Chapter 1

Vector-valued Functions

In the last chapter, you calculated the flight of the shell. For any time t, you could find a vector $[\mathtt{distance}, \mathtt{height}]$. This can be thought of as a function f that takes a number and returns a 2-dimensional vector. We call this a *vector-valued* function from $\mathbb{R} \rightarrow \mathbb{R}^2 \rightarrow \mathbb{R}^3$[1].

1.1 Vector-valued functions: position

We often make a vector-valued function by defining several real-valued functions. For example, if you threw a hammer with an initial upward speed of 12 m/s and a horizontal speed of 4 m/s along the x axis from the point $(1, 6, 2)$, its position at time t (during its flight) would be given by:

$$f(t) = [4t + 1,\ 6,\ -4.8t^2 + 12t + 2]$$

In other words, x is increasing with t, y is constant, and z is a parabola.

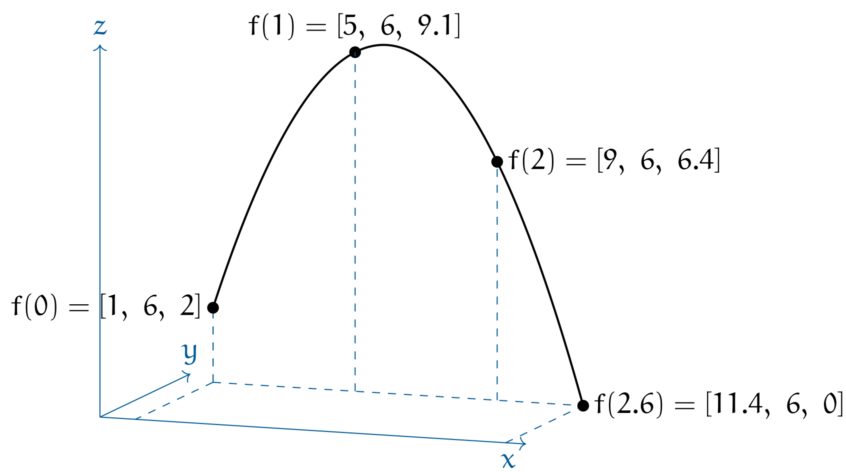

Figure 1.1: An example of a vector-valued function.

[1]the $\mathbb{R}$ symbol represents the set of all real numbers; the $\mathbb{R}^2$ symbol represents the set of all 2-dimensional vectors, and $\mathbb{R}^3$ represents the set of all 3-dimensional vectors

1.2 Finding the velocity vector

Now that we have its position vector, we can differentiate each component separately to get its velocity as a vector-valued function:

$$f'(t) = [4, 0, -9.8t + 12]$$

In other words, the velocity is constant along the x-axis, zero along the y-axis, and decreasing with time along the z axis.

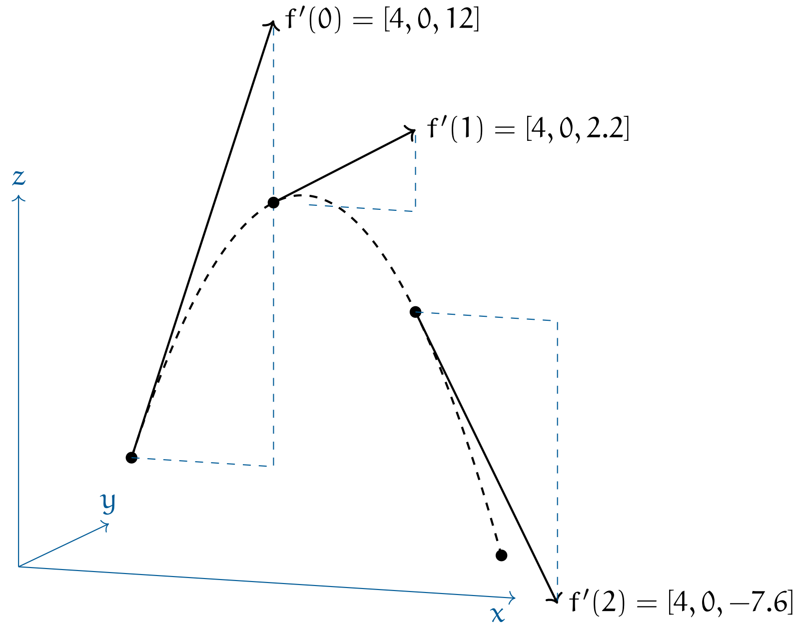

Figure 1.2: The derivatives of the position function (velocity) with respect to time.

1.3 Finding the acceleration vector

Now that we have its velocity, we can get its acceleration as a vector-valued function:

$$f''(t) = [0, 0, -9.8]$$

There is no acceleration along the x or y axes. It is accelerating down at a constant $9.8\text{m}/\text{s}^2$.

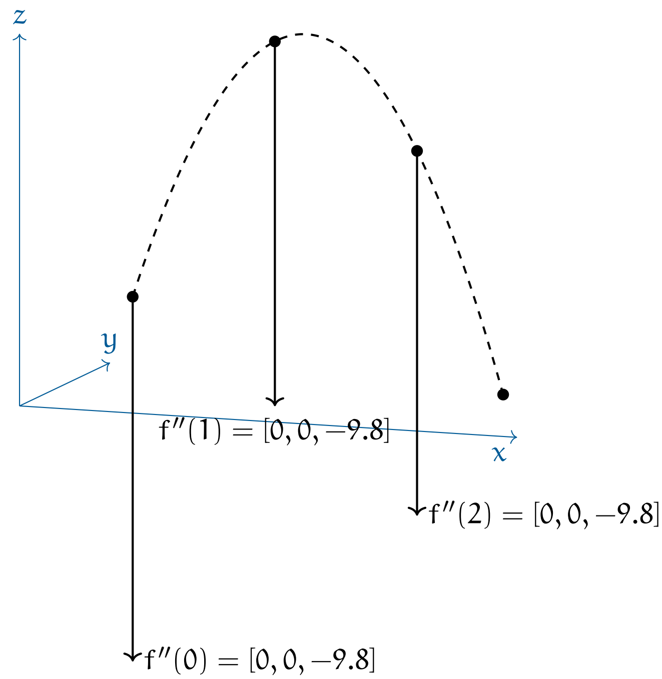

Figure 1.3: The acceleration vector is constant and points downward.

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises