# Chapter 1

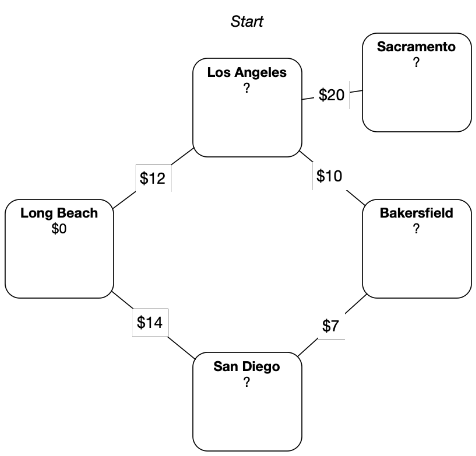
# Dijkstra's Algorithm

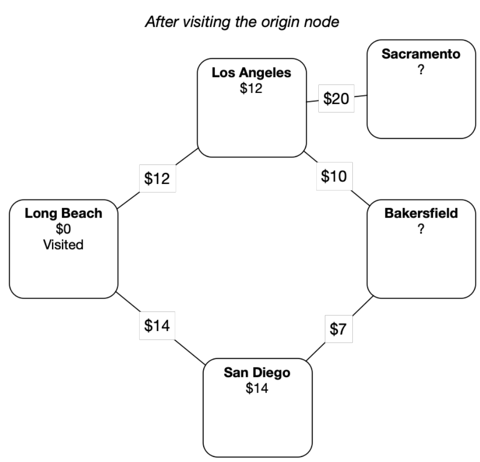
Edsger W. Dijkstra was a great Dutch computer scientist. He came up with an algorithm for finding the cheapest path through a graph with weighted edges. Today it is known as *Dijkstra's Algorithm*. It is used in a wide variety of common problems, and is also both simple and elegant.

## 1.1 Algorithm Description

You are going to mark each node with how much it would cost to get there from some origin node. For example, if you are shipping a container from Long Beach, you will mark each city with the cost of getting the container to that city.

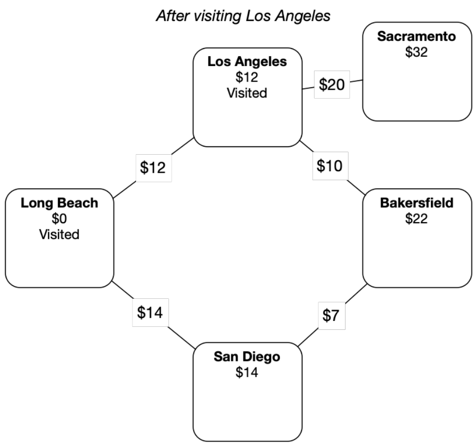
You start by marking the price for Long Beach to zero (the container is already there). You then mark each adjacent city with the cost on the edge. Next, you declare Long Beach to be "visited".

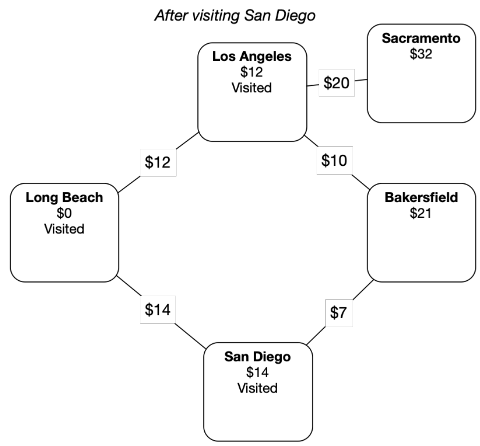



After that, you find the cheapest of the unvisited nodes. In this case, Los Angeles is cheaper than San Diego, so that is the node you will visit next.

You mark all of the unvisited nodes adjacent to Los Angeles, with the price to ship it to Los Angeles plus the cost of shipping the container from Los Angeles to that city. Note that Bakersfield is marked with $22.




Now, the cheapest unvisited node is San Diego, so you mark its neighbors with the cost to ship to San Diego, plus the price to ship from San Diego to the neighbor.

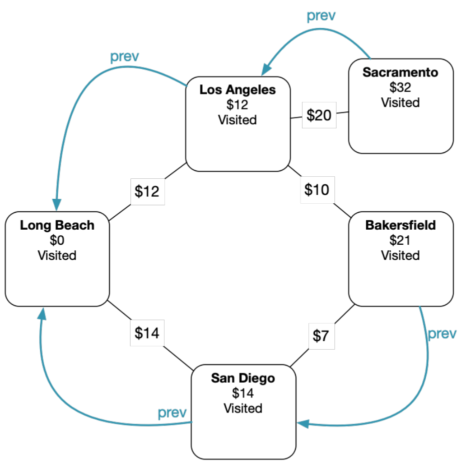
Notice that Bakersfield is already labeled with $22 from a route through Los Angeles. But the price would be $21 if you shipped it to Bakersfield via San Diego. Because the new route is cheaper, you change the price to the lower value.

(What does it mean that a node is "visited"? If a node is marked visited, it is marked with a price that won't get any smaller.)

You continue visiting the cheapest unvisited node until all the nodes have been visited. Then you know every node has been marked with its lowest price.

In a big graph, each node may be marked several times in this process — each time with a lower price from a cheaper router.

Of course, once you have the price, you will ask, "What is the route that gets me that price?" So, we will also mark each node with the neighbor from which it would receive the shipment — the previous node. This is easy to do as we execute the algorithm.




Now, to figure out the cheapest route from San Diego to Bakersfield, we start at the destination and follow the `prev` pointer back through San Diego, then to Long Beach.

## 1.2 Implementation

We do not actually want to sully our graph objects with the three additional pieces of information we need:

- The current minimal cost from the origin node. This is usually called the `dist`, from “distance”.
- The neighbor who gives us the current minimal cost. This is usually called `prev`, from “previous”.
- Whether the city is visited or now.

So, we will keep them in collections external to the graph.

For example, to keep track of the `dist`, we will have a `dist` dictionary. Each node will be a key, the current minimal cost will be the value. If the node hasn’t received even a first cost, we will put in infinity as the cost.

(After the algorithm is run, if the cost of a node is still infinity, that means that it cannot be reached from the origin node.)

We will also have a `prev` dictionary. The final node will be the key, and its previous neighbor will be the value.

Finally, the graph has a list of all the nodes, so we can just keep a set of the unvisited nodes.

Add a method to the `Graph` class that implements Dijkstra's algorithm:

```
def cost_from_node(self, origin_node):
    # Cost of cheapest path from origin node discovered so far
    # Initially the origin is zero and all the other are infinity
    dist = {k: math.inf for k in self.nodes}
    dist[origin_node] = 0.0

    # The previous city on that cheapest path
    prev = {}

    # All the nodes start as unvisited
    unvisited = set(self.nodes)

    # While there are still unvisited nodes
    while unvisited:

        # Find unvisited node with lowest cost
        min_cost = math.inf
        for u in unvisited:
            if dist[u] < min_cost:
                current_node = u
                min_cost = dist[u]

        # If none are less than inf, we are done
        # This happens in graphs that are not connected
        if min_cost == math.inf:
            return (dist, prev)

        # Remove the lowest cost node from the unvisited list
        unvisited.remove(current_node)

        # Update all the unvisited neighbors
        for edge in current_node.edges:

            # What node is at the other end of this edge?
            v = edge.other_end(current_node)

            # Visited nodes are already minimized, skip them
            if v not in unvisited:
                continue

            # Is this a shorter route?
            alt = dist[current_node] + edge.cost
            if alt < dist[v]:

                # Update the distance and prev dicts
                dist[v] = alt
                prev[v] = current_node

    return (dist, prev)
```

Append some code to your `cities.py` that tests this method:

```
(cost_from_long_beach, prev) = network.cost_from_node(long_beach)
print(f"\nMinimum costs from Long Beach = {cost_from_long_beach}")
print(f"\nLast city before = {prev}")

nyc_cost = cost_from_long_beach[nyc]

if nyc_cost < math.inf:
    print(f"\n*** Total cost from Long Beach to NYC: ${nyc_cost:.2f} ***")
else:
    print("You can't get to NYC from Long Beach")
```

When you run it, you should get a list of how much it costs to ship a container to each city from Long Beach:

```
Minimum costs from Long Beach = {(node:Long Beach, edges:1): 0.0,
(node:Los Angeles, edges:3): 12.0, (node:Denver, edges:3): 35.0,
(node:Pheonix, edges:3): 31.0, (node:Louisville, edges:3): 49.0,
(node:Cleveland, edges:4): 44.0, (node:Boston, edges:2): 57.0,
(node:New York City, edges:3): 52.0}
```

You will also get a collection of node pairs. What are these? For each node, you get the node that you would pass through on the cheapest route from Long Beach:

```
Last city before = {(node:Los Angeles, edges:3):(node:Long Beach, edges:1),
(node:Denver, edges:3):(node:Los Angeles, edges:3),
(node:Pheonix, edges:3):(node:Los Angeles, edges:3),
(node:Louisville, edges:3):(node:Denver, edges:3),
(node:Cleveland, edges:4):(node:Denver, edges:3),
(node:New York City, edges:3):(node:Cleveland, edges:4),
(node:Boston, edges:2): (node:Cleveland, edges:4)}
```

Your users will not want to read this; give them the shortest path as a list. Add a function to `graph.py` that turns the `prev` table into a path of nodes that lead from the origin to the destination:

```
def shortest_path(prev, destination):

    # Include the destination in the path
    path = [destination]
    current_node = destination

    # Keep stepping backward in the path
    while current_node in prev:

        # What node should come before the current node?
        previous_node = prev[current_node]
```

```
        # Insert it at the start of the list
        path.insert(0, previous_node)
        current_node = previous_node

    return path
```

Test that out:

```
if nyc_cost < math.inf:
    print(f"*** Total cost from Long Beach to NYC: $nyc_cost:.2f ***")

    path_to_nyc = graph.shortest_path(prev, nyc)
    print(f"*** Cheapest path from Long Beach to NYC: path_to_nyc ***")
else:
    print("You can't get to NYC from Long Beach")
```

This should look like this:

```
*** Cheapest path from Long Beach to NYC: [(node:Long Beach, edges:1),
(node:Los Angeles, edges:3), (node:Denver, edges:3), (node:Cleveland, edges:4),
(node:New York City, edges:3)] ***
```

## 1.3 Making it faster

On really big networks, doing a full Dijkstra's algorithm would take too long; luckily, there are many methods for getting similar results quickly. When you ask for directions from Google Maps, it does not do a full Dijkstra's Algorithm for every possible route — it would just take too long.

However, there is a way to speed up this implementation. Look at this snippet:

```
            # Find unvisited node with lowest cost
            min_cost = math.inf
            for u in unvisited:
                if dist[u] < min_cost:
                    current_node = u
                    min_cost = dist[u]
```

We are scanning through the list of all unvisited nodes, one by one, looking for the one with the lowest cost. If we kept this list sorted by cost, then the next one to visit would always be the first one in the list. This is done with a *priority queue* – a list that keeps itself sorted by some priority number – in this case the cost. In python, the standard priority queue is `heapq`.

(So why didn't I implement this using `heapq`? For Dijkstra's Algorithm, the nodes' priority – the current cost – changes as we find cheaper routes. `heapq` doesn't handle the changing priority very gracefully.)

In the next chapter, you will make a priority queue class that will work in this case.

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Appendix A

# Answers to Exercises

# Index