Chapter 1

Multiplying Polynomials

Watch Khan Academy's **Multiplying monomials** at `https://youtu.be/Vm7H0VTlIco`.

To review, when you multiply two monomials, you take the product of their coefficients and the sum of their degrees:

$$(2x^6)(5x^3) = (2)(5)(x^6)(x^3) = 10x^9$$

If you have a product of more than two monomials, multiply *all* the coefficients and sum *all* the exponents:

$$(3x^2)(2x^3)(4x) = (3)(2)(4)(x^2)(x^3)(x^1) = 24x^6$$

Exercise 1 Multiplying monomials

Working Space

Multiply these monomials

1. $(3x^2)(5x^3)$
2. $(2x)(4x^9)$
3. $(-5.5x^2)(2x^3)$
4. $(\pi)(-2x^5)$
5. $(2x)(3x^2)(5x^7)$

Answer on Page 7

1.1 Multiplying a monomial and a polynomial

Watch Khan Academy's **Multiplying monomials by polynomials** at https://youtu.be/pD2-H15ucNE.

When multiplying a monomial and a polynomial, you use the the distributive property.

Then it is just multiplying several pairs of monomials:

$$\begin{aligned}(3x^2)(4x^3 - 2x^2 + 3x - 7) \\ = (3x^2)(4x^3) + (3x^2)(-2x^2) + (3x^2)(3x) + (3x^2)(-7) \\ = 12x^5 - 6x^4 + 9x^3 - 21x^2\end{aligned}$$

Exercise 2 Multiplying a monomial and a polynomial

Working Space

Multiply these monomials

1. $(3x^2)(5x^3 - 2x + 3)$
2. $(2x)(4x^9 - 1)$
3. $(-5.5x^2)(2x^3 + 4x^2 + 6)$
4. $(\pi)(-2x^5 + 3x^4 + x)$
5. $(2x)(3x^2)(5x^7 + 2x)$

Answer on Page 7

1.2 Multiplying polynomials

Watch Khan Academy's **Multiplying binomials by polynomials** video at https://youtu.be/D6mivA_8L8U

When you are multiplying two polynomials, you will use the distributive property several times to make it one long polynomial. Then you will combine the terms with the same degree. For example,

$$\begin{aligned}
&(2x^2-3)(5x^2+2x-7)\\
&= (2x^2)(5x^2+2x-7)+(-3)(5x^2+2x-7)\\
&= (2x^2)(5x^2)+(2x^2)(2x)+(2x^2)(-7)+(-3)(5x^2)+(-3)(2x)+(-3)(-7)\\
&= 10^4+4x^3+-14x^2+-15x^2+-6x+21 = 10^4+4x^3+-29x^2+-6x+21
\end{aligned}$$

One common form that you will see is multiplying two binomials together:

$$(2x+7)(5x+3) = (2x)(5x+3)+(7)(5x+3) = (2x)(5x)+(7)(5x)+(2x)(3)+(7)(3)$$

Notice the product has become the sum of four parts: the firsts, the inners, the outers, and the lasts. People sometimes use the mnemonic FOIL to remember this pattern, but there is a general rule that works for all product of polynomials, not just binomials. Here it is: Every term in the first will be multiplied by every term in the second, and then just add them together.

So, for example, if you have a polynomial s with three terms and you multiply it by a polynomial t with five terms, you will get a sum of 15 terms – each term is a product of two monomials, one from s and one from t. (Of course, several of those terms might have the same degree, so they will be combined together when you simplify. Thus you typically end up with a polynomial with less than 15 terms.)

Using this rule, here is how I would multiply $2x^2-3x+1$ and $5x^2+2x-7$:

$$\begin{aligned}
(2x^2-3x+1)(5x^2+2x-7) &= \begin{array}{ccccccc}
(2x^2)(5x^2) & + & (2x^2)(2x) & + & (2x^2)(-7) & + \\
(-3x)(5x^2) & + & (-3x)(2x) & + & (-3x)(-7) & + \\
(1)(5x^2) & + & (1)(2x) & + & (1)(-7) &
\end{array}\\
&= 10x^4+4x^3+(-14)x^2+(-15)x^3+(-6)x^2+21x+5x^2+2x+(-7)\\
&= 10x^4+(4-15)x^3+(-14-6+5)x^2+(21+2)x+(-7)\\
&= 10x^4-11x^3-15x^2+23x-7
\end{aligned}$$

Note that the product (before combining terms with the same degree) has $3 \times 3 = 9$ terms – every possible combination of a term from the first polynomial and a term from the second polynomial.

One common source of error: losing track of the negative signs. You will need to be really careful. I have found that it helps to use + between all terms, and use negative coefficients to express subtraction. For example, if the problem says $4x^2 - 5x - 3$, you should work with that as $4x^2 + (-5)x + (-3)$

Exercise 3 Multiplying polynomials

Multiply the following pairs of polynomials:

1. $2x + 1$ and $3x - 2$
2. $-3x^2 + 5$ and $4x - 2$
3. $-2x - 1$ and $-3x - \pi$
4. $-2x^5 + 5x$ and $3x^5 + 2x$

Working Space

Answer on Page 7

Exercise 4 Observations

Let's say I have two polynomials, p_1 and p_2. p_1 has degree 23. p_2 has degree 12. What is the degree of their product?

Working Space

Answer on Page 8

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

$(3x^2)(5x^3) = 15x^5$

$(2x)(4x^9) = 8x^{10}$

$(-5.5x^2)(2x^3) = -11x^5$

$(\pi)(-2x^5) = -2\pi x^5$

$(2x)(3x^2)(5x^7) = 30x^{10}$

Answer to Exercise 2 (on page 3)

$(3x^2)(5x^3 - 2x + 3) = 15x^6 - 6x^3 + 6x^2$

$(2x)(4x^9 - 1) = 8x^{10} - 2x$

$(-5.5x^2)(2x^3 + 4x^2 + 6) = 11x^5 - 22x^4 + 33x^2$

$(\pi)(-2x^5 + 3x^4 + x) = -2\pi x^5 + 3\pi x^4 + \pi x$

$(2x)(3x^2)(5x^7 + 2x) = 30x^{10} + 12x^4$

Answer to Exercise 3 (on page 5)

$(2x + 1)(3x - 2) = 6x^2 - x - 2$

$(-3x^2 + 5)(4x - 2) = -12x^3 + 6x^2 + 20x - 10$

$(-2x - 1)(-3x - \pi) = 6x^2 + (4 + 2\pi)x + \pi$

$(-2x^5 + 5x)(3x^5 + 2x) = -6x^{10} + 12x^6 + 10x^2$

Answer to Exercise 4 (on page 5)

The degree of the product is determined by the term that is the product of the highest degree term in p_1 and the highest degree term in p_2. Thus, the product of a degree 23 polynomial and a degree 12 polynomial has degree 35.

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