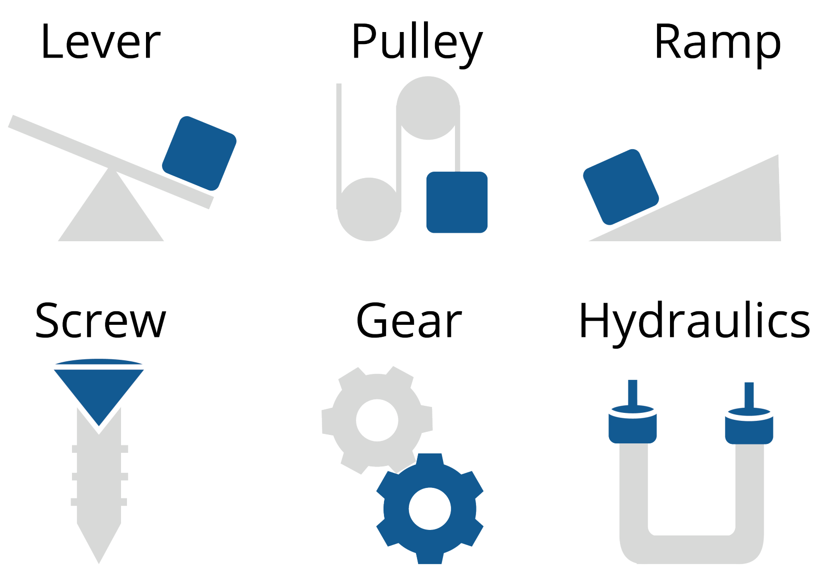
Chapter 1

Simple Machines

As mentioned earlier, physicists define work to be the force applied times the distance it is applied over. So, if you pushed your car 100 meters with 17 newtons of force, you have done 1700 joules of work.

Humans have always had to move really heavy things, so many centuries ago we developed simple machines to decrease the amount of force necessary to execute those tasks. These include things like:

- Levers
- Pulleys
- Ramps
- Gears
- Hydraulics
- Screws



While these machines can decrease the force needed, they don't change the amount of work that must be done. So if the force is decreased to a third, the distance that you must apply the force is increased by a factor of three.

"Mechanical gain" is what we call the increase in force.

1.1 Levers

A lever rotates on a fulcrum. To decrease the necessary force, the load is placed nearer to the fulcrum than where the force is applied.

In particular, physicists talk about the *torque* created by a force. When you push on a lever, the torque is the product of the force you exert and the distance from the point of rotation.

Torque is typically measured in newton-meters.

To balance two torques, the products must be the same. So, assuming that the forces are applied in the proper direction,

$$R_L F_L = R_A F_A$$

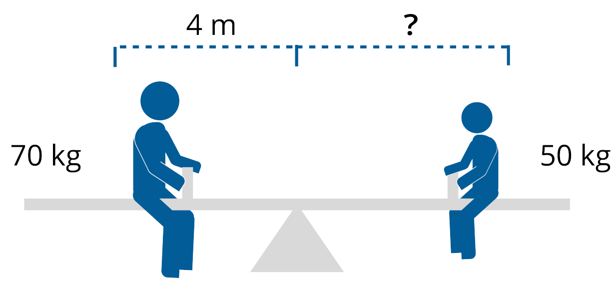
where R_L and R_A are the distance from the fulcrum to the where the load's force and the applied force (respectively) are applied, and F_L and F_A are the amounts of the forces.

Exercise 1 Lever

Paul, who weighs 70 kilograms, sits on a see-saw 4 meters from the fulcrum. Jan, who weighs 50 kilograms, wants to balance. How far should Jan sit from the fulcrum?

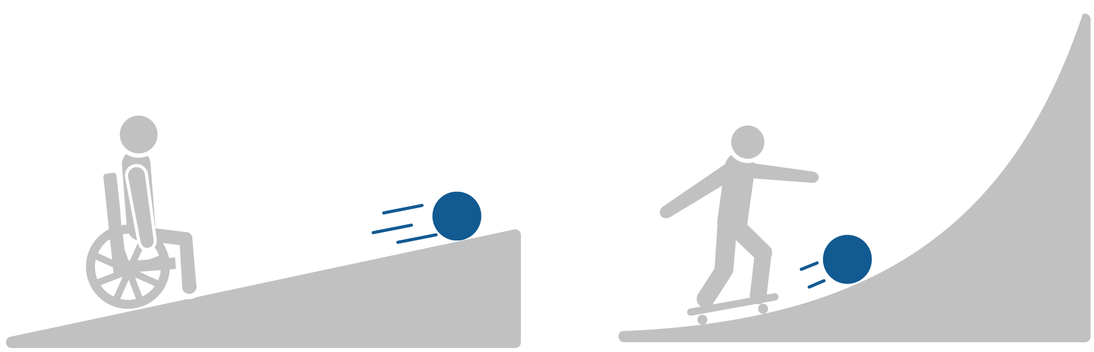
Working Space

Answer on Page 9



1.2 Ramps

Ramps, or incline planes, let you roll or slide objects up to a higher level. Steeper ramps give you less mechanical gain. For example, it is much easier to roll a ball up a wheelchair ramp than on a skateboard ramp.



Assuming the ramp has a constant steepness, the mechanical gain is equal to the ratio of the length of the ramp divided by the amount that it rises.

If you assume there is no friction, the force that you push a weight up the ramp will be:

$$F_A = \frac{V}{L} F_G$$

Where F_A is the force you need to push. L is the length of the ramp, V is the amount of vertical gain and F_G is the force of gravity on the mass.

(We haven't talked about the sine function yet, but in case you already know about it: Note that

$$\frac{V}{L} = \sin\theta$$

where θ is the angle between the ramp and level.)

Exercise 2 Ramp

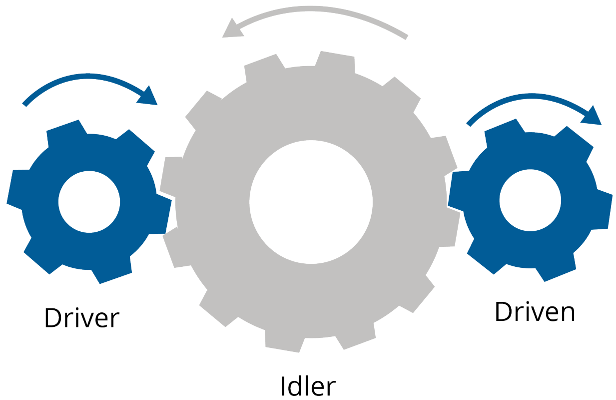
A barrel of oil weighs 136 kilograms. You can push with a force of up to 300 newtons. You have to get the barrel onto a platform that is 2 meters. What is the shortest board that you can use as a ramp?

Working Space

Answer on Page 9

1.3 Gears

Gears (which might have a chain connecting them like on a bicycle) have teeth and come in pairs. You apply torque to one gear, and it applies torque to another. The torque is increased or decreased based on the ratio between the teeth on the gears.



If N_A is the number of teeth on the gear you are turning with a torque of T_A, and N_L is the number of teeth on the gear it is turning, the resulting torque is:

$$T_L = \frac{N_A}{N_L} T_A$$

Exercise 3 Gears

Working Space

The bicycle is an interesting case because we are not trying to get mechanical gain. We want to spin the pedals slower with more force.

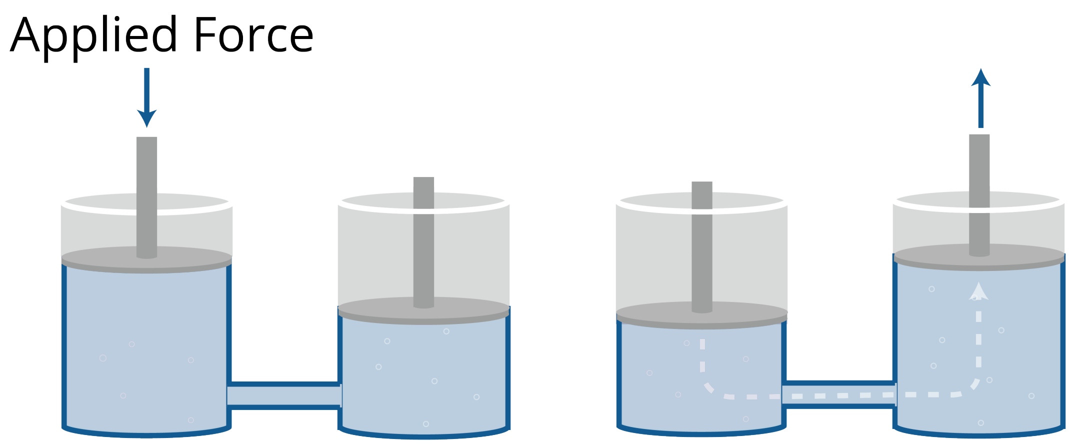
You like to pedal your bike at 70 revolutions per minute. The chainring that is connected to your pedals has 53 teeth. The circumference of your tire is 2.2 meters. You wish to ride a 583 meters per minute.

How many teeth should the rear sprocket have?

Answer on Page 9

1.4 Hydraulics

In a hydraulic system, like the braking system of a car, you exert force on a piston filled with fluid. The fluid carries that pressure into another cylinder. The pressure of the fluid pushes the piston in that cylinder out.



The pressure in the hose can be measured in pounds per square inch (PSI) or newtons per square meter (Pascals or Pa). We will use Pascals.

To figure out how much pressure you create, you divide the force by the area of the piston head you are pushing.

To figure out how much force that creates on the other end, you multiply the pressure times the area of the piston head that is pushing the load.

Exercise 4 Hydraulics

Working Space

Your car has disc brakes. When you put 2,500,000 pascals of pressure on the brake fluid, the car stops quickly. As the car designer, you would like that to require 12 newtons of force from the driver's foot.

What should the radius of the master cylinder (the one the driver is pushing on) be?

Answer on Page 9

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

Paul is exerting $(70)(9.8)$ newtons of force at 4 meters from the fulcrum, so he is creating a torque of 2,744 newton-meters of torque on the see-saw. Jan is creating $(50)(9.5) = 490$ newtons of force.

If r is the distance from the fulcrum to Jan's seat, to balance $490r = 2744$, so $r = 5.6$ meters.

Answer to Exercise 2 (on page 4)

To lift the barrel would require $136 \times 9.8 = 1,332.8$ newtons of force.

Letting L be the length of the ramp:

$$300 = \frac{2}{L}1332.8$$

So $L = 8.885$ meters.

Answer to Exercise 2 (on page 6)

$$583 = (70)(2.2)\frac{53}{n}$$

Thus $n = 14$ teeth.

Answer to Exercise 4 (on page 7)

We are looking for r, the radius of the piston head in meters. The area of the piston head is πr^2.

The pressure in pascals of the brake fluid is given by $12/(\pi r^2)$.

$$2,500,000 = \frac{12}{\pi r^2}$$

So $r = \sqrt{\frac{12}{\pi \times 2.5 \times 10^6}} = 0.001236077446474$ meters.