Chapter 1

Conic Sections

In mathematics, conic sections (or simply conics) are curves obtained as the intersection of the surface of a cone with a plane. The three types of conic section are the hyperbola, the parabola, and the ellipse. The circle is a special case of the ellipse, though historically it was sometimes called a fourth type. All of the equations below can be graphed on programs like Desmos.

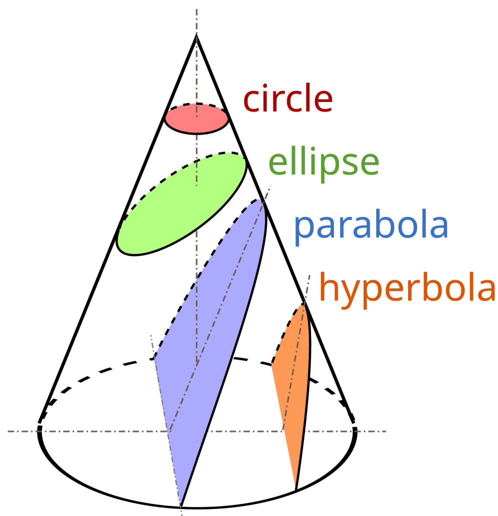


Figure 1.1: Visualization of conic sections.

Source: Wikimedia Commons, Public Domain: https://upload.wikimedia.org/wikipedia/commons/thumb/1/11/Conic_Sections.svg/1920px-Conic_Sections.svg.png

1.1 Definitions

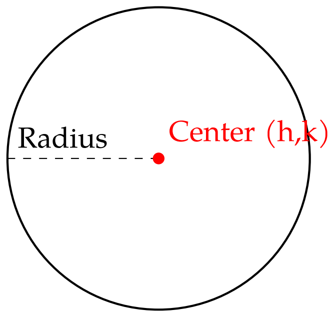
Each type of conic sections can be defined as follows:

1.1.1 Circle

A circle is the set of all points in a plane that are at a given distance (the radius) from a given point (the center). The standard equation for a circle with center (h, k) and radius

r is:

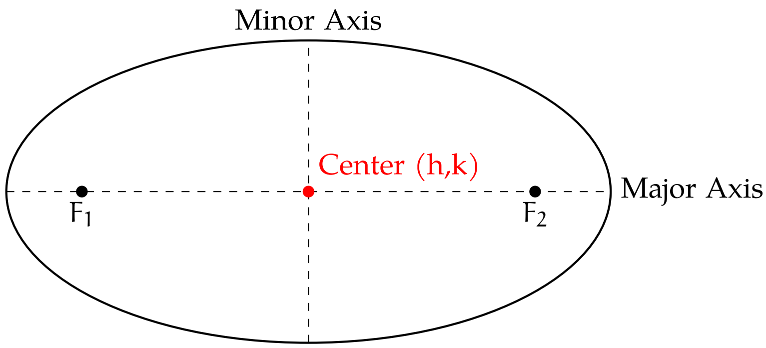
$$(x-h)^2 + (y-k)^2 = r^2 \tag{1.1}$$



1.1.2 Ellipse

An ellipse is the set of all points such that the sum of the distances from two fixed points (the foci) is constant. The standard equation for an ellipse centered at the origin with semi-major axis a and semi-minor axis b is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1.2}$$



1.1.3 Hyperbola

A hyperbola is the set of all points such that the absolute difference of the distances from two fixed points (the foci) is constant. A hyperbola is formed from slicing a *double-cone* — two cones placed tip-to-tip — parallel to or angled off of the central axes. The standard equation for a hyperbola centered at the origin is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{1.3}$$

or

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \tag{1.4}$$

depending on the orientation of the hyperbola.

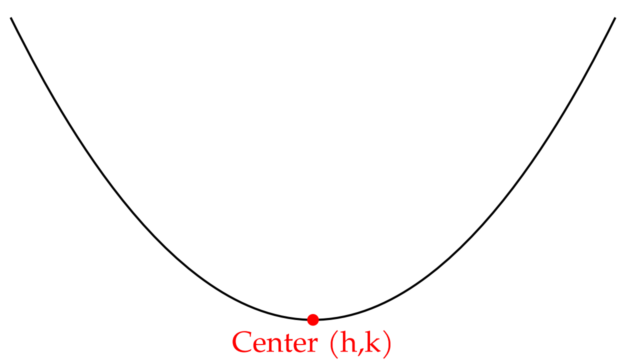
1.1.4 Parabola

A parabola is the set of all points that are equidistant from a fixed point (the focus) and a fixed line (the directrix). The standard equation for a parabola that opens upwards or downwards is:

$$y = a(x - h)^2 + k \tag{1.5}$$

and that opens leftwards or rightwards is:

$$x = a(y - k)^2 + h \tag{1.6}$$



where (h, k) is the vertex of the parabola, and a is a scalar.

Note that only the parabola out of these four is a function, as passes vertical line test. The other three cannot be expressed as functions, only equations.

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

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