

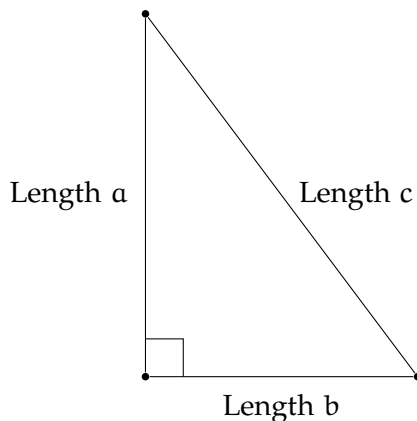
## CHAPTER 1

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# Pythagorean Theorem

Watch's Khan Academy's Intro to the Pythagorean Theorem video at <https://youtu.be/AA6RfgP-AHU>.

If you have a right triangle, the edges that touch the right angle are called *the legs*. The third edge, which is always the longest and opposite the right angle, is known as *the hypotenuse*. The Pythagorean Theorem gives us the relationship between the length of the legs and the length of the hypotenuse.



The Pythagorean Theorem tells us that  $a^2 + b^2 = c^2$ , given that  $c$  is the hypotenuse.

For example, if one leg has a length of 3 and the other has a length of 4, then  $a^2 + b^2 = 3^2 + 4^2 = 25$ . Thus,  $c^2$  must equal 25. This means you know the hypotenuse must be of length 5. This works for any right triangle

In reality, it rarely works out to be such a tidy number. For example, what is the length of the hypotenuse if the two legs are 3 and 6?  $a^2 + b^2 = 3^2 + 6^2 = 45$ . The length of the hypotenuse is the square root of that:  $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ , which is approximately 6.708203932499369.

Common side lengths for these triangles are referred to as *Pythagorean triples*, meaning they evaluate to a whole number. Some common examples are (3, 4, 5), (5, 12, 13), and (8, 15, 17). Multiples of right triangles are also triangles ie.  $(3, 4, 5) \implies (6, 8, 10)$ , which we will touch on next chapter.

There are also angle-based right triangles, consisting of ratios of the angles of the triangles. The most common ones are  $45^\circ$ - $45^\circ$ - $90^\circ$  and the  $30^\circ$ - $60^\circ$ - $90^\circ$ . We will discuss these further

in depth, but know for now that they are vital in trigonometry, and consist of Pythagorean triples as side lengths.

### Exercise 1 Find the Missing Length

What is the missing measure? All missing values should be whole numbers, except  $d$  is an irrational number; answer should be a decimal approximation.

Leg 1 = 6, Leg 2 = 8, Hypotenuse =  $a$

Leg 1 = 5, Leg 2 =  $b$ , Hypotenuse = 13

Leg 1 =  $c$ , Leg 2 = 15, Hypotenuse = 17

Leg 1 = 3, Leg 2 = 3, Hypotenuse =  $d$

*Working Space*

*Answer on Page 5*

A square's diagonal is a special case of the Pythagorean Theorem such that  $c = \sqrt{a^2 + b^2} = \sqrt{2s^2}$ .

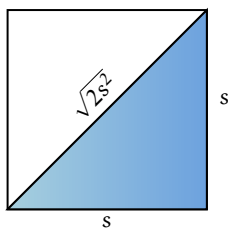
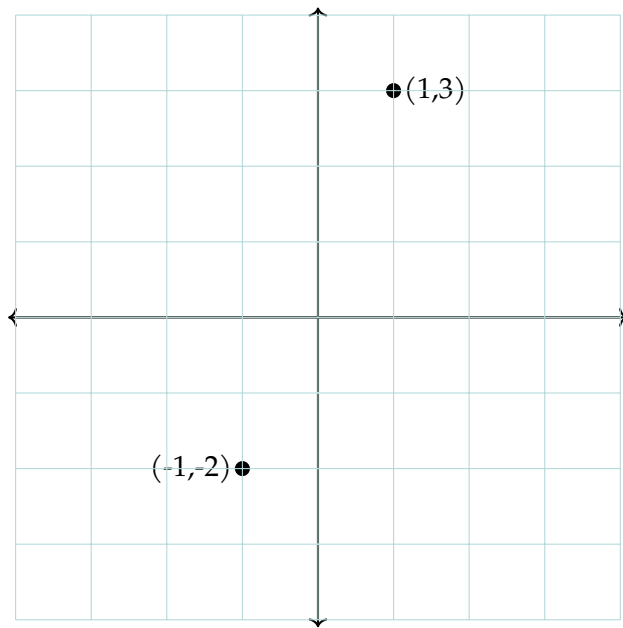


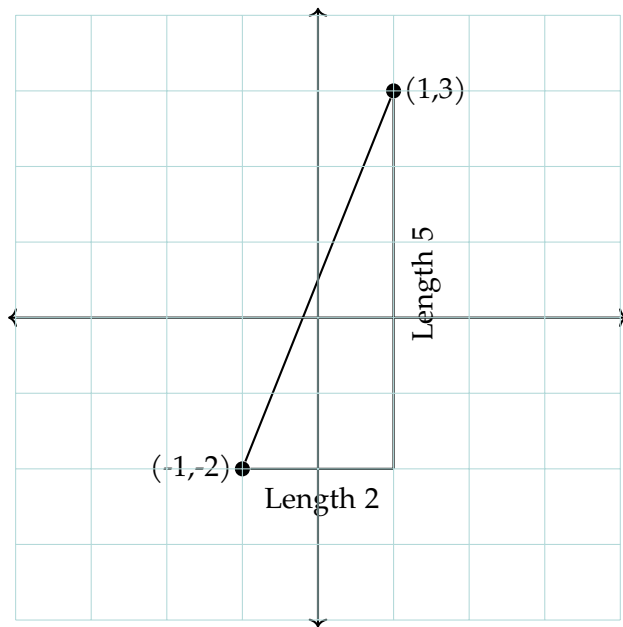
Figure 1.1: A special case of the Pythagorean Theorem where each side is side length  $s$  and the hypotenuse is  $\sqrt{2s^2}$ .

## 1.1 Distance between Points

What is the distance between these two points?



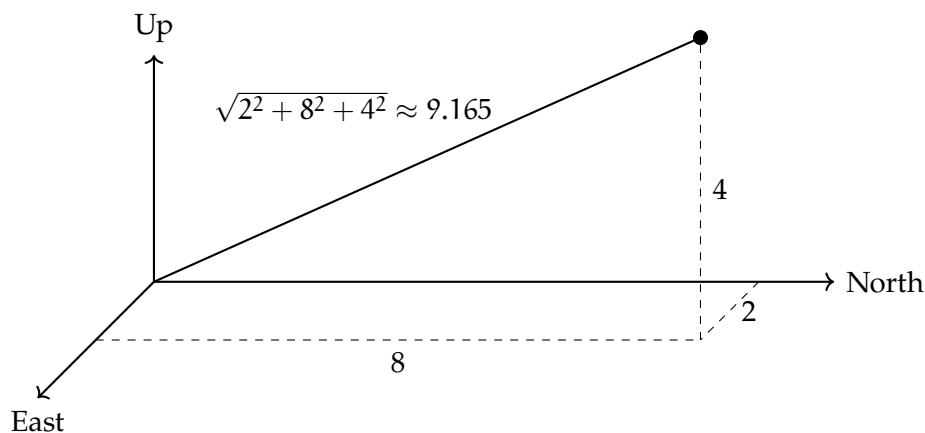
We can draw a right triangle and use the Pythagorean Theorem:



The distance between the two points is  $\sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.385165$ . In other words, you square the change in  $x$  and add it to the square of the change in  $y$ . The distance is the square root of that sum.

## 1.2 Distance in 3 Dimensions

What if the point is in three-dimensional space? For example, you move 2 meters East, 8 meters North, and 4 meters up in the air. How far are you from where you started? You just square each, sum them, and take the square root:  $\sqrt{2^2 + 8^2 + 4^2} = \sqrt{84} = 2\sqrt{21} \approx 9.165$  meters.



This leads us to a formal definition of the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Or in 3D space:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*

# Answers to Exercises

## Answer to Exercise 1 (on page 2)

a: 10 because  $6^2 + 8^2 = 10^2$

b: 12 because  $5^2 + 12^2 = 13^2$

c: 8 because  $8^2 + 15^2 = 17^2$

d:  $3\sqrt{2} \approx 4.24$  because  $3^2 + 3^2 = (3\sqrt{2})^2$





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