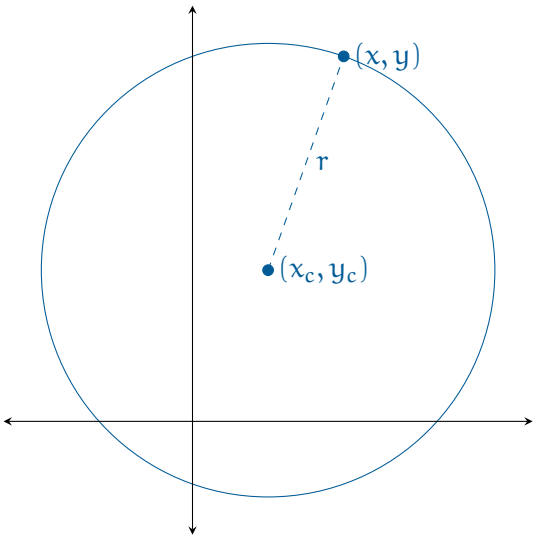
CHAPTER 1

# Circles

A circle is the set of points $(x, y)$ that are a particular distance $r$ from a particular point $(x_c, y_c)$. We say that $r$ is the *radius* and $(x_c, y_c)$ is the *center*




**Area and Radius**

If the radius of a circle is $r$, the area of its interior $(a)$ is given by

$$a = \pi r^2$$

## *Exercise 1* Area of a Circle

*Working Space*

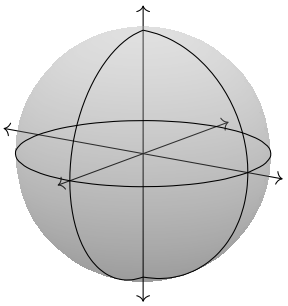
The paint you have says "One liter covers 6 square meters."

You are painting the top of a circular table with a radius of 3 meters.

How much paint will you need?

*Answer on Page 9*

Note that a circle lives in a particular plane. The points $(x, y, z)$ that are a particular distance $r$ from a particular point $(x_c, y_c, z_c)$ are a sphere:



The distance all the way across the middle of a circle (or a sphere) is its *diameter*. The diameter is always twice the radius.

For the rest of the chapter, we are talking about circles, points, and lines *in a plane*.

**Circumference and Diameter**

The circumference $(c)$ of a circle is the distance around the circle. If the diameter is $d$,

$$c = \pi d$$

## *Exercise 2* Circumference

*Working Space*

Using a tape measure, you figure out that the circumference of a tree in your yard is 64 cm.

Assuming the trunk is basically circular, what is its diameter?

*Answer on Page 9*

## *Exercise 3* Splitting a Pie

*Working Space*

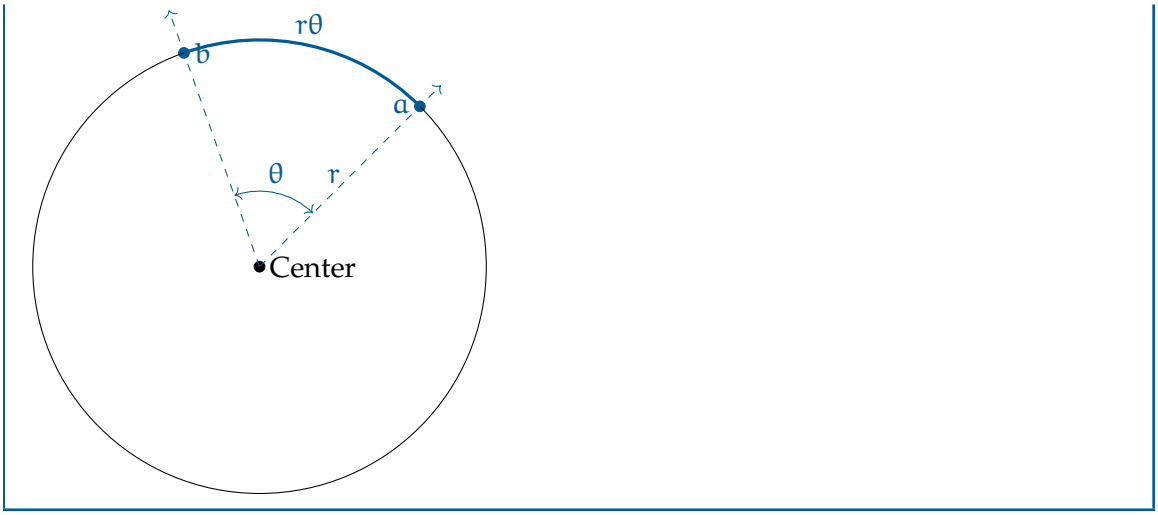
A pie has a radius of 13 cm. 7 friends all want equal sized wedges. You have a tape measure.

How many centimeters will each outer crust be?

*Answer on Page 9*

**Length of an Arc**

If you have two points $a$ and $b$ on a circle, the ray from the center through $a$ and the ray from the center through $b$ form an angle. If $\theta$ is the angle in radians and $r$ is the radius of the circle, the distance from $a$ to $b$ on the circle is $r\theta$.




## *Exercise 4* Arc Length

*Working Space*

You have been asked to find the radius of a very large cylindrical tank. You have a tape measure, but it is only 15 meters long and doesn't reach all the way around the tank.

However, you have a compass. So you stick one end of the tape measure to the side of the tank and measure the orientation of the wall at that point. Then you walk the 15 meters and measure the orientation of the wall there.

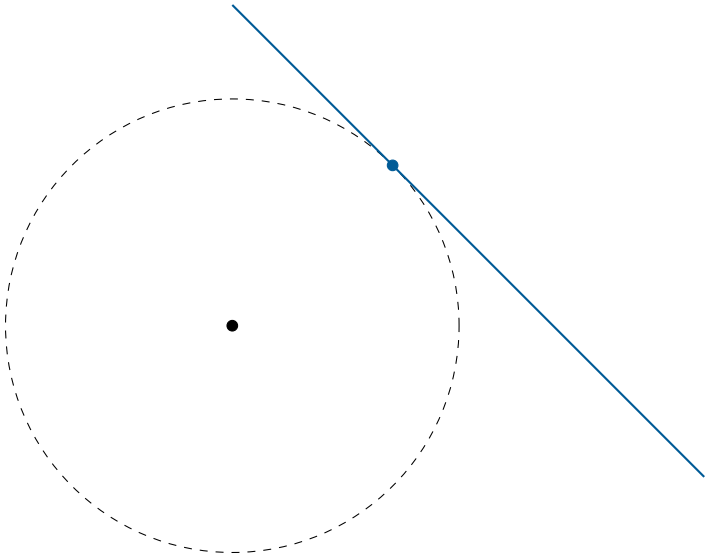
You find that 15 meters represents 72 degrees of arc.

What is the radius of the tank in meters?

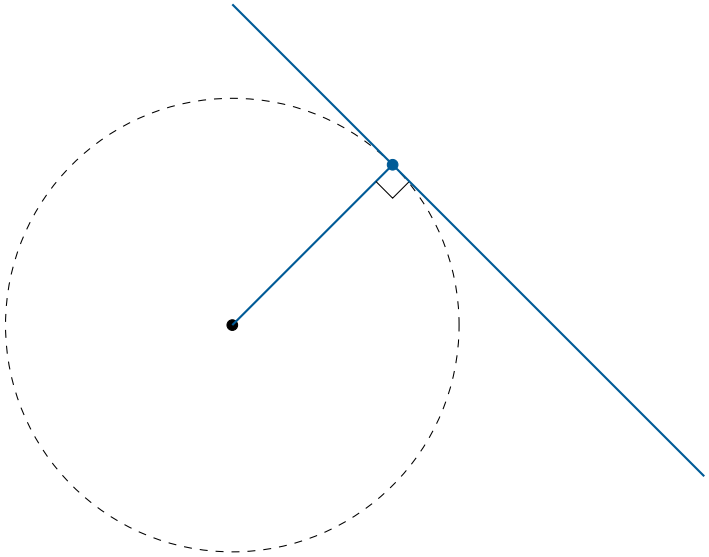
*Answer on Page 10*

## 1.1 Tangents

A line that is *tangent* to a circle touches it at exactly one point:



The tangent line is always perpendicular to the radius to the point of tangency:



## *Exercise 5* Painting a Comet

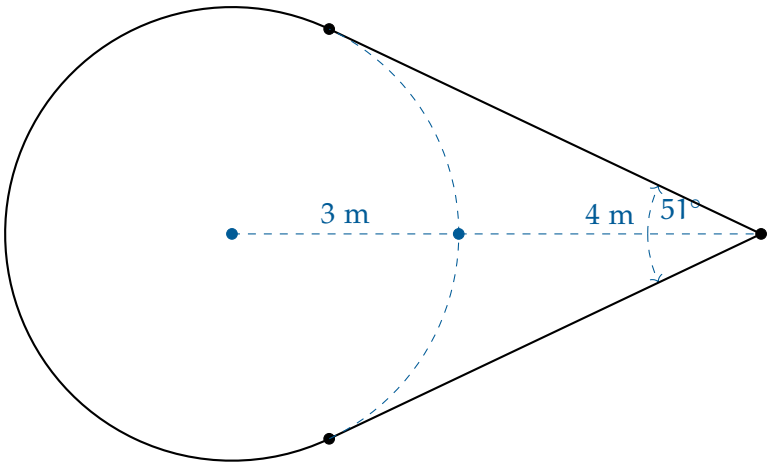
*Working Space*

You have been asked to paint a comet and its tail in yellow on the floor of a gymnasium.

A liter of yellow paint covers 6 square meters.

First you draw a circle with a radius of 3 meters. Then you mark a point D on the floor 7 meters from the center of the circle. Then you draw two tangent lines that pass through D.

You use a protractor to measure the angle at which the tangent lines meet: about 51°




Before you paint the area contained by the circle and the two tangent lines, how much paint will you need?

*Answer on Page 10*

## 1.2 Radians and circles

Previously, you learned that angles can be measured in degrees and radians. A circle is $360^o$ (see figure 1.1).

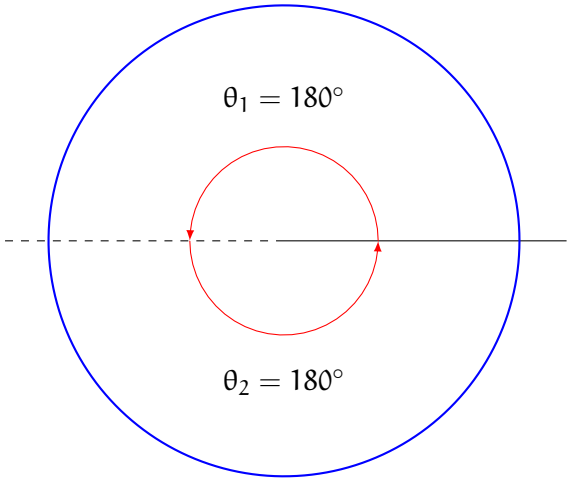



Figure 1.1: The total internal angle of a circle is $\theta_1 + \theta_2 = 360°$

This means a circle is also $2\pi$ radians:

$$360° \cdot \frac{\pi}{180°} = 2\pi$$

You may have been wondering: why is it that there are $\pi$ radians in a $180°$ angle? A radian is defined such that one radian is the angle at the center of a circle which defines an arc of the circumference equal to the radius of the circle (see figure 1.2).

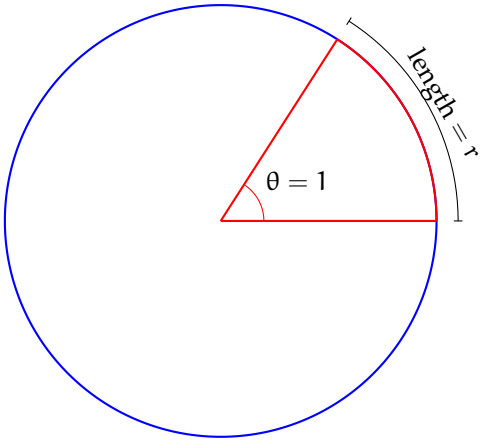



Figure 1.2: When the center angle is 1 radian, the length of the arc is equal to the radius of the circle

This makes it very straightforward to find the lengths of arcs if we know the center angle in radians. The arc length is just $\theta r$, where $\theta$ is the center angle in radians.

*This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.*

Appendix A

# Answers to Exercises

## Answer to Exercise 1 (on page 2)

The table has a radius of 3 meters.

So the area of its top is $3^2\pi \approx 28.27$.

$$28.27 \text{ square meters} \left( \frac{1 \text{ liter}}{6 \text{ square meters}} \right) = 4.72 \text{ liters}$$

## Answer to Exercise 2 (on page 3)

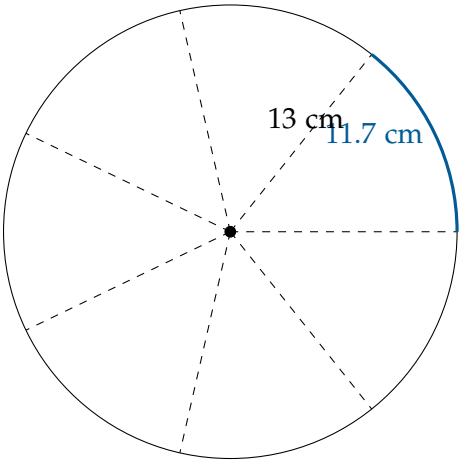
The diameter is

$$\frac{c}{\pi} = \frac{64}{\pi} \approx 20.37 \text{ centimeters}$$

## Answer to Exercise 3 (on page 3)

The circumference of the pie is $26\pi \approx 81.7$ centimenters.

The length of the crust for each piece would be about $\frac{81.7}{7} = 11.7$ cm.

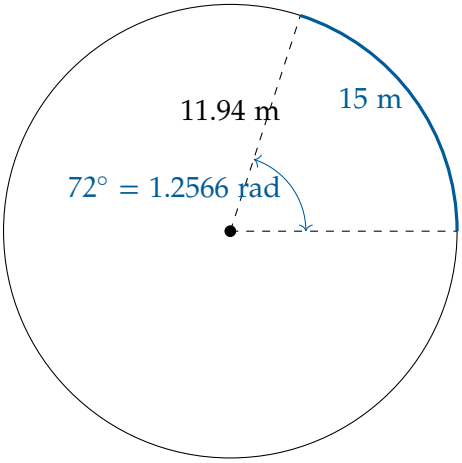



## Answer to Exercise 4 (on page 4)

$$72 \text{ degrees} \left( \frac{2\pi \text{ radians}}{360 \text{ degrees}} \right) \approx 1.2566 \text{ radians}$$

$$15 = 1.2566r$$

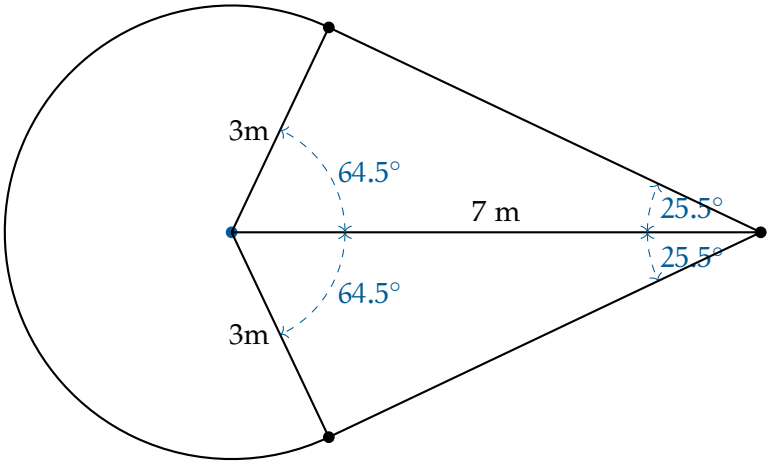
$$r = 11.94 \text{ meters}$$




## Answer to Exercise 5 (on page 6)

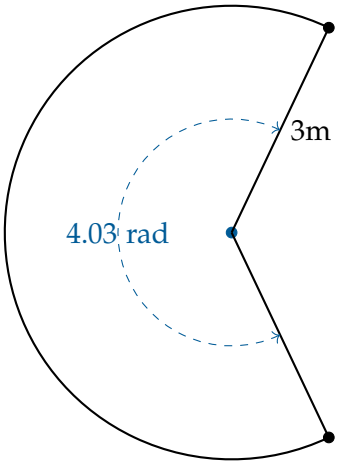
The trick here is to take advantage of the fact that the tangent is perpendicular to the

radius to make right triangles:



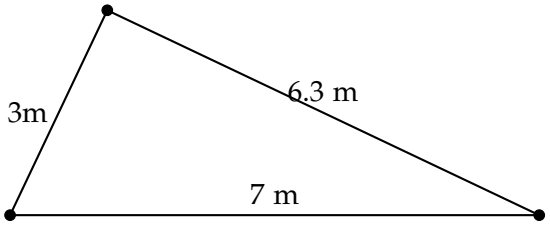

The wedge has radius 3 and represents $360 - 2(64.5) = 231° \approx 4.03$ radians.

We are finding the area of this piece:




The area of this piece is $(4.03)(3^2) = 36.27$ square meters.

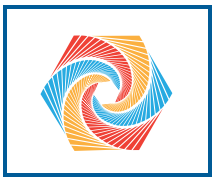
If a right triangle has a hypotenuse of 7m and one leg is 3m, the other leg is $\sqrt{7^2 - 3^2} = 2\sqrt{10} \approx 6.3$ m.




A right triangle with legs of 3m and 6.3m has an area of 9.45 square meters.

There are two of them, so the total area is $36.27 + 2(18.9) = 74.07$ square meters.

Six square meters per liter, so you need $\frac{74.07}{6} = 12.35$ liters of paint.



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