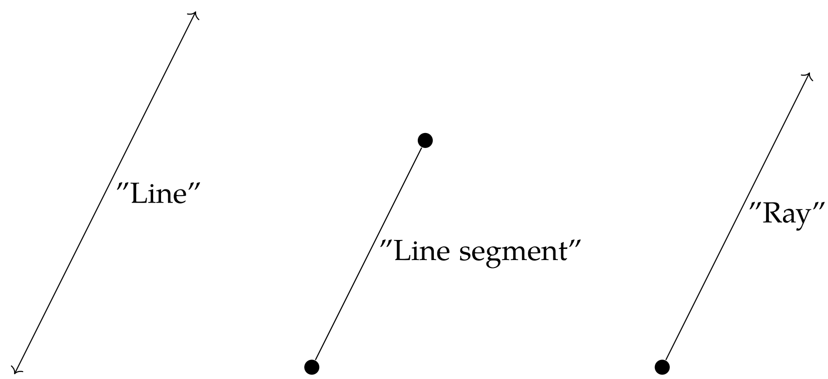
Chapter 1

Angles

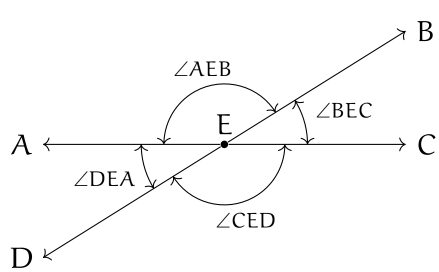
In the following recommend videos, the narrator talks about lines, line segments, and rays. When mathematicians talk about *lines*, they mean a straight line that goes forever in two directions. If you pick any two points on that line, the space between them is a *line segment*. If you take any line, pick a point on it, and discard all the points on one side of the point, that is a *ray*. All three have no width.



Watch the following videos from Khan Academy:

- Introduction to angles: `https://youtu.be/H-de6Tkxej8`
- Measuring angles in degrees: `https://youtu.be/92aLiyeQj0w`

When two lines cross, they form four angles:



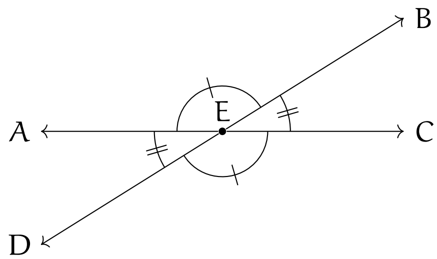
What do we know about those angles? (Note that $m\angle$ means "the measure of angle")

- The sum of any two adjacent angles adds to be 180° So, for example, $m\angle AEB +$

$m\angle BEC = 180°$. We use the phrase "adds to be 180°" so often that we have a special word for it: *supplementary*.

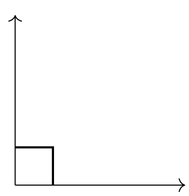
- The sum of all four angles is 360°.
- Angles opposite each other are equal. So, for example, $m\angle AEB = m\angle CED$.
- On any *horizontal* line segment with angles in between it, the sum of the angles must add up to 180°. For example, $\overline{AC}$ is a horizontal line, so angles between it must sum to 180°.

In a diagram, to indicate that two angles are equal we often put hash marks in the angle:



Here, the two angles with a single hash mark are equal, and the two angles with double hash marks are equal.

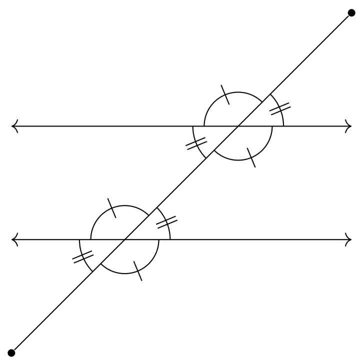
When two lines are perpendicular, the angle between them is 90°, and we say they meet at a *right angle*. When drawing diagrams, we indicate right angles with an elbow:



When an angle is less than 90°, it is said to be *acute*. When an angle is more than 90°, it is said to be *obtuse*.

Just as two angles which add up to 180° are referred to as supplementary angles, two (or more) angles which add up to 90° are *complementary* angles. For example, 2 45° angles are complementary, as well as 30° and 60°. If an angle is 65° degrees, we say that its *complement* is $90° - 65° = 25°$.

If two lines are parallel (never intersecting lines with points all the same distance apart), line segments that intersect both lines form the same angles with each line:



1.1 Radians

As you've seen above, angles can be measured in degrees. Just like you can measure length in more than one unit (inches, meters, etc.), there is more than one unit to measure angles in. Angles can also be measured in *radians*. Radians are unitless (that is, you don't have to put a letter after the number) and there are π radians across a straight line. On diagrams and equations, unknown angles in radians are usually represented with the greek letter θ. This means 180° is the same as π radians. Radians come from comparing the length of an arc to the radius of a circle–which we will explore in a future chapter.

Generally, you can use the following formula for your conversions:

$$\text{angle in degrees} = \text{angle in radians} \cdot \frac{180^\circ}{\pi}$$

Example: An angle is measured to be $\frac{\pi}{2}$ radians. What is the angle in degrees?

Solution: Since we know that π radians is the same as 180°, we can set up the unit conversion:

$$\frac{\pi}{2} \cdot \frac{180^\circ}{\pi} = 90^\circ$$

Therefore, a $\frac{\pi}{2}$ angle is 90°.

Exercise 1

Convert the following angles from degrees to radians, or from radians to degrees.

1. $360°$
2. $\frac{\pi}{3}$
3. $225°$
4. $\frac{3\pi}{4}$
5. $30°$
6. $45°$

Working Space

Answer on Page 5

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 4)

1. 2π
2. $60°$
3. $\frac{5\pi}{4}$
4. $135°$
5. $\frac{\pi}{6}$
6. $\frac{\pi}{4}$

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