WINNER DETERMINATION, MANIPULATION AND BRIBERY IN VOTING MECHANISMS; A SURVEY OF RESULTS IN COMPUTATIONAL SOCIAL CHOICE THEORY

A dissertation submitted to Birkbeck, University of London
for the degree of MSc in Mathematics.

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30st August 2024

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Abstract

BIRKBECK, UNIVERSITY OF LONDON

ABSTRACT OF DISSERTATION submitted by **Konstantinos Bampalis** and entitled **Winner Determination, Manipulation and Bribery in Voting Mechanisms; A Survey of Results in Computational Social Choice Theory** .

Date of Submission: 30st August 2024

Declaration

This dissertation is submitted under the regulations of Birkbeck, University of London as part of the examination requirements for the MSc degree in Mathematics. Any quotation or excerpt from the published or unpublished work of other persons is explicitly indicated and in each such instance a full reference of the source of such work is given. I have read and understood the Birkbeck College guidelines on plagiarism and in accordance with those requirements submit this work as my own.

Chapter 1

Introduction

1.1 Preference Aggregation

1.2 COMSOC, Motivation and Main Problems

We will focus on *Algorithmic* and *Computational* aspects of Social Choice Theory - a rich and interdisciplinary field known as *Computational Social Choice Theory*[1], at the intersection of Theoretical Computer Science, Mathematics and Economics. This raises the question as to why would a Computer Scientist care about Social Choice and why would a Social Choice Theorist care about Computer Science. It turns out that there is a significant interplay between the fields and a vast transfer of knowledge in both directions;

- From Computer Science to Social Choice Theory:
- From Social Choice Theory to Computer Science:

Overall, there is wide research on COMSOC spanning Economics, Game Theory, Decision Theory and Social Choice on the one hand and AI, Logic, Complexity Theory and Optimisation on the other hand. We refer the interested reader to: [13] and more*.

We can 'split' topics in COMSOC by (a) considering the problem that social choice theory studied and (b) the formal[2] techniques applied/studied. In this dissertation, we will consider the following:

- Winner; Here the social theoretic problem is *preference aggregation* and the design of

[1] To be referred to as COMSOC

[2] By formal, we mean mathematical or CS technique

voting rules. The computational aspect here involves the distinction between computationally *hard* and *easy* rules. In most cases, it is easy to determine a winner, however, there are voting rules which are much more complex.

- Manipulation; Here the social theoretic problem stems from the fact that all voting rules[3] are manipulable - it is theoretically impossible to make a rule manipulation-free. What we can do however is make it less *efficient* to do so. This is where the computational aspects come in - we use computational techniques as a shield against manipulation.

1.3 Goals, Achievements and Structure

This Dissertation aims to synthesize and survey important results on Computational Social Choice Theory for the main problems mentioned above. The remaining paper is structured as follows; Chapter 2 presents the main mathematical tools to tackle the dissertation, mainly Computational Complexity Theory. Chapter 3 gives an overview of Social Choice Theory, starting from basic voting systems, arriving at one of the most celebrated results in Social Choice Theory, the *Gibbard-Satterthwaite Theorem.* This result motivates our transition to the Computational Aspects of Social Choice Theory. The main meat of the dissertation is Chapter 4 where the main complexity results are presented. A contribution of this thesis is that it provides Polynomial Time Results for the Winner Problem, that could not be found elsewhere as well as some different proofs for Complexity Results on the Bribery Problem.

[3]subject to requirements

Chapter 2

Preliminaries

2.1 Computational Complexity Theory

2.2 Graph Theory and Tournaments

Chapter 3

Social Choice Theory

3.1 Voting and Voting Systems

At its core, Social Choice Theory studies the preferences of individuals over alternatives and methods for aggregating these preferences into a collective outcome. Given a set C of m alternatives and a set $N = [n]$ of voters, each $i \in N$ has a preference ordering V_i over C. Let $V = \{(V_1, ..., V_m) : V_i \text{ is a preference ordering over } C\}$. a *Voting System* is simply a rule that determines how to aggregate the individual preferences into a collective outcome.

Definition 3.1.1 (Voting System)**.** A Voting System is a mapping $f : V \to 2^C$.

Several different voting mechanisms have been developed and proposed. They are classified based on different ideas. We consider the most important ones in the following subsections.

3.1.1 Scoring Rules

Let m be the number of candidates. We define a *scoring vector*, $\alpha \in \mathbb{R}^m$, that determines the number of points that each candidate obtains by taking a certain position in a vote and the candidate with the most points wins. The basic idea is that if a candidate is at the ith position of a voter, then they receive α_i points. The difference in mechanisms based on Scoring Rules lies in how the α_i's are given.

Definition 3.1.2 (Plurality)**.** Consider a voter from N and their preference ordering. Candidate i receives α_i points as follows:

$$\alpha_i := \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

Definition 3.1.3 (k-Approval)**.** Consider a voter from N, their preference ordering and fix $k \leq m$. Candidate i receives α_i points as follows:

$$\alpha_i := \begin{cases} 1 & \text{if } 1 \leq k \leq m \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

Definition 3.1.4 (Borda Count)**.** Consider a voter from N, their preference ordering and fix $k \leq m$. Candidate i receives α_i points as follows:

$$\alpha_i := m - i \tag{3.3}$$

3.1.2 Pairwise Comparisons

3.1.3 Rules Proceeding in Stages

3.2 Properties of Voting Systems

So far, all the voting systems we have seen aim to do the same thing; Aggregate preferences to determine a winner. It is therefore natural to ask, which system is the *correct* one. This question *does not* have an answer - there is no universally right or wrong voting system. Instead, we focus on specific properties and combinations thereof and what we study is which voting systems satisfy these *combinations*, *simultaneously.* The keywords here are *combinations* and *simultaneously*; We want our voting systems to satisfy certain reasonable criteria. It turns out, that this is not always possible and some combinations are incompatible leading to several *Impossibility Results*[1].

- Non-Dictatorship: if the outcome is not dependent on a single voter.
- Resoluteness: if it always chooses a single winner.
- Sovereignty: if in principle every candidate can be made a winner.
- Strategy-Proofness: if no voter can benefit from reporting an untruthful vote.

3.3 Strategy-Proofness and Gibbard - Satterwhaite

Theorem 3.3.1 (Gibbard and Satterthwaite)**.** *If there are at least three candidates, there is no preference-based voting system that simultaneously satisfies: non-dictatorship, resoluteness, sovereignty and strategy-proofness.*

[1] It is worth noting that there are many properties that we could have discussed and many Impossibility Theorems that are of great importance, for example, [**?**]. For our purposes, these are enough. The interested reader can consult [13]

3.4 Context

It would be impossible to give a whole account of Social Choice Theory in a single chapter. Instead, we focused on the most important things; Voting Mechanisms and the Gibbard - Satterwhaite Theorem. As mentioned, our focus is on *Algorithmic* Aspects of Social Choice Theory, so in closing this chapter, we focus on these. All voting mechanisms we have seen have one goal; Aggregate preferences and obtain a (single) winner. It is desirable that this is done *efficiently*. As far as, the Gibbard - Satterwhaite goes, we obtained a rather bleak result. Since all of the voting mechanisms we have seen always lead to a winner, are non-dictatorial and do not exclude the possibility for any candidate to be elected, this leaves us with the fact that all systems are subject to strategic voting and manipulation. It turns out that this result can be circumvented and computational *hardness* provides us with a shield against manipulation.

Chapter 4

Computational Complexity Results on Preference Aggregation

4.1 The Winner Problem

$\mathcal{E}$-WINNER	
Input:	Preference Orderings V, Candidate $c \in C$ and Voting System $\mathcal{E}$
Question:	Is c a winner according to $\mathcal{E}$?

4.1.1 Polynomial Time Results

The main approach here is to transform the description of each protocol into pseudocode and analyze the running times.

Theorem 4.1.1. $\mathcal{E} \in \mathbf{P}$ *if* $\mathcal{E}$ *is a Scoring Protocol.*

Proof. □

Theorem 4.1.2. $\mathcal{E} \in \mathbf{P}$ *if* $\mathcal{E}$ *is the Condorcet Protocol.*

Proof. □

Theorem 4.1.3. $\mathcal{E} \in \mathbf{P}$ *if* $\mathcal{E}$ *is in the Copeland Protocol.*

Proof. □

Theorem 4.1.4. $\mathcal{E} \in \mathbf{P}$ *if* $\mathcal{E}$ *is the SVT Protocol.*

Proof. □

4.1.2 Hardness Results

4.2 The Manipulation Problem

$\mathcal{E}$-Manipulation

Input:

Question:

4.3 The Bribery Problem

$\mathcal{E}$-Bribery

Input:

Question:

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