

# Work and Energy

In this chapter, we are going to talk about how engineers define work and energy. It frequently takes force to get work done. Let's start with thinking about the relationship between force and energy. As we learned earlier, Force is measured in newtons, and one newton is equal to the force necessary to accelerate one kilogram at a rate of  $1\text{m/s}^2$ .

When you lean on a wall, you are exerting a force on the wall, but you aren't doing any work. On the other hand, if you push a car for a mile, you are clearly doing work, as in Figure ???. Work, to an engineer, is the force you apply to something, as well as the distance that something moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force applied over one meter.



Figure 1.1: Pushing on a wall does no work while pushing a car does work! Why?

For example, if you push a car with a force of 10 newtons for 12 meters, you have done 120 joules of work.

### Formula for Work

If a force applied is *constant*, the formulas for work is:

$$W = F \cdot d$$

where  $W$  is the work in joules ( $\text{N} \cdot \text{m}$ ),  $F$  is the *force* in newtons, and  $d$  is the distance in meters.

If the force is not in the same direction as the distance, we can use the cosine of the angle between the force and the distance:

$$W = F \cdot d \cdot \cos(\theta)$$

where  $\theta$  is the angle between the force and the distance. If the formula for work is *not constant*, the formula becomes:

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

where  $d\mathbf{r}$  is an infinitesimal amount of displacement. An example of this kind of force is like a spring force, where the force varies with displacement.

### Exercise 1 Book done by a constant force

This question is from an AP Physics C Review Book. You slowly lift a 2 kg book at a *constant velocity* through a distance of 3 m. How much work is done on the book?

Working Space

Answer on Page ??

The work-energy theorem (or work-energy principle) states that the net work done on an object is equal to the **change in its energy**. In other words, if you do work on an object, you are changing its energy. This is derived from Newton's second law of motion, covered in Chapter 1.

$$W = \Delta E$$

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs (British Thermal Unit), and calories.

Let's go over a few different forms that energy can take.

## 1.1 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat
- Electricity
- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

There are also other forms of energy such as spring potential energy, which we will cover in the oscillations chapter.

### 1.1.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

When we talk about heat energy, we will commonly be talking about *friction*. When a force does frictional work, it is output onto as a change in heat energy within the system.

### 1.1.2 Electricity

Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let's say your lightbulb consumes 60 *watts* of electricity, and you leave it on for 24 hours. We would say that you have consumed 1.44 kilowatt hours, or 3,600,000 joules. This comes into play when rating house electricity systems, lightbulbs, or electric cars. The electric energy stored in a battery of an electric car gets converted to kinetic energy and thermal energy taken out by friction.

### 1.1.3 Chemical Energy

As mentioned earlier, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosynthesis is a process that consumes energy. Therefore, the sugar molecule has more chemical

energy than the carbon dioxide and water molecules that were used in its creation.

Recall the balanced chemical equation for photosynthesis is:



In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

(You may find it helpful that the chapter on unit conversions appears earlier in this text, so that the calorie-joule relationship can be placed in proper context.)

Here is where things get tricky: People who work with food got tired of saying “kilocalories”, so they just started using “Calorie” to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, “Calorie” probably means kilocalorie.

### 1.1.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the brakes, the energy from the motion of the car is converted into heat in the brake pads and under the tires. This stored energy of motion is known as *kinetic energy*.

But how much energy does a moving object actually have?

#### Formula for Kinetic Energy

$$\text{KE} = \frac{1}{2}mv^2$$

where KE is the kinetic energy in joules,  $m$  is the mass in kilograms, and  $v$  is the speed in meters per second.

This equation tells us two important things:

- Kinetic energy increases *linearly* with mass. A heavier object has more energy at the same speed.
- Kinetic energy increases *quadratically* with velocity. This means that doubling the speed results in *four times* the kinetic energy.

Why does the velocity term appear squared? One way to understand this is to consider

the work required to accelerate an object. To speed something up, a force must act over a distance. As the object moves faster, each additional meter of acceleration takes less time but still increases the kinetic energy significantly. The mathematical result of this relationship is the  $v^2$  dependence. Kinetic energy ultimately tells us how powerful an object is. Real life calculations involve calculating car crash preventions, motor speeds, rocket propulsions, battery requirements, and gyroscopic forces.

## Exercise 2 Kinetic and Thermal Energy

A 1,200 kg car traveling at 22m/s comes to a stop using its brakes. Assume all the initial kinetic energy is converted into thermal energy in the brakes and tires.

Working Space

- (a) How much thermal energy,  $Q$  is produced?
- (b) If the break system has a 15 kg and a specific heat of  $c = 460(\text{J/kg}^\circ\text{C})$ , estimate the temperature increase of the brakes.

Also, you are given that the change in heat,  $\Delta T = \frac{Q}{m_{\text{brake}}c}$ .

Answer on Page ??

### 1.1.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

#### Formula for Gravitational Potential Energy

The formula for gravitational potential energy is

$$E = mgh$$

where  $E$  is the energy in joules,  $m$  is the mass of the object you lifted,  $g$  is acceleration due to gravity, and  $h$  is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects in free-fall near Earth's surface accelerate at  $9.8\text{m/s}^2$ .

There are various kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy. When you compress a spring, the energy put into the spring is proportional to the spring constant and distance it is compressed,  $\frac{1}{2}kx^2$ . We will dive deeper into this in the springs and oscillations chapter.

## 1.2 Conservation of Energy

The first law of thermodynamics says “Energy is neither created nor destroyed.”

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. A falling object converts potential energy into kinetic energy, causing it to speed up. However, the total amount of energy in a closed system stays constant.

The sum of mechanical energy (usually Kinetic and Potential) is always constant:

$$W_{\text{net,ext}} = \Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

Choosing a specific system or environment will benefit your problem, especially if the net external work done is 0. Problems that involve energy transformation are similar to accounting problems, the work before and after must be equal, and must equal some constant net value.

**Exercise 3      The Energy of Falling**

A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

*Working Space*

*Answer on Page ??*

**Exercise 4      Frictional Losses**

A 4.0 kg block is released from rest at the top of a rough incline of height 2.5 m. By the time it reaches the bottom, its speed is 4.2 m/s.

Determine the total mechanical energy lost to thermal energy.

You may use  $10 \text{ m/s}^2$  for  $g$

*Working Space*

*Answer on Page ??*

**Exercise 5      Blast Off!**

A rocket sled of mass 150 kg uses a small fuel-powered booster that releases  $1.2 \times 10^5$  J of chemical energy. During the burn, 65% of the energy is converted into useful kinetic energy for the sled, the rest is lost as heat or sound. Assume the sled travels on frictionless, smooth surface.

Calculate the final speed of the rocket sled.

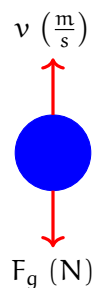
*Working Space*

*Answer on Page ??*

**1.3 Work and Kinetic Energy**

As stated above, the work-energy theorem tells us that the change in an object's kinetic energy is equal to the work done on that object. For now, we will only consider examples where the force and the direction of motion are parallel or perpendicular. When you learn about vectors, we will expand this to include forces that are skew to the direction of motion.

Consider what happens when you toss a ball in the air: once the ball leaves your hand, the only force acting on it is gravity. Initially, the ball is moving upwards while gravity points downwards:



Intuitively, we know that the ball will slow down (lose kinetic energy) as it moves upwards:



$$\Delta KE < 0$$

Since  $W = \Delta KE$ , we also know that gravity must be doing *negative work*. Whenever the direction of the force is opposite the direction of the motion, the work done by that force is negative.

### Exercise 6      A ball thrown upwards

If the ball has a mass of 0.5 kg, how much kinetic energy does it lose as it moves upwards by 1 m?

Working Space

Answer on Page ??

### Exercise 7      How far will you slide?

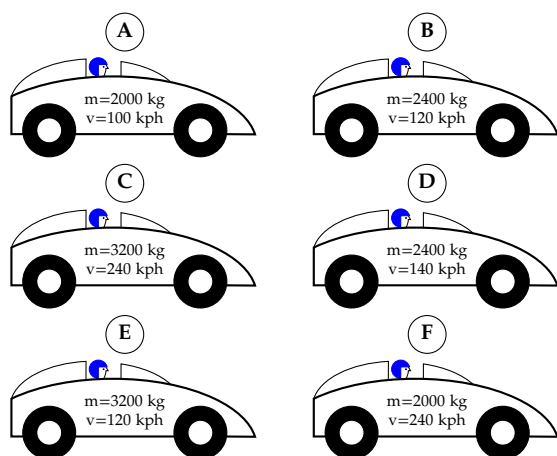
You are playing softball and have to slide into home. If you sprint at a maximum of 10 m/s and the force of friction between you and the ground is 0.3 times your weight, how far from the base can you start your slide and still reach home?

Working Space

Answer on Page ??

**Exercise 8 Ranking Stopping Force**

In drag racing, cars can reach speeds of 150 miles per hour (approximately 240 kilometers per hour). In order to be able to stop quickly and safely, drag racing cars are built with parachutes that deploy at the end of the race. Consider a drag race where cars of different masses reach different maximum speeds. There is 100 meters between the finish line and the fence surrounding the race track. If all the race cars deploy their parachutes at the finish line while going their maximum speed, rank the force needed from the parachute to stop each car in the required distance from least to greatest:

*Working Space**Answer on Page ??***1.3.1 Forces that do no work**

If the object you are pushing doesn't move, or the applied force is perpendicular to the direction of motion, that force does no work. Let's look at a few examples:

### Pushing Against an Immobile Object

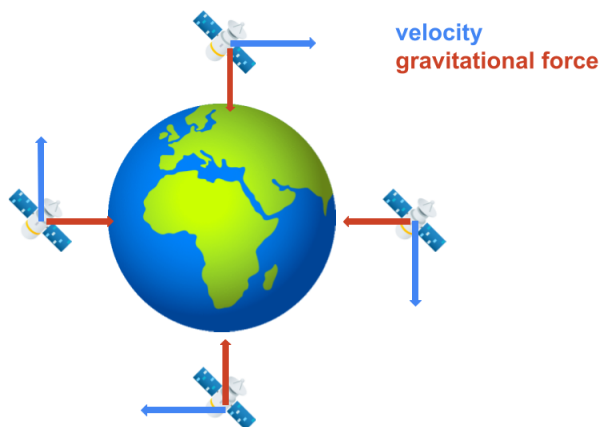
At the beginning of the chapter, we said that when you push on a wall, you don't do any work. Why is this? Well, if the wall is a good wall (that is, strong enough to not be pushed over by a person), the wall won't move while you push on it. This means the distance over which your push is applied is zero, and therefore the work done ( $F \cdot d = F \cdot 0 = 0$ ) is zero joules.

### Walking Across a Room with a Book

Imagine holding a book flat on your hands and walking at a constant velocity. Your hand is applying an upwards force to the book, but the book is moving horizontally. This means the force and direction of motion are *perpendicular*. Recall from the beginning of the chapter that if the force and distance are not parallel, then the work is given by  $W = F \cdot d \cos(\theta)$ . (When the vectors are parallel,  $\theta = 0$  and  $\cos(\theta) = 1$ , while when the vectors point in opposite directions,  $\theta = 180^\circ$  and  $\cos(\theta) = -1$ .) When the vectors are perpendicular, then  $\theta = 90^\circ$  and  $\cos(\theta) = 0$ . Therefore,  $W = 0$  as well and the upward force of your hands does no net work.

### Circular Motion

We will discuss circular motion further in a subsequent chapter. For now, know that constant-speed circular motion is caused by a constant-magnitude force that always points to the center of the circle the object is moving in. For example, you can take a weight on the end of a string and spin it. The tension in the string spins the weight, and the string always points from the object to your hand (the center of the weight's circular path). For a satellite, that force is gravitational attraction to the Earth.



As a result, the force changes the *direction*, but not the *magnitude* of the satellite's velocity. Let's re-examine the equation for kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Since the velocity is squared, the direction of motion doesn't affect the kinetic energy (a ball moving at 5 m/s upwards has the same kinetic energy as if the ball were moving at 5 m/s downwards). So, a force that causes circular motion doesn't change a circling object's kinetic energy, and therefore does no work (as expected when force and direction of motion are perpendicular)!

## 1.4 Efficiency and Power

Although energy is always conserved as it moves through different forms, scientists aren't always that good at controlling it.

In terms of an equation, efficiency is the ratio of the useful energy output to the total energy input. It is usually expressed as a percentage.

### Formula for Efficiency

$$\text{Efficiency} = \frac{\text{Useful Energy Output}}{\text{Total Energy Input}} \times 100\%$$

where the useful energy output is the energy that is actually used to do work or complete a task, and the total energy input is the total energy consumed by the system.

A machine is considered 100% efficient only if all the input work is converted into useful output work, with no energy lost to heat, friction, or sound. 100% efficient process don't exist in real-life: every process loses some useful energy to heat.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

In general, if you are trying to increase efficiency in any system, the solution is usually easy to identify by the heat that is produced. Reduce the heat, increase the efficiency.

Light bulbs are an interesting case. To get the same amount of light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just “inefficient at making light” — it is actually counteracting the air conditioner!

### Exercise 9 Unit Costs

A machine is rated to produce 500 bolts per hour, but in actual operation it runs at only 82% efficiency due to loss and scraps. The machine requires \$24 of electricity per hour to run.

- (a) What is the actual output of bolts per hour?
- (b) What is the unit cost (cost per bolt) of the production?

*Working Space*

*Answer on Page ??*

Another measure of Work and Energy is *Power*. Power is the rate at which work gets done (or energy gets transferred). Suppose I do work in 1000 J of work in 10 minutes but you do the same amount of work in 5 minutes. Because you did the same amount of work but in a quick amount of time, you were more *powerful*.

#### Formula for Power

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \implies P = \frac{W}{t}$$

Power is represented by the unit watt (W), defined as one joule per second (J/s).

Since power is a rate, we can also express it using calculus:

$$P = \frac{dW}{dt}$$

Equivalently, for work done by constant forces, this simplifies to  $P = \frac{F \cdot r}{t} = F \cdot v$ , since

Power is measured in the unit **watt**, where one watt is one Joule per second:  $1W = 1J/s$

### Exercise 10 Hydraulic Piston Problem

A hydraulic piston pushes a crate horizontally across the floor with a constant force of 450 N. During operation, the piston moves the crate 1.8 meters in 0.75 seconds.

- (a) How much work does the piston do on the crate?
- (b) What is the power output of the piston during this motion?

Working Space

Answer on Page ??

### Exercise 11 Spring Power

A compressed spring launches a small block. The spring delivers 120 W of power while releasing its stored energy over 0.40 seconds. The spring constant is  $k = 800$  N/m.

How far was the spring compressed?

Working Space

Answer on Page ??

**Exercise 12**      **Elevator Lift**

An elevator motor lifts a 750 kg cabin vertically upward a distance of 18 m in 12 s. The motor consumes electrical energy at a rate of 18 kW.

- (a) Compute the total electrical energy consumed during the lift. Note that  $1 \text{ kW} = 1000 \text{ watts} = 1000 \text{ J/s}$
- (b) Determine the gravitational potential energy gained by the elevator.
- (c) Calculate the efficiency of the motor during this lift.

*Working Space*

*Answer on Page ??*

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*





# Answers to Exercises

### Answer to Exercise ?? (on page ??)

In the case, the force you exert must balance the weight of the book, so  $F = mg = 2(9.8) = 19.6\text{N}$ . Since this force is straight upward and the displacement is in the same direction,  $W = F \cdot d = 19.6 \cdot 3 = 59.8\text{J}$ .

### Answer to Exercise ?? (on page ??)

(a) Since all kinetic energy is transferred into thermal energy, we can say:

$$Q = KE = \frac{1}{2}(1200)(22)^2 = 290,400\text{ J}$$

(b) The rise in heat is given by  $\Delta T = \frac{Q}{mc} = \frac{290,400}{15 \times 460} \approx 42^\circ\text{C}$

### Answer to Exercise ?? (on page ??)

At the top of the ladder, the cannonball has  $(9.8)(5)(3) = 147\text{ J}$  of potential energy.

At the bottom, the kinetic energy  $\frac{1}{2}(5)v^2$  must be equal to 147 joules. So  $v^2 = \frac{294}{5}$ . This means it is going about 7.7 m/s.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is negligible.)

### Answer to Exercise ?? (on page ??)

The initial total gravitational potential energy is  $mgh = 4(10)(2.5) = 100\text{ J}$ . The final kinetic energy is  $\frac{1}{2}mv^2 = \frac{1}{2}(4)(4.2)^2 = 35.28$ . So, the rest of the energy must be lost due to friction:  $100 - 35.28 = 64.72\text{ J}$ .

### Answer to Exercise ?? (on page ??)

The total useful energy can be calculated as  $0.65(1.2 \times 10^5) = 7.8 \times 10^4$  J. Then, we can set the final KE  $\frac{1}{2}mv^2 = 7.8 \times 10^4$  and solve for  $v$

$$\begin{aligned}\frac{1}{2}(150)v^2 &= 7.8 \times 10^4 \\ (150)v^2 &= 15.6 \times 10^4 \\ v^2 &= 1,040 \\ v &\approx 32.249 \text{ m/s}\end{aligned}$$

### Answer to Exercise ?? (on page ??)

The force acting on the ball is its weight,  $F_w = mg$ , and we will designate this as negative since weight points downwards. Using the work-energy theorem,

$$\Delta KE = F \cdot d = (mg) \cdot d = (0.5\text{kg}) \left(-9.8 \frac{\text{m}}{\text{s}^2}\right) (1\text{m}) = -4.9\text{J}$$

Therefore, the ball loses 4.9 joules of kinetic energy for every 1 meter it moves upwards (the fact it is *losing kinetic energy* is represented by the result being negative). Note that this is also the formula for potential energy, so it gained 4.9 J of potential energy.

### Answer to Exercise ?? (on page ??)

$$F_f \cdot d = \Delta KE = KE_f - KE_i$$

You'll reach the maximum distance you can slide when you stop moving, so we will use a final velocity of zero, which means a final kinetic energy of zero:

$$F_f \cdot d = -KE_i = \frac{1}{2}mv^2$$

Since the force of friction is 0.3 times your weight, we know that:

$$F_f = 0.3F_w = 0.3mg$$

Substituting and canceling the mass:

$$(0.3mg) \cdot d = \frac{1}{2}mv^2$$

$$0.3g \cdot d = \frac{1}{2}v^2$$

Since we know  $g$  and  $v$ , we can solve for  $d$ :

$$d = \frac{v^2}{0.6g} = \frac{(10 \frac{m}{s})^2}{0.6(9.8 \frac{m}{s^2})} \approx 1.7m$$

So, if you want to reach home base, you should start your slide no more than 1.7 m from home.

### Answer to Exercise ?? (on page ??)

Since all the cars need to stop in the same distance, the cars with the greatest kinetic energy will take the most force to stop. Calculating the kinetic energies (we won't change the units from kilometers per hour to meters per second, since we're just comparing the values):

Car	Mass [kg]	Max speed [kph]	KE [kg (kph) <sup>2</sup> ]
A	2000	100	$1 \times 10^7$
B	2400	120	$1.728 \times 10^7$
C	3200	240	$9.216 \times 10^7$
D	2400	140	$2.352 \times 10^7$
E	3200	120	$2.304 \times 10^7$
F	2000	240	$5.67 \times 10^7$

The correct ranking is A, B, E, D, F, C.

### Answer to Exercise ?? (on page ??)

(a)  $500 \times 0.82 = 410$  bolts

(b)  $\frac{\$24}{410} \approx \$0.06$

### Answer to Exercise ?? (on page ??)

(a)

$$W = F \cdot d = 450 \cdot 1.8 = 810 \text{ J}$$

(b)

$$P = \frac{W}{t} = \frac{810}{.75} = 1080 \text{ W}$$

Alternatively,

$$v = \frac{d}{t} = \frac{1.8}{0.75} = 2.4 \text{ m/s}$$
$$P = F \cdot v = 450 \cdot 2.4 = 1080 \text{ W}$$

### Answer to Exercise ?? (on page ??)

$$P = \frac{E}{t}$$

$$E = P \cdot t = 120(0.4) = 48 \text{ J} \qquad = 48 \text{ J}$$

So 48 J of energy is stored into the spring. We can find the spring's PE converted to distance by:

$$E = \frac{1}{2}kx^2$$
$$48 = \frac{1}{2}(800)x^2$$
$$x^2 = \frac{48}{400}$$
$$x = \sqrt{0.12} \approx 0.346 \text{ m}$$

So the spring had to have been compressed 34.6 cm.

### Answer to Exercise ?? (on page ??)

(a) Electrical energy input can be calculated as  $E = Pt = 18 \text{ kW} \times 1000 \text{ J/s} \times 12 \text{ sec} = 216,000 \text{ J}$ .

(b) The elevator travels 18 m upwards, so the total gravitational potential energy is  $mgh = 750(9.8)(18) = 132,300 \text{ J}$ .

(c) The efficiency can be calculated as  $\frac{\text{Useful Output}}{\text{Total Input}} = \frac{132,000}{216,000} \approx 61.1\%$



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