

We Compute the mean and the variance of the daily log-returns of the stocks through the sample. In particular, compute the variance through two methods: sample variance, EWMA with $\lambda=0.95$.

Table 3.1 Mean and Variance of daily log-returns compared to EWMA Variance at 0.95%

	Mean Log Return	Sample Variance	EWMA Variance
ISP = Intesa San Paolo	0.00126	0.00024	0.00018
BBVA = BBVA Bank	0.00160	0.00034	0.00026
UCG = UniCredit	0.00215	0.00040	0.00024
BNP = BNP Paribas	0.00046	0.00029	0.00029
DB = Deutsche Bank	0.00172	0.00045	0.00034

*Rounded to 5 decimals

Starting from the mean of the daily log-returns of the five chosen stocks, it can be seen that UCG shows the highest mean of 0.00215. This translates to an average daily return of 0.215% over the last three years, which indicates relatively stronger growth compared to the other stocks. ISP, BBVA, and DB show modest growth, with the daily average between 0.126%–0.160%. BNP shows the lowest growth, with an average daily return of only 0.046%.

To further compare the assets, we analysed the sample variance, which is also included in the table, as a mean daily return does not say anything without the relevant risk to its return. The historical variability of all assets is in general closer to each other, with DB being the highest at 0.00045 and ISP being the lowest at 0.00024. This shows DB has the highest daily fluctuations and is therefore the riskiest asset historically, and ISP the lowest, which means it is the safest based on past volatility.

When observing the EWMA variance ($\lambda = 0.95$), in the above table, which assigns more weight to recent data, we can spot an interesting trend. Per stock, it can be concluded whether volatility is declining or increasing when comparing the EWMA variance to the sample variance. The EWMA variance of ISP, BBVA, UCG, and DB is lower than their corresponding sample variance. This indicates recent volatility is lower than the historical average, suggesting a calming of market movements. On the other hand, BNP is an outlier, with its EWMA being nearly identical to its sample variance, implying that its recent volatility has remained consistent over the entire reference period, unlike the other banks which have seen volatility decrease recently.

We compute the VaR at 0.01 of each asset at one day and ten days. under the assumption of Normality and i.i.d. returns

Table 3.2 VaR of each stock for 1 day and 10 days

	VaR 1 day	VaR 10 days
ISP	0.0351	0.1111
BBVA	0.0413	0.1307
UCG	0.0446	0.1409
BNP	0.0388	0.1228
DB	0.0477	0.1507

*Rounded to 4 decimals

In the table above, it can be seen what the 1% chance is that the stock will lose more than that percentage on a one-day and ten-day basis. Vice versa, this means that with 99% confidence it can be said the stock will not lose more than this percentage.

For the one-day VaR, DB is the highest with 0.0477, which means there is a 1% probability that DB's return will be worse than -4.77% on a given day. The rest of the stocks have a one-day VaR at 1% of UCG 4.46%, BBVA 4.13%, BNP 3.88%, and ISP having the lowest with 3.51%.

Moreover, the ten-day VaR is the one-day VaR scaled by $\sqrt{10}$. Again, at a 1% chance, the VaR of DB remains the highest with 15.07%, which indicates there is a 1% probability that the return will be lower than -15.07% of the position value after ten days. Furthermore, the risk ranking stays the same as with the one-day VaR, with the respective VaRs being UCG 14.09%, BBVA 13.07%, BNP 12.28%, and ISP 11.11%. This ranking aligns with what has been concluded before in both variances, as DB is the riskiest stock of the five and ISP the least risky stock out of the five.

Under the hypothesis of normality, we compute the one-day VaR at 0.01 of the portfolio (€1) made up of 50% UniCredit and 50% Intesa Sanpaolo.

In this scenario, a portfolio of exactly one euro is computed with 50% UCG and 50% ISP. When computing the VaR at 1% of this portfolio, it comes out to 0.0375. This means there is a 1%

probability that the portfolio will lose more than 3.75% of its value in a single day. It can also be stated that with 99% confidence the loss tomorrow will not exceed €0.0375 on our €1 portfolio.

The portfolio VaR of 3.75% is lower than the weighted average of the individual VaRs (UCG 4.46% and ISP 3.51%), which would be 3.99%. This indicates that the portfolio is less risky than holding the assets separately. The reason for this is the diversification principle: given that the asset returns are not perfectly correlated, combining them reduces the overall variance while maintaining the weighted average return.

Under the hypothesis of normality, we compute the one-day VaR at 0.01 of the portfolio (€1) made up of 20% UniCredit, 20% BBVA, 20% BNP Paribas, 20% Intesa Sanpaolo, and 20% Deutsche Bank.

In this scenario, a portfolio of exactly one euro is constructed with all five assets equally weighted at 20%. When computing the VaR at 1%, the result is 0.0336. This means there is a 1% probability that the portfolio will lose more than 3.36% of its value in a single day. With 99% confidence, it can also be said that the loss on our portfolio of €1 will not exceed €0.0336.

All assets individually had a VaR between 3.51% and 4.77%, and the first portfolio had a combined VaR of 3.75%. This shows that diversification significantly reduces risk, as this new portfolio has a VaR of only 3.36%. The combined VaR is even lower than the lowest individual VaR of ISP at 3.51%. It also smooths the tail risk and mitigates the high risk contribution of DB at 4.77%. This confirms that the returns of the five assets are not perfectly correlated. By combining the five assets equally, the overall variance is lowered, which clearly demonstrates portfolio theory.

We estimate VaR at 0.01 and 0.05 through historical simulation (simple and weighted historical simulation) for each of the five asset with daily observations.

Table 3.3 Simple VaR compared to Weighted VaR at 1% and 5%

	Simple VaR 1%	Simple VaR 5%	Weighted VaR 1%	Weighted VaR 5%
ISP	0.0531	0.0248	0.0315	0.0300
BBVA	0.0513	0.0276	0.0286	0.0263
UCG	0.0533	0.0285	0.0455	0.0229
BNP	0.0517	0.0281	0.0404	0.0239
DB	0.0627	0.0309	0.0503	0.0308

**Rounded to 4 decimals*

In the table, the VaRs are based on historical simulation rather than normality like before. With simple historical simulation over the last three years, the worst 1% or 5% of losses exceeded the percentages above. For the weighted VaR, the most recent results have been given more importance but are read in the same way.⁴

In the simple VaR, it can be stated from the results that DB is the most significant outlier, which again has the highest VaR for 1% and 5%, with losses exceeding 6.27% and 3.09%, respectively. The other assets are all around the same percentages, with VaR at 1% in the range (5.13%–5.33%) and with a VaR at 5% in the range (2.48%–2.85%). This shows that historically, over the three-year window, DB is the riskiest stock and the other four are similar in tail risk.

Additionally, in the weighted VaR, DB is still the riskiest stock, with a VaR 1% of 5.03% and a VaR 5% of 3.08%. With BBVA and ISP, the weighted VaRs have relatively low recent tail risks, with the VaR 1% being 2.86% and 3.15% and VaR 5% being 2.63% and 3.00%. Further, UCG and BNP show that they experience severe losses at VaR 1% with 4.55% and 4.04%, but more moderate losses at VaR 5% with 2.29% and 2.39%.

Furthermore, when comparing the simple VaR against the weighted VaR, it shows that market movements are becoming calmer, as most weighted VaRs are lower compared to their simple VaRs. For the VaR 1%, a significant drop can be seen for the following stocks: ISP from 5.31% to 3.15% and BBVA from 5.13% to 2.86%. UCG, BNP, and DB also see a drop of around 1% but remain high between 4% and 5%. This indicates ISP and BBVA have seen subdued volatility in the most recent period regarding extreme risk. However, at the 5% level, ISP presents an exception where the weighted VaR 3.00% is higher than the simple VaR 2.48%, suggesting that while extreme crashes have decreased, recent moderate volatility has slightly increased.

Comparing these historical measures with the parametric VaR estimated under normality shows that the normal-based 1% VaRs (3.5%–4.8%) are consistently lower than the simple historical VaRs for all assets. This indicates that actual return distributions exhibit heavier tails and more extreme losses than predicted by the normal distribution. Weighted historical VaR lies between these two extremes: for calmer stocks like ISP and BBVA it falls below the parametric VaR, while for riskier stocks like UCG, BNP, and DB it remains above or close to the parametric level. Overall, the comparison highlights three key insights: historical returns contain heavier downside tails than the normal model implies, recent volatility has declined for some assets but not all, and Deutsche Bank remains the most volatile and tail-risk-intensive stock regardless of the method applied.

