Chapter 1

Kinematics

How can we describe the motion of objects other than falling bodies? Kinematics is the description of motion.

Kinematic Equations

$$x_f = x_0 + v_0 t + \frac{1}{2} a t^2 \tag{1.1}$$

$$v_f = v_0 + at \tag{1.2}$$

$$x_f = x_0 + \frac{1}{2}\left(v_f + v_0\right) t \tag{1.3}$$

$$v_f^2 = v_0^2 + 2a\left(x_f - x_0\right) \tag{1.4}$$

$$\Delta x - x_0 = v_{avg} t \tag{1.5}$$

Note that v_0 and v_i are synonomous, some professors will use v_0 to mean "velocity not" or inital velocity.

Note that each equation is missing a certain variable (or two)

- (5.1) is missing v_f.
- (5.2) is missing x_i or x_f
- (5.3) is missing a
- (5.4) is missing t
- (5.5) is missing a

It is important to note that these equations only work when the acceleration is held constant, which is referred to as *uniformly accelerated motion*.

Example: Terri and Jerry are running a race. Terri has a maximum acceleration of 3.4 m/s^2 and a top speed of 9.2 m/s. Jerry has a maximum acceleration of 3.7 m/s^2 and a top speed of 8.7 m/s. If the race is 200 m long, who will win? Assume that each runner has maximum acceleration until they reach their top speed and that they will maintain that top speed once they reach it.

Solution: We want to know how long it takes each runner to complete the 200 m race. For each runner, we will divide their run into two sections:

1. The time they are accelerating to their top speed
2. The time they are maintaining their top speed

We can use a table to track the results of our calculations:

Runner	Terri	Jerry
Leg 1 (s)		
Leg 2 (s)		
Total (s)		

We'll begin with Terri's first leg. Taking the starting line as $x = 0$, we know that:

$$x_0 = 0 \text{ m}$$

$$v_0 = 0 \ \frac{\text{m}}{\text{s}}$$

$$a = 3.4 \ \frac{\text{m}}{\text{s}^2}$$

And since we want to know how long it takes Terri to reach her top speed, we also know that $v_f = 9.2$ m/s. Since we don't know how far Terri will run before she reaches her top speed (and we're not looking for that quantity), we need to select an equation that does not include x:

$$v_f = v_0 + at$$

$$9.2\frac{\text{m}}{\text{s}} = 0\frac{\text{m}}{\text{s}} + \left(3.4\frac{\text{m}}{\text{s}^2}\right) t$$

$$t = \frac{9.2\frac{\text{m}}{\text{s}}}{3.4\frac{\text{m}}{\text{s}^2}} \approx 2.7 \text{ s}$$

With a similar method, we can find how long it takes Jerry to reach his top speed:

$$t = \frac{v_f}{a} = \frac{8.7\frac{\text{m}}{\text{s}}}{3.7\frac{\text{m}}{\text{s}^2}} \approx 2.4 \text{ s}$$

Let's go ahead and record this in our table:

Runner	Terri	Jerry
Leg 1 (s)	2.7	2.4
Leg 2 (s)		
Total (s)		

Now that we know how much time it takes each runner to reach their top speed, we need to figure out how much time it takes them to complete the race from the point at which each reaches their top speed. To do this, we will first have to find *where* each runner hits their top speed. (This is because we can't use $v_f = v_0 + at$ to find a time anymore, since from now on the runners' accelerations are zero, and all the other equations involve x.) For each runner, we know a, v_0, v_f, x_0, and t. You could choose any equation, but we will use this one:

$$x_f = x_0 + \frac{1}{2}(v_f + v_0)\,t$$

Since each runner begins on the starting line, $x_0 = 0$ m and $v_0 = 0$ m/s:

$$x_f = \frac{v_f \cdot t}{2}$$

For Terri:

$$x_f = \frac{\left(9.2\frac{m}{s}\right)(2.7\ s)}{2} \approx 12.4\ m$$

For Jerry:

$$x_f = \frac{\left(8.7\frac{m}{s}\right)(2.4\ s)}{2} \approx 10.2\ m$$

Now that we know where they reach their top speed, we can take that position as x_0 and find how long it takes each runner to reach the finish line at $x_f = 200$ m. Since $a = 0$ m/s^2, we can use:

$$x_f = x_0 + v_0 t$$

Rearranging to solve for t:

$$t = \frac{x_f - x_0}{v_0}$$

For Terri:

$$t = \frac{200\ m - 12.4\ m}{9.2\ \frac{m}{s}} \approx 20.4\ s$$

And for Jerry:

$$t = \frac{200\ m - 10.2\ m}{8.7\ \frac{m}{s}} \approx 21.8\ s$$

Completing our table, we see that Terri will win the race by finishing in the least amount of time:

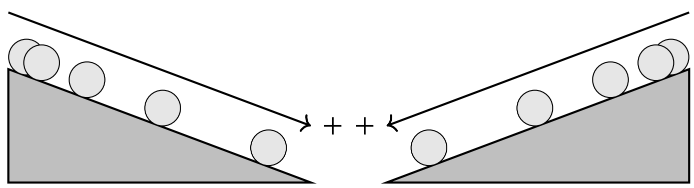
Runner	Terri	Jerry
Leg 1 (s)	2.7	2.4
Leg 2 (s)	20.4	21.8
Total (s)	23.1	24.2

1.1 Graphing Motion

1.1.1 Motion Diagrams

Example: Create a motion diagram of a ball rolling with a constant acceleration down an incline (the acceleration is in the same direction as motion).

Solution: The ball is accelerating down the ramp, so it will cover more distance each second. You could draw your ramp going left or right, as long as the distance covered each time interval increases as time passes.



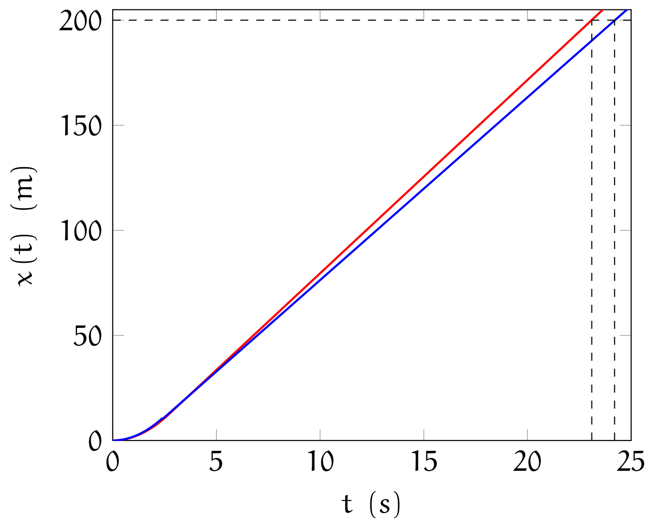
1.1.2 Position-Time and Velocity-Time Graphs

In the example problem above, graphs of each runner's motion would allow us to immediately see who would win. We can describe Terri's and Jerry's runs with piecewise functions:

$$x_{\text{Terri}}(t) = \begin{cases} \left(1.7\ \frac{\text{m}}{\text{s}^2}\right)t^2 & \text{if } 0 \le t < 2.7\ \text{s} \\ 12.4\ \text{m} + \left(9.2\ \frac{\text{m}}{\text{s}}\right)(t - 2.7\ \text{s}) & \text{if } t \ge 2.7\ \text{s} \end{cases}$$

$$x_{\text{Jerry}}(t) = \begin{cases} \left(1.85\ \frac{\text{m}}{\text{s}^2}\right)t^2 & \text{if } 0 \le t < 2.4\ \text{s} \\ 10.2\ \text{m} + \left(8.7\ \frac{\text{m}}{\text{s}}\right)(t - 2.4\ \text{s}) & \text{if } t \ge 2.4\ \text{s} \end{cases}$$

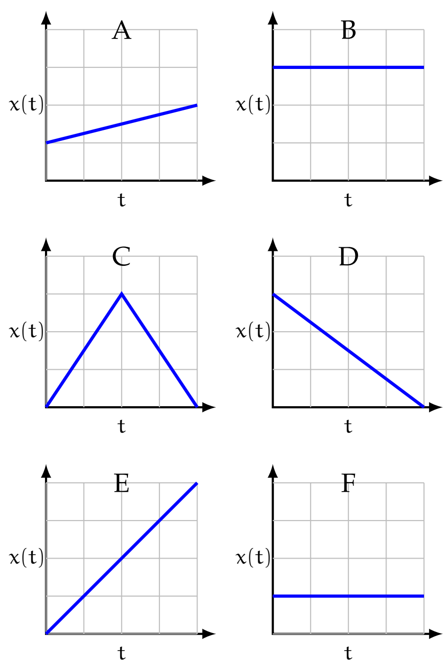
Graphing Terri in red and Jerry in blue:



Since the red function (Terri) crosses $x(t) = 200$ m first, we see that Terri will win.

Exercise 1

The following graphs show the position of an object from $t = 0$ s to $t = 10$ s. The scales on the y-axes are all the same.



Rank the objects from least to greatest (all negative values are lower than all positive values) in terms of:

1. displacement from $t = 0$ s to $t = 10$ s.
2. instantaneous velocity at $t = 7.5$ s.
3. distance traveled from $t = 0$ s to $t = 10$ s

Working Space

Answer on Page 9

1.2 Separation of Components

In the current chapter, we have learned how to use kinematics to describe one-dimensional motion. In the next chapter, you will learn to describe two-dimensional motion. It turns out that you can treat the different dimensions (horizontal and vertical motion) separately! Consider this scenario: A cannonball is shot from a cliff and the same instant an identical cannonball is dropped from the same cliff (see...). If the cannon is aimed horizontally, which cannonball will hit the ground first?

Go ahead and jot down what you think would happen: would the dropped ball hit first, the launched ball hit first, or would they both reach the ground below at the same time? Then, take a look at this video: `https://www.youtube.com/watch?v=zMF4CD7i3hg`. In it, two balls are released from the same height at the same time. One is dropped from rest while the other is launched horizontally (that is, its initial velocity is entirely in the x-direction). Based on the video, was your cannonball prediction correct? We'll learn to explain this phenomenon in the next chapter.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 6)

1. D, B/C/F, A, E; displacement is the difference in position between the starting and ending points. Object D moves backwards and therefore has a negative displacement. Objects B, C, and F all end in the same position they started and therefore have zero displacement. Objects A and E both move forward and have positive displacement. Object E moves forward 4 units while Object A only moves forward by 1 unit. Therefore, object E has a greater positive displacement than object A.

2. C, D, B/F, A, E; instantaneous velocity is given by the slope of a position-time graph at the indicated time (in this case, $t = 7.5s$). Since the graphs show 10 seconds of motion and there are 4 tick marks on each x-axis, each unit on the x-axis represents 2.5 seconds of time. Therefore, $t = 7.5s$ is the third tick mark on the x-axis. At $t = 7.5s$, the graphs of objects C and D have negative slopes and therefore negative velocities. Since the slope for object C is steeper than the slope for object D, object C's *speed* is greater than object D's, and object C's *velocity* is more negative than object D's. The graphs of object's B and F are horizontal at $t = 7.5s$ and therefore their velocities are zero. Graphs A and E have positive slopes, and since E is steeper, object E has a greater speed and more positive velocity than object A.

3. B/F, A, D, E, C; distance is a scalar and therefore always positive. B and F do not change position, and therefore travel a distance of 0. A moves forward 1 unit. D moves backwards 3 units (for a *distance* of 3). E moves forward 4 units. C moves forwards 3 units, then backwards 3 additional units, for a total of 6 units of distance.