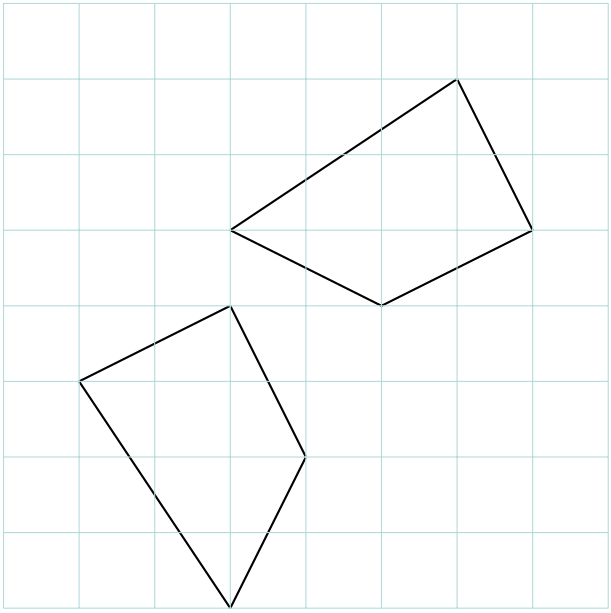
Chapter 1

Congruence

Look at this picture of two geometric figures.



They are the same shape, right? If you cut one out with scissors, it would lay perfectly on top of the other. In geometry, we say they are *congruent*.

What is the official definition of "congruent"? Two geometric figures are congruent if you can transform one into the other using only rigid transformations.

You might be wondering now, what are rigid transformations? A transformation is *Rigid* if it doesn't change the distances between the points or the measure of the angles between the lines, they form. These are all rigid transformations:

- Translations
- Rotations
- Reflections

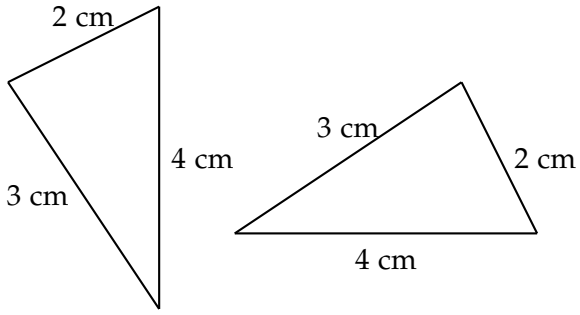
Once again imagine cutting out one figure with scissors and trying to match it with the second figure, your actions are rigid transformations:

- Translations - sliding the cutout left and right and up and down
- Rotations - rotating the cutout clockwise and counterclockwise
- Reflection - flipping the piece of paper over

A transformation is rigid if it is some combination of translations, rotations, and reflections.

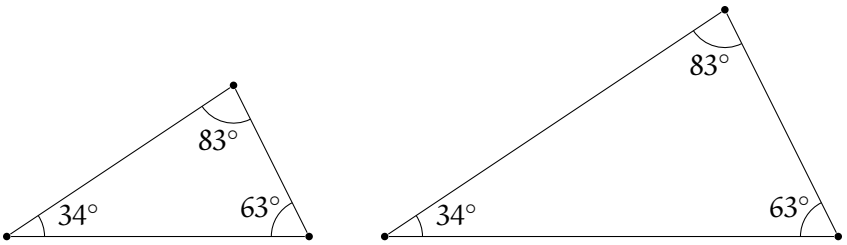
1.1 Triangle Congruency

If the sides of two triangles have the same length, the triangles must be congruent:



To be precise, the Side-Side-Side Congruency Test says that two triangles are congruent if three sides in one triangle are the same length as the corresponding sides in the other. We usually refer to this as the SSS test.

Note that two triangles with all three angles equal are not necessarily congruent. For example, here are two triangles with the same interior angles, but they are different sizes:

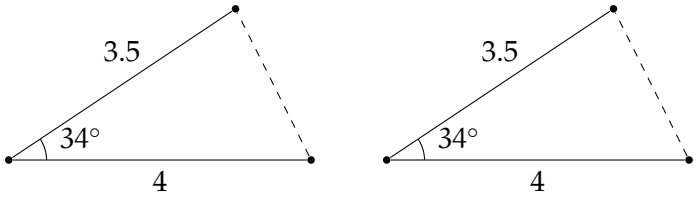


These triangles are not congruent, but they are *similar*. Meaning they have the same shape, but are not necessarily the same size.

Therefore, if you know two angles of a triangle, you can calculate the third. So it makes sense to say "If two triangles have two angles that are equal, they are similar triangles." And if two similar triangles have one side that is equal in length, they must be the same size – so they are congruent. Thus, the Side-Angle-Angle Congruency Test says that two

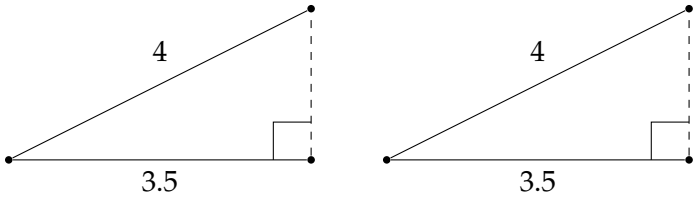
triangles are congruent if two angles and one side match.

What if you know that two triangles have two sides that are the same length and that the angle between them is also equal?



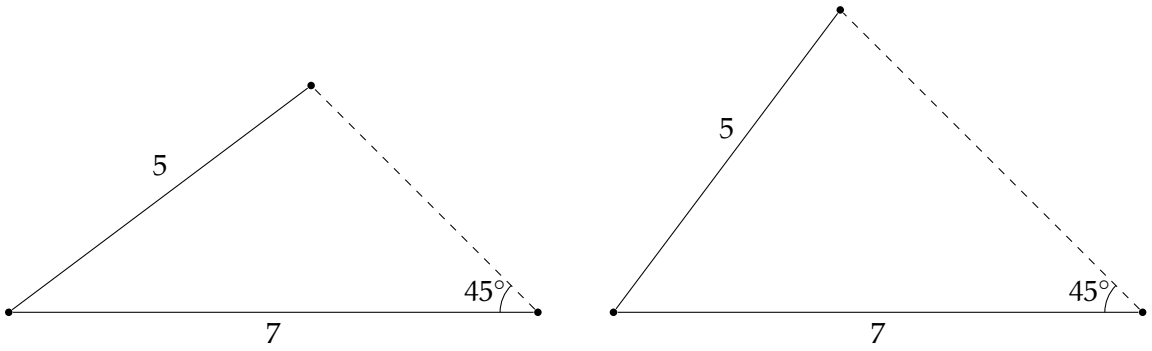
Yes, they must be congruent. This is the Side-Angle-Size Congruency Test.

What if the angle isn't the one between the two known sides? If it is a right angle, you can be certain the two triangles are congruent. (How do I know? Because the Pythagorean Theorem tells us that we can calculate the length of the third side. There is only one possibility, thus all three sides must be the same length.)

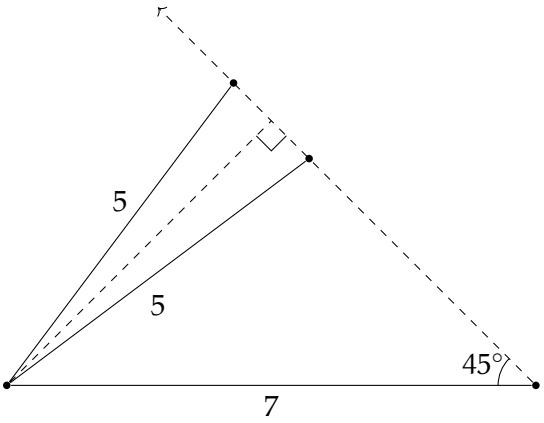


In this case, the third side of each triangle must be $\sqrt{4^2 - 3.5^2} \approx 1.9$.

What if the know angle is less than 90°? *The triangles are not necessarily congruent.* For example, let's say that there are two triangles with sides of length 5 and 7 and that the corresponding angle (at the end of the side of length 7) on each is 45°. Two different triangles satisfy this:



Let's see this another way by laying one triangle on top of the other:



So there is *not* a general Side-Side-Angle Congruency Test.

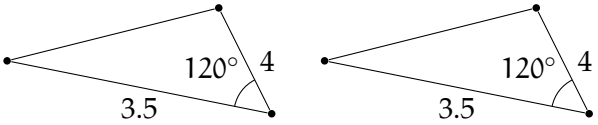
Here, then, is the list of common congruency tests:

- Side-Side-Side: All three sides have the same measure
- Side-Angle-Angle: Two angles and one side have the same measure
- Side-Angle-Side: Two sides and the angle between them have the same measure
- Side-Side-Right: They are right triangles and have two sides have the same measure

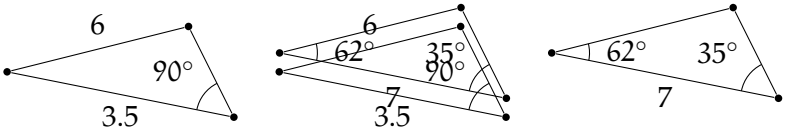
Exercise 1 Congruent Triangles

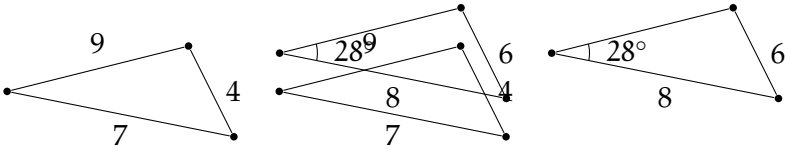
Working Space

Ted is terrible at drawing triangles: he always draws them exactly the same. Fortunately, he has marked these diagrams with the sides and angles that he measured. For each pair of triangles, write if you know them to be congruent and which congruency test proves it. For example:



(These drawings are clearly not accurate, but you are told the measurements are.) The answer is "Congruent by the Side-Angle-Side test."





Answer on Page 7

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 5)

Congruent by the Side-Side-Right Congruency Test.

Congruent by the Side-Side-Side Congruency Test.

Congruent by the Side-Angle-Angle Congruency Test.

We don't know if they are congruent. The measured angle is not between the measured sides.