

We Compute the mean and the variance of daily returns:

After obtaining the daily price for each of the asset between 2022 and 2024 the transformation _____ to get the daily returns of each asset. From said daily returns the following variance and expected($P(Pt(-t)1) - \text{returns}1$) where calculated:

Stock	AAPL	AMZN	META	GM	WMT	JPM
Mean return (%)	0.057	0.063	0.121	0.008	0.092	0.065
Variance	0.0003	0.0006	0.0009	0.0005	0.0002	0.0002

We evaluate the Normality of the stock returns :

Using the Shapiro test, where H_0 stands for a normal distribution, we obtained the next results:

p-values					
AAPL	AMZN	META	GM	WMT	JPM
5.194996e-11	3.771648e-14	6.007336e-28	1.929832e-08	2.218195e-24	2.735112e-16

As all the p-values are lower than the significance level of 5%, the null hypothesis is rejected in each asset. To further understand the distribution of the data, skewness and kurtosis were obtained:

Stock	AAPL	AMZN	META	GM	WMT	JPM
Skewness	0.216	0.073	-0.339	0.118	-1.109	0.397
Kurtosis	5.492	7.324	23.502	4.699	17.345	8.605

The stocks exhibit excess kurtosis implying thinner, taller bell with fat tails and a higher presence of outliers which complicates predictability. META and WMT stand out with particularly high kurtosis, while their negative skewness, which is significantly high for WMT, signals the presence of downside events. AAPL, AMZN and JPM show lower skewness but paired with significant excess kurtosis, which means that while the outliers are more symmetrically distributed they are still exposed to extreme outcomes. Lastly, GM is the one that most resembles a normal distribution, however, being paired with lower mean returns makes it a less attractive asset.

We apply the Mean Variance criterion:

The Mean-Variance criterion states that having two assets one is preferred over the other only if it presents simultaneously a higher expected return and lower risk, everything that doesn't comply is inconclusive. Based on that:

	AAPL	AMZN	META	GM	WMT	JPM
AAPL				AAPL BUY	WMT BUY	JPM BUY
AMZN					WMT BUY	JPM BUY
META						
GM	AAPL BUY				WMT BUY	JPM BUY
WMT	WMT BUY	WMT BUY		WMT BUY		WMT BUY
JPM	JPM BUY	JPM BUY		JPM BUY	WMT BUY	

The matrix presented highlights how WMT dominates over most of the assets, combining higher mean with one of the lowest risks. META, despite having the highest mean, fails to dominate due to its highest risk shown both by its variance and high kurtosis, making comparisons inconclusive. JPM follows behind WMT in terms of dominance, while AAPL dominates over GM, exposing the latter asset as the least attractive once again. Overall, the matrix confirms that Mean-variance criterion ranking is limited, making comparison for some assets impossible unless the investor preference is included.

We explore the mean variance utility with $u(x) = E(x) - (a/2)\sigma^2(x)$:

For this, Utility for two types of investors are estimated, one being less risk averse ($a=0.05$) and the other much more risk averse ($a=0.5$):

Mean-Variance Utility (U)						
Stock	AAPL	AMZN	META	GM	WMT	JPM
Utility $a=0.05$	0.00056	0.00061	0.00118	0.00007	0.00092	0.00064
Utility $a=0.5$	0.00050	0.00048	0.00097	-0.00005	0.00088	0.00059

Contrary to the Mean-Variance comparison, META yields the highest U, indicating that its high mean compensates for its elevated risk for both investors. For the less risk averse investor, all assets render positive U, performing well due to their favourable trade-off between mean and variance, although GM performs relatively poorly due to its low expected return. At a highest risk aversion level, U decreases along all assets, reflecting a stronger penalization of variance. Becoming more noticeable in the fact that GM shows negative U, as its low mean fails to compensate for its variance at a higher level of avoidance of risk.

We compute the covariance between the assets through a Covariance Matrix:

To accurately evaluate the relationship between assets the Covariance matrix is calculated:

	AAPL	AMZN	META	GM	WMT	JPM
AAPL	2.912631e-04	2.319824e-04	2.634503e-04	1.635298e-04	5.359174e-05	9.445507e-05
AMZN	2.319824e-04	5.830793e-04	4.437900e-04	2.053234e-04	8.134349e-05	1.335611e-04
META	2.634503e-04	4.437900e-04	9.419790e-04	2.271883e-04	5.785298e-05	1.306311e-04
GM	1.635298e-04	2.053234e-04	2.271883e-04	5.387306e-04	5.490257e-05	1.940895e-04
WMT	5.359174e-05	8.134349e-05	5.785298e-05	5.490257e-05	1.690669e-04	3.507324e-05
JPM	9.445507e-05	1.335611e-04	1.306311e-04	1.940895e-04	3.507324e-05	2.492930e-04

All assets exhibit a positive relationship, meaning they move in the same direction. Tech stocks (AAPL, META and AMZN) have the highest covariance, indicating stronger similarity in their movements, while GM show moderate relationship with tech. WMT and JPM have low covariance with the rest. From the diversification perspective, this offers an opportunity to reduce overall portfolio risk.

We compute the MVP of 2 frontiers (AAPL,AMZN, META), (GM,JPM,WMT) and compare them:

Calculating MVP for two portfolios:

Portfolio 1		Portfolio 2	
Stocks	weight	Stocks	weight
AAPL	86.05%	GM	2.36%
AMZN	15.15%	JPM	36.63%
META	-1.20%	WMT	61.01%

MVP minimises return variance. As a result, it can produce highly concentrated allocations. For instance, in Portfolio 1, AAPL represents about 86% of the portfolio, being the tech asset with the least amount of variance. Estimates are based on past data and do not guarantee similar future behaviour. Although the MVP minimises statistical variance, it exposes the investor to significant idiosyncratic risk, making the portfolio less diversified and potentially fragile from an economic risk perspective.

Portfolio	Expected Return	Variance
AAPL, AMZN, META	0.0571	0.0003
GM, JPM, WMT	0.0801	0.0001

Under the mean–variance criterion, Portfolio 2 strictly dominates Portfolio 1, as it achieves a more efficient risk–return trade-off.

We compute weights of the MVP and 0 correlation MVP with all assets :

Weights MVP, 6 assets:

AAPL	AMZN	META	GM	WMT	JPM
0.2108	-0.0407	0.0029	-0.0153	0.5313	0.3111

In this portfolio, JPM and WMT receive high positive weights due to their low variance and covariance with other assets. Tech assets exhibit higher covariance between each other and higher volatility. Among tech assets, AAPL has the lowest individual variance and therefore is assigned a relatively higher weight, benefiting the diversification principle (different sector). The low negative weights on the shorted stocks denote there is minimal gain to be made from the insurance principle.

Weights MVP, 6 assets, 0 correlation:

AAPL	AMZN	META	GM	WMT	JPM
0.1908	0.0953	0.0590	0.1032	0.3287	0.2229

When $\rho = 0$, indicating no correlation between assets, taking long positions across multiple assets benefits the investor through the diversification principle. The insurance principle is equal to 0 thus we observe only positive weights. In this case, asset weights are driven primarily by individual variances, with lower-variance assets receiving higher weights.

Comparing portfolios:

Portfolio	Expected Return	Variance
Correlation	0.0787	1.080509e-04
Zero Correlation	0.0696	5.557925e-05

We observe a much higher sharpe ratio for the theoretical portfolio which assumes 0 correlation due to the positive effect of long positions on all assets on volatility through the diversification principle. On the other hand, the correlated portfolio can neither fully take advantage of the diversification principle (assets have all positive correlation), nor exhibits considerable gains from the insurance principle (all assets are correlated and the shorted weights are small).

We compute the weights of the tangeant portfolio :

AAPL	AMZN	META	GM	WMT	JPM
0.0901	-0.1189	0.1676	-0.2438	0.7173	0.3869

In the tangency (maximum Sharpe ratio) portfolio, the allocation shifts toward assets with the strongest risk-adjusted returns. WMT becomes the dominant long position, as it delivers the highest Sharpe ratio, while META also receives a meaningful positive weight, reflecting a willingness to take on more risk to increase expected return. Compared to the MVP, short positions increase (notably in AMZN and GM), which are stocks that are poor mean-variance optimizers. However, the resulting allocation remains highly concentrated, highlighting a limitation of the mean–variance framework.