

# Inverse Trigonometric Functions

Recall from the chapter on functions that an inverse of a function is a machine that turns  $y$  back into  $x$ . The inverses of trigonometric functions are essential to solving certain integrals (you will learn in a future chapter why integrals are useful — for now, trust us that they are!). Let's begin by discussing the sin function and its inverse,  $\sin^{-1}$ , also called arcsin. (Note: this is not saying sin to the power of  $-1$ , it is a method of stating the inverse of the function).

Examine the graph of  $\sin x$  in figure 1.1. See how the dashed horizontal line crosses the function at many points? This means the function  $\sin x$  is not one-to-one. In other words, there is not a unique  $x$ -value for every  $y$ -value. This means that if we do not restrict the domain of  $\arcsin x$ , the result will not be a function (see figure 1.2). In figure 1.2, you can see that just reflecting the graph across  $y = x$  fails the vertical line test: an  $x$  value has more than one  $y$  value.

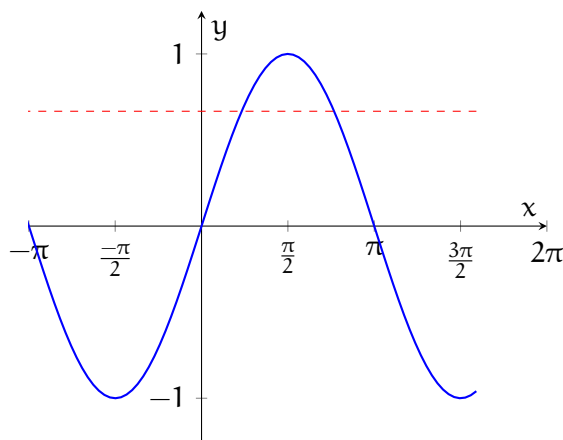


Figure 1.1: The horizontal line  $y = \frac{2}{3}$  crosses  $y = \sin x$  more than once

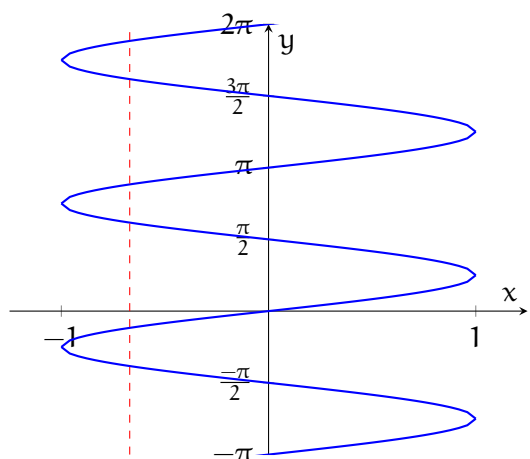


Figure 1.2: The inverse of an unrestricted sin function fails the vertical line test

## 1.1 Derivatives of Inverse Trigonometric Functions

$f$	$f'$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arccsc} x$	$-\frac{1}{x\sqrt{x^2-1}}$
$\operatorname{arcsec} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$

## 1.2 Practice

### Exercise 1

Find the  $f'$ . Give your answer in a simplified form.

- $f(x) = \arctan x^2$
- $f(x) = x \operatorname{arcsec}(x^3)$
- $f(x) = \arcsin \frac{1}{x}$

Working Space

Answer on Page 5

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*



# Answers to Exercises

## Answer to Exercise 1 (on page 2)

1. By the chain rule,  $f'(x) = 2 \arctan x \times \frac{d}{dx} \arctan x = 2 \arctan x \frac{1}{1+x^2}$
2. By the Product rule,  $f'(x) = x \frac{d}{dx} \operatorname{arcsec}(x^3) + \operatorname{arcsec}(x^3)$ . Further, by the chain rule,  $\frac{d}{dx} \operatorname{arcsec}(x^3) = \frac{1}{(x^3)\sqrt{(x^3)^2-1}} \times \frac{d}{dx}(x^3) = \frac{3x^2}{x^3\sqrt{x^6-1}}$ . Therefore,  $f'(x) = \frac{3}{\sqrt{x^6-1}} + \operatorname{arcsec}(x^3)$
3. By the chain rule,  $f'(x) = \frac{1/x}{\sqrt{1-(1/x)^2}} \times -\frac{1}{x^2} = -\frac{1}{x^3\sqrt{1-\frac{1}{x^2}}}$

