

## CHAPTER 1

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# Solving Quadratics

A quadratic function has three terms:  $ax^2 + bx + c$ .  $a$ ,  $b$ , and  $c$  are known as the *coefficients*. The coefficients can be any constant, except that  $a$  can never be zero. (If  $a$  is zero, it is a linear function, not a quadratic.)

When you have an equation with a quadratic function on one side and a zero on the other, you have a quadratic equation. For example:

$$72x^2 - 12x + 1.2 = 0$$

How can you find the values of  $x$  that will make this equation true?

You can always reduce a quadratic equation so that the first coefficient is 1, so that your equation looks like this:

$$x^2 + bx + c = 0$$

For example, if you are asked to solve  $4x^2 + 8x - 19 = -2x^2 - 7$

$$4x^2 + 8x - 19 = -2x^2 - 7$$

$$6x^2 + 8x - 12 = 0$$

$$x^2 + \frac{4}{3}x - 2 = 0$$

Here,  $b = \frac{4}{3}$  and  $c = -2$ .

$x^2 + bx + c = 0$  when

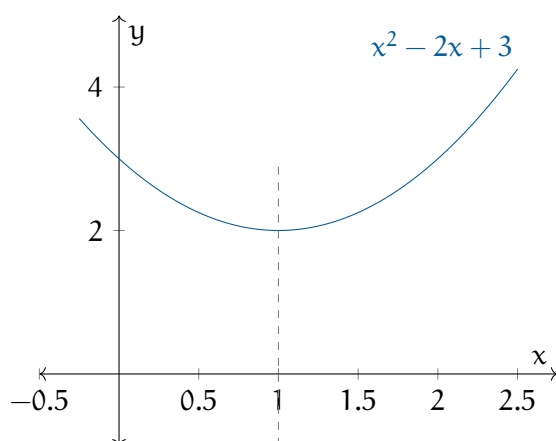
$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

Only when  $a = 1$

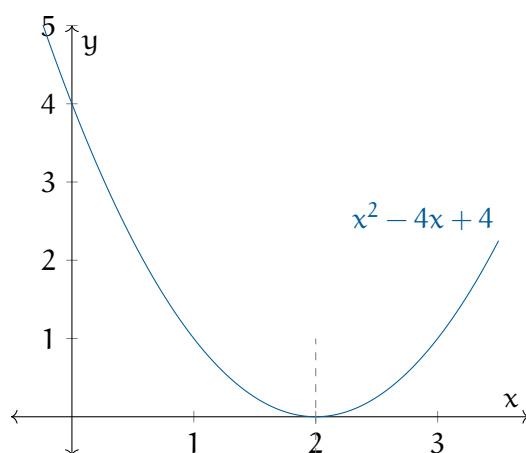
What does this mean?

For any  $b$  and  $c$ , the graph of  $x^2 + bx + c$  is a parabola that goes up on each end. Its low point is at  $x = -\frac{b}{2a}$ . This is referred to as the vertex formula.

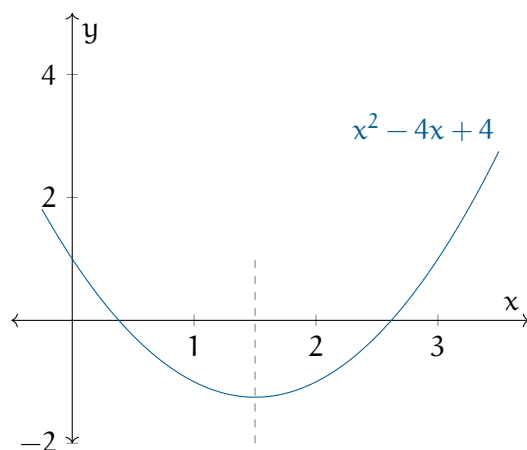
If there are no real roots ( $b^2 - 4c < 0$ ), which means the parabola never gets low enough to cross the  $x$ -axis:



If there is one real root ( $b^2 - 4c = 0$ ), it means that the parabola only touches the  $x$ -axis.



If there are two real roots ( $b^2 - 4c > 0$ ), it means that the parabola crosses the  $x$ -axis twice as it dips below and then returns:



### Exercise 1 Roots of a Quadratic

Working Space

In the last chapter, you found that the function for the height of your flying hammer is:

$$p = -\frac{1}{2}9.8t^2 + 12t + 2$$

At what time will the hammer hit the ground?

Answer on Page 7

## 1.1 The Traditional Quadratic Formula

If the last explanation was a little tricky to understand, the quadratic formula is a nifty tool.

### The Quadratic Formula

$ax^2 + bx + c = 0$  when

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## 1.2 Factoring Quadratics

Sometimes, the quadratic gets a bit clunky and hard to solve, especially without a calculator. That's when factoring quadratics becomes a good tool to use as well.

Factoring quadratics is a way to separate a quadratic, usually in the form  $x^2 + bx + c$  where ( $a = 1$ ), into two multiplied equations in the form  $(px + q)(rx + s) = 0$ . Solving these for  $x$  also gives the roots of the equation.

### Types of Factoring Techniques

1. Factoring out the GCF (Greatest Common Factor)

Always check for a common factor first:

$$2x^2 + 4x = 2x(x + 2)$$

2. Factoring Trinomials:  $a = 1$

For expressions like  $x^2 + bx + c$ , find two numbers that:

- Multiply to  $c$
- Add to  $b$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

3. Factoring Trinomials:  $a \neq 1$

Use the "AC method":

- (a) Multiply  $a \times c$
- (b) Find two numbers that multiply to  $ac$  and add to  $b$
- (c) Split the middle term and factor by grouping

$$6x^2 + 11x + 4 = 6x^2 + 3x + 8x + 4 = (3x + 2)(2x + 2)$$

4. Special Cases:

- Perfect Square Trinomials:  
 $a^2 + 2ab + b^2 = (a + b)^2$   
 $x^2 + 6x + 9 = (x + 3)^2$
- Difference of Squares:  
 $a^2 - b^2 = (a - b)(a + b)$   
 $x^2 - 16 = (x - 4)(x + 4)$

### Tips

- Always factor out the GCF first.

- Check your result by expanding (FOIL). The inverse of factoring is expanding using the FOIL method.
  - Not all quadratics are factorable over real integers.
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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*



# Answers to Exercises

## Answer to Exercise 1 (on page 3)

For what  $t$  is  $-4.9t^2 + 12t + 2 = 0$ ? Start by dividing both sides of the equation by  $-4.9$ .

$$t^2 - 2.45t - 0.408 = 0$$

The roots of this are at

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2} = -\frac{-2.45}{2} \pm \frac{\sqrt{(-2.45)^2 - 4(-0.408)}}{2} = 1.22 \pm 1.36$$

We only care about the root after we release the hammer ( $t > 0$ ).

$1.22 + 1.36 = 2.58$  seconds after releasing the hammer, it will hit the ground.







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