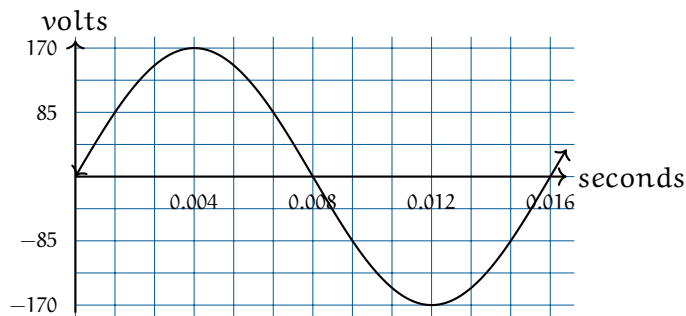


Alternating Current

We have discussed the voltage and current created by a battery. A battery pushes the electrons in one direction at a constant voltage; this is known as *Direct Current* or DC. A battery typically provides between 1.5 and 9 volts.

The electrical power that comes into your home on wires is different. If you plotted the voltage over time, it would look like this:



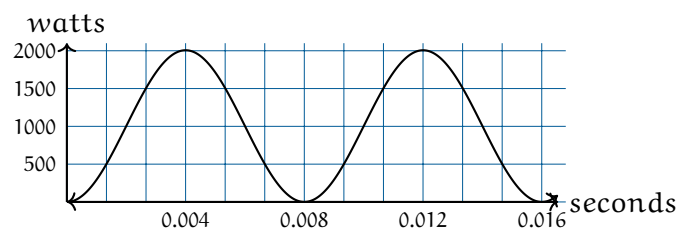
The x axis here represents ground. When you insert a two-prong plug into an outlet, one is “hot” and the other is “ground”. Ground represents 0 volts and should be the same voltage as the dirt under the building.

The voltage is a sine wave at 60Hz. Your voltage fluctuates between -170v and 170v. Think for a second what that means: The power company pushes electrons at 170v, then pulls electrons at 170v. It alternates back and forth this way 60 times per second.

1.1 Power of AC

Let’s say you turn on your toaster, which has a resistance of 14.4 ohms. How much energy (in watts) does it change from electrical energy to heat? We know that $I = V/R$ and that watts of power are IV . So, given a voltage of V , the toaster is consuming V^2/R watts.

However, V is fluctuating. Let’s plot the power the toaster is consuming:



Another sine wave! Here is a lesser-known trig identity: $(\sin(x))^2 = \frac{1}{2} - \frac{1}{2} \cos(2x)$

This is actually a cosine wave flipped upside down, scaled down by half the peak power, and translated up so that it is never negative. Note that it is also twice the frequency of the voltage sine wave.

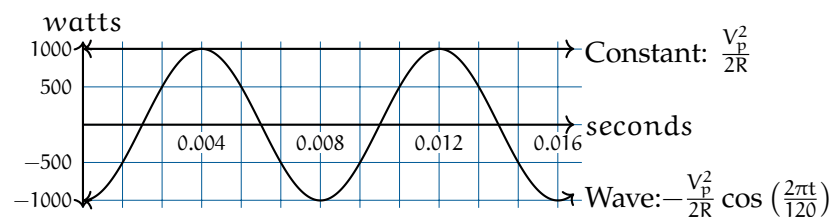
If we say the peak voltage is V_p and the resistance of the toaster is R , the power is given by

$$\frac{V_p^2}{2R} - \frac{V_p^2}{2R} \cos\left(\frac{2\pi t}{120}\right)$$

As a toaster user and as someone who pays a power bill, you are mostly interested in the average power. To get the average power, you take the area under the power graph and divide it by the amount of time.

We can think of the area under the curve as two easy-to-integrate quantities summed:

- A constant function of $y = \frac{V_p^2}{2R}$
- A wave $y = -\frac{V_p^2}{2R} \cos\left(\frac{2\pi t}{120}\right)$



When we integrate that constant function, we get $\frac{tV_p^2}{2R}$.

When we integrate that wave for a complete cycle we get...zero! The positive side of the wave is canceled out by the negative side.

So, the average power is $\frac{V_p^2}{2R}$ watts.

Someone at some point said, “I’m used to power being V^2/R . Can we define a voltage measure for AC power such that this is always true?”

So we started using V_{rms} which is just $\frac{V_p}{\sqrt{2}}$. If you look on the back of anything that plugs into a standard US power outlet, it will say something like “For 120v”. What they mean is, “For 120v RMS, we expect the voltage to fluctuate back and forth from 170v to -170v.”

Notice that this is the same Root-Mean-Squared that we defined earlier, but now we know that if $y = \sin(x)$, the RMS of y is $1/\sqrt{2} \approx 0.707$.

For current, we do the same thing. If the current is AC, the power consumed by a resistor is $I_{\text{RMS}}^2 R$, where I_{RMS} is the peak current divided by $\sqrt{2}$.

1.2 Power Line Losses

A wire has some resistance. Thinner wires tend to have more resistance than thicker ones. Aluminum wires tend to have more resistance than copper wires.

Let’s say that the power that comes to your house has to travel 20 km from the generator in a cable that has about 1Ω of resistance per km. Let’s also say that your home is consuming 12 kilowatts of power.

If that power is 120v RMS from the generator to your home, what percentage of the power is lost heating the power line? 10 amps RMS flow through your home. When that current goes through the wire, $I^2 R = (100)(20) = 2000$ watts is lost to heat.

This means the power company would need to supply 14 kilowatts of power, knowing that 2 kilowatts would be lost on the wires.

What if the power company moved the power at 120,000 volts RMS? Now only 0.01 amps RMS flow through your home. When that current goes through the wire $I^2 R = (0.0001)(20) = .002$ watts of power are lost on the power lines.

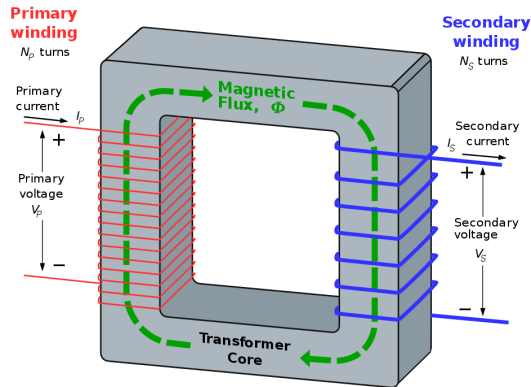
This is much, much more efficient. The only problem is that 120,000 volts would be incredibly dangerous. So the power company moves power long distances at very high voltages, like 765 kV. Before the power is brought into your home, it is converted into a lower voltage using a *transformer*.

1.3 Transformers

A transformer is a device that converts electrical power from one voltage to another. A good transformer is more than 95% efficient. The details of magnetic fields, flux, and inductance are beyond the scope of this chapter, so we are going to give a relatively

simple (and admittedly incomplete) explanation for now.

A transformer is a ring with two sets of coils wrapped around it.



(Diagram from Wikipedia)

When alternating current is run through the primary winding, it creates magnetic flux in the ring. The magnetic flux induces current in the secondary winding.

If V_P is the voltage across the primary winding and V_S is the voltage across the secondary winding, they are related by the following equation:

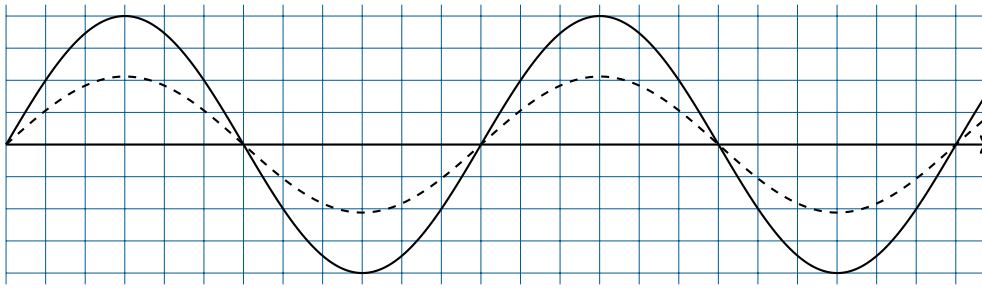
$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

where N_P and N_S are the number of turns in the primary and secondary windings.

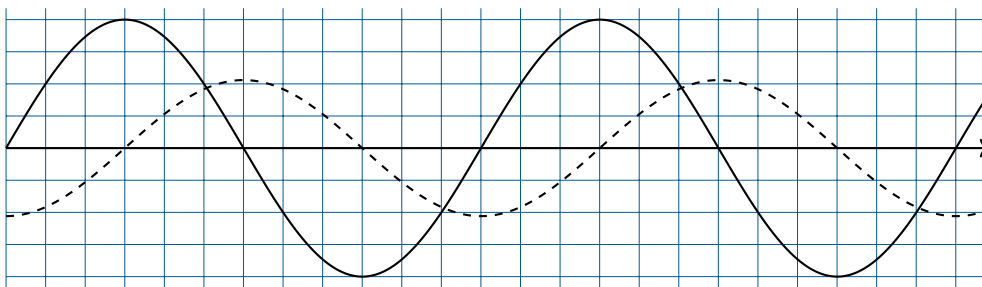
There are usually at least two transformers between you and the very high voltage lines. There are transformers at the substation that make the voltage low enough to travel on regular utility poles. On the utility poles, you will see cans that contain smaller transformers. Those step the voltage down to make the power safe to enter your home.

1.4 Phase and 3-phase power

If two waves are “in sync”, we say they have the same *phase*.

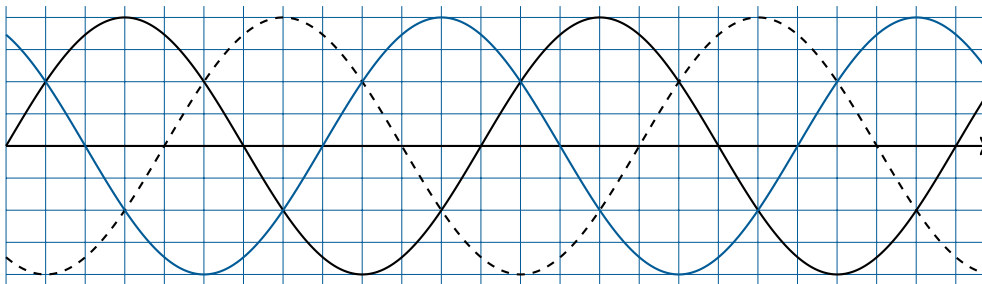


If they are the same frequency, but are not in sync, we can talk about the difference in their phase.



Here, we see that the smaller wave is lagging by $\pi/2$ or 90° .

In most power grids, there are usually three wires carrying the power. The voltage on each is $2\pi/3$ out of phase with the other two:



This is nice in two ways:

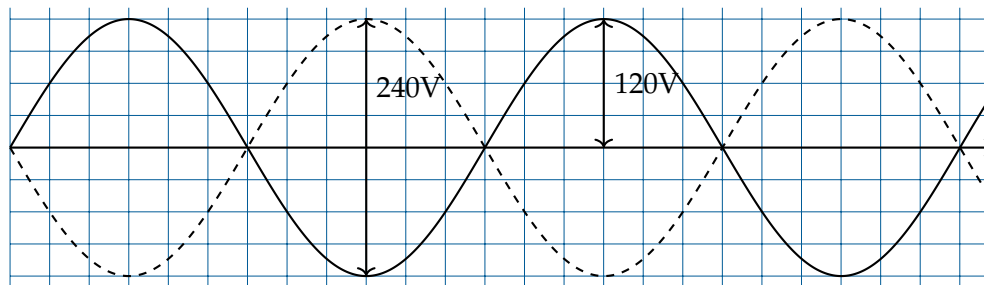
- While the power in each wire is fluctuating, the total power is not fluctuating at all.
- While the power plant is pushing and pulling electrons on each wire, the total number number of electrons leaving the load is zero.

(Both these assume that there each wire is attached to a load with the same constant resistance.)

In big industrial factories, you will see all three wires enter the building. Large amounts

of smooth power delivery means a great deal to an industrial user.

In residential settings, each home gets its power from one of the three wires. However, two wires typically carry power into the home. Each one carries 120V RMS, but they are out of phase by 180 degrees. Lights and small appliances are connected to one of the wires and ground, so they get 120V RMS. Large appliances, like air conditioners and washing machines, are connected across the two wires, so they get 240V RMS.



How do you get two circuits, 180 degrees out of phase, from one circuit? Using a center-tap transformer.

FIXME: Diagram here

This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.

Answers to Exercises

