

CHAPTER 1

Vector-valued Functions

In the last chapter, you calculated the flight of the shell. For any time t , you could find a vector [distance, height]. This can be thought of as a function f that takes a number and returns a 2-dimensional vector. We call this a *vector-valued* function from $\mathbb{R} \rightarrow \mathbb{R}^2 \rightarrow \mathbb{R}^3$ ¹.

1.1 Vector-valued functions: position

We often make a vector-valued function by defining several real-valued functions. For example, if you threw a hammer with an initial upward speed of 12 m/s and a horizontal speed of 4 m/s along the x axis from the point $(1, 6, 2)$, its position at time t (during its flight) would be given by:

$$f(t) = [4t + 1, 6, -4.8t^2 + 12t + 2]$$

In other words, x is increasing with t , y is constant, and z is a parabola.

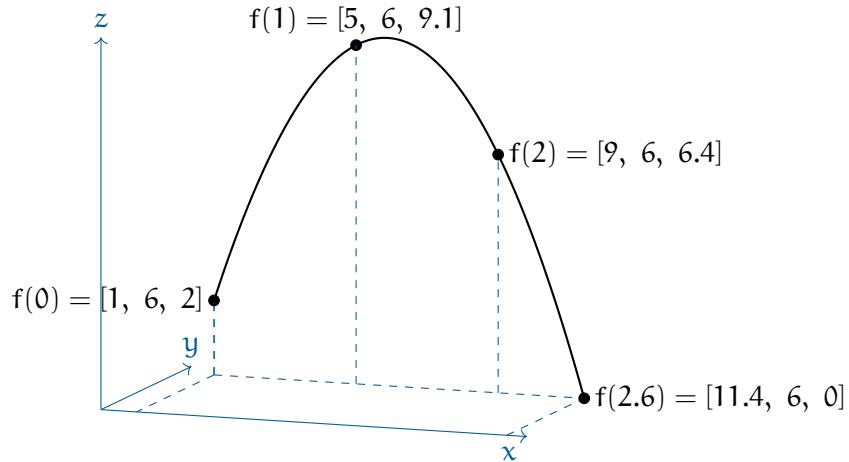


Figure 1.1: An example of a vector-valued function.

¹the \mathbb{R} symbol represents the set of all real numbers; the \mathbb{R}^2 symbol represents the set of all 2-dimensional vectors, and \mathbb{R}^3 represents the set of all 3-dimensional vectors

1.2 Finding the velocity vector

Now that we have its position vector, we can differentiate each component separately to get its velocity as a vector-valued function:

$$\mathbf{f}'(t) = [4, 0, -9.8t + 12]$$

In other words, the velocity is constant along the x -axis, zero along the y -axis, and decreasing with time along the z axis.

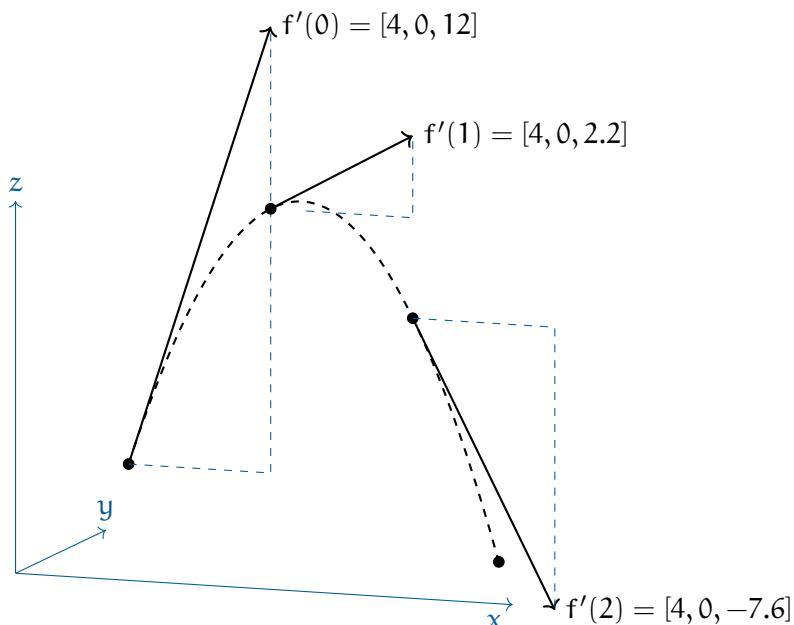


Figure 1.2: The derivatives of the position function (velocity) with respect to time.

1.3 Finding the acceleration vector

Now that we have its velocity, we can get its acceleration as a vector-valued function:

$$\mathbf{f}''(t) = [0, 0, -9.8]$$

There is no acceleration along the x or y axes. It is accelerating down at a constant 9.8m/s^2 .

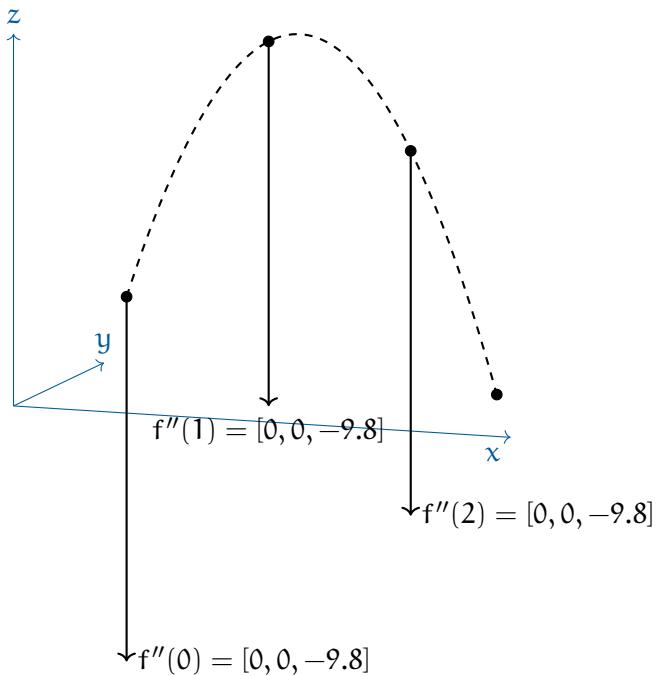


Figure 1.3: The acceleration vector is constant and points downward.

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APPENDIX A

Answers to Exercises

