Chapter 1

Drag

The very first computers were created to do calculations of how artillery would fly when shot at different angles. The calculations were similar to the ones you just did for the flying hammer, with two important differences:

- They were interested in two dimensions: the height and the distance across the ground.
- However, artillery flies a lot faster than a hammer, so they also had to worry about drag from the air.

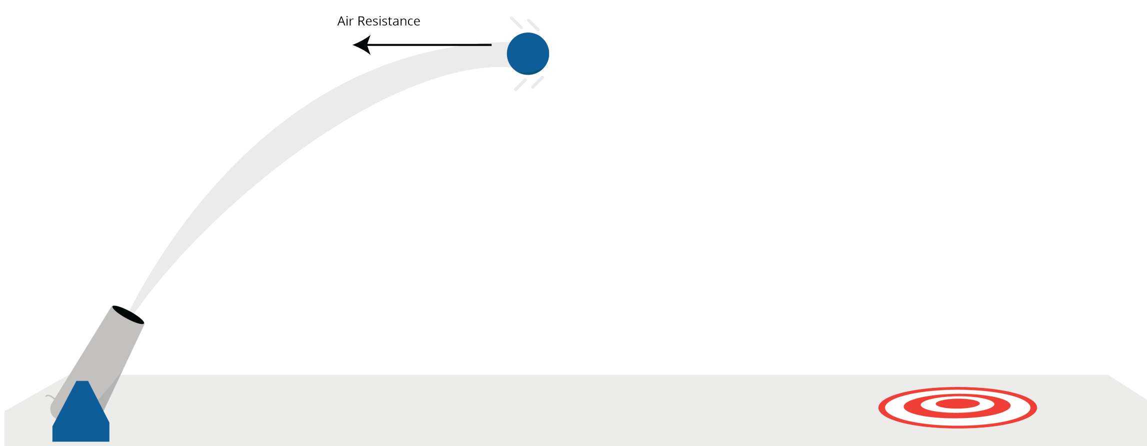


Figure 1.1: A cannonball shot has a parabolic trajectory.

1.1 Wind resistance

The first thing they did was put one of the shells in a wind tunnel. They measured how much force was created when they pushed 1 m/s of wind over the shell. Let's say it was 0.1 newtons.

One of the interesting things about the drag from the air (often called *wind resistance*) is that it increases with the *square* of the speed. Thus, if the wind pushing on the shell is 3 m/s, instead of 1 m/s, the resistance is $3^2 \times 0.1 = 0.9$ newtons.

(Why? Intuitively, three times as many air molecules are hitting the shell and each molecule is hitting it three times harder.)

So, if a shell is moving with the velocity vector v, the force vector of the drag points in the exact opposite direction. If μ is the force of wind resistance of the shell at 1 m/s, then the magnitude of the drag vector is $\mu|v|^2$ with μ being the wind resistance force.

1.2 Initial velocity and acceleration due to gravity

Let's say a shell is shot out of a tube at s m/s, and the tube is tilted θ radians above level. The initial velocity will be given by the vector $[s\cos(\theta), s\sin(\theta)]$

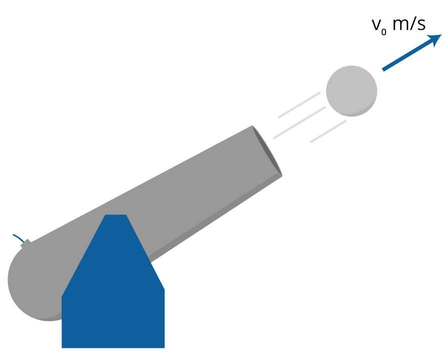


Figure 1.2: The initial velocity vector of the shell.

(The velocity of the shell is actually a 3-dimensional vector, but we are only going to worry about height and horizontal distance; we are assuming that the operator pointed it in the right direction.)

To figure out the path of the shell, we need to compute its acceleration. We remember that

$$F = ma$$

(Note that F and a are vectors.) Dividing both sides by m, we get:

$$a = \frac{F}{m}$$

Let's figure out the net force on the shell, so that we can calculate the acceleration vector.

If the shell has a mass of b, the force due to gravity will be in the downward direction, with a magnitude of $9.8b$ newtons.

To get the net force, we will need to add the force due to gravity with the force due to wind resistance.

1.3 Simulating artillery in Python

Create a file called `artillery.py`.

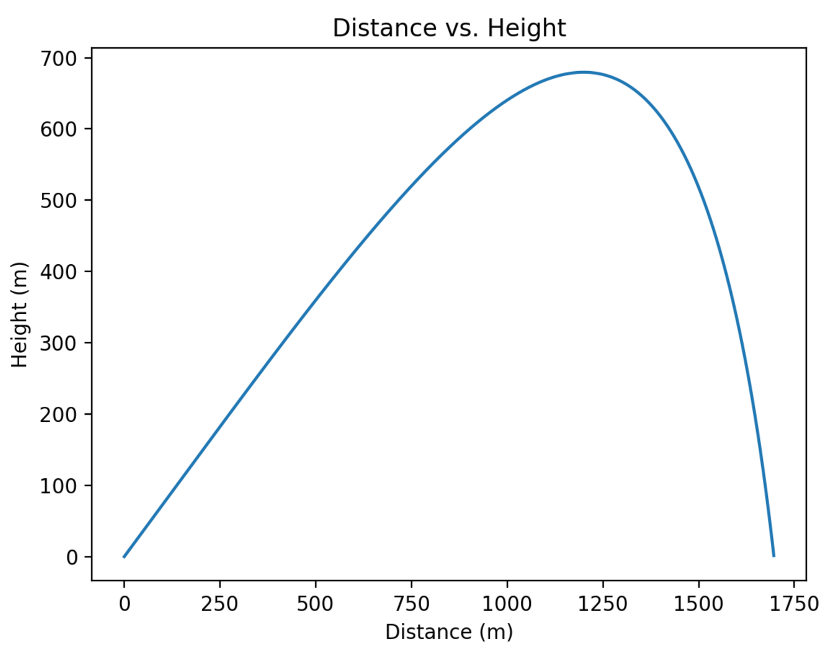
```
import numpy as np
import matplotlib.pyplot as plt

# Constants
mass = 45 # kg
start_speed = 300.0 # m/s
theta = np.pi/5 # radians (36 degrees above level)
time_step = 0.01 # s
wind_resistance = 0.05 # newtons in 1 m/s wind
force_of_gravity = np.array([0.0, -9.8 * mass]) # newtons

# Initial state
position = np.array([0.0, 0.0]) # [distance, height] in meters
velocity = np.array([start_speed * np.cos(theta), start_speed * np.sin(theta)])
time = 0.0 # seconds

# Lists to gather data
distances = []
heights = []
times = []

# While shell is aloft
while position[1] >= 0:
    # Record data
    distances.append(position[0])
    heights.append(position[1])
    times.append(time)

    # Calculate the next state
    time += time_step
    position += time_step * velocity

    # Calculate the net force vector
```

```
        force = force_of_gravity - wind_resistance * velocity**2

        # Calculate the current acceleration vector
        acceleration = force / mass

        # Update the velocity vector
        velocity += time_step * acceleration

    print(f"Hit the ground {position[0]:.2f} meters away at {time:.2f} seconds.")

    # Plot the data
    fig, ax = plt.subplots()
    ax.plot(distances, heights)
    ax.set_title("Distance vs. Height")
    ax.set_xlabel("Distance (m)")
    ax.set_ylabel("Height (m)")
    plt.show()
```

When you run it, you should get a message like:

```
Hit the ground 1696.70 meters away at 20.73 seconds.
```

You should also see a plot of the shell's path:



1.4 Terminal velocity

If you shot the shell very, very high in the sky, it would keep accelerating toward the ground until the force of gravity and the force of the wind resistance were equal. The speed at which this happens is called the *terminal velocity*. The terminal velocity of a falling human is about 53 m/s.

Note that kinematic equations do not apply to terminal velocity, because the acceleration is not constant. Instead, we can use the fact that at terminal velocity, the force of wind resistance equals the force of gravity.

Exercise 1 Terminal velocity

What is the terminal velocity of the shell described in our example?

Working Space

Answer on Page 7

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 5)

The force of gravity is $9.8 \times 45 = 441$ newtons.

At any speed s, the force of wind resistance is $0.05 \times s^2 = 0.05s^2$ newtons.

At terminal velocity, $0.05s^2 = 441$.

Solving for s, we get $s = \sqrt{\frac{441}{0.05}}$

Thus, terminal velocity should be about 94 m/s.

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