

# The Multivariate Normal Distribution

The multivariate normal distribution is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions. It is used in statistics to describe any set of correlated real-valued random variables.

## 1.1 Multivariate Normal Distribution

A random vector  $\mathbf{X} = [X_1, X_2, \dots, X_n]^T$  follows a multivariate normal distribution if every linear combination of its components has a univariate normal distribution. The distribution is parameterized by a mean vector and a covariance matrix.

The probability density function (pdf) of an  $n$ -dimensional multivariate normal distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{\sqrt{(2\pi)^n |\boldsymbol{\Sigma}|}} \exp \left( -\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right)$$

where:

- $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$  is the point up to which the function is integrated,
- $\boldsymbol{\mu} = [\mu_1, \mu_2, \dots, \mu_n]^T$  is the mean vector,
- $\boldsymbol{\Sigma}$  is the covariance matrix,
- $|\boldsymbol{\Sigma}|$  denotes the determinant of the covariance matrix,
- $T$  denotes the matrix transpose.

## 1.2 Covariance Matrix

The covariance matrix,  $\boldsymbol{\Sigma}$ , is a symmetric matrix that contains information about the variance of each variable and the covariance between every pair of variables in the distribution.

The element  $\Sigma_{ij}$  is the covariance between the  $i$ -th and the  $j$ -th random variable, and  $\Sigma_{ii}$

is the variance of the  $i$ -th random variable.

The covariance matrix provides a measure of how much each of the dimensions varies from the mean with respect to each other. A positive covariance between two variables indicates that the variables increase or decrease together, whereas a negative covariance indicates that one variable increases when the other decreases.

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*

# Answers to Exercises





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# INDEX

covariance matrix, [1](#)

Gaussian distribution, [1](#)

multivariate normal, [1](#)