Chapter 1

Logarithms

After the world created exponents, it needed the opposite. We could talk about the quantity $? = 2^3$, that is, "What is the product of 2 multiplied by itself three times?" We needed some way to talk about $2^? = 8$, that is, "2 to the what is 8?" This is why we developed the logarithm.

Here is an example:

$$\log_2 8 = 3$$

In English, you would say, "The logarithm base 2 of 8 is 3."

The base (2, in this case) can be any positive number. The argument (8, in this case) can also be any positive number.

Try this one: What is the logarithm base 2 of 1/16?

You know that $2^{-4} = \frac{1}{16}$, so $\log_2 \frac{1}{16} = -4$.

1.1 Logarithms in Python

Most calculators have pretty limited logarithm capabilities, but python has a nice `log` function that lets you specify both the argument and the base. Start python, import the math module, and try taking a few logarithms:

```
>>> import math
>>> math.log(8,2)
3.0
>>> math.log(1/16, 2)
-4.0
```

Let's say that a friend offers you 5% interest per year on your investment for as long as you want. You wonder, "How many years before my investment is 100 times as large?" You can solve this problem with logarithms:

```
>>> math.log(100, 1.05)
```

```
94.3872656381287
```

If you leave your investment with your friend for 94.4 years, the investment will be worth 100 times what you put in.

1.2 Logarithm Identities

The logarithm is defined this way:

$$\log_b a = c \iff b^c = a$$

Notice that the logarithm of 1 is always zero, and $\log_b b = 1$.

The logarithm of a product:

$$\log_b ac = \log_b a + \log_b c$$

This follows from the fact that $b^{a+c} = b^a b^c$. What about a quotient?

$$\log_b \frac{a}{c} = \log_b a - \log_b c$$

Exponents?

$$\log_b \left(a^c\right) = c \log_b a$$

Notice that because logs and exponents are the opposite of each other, they can cancel each other out:

$$b^{\log_b a} = a$$

and

$$\log_b \left(b^a\right) = a$$

1.3 Changing Bases

We mentioned that most calculators have pretty limited logarithm capabilities. Most calculators don't allow you to specify what base you want to work with. All scientific calculators have a button for "log base 10". So, you need to know how to use that button to get logarithms for other bases. Here is the change-of-base identity:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

For example, if you wanted to find $\log_2 8$, you would ask the calculator for $\log_{10} 8$, then divide that by $\log_{10} 2$. You should get 3.

1.4 Natural Logarithm

When you learn about circles, you are told that the circumference of a circle is about 3.141592653589793 times its diameter. Because we use this unwieldy number a lot, we give it a name: We say "The circumference of a circle is π times its diameter."

There is a second unwieldy number that we will eventually use a great deal in solving problems. It is about 2.718281828459045 (but the digits actually go on forever, just like π). We call this number e. (We are not going to talk about why e is special quite yet, but we will soon.)

Most calculators have a button labeled "ln". That is the *natural logarithm* button. It takes the log in base e.

Similarly, in python, if you don't specify a base, the logarithm is done in base e:

```
>>> math.log(10)
2.302585092994046
>>> math.log(math.e)
1.0
```

1.5 Logarithms in Spreadsheets

Spreadsheets have three log functions:

- `LOG` takes both the argument and the base. `LOG(8,2)` returns 3.
- `LOG10` takes just the argument and uses 10 as the base.

- `LN` takes just the argument and uses e as the base.

Here is a plot from a spreadsheet of a graph of $y = \mathrm{LOG}(x, 2)$.

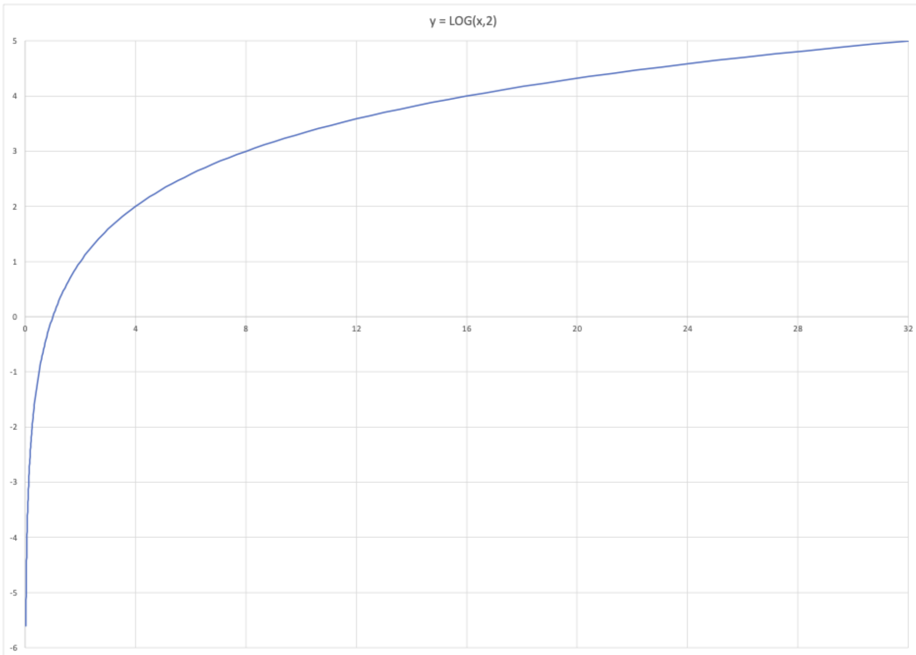

Figure 1.1: A graph of log base 2.

Spreadsheets also have the function `EXP(x)`, which returns e^x. For example, `EXP(2)` returns 7.38905609893065.

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

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