Chapter 1

Multiplying Polynomials in Python

At this point, you have created a nice toolbox of functions for dealing with lists of coefficients as polynomials. Create a file called `poly.py` and copy the folowing functions into it:

- `evaluate_polynomial`
- `polynomial_to_string`
- `add_polynomials`
- `scalar_polynomial_multiply`
- `subtract_polynomial`

Now, create another file in the same directory called `test.py`. Type this into that file:

```
import poly

polynomial_a = [9.0, -4.0, 3.0, -5.0]
print('Polynomial A =', poly.polynomial_to_string(polynomial_a))

polynomial_b = [-9.0, 0.0, 4.0, 2.0, 1.0]
print('Polynomial B =', poly.polynomial_to_string(polynomial_b))

# Evaluation
value_of_b = poly.evaluate_polynomial(polynomial_b, 3)
print('Polynomial B at 3 =', value_of_b)

# Adding
a_plus_b = poly.add_polynomials(polynomial_a, polynomial_b)
print('A + B =', poly.polynomial_to_string(a_plus_b))

# Scalar multiplication
b_scalar = poly.scalar_polynomial_multiply(-3.2, polynomial_b)
print('-3.2 * Polynomial B =', poly.polynomial_to_string(b_scalar))

# Subtraction
```

```
a_minus_b = poly.subtract_polynomial(polynomial_a, polynomial_b)
print('A - B =', poly.polynomial_to_string(a_minus_b))
```

When you run it, you should get the following:

```
Polynomial A = -5.0x^3 + 3.0x^2 + -4.0x + 9.0
Polynomial B = 1.0x^4 + 2.0x^3 + 4.0x^2 + -9.0
Polynomial B at 3 = 162.0
A + B = 1.0x^4 + -3.0x^3 + 7.0x^2 + -4.0x
-3.2 * Polynomial B = -3.2x^4 + -6.4x^3 + -12.8x^2 + 28.8
A - B = -1.0x^4 + -7.0x^3 + -1.0x^2 + -4.0x + 18.0
```

You are now ready to implement the multiplication of polynomials. The function will look like this:

```
def multiply_polynomials(a, b):
  ...Your code here...
```

It will return a list of coefficients.

In an exercise in the last chapter, you were asked " Let's say I have two polynomials, p_1 and p_2. p_1 has degree 23. p_2 has degree 12. What is the degree of their product?" The answer was $23 + 12 = 35$.

In our implementation, a polynomial of degree 23 is held in a list of length 24.

In Python, we wil be trying to multiply a polynomial `a` and a polynomial `b` represented as lists. What is the degree of that product?

```
result_degree = (len(a) - 1) + (len(b) - 1)
```

Now, we need to create an array of zeros that is one longer than that. Here is a cute Python trick: If you have a list, you can replicate it using the * operator.

```
a = [5,7]
b = a * 4
print(b)
# [5, 7, 5, 7, 5, 7, 5, 7]
```

Here iss how you will get a list of zeros:

```
result = [0.0] * (result_degree + 1)
```

We will step through `a`, getting the index and value of each entry. You can do this in one line using `enumerate`:

```
    for a_degree, a_coefficient in enumerate(a):
```

For each of those, we will step through the entire `b` polynomial. As you multiply together each term, you will add it to the appropriate coefficient of the result.

Here is the whole function:

```
def multiply_polynomials(a, b): # What is the degree of the resulting
polynomial?  result_degree = (len(a) - 1) + (len(b) - 1)

    # Make a list of zeros to hold the coefficents result = [0.0] *
    (result_degree + 1)

    # Iterate over the indices and values of a for a_degree,
    a_coefficient in enumerate(a):

        # Iterate over the indices and values of b for b_degree,
        b_coefficient in enumerate(b):

            # Calculate the resulting monomial coefficient =
            a_coefficient * b_coefficient degree = a_degree + b_degree

            # Add it to the right bucket
            result[degree] = result[degree] + coefficient

    return result
```

Take a long look at that function. When you understand it, type it into `poly.py`.

In `test.py`, try out the new function:

```
# Multiplication
a_times_b = poly.multiply_polynomials(polynomial_a, polynomial_b)
print('A x B =', poly.polynomial_to_string(a_times_b))
```

This is an example of a *nested loop*. The outer loop steps through the polynomial `a`. For each step it takes, the inner loop steps through the entire polynomial `b`.

1.1 Something surprising about lists

You can imagine that you might want to create two very similar polynomials. Let's say polynomial c is $x^2 + 2x + 1$ and polynomial d is $x^2 - 2x + 1$. You might think you are very clever to just alter that degree 1 coefficient like this:

```
c = [1.0, 2.0, 1.0]
d = c
d[1] = -2.0
```

If you printed out c, you would get $[1.0, -2.0, 1.0]$. Why? You assigned two variables (c and d) to the *the same list*. So, when you use one reference (d) to change the list, you see the change if you look at the list from either reference. *FIXME: Diagram of two references to the same list here.*

To create two separate lists, you would need to explicitly make a copy:

```
c = [1.0, 2.0, 1.0]
d = c.copy()
d[1] = -2.0
```

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises