CHAPTER 1

Multivariate Functions and Partial Derivatives

A real-valued multivariate function is a function that takes multiple real variables as input and produces a single real output. We generally denote such a function as $f : \mathbb{R}^n \to \mathbb{R}$, where $\mathbb{R}^n$ is the domain and $\mathbb{R}$ is the co-domain, (ie. $\mathbb{R}$ is the domain of one variable and $\mathbb{R}^2$ is the domain of a 2 variable function)

For example, consider a function f that takes two variables, x and y:

$$f(x, y) = x^2 + y^2$$

Here, $f : \mathbb{R}^2 \to \mathbb{R}$ takes an ordered pair (x, y) from the 2-dimensional real coordinate space, squares each, and adds them to produce a real number.

In a similar way, a function $g : \mathbb{R}^3 \to \mathbb{R}$ could take three variables, x, y, and z, and might be defined as:

$$g(x, y, z) = x^2 + y^2 + z^2$$

FIXME graphic of this showing area splotch mapping to line segment Here, the function squares each of the input variables, then adds them to produce a real number.

These functions are "real-valued" because their outputs are real numbers, and "multivariate" because they take multiple variables as inputs.

The concepts of limits, continuity, differentiability, and integrability can all be extended to multivariate functions, although they become more complex because we now have to consider different directions in which we approach a point, not just from the left or right, as in the univariate case. FIXME expand this?

For example, the partial derivative is the derivative of the function with respect to one variable, holding the others constant. It is one of the basic concepts in the calculus of multivariate functions.

For example, given the function $f(x, y) = x^2 + y^2$, the partial derivatives of f are computed as:

$$\frac{\partial f}{\partial x}(x, y) = 2x$$

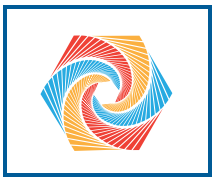
$$\frac{\partial f}{\partial y}(x, y) = 2y$$

FIXME expand on parametric funcs

We will expand on these partial derivatives in the next chapter.

Appendix A

Answers to Exercises



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