solution space. The formula required for this task is

$$\langle INDEX(I\$103 : I\$109; \$D54 + 1; 1) * 3600 \| T54 : T60 \rangle,$$

while factor 3600 ensures that the start of the time window is translated into seconds.

25. **Index upper time window bound:** This step is the same as Step 24. That is the upper bound of the time window is indexed onto the solution space. Herby,

$$\langle INDEX(J\$103 : J\$109; \$D54 + 1; 1) * 3600 \| U54 : U60 \rangle,$$

is used. Moreover, the factor 3600 ensures that the start of the time window, which is given in hours is translated into seconds.

26. **Calculate the visiting time:** If one intends to measure the flow of time in relation to the vehicles position on the trip the moment where the vehicle leaves the terminal point makes an ideal starting point. Hence, the cell attributed to the terminal point $\langle S53 \rangle$ is set to 0. The time at point j is the sum of the travel time of every previous arc and of service time at every previous point. That means the time is

$$\forall j \in \bar{J} : T_{0j_s} = T_{\omega(0)\omega(s)} \quad = \quad \sum_{i=1}^{n} T_{\omega(i-1)\omega(i)} + \sum_{i=1}^{n-1} \acute{t}_{\omega(i)} \quad n := \{1, ..., s\} \subset S$$

which translated into Excel this is

$$\langle S53 + I54 + V54 \| S54 : S60 \rangle.$$

With this the very foundation of the model is created. By having these calculations in place one can already investigate the behaviour of the model, by adjusting the input parameter. For example, it is already possible to search the best possible trip, by simply altering the values of the cells, which define the sequence of stops, i.e. J. However, at the current state this can only be done manually. Hence, the next step is to include the Excel solver into the implementation, since the main intention for the calculations presented above is to construct a model, which can be used for finding a sequence of stops that optimises an objective and the Excel solver can be used to find such an optimal solution for the model. In this case configuring the solver requires three steps. Firstly, an objective has to be defined. Secondly, it has to be defined what values have to be altered in order to achieve the previously defined objective. Thirdly, the borders of the problem have to be defined. That is, the conditions that restrict the optimisation process have to be implemented.

1. **Set objective cell:** The first step is to define the value that has to be minimised, i.e. the objective. In this case the most common objectives are distance, load, energy and cost.
   a) **Select Od:** The relevant objective cell is $\langle \$E\$61 \rangle$
   b) **Select Of:** The relevant objective cell is $\langle \$G\$61 \rangle$

c) **Select Oe:** The relevant objective cell is $\langle \$P\$74 \rangle$

d) **Select Oc:** The relevant objective cell is $\langle \$R\$61 \rangle$

2. **Define variable cells:** The next step is to define, which values can be altered by the solver during the optimisation process. The selection of which depends on the problem at hand.

   a) **Implementation with fixed Speed:** The relevant variable cells are only the cells, which contain the path. That is,

   $$\langle \$D\$54 : \$D\$59 \rangle$$

   is selected.

   b) **Implementation with variable Speed:** This implementation requires two variable inputs. Firstly, the path, which is the same as in Variant 2a, i.e.

   $$\langle \$D\$54 : \$D\$59 \rangle.$$

   Secondly, the speed can also be altered thus the range

   $$\langle \$E\$133 : \$K\$139 \rangle$$

   has to be additionally selected.

3. **Introduce constraints:** Having the objective and the decision variables defined the last step is the introduction of constraints. The nature of the problem, dictates the kind of restrictions that have to be imposed onto the optimisation process. For each of the most common variants encountered in the case the complete set of constraints is introduced. Hence, they are to some extent repetitive. Additionally, for all implementations it is important to ensure negative values wont occur.

   a) **Implementation with fixed Speed and without time windows:** In this case only the AllDifferent constraint is required, for executing the optimisation. Hence, the collection of constraints is:

   $$\langle \$D\$54 : \$D\$59 = AllDifferent \rangle$$

   b) **Implementation with variable Speed and without time windows:**

Additional to the constraints found in 3a, one has to introduce speed limits. These speed limits are found in range $\langle E113 : K119\rangle$ and range $\langle E123 : K129\rangle$, with the first one representing L while the second one represents U, i.e. the lower and upper bound of $\widetilde{LU}$. Therefore the collection of constraints looks as follows:

$$\begin{aligned}\langle \$D\$54 : \$D\$59 &= AllDifferent\rangle \\ \langle \$E\$133 : \$K\$139 &<= \$E\$123 : \$K\$129\rangle \\ \langle \$E\$133 : \$K\$139 &>= \$E\$113 : \$K\$119\rangle\end{aligned}$$

c) **Implementation with fixed Speed and with time windows:** This implementation requires additionally to the constraints found in Variant 3a a method for enforcing time windows. This is done by imposing the requirements that the travelling time T_{0j} has to be greater than the lower bounds of the time window, while at the same time it has to be lower than the upper bounds of the time window, which means $a_j \leq T_{0j} \leq b_j$. Therefore, this collection of constraints is:

$$\begin{aligned}\langle \$D\$54 : \$D\$59 &= AllDifferent\rangle \\ \langle \$S\$54 : \$S\$59 &<= \$U\$54 : \$U\$59\rangle \\ \langle \$S\$54 : \$S\$59 &>= \$T\$54 : \$T\$59\rangle\end{aligned}$$

d) **Implementation with variable Speed and with time windows:** No new constraints are added, since this implementation is virtually the same as Variant 3c. However, this time around it builds on Variant 3b, thus its set of constraints is:

$$
\begin{aligned}
\langle \$D\$54 : \$D\$59 &= AllDifferent\rangle \\
\langle \$E\$133 : \$K\$139 &<= \$E\$123 : \$K\$129\rangle \\
\langle \$E\$133 : \$K\$139 &>= \$E\$113 : \$K\$119\rangle \\
\langle \$S\$54 : \$S\$59 &<= \$U\$54 : \$U\$59\rangle \\
\langle \$S\$54 : \$S\$59 &>= \$T\$54 : \$T\$59\rangle
\end{aligned}
$$

These various solver configurations can be summarised as seen in Table 3.1. Herby, each cell represents a type of solver configuration.

	No Time Window (TW_{False})	Time Window (TW_{True})
Fixed Speed ($Speed_{Fixed}$)	$Solver(2a, 3a)$	$Solver(2a, 3c)$
Variable Speed ($Speed_{Variable}$)	$Solver(2b, 3a)$	$Solver(2b, 3c)$

Table 3.1: Guideline - Solver configurations

If these configurations are now combined with the model described in the step by step guideline, Table 3.2 can be created. This table visualises the minimum of steps required for implementing the model when optimising for the objectives Distance, Load, Energy and Cost. Additional, the differences between an implementation without time windows *(No TW)* and a implementation with time windows *(TW)* is also included in this table. In Table 3.2 a "X" refers to a mandatory step, and "-" indicates a optional step.

By combining Table 3.1 and Table 3.2, a notation for referencing different implementation variants can be created. Such a notation can later be used for quickly referencing the implementation. That is,

$$\mathscr{I}_{x,y,z} := Implementation(x, Speed_y, TW_z)$$

		Objectives							
		Distance		Load		Energy		Cost	
Step	Action	No TW	TW	No TW	TW	No TW	TW	No TW	TW
1	Positioning the Input	X	X	X	X	X	X	X	X
2	Prepare path selection	X	X	X	X	X	X	X	X
3	Calculating the Distance	X	X			X	X	X	X
4	Indexing the distance	X	X			X	X	X	X
5	Indexing the demand			X	X	X	X	X	X
6	Calculating the load in weight			X	X	X	X	X	X
7	Indexing the Speed		X		X	X	X	X	X
8	Calculate the travel time		X		X		X	X	X
9	Calculate alpha					X	X	X	X
10	Index alpha					X	X	X	X
11	Calculate beta					X	X	X	X
12	Calculate energy required to move the empty vehicle					X	X	X	X
13	Calculate energy required to move the load					X	X	X	X
14	Calculate energy required to move at a certain speed					X	X	X	X
15	Calculate total energy required					X	X	X	X
16	Energy unit translation					-	-	X	X
17	Translate the Energy into fuel consumption					-	-	X	X
18	Translate the fuel consumption into GHG emissions				-	-		X	X
19	Calculate the driver costs							X	X
20	Calculate the fuel costs							X	X
21	Calculate the cost of emissions							X	X
22	Calculate the total cost							X	X
23	Index servicing time		X		X		X		X
24	Index lower time window bound		X		X		X		X
25	Index upper time window bound		X		X		X		X
26	Calculate the visiting time		X		X		X		X

Table 3.2: Guideline - Step by Step Implementation Overview

with

$$x \in \{Distance,\ Load,\ Emissions,\ Cost\}$$
$$y \in \{Fixed,\ Variable\} z \in \{True,\ False\}.$$

For example,

$$\mathscr{I}_{Od} := Implementation(Distance, Speed_{Fixed}, TW_{False})$$

would reference an implementation ranging from Step 1 to Step 4 with a solver configuration of $Solver(Speed_{Fixed}, TW_{False})$, which then indicates that the variable cells defined in 2a and the constraints in 3a are required for solving the problem. Finally, with this all the necessary tools for solving the case presented in Chapter 4, as well as the required notation for understanding its analysis in Chapter 5 are introduced.

Chapter 4

The Case Study and Teaching Note

This chapter is used to introduce the case on the basis of which the analysis in Chapter 5 is performed. Said case introduces a narrative, a set of problems, as well as additional data. Moreover, this chapter also contains a teaching note. The teaching note includes the motivation behind the case as well as its teaching objectives. Furthermore, it briefly summarises the narrative and more importantly proposes a brief solution for the case.
*Disclaimer: Values that are marked with * are assumed. Their range out of which they are selected is rooted in reality. However, they are not specific to the presented vehicle.*

4.1 The Case

```
  Hello,
How are you doing?  I hope you have been well!  I wish you could
have been at the launch party.
I assume that by now you already had your last exam.  That is
why I want to ask you again whether you can help us in out endeavour
and since you are now finished with your studies, there is no
excuse to postpone your decision :-).  Currently we have no one
who can coordinate and plan the distribution of our goods.  Thus
it would be awesome if we could rely on your expertise if questions
regarding this matter arise.  We all would rejoice in having
you with us.  So what do you say?
Thomas
```

```
PS.: We have a new Homepage. You have to check it out. It
is awesome!
```

4.1.1 Our Website

Hello, we are Thomas Bauer, Sarah Maier, John and Jill Jeromy! We are the founders of Bijection, a recently founded company that produces alternative building materials. By doing so we want to shift our society closer to a sustainable one. Welcome to our website, here you can find not only information about our products, but also the background of the company, our core principles, our team and above how we contribute to our overall objective.

Our Team

We are Bijection. We make alternative building materials of any kind. We are committed to the principle of sustainability. We want to improve the society in which we are living. But who are We.

- Hello I am Thomas Bauer and I was born on 30.1.1983. My part to play in this company is the part of manager. But what am I actually doing. Well. For the most part I am responsible for budgeting, organising and planning.
- Hello, Sarah Maier is my name. I was born on 2.5.1987 and I am currently trying to create a new and exiting product, or to put it more formally, I am in charge of research and development.
- John Jeromy born on 8.8.1986. Head of Production. My area of expertise is the development and the oversight of the production process. Furthermore, I am involved in improving the already existing products.
- Hi, Jill Jeromy I entered the world on 8th of August in the year 1986. I live and breathe sales. Therefore, I am currently responsible for, you guessed it, sales.

Our Background

We want you to know why this company does what it does and how everything started. Therefore, we will take you back in time to the year 2010. Back then we where all still students. Thankfully, otherwise we would have not been confronted with the environmental impact of the construction industry. Did you know, the construction industry is responsible of 24% of the global material extraction? But wait there is more. The concrete industry alone is also responsible for about 38.5% of the European industrial greenhouse gas emissions (Mehta, 2001). So why is this important? Well we were so shocked by this that we decided to change that. Hence we started researching alternative building materials, which could complement more traditional building materials. And we found something. This something, however, is a highly interesting but also a rather strange something. Ready, here it comes! Making bricks and insulation material out of the mycelium of mushrooms. We know it sounds a little bit crazy, but stick with us! And don't be afraid it took John a very long time to convince the rest of us too. To be honest some of us were even sceptical until they had the first prototype in their hand and experienced its remarkable properties. After a long period of research we find ourself at the beginning of 2013. At this point in time we had just succeeded in creating a brick made out of mushrooms. As interesting as this already sounds, the properties of this brick that are really intriguing. Because this brick has by weight the strength of concrete, is non-toxic, mould and water resistant, fire proof and can also be used as insulation. naturally, after spending such an extensive amount of time and energy on developing these mycelium bricks. we decided to share our new product with the world (Travaglini et al., 2013, Mycoworks, 2016).

Our Core Principles

We founded the company Bijection in order to help people in their pursuit of building their own homes. However, it is of immense importance for us to do so in a sustainable and environmental friendly manner. Therefore, we founded this company on the following principles.

Purpose: Our purpose is to support the transition into a more sustainable future by developing new building materials. With these materials we intend to provide possible solutions for building homes in an environmentally friendly manner, while at the same time ensuring that resource depletion and exploitation can be kept at a minimum.

Efficiency: In order to ensure that our products benefit the environment, we spare no effort in transforming the waste produced by various industries into useful products. These products should not only be produced in an energy conserving manner, but they should also exhibit additional properties that benefit the environment.

Trade-offs: At the core of our business resides the challenge of finding a balance between generating the profits we need for continuing our business and the fulfilment of our success criteria.

Criteria: At the end of a year many of us recapitulate what they have achieved. We want to do the same! Therefore, we measure the success of our company by the following mix of qualitative and quantitative criteria:

- To what extent could we improve our old products?
- Have we made some progress in developing a new product?
- Have we managed to be CO2 neutral?
- Is the existence of the company secured?
- In what way did we improve society?

Inducements: The allocation of additional benefits, to our most important stakeholder is a top priority. This stakeholder being, the environment. Therefore, we calculate the costs associated with our greenhouse gas emissions output and donate an equivalent amount of money to a charity or aid organisation of our choice.

These principles are at the core of our very hearts, and therefore by extension at the center of our business. With these principles we want to ensure that this business is and remains a sustainable business (Parrish, 2010). This moral code is apparent throughout our whole Supply Chain and beyond. That is, for us it is of great importance that all the resources required for our products are sourced sustainably, every transport or delivery task is done with caution to the environment and that every product is produced in an environmentally friendly manner.

Our Products

Currently, we only managed to create one product that satisfies our standards. However, you can be ensured, we are dedicated to increase our product portfolio. Since great work requires time, we hope that in the meantime our M.Brick can help you in achieving your objective. Therefore, we want to show you a summary of its properties as well as give you some insight into the production process.

The mighty M.Brick: It is produced by combining agricultural waste such as corn husks or wood chips, with one to several different strains of fungi. Then the cultivation process starts. During this time, the fungus feeds on the wast products and multiplies. Thereafter, the mass is moulded into the desired shape. In our case a brick shape. Then the fungus is given some time to adapt to the shape. After that the brick is removed from its form. Following this some, additional time is required for the hardening process. As a final step, the brick is subjected to higher temperatures to stop the growing process, or to put it more bluntly "kill the mushroom". And voila you have a mycelium brick. In our case a 240 x 115 x 71 mm big and 623,1 g heavy M.Brick. But what is it good for? This material can be used for insulating (not only temperature but also sound), building or even packaging. Thus, we recommend its use in sandwich panels, for constructing non-load bearing walls or for constructing acoustic panels. The beauty in this process is that it is easily customisable. You want different properties or a different shape just contact us and we will tell you what is possible (Travaglini et al., 2013, Mycoworks, 2016).

What's next? Work in progress, we are currently on researching for more. So be curious and let us to surprise you!

Our Contribution

Even though we try to design our products in a way that minimises the environmental impact, it is nearly impossible to mitigate all negative externalities. May it be the production process or the act of distribution, most processes require energy and lead, even with mitigation efforts, to the release of greenhouse gases into the planets atmosphere. Therefore, we decided to calculate the costs of our CO2 emissions and donate an equivalent amount of money to Union of Concerned Scientists. But how do we price carbon? Easy! Or may be not? You decide. We offer you three choices.

No. 1: Simple, since we are part of the EU we would set our carbon price based on EEA (European Emission Allowances). However, to prevent massive volatility we would adjust the price quarterly.

No. 2: A bit more complicated. Based on the reports of the World Bank we calculate the average of the global carbon prices.

No. 3: And finally we are going to increment our carbon price until we reach the recommended price of 82 Euro in 2030. That is, starting from 0 at the year 2013, we are increasing our carbon price every quarter in order to end up at a price of 82 Euro in 2030 (Bram, 2015).

However, if you don't want to choose between either of them, we simply select the highest one.

4.1.2 Problems

Problem One: 9.7.2013

A rather dreadful day, however, to be honest not much better than the previous ones. No word from Thomas. Up until now not a single order was placed. Fortunately, the daily monotony was broken by an exited call from Thomas. "Six Accounts!!!! Jill managed to secure six orders. Isn't that great. We have already started to ramp up production to satisfy the demand. Hey, now that we finally have customers, I need your help. I recently got approached by two transportation companies. The cool thing about them is that both buy EEA. Could you figure out which one would suit us best? I will send you the details today."

`Hi, here is the information about the two delivery services (and the orders of course. I even marked the destinations on a map so it is easier to visualise the orders. ):`

`Which one would you suggest? Can you prepare a sound economical as well as ecological argumentation for the respective choice. Probably you don't need the reminder, but it is vital that the corporate philosophy is uphold under every circumstance. By the way, I am currently trying to build business connections in Germany, Italy, Hungary and the Czech Republic. Just to be sure, I have constructed three additional scenarios for you to test.`

Name	Maier's LKW Service	Name	Delivery-Max
Objective Price (domestic) Price (international)	Load 0,03 [$Euro/kg$] 0,06 [$Euro/kg$]	Objective Price	Distance 1,2 [$Euro/km$]
Truck	Mercedes 4160 8x4	Truck	MAN 41.663 8 x 4
L x W x H [mm] Curb Weight Legal Weight HP Type Fuel Type Engine Efficiency	8134 x 2500 x 3850 13,52 t 41 t 610 Heavy articulated* Diesel* 32%*	L x W x H [mm] Curb Weight Legal Weight HP Type Fuel Type Engine Efficiency	8390 x 2550 x 3900 13,5 t 41 t 663 Heavy articulated* Diesel* 34%*
Trailer	Krone Profi Liner EasyTrap SDP 27 eLB4-PZ	Trailer	Krone Mega Liner SDP 27 eLG4-PZ
L x W x H [mm] Coupling Hight Weight Legal Weight	13620 x 2550 x 2680 975 mm 6290 kg 39000 kg	L x W x H [mm] Coupling Hight Weight Legal Weight	13620 x 2550 x 2860 1050 mm 6710 kg 39000 kg

Table 4.1: Case - Problem One: Delivery Services

(Krone-Trailer, 2016, Felbermayr, 2016)

BO1							
Stops	Graz	Bregenz	Klagenfurt	Insbruck	Leoben	Linz	Vienna
Demand [$Units$]	0	4700	6000	5500	4300	5700	6700

Table 4.2: Case - Problem One: Order 1 (BO1)

TS1							
Stops	Graz	Vienna	Linz	Salzburg	Bregenz	Villach	Eisenstadt
Demand [$Units$]	0	20000	1000	1500	2000	2000	6000
TS2							
Stops	Graz	Hamburg	Frankfurt	Florence	Turin	Budapest	Prague
Demand [$Units$]	0	10500	6000	9000	2250	4500	750
TS3							
Stops	Graz	Debrecen	Ostrava	Stuttgart	Berlin	Munich	Milan
Demand [$Units$]	0	1400	4300	5700	4300	7174	10000

Table 4.3: Case - Problem One: Trips (TS1,TS2,TS3)

PS.: It is ridiculous to even mention that, but since you are prone to turn simple problems into complex ones, it feels that I have to. Do not overcomplicate the matter, we don't have that much time. That means:

- I don't want you to approximate the average travelling speed of the vehicle on each specific road. Simply use the Austrian speed limits, i.e. 30 to 130 km/h and our travelling speed estimation of 80 km/h as generally applicable travelling speed.
- I don't want you to try and calculate the road angle of every point on each road.

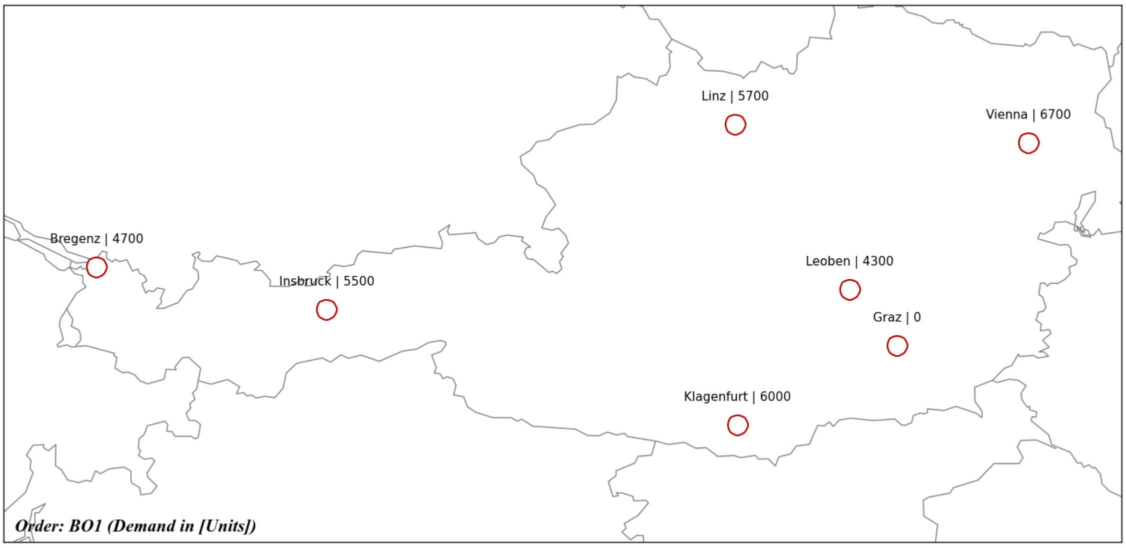


Figure 4.1: Case - Problem One: Order 1 (BO1)

Hence, please assume regarding this issue that we are dealing with a flat world, i.e. roads without slope.

Problem Two: 7.10.2013

After the transportation company was selected there was little to do. So the past month passed without any request from Thomas. However, in a surprising twist of events a E-Mail from Thomas arrived.

Hi

I recently got into a conversation with the head of our transportation company. Long story short, I told him about your fuel consumption model and he seemed interested. After some back and forth I got this response.

Dear Thomas Bauer!

We are intrigued by your idea. However, as much as we are admiring your environmental awareness, we are still a business. Hence, we are not willing to make compromises in favour of reducing emissions. Therefore, if the model you speak of can reduce our costs, we are more than willing to implement it. However, you need to know, we are a simple organisation, thus we are not interested in highly complicated computations. Implying that we would be grateful if said model requires as little information as possible. If it helps, we are mainly active in Austria, pay our drivers about 15

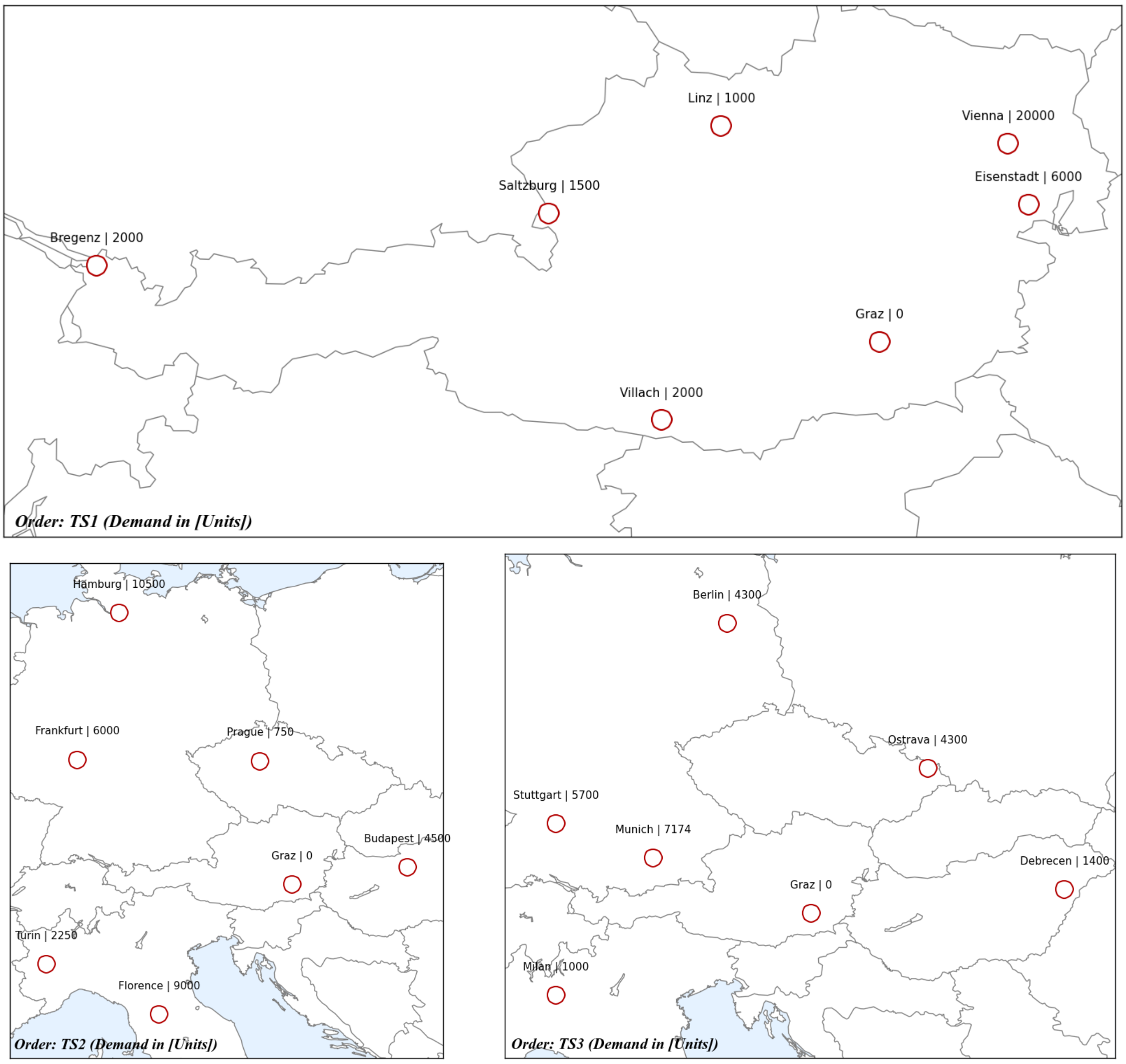


Figure 4.2: Case - Problem One: Trips (TS1,TS2,TS3)

> `% more than the average and primarily employ medium/heavy trucks, heavy trucks and heavy articulated ones. Could you show us the results of your model, based on your last order, i.e. Order 66.`

`Can you develop such a model? And can you compare it to our fuel consumption model? How and when do the results diverge? I have already attached Order 66. Additionally, I would suggest that also you use the test scenarios I gave you about two month ago.`
`Thomas`

BO66							
Stops	Graz	Klagenfurt	Linz	Eisenstadt	Leoben	Krems	St. Poelten
Demand [*Units*]	0	1500	1500	12000	5000	10000	2000

Table 4.4: Case - Problem Two: Order 66 (BO66)

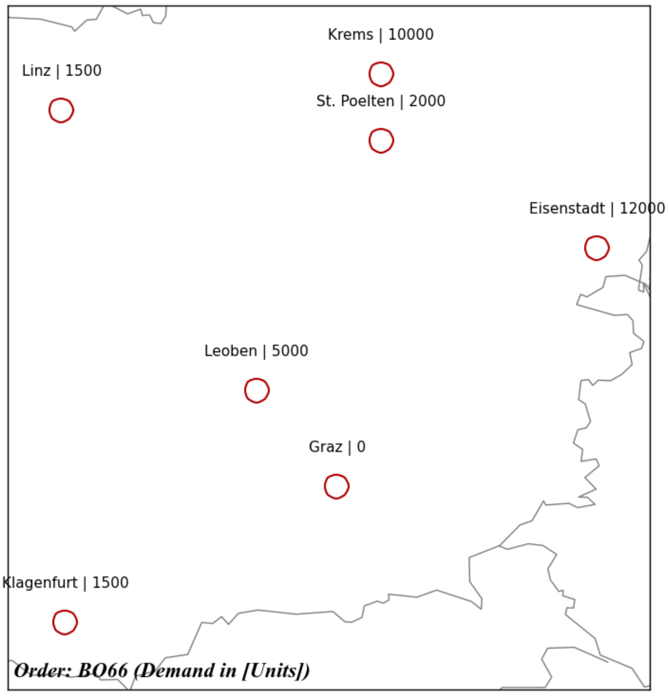


Figure 4.3: Case - Problem Two: Order 66 (BO66)

Problem Three: 2.6.2014

Hi

It has been some time. How are you?

Last week one of Sarah's friends, who once had a small transportation-company, decided that he is now moving into another industry. Therefore, he is selling his trucks and offered Sarah a friendship discount on one vehicle. This got me thinking. Would it be better if we would ship our goods on our own? Can you check that for me, against the test scenarios and the new order? I attached the information about both trucks as well as a new order.

Thanks, Thomas

PS: Before I forget, Jill said that she might know a suitable driver. Unfortunately, she is unsure about the amount of payment he requests. However, she assumes it will be around 10,50 Euro per hour.

Truck	Mercedes 4160 8x4	IVECO AS440S45T
L x W x H [mm]	6866 x 2500 x 3850	6074 x 2490 x 3710
Curb Weight [t]	11,96	7,4
Legal Weight [t]	41	18
HP	600	450
Type	Heavy articulated*	Medium/heavy truck*
Fuel Type	Diesel*	Gasoline*
Engine Efficiency	36%*	20%*
Trailer	Krone Profi Liner Ultra SDP 27 eLUBP-CS	City Liner CS Typ: SEP 10 zLNZ4-CS
L x W x H [mm]	13620 x 2550 x 2687	11025 x 2550 x 2700
Coupling Hight [mm]	1050	1150
Weight [t]	5980	4850
Legal Weight [kg]	39000	22000

Table 4.5: Case - Problem Three: Vehicles

(Krone-Trailer, 2016, Felbermayr, 2016)

BO542							
Stops	Graz	Vienna	St. Poelten	Salzburg	Munich	Insbruck	Zuerich
Demand [*Units*]	0	1500	1500	2000	4000	18000	4000

Table 4.6: Case - Problem Three: Order 542 (BO542)

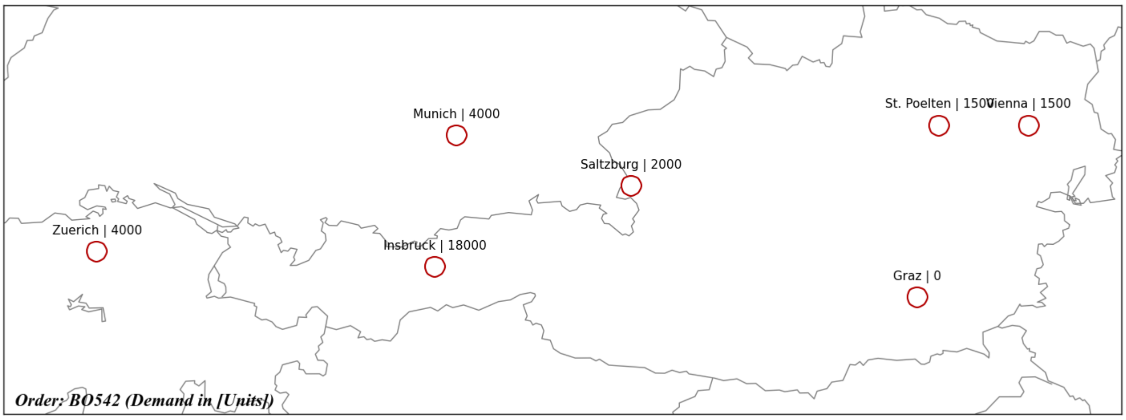


Figure 4.4: Case - Problem Three: Order 542 (BO542)

Problem Four: 25.7.2014

Hi

I recently talked with John about your model. I was rather surprised after he asked me, why we are always comparing everything against the objective that minimises GHG emissions, if we are already assigning a monetary value to the GHG output. Can you check if it makes sense to optimise for costs. Besides that, the conversation made me think. I mean why do we have to set the speed at 80 km/h. If possible wouldn't it be better if the model can set the travelling speed by itself. Before I start to change anything, I would like to know how this will impact the model and its results. You know the usual stuff. Oh and don't only test the new objective against the usual test scenarios but also against Order 600.

Thanks, Thomas Update: Three weeks ago Jill introduced me to the driver, his name is Peter and he charges 11 Euro per hour.

BO600							
Stops	Graz	Villach	Krems	Leoben	Budapest	Bregenz	Salzburg
Demand [*Units*]	0	1400	2800	4400	4300	6000	14000

Table 4.7: Case - Problem Three: Order 600 (BO600)

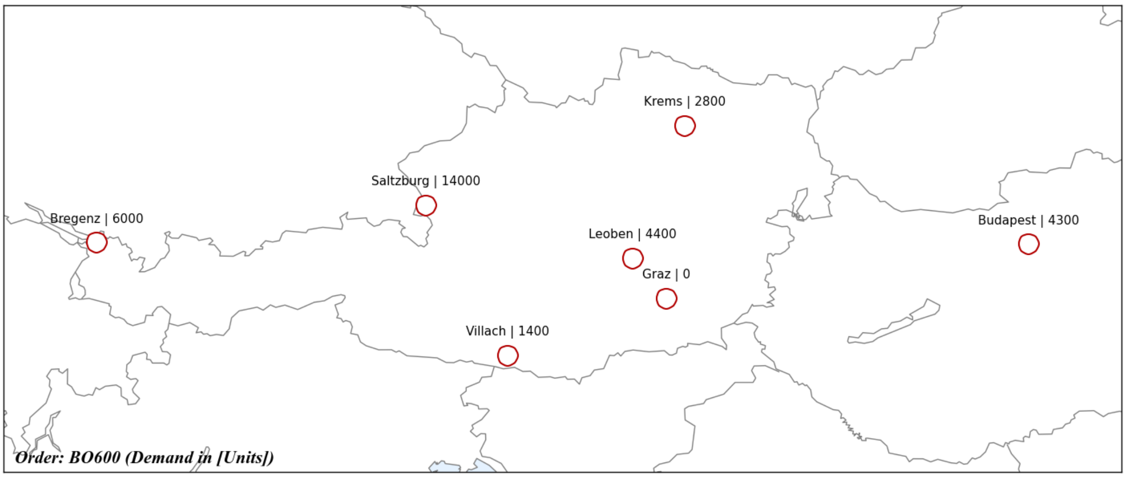


Figure 4.5: Case - Problem Four: Order 600 (BO600)

Problem Five: 23.2.2015

```
Hi
Have you already seen the customer survey results? I am glad the survey included some
questions about the customer's satisfaction concerning the delivery. It seems that
most of our customers like our product, but are very unsatisfied with our service.
The most common complaint about us, was that we seem to ignore every appointment made
when it comes to delivery. I talked with Jill and she said that normally she promises
the customer that we are going to deliver the goods at around a certain time. Based
on Jills suggestions I expanded the test scenarios to include such promises. That
is, I have added an earliest and a latest due date, as well as a serving time and inserted
some random values. Can you bend the model to accommodate these kinds of promises
as well? Maybe driving faster could help? When you are finished can you tell me how
this influenced your model and its result. I mean I have to be at least up to date
with the current capabilities of your model :-). Oh, just to be sure that you are
up to date Peter got a raise, he now earns 11,35 Euro per hour
Thomas
```

TS1						
Stops	Vienna	Linz	Salzburg	Bregenz	Villach	Eisenstadt
Earliest due date [h]	1	4	5	9	6	2
Latest due date [h]	10	10	15	17	13	7
Serving Time [h]	0,12	0,24	0,6	0,36	0,6	0,84
TS2						
Stops	Hamburg	Frankfurt	Florence	Turin	Budapest	Prague
Earliest due date [h]	20,5	0	0	0	0	0
Latest due date [h]	26	70	70	70	70	70
Serving Time [h]	0,57	0,33	0,49	0,12	0,25	0,1
TS3						
Stops	Debrecen	Ostrava	Stuttgart	Berlin	Munich	Milan
Earliest due date [h]	1	1	1	7	1	1
Latest due date [h]	50	50	50	9	50	50
Serving Time [h]	0,1	0,24	0,31	0,24	0,39	0,55

Table 4.8: Case - Problem One: Trips (TS1,TS2,TS3)

4.1.3 Appendix

Company Name	Bijection Materials GmbH
Legal Form	GmbH
Registered Seat	Graz
Business Address	Alte Poststrasse 300,8020
Scope of Business	Production of sustainable and innovative building materials
Date of Inscription	20.6.2013
Capital	70.000 Euro
Founder	Thomas Bauer born 30.1.1983, Sarah Maier born 2.5.1987, John Jeromy born 8.8.1986, Jill Jeromy born 8.8.1986
Statutory Body	Thomas Bauer born 30.1.1983

Table 4.9: Case - Bijection commercial register entry

Date	Price [Euro]
03.01.13	6,19
09.04.13	4,98
01.07.13	4,19
02.10.13	5,30
02.01.14	4,86
01.04.14	4,60
01.07.14	5,81
01.10.14	5,82
02.01.15	7,16
01.04.15	7,13
01.07.15	7,46
01.10.15	8,14

Table 4.10: Case - Prices of European Emission Allowances on the Global Environmental Exchange

(EEX, 2015)

Date	Price [Euro]
07.01.13	1,371
08.04.13	1,386
01.07.13	1,341
07.10.13	1,350
06.01.14	1,333
07.04.14	1,311
07.07.14	1,309
06.10.14	1,303
05.01.15	1,144
13.04.15	1,145
06.07.15	1,173
05.10.15	1,071

Table 4.11: Case - Diesel Fuel Prices in Austria

(bmwfw, 2015)

Date	Price [Euro]
Year	Salary [Euro/h]
2013	8,2
2014	8,4
2015	8,6

Table 4.12: Case - Average Driver Salary

(WKO, 2016a,b)

Date	Price [USD]
Sweden carbon tax	130
Finland carbon tax (transport fuels)	64
Switzerland carbon tax	62
Norway carbon tax (upper)	52
Finland carbon tax (heating fuels)	48
Tokyo CaT	36
UK carbon price floor	28
Denmark carbon tax	25
BC carbon tax	23
Ireland carbon tax	22
Slovenia carbon tax	19
France carbon tax	16
California CaT	13
Quebec CaT	13
Alberta SGER	12
Switzerland ETS	9
Korea ETS	9
EU ETS	9
Iceland carbon tax	8
Beijing Pilot ETS	7
RGGI	7
Portugal carbon tax	6
Shenzhen Pilot ETS	5
New Zealand ETS	5
Hubei Pilot ETS	4
Latvia carbon tax	4
Mexico carbon tax (upper)	3
Norway carbon tax (lower)	3
Tianjin Pilot ETS	3
Guangdong Pilot ETS	2
Chongqing Pilot ETS	2
Kazakhstan ETS	2
Estonia carbon tax	2
Shanghai Pilot ETS	2
Japan carbon tax	2
Mexico carbon tax (lower)	1
Poland carbon tax	1
Exchange Rate EUR/USD 1,0967 on 31.7.2015	

Table 4.13: Case - Established Carbon Prices on 1.8.2015

(Bram, 2015, Bank, 2015)

Date	Price [USD]
Sweden carbon tax	168
Tokyo CaT	95
Norwegian carbon tax (upper)	69
Swiss carbon tax	68
Finnish carbon tax	48
Danish carbon tax	31
British Columbia carbon tax	28
Irish carbon tax	28
Australia CPM	22
UK carbon price floor	16
California CaT	11
Shenzhen Pilot ETS	11
French carbon tax	10
Icelandic carbon tax	10
Guangdong Pilot ETS	10
QuÃ©bec CaT	10
Beijing Pilot ETS	9
EU ETS	9
South African carbon tax	5
Shanghai Pilot ETS	5
Mexican carbon tax (upper)	4
Tianjin Pilot ETS	4
Norwegian carbon tax (lower)	4
RGGI	3
Japanese carbon tax	2
Mexican carbon tax (lower)	1
New Zealand ETS	1
Exchange Rate EUR/USD 1,385 on 30.4.2014	

Table 4.14: Case - Established Carbon Prices on 1.5.2014

(Kossoy et al., 2014, Bank, 2015)

Date	Price [USD]
Sweden CO2 Tax	163
Finland CO2 Tax (upper)	78
Norway CO2 Tax (upper)	71
Finland CO2 Tax (lower)	39
Swiss CO2 Tax	38
British Columbia Revenue Neutral Carbon Tax	29
Ireland Natural Gas Carbon Tax	26
Mineral Oil Tax: Carbon Charge and Solid Fuel Carbon Tax	26
Denmark CO2 Tax	26
Australia Carbon Pricing Mechanism	24
Swiss Emissions Trading Scheme	19
California's Cap-and-Trade Program	14
South African Carbon Tax	13
United Kingdom Carbon Price Floor	7
Norway CO2 Tax (lower)	4
Japan Tax for Climate Change Mitigation	3
Regional Greenhouse Gas Initiative	2
New Zealand Emissions Trading Scheme	0,85
Exchange Rate EUR/USD 1,3214 on 30.4.2013	

Table 4.15: Case - Established Carbon Prices on 1.5.2013

(Kossoy et al., 2013, Bank, 2015)

City	UTM	Altitude [m]	WGS84	Geo URI
Bregenz	32T 556417 5261557	427	$47^{\circ}30'18''$ N, $9^{\circ}44'57''$ E	47.505, 9.749167
Insbruck	32T 680287 5237553	574	$47^{\circ}16'0''$ N, $11^{\circ}23'0''$ E	47.266667, 11.383333
Salzburg	33T 352732 5295944	424	$47^{\circ}48'0''$ N, $13^{\circ}2'0''$ E	47.8, 13.033333
Linz	33U 446851 5349893	266	$48^{\circ}18'0''$ N, $14^{\circ}17'0''$ E	48.3, 14.283333
Leoben	33T 507339 5247583	541	$47^{\circ}22'54''$ N, $15^{\circ}5'50''$ E	47.381667, 15.097222
Krems	33U 545629 5362796	203	$48^{\circ}25'0''$ N, $15^{\circ}37'0''$ E	48.416667, 15.616667
Vienna	33U 601551 5339433	151	$48^{\circ}12'0''$ N, $16^{\circ}22'0''$ E	48.2, 16.366667
Eisenstadt	33T 613463 5300742	182	$47^{\circ}51'0''$ N, $16^{\circ}31'0''$ E	47.85, 16.516667
Graz	33T 532903 5212664	353	$47^{\circ}4'0''$ N, $15^{\circ}26'0''$ E	47.066667, 15.433333
Klagenfurt	33T 446403 5162805	446	$46^{\circ}37'0''$ N, $14^{\circ}18'0''$ E	46.616667, 14.3
Villach	33T 411948 5163209	501	$46^{\circ}37'0''$ N, $13^{\circ}51'0''$ E	46.616667, 13.85
St. Poelten	33U 545822 5338714	267	$48^{\circ}12'0''$ N, $15^{\circ}37'0''$ E	48.2, 15.616667
Hamburg	32U 566322 5935622	6	$53^{\circ}33'55''$ N, $10^{\circ}0'5''$ E	53.565278, 10.001389
Berlin	33U 390301 5819734	34	$52^{\circ}31'0''$ N, $13^{\circ}23'0''$ E	52.516667, 13.383333
Duesseldorf	32U 345247 5678108	38	$51^{\circ}14'0''$ N, $6^{\circ}47'0''$ E	51.233333, 6.783333
Stuttgart	32U 513468 5403387	247	$48^{\circ}47'0''$ N, $9^{\circ}11'0''$ E	48.783333, 9.183333
Munich	32U 690961 5334306	519	$48^{\circ}8'0''$ N, $11^{\circ}34'0''$ E	48.133333, 11.566667
Frankfurt	32U 477360 5551650	112	$50^{\circ}7'0''$ N, $8^{\circ}41'0''$ E	50.116667, 8.683333
Zurich	32T 466023 5246010	408	$47^{\circ}22'0''$ N, $8^{\circ}33'0''$ E	47.366667, 8.55
Bern	32T 382051 5200774	542	$46^{\circ}57'0''$ N, $7^{\circ}27'0''$ E	46.95, 7.45
Florence	32T 681049 4850269	50	$43^{\circ}47'0''$ N, $11^{\circ}15'0''$ E	43.783333, 11.25
Milan	32T 514332 5034810	120	$45^{\circ}28'0''$ N, $9^{\circ}11'0''$ E	45.466667, 9.183333
Turin	32T 397659 4991178	240	$45^{\circ}4'0''$ N, $7^{\circ}42'0''$ E	45.066667, 7.7
Brno	33U 617771 5450947	190	$49^{\circ}12'0''$ N, $16^{\circ}37'0''$ E	49.2, 16.616667
Prague	33U 458267 5548059	177	$50^{\circ}5'0''$ N, $14^{\circ}25'0''$ E	50.083333, 14.416667
Ostrava	34U 305312 5523864	335	$49^{\circ}50'8''$ N, $18^{\circ}17'33''$ E	49.835556, 18.2925
Budapest	34T 353225 5261736	102	$47^{\circ}29'33''$ N, $19^{\circ}3'5''$ E	47.4925, 19.051389
Debrecen	34T 548110 5264258	121	$47^{\circ}31'47.89''$ N, $21^{\circ}38'20.98''$ E	47.52997, 21.63916

Table 4.16: Case - Geographical information

(Wikipedia, 2016d)

Altitude	Pressure [kPa]
0	101.3
75	100.4
150	99.5
250	98.5
300	97.7
450	95.9
600	94.1
750	92.4
900	90.8
1000	89.1
1200	87.5
1350	85.9
1500	84.3
1800	81.2
2100	78.1
2400	75.2

Table 4.17: Case - Air pressure by altitude

(ToolBox, 2016a)

Month	Germany	Austria	Italy	Czech Republic	Hungary
January	0,7	-2,1	5	-1,5	-0,6
February	1,5	-1,1	5,4	-0,3	1,1
March	5	2,8	8,1	3,8	6
April	9	6,7	10,9	8,7	11,3
May	13	11,6	15,5	13,3	16,2
June	16,1	14,6	19,4	16,3	19,5
July	18,2	16,5	22	18,3	21,2
August	18,1	16,5	22,2	18,3	21,1
September	14,1	12,2	18,4	13,7	16,2
October	9,5	7,9	14,4	8,7	10,9
November	4,8	2,6	9,8	3,4	5,5
December	1,2	-1,7	5,9	-0,9	0

Table 4.18: Case - Average temperatures from 1990 to 2012

(Bank, 2016)

Parameter	Value
Gravity	9.81 $[m/s^2]$
Average radius of the earth	6371 km
Specific gas constant for dry air	287.058 $[J/(kg \cdot K)]$
Average air density at sea level (15°)	1.225 $[kg/m^3]$
Rolling Resistance Coefficient:	
truck tire on asphalt	0,006 - 0,01
Energy Density:	
Gasoline	0,113200156
Diesel	0,100525841
Carbon Content:	
Gasoline	2,32154509
Diesel	2,663912231

Table 4.19: Case - Various Values

(ToolBox, 2015, Wikipedia, 2016a,b, Facts, 2005, Center, 2013)

Vehicle Type	c_d
Rolling Resistance	0,5
Small Car	0,53
Medium Car	0,55
Van	0,62
Light rigid	0,66
Light/Medium rigid	0,7
Medium rigid	0,72
Medium/heavy truck	0,77
Heavy truck	0,82
Heavy articulated	0,86

Table 4.20: Case - Drag Coefficient

(Akcelik and Besley, 2003)

4.2 Teaching Note

The teaching note complements the case presented in Section 4.1. It is separated into two parts. The first part serves as an introduction. It provides the motivation behind the case, the teaching objectives, as well as a summary of the case. The second part is mainly concerned with introducing a possible set of suggested solutions. This is done in a very brief manner, i.e. only the most important arguments are covered, in order to keep the teaching note within an appropriate length. However, Chapter 5 will expand upon the solution provided in the teaching note.

The case has two overarching themes, the Green Vehicle Routing Problem and sustainability. Firstly, sustainability in freight transport. May it be reducing green house gas emissions or other harmful aspects of freight transport, the issue of sustainability is of vital importance in light of the problems ahead (see Chapter 2). Therefore, measures have to be investigated, which help the reduction of natural resource consumption. One of these measures is the VRP variant, called GVRP, which focus on reducing the negative consequences of freight transport. One subgroup of the GVRP is the Pollution Routing Problem or PRP presented by Bektaş and Laporte (2011), which serves as the foundation for the case in question (Lin et al., 2014). That means, the case investigates freight transport related problems through an environmental lens by utilising the mentioned PRP.

By applying the PRP to solve the set of problems presented in the case, the student should be familiarised with sustainable decision support modelling for vehicle routing. In this particular instance this is done by investigating the differences between various objective functions, by assessing the behaviour of different objective functions, by investigating the extent to which the vehicle influences the overall outcome and by discussing the impact of variable speed and time windows. The case contains two parts. The first part introduces information about the company, i.e. its philosophy and its products. In the second part of the case various scenarios and problems are presented, while in the last part additional information for solving the case can be found. Both the narrative and the data provided in this case are constructed. Even though this is the case, most of the data is still rooted in reality, while the narrative is completely fictional.

In its most basic form the narrative revolves around a relatively small and newly founded business and its challenges regarding transportation. These challenges are presented through a rather narrow lens, which is done to direct the reader towards the learning objectives. The narrative can roughly be summarised as follows:

You are approached by a friend who recently founded together with three of his friends a company. He asks you if you could aid the company by solving some problems related to the transportation of their goods. As time passes you are repeatedly approached by your friend with new problems. May it be deciding between delivery services, assessing if it is feasible to in-source the delivery process or trying to improve customer satisfaction. These problems force you to improve upon previous iterations of the model, thus slowly creating a more realistic VRP. Moreover, sustainability and social responsibility are highly prioritised in this company. Therefore, you have to find out how these alterations and decisions impact the economic and environmental aspects of the company. Thus, it is upon you to analyse the problems at hand and discuss the results in an environmental and economical context.

In order to familiarise the student with the topic it is suggested that Bektaş and Laporte (2011) is read, as it provides the foundation for the model presented. However, if one intends to expand upon the tasks presented in the case or wants to adjust the model, additional sources such as Lin et al. (2014), Demir et al. (2014) and Cullinane and Edwards (2010) are recommended. Firstly, if the intent exists to adapt the capabilities of underlying VRP, the reader may be referred to Lin et al. (2014). This survey paper, contains an excellent description of various VRP variants as well as GVRP variants. Secondly, the desire may arise that the fuel consumption model has to be replaced. The first step towards satisfying this desire it to consult Demir et al. (2014), which contains various fuel consumption models to choose from. Lastly, Cullinane and Edwards (2010), contains an overview over a variety of different negative externalities related to freight traffic. Hence, it could be used as further inspiration for finding new applications for a VRP. Anyhow, the reader may also be informed that Chapter 3 contains a basic introduction into the model as well as a step by step guideline for implementing this version of the PRP. Furthermore, Chapter 5 expands upon the solutions provided in the following subsections

4.2.1 Problem One

In Problem One it has to be assessed whether company Maier's LKW Service (=: *Company* − *A*) or Delivery-Max (=: *Company* − *B*) provides the more desirable service. Even though, it is not possible to definitively state one preferable solution, it is suggested that Company-B is to be selected. This is due to the fact that if one simply compares the two companies based on their emission output, Company-B performs in most cases significantly better than Company-B. This can be observed in Table 4.21, which compares the GHG output for each company. The only exception in this case is TS1 in which due to vast differences in load Company-A has a lower GHG output. Furthermore, when looking at Table 4.22, which was constructed

Emissions [kg]	$P_{\mathscr{I}_{Of}(h)_A}$	$P_{\mathscr{I}_{Od}(h)_B}$	ΔP	$\Delta P[\%]$
TS1	1455,05	1509,97	54,92	3,77%
TS2	6703,55	3612,62	-3090,93	-46,11%
TS3	4145,38	3620,08	-525,31	-12,67%
BO1	1741,91	1413,27	-328,63	-18,87%
Average	3511,47	2538,98	-972,49	-27,69%

Table 4.21: Teaching Note - Problem 1 - Emissions comparison

by comparing the ideal solutions for each company, i.e. Oe, it becomes apparent that given the same trip Company-B will always perform around $4,25[\%]$ better than Company-A. The last argument regarding emission output can be constructed

Emissions [kg]	$P_{\mathscr{I}_{Oe}(h)_A}$	$P_{\mathscr{I}_{Oe}(h)_A}$	ΔP	$\Delta P[\%]$
TS1	1292,41	1239,01	-53,40	-4,13%
TS2	3774,86	3612,62	-162,25	-4,30%
TS3	3412,74	3268,56	-144,18	-4,22%
BO1	1426,61	1365,51	-61,10	-4,28%
Average	2476,66	2371,42	-105,23	-4,25%

Table 4.22: Teaching Note - Problem 1 - Truck comparison based on O_e

by looking at Table 4.23, which compares the company specific values against the results obtained by optimising for emissions. Company-A diverges from the optimal result in the best case by 13[%] and in the worst case by 78[%], while for Company-B the values 0[%] and 22[%] are observed. Hence, it can be concluded that Company-B operates closer to the ideal solution provided by Oe.

$\mathscr{I}_{Oe}$	TS1	TS2	TS3	BO1
$\Delta P_{\mathscr{I}_{Of}}[kg]$	162,63	2928,69	732,64	315,30
$\Delta P_{\mathscr{I}_{Od}}[kg]$	270,96	0,00	351,51	47,77
$\Delta P_{\mathscr{I}_{Of}}[\%]$	13%	78%	21%	22%
$\Delta P_{\mathscr{I}_{Od}}[\%]$	22%	0%	11%	3%

Table 4.23: Teaching Note - Problem 1 - Detailed emission comparison

However, if the costs are compared a different picture arises. That is, as seen in Table 4.24, which contrasts the costs between Company-A and Company-B, for all Scenarios but BO1, Company-A has a cheaper service. This is mostly due to the fact that the pricing policy of Company-A becomes more advantageous as the distance of the trip increases. Nevertheless, the price difference is on average only $2,7[\%]$, which if compared to the difference in GHG output of $-27,68[\%]$ seems fairly minuscule. However, if the prices are adjusted based on the GHG output and the price for emissions, which was at that time $24,50[Euro/t]$, Company-A would still perform around $1,6[\%]$ better than Company-B.

Cost [Euro]	$C_{\mathscr{I}_{Of(h)_A}}$	$C_{\mathscr{I}_{Od(h)_B}}$	ΔC	$\Delta C[\%]$
TS1	1112,23	1352,56	240,32	21,6%
TS2	3000,23	3575,69	575,46	19,2%
TS3	3310,46	3382,42	71,97	2,2%
BO1	2002,02	1364,23	-637,79	-31,9%
Average	2356,23	2418,72	62,49	2,7%

Table 4.24: Teaching Note - Problem 1 - Cost comparison

In conclusion, with both the environmental and economical perspective analysed one can now state a preferable transport company. In a direct comparison Company-A provides given the four scenarios a better economic result. Even if the costs are adjusted by the cost of GHG emissions, Company-A still performs better. However, by solely comparing environmental aspects, Company-B would be the most preferable. That is, given the four scenarios Company-B produces less GHG emissions. Moreover, the results achieved by selecting Company-B are closer to the ones provided by the "ideal" trip. Lastly, given the same distance Company-B would always achieve a lower GHG output, as it employs a more efficient vehicle. Therefore, it can be concluded that Company-A has lower costs, while Company-B inflicts less environmental damage. Furthermore, it can be concluded that for the most part shorter trips should be executed by Company-B. Hence, depending on the idealogical stance regarding GHG emissions, either company can be selected. However, in this particular analysis it is proposed that Company-B provides, given the arguments stated above, the better service.

4.2.2 Problem Two

In order to solve Problem Two it is required to create an objective function, which mimics the behaviour of the fuel consumption model or at least provides better results than the objective currently used by the selected transportation company. In this particular solution the objective *Odf* with the objective function $d \cdot (f + w)$ is used. Furthermore, this objective has to be compared against the objective of the selected company. Firstly, given the set of scenarios *Odf* had the same output as *Oe*. Table 4.25 can be created by comparing the cost and the GHG output of the new objective with the objective of the selected company (In this solution both options are covered).

Company A	$\Delta P[kg]$	$\Delta P[\%]$	$\Delta C[Euro]$	$\Delta C[\%]$
TS1	-162,63	-11,2%	-100,41	-11,2%
TS2	-2928,69	-43,7%	-1792,08	-44,0%
TS3	-732,64	-17,7%	-449,92	-17,8%
BO66	-194,00	-20,1%	-119,16	-20,2%
Average	-1004,49	-30,3%	-615,39	-30,4%
Company B	$\Delta P[kg]$	$\Delta P[\%]$	$\Delta C[Euro]$	$\Delta C[\%]$
TS1	-270,96	-17,9%	-138,75	-15,3%
TS2	0,00	0,0%	0,00	0,0%
TS3	-351,51	-9,7%	-180,00	-8,2%
BO66	-5,37	-0,7%	-1,99	-0,4%
Average	-156,96	-6,6%	-80,18	-5,6%

Table 4.25: Teaching Note - Problem 2 - General comparison

Overall, the results suggest that switching to the new objective is for both companies beneficial. It provides not only an overall cost reduction, but also decreases the energy consumption. That is, for Company-A the GHG output would be reduced by $30,3[\%]$, while the cost would decrease by $30,4[\%]$. By contrast, for Company-B these results are less significant with a reduction in GHG output by $6,6[\%]$ and a reduction in costs by $5,6[\%]$. A final and even more compelling argument can be made by looking at the profit margins of the respective company, which are summarised in Table 4.26. In general the values generated do only further solidify the arguments

Company A $[Euro]$	$Income(Of)$	$Income(Odf)$	$Margin(Of)$	$Margin(Odf)$	$\Delta Margin$	
TS1	1112,23	1439,36	217,15	644,69	427,54	196,9%
TS2	3000,23	3897,49	-1076,09	1613,25	2689,34	-249,9%
TS3	3310,46	3416,11	780,70	1336,28	555,58	71,2%
BO66	1336,55	1710,41	745,56	1238,59	493,02	66,1%
Average	2189,87	2615,84	166,83	1208,20	1041,37	624,2%
Company B $[Euro]$	$Income(Of)$	$Income(Odf)$	$Margin(Of)$	$Margin(Odf)$	$\Delta Margin$	
TS1	1352,56	1352,56	446,48	585,23	138,75	31,1%
TS2	3575,69	3575,69	1374,53	1374,53	0,00	0,0%
TS3	3382,42	3382,42	1196,42	1376,42	180,00	15,0%
BO66	771,56	779,33	314,33	324,08	9,76	3,1%
Average	2270,56	2272,50	832,94	915,07	82,13	9,9%

Table 4.26: Teaching Note - Problem 2 - Profit margins

presented above. As on average the profit margin would increase by $1041,37[Euro]$ or $624,2[\%]$ for Company-A and by $82,13[Euro]$ or $9,9[\%]$ for Company-B. To conclude, it is for both companies economically and ecologically advantageous to switch to the new objective. However, it is especially important for Company-A, since its overall result would improve drastically.

4.2.3 Problem Three

In order to solve Problem Three two issues have to be resolved. Firstly, it has to be tested, which vehicle is the most preferable option and secondly, whether it would be a good idea to in-source the delivery process.
Beginning with the selection of the vehicle, Mercedes 4160 ($=: \mathscr{V}_C$) and IVECO ($=: \mathscr{V}_D$). A direct comparison of the GHG output is hindered by the variations in capacity. Since, by ignoring the capacity of the vehicle, a vehicle with less curb weight would be favoured. This is due to the correlation between capacity and the curb weight (Bektaş and Laporte, 2011). However, in this particular case the vehicle with the least capacity is so inefficient in the translating $[J]$ into $[kg]$ this advantage is more than compensated. That is,

$$conv := \frac{\omega_F}{\eta} \cdot \omega_{GHG}.$$

One can conclude that based on the same energy requirement i.e. $P_{\mathscr{V}_A} = P_{\mathscr{V}_B} = P_{\mathscr{V}_C} = P_{\mathscr{V}_D}$, Truck C will be always the most preferable choice. This is due to the fact that

$$\begin{aligned} conv_A &= 0,8369 \\ conv_B &= 0,7876 \\ conv_C &= 0,7439 \\ conv_D &= 1,3140, \end{aligned}$$

which indicates that the energy required by $\mathscr{V}_D$ has to be $56,54[\%]$ of the energy required by $\mathscr{V}_C$ in order for both to have the same GHG output. This massive difference can be observed in Figure 4.6 , which shows the performance of every vehicle in $[kWh]$ and $[l]$, i.e. without and with the conversion factor across the three major variable parameter. Here it becomes apparent that even though $\mathscr{V}_D$ requires less energy, the picture drastically shifts, if the GHG output is compared. Hence, it can be concluded that $\mathscr{V}_C$ would be the better option. This is further supported by the fact that as seen in Table 4.27 it also has the highest carrying capacity amongst all vehicles.

Capacity $[Units]$	Truck A	Truck B	Truck C	Truck D
K_v	47458	50688	47458	38732
K_w	34216	33365	37008	9228
K	34216	33365	37008	9228

Table 4.27: Teaching Note - Problem 3 - Vehicle capacity

Secondly, the decision whether the delivery process should be in-sourced is fairly straight forward. By observing Table 4.28 the following things can be concluded. Given the same trip the in-sourcing would provide always the better option, since $\mathscr{V}_C$ would be used. This can be observed in $TS1$, $TS2$ and $TS3$, as in these scenarios *Odf* performs equally good to Oe, thus resulting in a decreased GHG output of $14[\%]$ if compared against Company-A and of $10[\%]$ if compared against Company-B. As for $BO542$, *Odf* performed slightly worse than Oe. Hence, due to the truck and the objective it can be concluded that in-sourcing would provide an overall better result. Lastly, by comparing the costs that occur in the in-source option against the costs

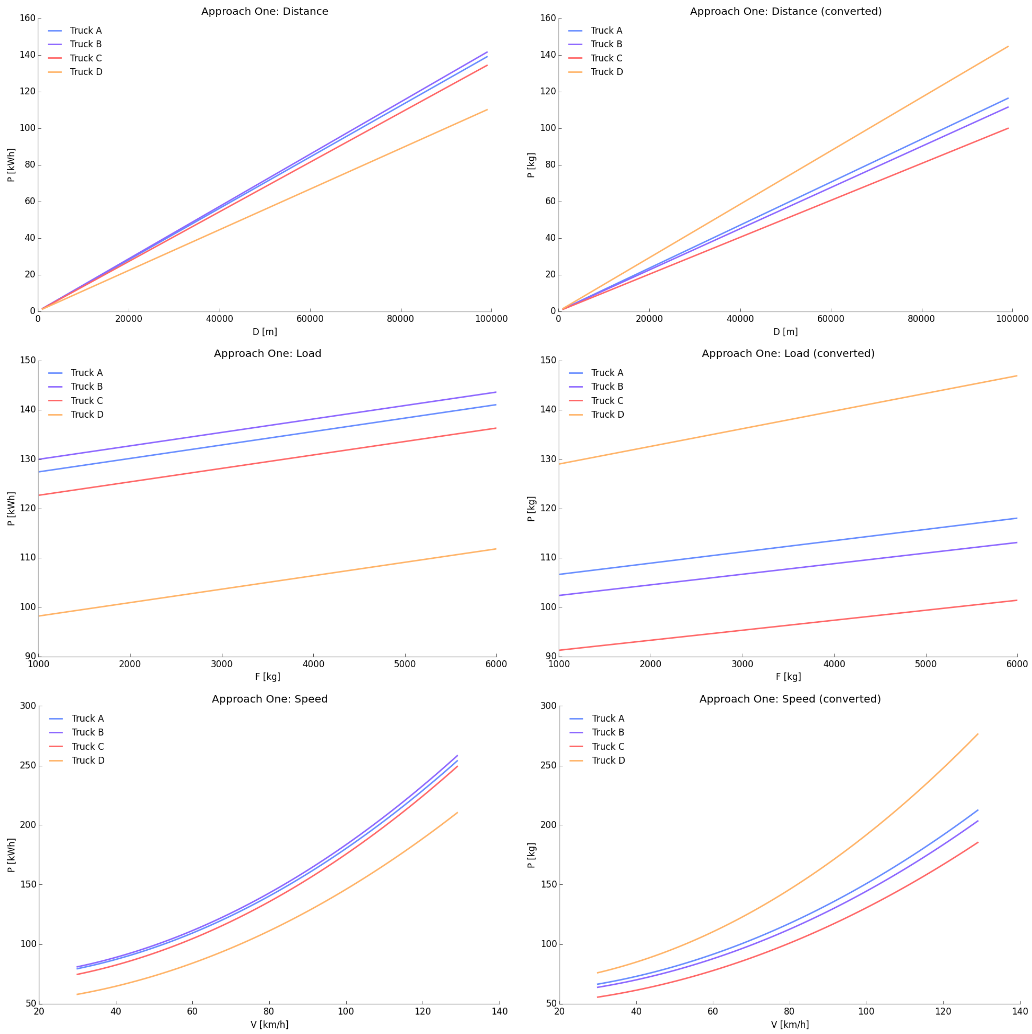


Figure 4.6: Teaching Note - Problem 3 - First approach

Emissions $[kg]$	$\Delta P_{\mathscr{V}_A}$	$\Delta P_{\mathscr{V}_B}$	$\Delta P_{\mathscr{V}_A}[\%]$	$\Delta P_{\mathscr{V}_B}[\%]$
TS1	-183,39	-129,99	-14,2%	-10,5%
TS2	-524,61	-362,36	-13,9%	-10,0%
TS3	-478,69	-334,51	-14,0%	-10,2%
BO542	-226,41	-160,45	-14,5%	-10,7%
Average	-353,27	-246,83	-14,1%	-10,3%

Table 4.28: Teaching Note - Problem 3 - Emission comparison

of the selected transport company, Table 4.29 can be created. The picture depicted shows high potential savings. That is, by in-sourcing the company could reduce its costs on average by $72,5[\%]$ if it switches away from Company-A and by $53,7[\%]$ if it switches away from Company-B.

Cost	C_A	C_B	C	ΔC_A	ΔC_B	$\Delta C_A[\%]$	$\Delta C_B[\%]$
TS1	1439,36	1352,56	713,95	-725,41	-638,61	-50,4%	-47,2%
TS2	3897,49	3575,69	2049,93	-1847,56	-1525,76	-47,4%	-42,7%
TS3	3416,11	3382,42	1867,41	-1548,70	-1515,01	-45,3%	-44,8%
BO542	3084,35	1548,59	848,75	-2235,60	-699,84	-72,5%	-45,2%
Average	2959,33	2464,81	1370,01	-1589,32	-1094,81	-53,7%	-44,4%

Table 4.29: Teaching Note - Problem 3 - Cost comparison

To conclude, the difference in GHG output, caused by having the better truck and by being able to employ a more suitable objective makes in-sourcing solely based on environmental arguments the more preferable option. Moreover, the massive variations observed by comparing the costs, which result mainly out of the differences in pricing, also provide a compelling economic argument for in-souring. Hence, in-sourcing would always provide a better option than the currently existing one.

4.2.4 Problem Four

Problem Four requires the investigation of two potential alterations to the model. Firstly, it has to be discussed how the behaviour of the model changes, when it is solved for cost rather than for GHG output. Secondly, it has to be addressed whether it makes sense if variable speed is introduced into the model.

Beginning with the first argument. For all test scenarios the results provided by Oc are the same as the results provided by Oe. This behaviour already indicates that some level of similarity between these two objectives exists. By contrast, in $BO600$, as seen in Table 4.30 which compares Oc against Oe, the results diverge. That is, with Oc provides a minuscule reduction in cost by $0,03[\%]$ or $0,32[Euro]$, while at the same time causing an increase in GHG output by $1,14[\%]$. This is due to the fact that in order to decrease Cd, T_{0j} has to be reduced, thus resulting in a decrease in distance by $4,72[\%]$, which unfortunately led to an increase in load by $20,83[\%]$ causing the rise in GHG output. Hence, it can be concluded that even though both objectives perform fairly similar Oc seems to favour the parameter distance more strongly than Oe. Furthermore, in the case it is implied that the high emission price

BO600	$X_{\mathscr{I}_{Oc}(BO600)}$	$X_{\mathscr{I}_{Oe}(BO600)}$	ΔX	$\Delta X[\%]$
D	1500,82	1571,65	70,84	4,72%
F	75,40	59,69	-15,70	-20,83%
P	1613,71	1595,23	-18,48	-1,14%
C	1028,74	1029,06	0,32	0,03%
T0j	21,52	22,41	0,89	4,11%
BO600	$P_{\mathscr{I}_{Oc}(BO600)}$	$P_{\mathscr{I}_{Oe}(BO600)}$	ΔP	$\Delta P[\%]$
Pw	546,21	571,99	25,78	4,72%
Pf	274,91	193,24	-81,67	-29,71%
Pv	792,59	830,00	37,41	4,72%
BO600	$C_{\mathscr{I}_{Oc}(BO600)}$	$C_{\mathscr{I}_{Oe}(BO600)}$	ΔC	$\Delta C[\%]$
Cd	206,36	216,10	9,74	4,72%
Cf	792,95	783,87	-9,08	-1,14%
Ce	29,43	29,09	-0,34	-1,14%

Table 4.30: Teaching Note - Problem 4 - Comparison of BO600

may be sufficient enough to justify the switch to the objective cost. Unfortunately, if the parameter cost is separated into its components, it becomes apparent that this assumption is wrong. Table 4.31, which relates the individual cost components to the total cost, indicates that the emission price has hardly any significance, i.e. it amounts on average to only $2,83[\%]$ of the total cost.

Hence, it can be concluded that the fact that Oc behaves relatively similar to Oe, is not due to the high $\acute{p}_e$, but rather because the fuel costs make up most of the costs. Therefore, if environmental thinking should be at the heart of the decisions made Oc is not recommended, if one is not willing to adjust carbon prices appropriately. Moving on to the decision of whether variable speed should be included into the model. Table 4.32 shows the travelling speed of the vehicle per arc. This table

Costs [%]	$\frac{Cd}{C}$	$\frac{Cf}{C}$	$\frac{Ce}{C}$
TS1	21,52%	75,67%	2,81%
TS2	19,83%	77,30%	2,87%
TS3	20,58%	76,57%	2,84%
BO600 (Oc)	20,06%	77,08%	2,86%
BO600 (Oe)	21,00%	76,17%	2,83%

Table 4.31: Teaching Note - Problem 4 - Detailed cost comparison

indicates that for Oe the speed will always be set to the lowest possible speed. By contrast, Oc seems to be set around a speed of $14,17[m/s]$, thus it could be assumed that the variable speed for Oc can be calculated without including variable speed into the optimisation process.

$\mathscr{I}_{Oc}[m/s]$	TS1	TS2	TS3	BO600	$\mathscr{I}_{Oe}[m/s]$	TS1	TS2	TS3	BO600
1. Stop	14,04	14,20	14,17	14,10	1. Stop	8,33	8,33	8,33	8,33
2. Stop	14,58	14,08	14,22	14,74	2. Stop	8,33	8,33	8,33	8,33
3. Stop	14,15	14,18	14,06	14,35	3. Stop	8,33	8,33	8,33	8,33
4. Stop	14,07	14,22	14,03	14,18	4. Stop	8,33	8,33	8,33	8,33
5. Stop	14,16	14,08	13,97	14,35	5. Stop	8,33	8,33	8,33	8,33
6. Stop	14,10	14,07	14,21	14,05	6. Stop	8,33	8,33	8,33	8,33
7. Stop	14,16	14,20	14,11	14,07	7. Stop	8,33	8,33	8,33	8,33

Table 4.32: Teaching Note - Problem 4 - Speed

The next step is to discuss the impact of speed onto the model. Therefore, Table 4.33 compares both objectives against their counterparts with variable speed. By doing so it becomes apparent that in both cases the variable speed produces better results. As for GHG output, Oe with variable speed leads on average to a reduction of $43,4[\%]$, while for Oc it only experienced a reduction by $29,7[\%]$. Furthermore, if the same is done for the parameter cost, Oe experiences only a reduction by $0,32[\%]$, while for Oc the reduction amounts to $12,06[\%]$. This difference can be explained due to the variations in speed. That is, the lower speed of Oe leads to the lowest possible GHG output, however, at the same time it causes an explosion in Cd. A trade-off which can only be considered by Oc.

Overall, it has to be concluded that using Oc may lead to similar results as Oe. This similarity is, however, not because of the high $\acute{p}_e$, but rather because of $\acute{p}_f$. Given the desire to have the environmental aspect a significant part in the decision

Emissions [kg]	$\Delta P_{\mathscr{I}_{Oc}}$	$\Delta P_{\mathscr{I}_{Oe}}$	$\Delta P_{\mathscr{I}_{Oc}}[\%]$	$\Delta P_{\mathscr{I}_{Oe}}[\%]$
TS1	-354,41	-511,54	-32,0%	-46,1%
TS2	-935,95	-1352,33	-28,8%	-41,6%
TS3	-889,36	-1279,23	-30,3%	-43,6%
BO600	-467,14	-713,28	-28,9%	-44,7%
Average	-661,72	-964,09	-29,7%	-43,4%
Cost [Euro]	$\Delta C_{\mathscr{I}_{Oc}}$	$\Delta C_{\mathscr{I}_{Oe}}$	$\Delta C_{\mathscr{I}_{Oc}}[\%]$	$\Delta C_{\mathscr{I}_{Oe}}[\%]$
TS1	-91,93	-2,39	-12,77%	-0,33%
TS2	-243,05	-6,32	-11,76%	-0,31%
TS3	-229,90	-5,97	-12,21%	-0,32%
BO600	-122,29	-3,33	-11,89%	-0,32%
Average	-171,79	-4,50	-12,06%	-0,32%

Table 4.33: Teaching Note - Problem 4 - General Comparison

process, this insight may indicate the need to elevate the price of emissions. However, the real advantage of Oc becomes apparent when variable speed is considered. Because of the nature of the cost function, Oc is able to select an optimal speed at which the individual parts of the cost function are balanced, thus leading overall to a more balanced result. Therefore, it is suggested that in any case the travelling speed should be set equal to the ideal speed of Oc. Moreover, the decision whether Oe or Oc should be selected, depends yet again on the ideological stance.

4.2.5 Problem Five

Problem Five requires the assessment of the restrictive capabilities of time windows as well as their impact on the output. Firstly, by simply contrasting the time required by the vehicle to reach a point against the time windows, Table 4.34 can be created. Furthermore, this is done for all objectives with and without variable speed across all scenarios. Moreover, every negative value depicted, is equivalent with a breach of the time window constraint, which is emphasised by bold font. By observing its content it becomes apparent that since no time windows are considered most instances experience some kind of time window violation. However, the most interesting behaviour depicted by this table is that even though $TS2$ and $TS3$ both have only one relatively narrow time window, $TS2$ allows the completion of the trips provided by Od, Oe and Oc without any violation, while in $TS3$ every trip has to be restructured. Thus indicating that even though time windows might reduce the set of possible solutions, good scheduling may be used to prevent undesired results. Furthermore, by focusing on *Oc* and *Oe* Table 4.35 could be created. It compares

[*h*]	$\mathscr{I}_{Ox}$				$\mathscr{I}_{Ox,V}$	
TS1	Od	Of	Oe	Oc	Oe	Oc
1. Stop	(**-4,37** | 11,37)	(0,8 | 8,2)	(**-0,51** | 5,51)	(**-0,51** | 5,51)	(1,97 | 3,03)	(0,22 | 4,78)
2. Stop	(**-2,7** | 10,7)	(0,43 | 4,57)	(1,84 | 7,16)	(1,84 | 7,16)	(5,16 | 3,84)	(2,81 | 6,19)
3. Stop	(4,77 | 5,23)	(0,62 | 7,38)	(0,89 | 5,11)	(0,89 | 5,11)	(7,44 | **-1,44**)	(2,83 | 3,17)
4. Stop	(7,72 | **-1,72**)	(8,05 | **-1,05**)	(1,48 | 8,52)	(1,48 | 8,52)	(10,29 | **-0,29**)	(4,09 | 5,91)
5. Stop	(12,89 | **-3,89**)	(11,47 | **-1,47**)	(1,19 | 6,81)	(1,19 | 6,81)	(15,16 | **-7,16**)	(5,33 | 2,67)
6. Stop	(12,52 | **-7,52**)	(14,42 | **-8,42**)	(8,62 | **-1,62**)	(8,62 | **-1,62**)	(29,38 | **-22,38**)	(14,78 | **-7,78**)
7. Stop	(16,85 | 9982,15)	(20,68 | 9978,32)	(16,85 | 9982,15)	(16,85 | 9982,15)	(40,33 | 9958,67)	(23,81 | 9975,19)
TS2	Od	Of	Oe	Oc	Oe	Oc
1. Stop	(6,12 | 63,88)	(**-10,27** | 15,77)	(6,12 | 63,88)	(6,12 | 63,88)	(16,32 | 53,68)	(9,19 | 60,81)
2. Stop	(10,56 | 59,44)	(24,45 | 45,55)	(10,56 | 59,44)	(10,56 | 59,44)	(27,34 | 42,66)	(15,51 | 54,49)
3. Stop	(17,76 | 52,24)	(34,07 | 35,93)	(17,76 | 52,24)	(17,76 | 52,24)	(46,34 | 23,66)	(26,2 | 43,8)
4. Stop	(2,51 | 2,99)	(44,56 | 25,44)	(2,51 | 2,99)	(2,51 | 2,99)	(39,3 | **-33,8**)	(13,29 | **-7,79**)
5. Stop	(29,73 | 40,27)	(56,21 | 13,79)	(29,73 | 40,27)	(29,73 | 40,27)	(76,76 | **-6,76**)	(43,48 | 26,52)
6. Stop	(35,39 | 34,61)	(65,72 | 4,28)	(35,39 | 34,61)	(35,39 | 34,61)	(91,7 | **-21,7**)	(51,95 | 18,05)
7. Stop	(39,11 | 9959,89)	(70,11 | 9928,89)	(39,11 | 9959,89)	(39,11 | 9959,89)	(101,18 | 9897,82)	(57,22 | 9941,78)
TS3	Od	Of	Oe	Oc	Oe	Oc
1. Stop	(2,91 | 46,09)	(5,4 | 43,6)	(2,91 | 46,09)	(2,91 | 46,09)	(9,44 | 39,56)	(4,82 | 44,18)
2. Stop	(7,65 | 41,35)	(10,3 | 38,7)	(7,65 | 41,35)	(7,65 | 41,35)	(21,42 | 27,58)	(11,71 | 37,29)
3. Stop	(12,81 | 36,19)	(13,06 | 35,94)	(12,81 | 36,19)	(12,81 | 36,19)	(34,26 | 14,74)	(19,1 | 29,9)
4. Stop	(13,49 | **-9,49**)	(21,75 | 27,25)	(13,49 | **-9,49**)	(13,49 | **-9,49**)	(45,56 | **-41,56**)	(22,91 | **-18,91**)
5. Stop	(25,4 | 23,6)	(21,66 | **-17,66**)	(25,4 | 23,6)	(25,4 | 23,6)	(66,92 | **-17,92**)	(37,65 | 11,35)
6. Stop	(30,08 | 18,92)	(38,01 | 10,99)	(30,08 | 18,92)	(30,08 | 18,92)	(79 | **-30**)	(44,5 | 4,5)
7. Stop	(37,06 | 9961,94)	(44,99 | 9954,01)	(37,06 | 9961,94)	(37,06 | 9961,94)	(95,79 | 9903,21)	(54,38 | 9944,62)

Table 4.34: Teaching Note -Problem 5 - Time Window infringement

the results obtained by imposing time windows with and without speed, against the results provided by solving the problem without a time window restriction. That is, the upper part uses the basic implementation as basis, while for the lower part is constructed based on the implementation that considers variable speed. By observing these values it can be concluded that an introduction of time windows without also including variable speed will always result in a less desired outcome. This, however, is not the case for time windows with variable speed. As in this case it even improves upon the result provided by the implementation with fixed speed and no time windows, in terms of GHG output. By only comparing the objective value of the implementation with time windows and variable speed against the one without time windows and without variable speed, it is not possible to definitively state which implementation will provide the better result. However, given the same comparison against the implementation without time windows and with variable speed, the results with time window constraints will always provide a less desirable solution.

To conclude, the introduction of time windows, will if not scheduled appropriately cause a deviation from the original trip, i.e. the trip which disregards time

$\mathscr{I}_{Ox}$	$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,V,TW}$			
Emissions [kg]	ΔP_{Oe}	ΔP_{Oc}	ΔP_{Oe}[%]	ΔP_{Oc}[%]	ΔP_{Oe}	ΔP_{Oc}	ΔP_{Oe}[%]	ΔP_{Oc}[%]
TS1	214,99	214,99	19,39%	19,39%	-145,66	-66,89	-13,13%	-6,03%
TS2	0,00	0,00	0,00%	0,00%	-865,97	-797,65	-26,64%	-24,54%
TS3	572,12	572,12	19,50%	19,50%	-59,70	-34,50	-2,03%	-1,18%
Average	262,37	262,37	10,79%	10,79%	-357,11	-299,68	-14,69%	-12,33%
Cost [Euro]	ΔC_{Oe}	ΔC_{Oc}	ΔC_{Oe}[%]	ΔC_{Oc}[%]	ΔC_{Oe}	ΔC_{Oc}	ΔC_{Oe}[%]	ΔC_{Oc}[%]
TS1	130,54	130,54	19,89%	19,89%	101,26	63,21	15,43%	9,63%
TS2	0,00	0,00	0,00%	0,00%	-148,03	-153,65	-7,88%	-8,18%
TS3	310,39	310,39	18,12%	18,12%	253,69	208,34	14,81%	12,16%
Average	146,98	146,98	10,38%	10,38%	68,97	39,30	4,87%	2,78%
$\mathscr{I}_{Ox,V}$	$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,V,TW}$			
Emissions [kg]	ΔP_{Oe}	ΔP_{Oc}	ΔP_{Oe}[%]	ΔP_{Oc}[%]	ΔP_{Oe}	ΔP_{Oc}	ΔP_{Oe}[%]	ΔP_{Oc}[%]
TS1	726,53	543,69	121,60%	69,67%	365,88	261,81	61,24%	33,55%
TS2	1352,33	861,07	71,25%	36,04%	486,36	63,42	25,63%	2,65%
TS3	1851,35	1391,43	111,88%	65,80%	1219,54	784,81	73,70%	37,11%
Average	1310,07	932,06	94,70%	52,92%	690,59	370,01	49,92%	21,01%
Cost [Euro]	ΔC_{Oe}	ΔC_{Oc}	ΔC_{Oe}[%]	ΔC_{Oc}[%]	ΔC_{Oe}	ΔC_{Oc}	ΔC_{Oe}[%]	ΔC_{Oc}[%]
TS1	93,02	198,63	13,41%	33,76%	63,75	131,30	9,19%	22,32%
TS2	-99,17	179,88	-5,02%	10,59%	-247,20	26,23	-12,50%	1,54%
TS3	216,58	480,67	11,98%	31,15%	159,87	378,62	8,85%	24,54%
Average	70,14	286,39	4,70%	22,44%	-7,86	178,71	-0,53%	14,00%

Table 4.35: Teaching Note -Problem 5 - Comparison against Implementations without Time Windows

windows. However, by introducing variable speed into the model it is indeed possible to achieve results, which deviate less from the optimal solution. Such a behaviour is caused by the fact that given variable speed it becomes possible to adjust the speed in a manner, which increases the set of possible solutions. Furthermore it can be said that by solely comparing the values of the respective objective function, the implementation without time windows and with variable speed will always provide the most beneficial solution, while the implementation with time windows and without variable speed will always provide the least desired outcome. This insight is sufficient enough to conclude that in order to receive the best result with time windows, it is important to include variable speed into the decision process. That is, simply increasing the speed is not sufficient as it may lead to an infringement of the upper bound of the time window. Moreover, in contrast to an environment which is not limited by time windows, it is not possible to calculated the desired speed in beforehand, as the speed on each arc has to be adjusted individually in order to ensure that the time window constraints are uphold.

Chapter 5

Analysis

This chapter is concerned with providing a detailed analysis of the problem set presented in Chapter 4. While the previously introduced set of solutions is only of descriptive nature, this chapter tries probe deeper into the behaviour of the PRP. However, since this is done based on the values presented in Chapter 4, the analytical parts of this chapter are interwoven with the solution of the case. Thereby, it is implied that any attempts of generalising the insights obtained may be taken with a grain of salt. This is, due to the fact that the sample size given by the case, against which the introduced ideas are tested, is relatively small. Anyhow, in contrast to Section 4.2 this chapter follows a more formal and structured approach. That is, the first part of each section includes the objective behind each task, as well as the context provided by the narrative of the case. The second part of each section introduces the input used for any further computation, as well as the Excel implementation used for creating the output. From there, the results obtained are analysed in greater detail. However, if necessary underlying concepts may be introduced at any part of the chapter. This liberty is taken since, the focus of this chapter is to investigate the behaviour of the PRP rather than to provide a brief solution for the case at hand. That is, generally speaking this chapter tries to answer the following research questions

- By comparing the objectives distance and load, is it possible to identify which objective produces results that are closer to the results obtained by optimising for GHG emissions.
- Is it possible to mimic the behaviour of the fuel consumption model, given a minimal amount of input?
- How can the most suitable vehicle be chosen and how does the choice of vehicle impact the output of the model?

- What are the differences between the objective cost and the objective emission?
- In which cases is it necessary to incorporate variable speed into the model and how does it effect the output of the model?
- How does the introduction of time window constraints impact the output of the model?

In order to answer these questions, each section tries to address one or two aspects concerning the behaviour of the PRP. Firstly, even though Section 5.1 tends to be a more descriptive section, it tries to investigate and compare the differences in behaviour between the objective distance, the objective load and the objective emissions. To be more precise, it tries to identify which objective performs closer to Oe. Secondly, Section 5.2. This section focuses on the behaviour of the fuel consumption model. However, at the same time tries to answer the question whether it is possible to approximate the behaviour of the fuel consumption model, given a decreased amount of information? In order to accomplish this goal a method for comparing two similar objective functions is also proposed. Thirdly, Section 5.3, is concerned with assessing the merit of a vehicle without executing the model. Furthermore, it investigates the extent to which the vehicle influences the outcome of the model. The forth section, i.e. Section 5.4, compares the fuel consumption model and the cost function, by investigating their behavioural differences. Moreover, it tries to determine how the implementation of variable speed influences the model and more importantly, when it is sensible to actually implement variable speed. Lastly, Section 5.5. This section, tends to be, similar to Section 5.1, relatively descriptive. Nevertheless, it tries to investigate the restrictive capabilities of time windows. Additionally, it expands on the question raised in Section 5.4, by discussing the utility of variable speed in an environment with time window constraints.

5.1 Problem One

Even though the sample size is minuscule Problem One is mainly concerned with depicting potential differences in behaviour between the objective Load (Of) and the objective Distance (Od) and Emission (Oe). By contrasting the results of Od and Of against the results obtained by optimising for the objective Emission (Oe), they can be compared against an "ideal" solution. Narrative wise one has to contrast and

decide between two companies based on economical and ecological aspects. These transport companies are Maier's LKW Service (=: $Company - A$) and Delivery-Max (=: $Company - B$). Both of these companies employ different vehicle as well as have different pricing and optimisation policies. Furthermore, four scenarios, i.e. Test-Scenario 1 (=: $TS1$),Test-Scenario 2 (=: $TS2$),Test-Scenario 3 (=: $TS3$) and Order 1(=: $BO1$) are given to contrast the two companies against. While $BO1$ and $TS1$ are created without any concerns for the outcome, $TS1$ and $TS2$ are specifically designed to better suit one specific objective. The intend behind this is to create a bigger picture within which one can better understand the differences between the objective load and distance and the conditions at which one is preferable.
The fist step for solving the problem is to identify the required data. As indicated the parameter group *World* remains equal throughout this problem. It is defined as $\mathscr{W} := \{c_r', \rho_{air}, g, r_s\}$, thus $\mathscr{W}_{P1} := \mathscr{W}_1$ with

$$\begin{aligned} c_r' &:= 0,01 \\ \rho_{air} &:= 1,225[kg/m^3] \\ g &:= 9,81[m/s^2] \\ a &:= 0[m/s^2] \\ r_s &:= 6371[km]. \end{aligned}$$

However, it has to be noted that it would be possible to calculate the parameter $\rho_{air} = \dfrac{p}{R_{air} \cdot T}$ based on the respective region. With p being absolute pressure in $[Pa]$, R_{air} being the gas constant for air measured in $[J/(kg \cdot K)]$ and T being the temperature in $[K]$ (Wikipedia, 2016a). To ensure that the focus resides on more relevant matters, this detail is neglected. Nevertheless, if this route is selected, ρ_{air} would become either a point of arc specific parameter. Anyhow, the next parameter group, i.e. *Product* does only include two entries. This is due to the fact that apart from the weight and size of the product no additional information is required. However, it is conceivable that given HVRP and a combination with different products additional information might be necessary. Thus, indicating that the information requirement for a product might vary between VRP variants. In this case however, this group is defined as $\mathscr{P} := \{\rho_{Product}, V_{Product}\}$ with $\mathscr{P}_{P1} := \mathscr{P}_1$, which is filled with

$$\begin{aligned} \rho_{product} &:= 0,6231[kg/Unit] \\ V_{product} &:= 0,0019596[m^3]. \end{aligned}$$

In this particular case it is not required to calculate the cost that would occur internally, as it is not necessary for the analysis. However, for the sake of completeness $\mathscr{C} := \{\acute{p_d}, \acute{p_f}, \acute{p_e}\}$ with $\mathscr{C}_{P1} := \mathscr{C}_{9.7.2013}$

$$\begin{aligned} \acute{p_d} &:= 8,2[Euro/h] \\ \acute{p_f} &:= 1,341[Euro/l] \\ \acute{p_e} &:= 4,19[Euro/t] \end{aligned}$$

Whereby, the prices are chosen based on the time frame at which the problem occurred. Finally, the only parameter group, which remains equal for both transport-companies is the group *Trip*. The parameter group *Trip* consists out of a family of points, i.e. $\mathscr{N} := (\acute{n}_i)_{i \in I}$, with each point being a set of parameter on its own, thus $\forall \acute{n} \in \mathscr{N} : \acute{n} := \{x, y, q, \widetilde{ab}, \acute{t}\}$. In this problem the parameter $\widetilde{ab}, \acute{t}$ are not required. Therefore, four scenarios are defined as

$$\begin{aligned} \mathscr{N}_{BO1} &:= \{Graz, Bregenz, Klagenfurt, Insbruck, Leoben, Linz, Vienna\} \\ \mathscr{N}_{TS1} &:= \{Graz, Vienna, Linz, Salzburg, Bregenz, Villach, Eisenstadt\} \\ \mathscr{N}_{TS2} &:= \{Graz, Hamburg, Frankfurt, Florence, Turin, Budapest, Prague\} \\ \mathscr{N}_{TS3} &:= \{Graz, Debrecen, Ostrava, Stuttgart, Berlin, Munich, Milan\}. \end{aligned}$$

All families presented are of the same cardinality therefore only one index set $I := \{0, 1, 2, 3, 4, 5, 6\}$ is required. To ensure easy referencing the collection of scenarios is defined as the set $H := \{\mathscr{N}_{BO1}, \mathscr{N}_{TS1}, \mathscr{N}_{TS2}, \mathscr{N}_{TS3}\}$. As for the elements in $\acute{n}$, the information for the geographical position of each individual point can be found in Table 4.16, thus $\forall h \in H : X_h \wedge Y_h$ are defined. The next relevant parameter is the demand of each point. Hence,

$$\begin{aligned} Q_{BO1} &:= \{0, 4700, 6000, 5500, 4300, 5700, 6700\} \\ Q_{TS1} &:= \{0, 20000, 1000, 1500, 2000, 2000, 6000\} \\ Q_{TS2} &:= \{0, 10500, 6000, 9000, 2250, 4500, 750\} \\ Q_{TS3} &:= \{0, 1400, 4300, 5700, 4300, 7174, 10000\} \end{aligned}$$

Hereby, the parameter represents a family with $i \in I$ being implicitly defined by the sequence at which the value is written. For example,

$$Q_{BO1} = (q_i)_{i \in I} = \{(0,0), (1,4700), (2,6000), (3,5500), (4,4300), (5,5700), (6,6700)\}$$

Additionally, arc specific information such as speed and road angle are required. With $\mathscr{A} := (\acute{a}_{ij})_{i,j \in I}$ with $\forall \acute{a} \in \mathscr{A} := \{\widetilde{LU}_h, V, \Theta\}$ [1] that is the set of arcs $\mathscr{A}_h$ is defined as

$$\begin{aligned} &\forall h \in H : \\ &\qquad V_h = \{v_{hij} | \forall i, j \in I : v_{h_{ij}} := 80[km/h]\} \\ &\qquad \Theta_h = \{\theta_{h_{ij}} | \forall i, j \in I : \theta_{h_{ij}} := 0[°]\} \end{aligned}$$

Thereby, for every scenario and every arc the speed $v_{h_{ij}} := 80[km/h]$ and the road angle $\theta_{h_{ij}} := 0[°]$ are defined. Having the similarities defined the next step is to identify the data that differentiates between Company-A and Company-B. These differences are mainly caused due to the variations between their vehicle. Hence, $\mathscr{V}_A$ and $\mathscr{V}_B$, which are instances of the parameter group $\mathscr{V} := \{K_v, w, K_w, A, \eta, \acute{c}_d, \omega_{GHG}, \omega_F\}$ are

[1] An arc has in the model far more attributes. In this case this refers only to the required input parameter for an arc.

defined as

$$
\begin{aligned}
K_{v_A} &:= 93[m^3] & K_{v_B} &:= 99,33[m^3] \\
w_A &:= 19680[kg] & w_B &:= 20210[kg] \\
M_{w_A} &:= 41000[kg] & M_{w_B} &:= 41000[kg] \\
A_A &:= 9,8175[m^2] & A_B &:= 9,97[m^2] \\
\eta_A &:= 0,32[\%] & \eta_B &:= 0,34[\%] \\
\acute{c}_{d_A} &:= 0,86 & \acute{c}_{d_A} &:= 0,86 \\
\omega_{GHG_A} &:= 2,664[kg/l] & \omega_{GHG_A} &:= 2,664[kg/l]. \\
\omega_{F_A} &:= 0,1005[l/kWh] & \omega_{F_B} &:= 0,1005[l/kWh].
\end{aligned}
$$

When compared to the data found in the case, some values presented above are already the result of some computation. The parameter K_v is the storage volume of the trailer, i.e. $K_v = width \cdot length \cdot hight$. The curb weight w is the sum of the weight of the truck and the weight of the trailer, i.e. $w = w_{Truck} + w_{Trailer}$. Finally, the parameter A is simply the frontal surface area calculated based on the hight and width of the truck and trailer, i.e. $A = max(width_{Truck}, width_{Trailer}) \cdot max(hight_{Truck}, hight_{Trailer})$. Anyhow, the two companies also diverge on their pricing strategy. As for Company-A they price based on the total weight carried with a price of $\acute{p}_{A1} := 30[Euro/t]$ for transporting goods in Austria and $\acute{p}_{A2} := 60[Euro/t]$ for any trip containing an international destination. In contrast Company-B prices based on the total distance travelled, with $\acute{p}_B := 1,2[Euro/km]$. After all the necessary input parameter are defined, the next step is to determine the required implementations. As mentioned earlier Of, Od and Oe are to be investigated and since variable speed and time windows are not mentioned, the implementations

$$
\begin{aligned}
\mathscr{I}_{Of} &:= Implementation(Load, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Od} &:= Implementation(Distance, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False})
\end{aligned}
$$

can be defined. While $\mathscr{I}_{Of}$ and $\mathscr{I}_{Od}$ represent the company A and B, $\mathscr{I}_{Oe}$ serves for calculating a best case scenario to compare the previous two against. The first step towards solving the problem is to execute the above mentioned implementations for every scenario. By doing so the values visible in Table 5.1 can be obtained.

Company A	$\mathscr{I}_{Of}(TS1)$	$\mathscr{I}_{Oe}(TS1)$	$\mathscr{I}_{Of}(TS2)$	$\mathscr{I}_{Oe}(TS2)$	$\mathscr{I}_{Of}(TS3)$	$\mathscr{I}_{Oe}(TS3)$	$\mathscr{I}_{Of}(BO1)$	$\mathscr{I}_{Oe}(BO1)$
Path	0-1-6-5-4-3-2-0	0-6-1-2-3-4-5-0	0-1-3-2-5-4-6-0	0-3-4-2-1-6-5-0	0-6-5-3-2-4-1-0	0-5-6-3-4-2-1-0	0-6-2-5-3-1-4-0	0-4-6-5-1-3-2-0
D	1272,46	1127,13	5460,11	2979,74	3452,91	2818,68	1380,49	1136,86
F	37,07	47,98	50,00	64,96	55,17	56,94	66,73	72,97
P	1455,05	1460,97	6703,55	4267,18	4145,38	3857,83	1741,91	1612,67
C	868,98	857,10	3962,28	2471,38	2458,05	2247,09	1025,66	935,10
Pw	571,52	572,27	2452,38	1512,88	1550,86	1431,11	620,04	577,21
Pf	127,54	131,71	1007,22	753,09	543,09	533,68	301,69	271,94
Pv	755,99	756,98	3243,95	2001,20	2051,44	1893,04	820,17	763,52
Company B	$\mathscr{I}_{Od}(TS1)$	$\mathscr{I}_{Oe}(TS1)$	$\mathscr{I}_{Od}(TS2)$	$\mathscr{I}_{Oe}(TS2)$	$\mathscr{I}_{Od}(TS3)$	$\mathscr{I}_{Oe}(TS3)$	$\mathscr{I}_{Od}(BO1)$	$\mathscr{I}_{Oe}(BO1)$
Path	0-5-4-3-2-1-6-0	0-6-1-2-3-4-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-1-2-4-3-6-5-0	0-5-6-3-4-2-1-0	0-2-3-1-5-6-4-0	0-4-6-5-1-3-2-0
$D[km]$	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	1136,86	1136,86
$F[t]$	93,78	47,98	64,96	64,96	86,45	56,94	70,53	72,97
$P[kg]$	1509,97	1400,60	3612,62	4083,77	3620,08	3694,85	1413,27	1543,60
$C[Euro]$	881,97	826,46	2139,13	2378,28	2126,41	2164,36	833,88	900,04
$Pw[kg]$	489,30	553,11	1293,53	1462,23	1223,62	1383,20	493,52	557,88
$Pf[kg]$	380,62	123,97	627,02	708,80	795,85	502,29	274,18	255,95
$Pv[kg]$	640,05	723,52	1692,06	1912,74	1600,61	1809,36	645,57	729,77

Table 5.1: Problem 1 - Output

The header of this table includes information about the implementation and the scenario used to calculate a column. The second row simply depicts the path at which the optimisation program converged. As a reminder the Family J containing the path is a collection of tupel $J \subseteq S \times I$, with $s \in S$ indicating the number of the stop. Thereby, the paths presented in Table 5.1 are represented as (s, i), with s being implicitly defined. For example, $0-1-6-5-4-3-2-0$ is actually

$$J := \{(0,0),(1,1),(2,6),(3,5),(4,4),(5,3),(6,2),(7,0)\}.$$

A more visual representation of the trips can be found in Figure 5.1, Figure 5.2, Figure 5.3 and Figure 5.4, with the width of the line serving as a representation of the carried load, i.e. $f_{\bar{j}} \in F_{\mathscr{I}_{Ox}(h)}$. The rows in the center of the table contain the values of core parameter, with the parameter cost being included for the sake of completeness. These parameter provide a sound basis for comparing the different instances. Furthermore, since GHG emissions can be divided into three parts they are included in the last section of the table.

In this case the economic aspect of the problem is investigated first. Therefore,

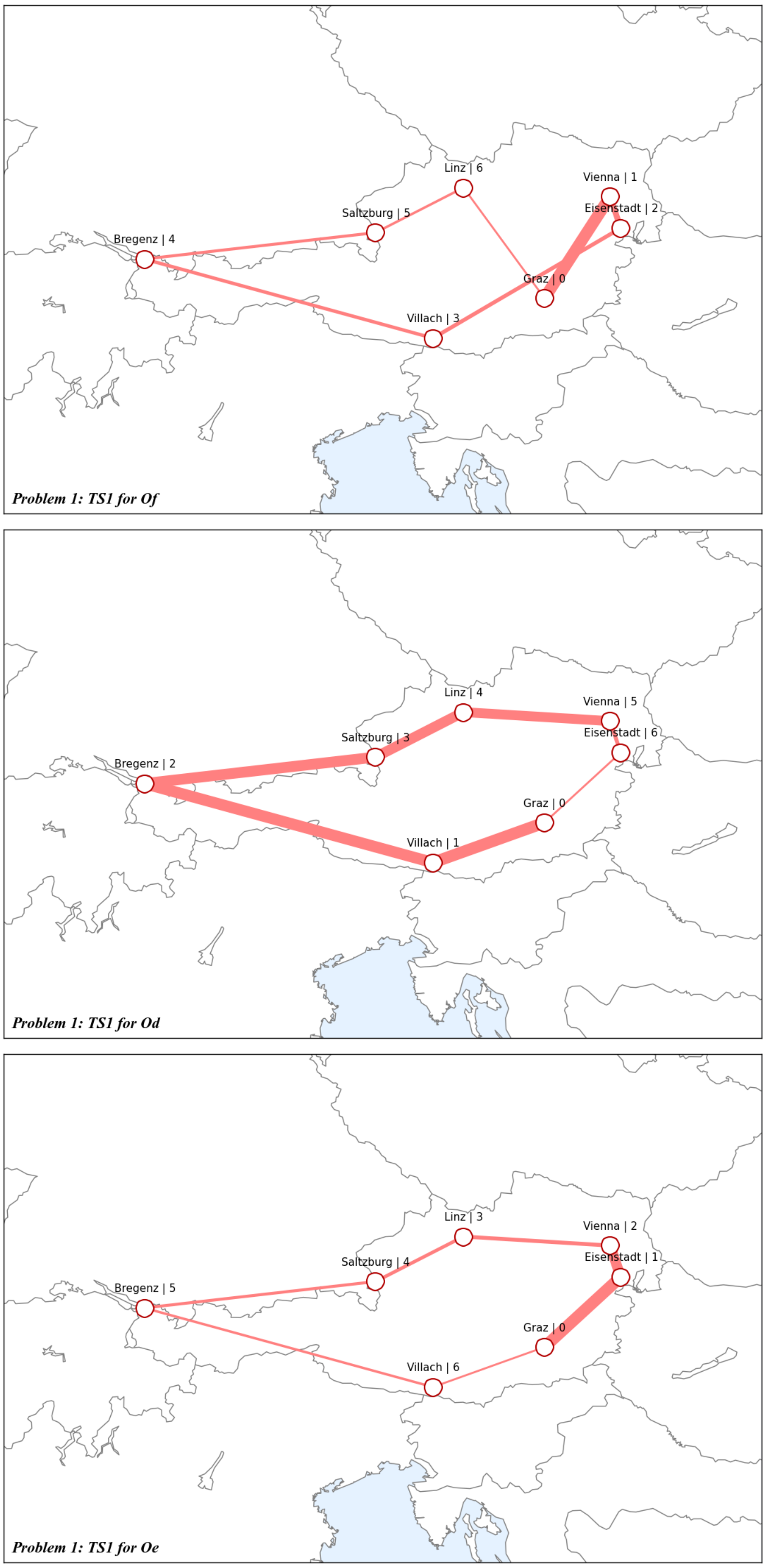


Figure 5.1: Problem 1 - Maps for TS1

Figure 5.2: Problem 1 - Maps for TS2

Figure 5.3: Problem 1 - Maps for TS3

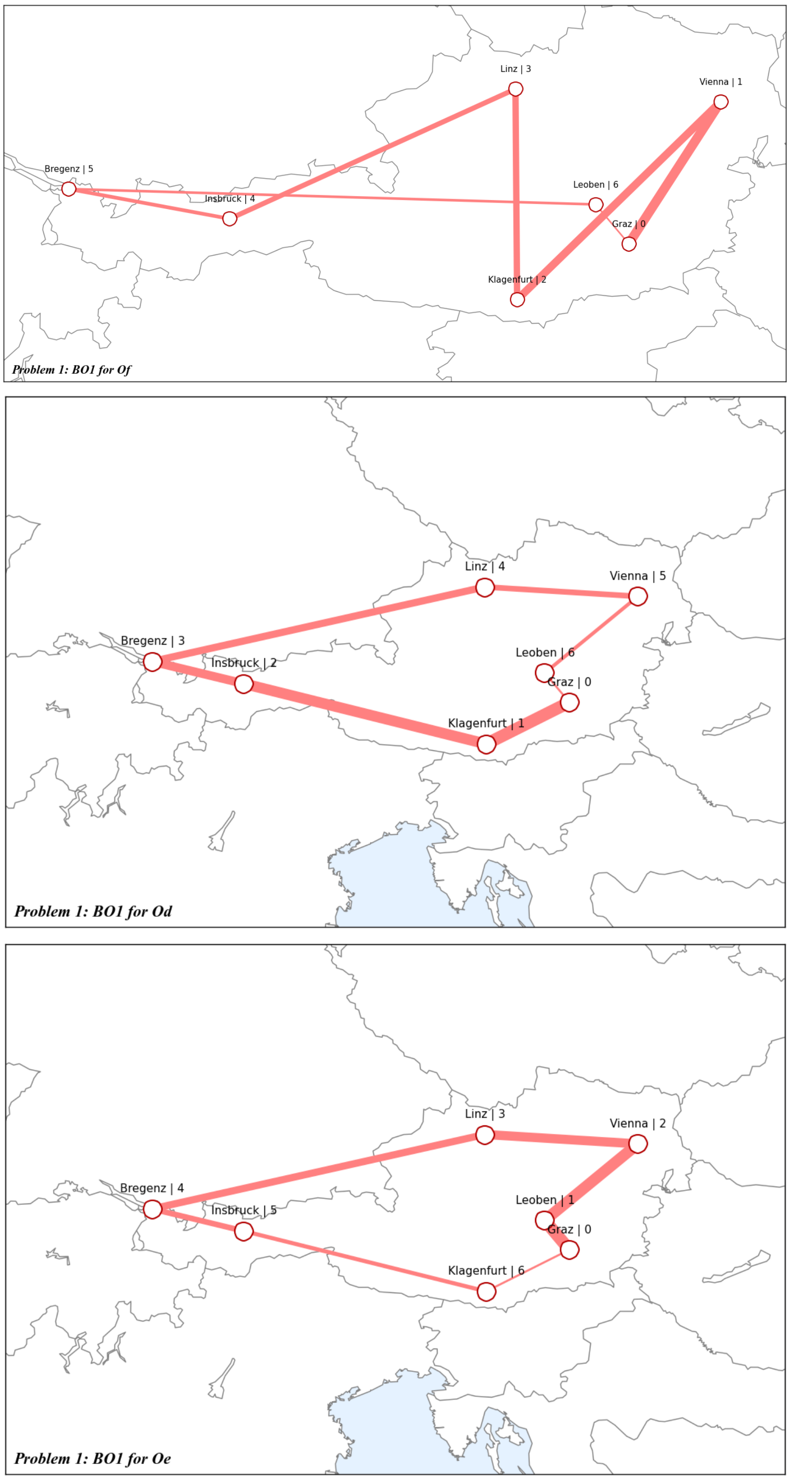


Figure 5.4: Problem 1 - Maps for BO1

the cost associated with each company has to be calculated. That is,

$$\begin{aligned} C_{A_h} &= p'_{Ax} \cdot F_{\mathscr{I}_{Of}(h)} = p'_{Ax} \cdot \sum_{\bar{j} \in \bar{J}_{\mathscr{I}_{Of}(h)}} f_{\bar{j}} \\ C_{B_h} &= p'_{B} \cdot D_{\mathscr{I}_{Od}(h)} = p'_{B} \cdot \sum_{\bar{j} \in \bar{J}_{\mathscr{I}_{Od}(h)}} d_{\bar{j}}. \end{aligned}$$

The result of these equations can be viewed in Table 5.2. Whereby each column of this table represents the cost that would occur, if the respective company was selected for a specific trip. The last two columns, however, describe the difference between the companies on the basis of $Company - B$, i.e.

$$\begin{aligned} \Delta C_{[Euro]} &:= C_{B_h} - C_{A_h} \\ \Delta C_{[\%]} &:= \frac{C_{B_h}}{C_{A_h}} - 1 = \frac{\Delta C_{[Euro]}}{C_{A_h}} \end{aligned}$$

The last row of Table 5.2 contains the average of the respective column [2]. These averages are are defined in general

$$m_{avr} := \frac{1}{\sharp M} \cdot \sum_{i=1}^{\sharp M} m_i \tag{5.1}$$

with M representing any set or family of values. In this case the average cost for one company across all $h \in H$ is calculated.
Returning to the results presented in Table 5.2. At the first glance this table would suggest that $Company - A$ would be the most sensible choice from a purely economic

[2] The average of column $\Delta C_{[\%]}$ is not calculated as presented in Equation 5.1 as this would lead to misrepresentation. Hence,

$$\Delta C_{[\%]_{average}} = \frac{C_{[Euro]_{average}}}{C_{A_{average}}}$$

, which accurately represents the percentage of the average deviation between Company-A and Company-B. Furthermore, this differs if one would simply take the average of the column in question, because

$$\frac{\sum_{i=1}^{n} \frac{a_i + b_i}{n}}{\sum_{i=1}^{n} \frac{b_i}{n}} = \frac{\sum_{i=1}^{n} a_i + b_i}{\sum_{i=1}^{n} b_i} \neq \sum_{i=1}^{n} \frac{a_i + b_i}{b_i \cdot n} = \sum_{i=1}^{n} \frac{c_i}{b_i \cdot n}$$

standpoint. This arises from the fact that the cost of delivery is lower in three out of four cases, with the outlier being $BO1$. This outlier, however, is not significant enough to turn the averages in favour of Company-B. Hence, the average costs of Company-A are lower than the ones that would occur by choosing Company-B. However, before any decisions are made the context of the situation should also be

Cost [Euro]	$C_{\mathscr{I}_{Of(h)_A}}$	$C_{\mathscr{I}_{Od(h)_B}}$	ΔC	$\Delta C[\%]$
TS1	1112,23	1352,56	240,32	21,6%
TS2	3000,23	3575,69	575,46	19,2%
TS3	3310,46	3382,42	71,97	2,2%
BO1	2002,02	1364,23	-637,79	-31,9%
Average	2356,23	2418,72	62,49	2,7%

Table 5.2: Problem 1 - Cost comparison

taken into consideration. Firstly, with an increase in distance C_A remains stable, while C_B increases. By contrast C_A increases with load, while C_B remains stable. Based on this one can easily calculate at which point, which company becomes more cost-effective. At this point

$$\Delta C = C_B - C_A = 0$$

From there one only has to test whether $\Delta C \geq 0$ or $\Delta C \leq 0$.

$$\begin{aligned}
\Delta C \leq 0 \quad &\Leftrightarrow \quad p'_B \cdot D_{\mathscr{I}_{Od}(h)} - p'_{Ax} \cdot F_{\mathscr{I}_{Of}(h)} \leq 0 \\
&\Leftrightarrow \quad F_{\mathscr{I}_{Of}(h)} \geq \frac{p'_B}{p'_{Ax}} \cdot D_{\mathscr{I}_{Od}(h)} \\
\Delta C \geq 0 \quad &\Leftrightarrow \quad p'_B \cdot D_{\mathscr{I}_{Od}(h)} - p'_{Ax} \cdot F_{\mathscr{I}_{Of}(h)} \geq 0 \\
&\Leftrightarrow \quad D_{\mathscr{I}_{Od}(h)} \geq \frac{p'_{Ax}}{p'_B} \cdot F_{\mathscr{I}_{Of}(h)}
\end{aligned}$$

With the given prices of $p'_{A1} := 30[Euro/kg]$ or $p'_{A2} := 60[Euro/kg]$ and $p'_B := 1,2[Euro/km]$ this equation can be used to assess the maximum distance at which Company-B is economically speaking still the better choice. Therefore, the average load is calculated. In this case the load was chosen as it is not directly influenced by the fact that a trip is domestic of international. Moreover, the range of this parameter is also vastly narrower, i.e. in this case $min(Q, K) \leq F_h \leq K \cdot (\sharp\mathscr{N} - 2)$.

That is, given Equation 5.1 and $\forall h \in H : M := \{\widehat{F}_{\mathscr{I}_{Of(h)_A}}, \widehat{F}_{\mathscr{I}_{Od(h)_B}}, \widehat{F}_{\mathscr{I}_{Oe(h)}}\}$ the value $63,96[t]$ is obtained. This results, based on the average load, in $1599,05[km]$ for domestic trips, which more that driving from Vienna to Bregenz and then back again to Vienna. As for international trips Company-A becomes more cost effective if the trip is longer than $3198,10[km]$. By comparing these values against the average length of a trip, i.e. $1196,82[km]$ domestically and $3418,31[km]$ internationally, the suggestion can be made that for domestic trips Company-B would in general outperform Company-A, while the reverse would be true for an international trip. However, since it is more likely for such a young company to initially operate within domestic borders, the first value can be considered as having a greater relevance. Unfortunately, it is not possible to compare the distance calculated based on the average values, against the respective scenarios given in Table 5.2. This is due to the fact that the load carried and the distance travelled vary, which indicates that concrete assessment has to be done on a case by case basis. Therefore, the calculated values can only be used as general approximation. That is, it can be concluded that given the load of an average trip, Company-B would be cost effective up until $1599,05[km]$ domestically or $3198,10[km]$ internationally. Unfortunately, due to a lack of data any suggestions implied by these values are far from definitive. Anyhow, in general it is possible to conclude that given only the results of four scenarios, Company-A would be the cheaper option.

After the economical argument the next aspect of this problem is the environmental

Emissions [kg]	$P_{\mathscr{I}_{Of(h)_A}}$	$P_{\mathscr{I}_{Od(h)_B}}$	ΔP	$\Delta P[\%]$
TS1	1455,05	1509,97	54,92	3,77%
TS2	6703,55	3612,62	-3090,93	-46,11%
TS3	4145,38	3620,08	-525,31	-12,67%
BO1	1741,91	1413,27	-328,63	-18,87%
Average	3511,47	2538,98	-972,49	-27,69%

Table 5.3: Problem 1 - Emissions comparison

one. It is described in Table 5.3, which has the same structure as Table 5.2. Based on these values it becomes apparent that in contrast to the previous results, the opposite picture is presented. That is, Company-A has only in one case, namely TS1, less GHG output than its competitor. Hence, one can assume that Company B provides, if solely measured in CO2 output, a more environmentally friendly service. However, for additional insight further investigations are required. Therefore, the values presented in Table 5.4 can be used. Since, it contrasts the parameter D, F and P of the two implementations $\mathscr{I}_{Of}$ and $\mathscr{I}_{Od}$, which are expressed in absolute and relative values. The relative ones use the results of $\mathscr{I}_{Of}$ as its basis. Out of the four scenarios TS1 and TS2 are the more interesting ones. Firstly, TS1 is the only scenario at which Company A, i.e. Of, results in less GHG output than Company-B, i.e. Od. To be precise $\Delta P_{[kg]} = 54,92[kg]$ or $\Delta P_{[kg]} = 3,77[\%]$. Hence, indicating only a slight variation. This difference is due to the fact that the total load carried $F_{\mathscr{I}_{Od}(TS1)_B} \cdot 2,529 = F_{\mathscr{I}_{Of}(TS1)_A}$, which is the greatest difference between two solutions across all scenarios. However, the difference in distance $\Delta D = -11,4[\%]$ is in comparison relatively marginal. Therefore, the reduction in distance was not significant enough to tilt the outcome in favour of Company-B. Even though, it is impressive that such a small difference in distance could nearly compensate for such a massive variation in load. As apparent in Figure 5.1 due to an unfortunate selection of $J_{\mathscr{I}_{Od}(TS1)_B}$ the vehicle of Company-B is forced to travel nearly the whole trip with the goods for the point "Vienna", which with $q_{TS1_1} := 20[t]$ amounts to nearly the complete carrying capacity of the vehicle by weight. In Figure 5.1 this is indicated by the width of the path. Furthermore, this visualisation makes a major flaw of Od apparent. Since if compared against the sequence $J_{\mathscr{I}_{Oe}(TS1)_B}$, the path selected for Od is the exact opposite of the "optimal" path, i.e. Oe. Hence, both trips have the same distance. However, since the objective distance does not consider the load of the vehicle at all, the algorithm could have only converged at this result at random.

The second interesting scenario is TS2. In this case the biggest difference between $P_{\mathscr{I}_{Of}}$ and $P_{\mathscr{I}_{Od}}$ can be observed, i.e. $\Delta P_{[kg]} = -46,1[\%]$. This is yet again due to an unfortunate path selection. However, this time it was caused by the improper integration of distance into the objective Of. Since, Of only measures distance in arcs. That is, an arc with $100[km]$ has in this implementation the same length as an arc with $1[km]$. Due to this it is entirely possible that ridiculously convoluted trips can be selected. An example of which can be found in Figure 5.2. In this case $\Delta D_{[km]} = -45,4[\%]$, which represents the highest difference in distance across all instances. However, in comparison to $TS1$ the reduction in weight was not sufficient enough to counter the higher distance. As for $TS3$ and $BO1$, both of them represent a middle ground, that is no extreme differences or unexpected behaviour between Of

	TS1	TS2	TS3	BO1
$\Delta D[km]$	-145,32	-2480,37	-634,23	-243,63
$\Delta F[t]$	56,70	14,95	31,28	3,80
$\Delta P[kg]$	54,92	-3090,93	-525,31	-328,63
$\Delta D[\%]$	-11,4%	-45,4%	-18,4%	-17,6%
$\Delta F[\%]$	152,9%	29,9%	56,7%	5,7%
$\Delta P[\%]$	3,8%	-46,1%	-12,7%	-18,9%
$\mathscr{I}_{Oe}$	TS1	TS2	TS3	BO1
$\Delta P_{\mathscr{I}_{Of}}[kg]$	162,63	2928,69	732,64	315,30
$\Delta P_{\mathscr{I}_{Od}}[kg]$	270,96	0,00	351,51	47,77
$\Delta P_{\mathscr{I}_{Of}}[\%]$	13%	78%	21%	22%
$\Delta P_{\mathscr{I}_{Od}}[\%]$	22%	0%	11%	3%

Table 5.4: Problem 1 - Detailed emission comparison

and Od can be detected. However, in both instances Company-B achieves better results. Thereby, one could conclude that Company-B is the preferable solution, when the decision is solely based on environmental concerns. Additionally, it has to be mentioned that the factor distance seems to have a greater influence over the overall GHG emissions. This assumption is made due to the fact that $\Delta D_{[\%]}$, seems to dictate $\Delta P_{[\%]}$. As $\Delta D_{[\%]}$ tends to be in closer proximity to $\Delta P_{[\%]}$ than $\Delta F_{[\%]}$. Thereby, it is indicated that Od might have a greater influence in calculating the GHG output, thus suggesting that the behaviour of Od is closer to the behaviour of Oe than Of. Apart from that another environmental argument can be made. That is, the performances of $\mathscr{I}_{Of}$ and $\mathscr{I}_{Od}$ can be directly compared against the respective ideal counterparts, i.e $\mathscr{I}_{Of}$. This comparison is depicted in the second part of Table 5.4, with $\Delta P_{\mathscr{I}_{Ox}}[\%]$ being calculated based on the respective $\Delta P_{[kg]_{\mathscr{I}_{Oe}}}$. Out of these values it can be deduced that the difference between the objective of the company and the ideal one tends to be smaller with Company-B. That is, apart form $TS1$ Company-B is at least 11% closer to the ideal result than Company-A. Moreover, the scenario in which Company-A has a better result than Company-B, is the result of the above mentioned problem of Od. That is, the path chosen is the exact same path as the optimal path, but in reverse. Hence, yet again the algorithm optimising the distance could have randomly converged on the ideal path as well. Thereby, implying the usefulness of including some measure of load into the objective Od (see Section 5.2). Overall, given the information obtained it is clear that when selecting Company-B the result will for the most part be closer to the most energy efficient solution. Nevertheless, yet another argument concerning the environmental impact can be made. The basis of which is the fact that both companies employ different trucks. Hence, one should detect a difference in fuel consumption and GHG emissions. However, due to the massive differences between $\mathscr{I}_{Of}$ and $\mathscr{I}_{Od}$ it is not sensible to compare the vehicles based on these implementations. Therefore, the respective $\mathscr{I}_{Oe}$ is chosen to investigate this matter. Thereby, it is also ensured that other parameter on a given trip stay the same. By doing so the results found in Table 5.5 can be created. A table which has the same structure as Table 5.2. Base on this it can be deduced that the emission of Truck-A are on average about $4,25\%$ higher than Truck-B. However, by comparing the two trucks based on the energy consumption a different picture arises, as in this case Truck-A requires about $1,74\%$ less energy than Truck-B. The reason behind this becomes apparent through the following implication.

$$\begin{aligned}
A \quad \uparrow \quad & \Rightarrow \beta \uparrow \Rightarrow Pv \uparrow \Rightarrow P \uparrow \Rightarrow P_{[l]} \downarrow \Rightarrow P_{[kg]} \downarrow \\
w \quad \uparrow \quad & \Rightarrow Pw \uparrow \Rightarrow P \uparrow \Rightarrow P_{[l]} \downarrow \Rightarrow P_{[kg]} \downarrow \\
\eta \quad \uparrow \quad & \Rightarrow P_{[l]} \downarrow \Rightarrow P_{[kg]} \downarrow \\
\Rightarrow \quad & P_{[kg]} \leq P_{[kg]} + \Delta_A P_{[kg]} + \Delta_w P_{[kg]} \\
\Rightarrow \quad & P_{[kg]} \leq P_{[kg]} + \Delta_A P_{[kg]} + \Delta_w P_{[kg]} + \Delta_\eta P_{[kg]} \\
& \vee P_{[kg]} \geq P_{[kg]} + \Delta_A P_{[kg]} + \Delta_w P_{[kg]} + \Delta_\eta P_{[kg]}
\end{aligned}$$

In every case presented in Table 5.5 $\Delta_\eta P_{[kg]} \geq \Delta_A P_{[kg]} + \Delta_w P_{[kg]}$. Thus indicating that without taking the efficiency of the vehicle into account the greater weight and bigger frontal surface area of Truck-B, lead to a higher energy requirement. However, due to a higher efficiency i.e. $\eta_A = 32[\%]$ and $\eta_B = 34[\%]$, this increase in energy demand is compensated.

Having now investigated economical as well as environmental factors, both of them can be used to construct one last argument. Namely, given the internal price of GHG emissions at that time, i.e. $\acute{p}_e = 24,50[Euro/t]$ and the GHG output of both companies, it is possible to adjust the cost that have been analysed above.

Emissions [kg]	$P_{\mathscr{I}_{Oe(h)A}}$	$P_{\mathscr{I}_{Oe(h)A}}$	ΔP	$\Delta P[\%]$
TS1	1292,41	1239,01	-53,40	-4,13%
TS2	3774,86	3612,62	-162,25	-4,30%
TS3	3412,74	3268,56	-144,18	-4,22%
BO1	1426,61	1365,51	-61,10	-4,28%
Average	2476,66	2371,42	-105,23	-4,25%
Emisions [kWh]	$P_{\mathscr{I}_{Oe(h)A}}$	$P_{\mathscr{I}_{Oe(h)A}}$	ΔP	$\Delta P[\%]$
TS1	1366,20	1391,61	28,72	1,86%
TS2	3990,38	4057,55	67,17	1,68%
TS3	3607,58	3671,12	63,54	1,76%
BO1	1508,06	1533,68	25,63	1,70%
Average	2618,05	2663,49	45,44	1,74%

Table 5.5: Problem 1 - Truck comparison based on O_e

That is,

$$C_{A_{new_h}} := \acute{p}_e \cdot P_{\mathscr{I}_{Of(H)A}} + C_{A_h}$$
$$C_{B_{new_h}} := \acute{p}_e \cdot P_{\mathscr{I}_{Od(H)A}} + C_{B_h},$$

thus resulting in the values presented in Table 5.6.

Cost [*Euro*]	$C_{A_{new}}$	$C_{A_{new}}$	ΔC_{new}	$\Delta C_{new_{[\%]}}$	ΔC	$\Delta C_{[\%]}$
TS1	1147,89	1389,56	241,67	21,1%	240,32	21,6%
TS2	3164,50	3664,22	499,72	15,8%	575,46	19,2%
TS3	3412,04	3471,13	59,09	1,7%	71,97	2,2%
BO1	2044,71	1398,86	-645,85	-31,6%	-637,79	-31,9%
Average	2442,28	2480,94	38,66	1,6%	62,49	2,7%

Table 5.6: Problem 1 - Adjusted Cost

By adjusting the cost for the GHG emissions produced, the difference between the cost of Company-A and the cost of Company-B reduces even further. That is, a reduction from $2,7[\%]$ to $1,6[\%]$. Thereby, weakening the economic argument for Company-A. However, at the same time the environmental argument for Company-B is also weakened, as now the economic costs have been adjusted based on the GHG output.

In conclusion, with both the environmental and economical perspective analysed one can now state a preferable transport company. In a direct comparison Company-A provides given the four scenarios a better economic result. Moreover, even if the costs are adjusted by the cost of GHG emissions, Company-A still performs better. However, by solely comparing the environmental aspects, Company-B would be the most preferable. That is, given the four scenarios Company-B produces less GHG emissions. Moreover, the results achieved by selecting Company-B are closer to the one provided by "ideal" trip. This is partly explained by the fact in the given examples, D tends to have a greater influence over P. Lastly, given the same distance Company-B would always achieve a lower GHG output, as it employs a more efficient vehicle. Therefore, it can be concluded that Company-A has lower costs, while Company-B inflicts less environmental damage. In order to resolve this problem, the average distance at which Company-A becomes more cost effective was calculated based on the average load. Said distance was greater than the average distance of domestic trips. Unfortunately, due to variations in load it is not useful to apply these result to individual problems. However,l it can be concluded that for the most part shorter trips should be executed by Company-B. Furthermore, the calculated distance suggest that in the usual case Company-B would outperform Company-A in domestic trips, while internationally the reverse would be true. Hence, depending on the idealogical stance regarding GHG emissions, either company can be selected. In this particular analysis it is proposed that Company-B provides, given the arguments stated above, the better service.

5.2 Problem Two

The second problem arises directly from an observation made in Section 5.1. Namely, that Od and Of can lead to unfortunate trips simply because both are more or less ignorant to the respective counterpart. Thus the question arises if it is possible to reduce said errors without relying on vast amounts of additional information, such as it is the case with the fuel consumption model. This exercise should improve the understanding of the fuel consumption model and especially the relationship between distance and load and their influence on the fuel consumption. Narrative wise one is asked to create a model, which under certain circumstances mimics the behaviour of the fuel consumption model used previously, while at the same time

requiring as little information as possible.
At first said circumstances have to be defined. The parameter used for solving Problem 1 are for the most part identical to the ones required here. That means,

$$\begin{aligned} \mathscr{W}_{P2} &:= \mathscr{W}_1 \\ \mathscr{P}_{P2} &:= \mathscr{P}_1 \end{aligned}$$

are identical, while the parameter group *Vehicle* is locked based on the decision made in Problem One. Therefore,

$$\mathscr{V}_{P2} := \begin{cases} \mathscr{V}_A & \textit{If Company A was selected} \\ \mathscr{V}_B & \textit{If Company B was selected} \end{cases}$$

Unfortunately, due to this an ambiguity arises. Hence, this section will encompass an analysis for both choices. That is, both objectives, i.e. Of and Od are covered. Hereby, Of is solved with $\mathscr{V}_{P2_A}$ and Od with $\mathscr{V}_{P2_B}$. However, apart from that some alteration to the input have to be made, most of which occur within the parameter group *Trip*. That is, the set of scenarios changed to $H := \{\mathscr{N}_{TS1}, \mathscr{N}_{TS2}, \mathscr{N}_{TS3}, \mathscr{N}_{BO66}\}$ with

$$\begin{aligned} \mathscr{N}_{TS1} &:= \{Graz, Vienna, Linz, Salzburg, Bregenz, Villach, Eisenstadt\} \\ \mathscr{N}_{TS2} &:= \{Graz, Hamburg, Frankfurt, Florence, Turin, Budapest, Prague\} \\ \mathscr{N}_{TS3} &:= \{Graz, Debrecen, Ostrava, Stuttgart, Berlin, Munich, Milan\} \\ \mathscr{N}_{BO66} &:= \{Graz, Klagenfurt, Linz, Eisenstadt, Leoben, Krems\} \end{aligned}$$

Even though, the composition of H changed the parameter V_h and Θ_h remain unchanged, i.e.

$$\begin{aligned} &\forall h \in H : \\ &\qquad V_h = \{v_{hij} | \forall i, j \in I : v_{h_{ij}} := 80[km/h]\} \\ &\qquad \Theta_h = \{\theta_{h_{ij}} | \forall i, j \in I : \theta_{h_{ij}} := 0[°]\} \end{aligned}$$

Similar to the previous problem all sets of points presented are of the same cardinality. Therefore, only one index set $I := \{0, 1, 2, 3, 4, 5, 6\}$ is required. Naturally, as $BO66$ is now introduced it has to be defined properly. The information for the geographical position of each individual point, i.e. X_{BO66} and Y_{BO66} can be found in Table 4.16. The demand on the other hand is,

$$Q_{BO66} := \{0, 1500, 1500, 12000, 5000, 10000\}.$$

Additionally, since the cost that accumulate during the trip are now relevant for the solution, the parameter group *Cost*, i.e. $\mathscr{C} := \{\acute{p_d}, \acute{p_f}, \acute{p_e}\}$ has to be defined. That means, based on the date of the Problem $\mathscr{C}_{P2} := \mathscr{C}_{7.10.2013}$, which equates to

$$\begin{aligned} \acute{p_d} &:= 9,43[Euro/h] \\ \acute{p_f} &:= 1,35[Euro/l] \\ \acute{p_e} &:= 5,3[Euro/t]. \end{aligned}$$

Even if the input is now defined properly, one can not simply proceed with solving for the desired objectives, as it was done in Problem One. This is due to the fact that part of the problem is to formulate a useful objective. However, it is already possible to define the implementations against which the new objective has to be compared.

$$\begin{aligned} \mathscr{I}_{Of} &:= Implementation(Load, Speed_{Fixed}, TW_{False}) \\ \mathscr{I}_{Od} &:= Implementation(Distance, Speed_{Fixed}, TW_{False}) \\ \mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False}) \end{aligned}$$

Therefore, the next step is to define the new objective. In order do so one has to understand the fuel consumption model, the situation dictated by the input and the problems implied by the objectives distance or load. Firstly, what is the main problem of the objective distance? As already stated above, *Od* neglects the total weight of the vehicle, thus errors as observed in Problem One (see Section 5.1) can arise. The most frequent problem was that *Od* suggested the same path as *Oe*, but with its sequence of stops being inverted, i.e. the vehicle travelled the optimal path backwards. Therefore, it can be assumed that a marginal incorporation of the parameter load into the objective could easily improve its outcome. As for Of, by only incorporating the arc as a measure of distance, Of becomes blind to the actual distance as soon as one arc has a different length than the other ones, i.e. $\exists j \in \bar{I} \forall i \in \bar{I} : d_j \neq d_i$. Thereby, indicating that the actual distance is not appropriately integrated. Therefore, it is likely that highly convoluted trips are created. Hence, its deficit could be resolved by including a more accurate measure of distance. Secondly, what is implied by the input parameters? The most important deduction one can make is that the road angle and the speed remain constant, i.e. $\forall h \in H \forall i \in \bar{I} : v_{h_i} = 80[km/h] \wedge \theta_{h_i} = 0 \wedge a_{h_i} = 0$. Thus with the fuel consumption equation in mind, the only variable parameters during a trip are D and F. Thereby, allowing more flexibility in defining the objective. Thirdly, the equation at the center of the fuel consumption model is

$$P = \alpha \cdot d \cdot f + \alpha \cdot d \cdot w + \beta \cdot d \cdot v^2.$$

This formulation consist of three parts with $Pf := \alpha \cdot d \cdot f$, $Pw := \alpha \cdot d \cdot w$ and $Pv := \beta \cdot d \cdot v^2$. Since each of them represent different factors and influence the outcome individually, it is reasonable to assume that the relationship between these three parts and their effect on the slope of the function harbour the key to the behaviour of the model. That means, if it is possible to create a function that mimics the behaviour of the model at any given point, i.e. constellation of load and distance, it should be possible to obtain similar results. In this case "similarly" means that an increase in distance or load leads to an increase in output proportionally to the increase experienced by the fuel consumption model. Thereby, it is implied that scaling is irrelevant. However, before this idea is further developed a quick example. $P_{100} := P \cdot 100 = (Pf + Pw + Pv) \cdot 100 = Pf \cdot 100 + Pw \cdot 100 + Pv \cdot 100$ still incorporates every factor equally into the end result. It might no longer accurately represent the energy consumption, but its optimal solution is still at the same parameter constellation. This is the reason why it is possible to translate the energy consumption from $[J]$ to $[kWh]$ without distortion. On the other hand, if the ratio is upset, i.e. $P'_{100} = Pf + Pw + Pv \cdot 100 = \alpha \cdot d \cdot f + \alpha \cdot d \cdot w + \beta \cdot d \cdot v^2 \cdot 100$, the optimal solution may vary, because the scaled parameter has now a different priority within the model. The next step in exploring this relationship is to investigate the partial derivatives of this function, which express the increase of the function at any given point. In this case the argument is constructed based on the example above.[3]

[3] In any future argument the scalar 100 can safely be generalised as $x \in \mathbb{R}$ with $x \neq 1 \wedge x \neq 0$.

Firstly, the derivatives are

$$\begin{aligned}\frac{\partial P}{\partial d} &:= \alpha(f+w)+\beta\cdot v^2\\ \frac{\partial P}{\partial f} &:= \alpha\cdot d\\ \frac{\partial P}{\partial v} &:= 2\cdot\beta\cdot d\cdot v.\end{aligned}$$

If adjusted for the example, the effect of the alterations can be observed. That is, the new partial derivatives are

$$\begin{aligned}\frac{\partial P_{100}}{\partial d} &:= 100\cdot(\alpha(f+w)+\beta\cdot v^2) & \frac{\partial \acute{P}_{100}}{\partial d} &:= \alpha(f+w)+100\cdot\beta\cdot v^2\\ \frac{\partial P_{100}}{\partial f} &:= 100\cdot\alpha\cdot d & \frac{\partial \acute{P}_{100}}{\partial f} &:= \alpha\cdot d\\ \frac{\partial P_{100}}{\partial v} &:= 100\cdot 2\cdot\beta\cdot d\cdot v & \frac{\partial \acute{P}_{100}}{\partial v} &:= 100\cdot 2\cdot\beta\cdot d\cdot v\end{aligned}$$

As the speed in this problem is fixed, the last line can be neglected. The next step is to investigate the relationship between the rate of change of the parameter distance and of the parameter load. In this example one can observe that distance and speed in $\acute{P}_{100}$ have a higher impact on the overall outcome than the factor load, when contrasted against the original version. It is assumed that if this is the case, the implication describing the behaviour of the optimisations algorithm

$$\frac{\partial \acute{P}_{100}}{\partial d}\uparrow\Rightarrow d\downarrow\Rightarrow \frac{\partial \acute{P}_{100}}{\partial f}\downarrow\Rightarrow f\uparrow\Rightarrow \frac{\partial \acute{P}_{100}}{\partial d}\uparrow$$

is true. If the change in the derivatives is equal, as seen with $\frac{\partial P_{100}}{\partial f}$ and $\frac{\partial P_{100}}{\partial d}$, the respective changes should balance each other out. Hence, to find an appropriate indicator, which maps the differences between the behaviour of the original model and the substitute model, the relationship between the influencing factors and between the models has to be established. This can be done by utilising the dividing ratio, which describes the position of three points on a line (Havlicek, 2006). Based on the insight obtained above, it is assumed that if the position of the individual elements of the new model is at any given time in proportion to the elements of the fuel consumption model, both of them have a similar behaviour. The use of the dividing ratio is possible, since all results are real numbers. Hence, each element can be understood as a point on a line. These points are then put into relationship based on a selected origin. Formally the dividing ratio is defined as

$$\begin{aligned}(ac : bc) = t \quad &\Leftrightarrow \quad a = b \cdot t + c \cdot (1 - t) \\ &\Leftrightarrow \quad (a - c) = (b - c) \cdot t \Leftrightarrow t = \frac{a - c}{b - c}\end{aligned}$$

In this case $c := 0$. Given this the indicator

$$r_{fuel} := \frac{\frac{\partial P_{fuel}}{\partial d}}{\frac{\partial P_{fuel}}{\partial f}} \qquad r_{new} := \frac{\frac{\partial P_{New}}{\partial d}}{\frac{\partial P_{New}}{\partial f}}$$

$$R := \frac{r_{new}}{r_{fuel}} \tag{5.2}$$

is proposed. It describes the relationship between the two influencing factors of each individual model. That is, how strongly the model reacts to an increase in one of these factors. Furthermore, the factor representing the new model is then put in relation to the factor of the old model. Thereby, the movement of each individual function is compared.

Based on this indicator one can now introduce alternative objectives. The most obvious one would be the combination of distance and load. However, even this simple approach already opens up two possible options, i.e.

$$\acute{Odf} := \acute{DF} = d \cdot f \quad and \quad Odf := DF = d \cdot (f + w)$$

Before applying Equation 5.2, some analysis is to be done. The main difference between the two equations is that

$$\frac{\partial \acute{Odf}}{\partial d} := f \quad and \quad \frac{\partial Odf}{\partial d} := f + w,$$

which indicates that $\acute{Odf}$ should favour the parameter load in comparison to Odf.

This is due to the fact that

$$\frac{\partial Odf}{\partial d} \geq \frac{\partial \acute{O}df}{\partial d} \wedge \frac{\partial Odf}{\partial f} = \frac{\partial \acute{O}df}{\partial f}.$$

Therefore, suggesting that $R(\acute{O}df) \leq R(Odf)$. In order to test the behaviour of the proposed indicator a scenario similar to the ones found in this problem is created. That is, the input used to create the heat-maps (Figure 5.5 to Figure 5.8) is

$$\begin{aligned} f &:= [1; 21,320)[t] \\ d &:= [10; 1000)[km] \\ \alpha &:= 0,0981 \\ \beta &:= 5,171368125 \\ v &:= 80[km/h] \\ w &:= 19,68[t] \\ A &:= 9,8175[m^2] \\ \omega_F &:= 0,1005[l/kWh] \\ \omega_{GHG} &:= 2,664[kg/l]. \end{aligned}$$

The results are presented in form of heat maps. The heat maps titled with "Proportional Absolute" have their absolute values put in relationship with its smallest result. The ones titled with "Relative" simply represent the results of Equation 5.2. The above made assumption about $\acute{O}df$ can be observed in Figure 5.6. As seen in the proportional absolute section of the figure, an increase in the parameter load, i.e. a step to the right, leads in proportion to its minimal value to a more significant increase in output than the same step in Figure 5.7. With Figure 5.7 representing Odf, it can be conclude that in $\acute{O}df$ load has a greater influence over the overall outcome. At the same time, however, an increase in the parameter distance leads to a similar increase in both cases. Furthermore, as suspected $R(\acute{O}df) \leq R(Odf)$ is true, which can be observed in the "Relative" parts of the figures. Hence, it is proposed that Odf behaves closer to the traditional fuel consumption model, since the behaviour of the new model is closer to the behaviour of the old one, if R is closer to 1. Unfortunately, based on this reasoning Odf still tilts the favours towards the parameter load. Therefore, the incentive exists to create a model, which has an even better fit. To do so the scaling of the parameter within the model has to be adjusted, thus it is wise to go back to the partial derivatives of function P and the Equation 5.2. That is,

$$\begin{aligned} r_{fuel} &= \frac{\alpha(f+w)+\beta\cdot v^2}{\alpha\cdot d} \\ &= \frac{\alpha(f+w+\frac{\beta}{\alpha}\cdot v^2)}{\alpha\cdot d} \\ &= \frac{f+w+\frac{\beta}{\alpha}\cdot v^2}{d} \\ &\Rightarrow P := \alpha\cdot d\cdot(f+w+\frac{\beta}{\alpha}v^2) \end{aligned}$$

Given this one only has to properly define the factor

$$\frac{\beta}{\alpha} = \frac{a+g\cdot sin(\theta)+g\cdot \acute{c_r}\cdot sin(\theta)}{0.5\cdot \acute{c_d}\cdot A\cdot\rho_{air}} = k.$$

The first step in doing so is to define the usual limits of each parameter. Firstly, since a flat world and no acceleration are assumed θ and a are constant. The same can be said for g. In contrast parameter A does indeed change, however, due to the fact that it is fairly easy to determine the surface area of the truck, a compromise will be made and A is included as variable input, i.e. $\frac{\beta}{\alpha} = k\cdot A \Rightarrow \frac{\beta_{\neg A}}{\alpha} = k$. Hence, the only parameter remaining are ρ_{air}, $\acute{c_d}$ and $\acute{c_r}$. Unfortunately, the case does not provide information about the average ρ_{air} in Austria, thus it has to be calculated.[4] Table 5.7 shows the lower and the upper bound, as well as the average values of each required parameter. The average values for temperature and altitude are calculated based on Table 4.18 and Table 4.16, while the pressure for each extreme and for the average was obtained by means of linear-interpolation based on the values found in Table 4.17 [5]. Furthermore, the values for $\acute{c_r}$ are taken from ToolBox (2016b)

4

$$\rho_{air} = \frac{p_\rho}{R_{air_\rho}\cdot T_\rho} \quad R_{air_\rho} := 287,058\,[J/kg\cdot K] \;\; and\; T_\rho := 273,15 + C^\circ[K]$$

(Wikipedia, 2016a)

5

$$y = y_0\cdot\frac{x_1-x}{x_1-x_0} + y_1\cdot\frac{x-x_0}{x_1-x_0}$$

(Wikipedia, 2016f)

and can be found in Table 4.19, while the ones for $\acute{c_d}$ are taken from Akcelik and Besley (2003) and are visible in Table 4.20. Moreover, since only medium/heavy trucks, heavy trucks and heavy articulated are used $\acute{c_d}$ the range can easily be limited. By calculating the ratio between $\frac{\beta_{\neg A}}{\alpha}$, with $\beta_{\neg A} := 0.5 \cdot \acute{c_d} \cdot \rho_{air}$ for every

Limits	Temperature [$C°$]	Altitude [m]	$\acute{c_r}$	$\acute{c_d}$	Pressure Pa
AV	7,20833333	361,25	0,008	0,815	96965
MIN	-2,1	151	0,006	0,77	99490
MAX	16,5	574	0,01	0,86	94412

Table 5.7: Problem 2 - Limits and Averages

permutation of the above values, the lower and upper bounds can be defined. That is, $\frac{\beta_{\neg A}}{\alpha} = [4,46\ ;\ 9,34]$ with its average being at $6,26$. In this case it was chosen to conform to the average, thus $k \simeq 6,26$. Having k defined, the new equation can be constructed as

$$\check{O}df := d \cdot (f + w + 6,26 \cdot A \cdot v^2) \simeq d \cdot (f + w + 1,75 \cdot A \cdot v^2) \quad v = [km/h] \quad (5.3)$$

Similar to the previous objectives, its relationship with the fuel consumption model can be described through Equation 5.2. As apparent through Figure 5.8, this equation is behaviour wise a close approximation of the fuel consumption model.

Anyhow, $\check{O}df$ may be a more precise approximation, however, for the sake of simplicity the objective Odf is selected for further analysis, as it provides reasonable improvements, while fulfilling the minimum information requirement. Hence the last implementation

$$\mathscr{I}_{Odf} := Implementation(Distance \cdot Load, Speed_{Fixed}, TW_{False})$$

can be defined. The now complete set of implementations has to be executed for its respective scenarios, thus generating the data found in Table 5.8. Its structure is in general the same as the one found in Table 5.1. However, in this case the results produced by $\mathscr{I}_{Oe}(h)$ and $\mathscr{I}_{Odf}(h)$ are merged into one column, as both of them generated the same sequence of stops. Thus partly supporting the claim that the behaviour of the new objective has more similarities with Oe than the other two.

The first step is to compare the costs and the emissions produced during a trip. Hereby, the values produced by $\mathscr{I}_{Od}$ and/or $\mathscr{I}_{Of}$ are contrasted against the ones

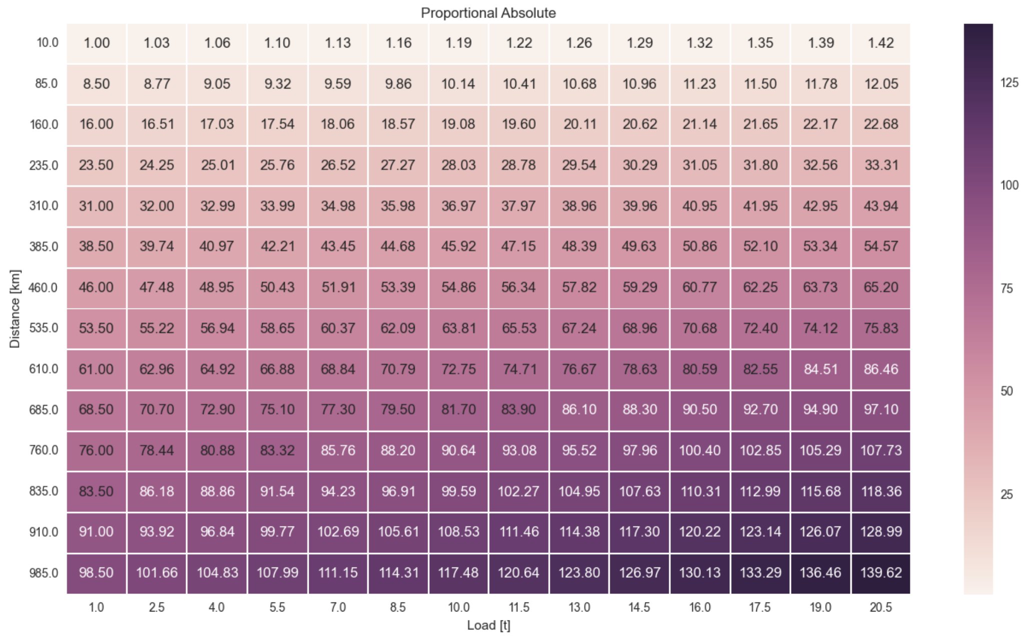


Figure 5.5: Problem 2 - Result of P

created by $\mathscr{I}_{Odf}$. That is, similarly to Section 5.1

$$
\begin{aligned}
\Delta P_h &:= P_{\mathscr{I}_{Odf}(h)_X} - P_{\mathscr{I}_x(h)_X} \\
\Delta C_h &:= C_{\mathscr{I}_{Odf}(h)_X} - C_{\mathscr{I}_x(h)_X} \\
\Delta P_h[\%] &:= \frac{P_{\mathscr{I}_{Odf}(h)_X}}{P_{\mathscr{I}_x(h)_X}} - 1 = \frac{\Delta P_{\mathscr{I}_x(h)_X}}{P_{\mathscr{I}_x(h)_X}} \\
\Delta C_h[\%] &:= \frac{C_{\mathscr{I}_{Odf}(h)_X}}{C_{\mathscr{I}_x(h)_X}} - 1 = \frac{\Delta C_{\mathscr{I}_x(h)_X}}{C_{\mathscr{I}_x(h)_X}}
\end{aligned}
$$

with $x \in \{Od, Of\}$ and X being the respective company. Based on Table 5.9, which contains the above defined values, it becomes apparent that switching to the new objective would not only decrease the overall GHG output, but also lower the costs of both companies. For Company A the costs would reduce dramatically. That is, in the given scenarios the company would save on average $615,39[Euro]$ or $30,4[\%]$. Nevertheless, it has to be mentioned that the outlier $\mathscr{I}_{Of}(TS2)_A$, dramatically increases the average savings of Company A. By comparison, Company B does not experience such a massive decrease in cost. However, it is for Company B still possi-

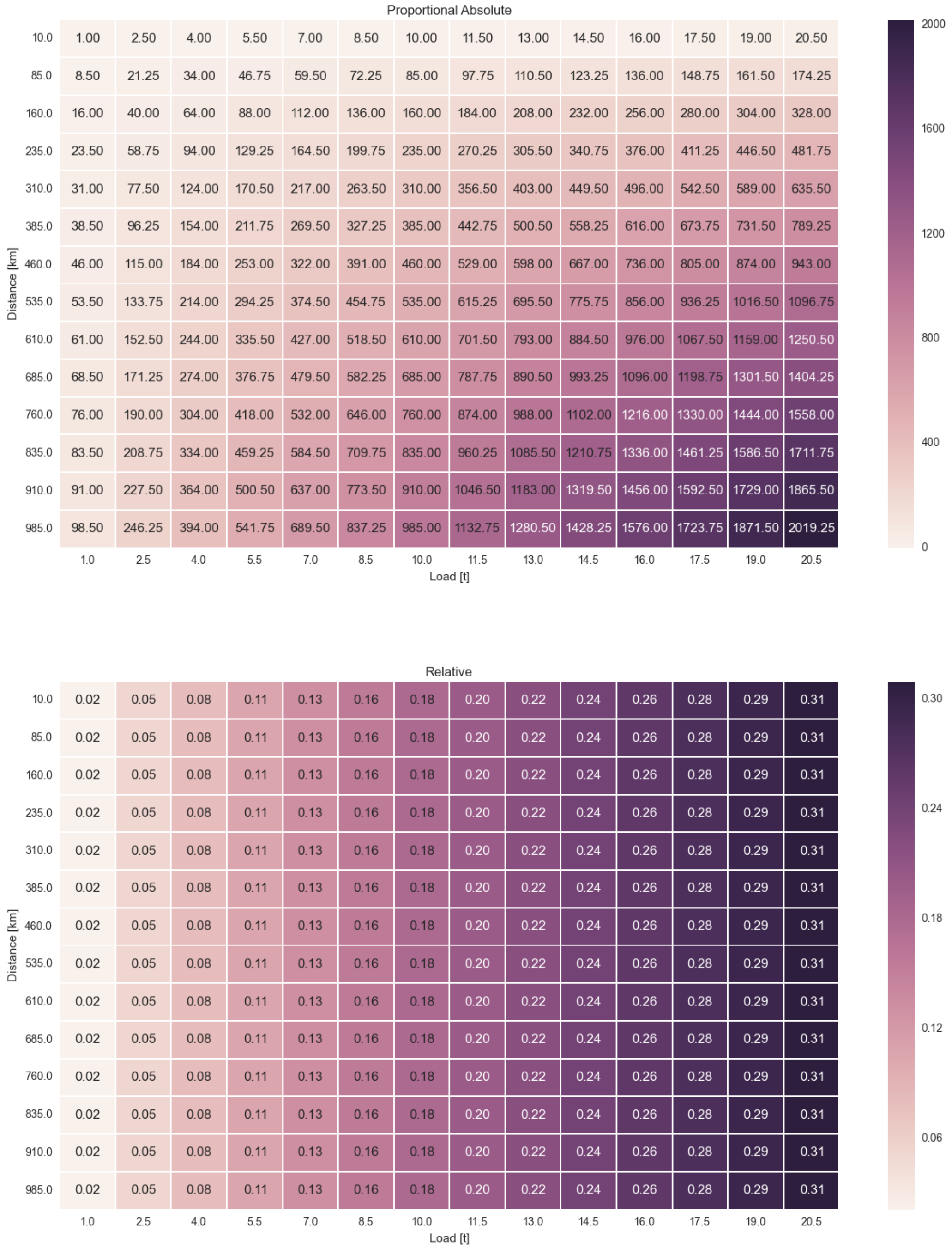


Figure 5.6: Problem 2 - Results of $\acute{O}df$

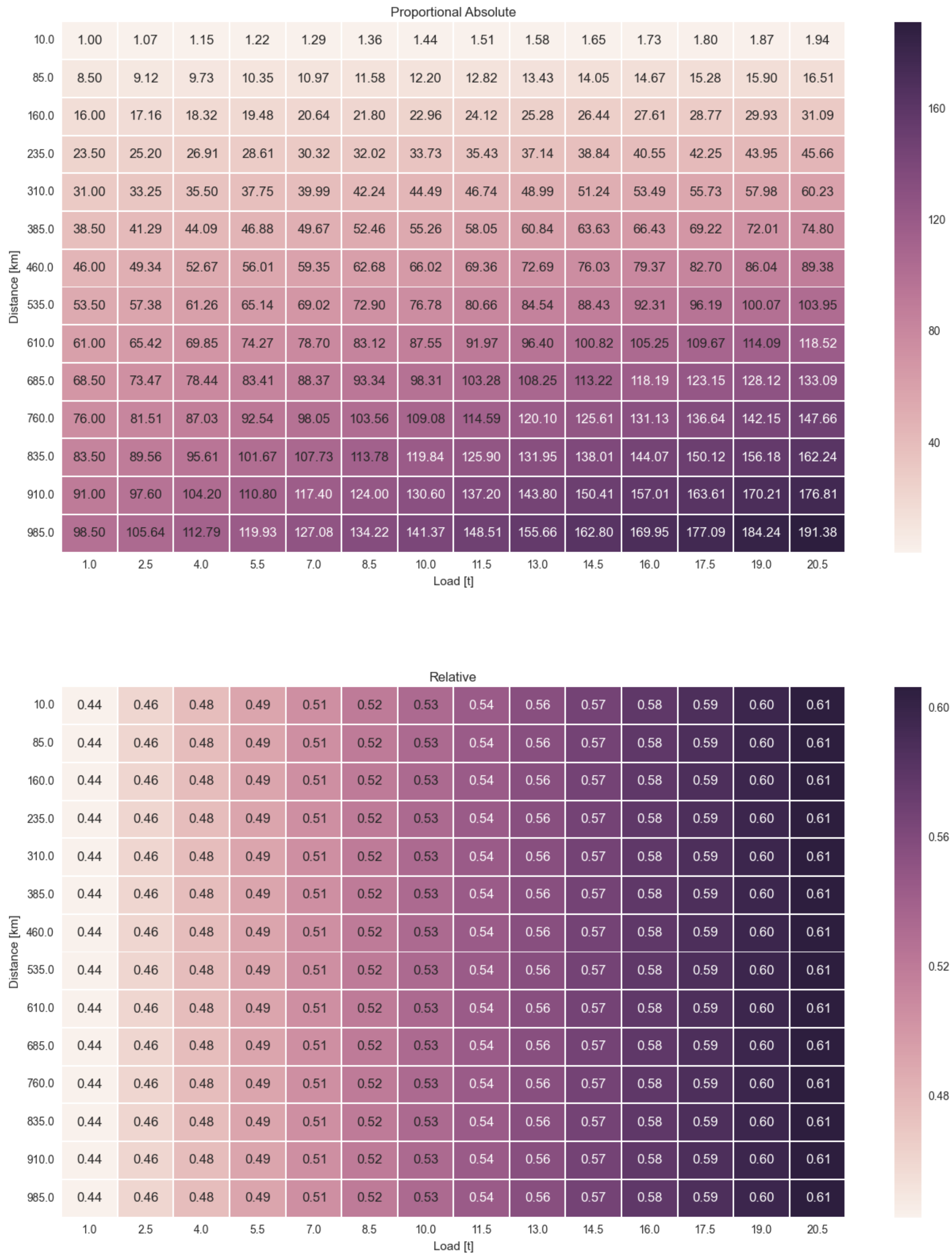
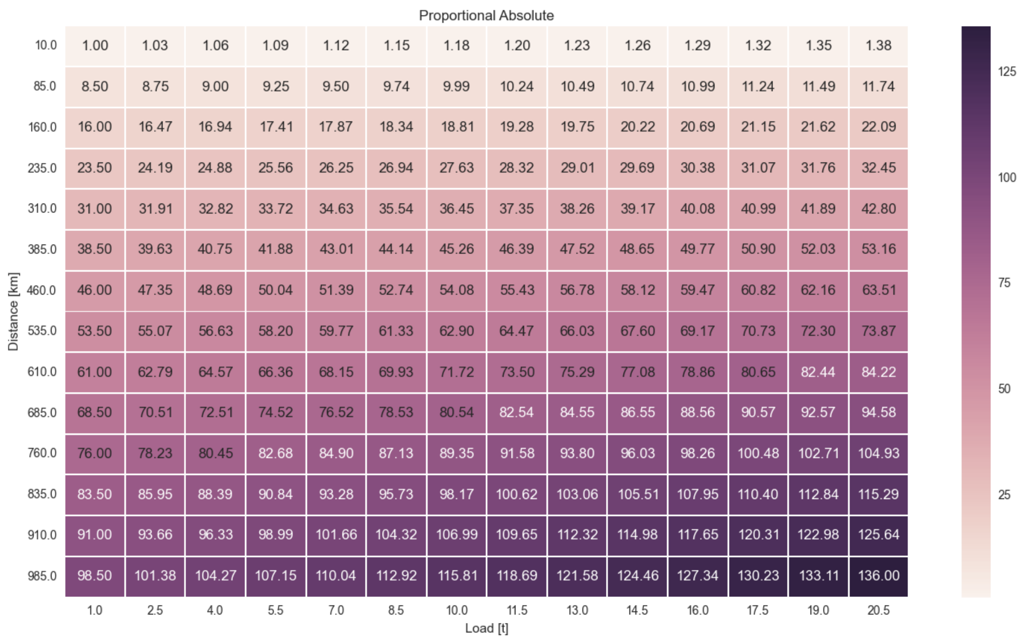


Figure 5.7: Problem 2 - Results of Odf



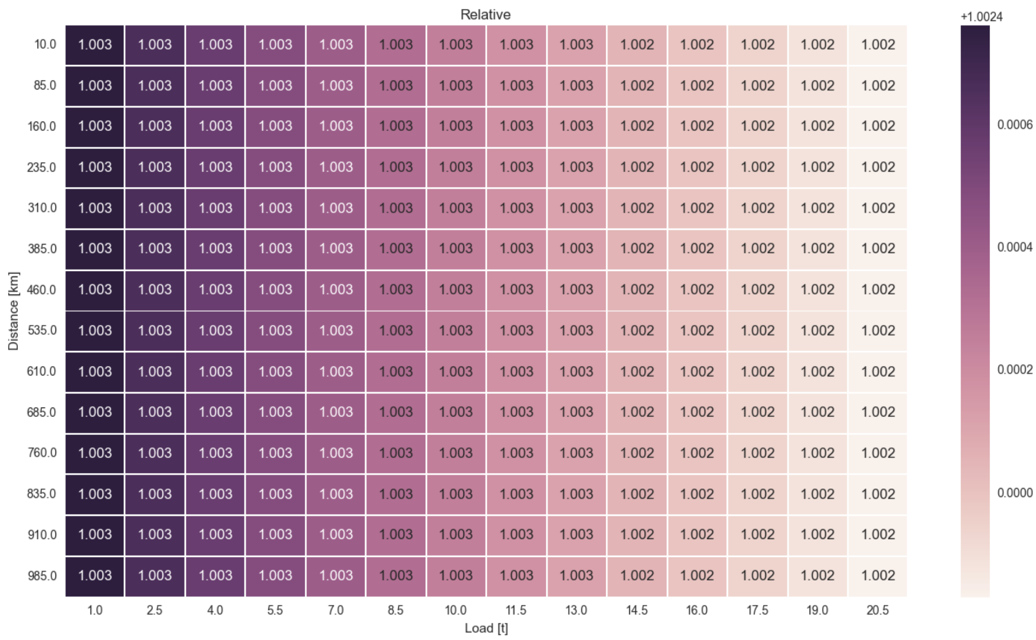


Figure 5.8: Problem 2 - Results of $\check{O}df$

Company A	$\mathscr{I}_{Of}(TS1)$	$\mathscr{I}_{Odf}(TS1)$	$\mathscr{I}_{Of}(TS2)$	$\mathscr{I}_{Odf}(TS2)$	$\mathscr{I}_{Of}(TS3)$	$\mathscr{I}_{Odf}(TS3)$	$\mathscr{I}_{Of}(BO66)$	$\mathscr{I}_{Odf}(BO66)$
Path	0-1-6-5-4-3-2-0	0-6-1-2-3-4-5-0	0-1-3-2-5-4-6-0	0-3-4-2-1-6-5-0	0-6-5-3-2-4-1-0	0-5-6-3-4-2-1-0	0-3-5-4-6-2-1-0	0-4-3-6-5-2-1-0
$D[km]$	1272,46	1127,13	5460,11	2979,74	3452,91	2818,68	817,62	649,44
$F[t]$	37,07	47,98	50,00	64,96	55,17	56,94	44,55	57,01
$P[kg]$	1455,05	1292,41	6703,55	3774,86	4145,38	3412,74	965,90	771,90
$C[Euro]$	895,08	794,67	4076,32	2284,24	2529,75	2079,83	590,99	471,82
$Pw[kg]$	571,52	506,25	2452,38	1338,34	1550,86	1266,00	367,23	291,69
$Pf[kg]$	127,54	116,52	1007,22	666,21	543,09	472,11	112,91	94,37
$Pv[kg]$	755,99	669,65	3243,95	1770,32	2051,44	1674,63	485,76	385,84
$Cd[Euro]$	149,99	132,86	643,61	351,24	407,01	332,25	96,38	76,55
$Cf[Euro]$	737,38	654,96	3397,18	1913,00	2100,77	1729,49	489,49	391,18
$Ce[Euro]$	7,71	6,85	35,53	20,01	21,97	18,09	5,12	4,09
$T_{0j}[h]$	18,67	16,85	70,11	39,11	44,99	37,06	12,98	10,88
Company B	$\mathscr{I}_{Od}(TS1)$	$\mathscr{I}_{Odf}(TS1)$	$\mathscr{I}_{Od}(TS2)$	$\mathscr{I}_{Odf}(TS2)$	$\mathscr{I}_{Od}(TS3)$	$\mathscr{I}_{Odf}(TS3)$	$\mathscr{I}_{Of}(BO66)$	$\mathscr{I}_{Odf}(BO66)$
Path	0-5-4-3-2-1-6-0	0-6-1-2-3-4-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-1-2-4-3-6-5-0	0-5-6-3-4-2-1-0	0-3-6-5-2-4-1-0	0-4-3-6-5-2-1-0
$D[km]$	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	642,97	649,44
$F[t]$	93,78	47,98	64,96	64,96	86,45	56,94	53,59	57,01
$P[kg]$	1509,97	1239,01	3612,62	3612,62	3620,08	3268,56	744,90	739,53
$C[Euro]$	906,07	767,32	2201,16	2201,16	2186,00	2006,00	457,23	455,25
$Pw[kg]$	489,30	489,30	1293,53	1293,53	1223,62	1223,62	279,12	281,93
$Pf[kg]$	380,62	109,66	627,02	627,02	795,85	444,34	100,67	88,82
$Pv[kg]$	640,05	640,05	1692,06	1692,06	1600,61	1600,61	365,11	368,79
$Cd[Euro]$	132,86	132,86	351,24	351,24	332,25	332,25	75,79	76,55
$Cf[Euro]$	765,21	627,90	1830,78	1830,78	1834,56	1656,42	377,50	374,78
$Ce[Euro]$	8,00	6,57	19,15	19,15	19,19	17,32	3,95	3,92
$T_{0j}[h]$	16,85	16,85	39,11	39,11	37,06	37,06	10,80	10,88

Table 5.8: Problem 2 - Output

ble to save $80,18[Euro]$ or $5,6[\%]$. A similar picture arises when the GHG output is observed. That is a massive decrease in emissions leads to a similar decrease in savings. Hence, for Company-A the GHG output decreases by $1004,49[kg]$ or $30,3[\%]$, while for Company-B that means a reduction by $156,96[kg]$ or $6,6[\%]$. The same behaviour can be observed in Figure 5.9, which contrasts the initial costs and GHG output (darker color) with the output of the new objective (lighter color). Hereby, the left side depicts the emissions, while the right side compares the costs.

The reason behind the relationship between the cost function and fuel consumption model will be investigated in depth in Section 5.4. However, generally speaking the fuel costs and emission costs are directly related to the fuel consumption of the vehicle, which is in itself is related to the overall energy consumption of the vehicle. Moreover, as implied by Figure 5.10 this Cf is by far the most influential one, thus explaining the relationship between the P and C in a very crude manner. The actual values on the basis of which Figure 5.10 was constructed can to some extend be found in Table 5.10. It simply compares the individual components of the parameter cost between the initial objective, which is used as a basis for the calculations, and the new objective.

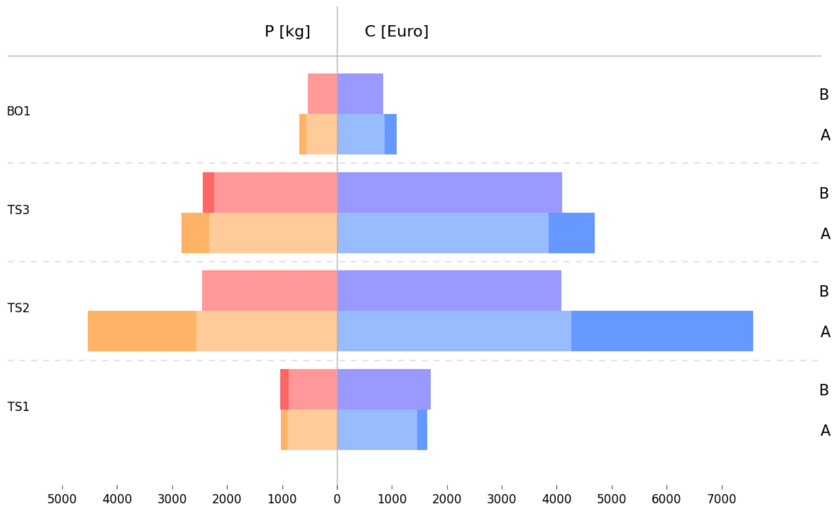


Figure 5.9: Problem 2 - General Comparison

Company A	$\Delta P[kg]$	$\Delta P[\%]$	$\Delta C[Euro]$	$\Delta C[\%]$
TS1	-162,63	-11,2%	-100,41	-11,2%
TS2	-2928,69	-43,7%	-1792,08	-44,0%
TS3	-732,64	-17,7%	-449,92	-17,8%
BO66	-194,00	-20,1%	-119,16	-20,2%
Average	-1004,49	-30,3%	-615,39	-30,4%
Company B	$\Delta P[kg]$	$\Delta P[\%]$	$\Delta C[Euro]$	$\Delta C[\%]$
TS1	-270,96	-17,9%	-138,75	-15,3%
TS2	0,00	0,0%	0,00	0,0%
TS3	-351,51	-9,7%	-180,00	-8,2%
BO66	-5,37	-0,7%	-1,99	-0,4%
Average	-156,96	-6,6%	-80,18	-5,6%

Table 5.9: Problem 2 - General comparison

By comparing the different cost components, it becomes apparent that scenario $BO66$ is the most interesting one, because it is the only one where ΔCd is positive, i.e. it increased. A behaviour which is especially interesting since in every other case the overall cost declined. This is due to the fact that $D_{\mathscr{I}_{Od}(BO66)} < D_{\mathscr{I}_{Odf}(BO66)}$. However, this slight increase in cost was easily compensated by the reduction in energy consumption. Furthermore, as apparent in Figure 5.1, Figure 5.2, Figure 5.3 and Figure 5.11 the new objective has a significant impact on J. Especially $J_{\mathscr{I}_{Of}(h)}$

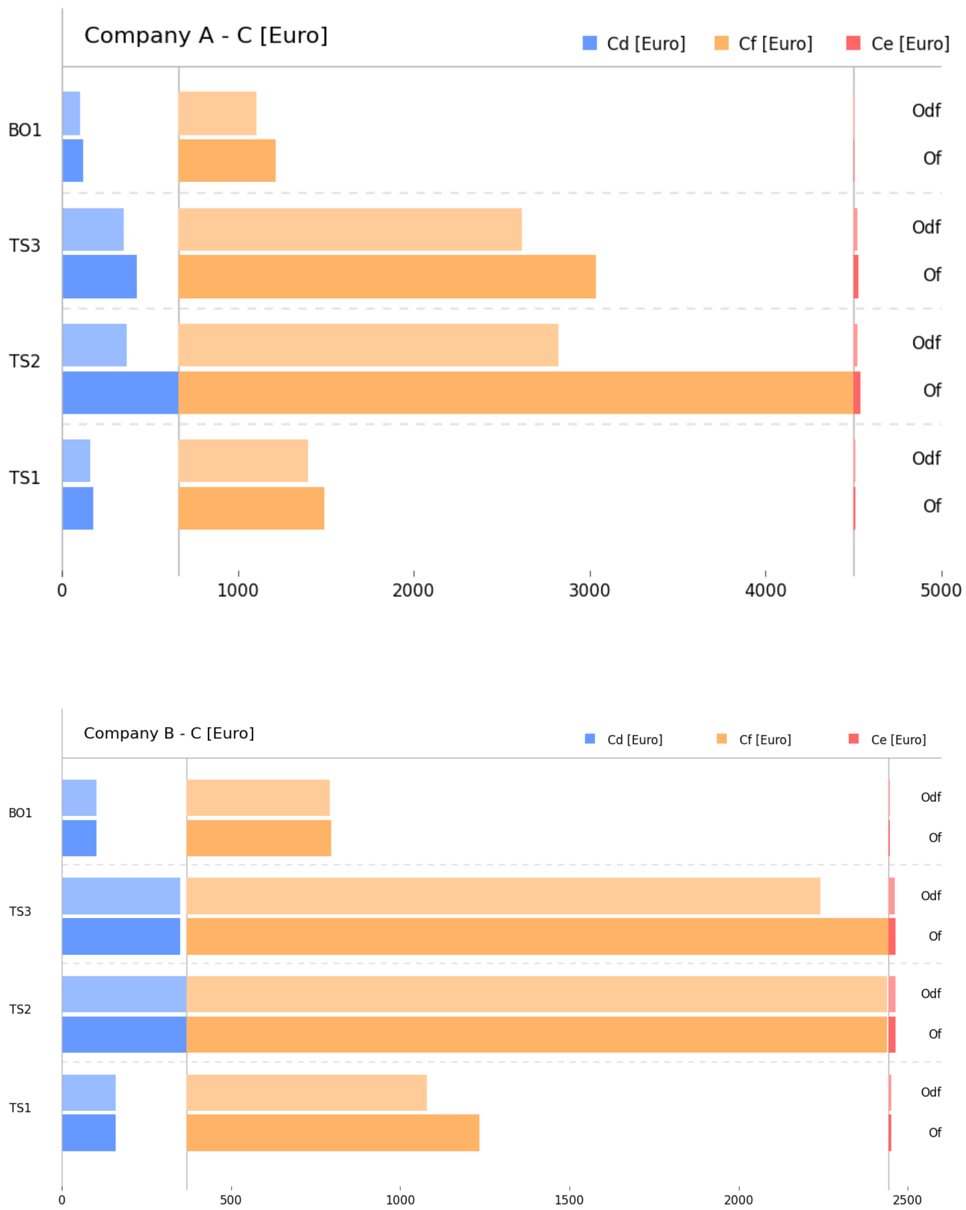


Figure 5.10: Problem 2 - Cost Comparison

Company A	$\Delta Cd[Euro]$	$\Delta Cf[Euro]$	$\Delta Ce[Euro]$	$\Delta Cd[\%]$	$\Delta Cf[\%]$	$\Delta Ce[\%]$
TS1	-17,13	-82,42	-0,86	-11,4%	-11,2%	-11,2%
TS2	-292,37	-1484,18	-15,52	-45,4%	-43,7%	-43,7%
TS3	-74,76	-371,28	-3,88	-18,4%	-17,7%	-17,7%
BO66	-19,82	-98,31	-1,03	-20,6%	-20,1%	-20,1%
Average	-101,02	-509,05	-5,32	-31,2%	-30,3%	-30,3%
Company B	$\Delta Cd[Euro]$	$\Delta Cf[Euro]$	$\Delta Ce[Euro]$	$\Delta Cd[\%]$	$\Delta Cf[\%]$	$\Delta Ce[\%]$
TS1	0,00	-137,31	-1,44	0,0%	-17,9%	-17,9%
TS2	0,00	0,00	0,00	0,0%	0,0%	0,0%
TS3	0,00	-178,14	-1,86	0,0%	-9,7%	-9,7%
BO66	0,76	-2,72	-0,03	1,0%	-0,7%	-0,7%
Average	0,19	-79,54	-0,83	0,1%	-6,6%	-6,6%

Table 5.10: Problem 2 - Detailed cost comparison

experienced massive changes. These changes are mostly caused by the inclusion of distance into the model. Thereby, the massive distance caused by highly convoluted trips was no longer justified, as it would lead to a massive increase in $d_{h_{\bar{j}}} \cdot f_{h_{\bar{j}}}$. By contrast, $J_{\mathscr{I}_{Od}}$ mainly experienced an inversion of its path. Hence, the flaw identified in Section 5.1 could be eradicated with the new objective.

Overall, the results suggest that switching to the new objective is beneficial for either company. As it not only provides an overall cost reduction, but also decreases the energy consumption. Hence, the energy-efficiency of the company increases. A final an even more compelling argument can be made by looking at the profit margins of the respective company. Given the prices introduced in Section 5.1, i.e.

$$\begin{aligned} \acute{p}_{A1} &:= 30[Euro/t] \\ \acute{p}_{A2} &:= 60[Euro/t \\ \acute{p}_{B} &:= 1,2[Euro/km], \end{aligned}$$

with $\acute{p}_{A1}$ being for domestic trips and $\acute{p}_{A2}$ for international ones, the profit margins as seen in Table 5.11 can be calculated. Here it becomes apparent that the pricing strategy of Company-A is in combination with its optimisation objective fairly suboptimal. As given the conventional approach the margin of Company-A in $TS2$ would be $-1076,09[Euro]$. The same scenario with the new objective

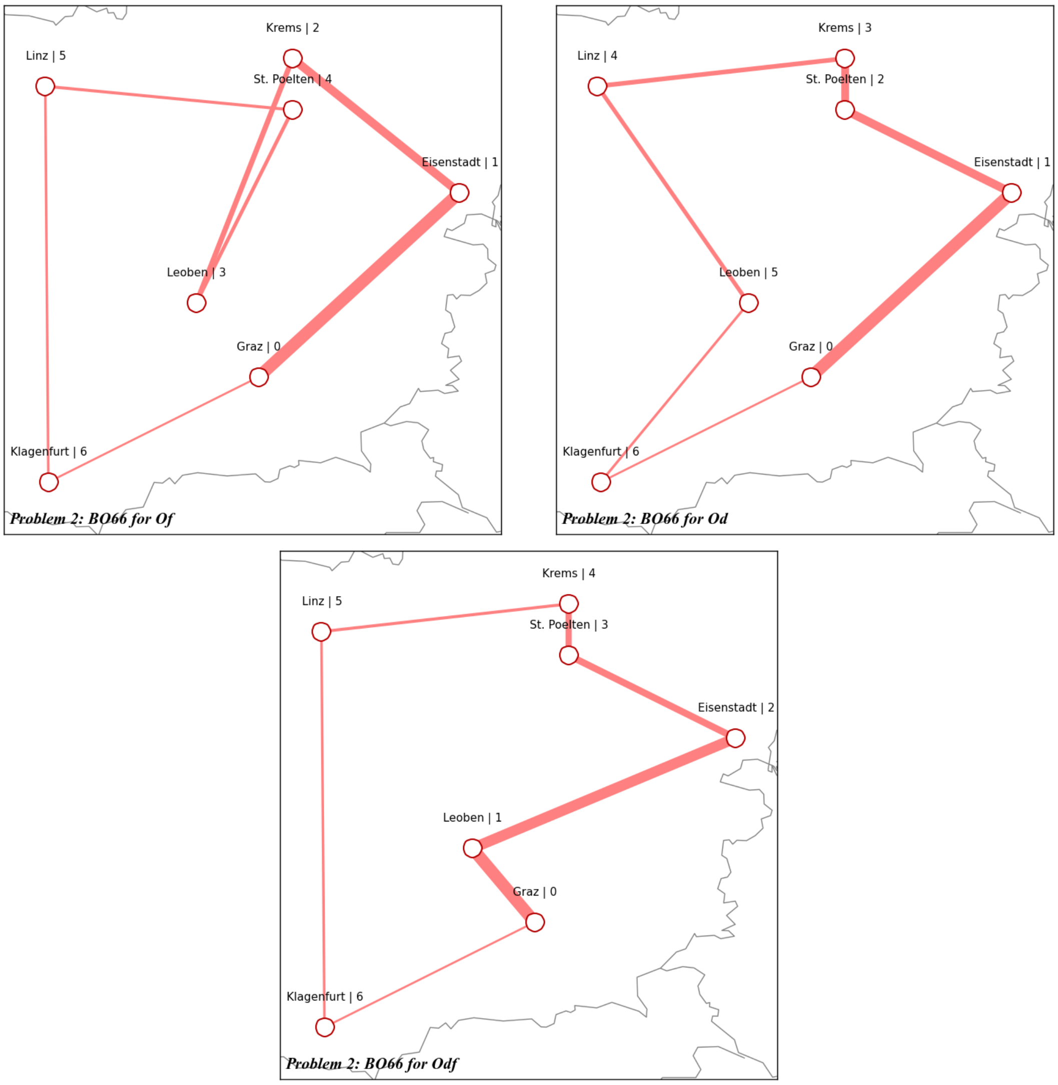


Figure 5.11: Problem 2 - Maps for BO66

Company A [$Euro$]	$Income(Of)$	$Income(Odf)$	$Margin(Of)$	$Margin(Odf)$	$\Delta Margin$	
TS1	1112,23	1439,36	217,15	644,69	427,54	196,9%
TS2	3000,23	3897,49	-1076,09	1613,25	2689,34	-249,9%
TS3	3310,46	3416,11	780,70	1336,28	555,58	71,2%
BO66	1336,55	1710,41	745,56	1238,59	493,02	66,1%
Average	2189,87	2615,84	166,83	1208,20	1041,37	624,2%
Company B [$Euro$]	$Income(Of)$	$Income(Odf)$	$Margin(Of)$	$Margin(Odf)$	$\Delta Margin$	
TS1	1352,56	1352,56	446,48	585,23	138,75	31,1%
TS2	3575,69	3575,69	1374,53	1374,53	0,00	0,0%
TS3	3382,42	3382,42	1196,42	1376,42	180,00	15,0%
BO66	771,56	779,33	314,33	324,08	9,76	3,1%
Average	2270,56	2272,50	832,94	915,07	82,13	9,9%

Table 5.11: Problem 2 - Profit margins

would provide a profit margin of $1613,25[Euro]$, thus amounting to an increase in profits of $2689,34[Euro]$. In general the values generated do only further solidify the arguments presented above. As on average the profit margin would increase by $1041,37[Euro]$ or $624,2[\%]$ for Company-A and by $82,13[Euro]$ or $9,9[\%]$ for Company-B.

In conclusion, it is economically and ecologically advantageous to reduce the drawbacks of the individual objectives load and distance by combining them together into one objective. Thereby, one of the trade-offs made by the fuel consumption model can roughly be approximated. However, due to its nature, this objective still dramatically oversimplifies the problem at hand. Hence, leading to a distortion of the relationship between D and F as indicated by Equation 5.2. Furthermore, the new objective completely ignores the factor V. Nevertheless, due to the fact that it performs relatively well, even though only a marginal amount of information is required, such drawbacks can be tolerated.

5.3 Problem Three

The focus of Problem Three is split, i.e. two individual aspects are investigated. The first part of section intents to create an understanding of the how differences between vehicles influences the overall outcome. This part also tries to investigate methods for assessing the merit of a vehicle without a concrete VRP to solve. To do so two additional vehicles are introduced into the narrative, which have to be contrasted against each other and against the vehicle used by the transport-company. The

narrative then requires that one vehicle is chosen for the next part. The second part, focuses on contrasting the results obtained by in-sourcing the delivery process against the ones obtained via the existing solution.
As usual, the available information has to be defined. Firstly the parameter groups *World* and *Product* stay the same, i.e.

$$\begin{aligned}\mathscr{W}_{P3} &:= \mathscr{W}_1.\\ \mathscr{P}_{P3} &:= \mathscr{P}_1\end{aligned}$$

Furthermore, the first instance of the parameter group *Vehicle* is locked based on the decision made in Problem One and is therefore

$$\mathscr{V}_{P3_E} := \begin{cases} \mathscr{V}_A & \textit{If Company A was selected} \\ \mathscr{V}_B & \textit{If Company B was selected} \end{cases}$$

Due to the differences between the two vehicle, this section will include both vehicles in its analysis. Therefore, the notation $\mathscr{V}_A$ and $\mathscr{V}_B$ is used. Fortunately, company specific objectives Of and Od have been harmonised in Problem Two (see Section 5.2). However, apart from that two additional instances of $\mathscr{V}$ have to be defined. That is, $\mathscr{V}_{P3_C} := \mathscr{V}_C$ and $\mathscr{V}_{P3_D} := \mathscr{V}_D$ are defined as

$$\begin{aligned}
K_{v_C} &:= 93[m^3] & K_{v_D} &:= 75,9[m^3]\\
w_C &:= 17940[kg] & w_D &:= 12250[kg]\\
K_{w_C} &:= 41000[kg] & K_{w_D} &:= 18000[kg]\\
A_C &:= 9,8175[m^2] & A_D &:= 9,58[m^2]\\
\eta_C &:= 0,36[\%] & \eta_D &:= 0,2[\%]\\
\acute{c}_{d_C} &:= 0,86 & \acute{c}_{d_D} &:= 0,77\\
\omega_{GHG_C} &:= 2,664[kg/l] & \omega_{GHG_D} &:= 2,3215[kg/l].\\
\omega_{F_C} &:= 0,1005[l/kWh] & \omega_{F_D} &:= 0,1132[l/kWh].
\end{aligned}$$

The first focus point of this section requires no additional information. This is due to the fact that it is intended to assess the advantages of a vehicle in general, i.e.

not bound to any trip. Such an approach is not only more advantageous, since it allows the formulation of a more inclusive statement, but also necessary since the implementation presented in the guideline (see Section 3.2) does not allow multiple trips. Hence, the trip specific parameter are defined after the analysis of the first focus point. Speaking of which, the first step in comparing the different vehicle is to identify what parameter are influenced if the vehicle is changed.

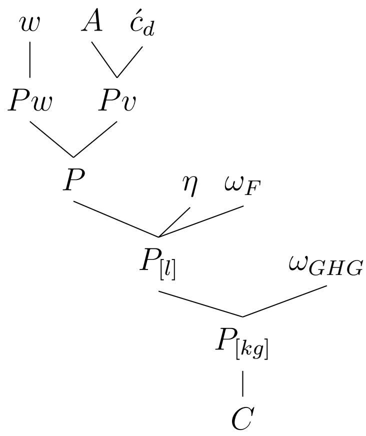


Figure 5.12: Problem 3 - Impact of the vehicle

The tree presented above provides an overview over the influence a vehicle parameter has on the GHG output. Firstly, the parameter involved in converting the required energy into GHG emissions can be compared easily. To do so every parameter that influenced by the vehicle after the calculation of P is combined into one, i.e.

$$conv := \frac{\omega_F}{\eta} \cdot \omega_{GHG}.$$

By computing this equation one can conclude that given the same energy requirement i.e. $P_{\mathcal{V}_A} = P_{\mathcal{V}_B} = P_{\mathcal{V}_C} = P_{\mathcal{V}_D}$, Truck C will always be the most preferable choice. This is due to the fact that

$$\begin{aligned} conv_A &= 0,8369 \\ conv_B &= 0,7876 \\ conv_C &= 0,7439 \\ conv_D &= 1,3140, \end{aligned}$$

which indicates that $P_{[kg]_C} = P_{[kWh]_C} \cdot conv_C = 0,944 \cdot P_{[kg]_B}$, with $P_{[kg]_B}$ being the vehicle with the second best conversion factor. As for the comparison between $P_{[kg]_C}$ and $P_{[kg]_D}$ this difference amounts to $P_{[kg]_C} = 0,566 \cdot P_{[kg]_D}$. Therefore, indicating that P_D has to be significantly lower than P_C in order to provide a better result than $\mathscr{V}_C$. Hence, the next step is to test, if the other influencing parameter result in such an overall reduction.
In order to test the influence of the vehicle on the raw energy consumption the other two branches have to be explored. The most intuitive step would be to calculate P for every vehicle under the same conditions. However, due to the variations in carrying capacity it is not as straight forward as it may seem. For example, a vehicle might have a lower curb weight, which can be an indicator for lower carrying capacity (Bektaş and Laporte, 2011). If this is the case it is conceivable that in order to serve the same amount of customer, the vehicle has to travel back to the terminal point. Thereby, additional distance has to be travelled, which could mitigate eventual benefits of a lower curb weight. Unfortunately, due to the limitations of the implementation, it is not possible to accurately test this. By contrasting the carrying capacity of each vehicle against the other ones, it is apparent that such a case, i.e. huge variations in carrying capacity, is given (see Table 5.12 with $K = min(K_v, K_w)$). Therefore, as indicated above a simple comparison becomes difficult.

Capacity $[Units]$	Truck A	Truck B	Truck C	Truck D
K_v	47458	50688	47458	38732
K_w	34216	33365	37008	9228
K	34216	33365	37008	9228

Table 5.12: Problem 3 - Vehicle capacity

Hence, three ideas are proposed. The first approach suggest to simply ignore the limitations resulting out of the carrying capacity, by simply plotting the fuel consumption function up to $f = min((K_i)_{i \in X})$ with $X := \{\mathscr{V}_A, \mathscr{V}_B, \mathscr{V}_C, \mathscr{V}_D\}$. By doing so one favours the vehicle with the lowest capacity, because of the correlation between curb weight and carrying capacity (Bektaş and Laporte, 2011). The following figures, i.e. Figure 5.13, Figure 5.14 and Figure 5.15, were obtained by using the values

$$\begin{aligned}
\alpha &:= 0,0981\\
\rho_{air} &:= 1.225\\
D_{\Delta} &:= [1,100)[km]\\
F_{\Delta} &:= [1;5,75][kg]\\
V_{\Delta} &:= [30,130)[km/h]\\
d &:= 100[km]\\
f &:= 5,75[kg]\\
v &:= 80[km/h].
\end{aligned}$$

The only missing values are the vehicle specific values. Based on Figure 5.13 it can easily be deduced that without converting the energy consumption into GHG output, $\mathscr{V}_D$ is superior. This is due to the fact that $\acute{c}_d$, w and A are smaller in $\mathscr{V}_D$ than in any other vehicle. However, by applying the conversion factor calculated above, this advantage is easily negated. In this case $\mathscr{V}_C$ provides the best result, as it has not only the best conversion factor, but also has the second best vehicle specific parameter. Unfortunately, due to the fact that apart form $\mathscr{V}_D$ every other vehicle only uses a significantly smaller proportion of its carrying capacity, the results are distorted in favour of $\mathscr{V}_D$. As a vehicle which operates at full capacity has the lowest curb weight-total weight ratio, i.e. $\frac{F}{w}$. Therefore, the second approach intents to investigate the merit of a vehicle in relation to its maximal carrying capacity.

The second approach was developed out of the idea of assessing the proportion between the curb weight of the vehicle and the load. For example, given $w = 5000[kg]$ and $f = 5000[Unit]$ would indicate that $1[kg/Unit]$ of vehicle is required to deliver one unit product. If the case arises that the actual carrying capacity of the vehicle is $K = 10000[Unit]$ the ratio would drop to $0,5[kg/Unit]$, thus its "weight-efficiency" would increase, as less weight would be required to deliver one unit of product. A similar argument can be constructed for P. That is, how much energy is required to carry one product may be expressed as

$$P_{\frac{1}{f}} := \frac{P}{f}.$$

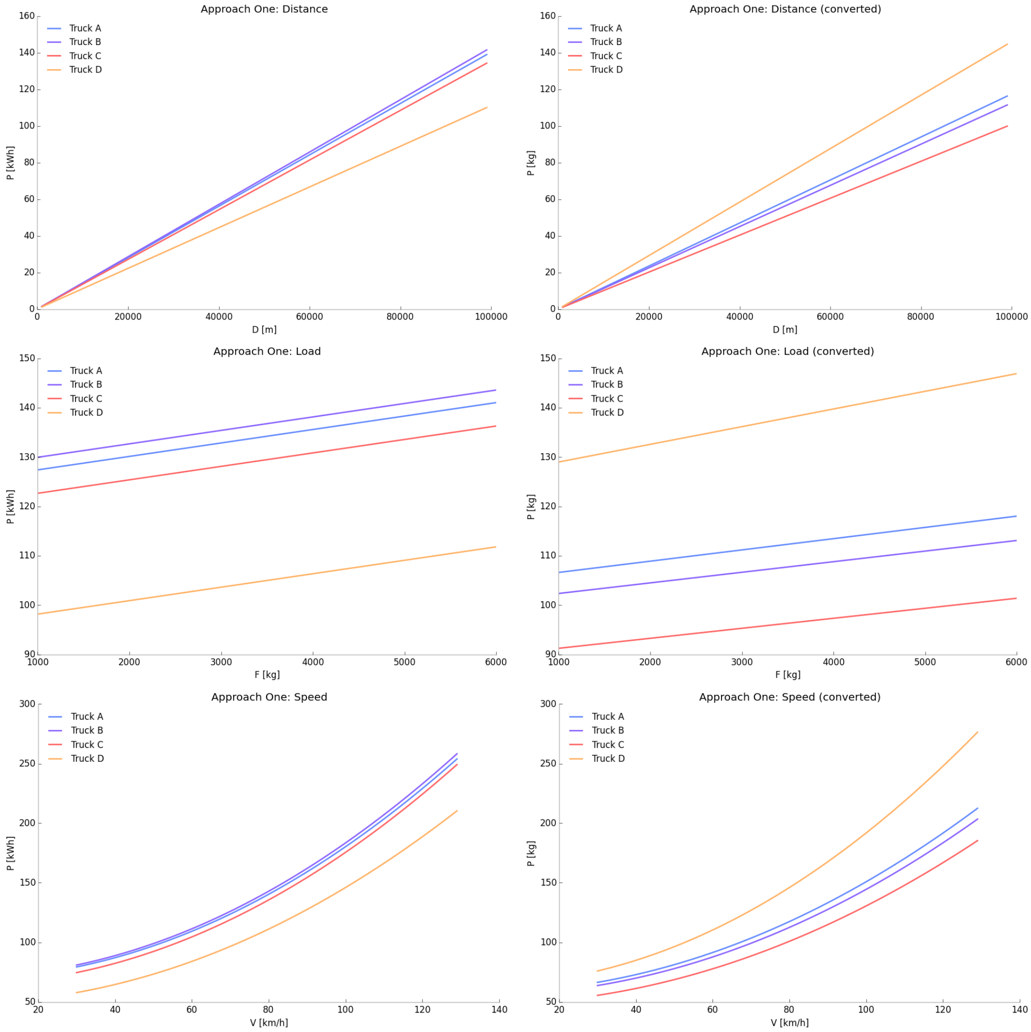


Figure 5.13: Problem 3 - First approach

Naturally, the parameter changes based on the amount of carried products. Hence, its values have to be investigated based on the range $F := [0, K_{\mathscr{V}_x})$, which results in the graphs found in Figure 5.14. The most interesting deduction to make is that even without the conversion factor $\mathscr{V}_D$ performs worst in comparison to the other vehicle. Said behaviour can partially be attributed to the unfortunate ratio between its curb weight and its capacity, which is at its best, i.e. maximal utilisation, at $1,33[kg/Unit]$. This, if compared against the ratio of $\mathscr{V}_C$, which is $0,48[kg/Unit]$, seems fairly inefficient. However, while this approach is able to express the proportional efficiency of the vehicle, it can not account for the changes in behaviour, which may result from a low carrying capacity.

The last approach tries to model such behaviour, by simulating a single arc with one terminal point and one customer. Given this and the premiss that if the demand is higher than the capacity of the vehicle, i.e. $Q > K$, multiple trips are required the equation

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} d \cdot (\alpha \cdot (f_i + w) + \beta \cdot v^2)$$

can be created. In this case n reflects the amount of times the arc has to be crossed in order to fulfil the demand. However, as on every second crossing, i.e. the way back to the terminal, $f_i = 0$, the equation can be adapted to

$$\sum_{i=1}^{2 \cdot n} d \cdot (\alpha \cdot w + \beta \cdot v^2) + d \cdot \alpha \cdot \sum_{i=1}^{n} f_i.$$

That implies that n is redefined as $n = \lceil \frac{Q}{K} \rceil$, which implies that even if $mod(Q, K) = 1[Unit]$ one additional trip has to be made to satisfy the demand. Therefore, the final equation

$$d \cdot (2 \lceil \frac{Q}{K} \rceil \cdot (\alpha \cdot w + \beta \cdot v^2) + \alpha \cdot Q)$$

can be created, which requires the vehicle to travel to the point with full capacity, then return to the terminal point with $f = 0$ and then repeat if the demand is not yet satisfied. The picture painted based on these assumptions can be observed in Figure 5.15, which further tilts the favour towards $\mathscr{V}_C$, as the energy requirement of $\mathscr{V}_D$ increases dramatically due to its low carrying capacity. To conclude, since $\mathscr{V}_C$ has proven superior across all approaches it seems reasonable to assume that it will be the most favourable option in a real vehicle routing scenario.

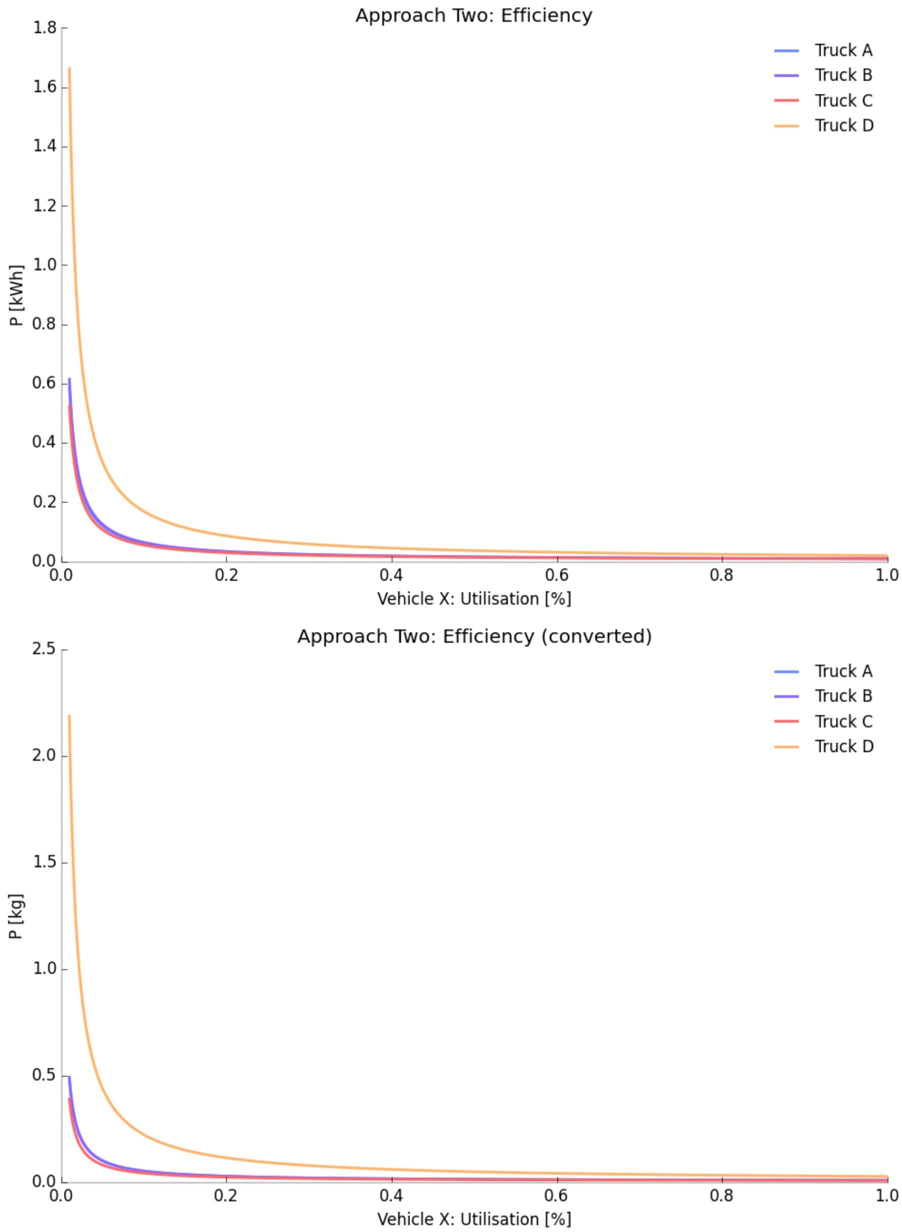


Figure 5.14: Problem 3 - Second approach

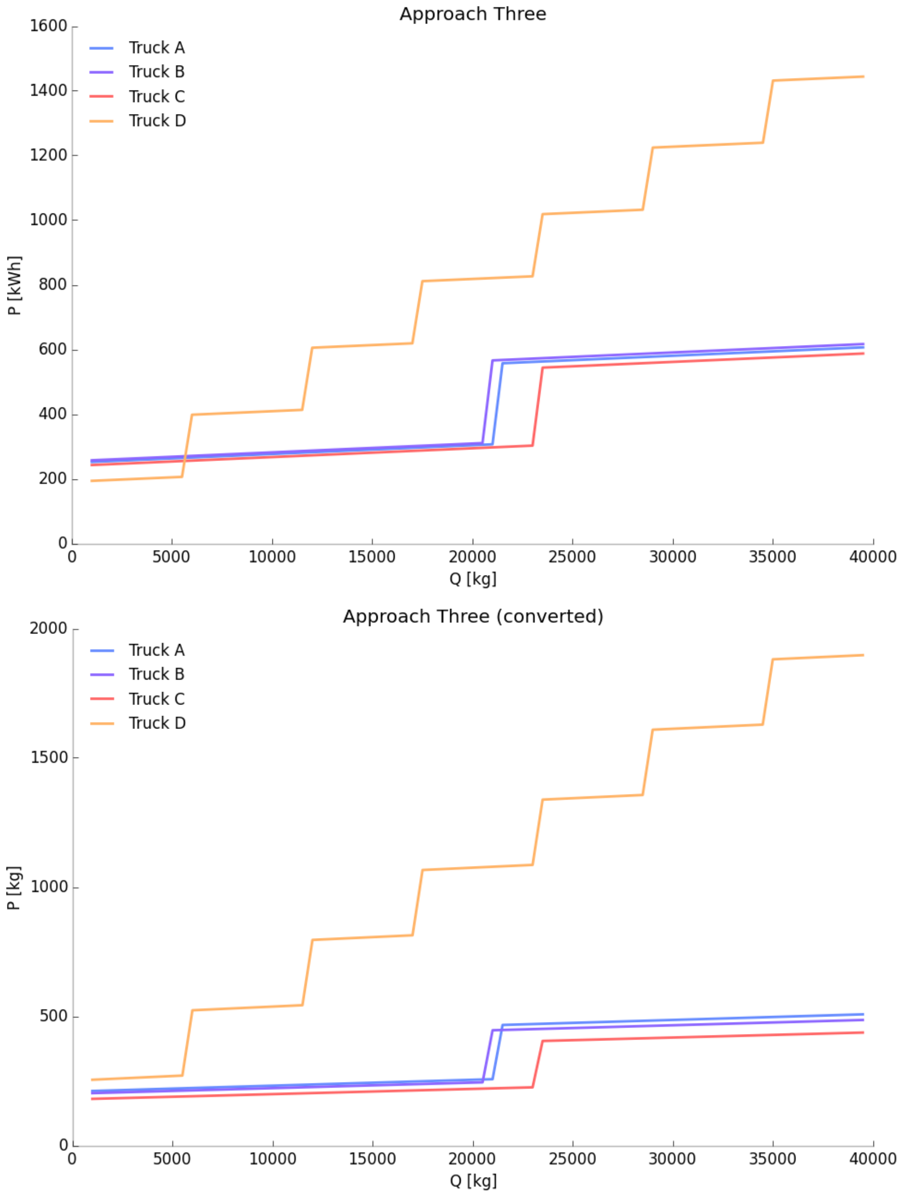


Figure 5.15: Problem 3 - Third approach

Anyhow, since $\mathscr{V}_C$ is now established as the most preferable option, it was chosen for the second part of this section. From there the next step is to determine, whether it is in this particular instance favourable to internalise the transportation of goods. However, to do so additional information is required. Firstly, a new instance of H has to be created. That is, $BO66$ is replaced with $BO542$, thus $H := \{\mathscr{N}_{TS1}, \mathscr{N}_{TS2}, \mathscr{N}_{TS3}, \mathscr{N}_{BO542}\}$ with

$$\mathscr{N}_{BO542} := \{Graz, Vienna, St.Poelten, Salzburg, Munich, Insbruck, Zuerich\}$$

and

$$Q_{BO542} := \{0, 2000, 1000, 2000, 4000, 18000, 4000\}.$$

Its geographical information, i.e. X_{BO542} and Y_{BO542} is taken from Table 4.16. Even though, the composition of H changed the parameter,V_h, Θ_h remain unchanged, i.e.

$$\begin{aligned}
&\forall h \in H : \\
&\qquad V_h = \{v_{hij} | \forall i, j \in I : v_{h_{ij}} := 80[km/h]\} \\
&\qquad \Theta_h = \{\theta_{h_{ij}} | \forall i, j \in I : \theta_{h_{ij}} := 0[°]\}
\end{aligned}$$

Similar to the previous problem all sets of points presented are of the same cardinality therefore only one index set $I := \{0, 1, 2, 3, 4, 5, 6\}$ is required. Additionally, the parameter group $Cost$ is defined based on the date of the problem. Therefore, $\mathscr{C}_{2.6.2014}$ is

$$\begin{aligned}
\acute{p}_d &:= 10,5[Euro/h] \\
\acute{p}_f &:= 1,31[Euro/l] \\
\acute{p}_e &:= 18,24[Euro/t].
\end{aligned}$$

However, to appropriately decide between the options presented the current cost for delivery is required. Therefore, depending on the choice made in Problem One (see

Section 5.1) additional prices have to be defined. That is,

$$\acute{p}_e := \begin{cases} \acute{p}_{A1} := 30[Euro/t] & \textit{If Company A} \ \wedge \textit{domestic} \\ \acute{p}_{A2} := 60[Euro/t] & \textit{If Company A} \wedge \textit{international} \\ \acute{p}_{B} := 1,2[Euro/km] & \textit{If Company B} \end{cases} .$$

However, since both options are covered, the usual notation is employed. Having the input defined, the last step is to identify the implementations. Since Problem Two harmonised the objective, in this case to *Odf*, it is no longer necessary to solve for either Od or Of. However, internally the objective Oe is utilised, thus

$$\begin{aligned} \mathscr{I}_{Odf} &:= Implementation(Distance, Speed_{Fixed}, TW_{False}) \\ \mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False}). \end{aligned}$$

By executing these implementations for all scenarios and vehicles the values found in Table 5.13 can be created.

A small glance at the output and it becomes apparent that, apart form $BO542$, $\forall h \in H \setminus \{BO542\} : J_{\mathscr{I}_{Oe}}(h) = J_{\mathscr{I}_{Odf}}$. Hence, the differences between these instances has to result from the different vehicles. In the case of $BO542$, however, the differences occurs not only due to the vehicles in use, but because of the selected objective. That is, even if the difference is marginal Oe still performs better than *Odf* when measured in emissions and cost. As mentioned in Section 5.2 this is mainly caused by the fact that *Odf* puts a higher emphasis on parameter load. However, in this case the parameter load is measured based on the overall distance the load is carried, which is not the load carried per arc. Therefore, the values provided by the summary may seem misleading. Anyhow, in order to reduce said factor it routes the vehicle over "Salzburg" (see Figure 5.16), thereby it was possible to reduce $d \cdot (f + w)$ by $62,56[km \cdot t]$ in scenario $BO542$. Unfortunately, this results in an increase in GHG output by $7,33[kg]$ or by $0,55[\%]$. This difference was caused because $D_{\mathscr{I}_{Odf}} > D_{\mathscr{I}_{Oe}}$, which resulted in $Pv \uparrow \wedge Pw \uparrow$ while $Pf \downarrow$. Unfortunately, this decrease is not sufficient enough to compensate for the increase of the other parameter, i.e. $|Pv \uparrow \wedge Pw \uparrow| \geq |Pf \downarrow|$. Moreover, since the internal cost function has a greater emphasis on the parameter distance than Oe (see Section 5.4) this effects the overall cost even more. Hence, resulting in an increase in cost by $5,88[Euro]$ or $0,69[\%]$ in scenario $BO542$. While the values presented are limited to $\mathscr{V}_C$ the same argument

Truck A	$\mathscr{I}_{Odf}(TS1)$	$\mathscr{I}_{Oe}(TS1)$	$\mathscr{I}_{Odf}(TS2)$	$\mathscr{I}_{Oe}(TS2)$	$\mathscr{I}_{Odf}(TS3)$	$\mathscr{I}_{Oe}(TS3)$	$\mathscr{I}_{Odf}(BO542)$	$\mathscr{I}_{Oe}(BO542)$
Path	0-6-1-2-3-4-5-0	0-6-1-2-3-4-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-5-6-3-4-2-1-0	0-5-6-3-4-2-1-0	0-5-6-4-3-2-1-0	0-3-5-6-4-2-1-0
D	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	1274,20	1290,49
F	47,98	47,98	64,96	64,96	56,94	56,94	38,94	51,41
P	1292,41	1292,41	3774,86	3774,86	3412,74	3412,74	1552,83	1561,73
C	807,55	807,55	2317,67	2317,67	2111,72	2111,72	959,76	966,44
Pw	506,25	506,25	1338,34	1338,34	1266,00	1266,00	572,30	579,62
Pf	116,52	116,52	666,21	666,21	472,11	472,11	223,51	215,41
Pv	669,65	669,65	1770,32	1770,32	1674,63	1674,63	757,03	766,70
Cd	147,94	147,94	391,09	391,09	369,95	369,95	167,24	169,38
Cf	636,04	636,04	1857,74	1857,74	1679,52	1679,52	764,20	768,58
Ce	23,57	23,57	68,84	68,84	62,24	62,24	28,32	28,48
T0j	16,85	16,85	39,11	39,11	37,06	37,06	18,69	18,89
Truck B	$\mathscr{I}_{Odf}(TS1)$	$\mathscr{I}_{Oe}(TS1)$	$\mathscr{I}_{Odf}(TS2)$	$\mathscr{I}_{Oe}(TS2)$	$\mathscr{I}_{Odf}(TS3)$	$\mathscr{I}_{Oe}(TS3)$	$\mathscr{I}_{Odf}(BO542)$	$\mathscr{I}_{Oe}(BO542)$
Path	0-6-1-2-3-4-5-0	0-6-1-2-3-4-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-5-6-3-4-2-1-0	0-5-6-3-4-2-1-0	0-5-6-4-3-2-1-0	0-3-5-6-4-2-1-0
D	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	1274,20	1290,49
F	47,98	47,98	64,96	64,96	56,94	56,94	38,94	51,41
P	1239,01	1239,01	3612,62	3612,62	3268,56	3268,56	1487,06	1495,76
C	780,29	780,29	2234,87	2234,87	2038,13	2038,13	926,19	932,77
Pw	489,30	489,30	1293,53	1293,53	1223,62	1223,62	553,14	560,21
Pf	109,66	109,66	627,02	627,02	444,34	444,34	210,36	202,74
Pv	640,05	640,05	1692,06	1692,06	1600,61	1600,61	723,56	732,81
Cd	147,94	147,94	391,09	391,09	369,95	369,95	167,24	169,38
Cf	609,76	609,76	1777,89	1777,89	1608,57	1608,57	731,83	736,12
Ce	22,60	22,60	65,89	65,89	59,61	59,61	27,12	27,28
T0j	16,85	16,85	39,11	39,11	37,06	37,06	18,69	18,89
Truck C	$\mathscr{I}_{Odf}(TS1)$	$\mathscr{I}_{Oe}(TS1)$	$\mathscr{I}_{Odf}(TS2)$	$\mathscr{I}_{Oe}(TS2)$	$\mathscr{I}_{Odf}(TS3)$	$\mathscr{I}_{Oe}(TS3)$	$\mathscr{I}_{Odf}(BO542)$	$\mathscr{I}_{Oe}(BO542)$
Path	0-6-1-2-3-4-5-0	0-6-1-2-3-4-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-5-6-3-4-2-1-0	0-5-6-3-4-2-1-0	0-5-6-4-3-2-1-0	0-3-5-6-4-2-1-0
D	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	1274,20	1290,49
F	47,98	47,98	64,96	64,96	56,94	56,94	38,94	51,41
P	1109,03	1109,03	3250,25	3250,25	2934,05	2934,05	1335,32	1342,65
C	713,95	713,95	2049,93	2049,93	1867,41	1867,41	848,75	854,63
Pw	410,21	410,21	1084,45	1084,45	1025,84	1025,84	463,73	469,66
Pf	103,57	103,57	592,19	592,19	419,66	419,66	198,67	191,48
Pv	595,24	595,24	1573,62	1573,62	1488,56	1488,56	672,91	681,51
Cd	147,94	147,94	391,09	391,09	369,95	369,95	167,24	169,38
Cf	545,79	545,79	1599,56	1599,56	1443,95	1443,95	657,15	660,76
Ce	20,23	20,23	59,28	59,28	53,51	53,51	24,35	24,49
T0j	16,85	16,85	39,11	39,11	37,06	37,06	18,69	18,89

Table 5.13: Problem 3 - Output

can be made for any other vehicle. This is especially relevant since the internal objective would be Oe, while the external is Odf. Hence, it can be concluded that solely based on the objective, in-sourcing already provides an advantage over the existing option.

The next step is to compare the GHG output between the various vehicle and scenarios. The values required to do so can be found in Table 5.14, which is constructed based on the same format as the comparison tables of previous sections. As already established, the differences in $TS1$, $TS2$ and $TS3$, are not the result of a different path, but rather due to selected vehicle. Fortunately, the conclusion made in the

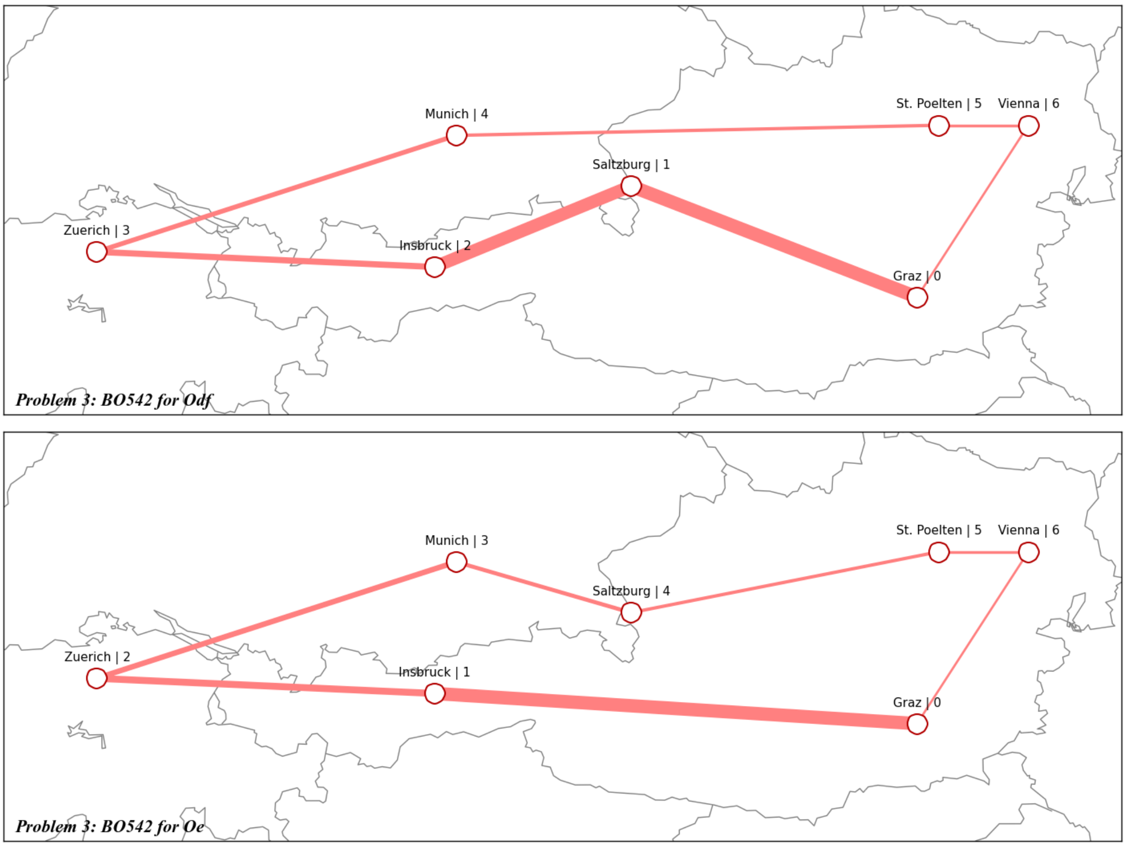


Figure 5.16: Problem 3 - BO542

first part of the section is supported by the values presented. That is by simply using $\mathscr{V}_C$ it is possible to reduce the GHG emissions for the same trip by at least 10[%] regardless of which company was chosen. Therefore, when simply comparing the different alternatives solely based on the GHG output the internal solution provides superior results, as the vehicle and the objective used in said instance outperform the other options.

Emissions $[kg]$	$\Delta P_{\mathscr{V}_A}$	$\Delta P_{\mathscr{V}_B}$	$\Delta P_{\mathscr{V}_A}[\%]$	$\Delta P_{\mathscr{V}_B}[\%]$
TS1	-183,39	-129,99	-14,2%	-10,5%
TS2	-524,61	-362,36	-13,9%	-10,0%
TS3	-478,69	-334,51	-14,0%	-10,2%
BO542	-226,41	-160,45	-14,5%	-10,7%
Average	-353,27	-246,83	-14,1%	-10,3%

Table 5.14: Problem 3 - Emission comparison

Lastly, it has to be tested if internalising the delivery process would result in a cost reduction. Therefore, the internal costs and the costs of the respective company has to be calculated. As a reminder

$$
\begin{aligned}
C_{A_h} &:= \acute{p}_{Ax} \cdot F_{\mathscr{I}_{Odf}(h)} \\
C_{B_h} &:= \acute{p}_{B} \cdot D_{\mathscr{I}_{Odf}(h)} \\
C_h &:= \acute{p}_e \cdot P_{[kg]_{\mathscr{I}_{Oe}(h)}} + \acute{p}_f \cdot P_{[l]_{\mathscr{I}_{Oe}(h)}} + \acute{p}_d \cdot T_{\mathscr{I}_{Oe}(h)}.
\end{aligned}
$$

Given these different cost functions the values found in Table 5.15 can be generated. Based on these one can conclude that there are massive potential savings to be obtained by in-sourcing the delivery process. That is, by switching away form Company A one could on average save up to $53,7[\%]$, while a switch away from Company B would result in average savings of $44,4[\%]$. Hereby, the one outlier can be detected. As $\Delta C_{A_{[\%]}}(BO542) = -72,5[\%]$ is significantly higher than in the other scenarios. The reason behind this occurrence it that $BO542$ is for an international trip relatively short with $\dfrac{D_{\mathscr{I}_{Odf}(BO542)}}{D_{\mathscr{I}_{Odf}(TS1)}} = 114,5[\%]$, i.e. $BO542$ is only $14,5[\%]$ bigger that the domestic scenario $TS1$. However, at the same time the pricing strategy requires a price of $\acute{p}_{A2} := 60[Euro/t]$. Hence, the advantage Company A has, i.e. long distance deliveries was not utilised (see Section 5.1).

Cost	C_A	C_B	C	ΔC_A	ΔC_B	$\Delta C_A[\%]$	$\Delta C_B[\%]$
TS1	1439,36	1352,56	713,95	-725,41	-638,61	-50,4%	-47,2%
TS2	3897,49	3575,69	2049,93	-1847,56	-1525,76	-47,4%	-42,7%
TS3	3416,11	3382,42	1867,41	-1548,70	-1515,01	-45,3%	-44,8%
BO542	3084,35	1548,59	848,75	-2235,60	-699,84	-72,5%	-45,2%
Average	2959,33	2464,81	1370,01	-1589,32	-1094,81	-53,7%	-44,4%

Table 5.15: Problem 3 - Cost comparison

In general it is suggested that the external costs primarily depend on the selection of the objectives and the pricing strategy of the company. However, since the internal price depends also on the vehicle in use, one can argue that this factor also influences the overall comparison. Nevertheless, the biggest influencing factor is the pricing strategy. A conclusion which can be made by calculating the cost of each company based on the same cost function used to assess the internal cost. By doing so the values found in Table 5.16 can be created. In which the greatest variation between an external company and the internal solution would be $-267,75[Euro]$. By contrast as the same instance in Table 5.15 lists a difference of $-1847,56[Euro]$. Therefore, it seems save to assume that the massive savings made by in-sourcing the transportation process mainly originate out of the pricing strategies employed by the respective company.

Internal Cost	ΔC_A	ΔC_B	$\Delta C_A[\%]$	$\Delta C_B[\%]$
TS1	-93,60	-66,34	-11,6%	-8,5%
TS2	-267,75	-184,94	-11,6%	-8,3%
TS3	-244,31	-170,72	-11,6%	-8,4%
BO542	-117,69	-84,02	-12,2%	-9,0%
Average	-180,84	-126,51	-11,7%	-8,5%

Table 5.16: Problem 3 - Internal cost comparison

To conclude, given the option between $\mathscr{V}_C$ and $\mathscr{V}_D$, $\mathscr{V}_C$ outperforms the latter one in nearly every test. May it be by comparing the GHG output per carried product at full capacity or by simulating the behaviour of a scenario at which $Q > K$ or even if the absolute GHG output is measured $\mathscr{V}_C$, remains the most preferable choice. This can, to some extent, be traced back to a rather efficient conversion of energy to GHG emission. However, the conversion factor only scales the overall GHG output, thus it does only effect the value of the result and not the behaviour of the model. By contrast, the parameter w, A and $\acute{c}_d$ impact the internal dynamics of the model. Furthermore, multiple trips are in this variation of the model not possible, thus the overall demand a trip can have is limited by the carrying capacity of the vehicle. Anyhow, by comparing the results obtained through in-souring the delivery process against the ones currently in use, several arguments could be made. Firstly, $\mathscr{V}_C$ not only outperforms $\mathscr{V}_D$, but also $\mathscr{V}_A$ and $\mathscr{V}_B$. Hence, given equal conditions in-sourcing would create less GHG emissions. Secondly, the transportation companies use *Odf* rather than *Oe*. Therefore, employing such a company will always result in a greater or equal GHG output. Thirdly, due to the above mentioned advantages and most importantly due to the pricing strategies used by the transport-companies, in-sourcing the delivery process would in this case result in a massive cost reduction.

5.4 Problem Four

Problem Four investigates, similar to Problem Three, two different aspects. The first one is to identify the differences between the objective cost and the objective emissions, i.e. *Oc* and *Oe*. In the narrative this is introduced by the question, whether it would be more advantageous for the company, if the trip was constructed with the incentive to minimise costs. This question was justified by the fact the company already prices CO2 emissions. Hence, it was requested to argue for one of those two approaches. The second objective is to investigate the impact of variable speed on the model. Narrative wise, this originated out of the question why up until now only a fixed speed was assumed. Before diving into the comparison between the *Oe* and *Oc*, the input has to be defined. Firstly the parameter groups *World* and *Product* stay the same, i.e.

$$\begin{aligned} \mathscr{W}_{P4} &:= \mathscr{W}_1. \\ \mathscr{P}_{P4} &:= \mathscr{P}_1 \end{aligned}$$

Secondly, the parameter group *Vehicle* is locked based on the decision made in Problem Three (see Section 5.3). Due to its overwhelming superiority only $\mathscr{V}_C$ it is selected for this problem, i.e.

$$\mathscr{V}_{P4} := \mathscr{V}_C$$

Thirdly, $BO542$ is replaced by $BO600$, thus $H := \{\mathscr{N}_{TS1}, \mathscr{N}_{TS2}, \mathscr{N}_{TS3}, \mathscr{N}_{BO600}\}$ with

$$\mathscr{N}_{BO600} \;:=\; \{Graz, Villach, Krems, Leoben, Budapest, Bregenz, Salzburg\}$$

and

$$Q_{BO600} \;:=\; \{0, 1400, 2800, 4400, 4300, 6000, 14000\}.$$

Its geographical information, i.e. X_{BO600} and Y_{BO600} is taken from Table 4.16. Similar to the previous problems all sets presented are of the same cardinality. Therefore only one index set $I := \{0, 1, 2, 3, 4, 5, 6\}$ is required. Even though, the composition of H changed the parameter V_h, Θ_h remain unchanged, i.e.

$$\begin{aligned}
&\forall h \in H : \\
&\qquad V_h = \{v_{hij} | \forall i, j \in I : v_{h_{ij}} := 80[km/h]\} \\
&\qquad \Theta_h = \{\theta_{h_{ij}} | \forall i, j \in I : \theta_{h_{ij}} := 0[°]\}
\end{aligned}$$

additionally, the parameter $\widetilde{LU_h}$ has to be introduced, with

$$\begin{aligned}
&\forall h \in H : \\
&\qquad \widetilde{LU_h} = \{\widetilde{lu_{h_{ij}}} = (l_{h_{ij}}, u_{h_{ij}}) | \forall i, j \in I : (l_{h_{ij}}, u_{h_{ij}}) := (30, 130)\} \\
&\qquad \Rightarrow l_{h_{ij}} := 30[km/h] \wedge u_{h_{ij}} := 130[km/h].
\end{aligned}$$

implying that on every arc in every scenario the lower speed limit is set to $30[km/h]$, while the upper one is set to $130[km/h]$. However, $\widetilde{LU_h}$ will only be required in the second part of this section. That is, as soon as V is no longer fixed. Additionally, the parameter group $Cost$ is for the most part defined based on the date of the problem. Therefore, $\mathscr{C}_{P4} := \mathscr{C}_{25.7.2014}$, which is

$$\begin{aligned}
\acute{p}_d &:= 10,5[Euro/h] \\
\acute{p}_f &:= 1,309[Euro/l] \\
\acute{p}_e &:= 18,24[Euro/t].
\end{aligned}$$

Having the input defined the last step is to identify the implementations. Since the differences between Oc and Oe have to be investigated, these are the only two implementations required for the first part of this section. That is,

$$\begin{aligned}
\mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oc} &:= Implementation(Cost, Speed_{Fixed}, TW_{False}).
\end{aligned}$$

Before however, any of these are executed, it is advantageous to compare the cost and the fuel consumption function on a more or less general level. As a reminder

$$\begin{aligned}
C &:= \acute{p_e} \cdot P_{[kg]} + \acute{p_f} \cdot P_{[l]} + \acute{p_d} \cdot T \\
P &:= \alpha \cdot d \cdot f + \alpha \cdot d \cdot w + \beta \cdot d \cdot v^2,
\end{aligned}$$

which can be expanded into

$$\begin{aligned}
C &:= d \cdot (\alpha \cdot (f + w) + \beta \cdot v^2) \cdot (\acute{p_e} \cdot \omega_{GHG} + \acute{p_f}) \cdot \left(2,78 \cdot 10^{-7} \cdot \frac{\omega_F}{\eta}\right) + \frac{d \cdot \acute{p_d}}{v} \\
P &:= d \cdot (\alpha \cdot (f + w) + \beta \cdot v^2).
\end{aligned}$$

For easy reference shall

$$z := (\acute{p_e} \cdot \omega_{GHG} + \acute{p_f}) \cdot \left(2,78 \cdot 10^{-7} \cdot \frac{\omega_F}{\eta}\right)$$

be the conversion factor. For this equation it is important that the prices match with the units of the corresponding values, e.g. $\acute{p_e} = [Euro/t] \wedge P_{[kg]} = [kg]$ demands that either the price has to be translated into in $[Euro/kg]$ or the emissions have to be translated into $[t]$. Anyhow, the expanded form of the cost function implies that the only difference between these two is the conversation factor z and $\frac{d \cdot \acute{p_d}}{v}$. However, the conversation factor can not be responsible for any behavioural differences one its own. Since, as investigated in Problem Two (see Section 5.2), it would only scale the result of the fuel consumption model. Hence, it can not cause a shift in the ratio, as defined in Equation 5.2. For example, when neglecting $\frac{d \cdot \acute{p_d}}{v}$ the ratio would be

1. Since

$$\begin{aligned}
\acute{C} &:= d \cdot (\alpha \cdot (f + w) + \beta \cdot v^2) \cdot z + \frac{d \cdot \acute{p_d}}{v} - \frac{d \cdot \acute{p_d}}{v} \\
&\Rightarrow \frac{\partial C}{\partial d} := (\alpha \cdot (f + w) + \beta \cdot v^2) \cdot z \\
&\Rightarrow \frac{\partial C}{\partial f} := \alpha \cdot d \cdot z,
\end{aligned}$$

which would result in

$$r_{\acute{C}} := \frac{\frac{\partial \acute{C}}{\partial d}}{\frac{\partial \acute{C}}{\partial f}} = \frac{(\alpha \cdot (f + w) + \beta \cdot v^2) \cdot z}{\alpha \cdot d \cdot z} = \frac{\alpha \cdot (f + w) + \beta \cdot v^2}{\alpha \cdot d} = r_{Fuel}$$

Hence, if a difference between the behaviour of those two function exists, it has to result from $\acute{C} + \frac{d \cdot \acute{p_d}}{v}$, which when looking at the partial derivatives of the whole cost function

$$\begin{aligned}
\frac{\partial P}{\partial d} &:= (\alpha \cdot (f + w) + \beta \cdot v^2) \cdot z + \frac{\acute{p_d}}{v} \\
\frac{\partial P}{\partial f} &:= \alpha \cdot d \cdot z,
\end{aligned}$$

seems to be the case, because

$$r_C := \frac{\frac{\partial C}{\partial d}}{\frac{\partial C}{\partial f}} = \frac{(\alpha \cdot (f + w) + \beta \cdot v^2) \cdot z + \frac{\acute{p_d}}{v}}{\alpha \cdot d \cdot z}$$

Therefore, by following Equation 5.2

$$R := \frac{\dfrac{(\alpha \cdot (f+w) + \beta \cdot v^2) \cdot z + \dfrac{\acute{p}_d}{v}}{\alpha \cdot d \cdot z}}{\dfrac{\alpha \cdot (f+w) + \beta \cdot v^2}{\alpha \cdot d}}$$

$$\Rightarrow R := 1 + \frac{\acute{p}_d}{v \cdot z \cdot (\alpha \cdot (f+w) + \beta \cdot v^2)}$$

can be created. Moreover, since in a realistic scenario no parameter can be smaller than 0 the only case left is $R \geq 1$, which as established in Section 5.2 implies that the parameter distance has in proportion a higher priority in the cost function as compared to the fuel consumption model. Hence, it can be concluded that the parameter distance will be favoured in the optimisation process. With this in mind the next step is to execute the implementations introduced above for $\forall h \in H$. Thereby, the values found in Table 5.17 can be created. As depicted in Table 5.17

Fixed Speed Path	$\mathscr{I}_{Oc}(TS1)$ 0-6-1-2-3-4-5-0	$\mathscr{I}_{Oe}(TS1)$ 0-6-1-2-3-4-5-0	$\mathscr{I}_{Oc}(TS2)$ 0-3-4-2-1-6-5-0	$\mathscr{I}_{Oe}(TS2)$ 0-3-4-2-1-6-5-0	$\mathscr{I}_{Oc}(TS3)$ 0-5-6-3-4-2-1-0	$\mathscr{I}_{Oe}(TS3)$ 0-5-6-3-4-2-1-0	$\mathscr{I}_{Oc}(BO600)$ 0-3-1-5-6-2-4-0	$\mathscr{I}_{Oe}(BO600)$ 0-3-6-5-1-2-4-0
$D[km]$	1127,13	1127,13	2979,74	2979,74	2818,68	2818,68	1500,82	1571,65
$F[t]$	47,98	47,98	64,96	64,96	56,94	56,94	75,40	59,69
$P[kg]$	1109,03	1109,03	3250,25	3250,25	2934,05	2934,05	1613,71	1595,23
$C[Euro]$	720,16	720,16	2066,11	2066,11	1882,82	1882,82	1028,74	1029,06
$Pw[kg]$	410,21	410,21	1084,45	1084,45	1025,84	1025,84	546,21	571,99
$Pf[kg]$	103,57	103,57	592,19	592,19	419,66	419,66	274,91	193,24
$Pv[kg]$	595,24	595,24	1573,62	1573,62	1488,56	1488,56	792,59	830,00
$Cd[Euro]$	154,98	154,98	409,71	409,71	387,57	387,57	206,36	216,10
$Cf[Euro]$	544,96	544,96	1597,12	1597,12	1441,74	1441,74	792,95	783,87
$Ce[Euro]$	20,23	20,23	59,28	59,28	53,51	53,51	29,43	29,09
$T0j[h]$	16,85	16,85	39,11	39,11	37,06	37,06	21,52	22,41

Table 5.17: Problem 4 - Output (fixed Speed)

only in one scenario the different objectives produce two different results. That is, $\forall h \in H \setminus \{BO600\} : J_{\mathscr{I}_{Oc}(h)} = J_{\mathscr{I}_{Oe}(h)}$. Hence, only $BO600$ will be investigated, as the other scenarios can not be used to create further insights, other than the fact that no better solution for the objective cost could be found. A behaviour which seems fairly obvious, since $\forall h \in H \setminus \{BO600\} : D_{\mathscr{I}_{Od}(h)} = D_{\mathscr{I}_{Oe}(h)}$. Thereby, leaving Oc, which has a higher emphasis on the factor distance, no better option than selecting the same result as presented by Oe. Fortunately, this not the case for $BO600$. Hence, it can be assumed that by contrasting its values, additional information may be obtained. Even though it is redundant the results of $BO600$ are listed in Table 5.18. However, the table also contains a comparison between these

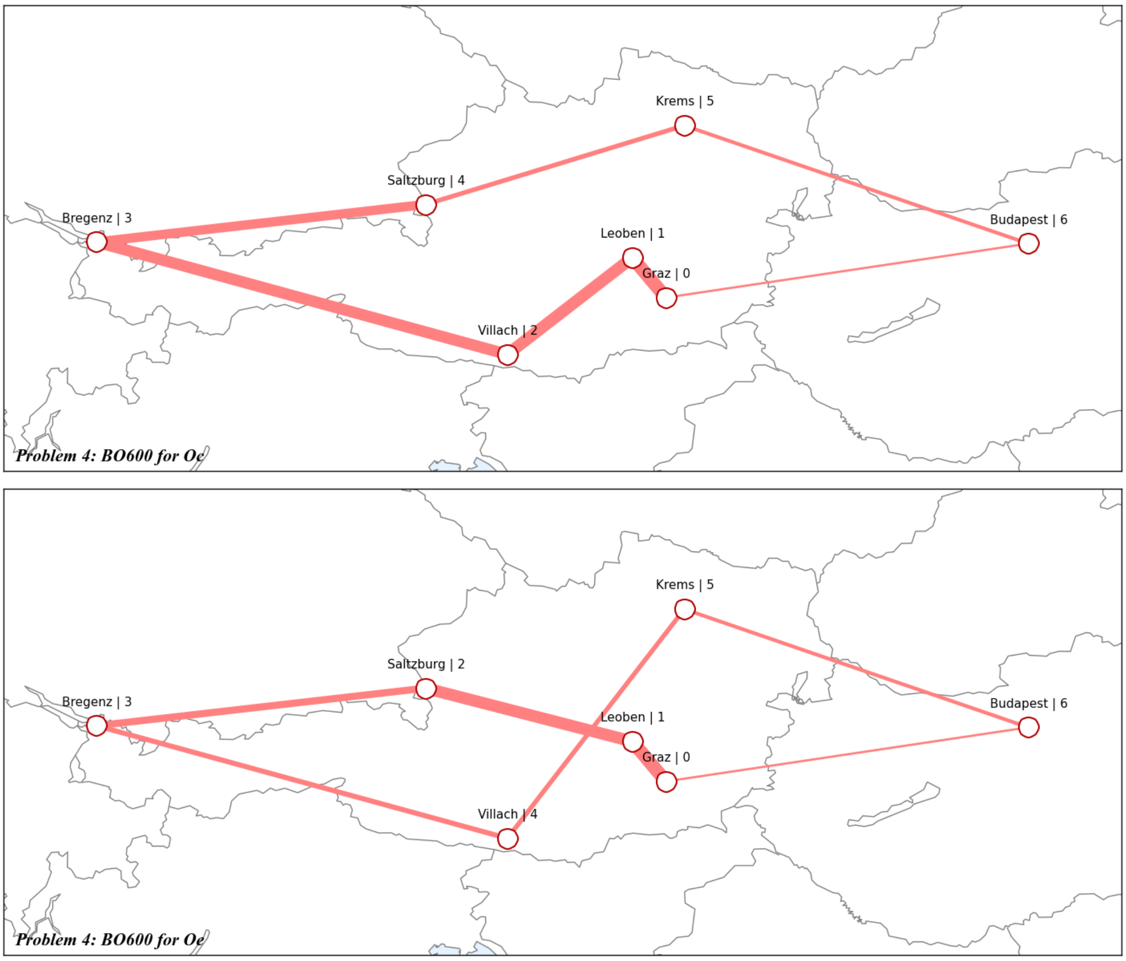


Figure 5.17: Problem 4 - BO600

values, which is constructed with Oe as a basis (see Section 5.1). In this case X represents the parameter of the respective row. Additionally, Figure 5.17 visualises the differences between $J_{\mathscr{I}_{Oe}(BO600)}$ and $J_{\mathscr{I}_{Oc}(BO600)}$.

The first deduction to make is that the resulting GHG output of Oe is about $1,14[\%]$ lower than the one from Oc. The main reason behind this is that even though Oc manages to create a path with a lower distance[6] than $D_{\mathscr{I}_{Oe}(BO600)}$, the massive increase in load is potent enough to not only negate the effect, but also shift the overall superiority regarding GHG output towards Oe. Obviously, this is not much of a surprise. However, in this case it beautifully depicts the trade-off between load an distance. That is, the choice made by the optimisations algorithm

6 $D_{\mathscr{I}_{Oc}(BO600)} = D_{\mathscr{I}_{Od}(BO600)}$

BO600	$X_{\mathscr{I}_{Oc}(BO600)}$	$X_{\mathscr{I}_{Oe}(BO600)}$	ΔX	$\Delta X[\%]$
D	1500,82	1571,65	70,84	4,72%
F	75,40	59,69	-15,70	-20,83%
P	1613,71	1595,23	-18,48	-1,14%
C	1028,74	1029,06	0,32	0,03%
T0j	21,52	22,41	0,89	4,11%
BO600	$P_{\mathscr{I}_{Oc}(BO600)}$	$P_{\mathscr{I}_{Oe}(BO600)}$	ΔP	$\Delta P[\%]$
Pw	546,21	571,99	25,78	4,72%
Pf	274,91	193,24	-81,67	-29,71%
Pv	792,59	830,00	37,41	4,72%
BO600	$C_{\mathscr{I}_{Oc}(BO600)}$	$C_{\mathscr{I}_{Oe}(BO600)}$	ΔC	$\Delta C[\%]$
Cd	206,36	216,10	9,74	4,72%
Cf	792,95	783,87	-9,08	-1,14%
Ce	29,43	29,09	-0,34	-1,14%

Table 5.18: Problem 4 - Comparison of BO600

to increase the total distance travelled by $4,72[\%]$ led to a overall reduction in load of $20,83[\%]$, which then resulted in a decrease in GHG output by $1,14[\%]$. The underlying reason behind this behaviour is that the increase in Pv and Pw, both of which directly correlate with D, was compensated by the decrease in Pf, which is effected by both D and F. Anyhow, this reduction in P led to a decrease in Ce and Cf as both of them directly relate to P. However, the increase in distance, which offset the overall decrease in GHG emissions, led to an increase in travelling time with $\Delta T := 4,72[\%]$ and $\Delta T_{0j} := 4,11[\%]$. Therefore, Cd also increased by $4,72[\%]$ and thus causing an overall increase in cost by $0,03[\%]$. To summarise, in order to save a total of $0,32[Euro]$ the overall GHG output was increased by $19,48[kg]$. That is, in this case only a slight alteration in prices, i.e. $\acute{p}_f \uparrow \vee \acute{p}_e \uparrow \vee \acute{p}_d \downarrow$ would have caused Oc to behave the same as Oe. The most shocking part, however, is that Ce has nearly no influence over the outcome. In this case for every litre consumed Ce only increases by $0,049[Euro]$, while Cf increases by $1,309[Euro]$ per litre. To put this in perspective, even though $\acute{p}_e$ is already $314[\%]$ greater than the European Emission Allowances at that time (see Table 4.10), Cf is sill $26,94$ times more influential than Ce. This becomes especially apparent when the individual components of the parameter costs are compared. Table 5.19 depicts this comparison, by putting these elements into proportion against the overall cost. Based on the results the notion, that Cf is vastly more influential than Ce, is supported. Especially with Cf being on average $76,72[\%]$ of the overall cost, thus making it the biggest cost factor. As for Cd, it is only responsible for $20,44[\%]$ of the total cost, while Ce is at marginal $2,85[\%]$.

Costs [%]	$\frac{Cd}{C}$	$\frac{Cf}{C}$	$\frac{Ce}{C}$
TS1	21,52%	75,67%	2,81%
TS2	19,83%	77,30%	2,87%
TS3	20,58%	76,57%	2,84%
BO600 (Oc)	20,06%	77,08%	2,86%
BO600 (Oe)	21,00%	76,17%	2,83%

Table 5.19: Problem 4 - Detailed cost comparison

Hence, it can be concluded that Oc's relatively similar behaviour to Oe, is not due to the high $\acute{p}_e$, but rather because the fuel costs make up most of the costs. Therefore, if environmental thinking should be at the heart of decision making Oc is not recommended, if one is not willing to adjust carbon prices appropriately. However, before any final judgement can be made, the impact of variable speed on the decision making process shall be investigated.

The second part of this section is concerned with introducing the concept of variable speed into the model. By introducing variable speed, three questions arise. Firstly, in which conditions is variable speed useful? Secondly, how is the "optimal" speed for a situation chosen? Thirdly, to what extend does variable speed change the dynamic in the fuel consumption model and the cost function. Before the required implementations can be defined it is vital to understand the extend of change variable speed induces. If V is changed the following parameter are affected

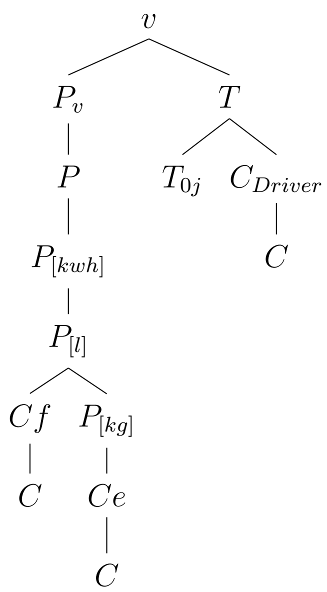


Figure 5.18: Problem 4 - Influence of Speed

Based on this tree one can deduce that an increase or decrease in speed would only tangentially influence the results provided by Od and Of, as both of them do not require values from the fuel consumption model. Hence, the left side of this tree is neglected for these objectives. However, by adjusting the speed one can influence the travelling time T. This, as depicted above, results in a change in operational costs and in the time required to reach a certain point, i.e. Cd and T_{0j}. As with the fuel consumption model, the costs are of no concern in both of these objectives. Therefore, without time windows the speed can not influence the outcome of Od or Of. However, if time windows are introduced (see Section 5.5) adjusting the travelling time can be used to reach a desired point within the required time frame, thus enhancing the overall outcome. Hence, it can be concluded that for both of these objectives variable speed could influence the resulting J, if time windows exist. By contrast Oe is impacted solely by the initial steps of the trees left side, while the results provided by Oc are influenced by every factor depicted above. In conclusion, it is reasonable to assume that variable speed is most useful when time windows have to be uphold. However, in a case without time windows only Oe and Oc seem to be influenced by the decision to implement variable speed. This statement, however, is explored in detail down below. Anyhow, above all it is save to assume that Od and Of, without time windows do not respond to a changes in speed, thus including these objectives in this section seems unnecessary. Equipped with this knowledge the required implementations can now be selected. That is,

$$\begin{aligned}
\mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oc} &:= Implementation(Cost, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oe,V} &:= Implementation(Emission, Speed_{Variable}, TW_{False}) \\
\mathscr{I}_{Oc,V} &:= Implementation(Cost, Speed_{Variable}, TW_{False})
\end{aligned}$$

are required.
However, before any results are computed, question two and three shall be investigated. Firstly, in order to determine the value at which the optimisation algorithm will set the speed one has to yet again investigate the fuel consumption equation. In this particular case the partial derivative

$$\frac{\partial P}{\partial v} := 2 \cdot \beta \cdot d \cdot v$$

is required. By equating this derivative to 0 the insight

$$\begin{aligned}
\frac{\partial P}{\partial v} &:= 2 \cdot \beta \cdot d \cdot v = 0 \\
&\Rightarrow v = 0
\end{aligned}$$

can be obtained. Hence, it can be concluded that Oe will if possible always set $v_{\bar{j}} = 0[m/s]$. If, however, $L_{\bar{j}} > 0$ the lower speed limit will determine the travelling speed in this solution. Such a behaviour has its roots in the fact that a simplified model is used. Said model directly translates a decrease in travelling speed into a reduction in energy consumption. However, it is highly important that this behaviour is not generalised, since this implementation is only based on a simplified fuel consumption model. For example, the original form of the model, namely CMEM, predicts an increase in energy consumption, if the travelling speed drops below a certain point. This can partly be attributed to different environmental conditions and the behaviour of the engine at lower speeds (Bektaş and Laporte, 2011, Demir et al., 2014). With Oc being the second objective influenced by variable speed, its optimal speed has to be assessed. In order to so the problem is approached in a similar fashion as before. Hence, if the partial derivative of the cost function

$$\frac{\partial C}{\partial v} := 2 \cdot \beta \cdot d \cdot v \cdot z - \frac{\acute{p_d}}{v^2}$$

is set equal to 0, the equation

$$v := \sqrt[3]{\frac{\acute{p_d}}{2 \cdot \beta \cdot z}} = \sqrt[3]{\frac{\acute{p_d}}{2 \cdot \beta \cdot (\acute{p_e} \cdot \omega_{GHG} + \acute{p_f}) \cdot \left(2,78 \cdot 10^{-7} \cdot \frac{\omega_F}{\eta}\right)}} \tag{5.4}$$

can be constructed. This equation describes the trade-off between the speed that minimises the emissions and the speed that minimises Cd. This is because the layer beneath the cost objective, i.e. the objective energy, would simply minimise the speed until the lower bound is reached. At the same time, however, Cd increases disproportionally with a decrease in speed, thus this part of the cost function tries to increase the speed until the upper bound is reached. To be more specific Cf and Ce are directly related to the energy consumption of the vehicle. Therefore, both will increase if the speed increases as well. On the other hand, the Cd is indirectly related to the speed of the vehicle. That means if the speed of the vehicle increases, the time required for the trip decreases, which then leads to a decrease in Cd.
The third question about the impact of variable speed is how it effects the relationship between the factor load and the factor distance. Similar to previous comparisons Equation 5.2 is used to determine, if the relationship shifts with an increase or a decrease in speed. The first equation to be investigated is the fuel consumption model. Hence, given

$$R := \frac{\frac{\alpha \cdot (f + w) + \beta \cdot v_1^2}{\alpha \cdot d}}{\frac{\alpha \cdot (f + w) + \beta \cdot v_2^2}{\alpha \cdot d}}$$

the implications

$$
\begin{aligned}
&If: R < 1\\
&\qquad \Rightarrow \quad \alpha \cdot (f + w) + \beta \cdot v_1^2 < \alpha \cdot (f + w) + \beta \cdot v_2^2 \Leftrightarrow \beta \cdot v_1^2 < \beta \cdot v_1^2 \Leftrightarrow |v_1| < |v_2|\\
&\qquad \quad\;\; since\ v \geq 0 \Rightarrow v_1 < v_2\\
&If: R > 1\\
&\qquad \Rightarrow \quad \alpha \cdot (f + w) + \beta \cdot v_1^2 > \alpha \cdot (f + w) + \beta \cdot v_2^2 \Leftrightarrow \beta \cdot v_1^2 > \beta \cdot v_1^2 \Leftrightarrow |v_1| > |v_2|\\
&\qquad \quad\;\; since\ v \geq 0 \Rightarrow v_1 > v_2
\end{aligned}
$$

can be made. Thereby, it is implied that if the speed increases the model starts to favour the distance over load and the other way round. Therefore, it could be possible that if variable speed is chosen the optimisation algorithm selects a path, which differs from the one with fixed speed. However, this assumes that the selected speed is not equal to the fixed speed. The second equation to test is the cost function, which formally is

$$
R := \frac{\dfrac{z \cdot (\alpha \cdot (f + w) + \beta \cdot v_1^2) + \dfrac{\acute{p}_d}{v_1}}{z \cdot \alpha \cdot d}}{\dfrac{z \cdot (\alpha \cdot (f + w) + \beta \cdot v_2^2) + \dfrac{\acute{p}_d}{v_2}}{z \cdot \alpha \cdot d}}
$$

Based on this the implications

$$
\begin{aligned}
&If: R < 1\\
&\qquad \Rightarrow \quad \alpha \cdot (f + w) + \beta \cdot v_1^2 + \frac{\acute{p}_d}{v_1} < \alpha \cdot (f + w) + \beta \cdot v_2^2 + \frac{\acute{p}_d}{v_2}\\
&\qquad \Leftrightarrow \quad \frac{\beta \cdot v_1^3 + \acute{p}_d}{v_1} < \frac{\beta \cdot v_2^3 + \acute{p}_d}{v_2}\\
&\qquad \Rightarrow \quad v_1 < v_2\\
&If: R > 1\\
&\qquad \Rightarrow \quad \alpha \cdot (f + w) + \beta \cdot v_1^2 + \frac{\acute{p}_d}{v_1} > \alpha \cdot (f + w) + \beta \cdot v_2^2 + \frac{\acute{p}_d}{v_2}\\
&\qquad \Leftrightarrow \quad \frac{\beta \cdot v_1^3 + \acute{p}_d}{v_1} > \frac{\beta \cdot v_2^3 + \acute{p}_d}{v_2}
\end{aligned}
$$

can be made. Unfortunately, it is not possible to definitively define in which direction the ratio will tilt. However, at smaller speeds the parameter $\acute{p}_d$ is significant, thus it is possible that $v_1 < v_2$ results in $\frac{\beta \cdot v_1^3 + \acute{p}_d}{v_1} > \frac{\beta \cdot v_2^3 + \acute{p}_d}{v_2}$ thereby tilting the focus towards distance. However, due to the cubic increase, $\acute{p}_d$ becomes quickly neglect able. Thus resulting in

$$\frac{\beta \cdot v^3 + \acute{p}_d}{v} \simeq \frac{\beta \cdot v^3}{v} = \beta \cdot v^2,$$

which implies that at higher speeds Oc starts to behave more like Oe. Therefore, even if it is possible to directly asses the ideal speed without implementing variable speed into the model, the optimisation algorithm may have selected a different path due to shifts in the internal dynamics of the model. Hence, it would be ideal to calculate the ideal speed in beforehand set said speed as a fixed speed and then execute the optimisation process.
Anyhow, having gathered all the necessary insights the next step is to solve for the previously defined implementations. Thereby the values found in Table 5.20 can be obtained.

Variable Speed Path	$\mathscr{I}_{Oc}(TS1)$ 0-6-1-2-3-4-5-0	$\mathscr{I}_{Oe}(TS1)$ 0-6-1-2-3-4-5-0	$\mathscr{I}_{Oc}(TS2)$ 0-3-4-2-1-6-5-0	$\mathscr{I}_{Oe}(TS2)$ 0-3-4-2-1-6-5-0	$\mathscr{I}_{Oc}(TS3)$ 0-5-6-3-4-2-1-0	$\mathscr{I}_{Oe}(TS3)$ 0-5-6-3-4-2-1-0	$\mathscr{I}_{Oc}(BO600)$ 0-3-1-5-6-2-4-0	$\mathscr{I}_{Oe}(BO600)$ 0-3-6-5-1-2-4-0
$D[km]$	1127.13	1127.13	2979.74	2979.74	2818.68	2818.68	1500.82	1571.65
$F[t]$	47.98	47.98	64.96	64.96	56.94	56.94	75.40	59.69
$P[kg]$	754.62	597.49	2314.30	1897.93	2044.69	1654.82	1146.57	881.95
$C[Euro]$	628.23	717.77	1823.06	2059.79	1652.92	1876.85	906.45	1025.73
$Pw[kg]$	410.21	410.21	1084.45	1084.45	1025.84	1025.84	546.21	571.99
$Pf[kg]$	103.57	103.57	592.19	592.19	419.66	419.66	274.91	193.24
$Pv[kg]$	240.84	83.71	637.66	221.29	599.20	209.33	325.45	116.72
$Cd[Euro]$	243.66	413.28	643.65	1092.57	610.90	1033.52	322.13	576.27
$Cf[Euro]$	370.81	293.59	1137.21	932.61	1004.73	813.15	563.40	433.37
$Ce[Euro]$	13.76	10.90	42.21	34.61	37.29	30.18	20.91	16.08
$T0j[h]$	24.91	40.33	60.37	101.18	57.37	95.79	32.04	55.15

Table 5.20: Problem 4 - Output (variable Speed)

However, in addition to the conventional output, one additional set of values has to be collected. That is, the speed of the vehicle at each arc. Hence, Table 5.21 contains said values. In this table a row represents the arc that has to be travelled to reach the stop. However, due to grouping the values based on the sequence of arcs rather than the specific arc, listing becomes possible. Unfortunately, a direct comparison between these values is only possible if both cases have the same J.

Fortunately, this is not necessary. As it is possible to draw the relevant con-

$\mathscr{I}_{Oc}[m/s]$	TS1	TS2	TS3	BO600	$\mathscr{I}_{Oe}[m/s]$	TS1	TS2	TS3	BO600
1. Stop	14,04	14,20	14,17	14,10	1. Stop	8,33	8,33	8,33	8,33
2. Stop	14,58	14,08	14,22	14,74	2. Stop	8,33	8,33	8,33	8,33
3. Stop	14,15	14,18	14,06	14,35	3. Stop	8,33	8,33	8,33	8,33
4. Stop	14,07	14,22	14,03	14,18	4. Stop	8,33	8,33	8,33	8,33
5. Stop	14,16	14,08	13,97	14,35	5. Stop	8,33	8,33	8,33	8,33
6. Stop	14,10	14,07	14,21	14,05	6. Stop	8,33	8,33	8,33	8,33
7. Stop	14,16	14,20	14,11	14,07	7. Stop	8,33	8,33	8,33	8,33

Table 5.21: Problem 4 - Speed

clusions without directly comparing two instances. Firstly, for every result created by executing Oe, the speed at which vehicle travels at any given arc is set to $8,33[m/s]$ or $30[km/h]$, which is equal to the lower speed limit. Formally that means $\forall h \in H \forall j \in \bar{J}_{\mathscr{I}_{Oe,V}} : v_j = l_j$. Therefore, it can be concluded that the behaviour of the model supports the previously proposed idea, namely optimal speed can be defined without implementing variable speed. However, the same argument to be tested for Oc. Unfortunately, the results of Oc present a less homogeneous picture, as slight variations between arcs can be detected. However, on average the v is set to $14,17[m/s]$. From there on Equation 5.4 can be used to calculate the optimal speed. In this case the optimal speed would be $14,1[m/s]$. Even though, it is not exactly the same as the speed provided by executing the implementations, it is close enough to support the assumption. Hence, the conclusion can be made that it is indeed possible to calculate the optimal speed, not only for Oe but also for Oc. Since it is required to identify the impact of variable speed on the model, the next step would be to compare the results with and without variable speed (see Table 5.22). Fortunately, for every scenario $J_{\mathscr{I}_{Ox}(h)} = J_{\mathscr{I}_{Ox,V}(h)}$, thus the elements Pw and Pf remain the same regardless of which objective is selected. Therefore, the overall decrease in GHG emissions, which amounts on average to a reduction by $43,4[\%]$ for Oe and $29,7[\%]$ for Oc is only caused by a reduction in Pv. Thereby being the source behind the difference between $P_{\mathscr{I}_{Oc}}$ and $P_{\mathscr{I}_{Oe}}$. That means, with either objective it is possible to drastically reduce the GHG output if variable speed is selected. The more interesting behaviour, however, is depicted in the second part of Table 5.22, as it depicts the overall cost of the trip. Since Cf and Ce directly relate to P one could expect that the reduction in GHG output may result in a similar reduction in cost. However, while $C_{\mathscr{I}_{Oe}(h)}$ only decreases marginally, i.e. $C_{\mathscr{I}_{Oe}} = 0,32[\%]$, $C_{\mathscr{I}_{Oc}(h)}$ decrease on average by $12,06[\%]$. This is due to the fact that even though a decrease in speed leads to a decrease in Ce and Cf, Cd increases disproportionally fast at lower speeds. A behaviour which can be observed in Figure 5.19. The upper part depicts the cost in range $V_\Delta = [1, 130)[km/h]$ and the lower one depicts the cost in range $V_\Delta = [30, 130)[km/h]$. The graph is constructed based on the values presented in this section. Additionally, D was set to $1500[km]$ and $F = 20[t]$. Said figure indicates that factor Cd is mostly significant at lower speeds, while the influence of Cf and Ce increase at higher speeds. However, as visible in Figure 5.19 Ce is nearly negligible. Hence, it can be concluded that the behaviour of Oc will be closer to the behaviour on Oe if a higher speed is chosen.

Emissions [kg]	$\Delta P_{\mathscr{I}_{Oc}}$	$\Delta P_{\mathscr{I}_{Oe}}$	$\Delta P_{\mathscr{I}_{Oc}}[\%]$	$\Delta P_{\mathscr{I}_{Oe}}[\%]$
TS1	-354,41	-511,54	-32,0%	-46,1%
TS2	-935,95	-1352,33	-28,8%	-41,6%
TS3	-889,36	-1279,23	-30,3%	-43,6%
BO600	-467,14	-713,28	-28,9%	-44,7%
Average	-661,72	-964,09	-29,7%	-43,4%
Cost [Euro]	$\Delta C_{\mathscr{I}_{Oc}}$	$\Delta C_{\mathscr{I}_{Oe}}$	$\Delta C_{\mathscr{I}_{Oc}}[\%]$	$\Delta C_{\mathscr{I}_{Oe}}[\%]$
TS1	-91,93	-2,39	-12,77%	-0,33%
TS2	-243,05	-6,32	-11,76%	-0,31%
TS3	-229,90	-5,97	-12,21%	-0,32%
BO600	-122,29	-3,33	-11,89%	-0,32%
Average	-171,79	-4,50	-12,06%	-0,32%

Table 5.22: Problem 4 - General Comparison

Overall, it has to be concluded that using Oc may lead to similar results as Oe, if the travelling speed is set at higher values. Thereby, it is to assume that regardless of which objective is selected, the results will be increasingly similar. This similarity is, however, not because of the high $\acute{p}_e$, but rather because of $\acute{p}_f$. Given the desire to have the environmental aspect a significant part in the decision process, this insight may indicate the need to elevate the price of emissions. However, the real advantage of Oc becomes apparent when variable speed is considered. Because of the nature of the cost function, Oc is able to select an optimal speed at which the individual parts of the cost function are balanced. This leads to a less significant reduction in GHG output, if compared against Oe. However, while Oe decreases the speed to such an extent that Cd starts to sore, Oc manages to significantly reduce the overall cost. Hence, it is suggested that due to the ability of balancing the environmental and economical aspects Oc should be selected. As for variable

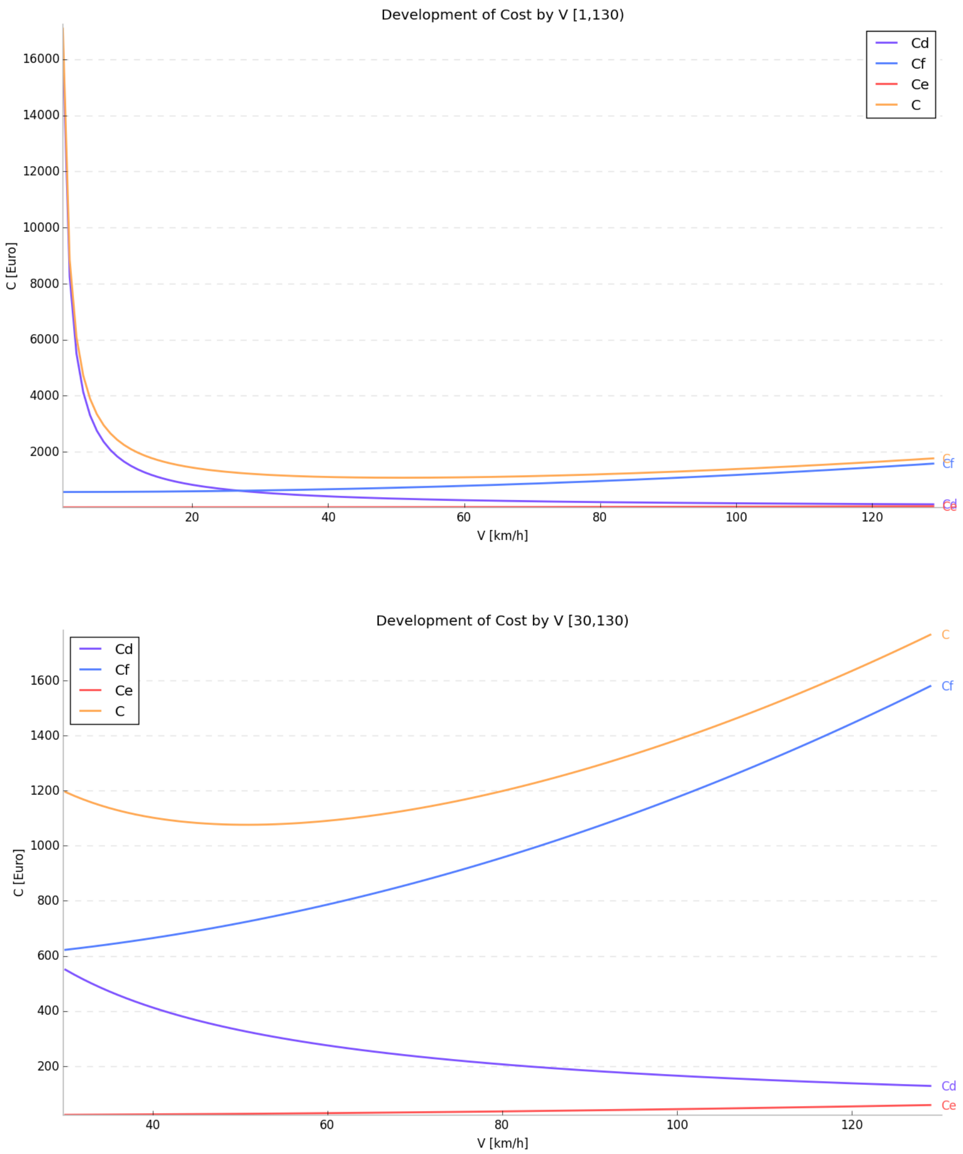


Figure 5.19: Problem 4 - Development Cost by Speed

speed. Since it is possible to calculate the optimal speed in beforehand, it is not sensible to use variable speed in the given context. However, this judgement is only valid for implementations without time windows. The next section, will investigate the case with time windows further.

5.5 Problem Five

The objective of Problem Six is to convey the importance of scheduling, by showing the restrictive capabilities of hard time windows. That is, it has to be investigated how strongly the results are effected, if time windows are introduced? Narrative wise, one is asked to investigate possibilities for increasing customer satisfaction, by decreasing the delay present in the current delivery process. Additionally, the question was raised, whether variable speed has a more significant impact on the results in an environment with time windows, as compared to the environment discussed in Problem Four (see Section 5.4).
As usual the required parameter are defined. Firstly the parameter groups *World* and *Product* stay the same, i.e.

$$
\begin{aligned}
\mathscr{W}_{P4} &:= \mathscr{W}_1. \\
\mathscr{P}_{P4} &:= \mathscr{P}_1
\end{aligned}
$$

Secondly, the parameter group *Vehicle* is locked based on the decision made in Problem Three (see Section 5.3). As established in Section 5.4 only $\mathscr{V}_C$ is discussed, i.e.

$$\mathscr{V}_{P4} := \mathscr{V}_C$$

Thirdly, other then the test scenarios, no additional set of nodes is required , thus $H := \{\mathscr{N}_{TS1}, \mathscr{N}_{TS2}, \mathscr{N}_{TS3}\}$ and by extension only one index set $I := \{0, 1, 2, 3, 4, 5, 6\}$

is required. The parameter $\widetilde{LU}_h, V_h, \Theta_h$ remain, i.e.

$$\begin{aligned}
&\forall h \in H: \\
&\qquad \widetilde{LU}_h = \{\widetilde{lu_{h_{ij}}} = (l_{h_{ij}}, u_{h_{ij}}) | \forall i, j \in I : (l_{h_{ij}}, u_{h_{ij}}) := (30, 130)\} \\
&\qquad\quad \Rightarrow l_{h_{ij}} := 30[km/h] \wedge u_{h_{ij}} := 130[km/h] \\
&\qquad V_h = \{v_{hij} | \forall i, j \in I : v_{h_{ij}} := 80[km/h]\} \\
&\qquad \Theta_h = \{\theta_{h_{ij}} | \forall i, j \in I : \theta_{h_{ij}} := 0[°]\}.
\end{aligned}$$

Furthermore, as this problem deals with time windows, the parameters $\widetilde{AB}_h$ and $\acute{T}_h$ have to be introduced. These are defined as

$$\begin{aligned}
\widetilde{AB}_{TS1} &:= \{(0\,;\,999), (1\,;\,10), (4\,;\,10), (5\,;\,15), (9\,;\,17), (6\,;\,13), (2\,;\,7)\} \\
\widetilde{AB}_{TS2} &:= \{(0\,;\,999), (20,5\,;\,25), (0\,;\,70), (0\,;\,70), (0\,;\,70), (0\,;\,70), (0\,;\,70)\} \\
\widetilde{AB}_{TS3} &:= \{(0\,;\,999), (1\,;\,50), (1\,;\,50), (1\,;\,50), (7\,;\,9), (1\,;\,50), (1\,;\,50)\}
\end{aligned}$$

$$\begin{aligned}
\acute{T}_{TS1} &:= \{0\,; 0,12\,;\, 0,24\,;\, 0,6\,;\, 0,36\,;\, 0,6\,;\, 0,84\} \\
\acute{T}_{TS2} &:= \{0\,; 0,57\,;\, 0,33\,;\, 0,49\,;\, 0,12\,;\, 0,25\,;\, 0,1\} \\
\acute{T}_{TS3} &:= \{0\,; 0,1\,;\, 0,24\,;\, 0,31\,;\, 0,24\,;\, 0,39\,;\, 0,55\}
\end{aligned}$$

Finally, the last parameter group *Cost* is for the most part defined based on the date of the problem. Therefore, $\mathscr{C}_{P4} := \mathscr{C}_{23.2.2015}$, which is

$$\begin{aligned}
\acute{p}_d &:= 11,35[Euro/h] \\
\acute{p}_f &:= 1,14[Euro/l] \\
\acute{p}_e &:= 18,24[Euro/t].
\end{aligned}$$

Having the input defined the last step is to identify the implementations. Narrative wise it would be sufficient to only solve for Oc and Oe. However, since one wants to test the restrictive effects of time windows on the model in general, it would be ideal solve for all objectives. Thus, implementations

$$
\begin{aligned}
\mathscr{I}_{Of} &:= Implementation(Load, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Od} &:= Implementation(Distance, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oe} &:= Implementation(Emission, Speed_{Fixed}, TW_{False}) \\
\mathscr{I}_{Oc} &:= Implementation(Cost, Speed_{Fixed}, TW_{False}) \\
\\
\mathscr{I}_{Of,TW} &:= Implementation(Load, Speed_{Fixed}, TW_{True}) \\
\mathscr{I}_{Od,TW} &:= Implementation(Distance, Speed_{Fixed}, TW_{True}) \\
\mathscr{I}_{Oe,TW} &:= Implementation(Emission, Speed_{Fixed}, TW_{True}) \\
\mathscr{I}_{Oc,TW} &:= Implementation(Cost, Speed_{Fixed}, TW_{True}) \\
\\
\mathscr{I}_{Of,V,TW} &:= Implementation(Load, Speed_{Variable}, TW_{True}) \\
\mathscr{I}_{Od,V,TW} &:= Implementation(Distance, Speed_{Variable}, TW_{True}) \\
\mathscr{I}_{Oe,V,TW} &:= Implementation(Emission, Speed_{Variable}, TW_{True}) \\
\mathscr{I}_{Oc,V,TW} &:= Implementation(Cost, Speed_{Variable}, TW_{True}) \\
\\
\mathscr{I}_{Oe,V} &:= Implementation(Emission, Speed_{Variable}, TW_{False}) \\
\mathscr{I}_{Oc,V} &:= Implementation(Cost, Speed_{Variable}, TW_{False})
\end{aligned}
$$

are required. since it is established that Od and Of are not effected by a scenario with variable speed and without time windows (see Section 5.4), $\mathscr{I}_{Ox,V}$ is limited to Oe and Oc. Anyhow, before one blindly executes the input for every objective it is advantageous to identify the values required for comparing the different implementations. In this case this becomes especially important, since if one intends to track the limitations imposed by the time windows a simple summary, will no longer be sufficient. Therefore, the usual summary, which is found in Table 5.23, is complemented by Table 5.24, which contains the time required to reach each point, as well as the travelling speed of the vehicle on each arc.

Unfortunately, before any further investigation are made a brief disclaimer is necessary. Given the environment[7] used to compute the results, the quality of the

[7] Hardware: MacBook Pro (13 Inch, Mid 2012); Operating-system: OSX El Captain 10.11.2

	$\mathscr{I}_{Ox}$				$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,TW,V}$				$\mathscr{I}_{Ox,V}$	
TS1	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
Path	0-5-4-3-2-1-6-0	0-1-6-4-5-3-2-0	0-6-1-2-3-4-5-0	0-6-1-2-3-4-5-0	0-1-6-2-3-5-4-0	0-1-6-5-2-3-4-0	0-1-6-2-3-5-4-0	0-1-6-2-3-5-4-0	0-6-1-2-5-3-4-0	0-6-1-2-5-3-4-0	0-1-6-2-3-5-4-0	0-1-6-2-5-3-4-0	0-6-1-2-3-4-5-0	0-6-1-2-3-4-5-0
$D[km]$	1127,13	1433,99	1127,13	1127,13	1368,83	1406,03	1368,83	1368,83	1329,26	1329,26	1368,83	1372,78	1127,13	1127,13
$F[t]$	93,78	37,07	47,98	47,98	39,26	38,32	39,26	39,26	47,67	47,67	39,26	38,94	47,98	47,98
$P[kg]$	1364,93	1411,03	1109,03	1109,03	1324,02	1361,73	1324,02	1324,02	1187,41	1187,41	963,36	1042,13	597,49	780,33
$C[Euro]$	770,96	835,14	656,40	656,40	786,94	809,10	786,94	786,94	842,19	842,19	757,67	719,61	693,92	588,31
$Pw[kg]$	410,21	521,89	410,21	410,21	498,17	511,71	498,17	498,17	483,77	483,77	498,17	499,61	410,21	410,21
$Pf[kg]$	359,47	131,84	103,57	103,57	102,96	107,49	102,96	102,96	101,17	101,17	102,96	105,76	103,57	103,57
$Pe[kg]$	595,24	757,30	595,24	595,24	722,88	742,53	722,88	722,88	602,47	602,47	362,23	436,77	83,71	266,54
$Cd[Euro]$	159,91	203,45	159,91	159,91	194,20	199,48	194,20	194,20	310,61	310,61	326,38	253,06	426,43	238,97
$Cf[Euro]$	586,16	605,96	476,26	476,26	568,59	584,79	568,59	568,59	509,93	509,93	413,71	447,54	256,59	335,11
$Ce[Euro]$	24,89	25,73	20,23	20,23	24,15	24,83	24,15	24,15	21,66	21,66	17,57	19,01	10,90	14,23
$T0j[h]$	16,85	20,68	16,85	16,85	19,87	20,34	19,87	19,87	30,13	30,13	31,52	25,06	40,33	23,81
TS2	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
Path	0-3-4-2-1-6-5-0	0-1-3-2-5-4-6-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-3-2-1-5-4-6-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0	0-3-1-2-5-4-6-0	0-5-6-1-2-4-3-0	0-5-6-1-2-4-3-0	0-3-4-2-1-6-5-0	0-3-4-2-1-6-5-0
$D[km]$	2979,74	5460,11	2979,74	2979,74	2979,74	4549,85	2979,74	2979,74	2979,74	4794,63	2979,74	2979,74	2979,74	2979,74
$F[t]$	64,96	50,00	64,96	64,96	64,96	53,74	64,96	64,96	64,96	50,94	78,98	78,98	64,96	64,96
$P[kg]$	3250,25	5765,98	3250,25	3250,25	3250,25	4703,96	3250,25	3250,25	3250,25	4413,05	2384,28	2452,61	1897,93	2389,18
$C[Euro]$	1877,83	3355,97	1877,83	1877,83	1877,83	2751,39	1877,83	1877,83	1877,83	2770,43	1729,80	1724,17	1977,00	1697,95
$Pw[kg]$	1084,45	1987,16	1084,45	1084,45	1084,45	1655,88	1084,45	1084,45	1084,45	1744,96	1084,45	1084,45	1084,45	1084,45
$Pf[kg]$	592,19	895,30	592,19	592,19	592,19	645,28	592,19	592,19	592,19	721,36	650,78	650,78	592,19	592,19
$Pv[kg]$	1573,62	2883,51	1573,62	1573,62	1573,62	2402,80	1573,62	1573,62	1573,62	1946,73	649,05	717,38	221,29	712,55
$Cd[Euro]$	422,75	774,65	422,75	422,75	422,75	645,51	422,75	422,75	422,75	794,78	662,40	626,19	1127,34	628,36
$Cf[Euro]$	1395,80	2476,16	1395,80	1395,80	1395,80	2020,09	1395,80	1395,80	1395,80	1895,16	1023,91	1053,26	815,05	1026,02
$Ce[Euro]$	59,28	105,16	59,28	59,28	59,28	85,79	59,28	59,28	59,28	80,48	43,48	44,73	34,61	43,57
$T0j[Euro]$	39,11	70,11	39,11	39,11	39,11	58,73	39,11	39,11	39,11	71,89	60,22	57,03	101,18	57,22
TS3	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
Path	0-5-6-3-4-2-1-0	0-6-5-3-2-4-1-0	0-5-6-3-4-2-1-0	0-5-6-3-4-2-1-0	0-4-3-6-5-2-1-0	0-4-6-5-3-2-1-0	0-4-3-6-5-2-1-0	0-4-3-6-5-2-1-0	0-4-3-6-5-2-1-0	0-4-5-6-3-2-1-0	0-4-3-6-5-1-2-0	0-4-5-6-3-2-1-0	0-5-6-3-4-2-1-0	0-5-6-3-4-2-1-0
$D[km]$	2818,68	3452,91	2818,68	2818,68	3201,16	3498,40	3201,16	3201,16	3201,16	3339,98	3332,10	3339,98	2818,68	2818,68
$F[t]$	56,94	55,17	56,94	56,94	64,99	61,39	64,99	64,99	64,99	63,15	66,79	63,15	56,94	56,94
$P[kg]$	2934,05	3562,90	2934,05	2934,05	3506,17	3847,29	3506,17	3506,17	3827,14	4285,20	2874,36	2899,56	1654,82	2114,74
$C[Euro]$	1713,42	2084,93	1713,42	1713,42	2023,81	2218,70	2023,81	2023,81	2156,79	2354,05	1967,11	1921,76	1807,23	1543,14
$Pw[kg]$	1025,84	1256,66	1025,84	1025,84	1165,03	1273,21	1165,03	1165,03	1165,03	1215,56	1212,69	1215,56	1025,84	1025,84
$Pf[kg]$	419,66	482,74	419,66	419,66	650,59	726,56	650,59	650,59	650,59	642,94	680,07	642,94	419,66	419,66
$Pv[kg]$	1488,56	1823,50	1488,56	1488,56	1690,55	1847,52	1690,55	1690,55	2011,51	2426,70	981,60	1041,05	209,33	669,25
$Cd[Euro]$	399,90	489,88	399,90	399,90	454,16	496,34	454,16	454,16	443,46	435,65	680,31	623,69	1066,40	596,41
$Cf[Euro]$	1260,01	1530,06	1260,01	1260,01	1505,70	1652,19	1505,70	1505,70	1643,54	1840,25	1234,37	1245,20	710,65	908,16
$Ce[Euro]$	53,51	64,98	53,51	53,51	63,94	70,17	63,94	63,94	69,80	78,15	52,42	52,88	30,18	38,57
$T0j[h]$	37,06	44,99	37,06	37,06	41,84	45,56	41,84	41,84	40,90	40,21	61,77	56,78	95,79	54,38

Table 5.23: Problem 5 - Output

solving process declined, when variable speed and time windows had to be considered. That is, it was not uncommon that the time window constraints where ignored. Moreover, in some cases it was even possible to easily construct a better

(15C50); Software: Microsoft Excel for Mac 2011, Version 1.4.6.0 (151221); Computation: Excel Solver preferably Evolutionary Algorithm (EA)

$T_{0j}[h]$	$\mathscr{I}_{Ox}$				$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,TW,V}$				$\mathscr{I}_{Ox,V}$	
TS1	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	1,63	1,80	1,49	1,49	1,80	1,80	1,80	1,80	2,13	2,13	2,15	2,44	3,97	2,22
2. Stop	6,30	2,43	2,84	2,84	2,43	2,43	2,43	2,43	3,87	3,87	2,93	3,23	6,16	3,81
3. Stop	9,77	9,62	4,89	4,89	5,43	6,31	5,43	5,43	5,52	5,52	6,44	6,73	11,44	6,83
4. Stop	11,72	14,05	6,48	6,48	7,03	9,29	7,03	7,03	11,35	11,35	8,46	9,90	15,29	9,09
5. Stop	13,89	16,47	10,19	10,19	9,44	10,88	9,44	9,44	13,58	13,58	11,16	12,63	24,16	14,33
6. Stop	14,52	18,42	14,62	14,62	14,12	14,58	14,12	14,12	16,28	16,28	16,78	16,99	35,38	20,78
7. Stop	16,85	20,68	16,85	16,85	19,87	20,34	19,87	19,87	30,13	30,13	31,52	25,06	40,33	23,81
TS2	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	6,12	10,23	6,12	6,12	6,12	6,12	6,12	6,12	6,12	6,44	5,23	5,23	16,32	9,19
2. Stop	10,56	24,45	10,56	10,56	10,56	15,74	10,56	10,56	10,56	23,99	14,99	13,72	27,34	15,51
3. Stop	17,76	34,07	17,76	17,76	17,76	21,00	17,76	17,76	17,76	29,19	25,59	22,93	46,34	26,20
4. Stop	23,01	44,56	23,01	23,01	23,01	33,18	23,01	23,01	23,01	43,30	33,98	30,79	59,80	33,79
5. Stop	29,73	56,21	29,73	29,73	29,73	44,83	29,73	29,73	29,73	57,80	44,80	41,61	76,76	43,48
6. Stop	35,39	65,72	35,39	35,39	35,39	54,34	35,39	35,39	35,39	67,08	50,66	47,47	91,70	51,95
7. Stop	39,11	70,11	39,11	39,11	39,11	58,73	39,11	39,11	39,11	71,89	60,22	57,03	101,18	57,22
TS3	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	3,91	6,40	3,91	3,91	7,79	7,79	7,79	7,79	8,78	8,42	8,66	8,66	10,44	5,82
2. Stop	8,65	11,30	8,65	8,65	14,41	18,55	14,41	14,41	16,52	14,45	17,69	17,73	22,42	12,71
3. Stop	13,81	14,06	13,81	13,81	19,33	23,45	19,33	19,33	20,75	20,03	24,25	24,02	35,26	20,10
4. Stop	20,49	22,75	20,49	20,49	24,22	26,21	24,22	24,22	26,27	24,97	30,50	30,82	52,56	29,91
5. Stop	26,40	28,66	26,40	26,40	31,18	34,90	31,18	31,18	31,27	31,11	43,05	42,31	67,92	38,65
6. Stop	31,08	39,01	31,08	31,08	35,86	39,58	35,86	35,86	34,77	36,35	49,49	48,28	80,00	45,50
7. Stop	37,06	44,99	37,06	37,06	41,84	45,56	41,84	41,84	40,90	40,21	61,77	56,78	95,79	54,38
$V[m/s]$	$\mathscr{I}_{Ox}$				$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,TW,V}$				$\mathscr{I}_{Ox,V}$	
TS1	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	15,56	15,56	18,61	16,39	8,33	14,94
2. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	12,49	12,49	16,94	16,94	8,33	14,92
3. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	28,06	28,06	18,06	18,06	8,33	14,82
4. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	9,44	9,44	16,94	18,06	8,33	14,88
5. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	24,72	24,72	19,17	18,89	8,33	14,87
6. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	32,78	32,78	18,06	18,33	8,33	14,87
7. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	8,89	8,89	8,33	15,56	8,33	14,85
TS2	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	21,11	14,72	14,72	8,33	14,80
2. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	17,78	13,00	15,00	8,33	15,07
3. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	23,61	13,00	15,00	8,33	14,87
4. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	16,39	14,00	15,00	8,33	15,08
5. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	17,78	15,00	15,00	8,33	14,98
6. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,78	15,28	15,28	8,33	14,77
7. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	20,28	15,00	15,00	8,33	15,33
TS3	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Od	Of	Oe	Oc	Oe	Oc
1. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	19,72	20,58	20,00	20,00	8,33	14,95
2. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	18,89	24,17	16,11	15,85	8,33	14,86
3. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	26,11	18,61	16,39	16,38	8,33	14,96
4. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	19,44	23,33	16,94	16,39	8,33	14,91
5. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	31,67	31,94	17,22	16,66	8,33	14,82
6. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	30,28	19,72	15,56	17,22	8,33	14,93
7. Stop	22,22	22,22	22,22	22,22	22,22	22,22	22,22	22,22	21,67	34,72	8,61	15,56	8,33	14,89

Table 5.24: Problem 5 - Extended Output

solution by hand, as compared to the solution provided by the algorithm. By switching from the Evolutionary Algorithm to the GRG Nonlinear the behaviour changed. The solver uphold the constraints given, but at the same time it seemed as if it converged at the first possible result rather than a "good" one. Due to these inconsistencies, further analysis into the behaviour within the optimisation process is reduced to a minimum. Hence, the remaining section is of a more descriptive nature.

The first step is to investigate potential changes in J. To accomplish this several visual aids are created. A type of which, Figure 5.20 to Figure 5.31 depict the time at which the vehicle arrives at a specific point T_{0j} compared to the bounds of the time window $\widetilde{AB}_h$. One figure represents one objective across all its variations, while the numbers represent the sequence of stops. These figures should help to visualise the restrictive abilities of time windows. Additionally, Table 5.25 complements said figures by depicting $\forall h \in H \forall i \in I : \Delta\widetilde{AB}_h := (T_{0i_h} - A_{h_i}, B_{h_i} - T_{0i_h})$ for every non time window implementation. Therefore, every negative value depicted, is equivalent with a breach of the time window constraint, which is emphasised by bold font. As for the implementations that considers time windows, such breaches should not exist, thus eliminating the need for inclusion. Lastly, as usual Figure 5.32, Figure 5.33 and Figure 5.34 visualise the changes in J. However, in order to prevent an even greater output volume, only the objectives Oe and Oc are contrasted.

<table>
<tr><th>[h]</th><th colspan="4">$\mathscr{I}_{Ox}$</th><th colspan="2">$\mathscr{I}_{Ox,V}$</th></tr>
<tr><td>TS1</td><td>Od</td><td>Of</td><td>Oe</td><td>Oc</td><td>Oe</td><td>Oc</td></tr>
<tr><td>1. Stop</td><td>(**-4,37** | 11,37)</td><td>(0,8 | 8,2)</td><td>(**-0,51** | 5,51)</td><td>(**-0,51** | 5,51)</td><td>(1,97 | 3,03)</td><td>(0,22 | 4,78)</td></tr>
<tr><td>2. Stop</td><td>(**-2,7** | 10,7)</td><td>(0,43 | 4,57)</td><td>(1,84 | 7,16)</td><td>(1,84 | 7,16)</td><td>(5,16 | 3,84)</td><td>(2,81 | 6,19)</td></tr>
<tr><td>3. Stop</td><td>(4,77 | 5,23)</td><td>(0,62 | 7,38)</td><td>(0,89 | 5,11)</td><td>(0,89 | 5,11)</td><td>(7,44 | **-1,44**)</td><td>(2,83 | 3,17)</td></tr>
<tr><td>4. Stop</td><td>(7,72 | **-1,72**)</td><td>(8,05 | **-1,05**)</td><td>(1,48 | 8,52)</td><td>(1,48 | 8,52)</td><td>(10,29 | **-0,29**)</td><td>(4,09 | 5,91)</td></tr>
<tr><td>5. Stop</td><td>(12,89 | **-3,89**)</td><td>(11,47 | **-1,47**)</td><td>(1,19 | 6,81)</td><td>(1,19 | 6,81)</td><td>(15,16 | **-7,16**)</td><td>(5,33 | 2,67)</td></tr>
<tr><td>6. Stop</td><td>(12,52 | **-7,52**)</td><td>(14,42 | **-8,42**)</td><td>(8,62 | **-1,62**)</td><td>(8,62 | **-1,62**)</td><td>(29,38 | **-22,38**)</td><td>(14,78 | **-7,78**)</td></tr>
<tr><td>7. Stop</td><td>(16,85 | 9982,15)</td><td>(20,68 | 9978,32)</td><td>(16,85 | 9982,15)</td><td>(16,85 | 9982,15)</td><td>(40,33 | 9958,67)</td><td>(23,81 | 9975,19)</td></tr>
<tr><td>TS2</td><td>Od</td><td>Of</td><td>Oe</td><td>Oc</td><td>Oe</td><td>Oc</td></tr>
<tr><td>1. Stop</td><td>(6,12 | 63,88)</td><td>(**-10,27** | 15,77)</td><td>(6,12 | 63,88)</td><td>(6,12 | 63,88)</td><td>(16,32 | 53,68)</td><td>(9,19 | 60,81)</td></tr>
<tr><td>2. Stop</td><td>(10,56 | 59,44)</td><td>(24,45 | 45,55)</td><td>(10,56 | 59,44)</td><td>(10,56 | 59,44)</td><td>(27,34 | 42,66)</td><td>(15,51 | 54,49)</td></tr>
<tr><td>3. Stop</td><td>(17,76 | 52,24)</td><td>(34,07 | 35,93)</td><td>(17,76 | 52,24)</td><td>(17,76 | 52,24)</td><td>(46,34 | 23,66)</td><td>(26,2 | 43,8)</td></tr>
<tr><td>4. Stop</td><td>(2,51 | 2,99)</td><td>(44,56 | 25,44)</td><td>(2,51 | 2,99)</td><td>(2,51 | 2,99)</td><td>(39,3 | **-33,8**)</td><td>(13,29 | **-7,79**)</td></tr>
<tr><td>5. Stop</td><td>(29,73 | 40,27)</td><td>(56,21 | 13,79)</td><td>(29,73 | 40,27)</td><td>(29,73 | 40,27)</td><td>(76,76 | **-6,76**)</td><td>(43,48 | 26,52)</td></tr>
<tr><td>6. Stop</td><td>(35,39 | 34,61)</td><td>(65,72 | 4,28)</td><td>(35,39 | 34,61)</td><td>(35,39 | 34,61)</td><td>(91,7 | **-21,7**)</td><td>(51,95 | 18,05)</td></tr>
<tr><td>7. Stop</td><td>(39,11 | 9959,89)</td><td>(70,11 | 9928,89)</td><td>(39,11 | 9959,89)</td><td>(39,11 | 9959,89)</td><td>(101,18 | 9897,82)</td><td>(57,22 | 9941,78)</td></tr>
<tr><td>TS3</td><td>Od</td><td>Of</td><td>Oe</td><td>Oc</td><td>Oe</td><td>Oc</td></tr>
<tr><td>1. Stop</td><td>(2,91 | 46,09)</td><td>(5,4 | 43,6)</td><td>(2,91 | 46,09)</td><td>(2,91 | 46,09)</td><td>(9,44 | 39,56)</td><td>(4,82 | 44,18)</td></tr>
<tr><td>2. Stop</td><td>(7,65 | 41,35)</td><td>(10,3 | 38,7)</td><td>(7,65 | 41,35)</td><td>(7,65 | 41,35)</td><td>(21,42 | 27,58)</td><td>(11,71 | 37,29)</td></tr>
<tr><td>3. Stop</td><td>(12,81 | 36,19)</td><td>(13,06 | 35,94)</td><td>(12,81 | 36,19)</td><td>(12,81 | 36,19)</td><td>(34,26 | 14,74)</td><td>(19,1 | 29,9)</td></tr>
<tr><td>4. Stop</td><td>(13,49 | **-9,49**)</td><td>(21,75 | 27,25)</td><td>(13,49 | **-9,49**)</td><td>(13,49 | **-9,49**)</td><td>(45,56 | **-41,56**)</td><td>(22,91 | **-18,91**)</td></tr>
<tr><td>5. Stop</td><td>(25,4 | 23,6)</td><td>(21,66 | **-17,66**)</td><td>(25,4 | 23,6)</td><td>(25,4 | 23,6)</td><td>(66,92 | **-17,92**)</td><td>(37,65 | 11,35)</td></tr>
<tr><td>6. Stop</td><td>(30,08 | 18,92)</td><td>(38,01 | 10,99)</td><td>(30,08 | 18,92)</td><td>(30,08 | 18,92)</td><td>(79 | **-30**)</td><td>(44,5 | 4,5)</td></tr>
<tr><td>7. Stop</td><td>(37,06 | 9961,94)</td><td>(44,99 | 9954,01)</td><td>(37,06 | 9961,94)</td><td>(37,06 | 9961,94)</td><td>(95,79 | 9903,21)</td><td>(54,38 | 9944,62)</td></tr>
</table>

Table 5.25: Problem 5 - Time Window infringement

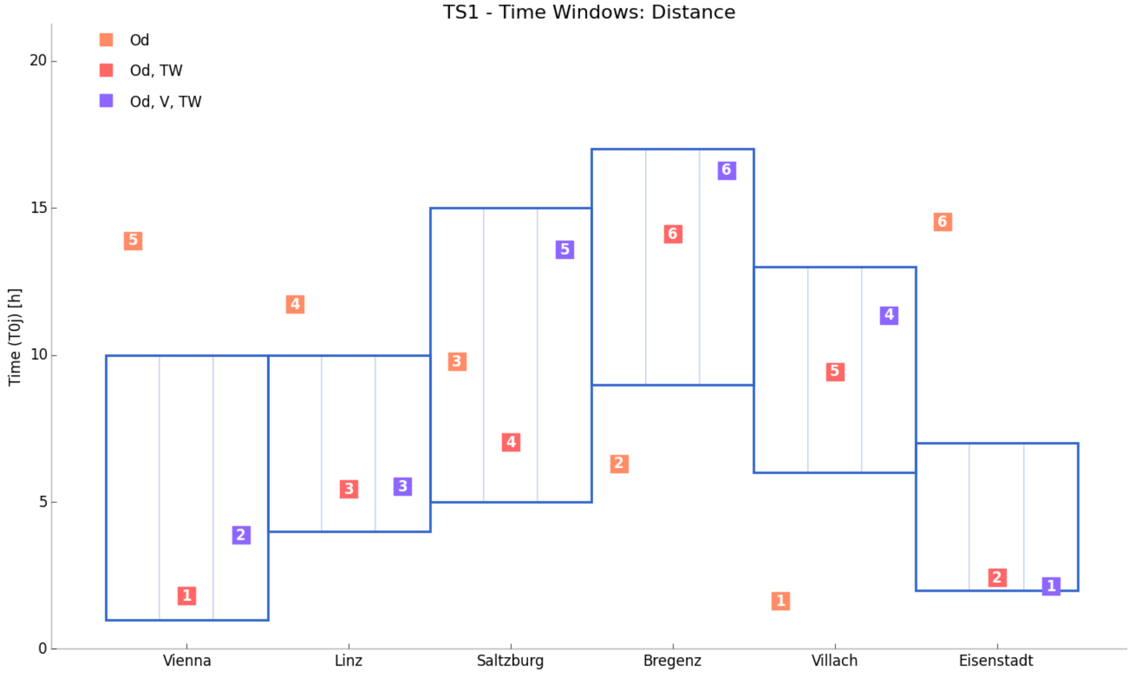


Figure 5.20: Problem 5 - Time windows: TS1 | Od

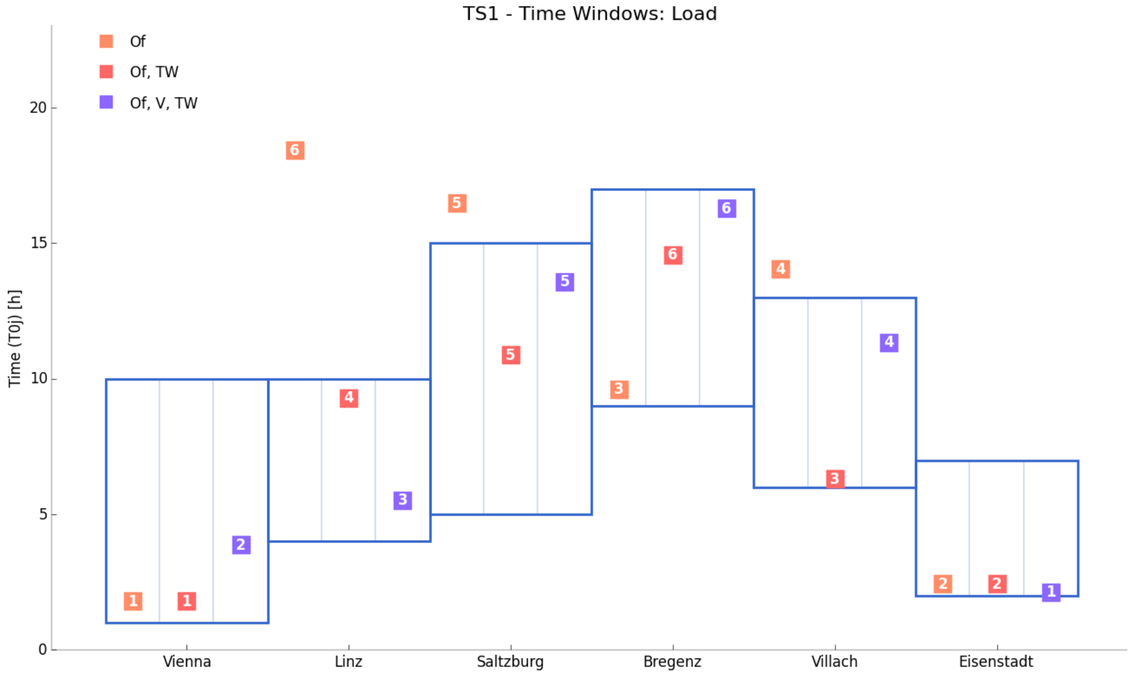


Figure 5.21: Problem 5 - Time windows: TS1 | Of

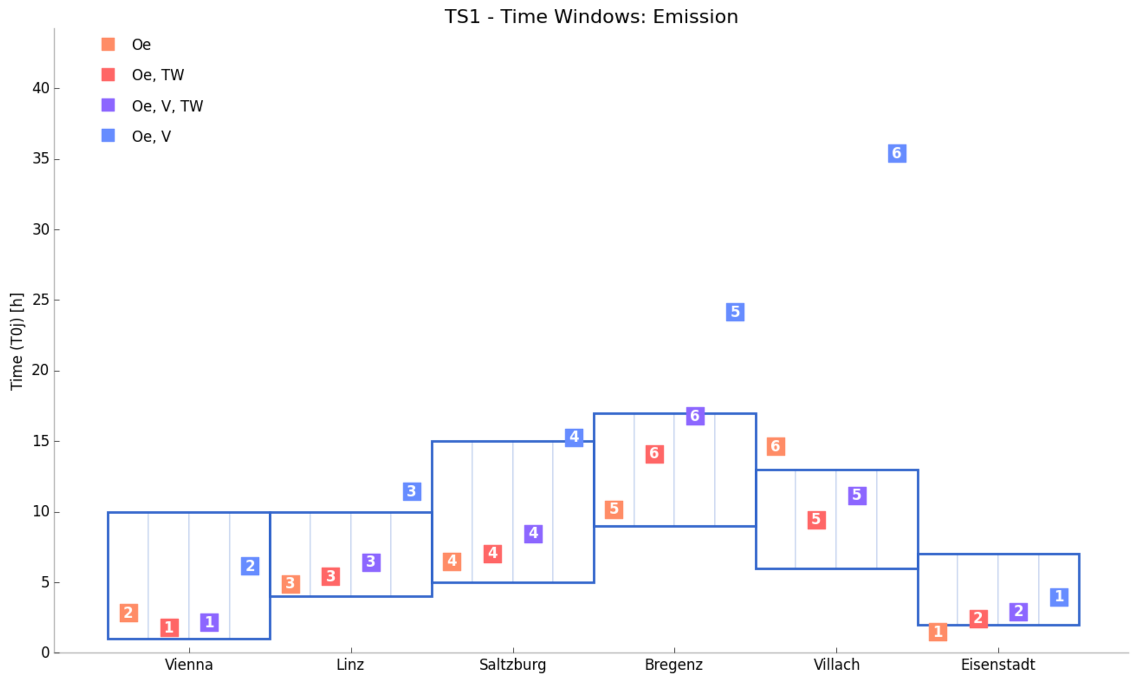


Figure 5.22: Problem 5 - Time windows: TS1 | Oe

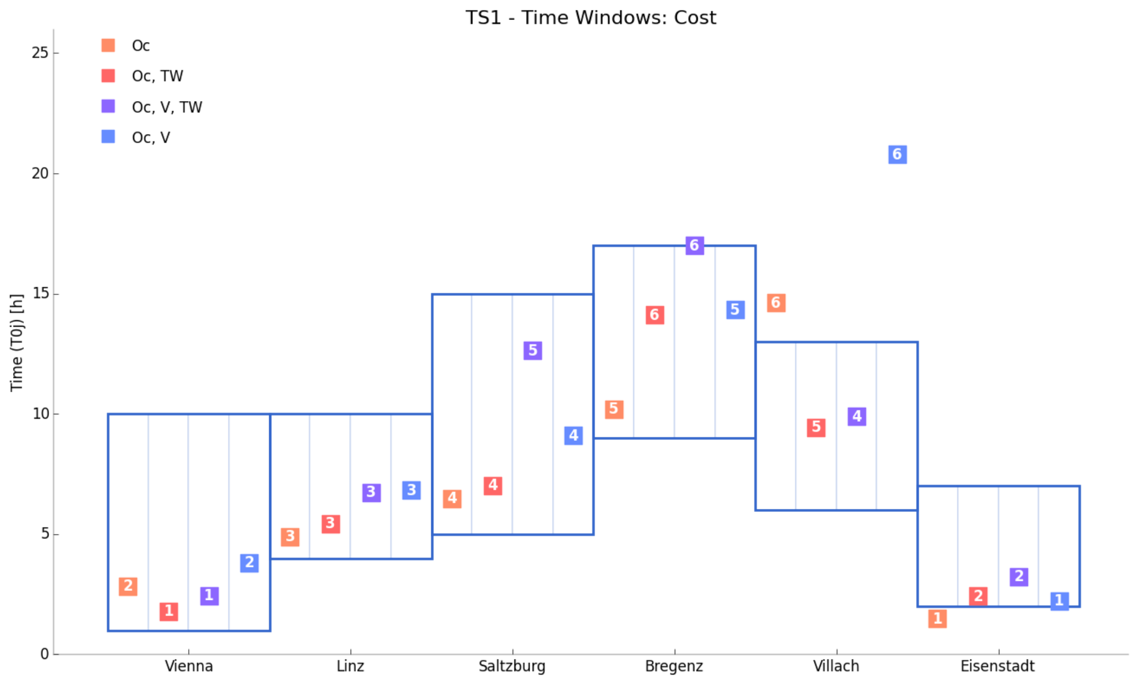


Figure 5.23: Problem 5 - Time windows: TS1 | Oc

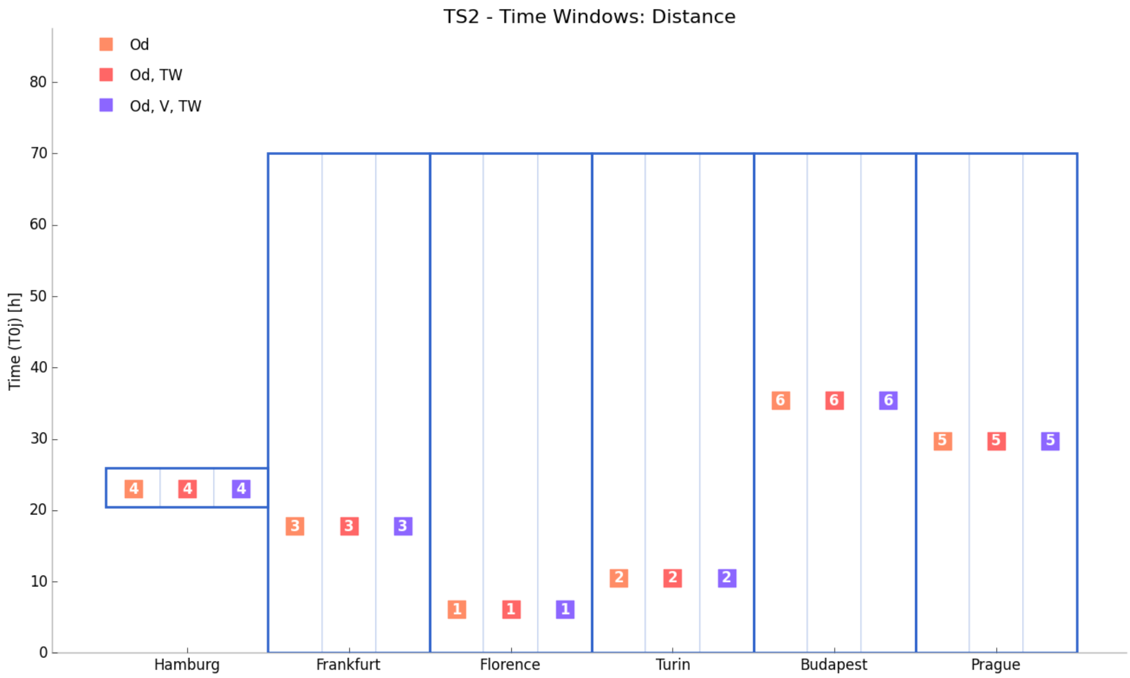


Figure 5.24: Problem 5 - Time windows: TS2 | Od

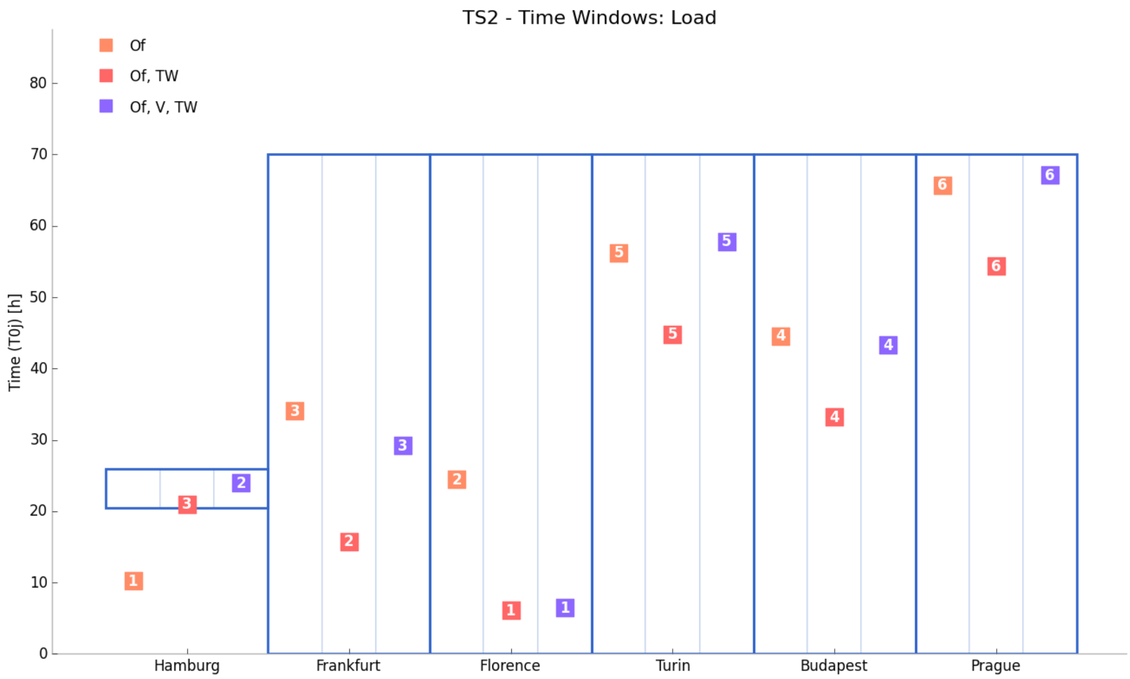


Figure 5.25: Problem 5 - Time windows: TS2 | Of

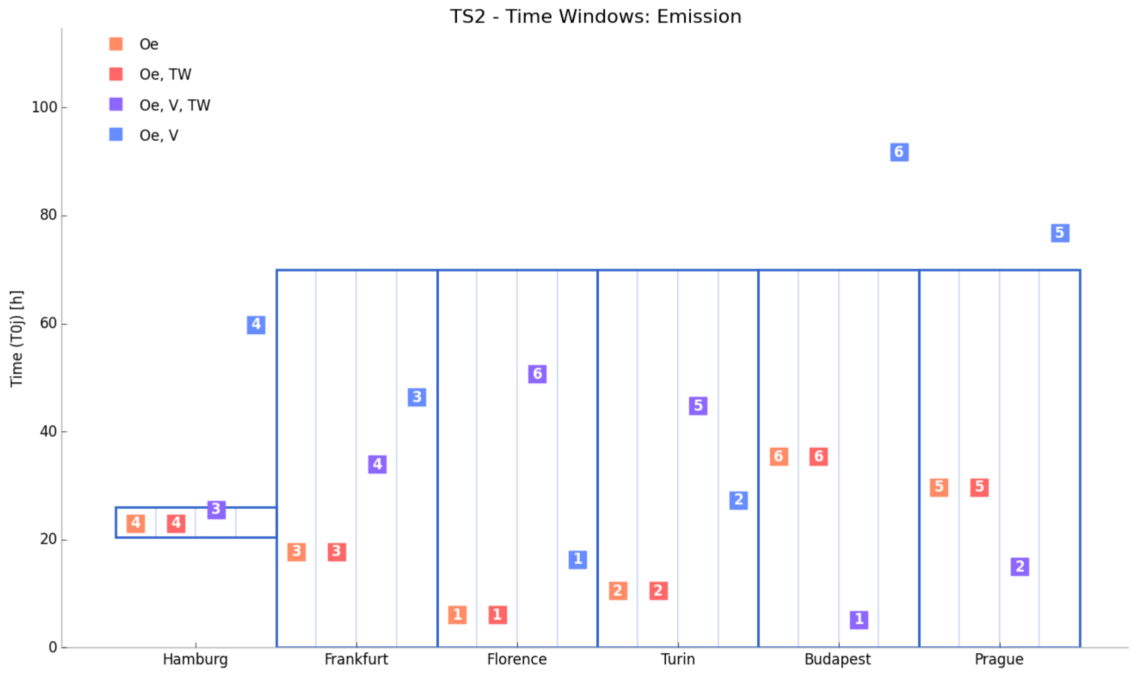


Figure 5.26: Problem 5 - Time windows: TS2 | Oe

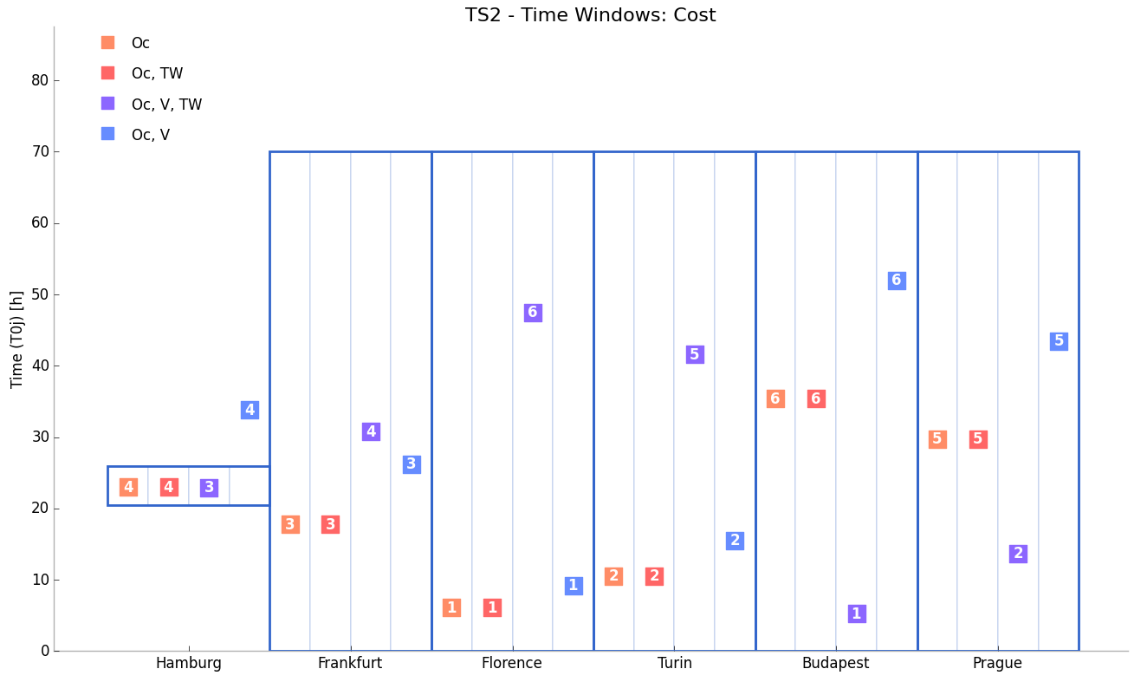


Figure 5.27: Problem 5 - Time windows: TS2 | Oc

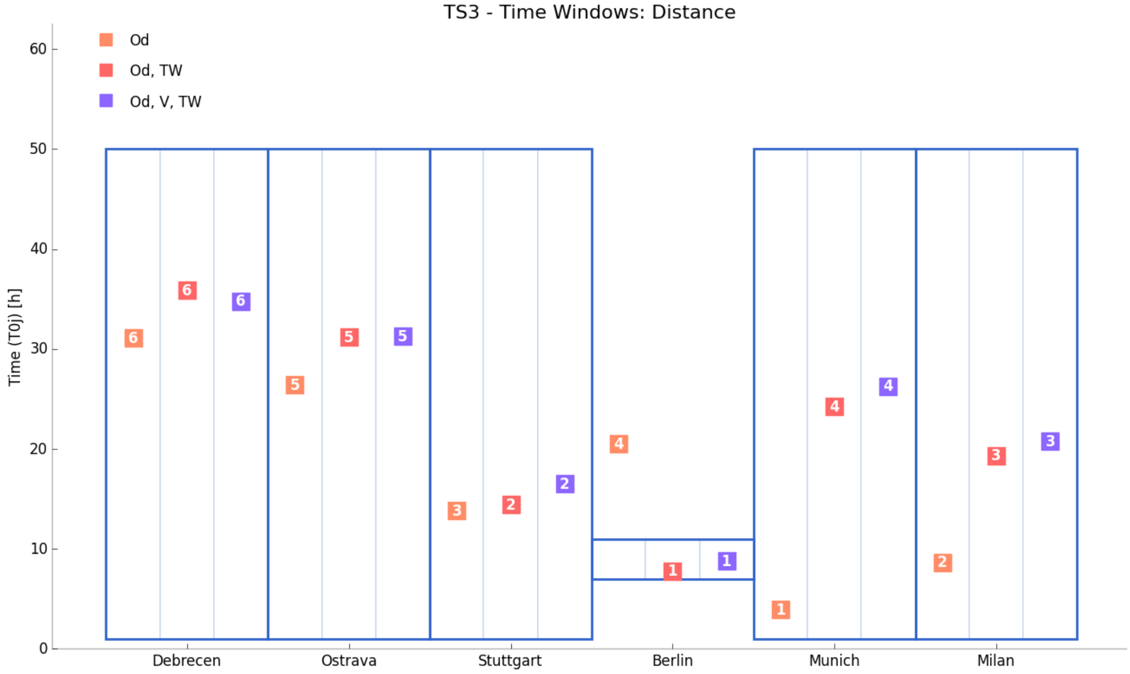


Figure 5.28: Problem 5 - Time windows: TS3 | Od

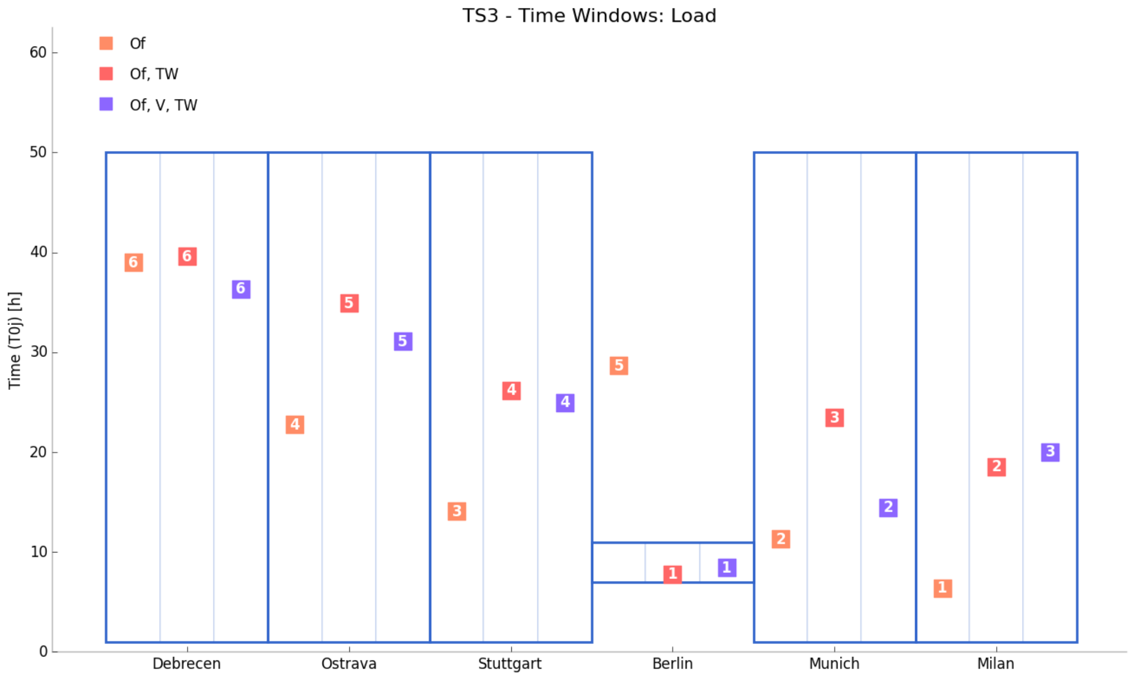


Figure 5.29: Problem 5 - Time windows: TS3 | Of

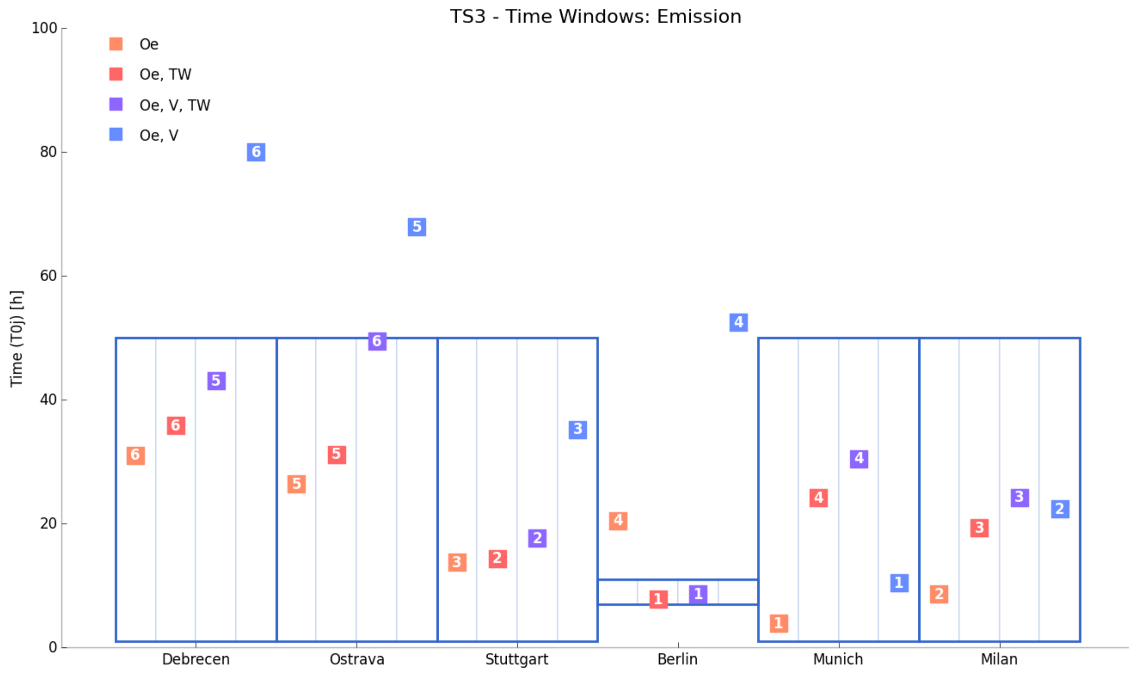


Figure 5.30: Problem 5 - Time windows: TS3 | Oe

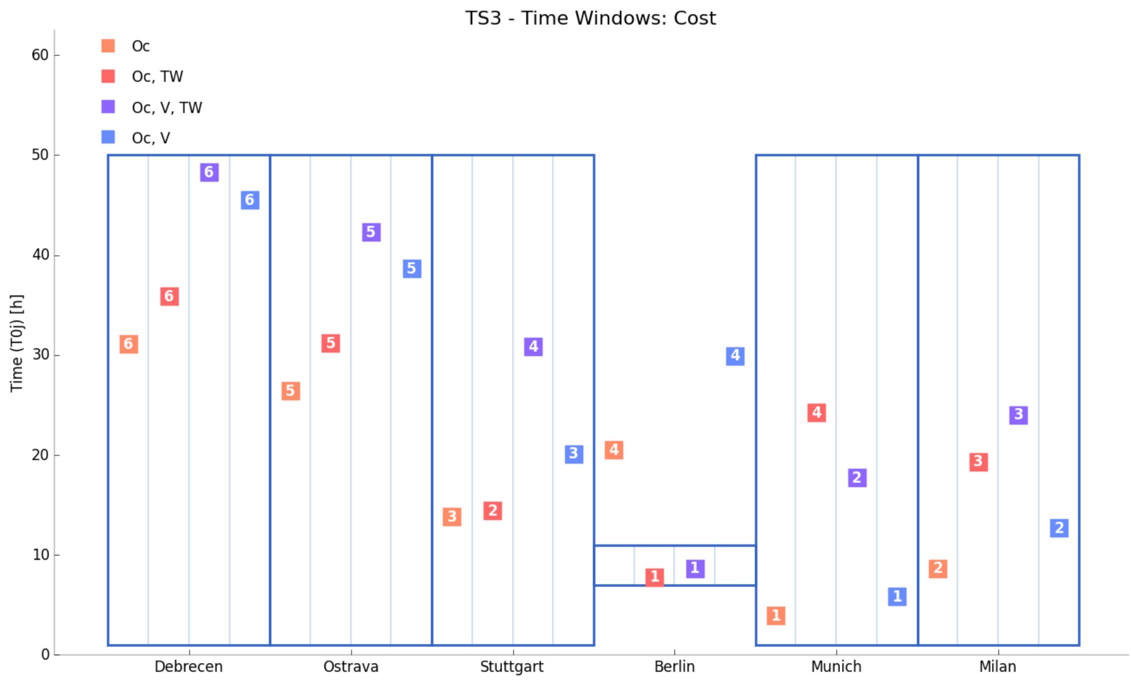


Figure 5.31: Problem 5 - Time windows: TS3 | Oc

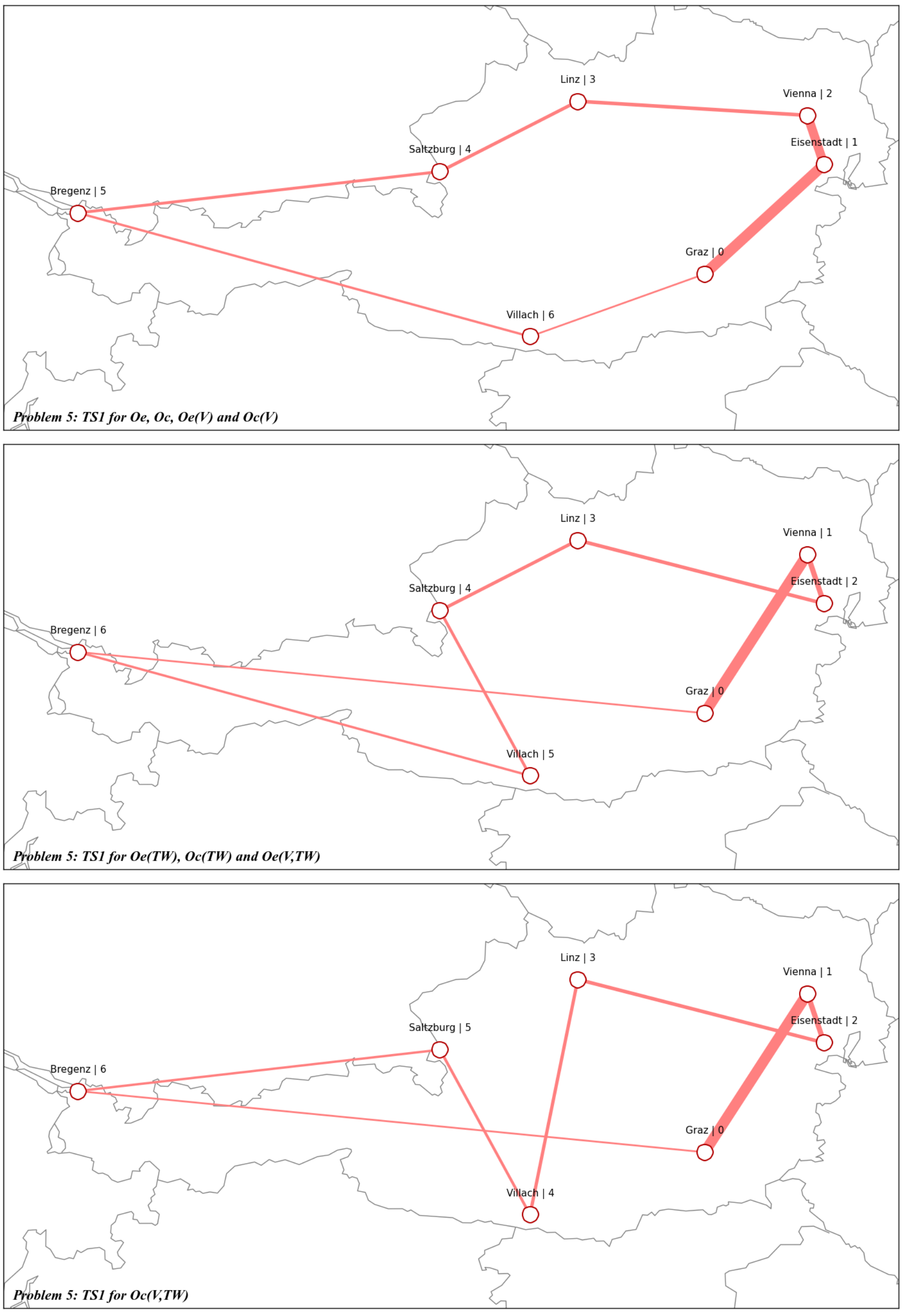


Figure 5.32: Problem 5 - Trip: TS1

At first, by briefly observing the output, one can conclude that apart from $I_{\mathscr{I}_{Od}(TS2)} = I_{\mathscr{I}_{Oe}(TS2)} = I_{\mathscr{I}_{Oc}(TS2)}$ every other implementation without time windows violated the time window constraints. Therefore, the above stated objectives should not experience any changes, if time windows are introduced. Anyhow, for further analysis

Od and Of shall be investigated fist. In the solution provided by $\mathscr{I}_{Od}(TS1)$, which is depicted in Figure 5.20, only one point is reached during the desired time frame. By introducing time window constraints a feasible solution was possible. However, restructuring J led to an increase in D by $21,4[\%]$. If $\mathscr{I}_{Od,V,TW}(TS1)$ is now considered and compared to $\mathscr{I}_{Od}(TS1)$ D only increased by $17,9[\%]$. Thereby, it is implied that variable speed expands the set of possible solution, which might lead to a better result. When looking at Of it is indeed true that $\mathscr{I}_{Od,TW}(TS1)$ induces an increase in load by $3,4[\%]$. However, the strange result is that by switching to $\mathscr{I}_{Od,V,TW}(TS1)$, the load increased by $28,6[\%]$, if compared to the original solution. This behaviour may be the result of the problems described earlier since, given simple logic the statement

$$\forall x \in \{Od, Of\} : \ \mathscr{I}_x(h) \leq \mathscr{I}_{x,V,TW}(h) \leq \mathscr{I}_{x,TW}(h)$$

has to be true. As for objectives Oe and Oc this is not as easily defined, since both of their objective values are influenced by speed. Anyhow, as for $TS2$, which is depicted in Figure 5.24 and Figure 5.25. It is apparent that even though, n_1, i.e. Hamburg, has a relatively narrow time window with $\widetilde{AB}_1 = (20,5\,;\,26)$, due to the fortunate placement of said time window no instance of Od is influenced by the newly imposed restrictions. By contrast, Of had to severely restructure the trip, as it was not possible to directly head for the point with the highest demand. To uphold the lower bound of the time window in a scenario with fixed speed, Hamburg had to be the third point in the trip, thus leading to an increase in load by $7,5[\%]$. Furthermore, as it should be the result from $\mathscr{I}_{Of,V,TW}(TS2)$ produced a better result with only a slight increase of $1,9[\%]$. Finally, $TS3$, which as visible in Figure 5.28 and Figure 5.29 has similar to $TS2$ also a rather narrow time window, i.e. $\widetilde{AB}_4 = (7\,;\,11)$ with $n_4 = Berlin$. However, since $\mathscr{I}_{Od}(TS3)$ and $\mathscr{I}_{Of}(TS3)$ arranged this point at a relativity late state of the trip, both of them have to significantly restructure the early part of the trip in order to accommodate the one time window. This had to be done since with fixed speed it would not have been possible to reach n_4 in time, which resulted in $j_1 \in J_{\mathscr{I}_{x,TW}(TS3)} : j_1 := (1,4)$. Furthermore, $J_{\mathscr{I}_{Od,TW}(TS3)} = J_{\mathscr{I}_{Od,V,TW}(TS3)}$, thus for both of them $\Delta D = 14,1[\%]$. However, the optimisation algorithm increased the speed in $\mathscr{I}_{Od,V,TW}(TS3)$ on some arcs, which led due to the quadratic nature of the fuel consumption model to an increase in GHG output by $30,4[\%]$. If compared to $\mathscr{I}_{Od,TW}(TS3)$ in which the increase was only

$19,5[\%]$, the major impact of speed can be observed. More importantly, however, it indicates that due to the fact that the speed is not part of the objective function, the speed can be set to any value, which is tolerated by the time window constraints. Lastly, for Of the two implementation differ, with $\Delta F_{\mathscr{I}_{Of,TW}(TS3)} = 11,3[\%]$ and $\Delta F_{\mathscr{I}_{Of,V,TW}(TS3)} = 14,5[\%]$, which yet again is a highly contradictory result. Therefore, further lowering the credibility of the results obtained by this method.

With Od and Of covered the next step is to assess Oe and Oc. At first, due

$\mathscr{I}_{Ox}$	$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,V,TW}$			
Emissions [kg]	ΔP_{Oe}	ΔP_{Oc}	$\Delta P_{Oe}[\%]$	$\Delta P_{Oc}[\%]$	ΔP_{Oe}	ΔP_{Oc}	$\Delta P_{Oe}[\%]$	$\Delta P_{Oc}[\%]$
TS1	214,99	214,99	19,39%	19,39%	-145,66	-66,89	-13,13%	-6,03%
TS2	0,00	0,00	0,00%	0,00%	-865,97	-797,65	-26,64%	-24,54%
TS3	572,12	572,12	19,50%	19,50%	-59,70	-34,50	-2,03%	-1,18%
Average	262,37	262,37	10,79%	10,79%	-357,11	-299,68	-14,69%	-12,33%
Cost [Euro]	ΔC_{Oe}	ΔC_{Oc}	$\Delta C_{Oe}[\%]$	$\Delta C_{Oc}[\%]$	ΔC_{Oe}	ΔC_{Oc}	$\Delta C_{Oe}[\%]$	$\Delta C_{Oc}[\%]$
TS1	130,54	130,54	19,89%	19,89%	101,26	63,21	15,43%	9,63%
TS2	0,00	0,00	0,00%	0,00%	-148,03	-153,65	-7,88%	-8,18%
TS3	310,39	310,39	18,12%	18,12%	253,69	208,34	14,81%	12,16%
Average	146,98	146,98	10,38%	10,38%	68,97	39,30	4,87%	2,78%

$\mathscr{I}_{Ox,V}$	$\mathscr{I}_{Ox,TW}$				$\mathscr{I}_{Ox,V,TW}$			
Emissions [kg]	ΔP_{Oe}	ΔP_{Oc}	$\Delta P_{Oe}[\%]$	$\Delta P_{Oc}[\%]$	ΔP_{Oe}	ΔP_{Oc}	$\Delta P_{Oe}[\%]$	$\Delta P_{Oc}[\%]$
TS1	726,53	543,69	121,60%	69,67%	365,88	261,81	61,24%	33,55%
TS2	1352,33	861,07	71,25%	36,04%	486,36	63,42	25,63%	2,65%
TS3	1851,35	1391,43	111,88%	65,80%	1219,54	784,81	73,70%	37,11%
Average	1310,07	932,06	94,70%	52,92%	690,59	370,01	49,92%	21,01%
Cost [Euro]	ΔC_{Oe}	ΔC_{Oc}	$\Delta C_{Oe}[\%]$	$\Delta C_{Oc}[\%]$	ΔC_{Oe}	ΔC_{Oc}	$\Delta C_{Oe}[\%]$	$\Delta C_{Oc}[\%]$
TS1	93,02	198,63	13,41%	33,76%	63,75	131,30	9,19%	22,32%
TS2	-99,17	179,88	-5,02%	10,59%	-247,20	26,23	-12,50%	1,54%
TS3	216,58	480,67	11,98%	31,15%	159,87	378,62	8,85%	24,54%
Average	70,14	286,39	4,70%	22,44%	-7,86	178,71	-0,53%	14,00%

Table 5.26: Problem 5 - Comparison against Implementations without Time Windows

to $\forall h \in H \forall x \in \{Oe, Oc\} : J_{\mathscr{I}_x(h)} = J_{\mathscr{I}_{x,V}(h)}$ any difference between these two results from variations between the fixed and the optimal speed. For Oe especially, it is highly apparent that the low optimal speed of $30[km/h]$ results in massive delays. Beginning with $TS2$, which is depicted in Figure 5.33, no change between $J_{\mathscr{I}_x(TS2)} = J_{\mathscr{I}_{x,TW}(TS2)}$ occurred. Thereby, it is yet again illustrated that even though only a small time window is given, which by extension massively restricts the possible set of solutions, the negative impact of such a restriction can be mitigated by ensuring that the time window is properly placed. The only, change experienced in $TS2$ for these objectives occurs when implementing variable speed. Table 5.26, which compares the emission produced by $\mathscr{I}_{Oc,TW}(h)$ and $\mathscr{I}_{Oc,V,TW}(h)$ against the respective counterparts $\mathscr{I}_{Oc}(h)$ and $\mathscr{I}_{Oc,V}(h)$ can be used for a more detailed analysis. Hence, as visible in said table, if $P_{\mathscr{I}_{Ox}(TS2)}$ and $P_{\mathscr{I}_{Ox}(TS2,V,TW)}$ are contrasted, a reduction by 27[%] for Oe and by 25[%] for Oc can be observed. However, as apparent by the increase of F in $\mathscr{I}_{Ox}(TS2, V, TW)$ and the fact that $D_{\mathscr{I}_{Ox}(TS2)} = D_{\mathscr{I}_{Ox}(TS2,V,TW)}$, this solution does only provide a better result due to the reduction in speed, because given the same speed $J_{\mathscr{I}_{Ox,V,TW}}$ would perform significantly worst. By comparing $P_{\mathscr{I}_{Ox,V,TW}}$ against $P_{\mathscr{I}_{Ox,V}}$ the result with time windows produced significantly more GHG output. However, if $C_{\mathscr{I}_{Oe,V,TW}}$ and $C_{\mathscr{I}_{Oe,V}}$ are compared, it is easy to detect that in this single case $\mathscr{I}_{Oe,V,TW}$ outperforms $\mathscr{I}_{Oe,V}$. This can be attributed to the narrow perspective of Oe regarding variable speed, which leads to massive travelling times, as suspected when looking at Figure 5.26. However, to be precise the travelling time between these two objectives differs by $40,96[h]$, thus resulting in the massive $Cd_{\mathscr{I}_{Oe,V}(TS2)}$. Moving on, $\mathscr{I}_{Oe,TW}(TS1)$ provides the same results as $\mathscr{I}_{Oc,TW}(TS1)$, if both are compared against $\mathscr{I}_{Ox}$. That is, $\Delta P_{\mathscr{I}_{Oe,TW}(TS1)} = \Delta P_{\mathscr{I}_{Oc,TW}(TS1)} = 19,39[\%]$ and $\Delta C_{\mathscr{I}_{Oe,TW}(TS1)} = \Delta C_{\mathscr{I}_{Oc,TW}(TS1)} = 19,89[\%]$. The reason behind this increase is that in both instances the distance increased by 21[%], while the load decreased by 18[%], thus implying the greater influence of the parameter distance over the GHG output. Furthermore, since the cost function reacts more strongly to an increase in distance, the stronger increase in costs is explained. However, if the comparison is extended to $\mathscr{I}_{Ox,V,TW}(TS1)$ it becomes apparent that $J_{\mathscr{I}_{Oe,V,TW}(TS1)} \neq J_{\mathscr{I}_{Oc,V,TW}(TS1)}$ (see Figure 5.32). Thereby, indicating that the variations between these two originate not only from differences in speed. The reason between $J_{\mathscr{I}_{Oe,V,TW}(TS1)} \neq J_{\mathscr{I}_{Oc,V,TW}(TS1)}$ will be investigated later on in this section. However, by simply comparing the results against $\mathscr{I}_{Ox}(TS1)$ it becomes apparent that while emission output decreases, the costs increase. The reason behind the increase in cost lies in the rise of Cd by $104,10[\%]$. By contrast, to TS2 this increase in cost was high enough to result in greater cost than $C_{\mathscr{I}_{Ox,V}}$. That is generally speaking, both $C_{\mathscr{I}_{Ox,TW}}$ and $C_{\mathscr{I}_{Ox,V,TW}}$ provide a less desired outcome, whether they are compared against $C_{\mathscr{I}_{Ox}}$ or $C_{\mathscr{I}_{Ox,V}}$. However, if P is compared $P_{\mathscr{I}_{Ox,V,TW}} < P_{\mathscr{I}_{Ox}}$. A similar case can be made for TS3. That is, as in TS1 $J_{\mathscr{I}_{Ox,TW}(TS3)} = J_{\mathscr{I}_{Ox}(TS3)}$, with $\Delta P_{\mathscr{I}_{Ox,TW}(TS3)} = 19,5[\%]$ and $\Delta C_{\mathscr{I}_{Ox,TW}(TS3)} = 18,12[\%]$. The increase in GHG output results from an increase in distance by $13,6[\%]$ and in load by $14,1[\%]$, which also explains why in comparison to TS1 the costs increase less significantly than the GHG output. If $\mathscr{I}_{Ox,V,TW}(TS3)$ is included into the comparison, the first thing to be noticed is that similar to TS1 $J_{\mathscr{I}_{Oe,V,TW}(TS3)} \neq J_{\mathscr{I}_{Oc,V,TW}(TS3)}$. The reason behind this will be discussed later. Apart from that the remaining deduction are fairly obvious. The GHG output decreases due to the variable speed, while the combination of a reduction in speed and an increase in distance results in a rise of Cd. Moreover, this increase in Cd leads to an increase in C. To summarise given $h \in H \setminus \{TS2\}$:

$$
\begin{aligned}
P_{\mathscr{I}_{Ox,V}(h)} &\leq P_{\mathscr{I}_{Ox,V,TW}(h)} \leq P_{\mathscr{I}_{Ox}(h)} \leq P_{\mathscr{I}_{Ox,TW}(h)} \\
C_{\mathscr{I}_{Oe}(h)} &\leq C_{\mathscr{I}_{Oe,V}(h)} \leq C_{\mathscr{I}_{Oe,V,TW}(h)} \leq C_{\mathscr{I}_{Oe,TW}(h)} \\
C_{\mathscr{I}_{Oc,V}(h)} &\leq C_{\mathscr{I}_{Oc}(h)} \leq C_{\mathscr{I}_{Oc,V,TW}(h)} \leq C_{\mathscr{I}_{Oc,TW}(h)}
\end{aligned}
$$

and for TS2

$$
\begin{aligned}
P_{\mathscr{I}_{Ox,V}(TS2)} &\leq P_{\mathscr{I}_{Ox,V,TW}(TS2)} \leq P_{\mathscr{I}_{Ox}(TS2)} \leq P_{\mathscr{I}_{Ox,TW}(TS2)} \\
C_{\mathscr{I}_{Oe,V,TW}(TS2)} &\leq C_{\mathscr{I}_{Oe}(TS2)} \leq C_{\mathscr{I}_{Oe,TW}(TS2)} \leq C_{\mathscr{I}_{Oe,V}(TS2)} \\
C_{\mathscr{I}_{Oc,V}(TS2)} &\leq C_{\mathscr{I}_{Oc,V,TW}(TS2)} \leq C_{\mathscr{I}_{Oc}(TS2)} \leq C_{\mathscr{I}_{Oc,TW}(TS2)}
\end{aligned}
$$

At first P shall be investigated. Regardless of the set of solutions and given $\forall j \in \bar{J} : \acute{v}_j < v_j$ with $\acute{v} \in V_{\mathscr{I}_{Ox,V}(h)}$ and $v \in V_{\mathscr{I}_{Ox}(h)}$ the conclusion can be drawn that regardless of the objective $P_{\mathscr{I}_{Ox,V}(h)}$ will always be the most preferable, while $P_{\mathscr{I}_{Ox,TW}(h)}$ will always be the least desirable solution. However, the arrangement between $P_{\mathscr{I}_{Ox,V,TW}(h)}$ and $P_{\mathscr{I}_{Ox}(h)}$ depends on the time windows and $V_{\mathscr{I}_{Ox}(h)}$. The same argument can be made for C when executed for Oc. The exception, however, is C with Oe since the rage of which can not explicitly stated. However, in general this insight is sufficient enough to conclude that in order to receive the best result with time windows, it is highly important to include variable speed into the decision process. Apart from that it has to discussed why $\forall h \in H \setminus \{TS2\} : J_{\mathscr{I}_{Oe,V,TW}(h)} \neq J_{\mathscr{I}_{Oc,V,TW}(h)}$. The first and most interesting difference between these instances is that for both scenarios $D_{\mathscr{I}_{Oe,V,TW}(h)} < D_{\mathscr{I}_{Oc,V,TW}(h)}$, which seems to contradict the assumption made in Section 5.2 that distance has a higher significance in Oc than in Od. However, if one considers that due to the restrictions imposed by the time windows, Oc can not freely select its desired speed of $14,88[m/s]$ (see Equation 5.4), the function has to compromise. As

$$\begin{aligned} Cd &:= \frac{\acute{p}_d \cdot d_j}{v_j} = \acute{p}_d \cdot t_j \\ &\Rightarrow \vee \uparrow \vee D \downarrow \Rightarrow Cd \downarrow \end{aligned}$$

It would be possible to compensate for an increase in distance by increasing the travelling speed as well. As indicated by the reduction in T_{0j}, which is $T_{0j} := T + \acute{T}$, this seems to be the case. Additionally, this is supported by the fact that the average speed is $V_{\mathscr{I}_{Oe,V,TW}(h)} < V_{\mathscr{I}_{Oc,V,TW}(h)}$. Furthermore, it has to be mentioned that in both scenarios $F_{\mathscr{I}_{Oe,V,TW}(h)} > F_{\mathscr{I}_{Oc,V,TW}(h)}$. However, this difference is especially in $TS1$ relatively minuscule. Anyhow, given this information the statements

$$\begin{aligned} F \downarrow \wedge D \uparrow \wedge V \uparrow &\Rightarrow P \uparrow \Rightarrow Cf \uparrow Ce \uparrow \\ &\Rightarrow T \downarrow \Rightarrow Cd \downarrow \end{aligned}$$

can be made. Furthermore, since

$$\begin{aligned} P_{\mathscr{I}_{Oe,V,TW}(h)} &< P_{\mathscr{I}_{Oc,V,TW}(h)} \wedge C_{\mathscr{I}_{Oe,V,TW}(h)} > C_{\mathscr{I}_{Oc,V,TW}(h)} \\ &\Rightarrow |Cf \uparrow Ce \uparrow| < |Cd \downarrow| \end{aligned}$$

is the behaviour shown by the model, it can be assumed that by selecting the shorter path the speed has to be reduced in order to conform with the time window constraints. This reduction of speed could have led to a disproportionate increase in Cd, thus preventing the shorter path from being feasible. Having now discussed the effect of time windows on a case by case basis, a general perspective of time windows and their impact has to be discussed. Hereby, the "generalised" view is limited to the averages of the results provided in Table 5.26. That is, if one intends to introduce time window into a model, which operates based on a fixed speed of $80[km/h]$ the GHG output will increase on average by $10,8[\%]$, while at the same time an increase in costs of $10,3[\%]$ can be expected. In most cases the result should be improved by introducing variable speed into the model. That is, for Oe the GHG output decreases by $14,7[\%]$, while costs increase on average by $4,87[\%]$, which is significantly less that the results provided by $\mathscr{I}_{Oe,TW}(h)$. As for the results of Oc, the GHG output decreased this case by $12,33[\%]$, while the costs only increased

by $2,78[\%]$. Therefore, it can be concluded that in this case the possibility of adjusting the speed during the optimisation process is potent enough to compensate for the limitations introduced by the time window constraint. Additionally, if the comparison is based on $\mathscr{I}_{Ox,V}(h)$ it becomes apparent that $\mathscr{I}_{Ox,V}(h)$ outperforms any other implementation, if GHG output is concerned. Therefore, the differences between $P_{\mathscr{I}_{Ox,V}(h)}$, $P_{\mathscr{I}_{Ox,TW}(h)}$ and $P_{\mathscr{I}_{Ox,V,TW}(h)}$ are even greater. However, due to the low travelling speed in $\mathscr{I}_{Oe,V}(h)$ the restrictions imposed by the time windows might even prevent the explosion of Cd. In this case, the average costs of $\mathscr{I}_{Oe,V,TW}(h)$ are even lower than the ones of $\mathscr{I}_{Oe,V}(h)$.
To conclude, the introduction of time windows, will if not scheduled appropriately cause deviation from the original trip, i.e. the trip which disregards time windows. If only the objectives Od and Of are considered this will always result in a worst or at best an equally good solution. This is due to the fact that neither D nor F are influenced by the speed of the vehicle. However, by introducing variable speed into the model it is indeed possible to achieve results, which deviate less from the optimal solution. Such a behaviour is caused by the fact that given variable speed it becomes possible to adjust the speed in a manner, which increases the set of possible solutions. For Oe and Oc a similar behaviour can be observed. However, since both objectives include the speed of the vehicle into their function, constructing a definitive alignment is not possible. However, it can be said with confidence that given $x \in \{Oe, Oc\}$ and with X being the respective value, i.e. P and C, as well as $\forall j \in \bar{J} : \acute{v}_j < v_j$ with $\acute{v} \in V_{\mathscr{I}_{Ox,V}(h)}$ and $v \in V_{\mathscr{I}_{Ox}(h)}$ that

$$X_{\mathscr{I}_{x,V}(h)} \leq X_{\mathscr{I}_{x}(h)} \leq X_{\mathscr{I}_{x,TW}(h)} \ \wedge \ X_{\mathscr{I}_{x,V}(h)} \leq X_{\mathscr{I}_{x,V,TW}(h)} \leq X_{\mathscr{I}_{x,TW}(h)}.$$

Unfortunately, it is not possible to put $X_{\mathscr{I}_{x}(h)}$ and $X_{\mathscr{I}_{x,V,TW}(h)}$ in order. However, this insight is sufficient enough to conclude that in order to receive the best result with time windows, it is recommended to include variable speed into the decision process. Moreover, in contrast to an environment which is not limited by time windows, it is not possible to calculated the desired speed in beforehand, as each $\forall j \in \bar{J} : v_j$ has to be individually adjusted in order to ensure that the time window constraints are uphold.

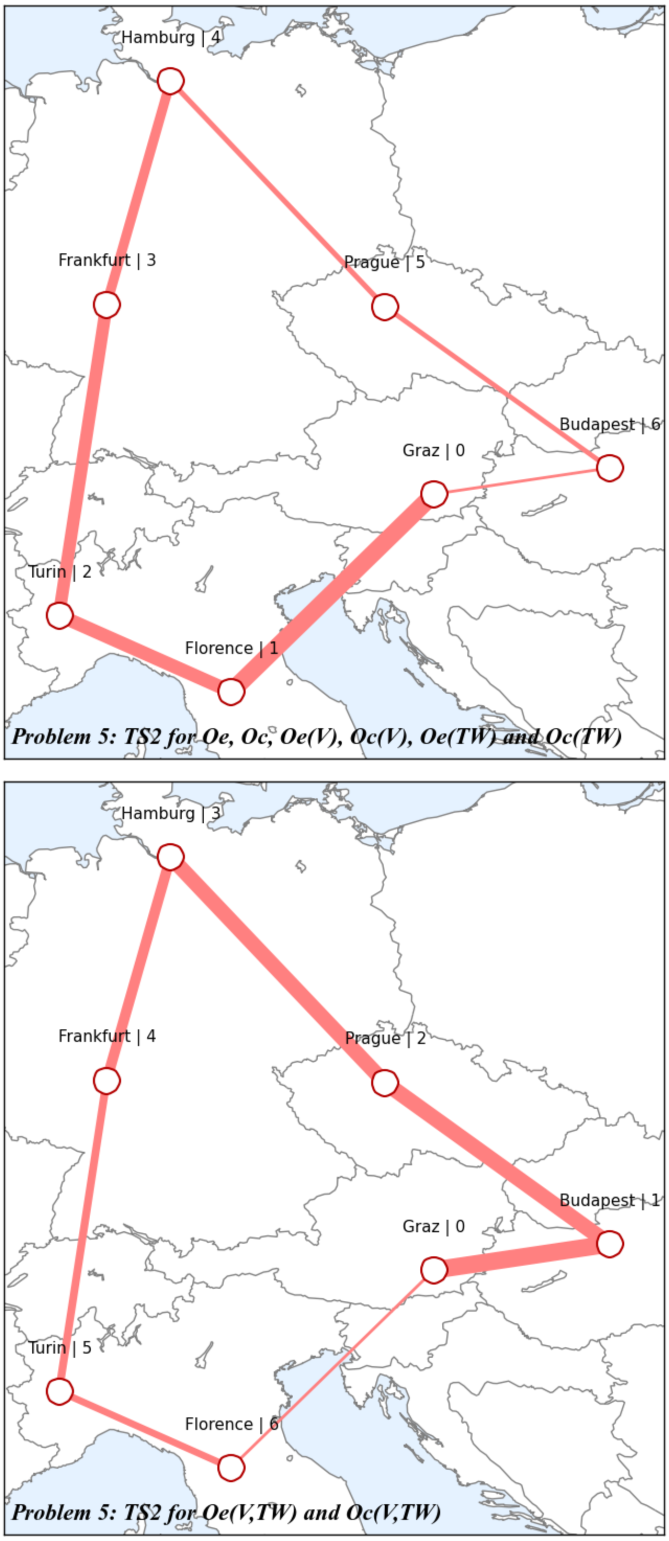


Figure 5.33: Problem 5 - Trip: TS2

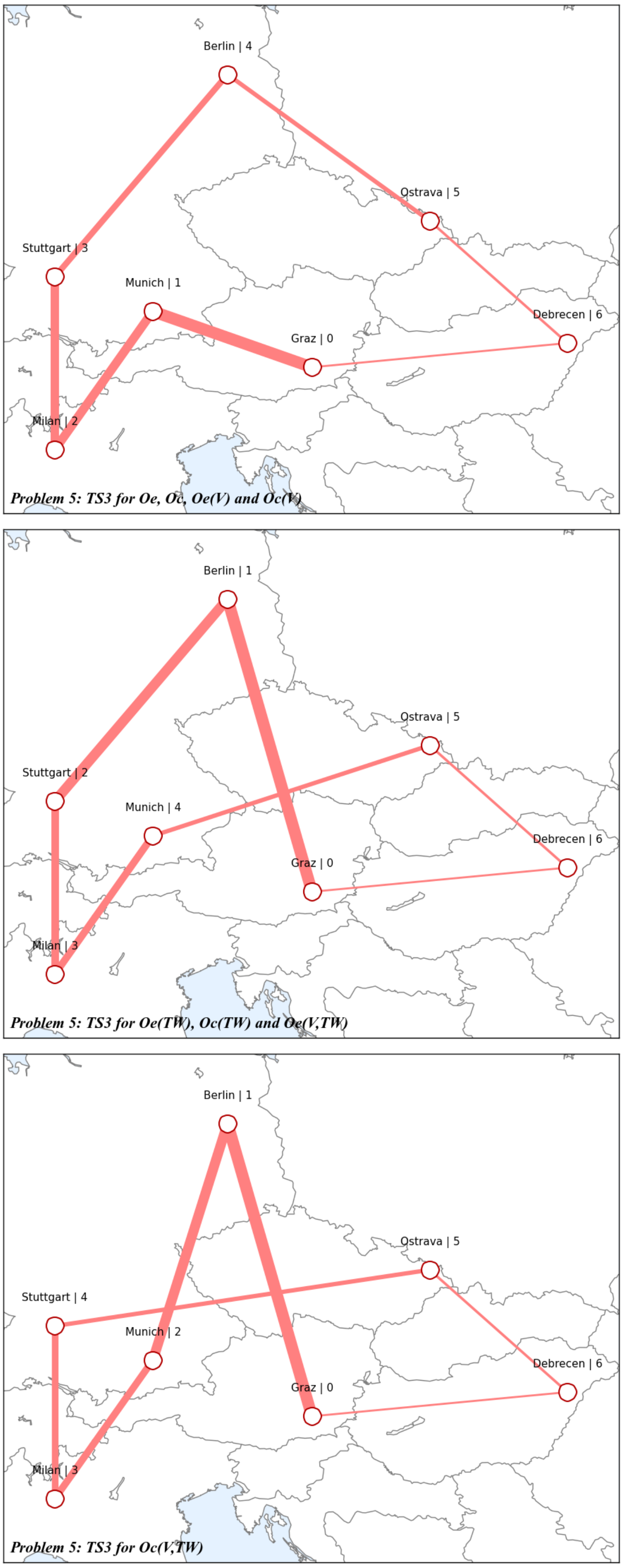


Figure 5.34: Problem 5 - Trip: TS3

Chapter 6

Summary

To summarise, this paper served as an introduction into the topic of the PRP. At first it investigated the environment in which the PRP exists. Thereby, putting the PRP into the context. From there, a single version of a PRP is introduced. This version is based on the PRP presented in Bektaş and Laporte (2011), with its parameter and components being introduced in Chapter 3. Additionally, a step by step guide for implementing this model into Excel was provided. This implementation was then used to solve the case provided in Chapter 4, which also contains a teaching note. Lastly, some behavioural aspects of the model were discussed. Furthermore, Chapter 2 and Chapter 5 try to answer a set of research questions. The results of these questions, which were provided throughout this paper are briefly addressed below.

- *Why is it important to introduce environmental factors into the VRP?*
  Freight transport causes a variety of different negative effects, namely resource consumption, land use, noise, accidents or emissions. All of which either negatively impact human health or the environment. The category emissions is even capable of causing negative environmental effects on a global scale. A particular kind of emissions, namely green house gases, contribute to climate change, which is predicted to destabilise the environment, thus causing humanitarian problems and negative economic effects. Since freight transport contributes to the global GHG output and thus to climate change, it is conceivable that introducing factors such as GHG output into the model may reduce the severity of climate change.

- *What is a GVRP, how is it structured and what is its relationship to the VRP?*

In general a GVRP can be defined as a VRP that minimises any kind of externalities that negatively impact the environment. May it be minimising the required energy or the GHG output, the collection of waste or even the routing of vehicles with alternative fuel sources. The GVRP group can roughly be differentiated into the Green-VRP (G-VRP), the Pollution Routing Problem (PRP) and the VRP in reverse logistics (VRPRL). Furthermore, since many GVRP have to approximate the energy consumption of the vehicle the GVRP is closely related to fuel consumption models. These models require various parameter to assess the fuel consumption of the vehicle. This can be done on a macroscopic and on a microscopic scale. Lastly, since the GVRP is above all also a VRP, it requires an optimisation algorithm in order to be solved. Here three prominent families exist, namely Exact Algorithms, Heuristics and Meta-Heuristics.

- *How can a GVRP be recognised and categorised?*
  Since all GVRP are also VRP, it can be concluded that these problems are also VRP variants. Furthermore, due to the concept of the Rich VRP it is possible that one instance of a GVRP could be classified under multiple VRP variants. Hence, it is best to distinguish a GVRP on three different levels. Firstly, it has to be assessed how many traits of other VRP variants are included. Then it has to be separated into G-VRP, PRP and VRPRL. Lastly, the type of fuel consumption model has to be determined. Here the microscopic model distinguishes itself from the macroscopic model by containing a higher amount of high quality parameter that describe the environment, the vehicle and the traffic.

- *By comparing the objectives distance and load, is it possible to identify which objective produces results that are closer to the results obtained by optimising for GHG emissions?*
  It was not entirely possible to answer this question. However, given the sample size, it was possible to conclude that in most instances Od will perform closer to Oe than Of. This conclusion could be made, since in a higher amount of observed cases $J_{I_{Od}} = J_{I_{Oe}}$. Furthermore, for nearly all cases $D_{I_{Od}} = D_{I_{Oe}}$ is true, thus indicating that the cases presented in which $J_{I_{Od}} \neq J_{I_{Oe}}$ are due to

the fact Od routed the vehicle through the same sequence, but in an opposite direction. An additional argument was made. That is, by comparing the differences between the respective objective and Oe, the difference in distance had a more significant effect on the variation in GHG output.

- *Is it possible to mimic the behaviour of the fuel consumption model, given a minimal amount of input?*

  In order to test the differences between the new function and the fuel consumption model the equation 5.2 was proposed. Based on the insight provided by this function and by the results obtained, it can be concluded that yes it is indeed possible. One contender is $d \cdot (f + w)$, which performs better than Od or Of respectively. However, due to its nature, this objective still dramatically oversimplifies the problem at hand. Hence, leading to a distortion of the relationship between D and F, i.e. it favours F. Furthermore, the new objective completely ignores the factor V. By contrast the function

  $$d \cdot (f + w + 1,75 \cdot A \cdot v^2) \quad v = [km/h]$$

  would behave even closer to the fuel consumption model. However, it requires far more information than $d \cdot (f + w)$.

- *How can the most suitable vehicle be chosen and how does the choice of vehicle impact the output of the model?*
  Due to differences in capacity, it becomes difficult to assess whether a vehicle would perform better or worse in a VRP. Hence, the best step is to start by calculating how efficient the vehicle translates the required energy into GHG output. This indicates that given $P_{[J]\mathscr{V}_A} = P_{[J]\mathscr{V}_B}$, which vehicle would perform better. From there three approaches are proposed. Firstly, a simple comparison of $P_{[J]}$. This approach favours vehicle with a low curb weight. Secondly, putting the GHG output in relation to the goods carried. This approach favours a vehicle with a high storage weight ratio. Finally, simulating the behaviour experienced when the demand is higher than the capacity of the vehicle. That is, including the possibility of multiple trips. This approach, however, favours vehicle with a high capacity. By applying these three ap-

proaches, one should be able to determine the most preferable vehicle. The impact on the models output was assessed by discussing the impact of different vehicle specific parameter. Firstly, the conversion factor only scales the overall GHG output, thus it does only effect the value of the result and not the behaviour of the model. Moreover, the parameter w influences Pw and the parameter A and $\acute{c}_d$ impact Pv. Hence, these three parameter influence the internal dynamics of the model. Furthermore, multiple trips are in this variation of the model not possible, thus the overall demand a trip can have is limited to the carrying capacity of the vehicle.

- *What are the differences between the objective cost and the objective emission?*
Given $R := 1 + \dfrac{\acute{p}_d}{v \cdot z \cdot (\alpha \cdot (f + w) + \beta \cdot v^2)}$ it can be concluded that the factor distance will always be of higher importance in Oc than in Oe, with the only exception being $\acute{p}_d := 0[Euro]$. Hence, how strongly the Oe and Oc will differ, depends on Cd. Furthermore, it can be concluded that a higher travelling speed, will increase the similarities between Oe and Oc.

- *How does the implementation of variable speed impact the outcome and in which cases is it necessary to incorporate variable speed into the model?*
It can be concluded that variable speed is only necessary when the model is restricted by time windows. This is due to the fact that both Od and Of will not respond to changes in speed, if there are no time windows present. Even though, it is true that Oe and Oc both respond to changes in speed, it is in both cases possible to calculate the speed in beforehand. However, while the optimal speed can be calculated, the behaviour of the model changes with an increase or decrease in speed. For example, an increase in speed increases the priority of the factor distance in the fuel consumption model. Above all, it can be said that by selecting the optimal speed the results can be improved significantly.
- *How does the introduction of time window constraints impact the output of the model?*
It can be concluded that an implementation without time windows and with variable speed will always achieve the most desirable result. Moreover, an implementation with time windows and without variable speed will always

provide the least desirable results. For the other two permutations it is not so clear, since their order depends on the selected speed and the restrictions of the time windows. However, it can be said that in every case it is more preferable to include variable speed into the optimisation process, as it expands the set of possible solutions. Furthermore, while it is true that a narrow time window will heavily restrict the optimisation process, good scheduling can mitigate the potential negative effects of said time windows.

Bibliography

Ahn, K., Rakha, H., Trani, A., Van Aerde, M., 2002. Estimating vehicle fuel consumption and emissions based on instantaneous speed and acceleration levels. Journal of transportation engineering 128 (2), 182–190.

Akcelik, R., Besley, M., 2003. Operating cost, fuel consumption, and emission models in aasidra and aamotion. In: 25th Conference of Australian Institutes of Transport Research (CAITR 2003). pp. 1–15.

Alwakiel, H. N., 2011. Leveraging weigh-in-motion (wim) data to estimate link-based heavy-vehicle emissions. Ph.D. thesis, Portland State University.

Archetti, C., Bouchard, M., Desaulniers, G., 2011. Enhanced branch and price and cut for vehicle routing with split deliveries and time windows. Transportation Science 45 (3), 285–298.

Bank, E. C., 11 2015. Euro exchange rates usd. [Online; accessed 25.11.2015]. URL `https://www.ecb.europa.eu/stats/exchange/eurofxref/html/eurofxref-graph-usd.en.html`

Bank, W., 1 2016. Climate change knowledge portal. [Online; accessed 14.1.2016]. URL `http://sdwebx.worldbank.org/climateportal/index.cfm?page=global_map_region&ThisMap=EU`

Bauer, J., Bektaş, T., Crainic, T. G., 2010. Minimizing greenhouse gas emissions in intermodal freight transport: an application to rail service design. Journal of the Operational Research Society 61 (3), 530–542.

Bektaş, T., Laporte, G., 2011. The pollution-routing problem. Transportation Research Part B: Methodological 45 (8), 1232–1250.

Beltrami, E. J., Bodin, L. D., 1974. Networks and vehicle routing for municipal waste collection. Networks 4 (1), 65–94.

BMF, 8 2015. Fahrzeugklassen. [Online; accessed 30.08.2015].
URL `https://www.help.gv.at/Portal.Node/hlpd/public/content/6/Seite.061800.html#Klasse0`

bmwfw, 11 2015. Treibstoffpreismonitor. [Online; accessed 20.11.2015].
URL `http://www.bmwfw.gv.at/EnergieUndBergbau/Energiepreise/Seiten/MonitorTreibstoff.aspx?Report=9`

Bram, K. A. P. G. O. K. P. N. K. N. B. K. L. L. W. L. B., 2015. State and trends of carbon pricing 2015. Washington, D.C. : World Bank Group, 92.

Brandão, J., 2004. A tabu search algorithm for the open vehicle routing problem. European Journal of Operational Research 157 (3), 552–564.

Caceres-Cruz, J., Arias, P., Guimarans, D., Riera, D., Juan, A. A., 2014. Rich vehicle routing problem: Survey. ACM Computing Surveys (CSUR) 47 (2), 32.

Cao, E., Lai, M., 2010. The open vehicle routing problem with fuzzy demands. Expert Systems with Applications 37 (3), 2405–2411.

Carbone, G., Mangialardi, L., Mantriota, G., 2001. Fuel consumption of a mid class vehicle with infinitely variable transmission. SAE TRANSACTIONS 110 (3), 2474–2484.

Carter, C. R., Ellram, L. M., 1998. Reverse logistics: a review of the literature and framework for future investigation. Journal of business logistics 19 (1), 85.

Center, A. F. D., 2013. Alternative fuels data center fuel properties comparison. US Department of Energy–Energy Efficiency and Renewable Energy–Alternative Fuels Data Center.

Cheng, R., Gen, M., Tozawa, T., 1995. Vehicle routing problem with fuzzy due-time using genetic algorithms. Japan Society for Fuzzy Theory and Systems 7 (5), 1050–1061.

Choi, E., Tcha, D.-W., 2007. A column generation approach to the heterogeneous fleet vehicle routing problem. Computers & Operations Research 34 (7), 2080–2095.

Christofides, N., Beasley, J. E., 1984. The period routing problem. Networks 14 (2), 237–256.

Clarke, G. u., Wright, J. W., 1964. Scheduling of vehicles from a central depot to a number of delivery points. Operations research 12 (4), 568–581.

Committee for the National Tire Efficiency Study, U. S., 2006. Tires and passenger vehicle fuel economy. Special report, Transportation Research Board of the National Academies.

Cook, T. M., Russell, R. A., 1978. A simulation and statistical analysis of stochastic vehicle routing with timing constraints. Decision sciences 9 (4), 673–687.

Cordeau, J.-F., Gendreau, M., Hertz, A., Laporte, G., Sormany, J.-S., 2005. New heuristics for the vehicle routing problem. Springer.

Cullinane, S., Edwards, J., 2010. Assessing the environmental impacts of freight transport. Green Logistics: Improving The Environmental Sustainability Of Logistics., 31–48.

Dantzig, G. B., Ramser, J. H., 1959. The truck dispatching problem. Management science 6 (1), 80–91.

Demir, E., Bektaş, T., Laporte, G., 2012. An adaptive large neighborhood search heuristic for the pollution-routing problem. European Journal of Operational Research 223 (2), 346–359.

Demir, E., Bektaş, T., Laporte, G., 2014. A review of recent research on green road freight transportation. European Journal of Operational Research 237 (3), 775–793.

Den Boer, L., Schroten, A., 2007. Traffic noise reduction in europe. CE Delft 14, 2057–2068.

Dondo, R., Cerdá, J., 2007. A cluster-based optimization approach for the multi-depot heterogeneous fleet vehicle routing problem with time windows. European Journal of Operational Research 176 (3), 1478–1507.

Dror, M., Trudeau, P., 1989. Savings by split delivery routing. Transportation Science, 141–145.

Dror, M., Trudeau, P., 1990. Split delivery routing. Naval Research Logistics (NRL) 37 (3), 383–402.

Du Plessis, H. W., Visser, A. T., Curtayne, P. C., 1990. Fuel consumption of vehicles as affected by road-surface characteristics. Surface characteristics of roadways: International research and technologies 480, 496.

EEX, 11 2015. European emission allowances - global environmental exchange. [Online; accessed 7.11.2015].
URL `https://www.eex.com/en/market-data/emission-allowances/spot-market/european-emission-allowances#!/2015/11/06`

Eksioglu, B., Vural, A. V., Reisman, A., 2009. The vehicle routing problem: A taxonomic review. Computers & Industrial Engineering 57 (4), 1472–1483.

Erdoğan, S., Miller-Hooks, E., 2012. A green vehicle routing problem. Transportation Research Part E: Logistics and Transportation Review 48 (1), 100–114.

Facts, E., 2005. Average carbon dioxide emissions resulting from gasoline and diesel fuel. US EPA, February.

Fagerholt, K., Laporte, G., Norstad, I., 2010. Reducing fuel emissions by optimizing speed on shipping routes. Journal of the Operational Research Society 61 (3), 523–529.

Faulin, J., Juan, A., Lera, F., Grasman, S., 2011. Solving the capacitated vehicle routing problem with environmental criteria based on real estimations in road transportation: a case study. Procedia-Social and Behavioral Sciences 20, 323–334.

Felbermayr, 2 2016. Felbermayr sattelzugmaschinen. [Online; accessed 22.2.2016].
URL `http://www.felbermayr.cc/fileadmin/transporte_hebetechnik_pdfs/strasse_gereaeteuebersicht/Zugfahrzeuge.pdf`

Felipe, Á., Ortuño, M. T., Righini, G., Tirado, G., 2014. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. Transportation Research Part E: Logistics and Transportation Review 71, 111–128.

Francis, C. D., Ortega, C. P., Cruz, A., 2009. Noise pollution changes avian communities and species interactions. Current biology 19 (16), 1415–1419.

Gendreau, M., Laporte, G., Séguin, R., 1996. Stochastic vehicle routing. European Journal of Operational Research 88 (1), 3–12.

Goel, A., Vidal, T., 2013. Hours of service regulations in road freight transport: an optimization-based international assessment. Transportation science 48 (3), 391–412.

Hall, N., 5 2015a. The drag coefficient. [Online; accessed 30.8.2015].
URL `https://www.grc.nasa.gov/www/K-12/airplane/dragco.html`

Hall, N., 5 2015b. Shape effects on drag. [Online; accessed 15.9.2015].
URL `https://www.grc.nasa.gov/www/K-12/airplane/shaped.html`

Hall, N., 5 2015c. The weight equation. [Online; accessed 15.9.2015].
URL `https://www.grc.nasa.gov/www/K-12/airplane/wteq.html`

Havlicek, H., 2006. Lineare Algebra für Technische Mathematiker. Heldermann.

Helms, H., Pehnt, M., Lambrecht, U., Liebich, A., 2010. Electric vehicle and plug-in hybrid energy efficiency and life cycle emissions. In: 18th International Symposium Transport and Air Pollution, Session. Vol. 3. p. 113.

Hong, L., 2012. An improved lns algorithm for real-time vehicle routing problem with time windows. Computers & Operations Research 39 (2), 151–163.

Kanari, N., Pineau, J.-L., Shallari, S., 2003. End-of-life vehicle recycling in the european union. Jom 55 (8), 15–19.

Kara, I., Kara, B. Y., Yetis, M. K., 2007. Energy minimizing vehicle routing problem. In: Combinatorial optimization and applications. Springer, pp. 62–71.

Koetse, M. J., Rietveld, P., 2009. The impact of climate change and weather on transport: An overview of empirical findings. Transportation Research Part D: Transport and Environment 14 (3), 205–221.

Kossoy, A., Oppermann, K., Platonova-Oquab, A., Suphachalasai, S., Höhne, N., Klein, N., Gilbert, A., Lam, L., Toop, G., Wu, Q., et al., 2014. State and trends of carbon pricing 2014.

Kossoy, A., Oppermann, K., Reddy, R. C., Bosi, M., Boukerche, S., Höhne, N., Klein, N., Gilbert, A., Jung, M., Borkent, B., et al., 2013. Mapping carbon pricing initiatives: developments and prospects. The World Bank: Carbon Finance, 1–96.

Krone-Trailer, 2 2016. Krone-trailer produkte. [Online; accessed 25.2.2016]. URL http://www.krone-trailer.com/produkte/

Kuo, Y., 2010. Using simulated annealing to minimize fuel consumption for the time-dependent vehicle routing problem. Computers & Industrial Engineering 59 (1), 157–165.

Laporte, G., 2007. What you should know about the vehicle routing problem. Naval Research Logistics 54 (8), 811–819.

Li, X., Leung, S. C., Tian, P., 2012. A multistart adaptive memory-based tabu search algorithm for the heterogeneous fixed fleet open vehicle routing problem. Expert Systems with Applications 39 (1), 365–374.

Lin, C., Choy, K. L., Ho, G. T., Chung, S., Lam, H., 2014. Survey of green vehicle routing problem: Past and future trends. Expert Systems with Applications 41 (4), 1118–1138.

Marinakis, Y., Migdalas, A., 2007. Annotated bibliography in vehicle routing. Operational Research 7 (1), 27–46.

McKendry, P., 2002. Energy production from biomass (part 2): conversion technologies. Bioresource technology 83 (1), 47–54.

Mehta, P. K., 2001. Reducing the environmental impact of concrete. Concrete international 23 (10), 61–66.

Mendoza, J. E., Castanier, B., Guéret, C., Medaglia, A. L., Velasco, N., 2010. A memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands. Computers & Operations Research 37 (11), 1886–1898.

Mycoworks, 2 2016. Mycoworks. [Online; accessed 18.2.2016]. URL http://www.mycoworks.com/

Newbery, D. M., 1988. Road damage externalities and road user charges. Econometrica: Journal of the Econometric Society, 295–316.

OECD/ITF, 2010. Reducing transport greenhouse gas emissions trends and data 2010. OECD Publishing/ITF.

OECD/ITF, Jannuary 2015. Itf transport outlook 2015 (summary). OECD Publishing/ITF, 172.

Oliveira, J. F., Carravilla-FEUP, M. A., 2009. Heuristics and local search.

Palmer, A., 2007. The development of an integrated routing and carbon dioxide emissions model for goods vehicles.

Parragh, S. N., Doerner, K. F., Hartl, R. F., 2007. A survey on pickup and delivery problems. Part II: Transportation between pickup and delivery locations, to appear: Journal für Betriebswirtschaft.

Parrish, B. D., 2010. Sustainability-driven entrepreneurship: Principles of organization design. Journal of Business Venturing 25 (5), 510–523.

Pirkwieser, S., Raidl, G. R., 2009. Boosting a variable neighborhood search for the periodic vehicle routing problem with time windows by ilp techniques. In: Proceedings of the 8th metaheuristic international conference (MIC 2009), Hamburg, Germany. p. 33À35.

Powell, W. B., 1986. A stochastic model of the dynamic vehicle allocation problem. Transportation science 20 (2), 117–129.

Psaraftis, H. N., 1980. A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. Transportation Science 14 (2), 130–154.

Qureshi, A. G., Taniguchi, E., Yamada, T., 2010. Exact solution for the vehicle routing problem with semi soft time windows and its application. Procedia-Social and Behavioral Sciences 2 (3), 5931–5943.

Russell, R. A., 1977. Technical note—an effective heuristic for the m-tour traveling salesman problem with some side conditions. Operations Research 25 (3), 517–524.

Sariklis, D., Powell, S., 2000. A heuristic method for the open vehicle routing problem. Journal of the Operational Research Society, 564–573.

Savelsbergh, M. W., Sol, M., 1995. The general pickup and delivery problem. Transportation science 29 (1), 17–29.

Schellnhuber, H. J., Hare, B., Serdeczny, O., Schaeffer, M., Adams, S., Baarsch, F., Schwan, S., Coumou, D., Robinson, A., Vieweg, M., et al., 2013. Turn down the heat: climate extremes, regional impacts, and the case for resilience. Turn down the heat: climate extremes, regional impacts, and the case for resilience.

Schneider, M., Stenger, A., Goeke, D., 2014. The electric vehicle-routing problem with time windows and recharging stations. Transportation Science 48 (4), 500–520.

Solomon, M. M., 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. Operations research 35 (2), 254–265.

Speidel, V., 1976. Edp-assisted fleet scheduling in tramp and coastal shipping. In: Proceedings of the 2nd International Ship Operation Automation Symposium, Washington, DC, August 30-September 2, 1976. Proceedings expected to be available about December 1976. Vol. 5.

Thangiah, S. R., Potvin, J.-Y., Sun, T., 1996. Heuristic approaches to vehicle routing with backhauls and time windows. Computers & Operations Research 23 (11), 1043–1057.

Tillman, F. A., 1969. The multiple terminal delivery problem with probabilistic demands. Transportation Science 3 (3), 192–204.

ToolBox, E., 8 2015. Rolling friction and rolling resistance. [Online; accessed 30.8.2015].
URL `http://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html`

ToolBox, E., 1 2016a. Air pressure by altitude. [Online; accessed 10.1.2016].
URL `http://www.engineeringtoolbox.com/air-altitude-density-volume-d_195.html`

ToolBox, E., 1 2016b. Drag coefficient. [Online; accessed 6.1.2016].
URL `http://www.engineeringtoolbox.com/drag-coefficient-d_627.html`

Travaglini, S., Noble, J., Ross, P., Dharan, C., 2013. Mycology matrix composites. American Society for Composites — Twenty - Eighth Technical Conference.

Ubeda, S., Arcelus, F., Faulin, J., 2011. Green logistics at eroski: A case study. International Journal of Production Economics 131 (1), 44–51.

Wang, T., Lee, I.-S., Kendall, A., Harvey, J., Lee, E.-B., Kim, C., 2012. Life cycle energy consumption and ghg emission from pavement rehabilitation with different rolling resistance. Journal of Cleaner Production 33, 86–96.

Wikipedia, 8 2015a. Articulated vehicle. [Online; accessed 30.8.2015]. URL `https://en.wikipedia.org/wiki/Articulated_vehicle`

Wikipedia, 8 2015b. Fuel efficiency. [Online; accessed 30.8.2015]. URL `https://en.wikipedia.org/wiki/Fuel_efficiency`

Wikipedia, 8 2015c. Gravity of earth. [Online; accessed 30.8.2015]. URL `https://en.wikipedia.org/wiki/Gravity_of_Earth#cite_note-2`

Wikipedia, 8 2015d. Rolling resistance. [Online; accessed 27.8.2015]. URL `https://en.wikipedia.org/wiki/Rolling_resistance`

Wikipedia, 1 2016a. Density of air. [Online; accessed 6.1.2016]. URL `https://en.wikipedia.org/wiki/Density_of_air`

Wikipedia, 1 2016b. Earth radius. [Online; accessed 6.1.2016]. URL `https://en.wikipedia.org/wiki/Earth_radius`

Wikipedia, 1 2016c. Geographical distance. [Online; accessed 23.01.2016]. URL `https://en.wikipedia.org/wiki/Geographical_distance`

Wikipedia, 1 2016d. Geohack. [Online; accessed 4.1.2016]. URL `https://tools.wmflabs.org/geohack/`

Wikipedia, 1 2016e. Kilowatt hour. [Online; accessed 23.01.2016]. URL `https://en.wikipedia.org/wiki/Kilowatt_hour`

Wikipedia, 2 2016f. Linear interpolation. [Online; accessed 10.2.2016]. URL `https://en.wikipedia.org/wiki/Linear_interpolation`

Wikipedia, 2 2016g. Universal transverse mercator coordinate system. [Online; accessed 20.2.2016].
URL https://en.wikipedia.org/wiki/Universal_Transverse_Mercator_coordinate_system

Wikipedia, 2 2016h. World geodetic system. [Online; accessed 20.2.2016].
URL https://en.wikipedia.org/wiki/World_Geodetic_System

Wilson, H., Weissberg, H., 1967. Advanced dial-a-ride algorithms research project: Final report r76-20. Tech. rep., Technical Report Department of Civil Engineering. MIT Cambridge, MA.

WKO, 1 2016a. Wko lohntafel güterbeförderung 2014. [Online; accessed 8.1.2016].
URL https://www.wko.at/Content.Node/branchen/ooe/TransportVerkehr/Gueterbefoerderungsgewerbe/KV_506_Arbeiter_2014_-_Lohntafel.pdf

WKO, 1 2016b. Wko lohntafel güterbeförderung 2015. [Online; accessed 8.1.2016].
URL https://www.wko.at/Content.Node/Service/Arbeitsrecht-und-Sozialrecht/Kollektivvertraege/-KV-Struktur-/Sparte-Transport-und-Verkehr/Gueterbefoerderungsgewerbe/Lohn--und-Gehaltstafeln/Gueterbefoerderung_Lohn_2015.pdf

WolframMathWorld, 1 2016. Great circle. [Online; accessed 23.01.2016].
URL http://mathworld.wolfram.com/GreatCircle.html

Xiao, Y., Zhao, Q., Kaku, I., Xu, Y., 2012. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. Computers & Operations Research 39 (7), 1419–1431.

Xu, J., Yan, F., Li, S., 2011. Vehicle routing optimization with soft time windows in a fuzzy random environment. Transportation Research Part E: Logistics and Transportation Review 47 (6), 1075–1091.

Yang, S., Li, S., Lu, Y., 2010. Investigation on dynamical interaction between a heavy vehicle and road pavement. Vehicle System Dynamics 48 (8), 923–944.

Zegeye, S., De Schutter, B., Hellendoorn, J., Breunesse, E., Hegyi, A., 2013. Integrated macroscopic traffic flow, emission, and fuel consumption model for control purposes. Transportation Research Part C: Emerging Technologies 31, 158–171.