Chapter 1

Simple Machines

As mentioned earlier, physicists define work as the force applied times the distance over which it is applied. For example, if you push your car 100 meters with a force of 17 newtons, you have done 1700 joules of work.

Humans have long needed to move heavy objects, so many centuries ago, we developed simple machines to reduce the amount of force necessary to perform such tasks. These include:

- Levers
- Pulleys
- Inclined planes (ramps)
- Screws
- Gears
- Hydraulics
- Wedges

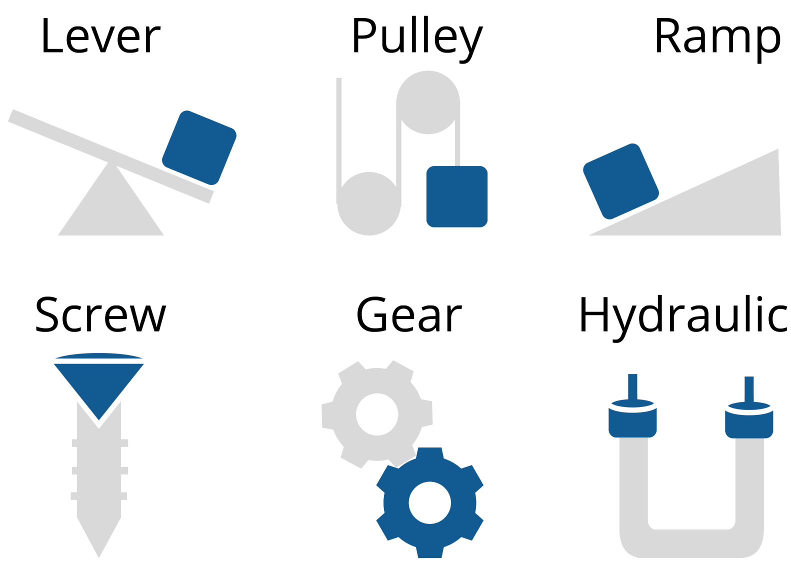


Figure 1.1: The 6 main simple machines: Levers, Pulleys, Ramps, Hydraulics, Screws, and Gears.

While these machines can reduce the force needed, they do not change the total amount of work that must be done. For instance, if the force is reduced by a factor of three, the distance over which the force must be applied *increases* by the same factor.

1.1 Mechanical Advantage

Mechanical advantage is the ratio between the force output by the machine and the force the user puts into the machine:

$$MA = \frac{F_{out}}{F_{in}} = \frac{\texttt{Load}}{\texttt{Effort}}$$

Since the input force is *applied* to the simple machine, sometimes the input force is called an applied force and abbreviated as F_a. For example, you only need to apply a relatively little force to your car's brakes in order for the hydraulic braking system to apply enough force to your tires to stop them spinning (we'll examine this further below).

1.1.1 What does it mean to work hard?

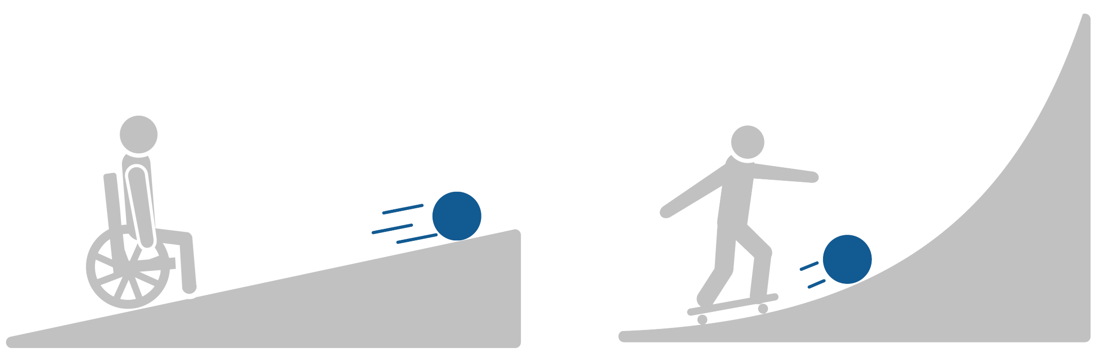
Humans use simple machines to "make work easier", but what does this mean in a physics sense? Does using a machine actually decrease the amount of work the user has to do?

When we say a task is easier, we usually mean *we have to apply less force*. You might say that it is "less work" to push something up a shallow incline than up a steep incline. But does the person pushing *actually do less work* (in a physics sense), or does that work simply require a smaller force? We'll answer this question by examining the physics of incline planes below, and the results will be true for all simple machines.

Ideally we want a higher Mechanical Advantage, which means we are putting in less input force for a greater output force.

1.2 Inclined Planes

Inclined planes, or ramps, allow you to roll or slide objects to a higher level. Steeper ramps require less mechanical advantage. For instance, it is much easier to roll a ball up a wheelchair ramp than a skateboard ramp.

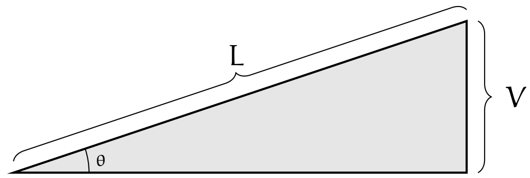


Assuming the incline has a constant steepness, the mechanical advantage is equal to the ratio of the length of the inclined plane to the height it rises.

If friction is neglected, the force required to push a weight up the inclined plane is given by:

$$F_A = \frac{V}{L} F_g \tag{1.1}$$

where F_A is the applied force, L is the length of the inclined plane, V is the vertical rise, and F_g is the gravitational force acting on the mass.



(We will discuss sine function later, but in case you're familiar with it, note that:

$$\frac{V}{L} = \sin\theta \tag{1.2}$$

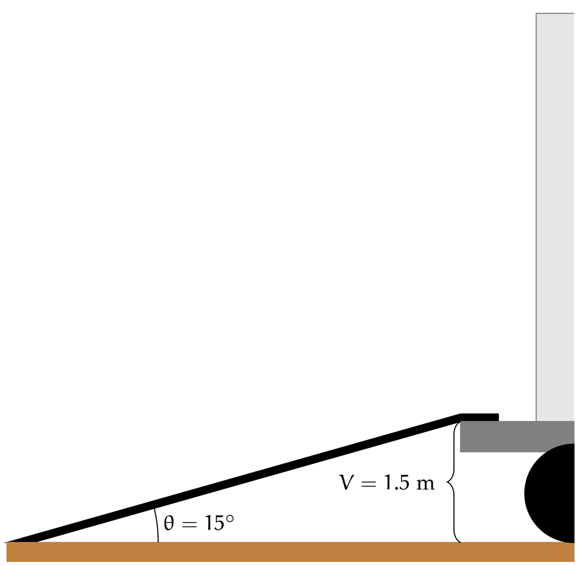
where θ is the angle between the inclined plane and the horizontal surface.)

Let's compare the force needed and work done when pushing a load up a ramp versus just lifting it vertically. Consider a family on moving day: there's a hand trolley loaded with 200 N (about 45 pounds) of boxes. If the bed of the moving truck is 1.5 m high, how much work would it take to lift the boxes straight up into the truck? What about with a ramp?

First, let's look at how much force and work is needed if you were to lift the entire 200 N load straight up into the air. You'd need to apply 200 N of force upwards for a distance of 1.5 m:

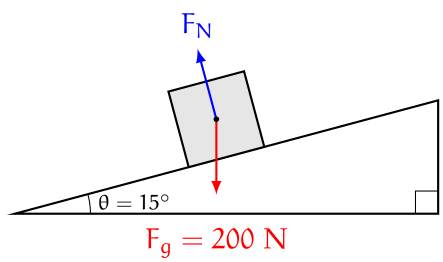
$$W = F \cdot d = (200\text{ N})(1.5\text{ m}) = 300\text{ J}$$

So, without a ramp, you would have to apply 200 N and do 300 J of work. Suppose your moving truck comes with a ramp that has an incline of 15 degrees:

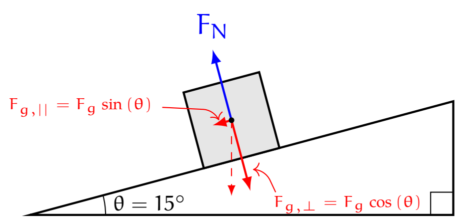


Since $\sin(\theta) = V/L$, we know that $L = V/\sin(\theta)$. You can use a calculator or search engine to find that the sine of $15^\circ \approx 0.26$. Therefore, the length of the ramp is approximately 5.8 meters. How much force does it take to move the load of boxes up the ramp? Intuitively, we know it is less force. We can use a *free body diagram* to determine the minimum force needed to push the box up the ramp. (A free body diagram is a simplified model showing all the forces acting on an object. You'll learn to create and use your own free body diagrams in a later chapter. For now, just follow along.)

Before you push it, there are two forces acting on the loaded hand trolley: its weight (F_g) and the normal force between the trolley and the ramp (F_N):

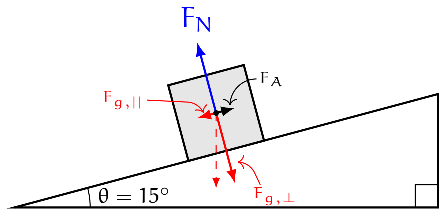


Notice that the normal force is perpendicular to the ramp! We want to know how much force it takes to push the load up the ramp, so we will "split" the weight force vector into two parts: one part parallel to the ramp ($F_{g,||}$) and one part perpendicular ($F_{g,\perp}$):



We did this by treating the weight vector as the hypotenuse of a right triangle with legs perpendicular and parallel to the ramp. You'll learn how to do this and why it works in the chapter on vectors. For now, just trust that the part of the hand trolley's weight that is perpendicular to the ramp is $F_g \cos(\theta)$ and the part that is parallel to the ramp is $F_g \sin(\theta)$.

What force do you need to overcome to push the hand trolley up the ramp? Just the part of the weight that is parallel to the ramp! You'll need to apply an equal force in the opposite direction (up the ramp) to move the hand trolley:



So, we know that you are pushing with an applied force of $F_A = F_{g,\parallel} = F_g \sin(\theta)$. Therefore, the work you would do pushing the hand trolley up the ramp is:

$$F_A \cdot L = F_g \sin(\theta) \cdot \left(\frac{V}{\sin(\theta)}\right) = F_g \cdot V = 300 \text{ J}$$

Therefore, when using a ramp, you still perform the same amount of work! This is a key property of simple machines: *the work done doesn't change*.

So what makes it "easier" to use a ramp to lift the hand trolley? The fact that you need to apply less *force* to move the hand trolley ($F_A < F_g$). Now, let's look at the mechanical advantage of the ramp. In this case, the mechanical advantage is given by:

$$MA = \frac{F_g}{F_A}$$

Substituting for F_A, we see that:

$$MA = \frac{F_g}{F_g \sin(\theta)} = \frac{1}{\frac{V}{L}} = \frac{L}{V}$$

So for a ramp whose length is L and vertical rise is V, the mechanical advantage is equal to the length divided by the rise.

Ramps

For a ramp, the mechanical advantage is equal to $\frac{L}{V}$ and the force needed to push an object with weight W up the ramp is given by $W \cdot \frac{V}{L} = W \cdot \sin(\theta)$, where L is the length of the ramp, V is the vertical rise of the ramp, and θ is the angle the ramp forms with the (horizontal) ground.

We have a whole inclined planes chapter coming up, but this was just a brief overview of their function as a mechanic

Exercise 1 Ramp

You need to lift a barrel of oil with a mass of 136 kilograms. You can apply a force of up to 300 newtons. You need to get the barrel onto a platform that is 2 meters high. What is the shortest length of inclined plane you can use?

Working Space

Answer on Page ??

1.3 Levers

A lever pivots on a fulcrum. To decrease the necessary force, the load is placed closer to the fulcrum than where the force is applied.

Physicists also discuss the concept of *torque* created by a force. When you apply force to a lever, the torque is the product of the force you exert and the distance from the point of rotation.

Torque is typically measured in newton-meters (N·m).

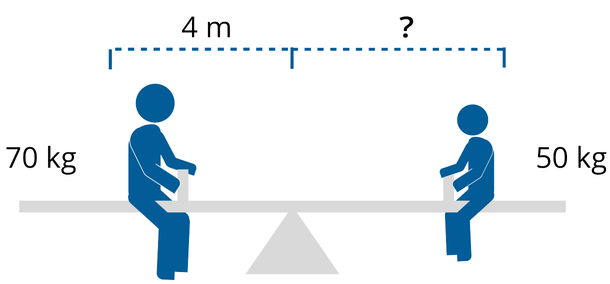
To balance two torques, the products of force and distance must be equal. Thus, assuming the forces are applied in the correct direction, the equation becomes:

$$R_L F_g = R_A F_A \tag{1.3}$$

where R_L and R_A represent the distances from the fulcrum to where the load's weight and the applied force are exerted, respectively, and F_g and F_A are the magnitudes of the forces.

Exercise 2 Lever

Paul, whose mass is 70 kilograms, sits on a see-saw 4 meters from the fulcrum. Jan, whose mass is 50 kilograms, wishes to balance the see-saw. How far should Jan sit from the fulcrum?



Working Space

Answer on Page ??

1.4 Gears

Gears are rotating parts of machines that transmit torque or other types of rotational motion through intertwined teeth. Often gears are meshed together; two or more meshed gears are refered to as a gear train.

Gears have teeth that mesh with each other. When you apply torque to one gear, it transfers torque to the other. The resulting torque is increased or decreased depending on the ratio of the number of teeth on the gears. We will go in depth on torque in a future chapter, but know that torque depends on the radius of the gear, and the radius of a gear is proportional to the number of teeth, the tooth count directly controls the torque.

1.4.1 Involute Gears

Gear teeth have a very unique shape. The vast majority of modern gears use what is known as an *involute gear profile*. An involute gear is one whose tooth shape is derived from the *involute of a circle*. This curve is generated by imagining a fixed string being unwrapped from the circumference of a circle. The path traced by the end of the string forms the involute.

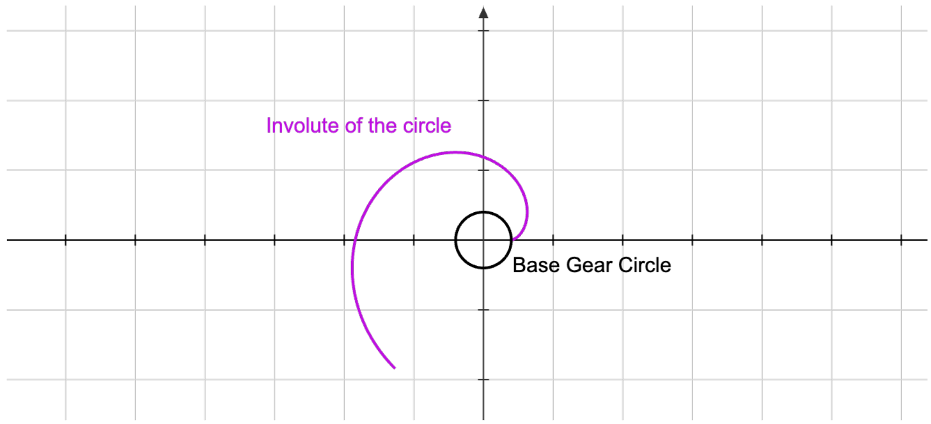


Figure 1.2: The involute of a circle shown. The purple line is revolved for 4π radians or 2 revolutions (for clarity).

Although the involute curve resembles a spiral, it is not an Archimedean spiral. Instead, its geometry is defined by the constant unwrapping of the string, which gives the involute several important mechanical properties.

The primary advantage of the involute profile shape is that it ensures smooth and reliable torque transfer between two or more meshing gears. When two involute gears engage, their teeth make contact at a single point that moves along a straight line known as the **line of action**. This line is tangent to the base circles of both gears. As the gears rotate, the point of contact travels along this line while maintaining a constant pressure angle, which allows the gears to transmit motion at a constant speed ratio even if the center distance between them varies slightly.

Some more gear construction terminology:

The pressure angle The acute angle between the line of action and a normal to the line connecting the gear centers. May vary depending on the involute shape, but most commonly 20°.

Addendum The difference between the pitch circle and its tooth tip circle.

Dedendum The difference between the radius of the pitch circle of a gear and its root circle

Tooth Height The distance between its root circle and the tip are called the tooth height (h).

Total Height of Tooth The total height of the gear is the sum of the addendum (ha = 1.00m) and the dedendum (hf = 1.25m).

FIXME Possible gear diagram in tikz wth labeled everything

Figure 1.3: Two gears meshing together diagram (not directly conjoined for clarity).

For part of the motion, Gear One pushes Gear Two down. Once the contact point passes the midpoint of the line of action, Gear One pushes up on Gear Two. This causes a constant transfer of torque.

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A parametric function, defined by $x(\theta)$ and $y(\theta)$ is given by

$$\begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} = r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + r\theta \begin{bmatrix} \sin(\theta) \\ -cos(\theta) \end{bmatrix}$$

For gears, a large torque makes it easier to start rotating an object or to continue rotating it against resistance, such as climbing a hill or accelerating a heavy load.

Power, on the other hand, measures how quickly work is being done. In rotational motion, power depends on both torque and rotational speed. A system can produce high torque at low speed, or lower torque at high speed, and still deliver the same power.

Gears allow a machine to adjust this balance between torque and speed without significantly changing the power being transmitted (neglecting losses). When gears increase torque, they reduce rotational speed, and visa versa.

A familiar example is riding a bicycle up a hill. When climbing, it becomes difficult to pedal in a high gear because the torque required at the pedals is large. Shifting to a lower gear increases the torque applied to the rear wheel for the same pedaling effort, making it easier to climb. However, the pedals must turn more times for each rotation of the wheel, so the bicycle moves more slowly. The rider's power output remains roughly the same, but it is delivered as higher torque at lower speed.

The same principle applies to the gears in a car. When starting from rest or climbing a steep incline, a low gear is used to provide high torque at the wheels, allowing the vehicle to overcome inertia and gravity. At highway speeds, higher gears are used to reduce torque and increase speed, improving efficiency and reducing engine wear. In both cases, the engine produces power, and the transmission adjusts how that power is distributed between torque and rotational speed.

In mechanical systems, gears therefore do not create power; instead, they shape how power is delivered.

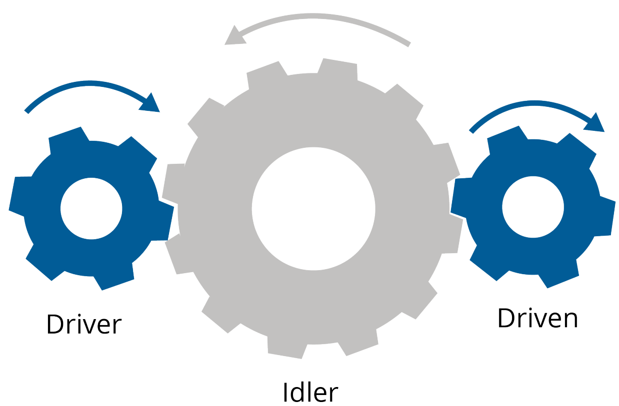


Figure 1.4: .

1.4.2 Teeth

If N_A is the number of teeth on the gear you are turning with a torque of T_A, and N_L is the number of teeth on the gear it is turning, the resulting torque is:

$$T_L = \frac{N_A}{N_L} T_A$$

Exercise 3 Gears

In a bicycle, the goal is not always to gain mechanical advantage, but to spin the pedals slower while applying more force.

You like to pedal your bike at 70 revolutions per minute. The chainring connected to your pedals has 53 teeth. The circumference of your tire is 2.2 meters. You want to ride at 583 meters per minute.

How many teeth should the rear sprocket have?

Working Space

Answer on Page ??

1.5 Hydraulics

In a hydraulic system, such as a car's braking system, you exert force on a piston filled with fluid. The fluid transmits this pressure into another cylinder, where it pushes yet another piston that moves the load. The pressure at each end of the hydraulic system must be the same.

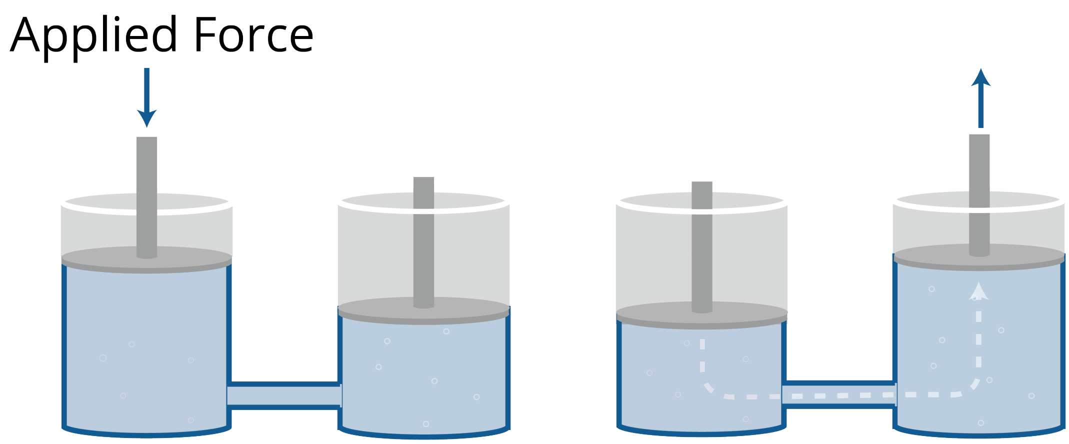


Figure 1.5: A diagram showing the transfer of fluid from one container to the other.

Pressure is force applied to an area; it is calculated by dividing the force by the area. The pressure in the fluid is typically measured in pascals (Pa), which is equivalent to N/m^2. We will use pascals for this calculation.

To calculate the pressure you create, divide the force applied F_a by the area of the piston head A. To determine the force on the other piston, multiply the pressure by the area of the second piston.

$$P = \frac{F_{a_1}}{A_1} = \frac{F_{a_2}}{A_2} \tag{1.4}$$

Exercise 4 Hydraulics

Your car has disc brakes. When you apply 2,500,000 pascals of pressure to the brake fluid, the car stops quickly. As the car designer, you want this to require only 12 newtons of force from the driver's foot.

What should the radius of the master cylinder (the piston the driver pushes) be?

Working Space

Answer on Page ??

1.6 Pulleys

Pulleys are anything that changes the direction of a force, typically by using a wheel and a rope. A single pulley can make things easier by allowing you to pull down instead of pushing up. This lets you use your body weight to help you pull rather than just your arm strength.

By attaching multiple pulleys, you can do more and actually reduce the force needed to lift a load. For example, if you use two pulleys, you can reduce the force needed to lift a load by half. However, you will have to pull twice as much rope to lift the load the same distance.

Each additional pulley you add to the system adds another segment of rope that supports the load. The mechanical advantage of a pulley system is equal to the number of rope segments supporting the load.

$$\mathrm{MA} = \mathrm{N}$$

where N is the number of rope segments supporting the load.

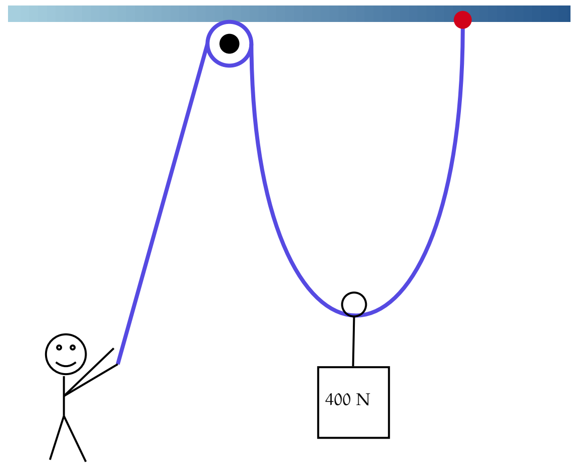


Figure 1.6: A 400 N weight being distributed by one pulley.

Here, a rope is carrying a 400 N weight. Attached to the weight is a movable pulley, while the top pulley is fixed to the ceiling. Assuming an ideal (massless) rope and frictionless pulleys, the tension is the same everywhere in the rope. The movable pulley is supported by two rope segments, so the upward force on the load is 2T. For the load to be held or lifted at constant speed,

$$2T = 400 \text{ N} \quad \Rightarrow \quad T = 200 \text{ N}$$

Assuming an ideal (massless) rope and frictionless pulleys, the tension is the same everywhere in the rope, so there is 200 N throughout the rope. Since there are two ropes supporting the load, the Mechanical Advantage is 2.

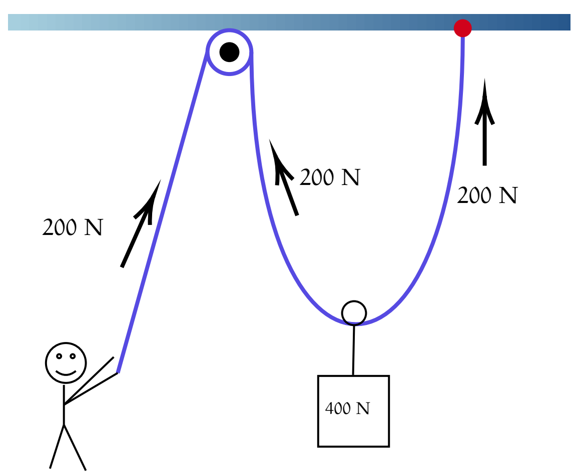


Figure 1.7: The tension in the rope is 200 N.

However, this does come at a cost! The weight of the load is 400 N so the pulley system must provide a total upward force of 400 N on the load. However, both sections of rope (although technically the same rope) pull at 200 N for a total of 400 N.

The work done, however, is still the same! When the person pulls the rope down by 2 m, each supporting rope segment shortens by 1 m, so the load rises by 1 m. This is all ignoring friction and assuming constant speed. Work done by the person:

$$W = \mathrm{F} \times \mathrm{d} = 200\,\mathrm{N} \times 2\,\mathrm{m} = 400\,\mathrm{J}$$

Work gained by the load:

$$W = 400\,\mathrm{N} \times 1\,\mathrm{m} = 400\,\mathrm{J}$$

This satisfies our work equation

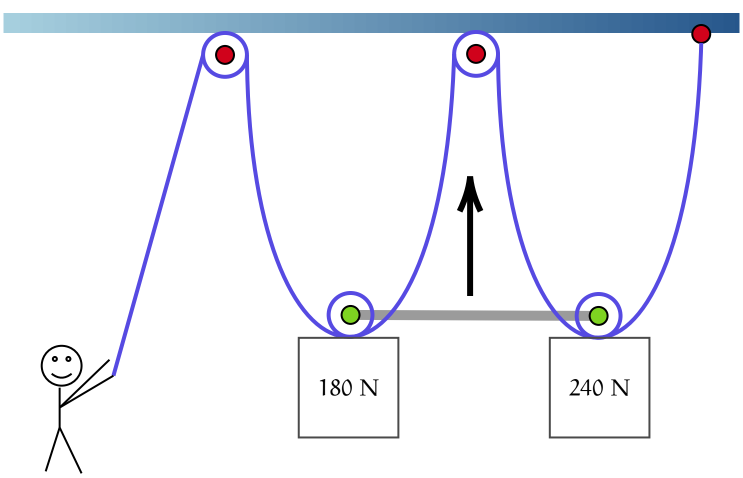
$$\mathrm{Work_{in}} = \mathrm{Work_{out}} \tag{1.5}$$

Note that we can then rewrite our mechanical advantage as a proportion of distance:

$$\mathrm{MA} = \frac{\mathrm{d_{effort}}}{\mathrm{d_{load}}} \tag{1.6}$$

Exercise 5 Pulleys

Working Space



A person is pulling 2 weights tensioned by 4 pulleys. There are 2 fixed pulleys (shown in red) and 2 movable pulleys (shown in green). The two movable pulleys are fixed such that they move together vertically. Calculate the input force that is needed to begin lifting the weights.

Answer on Page ??

1.7 Wedges

A wedge is a triangular shaped simple machine that converts forces from one direction to another. Often, it is from a vertical direction to a horizontal one.

Similar to an inclined plane, wedges have steep angles to manipulate the properties of triangles. The force applied to the top portion of a wedge is transfered acting perpendicular to the sloped sides of the wedge.

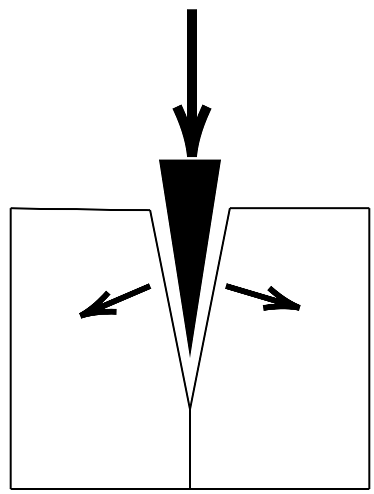


Figure 1.8: A wedge splitting a block of wood, with the forces acting perpendicular.

Similar to the inclined plane, the Mechanical Advantage is the proportion of the vertical rise or width, and the length of the sloped portion of the triangle.

$$\mathrm{MA} = \frac{\mathrm{L}}{\mathrm{V}} \tag{1.7}$$

The smaller the angle of a wedge, the greater the ratio of the length of its slope to its width, and the more mechanical advantage it will yield. This is a concept that comes directly from trigonometry.

Exercise 6 Wedges 1

An axe is used to split wood. The length of the wedge is 12 cm and the thickness is 3 cm. Calculate the mechanical advantage.

Working Space

Answer on Page ??

Exercise 7 Wedges 2

Explain why a flathead-screwdriver tip is a poor option for cutting a steak compared to a knife.

Working Space

Answer on Page ??

Appendix A

Answers to Exercises

Answer to Exercise ?? (on page ??)

The weight of the barrel is $136\ \text{kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} = 1332.8\ \text{N}$.

Let L be the length of the inclined plane. The force needed to push the barrel up is related by:

$$300\ \text{N} = \frac{2\ \text{m}}{L} \times 1332.8\ \text{N}$$

Solving for L, we find $L = \frac{2\ \text{m} \times 1332.8}{300} \approx 8.885\ \text{m}$.

Answer to Exercise ?? (on page ??)

Paul exerts a force of $70\ \text{kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} = 686\ \text{N}$ at a distance of 4 meters from the fulcrum, creating a torque of $686\ \text{N} \times 4\ \text{m} = 2744\ \text{N} \cdot \text{m}$. Jan exerts a force of $50\ \text{kg} \times 9.8\ \frac{\text{m}}{\text{s}^2} = 490\ \text{N}$.

Let r be the distance from the fulcrum to Jan's seat. To balance the torques:

$$490\ \text{N} \times r = 2744\ \text{N} \cdot \text{m}$$

Solving for r, we find $r = \frac{2744}{490} \approx 5.6$ meters.

Answer to Exercise ?? (on page ??)

The equation relating these quantities is:

$$583 = 70 \times 2.2 \times \frac{53}{n}$$

Solving for n, we find $n = 14$ teeth.

Answer to Exercise ?? (on page ??)

We are solving for the radius r of the piston. The area of the piston is πr^2, so the pressure is:

$$\text{Pressure} = \frac{12}{\pi r^2}$$

Setting the pressure equal to 2,500,000 pascals:

$$2,500,000 = \frac{12}{\pi r^2}$$

Solving for r, we find:

$$r = \sqrt{\frac{12}{\pi \times 2.5 \times 10^6}} \approx 0.00124 \text{ meters.}$$

Answer to Exercise ?? (on page ??)

There are 4 segments supporting the net weight total of 420 N. Each segment, then, splits the weight into $\dfrac{420\text{ N}}{4} = 105$ N. The input force required to move both weights is 105 N, which moves it 4 m. The output work matches as well: 420 N is moved 1 m.

Answer to Exercise ?? (on page ??)

The mechanical advantage of the wedge is $\frac{12\text{ cm}}{3\text{ cm}} = 4$.

Answer to Exercise ?? (on page ??)

A knife cuts better than a screwdriver tip because its edge is much thinner, giving it a greater mechanical advantage. This allows the applied force to be concentrated over a very small area, producing much higher pressure. As a result, less force is needed to cut the material. A screwdriver, on the other hand, has a very thick edge (in comparison), so its pressure and mechanical advantage are much less.

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