Chapter 1

Inverse Trigonometric Functions

Recall from the chapter on functions that an inverse of a function is a machine that turns y back into x. The inverses of trigonometric functions are essential to solving certain integrals (you will learn in a future chapter why integrals are useful — for now, trust us that they are!). Let's begin by discussing the sin function and its inverse, $\sin^{-1}$, also called arcsin.

Examine the graph of $\sin x$ in figure 1.1. See how the dashed horizontal line crosses the function at many points? This means the function $\sin x$ is not one-to-one. In other words, there is not a unique x-value for every y-value. This means that if we do not restrict the domain of $\arcsin x$, the result will not be a function (see figure 1.2). In figure 1.2, you can see that just reflecting the graph across $y = x$ fails the vertical line test: an x value has more than one y value.

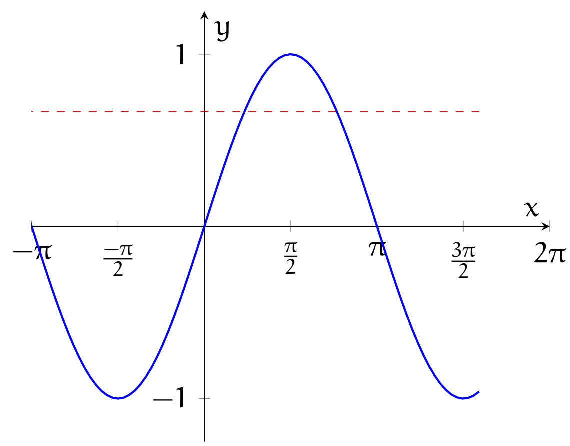

Figure 1.1: The horizontal line $y = \frac{2}{3}$ crosses $y = \sin x$ more than once

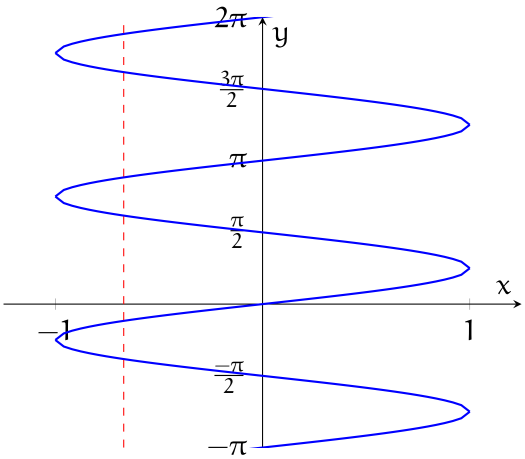

Figure 1.2: The inverse of an unrestricted sin function fails the vertical line test

1.1 Derivatives of Inverse Trigonometric Functions

f	f'
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arccsc} x$	$-\frac{1}{x\sqrt{x^2-1}}$
$\operatorname{arcsec} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$

1.2 Practice

Exercise 1

Find the f'. Give your answer in a simplified form.

1. $f(x) = \arctan x^2$
2. $f(x) = x\operatorname{arcsec}(x^3)$
3. $f(x) = \arcsin \frac{1}{x}$

Working Space

Answer on Page 5

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

1. By the chain rule, $f'(x) = 2\arctan x \times \frac{d}{dx}\arctan x = 2\arctan x\frac{1}{1+x^2}$

2. By the Product rule, $f'(x) = x\frac{d}{dx}\text{arcsec }(x^3) + \text{arcsec }(x^3)$. Further, by the chain rule, $\frac{d}{dx}\text{arcsec }(x^3) = \frac{1}{(x^3)\sqrt{(x^3)^2-1}} \times \frac{d}{dx}(x^3) = \frac{3x^2}{x^3\sqrt{x^6-1}}$. Therefore, $f'(x) = \frac{3}{\sqrt{x^6-1}} + \text{arcsec }(x^3)$

3. By the chain rule, $f'(x) = \frac{1/x}{\sqrt{1-(1/x)^2}} \times -\frac{1}{x^2} = -\frac{1}{x^3\sqrt{1-\frac{1}{x^2}}}$