Chapter 1

Solving Quadratics

A quadratic function has three terms: ax^2+bx+c. a, b, and c are known as the *coeffcients*. The coefficients can be any constant, except that a can never be zero. (If a is zero, it is a linear function, not a quadratic.)

When you have an equation with a quadratic function on one side and a zero on the other, you have a quadratic equation. For example:

$$72x^2 - 12x + 1.2 = 0$$

How can you find the values of x that will make this equation true?

You can always reduce a quadratic equation so that the first coefficient is 1, so that your equation looks like this:

$$x^2 + bx + c = 0$$

For example, if you are asked to solve $4x^2 + 8x - 19 = -2x^2 - 7$

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$$6x^2 + 8x - 12 = 0$$

$$x^2 + \frac{4}{3}x - 2 = 0$$

Here, $b = \frac{4}{3}$ and $c = -2$.

$x^2 + bx + c = 0$ when

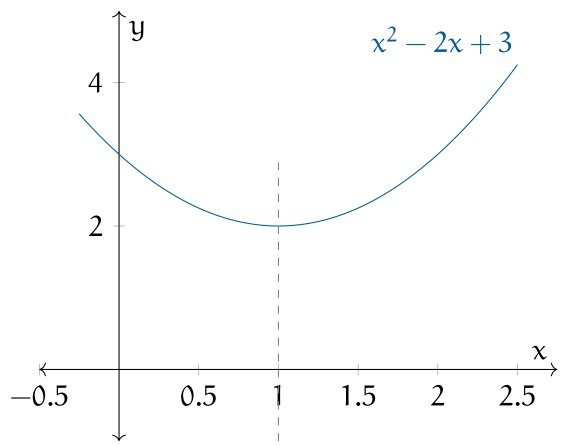
$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

Only when $a = 1$

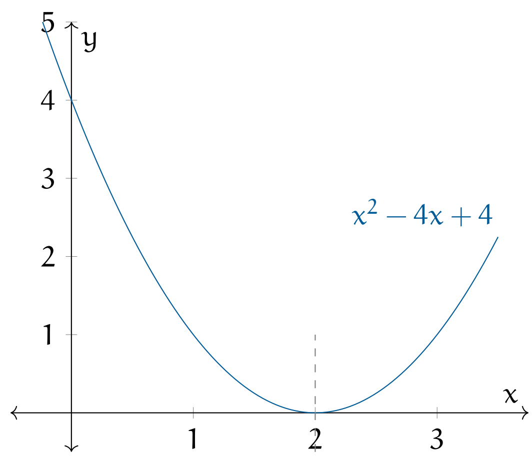
What does this mean?

For any b and c, the graph of $x^2 + bx + c$ is a parabola that goes up on each end. Its low point is at $x = -\frac{b}{2a}$. This is referred to as the vertex formula.

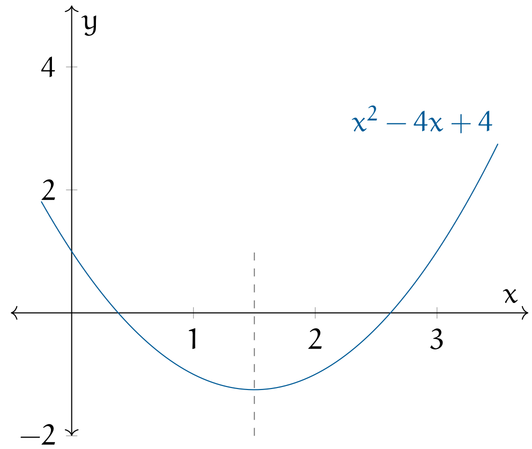
If there are no real roots ($b^2 - 4c < 0$), which means the parabola never gets low enough to cross the x-axis:



If there is one real root ($b^2 - 4c = 0$), it means that the parabola only touches the x-axis.



If there are two real roots ($b^2 - 4c > 0$), it means that the parabola crosses the x-axis twice as it dips below and then returns:



Exercise 1 Roots of a Quadratic

Working Space

In the last chapter, you found that the function for the height of your flying hammer is:

$$p = -\frac{1}{2}9.8t^2 + 12t + 2$$

At what time will the hammer hit the ground?

Answer on Page 7

1.1 The Traditional Quadratic Formula

If the last explanation was a little tricky to understand, the quadratic formula is a nifty tool.

The Quadratic Formula

$ax^2 + bx + c = 0$ when

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.2 Factoring Quadratics

Sometimes, the quadratic gets a bit clunky and hard so solve, especially without a calculator. That's when factoring quadratics becomes a good tool to use as well.

Factoring quadratics is a way to seperate a quadratic, usually in the form x^2+bx+c where $(a=1)$, into two multiplied equations in the form $(px+q)(rx+s)=0$. Solving these for x also gives the roots of the equation.

Types of Factoring Techniques

1. Factoring out the GCF (Greatest Common Factor)
   Always check for a common factor first:

$$2x^2+4x=2x(x+2)$$

2. Factoring Trinomials: $a=1$
   For expressions like x^2+bx+c, find two numbers that:
   - Multiply to c
   - Add to b

$$x^2+5x+6=(x+2)(x+3)$$

3. Factoring Trinomials: $a\neq 1$
   Use the "AC method":
   (a) Multiply $a\times c$
   (b) Find two numbers that multiply to ac and add to b
   (c) Split the middle term and factor by grouping

$$6x^2+11x+4=6x^2+3x+8x+4=(3x+2)(2x+2)$$

4. Special Cases:
   - Perfect Square Trinomials:
     $a^2+2ab+b^2=(a+b)^2$
     $x^2+6x+9=(x+3)^2$
   - Difference of Squares:
     $a^2-b^2=(a-b)(a+b)$
     $x^2-16=(x-4)(x+4)$

Tips

- Always factor out the GCF first.

- Check your result by expanding (FOIL). The inverse of factoring is expanding using the FOIL method.
- Not all quadratics are factorable over real integers.

APPENDIX A

Answers to Exercises

Answer to Exercise 1 (on page 3)

For what t is $-4.9t^2 + 12t + 2 = 0$? Start by dividing both sides of the equation by -4.9.

$$t^2 - 2.45t - 0.408 = 0$$

The roots of this are at

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2} = -\frac{-2.45}{2} \pm \frac{\sqrt{(-2.45)^2 - 4(-0.408)}}{2} = 1.22 \pm 1.36$$

We only care about the root after we release the hammer $(t > 0)$.

$1.22 + 1.36 = 2.58$ seconds after releasing the hammer, it will hit the ground.

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