Chapter 1

Orbits

Gravity is the force that attracts two bodies toward each other. It is responsible for the behavior of orbital motion. Gravity is described by Newton's law of universal gravitation, which states that every point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The formula for gravitational force is:

$$F_g = \frac{Gm_1 m_2}{r^2}$$

Where F_g is the gravitational force between two objects, G is the gravitational constant, m_1 and m_2 are the masses of the objects, and r is the distance between the centers of the two objects. G is a constant with a value of approximately $6.674 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$.

If we think about acceleration due to gravity where one mass is significantly larger than the other, we can rewrite the equation as:

$$A_g = \frac{Gm_1 m_2}{r^2 m_2} = \frac{Gm_1}{r^2}$$

Where m_1 is the mass of the more massive object and m_2 is the mass of the significantly less massive object. This cancelling of the m_2 mass is why the acceleration due to gravity is independent of the mass of the object in free fall.

A satellite stays in orbit around the planet because the pull of the planet's gravity causes it to accelerate toward the center of the planet.

The satellite must be moving at a very particular speed to keep a constant distance from the planet — to travel in a circular orbit. If it is moving too slowly, it will get closer to the planet. If it is going too fast, it will get farther from the planet.

The radius of a satellite in a low orbit is typically about 2 million meters above the ground. At that distance, the acceleration due to gravity is more like $6.8 m/s^2$, instead of the $9.8 m/s^2$ that we experience on the surface of the planet.

How fast does the satellite need to be moving in a circle with a radius of 8.37 million meters to have an acceleration of $6.8 m/s^2$? Real fast.

Recall that the acceleration vector is

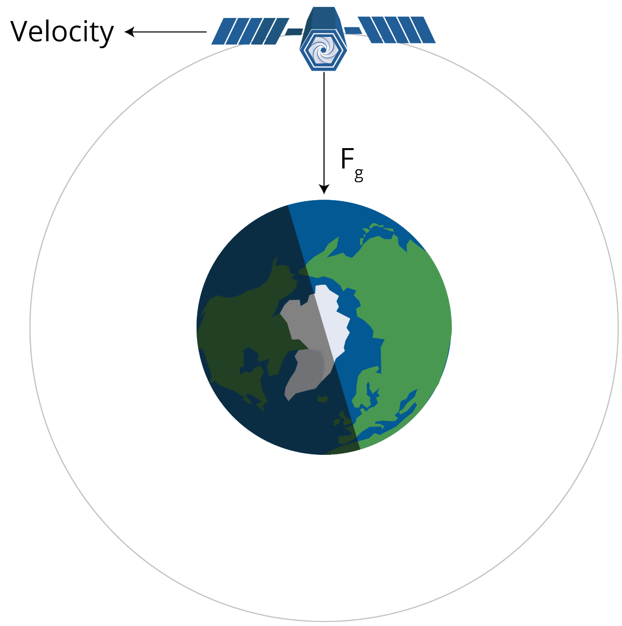

Figure 1.1: A satellite's centripetal force is gravity.

Figure 1.2: A diagram showing the required speeds for entering orbit.

$$a = \frac{v^2}{r}$$

Thus the velocity v needs to be:

$$v = \sqrt{ar} = \sqrt{6.8(8.37x10^6)} = 7,544 \text{ m/s}$$

(That's 16,875 miles per hour.)

When a satellite falls out of orbit, it enters the atmosphere at that 7,544 m/s. The air rushing by generates so much friction that the satellite gets very, very hot, and usually disintegrates.

1.1 Astronauts are not weightless

Some people see astronauts floating inside an orbiting spacecraft and think there is no gravity: that the astronauts are so far away that the gravity of the planet doesn't affect them. This is incorrect. The gravity might be slightly less (Maybe 6 newtons per kg instead of 9.8 newtons per kg), but the weightless they experience is because they and the spacecraft is in free fall. They are just moving so fast (in a direction perpendicular to gravity) that they don't collide with the planet.

Exercise 1 Mars Orbit

Working Space

The radius of Mars is 3.39 million meters. The atmosphere goes up another 11 km. Let's say you want to put a satellite in a circular orbit around Mars with a radius of 3.4 million meters.

The acceleration due to gravity on the surface of Mars is $3.721 \mathrm{m}/\mathrm{s}^2$. We can safely assume that it is approximately the same 11 km above the surface.

How fast does the satellite need to be traveling in its orbit? How long will each orbit take?

Answer on Page 7

1.2 Geosynchronous Orbits

The planet earth rotates once a day. Satellites in low orbits circle the earth many times a day. Satellites in very high orbits circle less than once per day. There is a radius at which a satellite orbits exactly once per day. Satellites at this radius are known as "geosynchronous" or "geostationary", because they are always directly over a place on the planet.

The radius of a circular geosynchronous orbit is 42.164 million meters. (About 36 km above the surface of the earth.)

A geosynchronous satellite travels at a speed of 3,070 m/s.

Geosynchronous satellites are used for the Global Positioning Satellite system, weather monitoring system, and communications system.

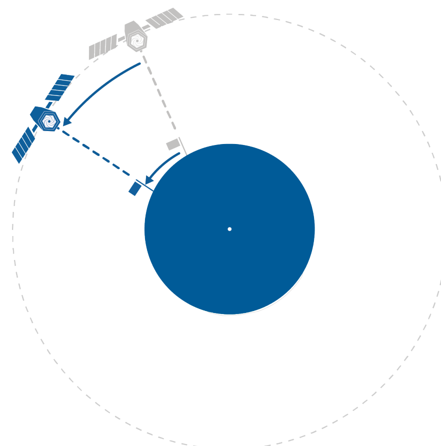

Figure 1.3: A satellite in geosynchronous orbit.

1.3 Escape velocity

FIXME: Add text for escape velocity

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

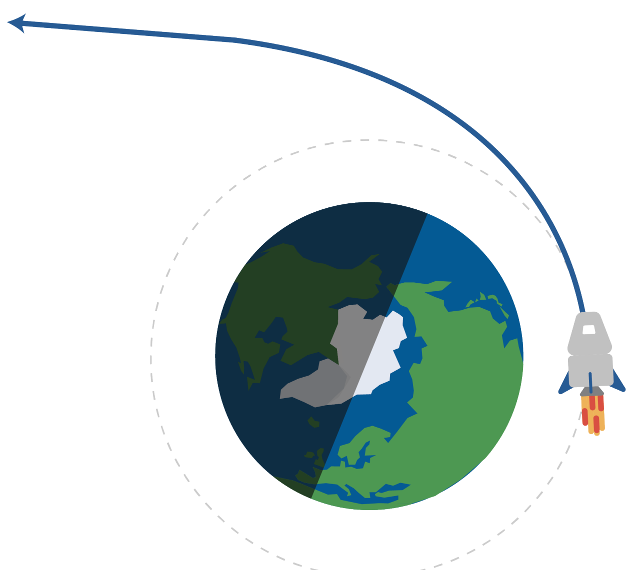

Figure 1.4: A satellite can reach a speed at which it ”escapes” earth’s orbit (centripetal force).

Appendix A

Answers to Exercises

Answer to Exercise ?? (on page 4)

$$v = \sqrt{3.721(3.4 \times 10^6)} = 3,557 \text{ m/s}$$

The circular orbit is $2\pi(3.4 \times 10^6) = 21.4 \times 10^6$ meters in circumference.

The period of the orbit is $(21.4 \times 10^6)/3,557 \approx 6,000$ seconds.

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