CHAPTER 1

Force, Mass, and Acceleration

To talk about forces, we first need to understand the concept of forces. A force is a push or pull on an object which (normally) influences its direction or speed. Forces are *vectors*, which means they have a direction and a magnitude, and it means that forces can be added together, even when in opposite directions. This leads us to Newton's Laws of Motion, which describe the relationship between forces and motion.

1.1 Newton's First Law

Newton's First Law of Motion states that an object at rest will stay at rest, and an object in motion will stay in motion with a constant velocity (same speed and direction) unless acted upon by a *net external force*. This means that if there is no net force on an object, it will not *accelerate*.

For example, a car that stops pressing the gas on a flat road will eventually stop due to friction, which is a force that acts opposite to the direction of motion. If there were no friction, the car would continue moving at a constant speed forever.

This law is sometimes also referred to as the law of *inertia*, which is the tendency of objects to resist changes in their motion.

1.2 Mass, Acceleration, and Newton's Second Law

Each atom has a mass, which means everything made up of those atoms has mass as well (and that's pretty much everything!). We measure mass in grams. A paper clip is about 1 gram of steel. An adult human can have a mass of 70,000 grams, so for larger things, we often talk about kilograms, which is 1000 grams.

The first interesting thing about mass is that objects with more mass require more force to accelerate. For example, pushing a bicycle so that it accelerates from a standstill to jogging speed in 2 seconds requires much less force than pushing a train so that it accelerates at the same rate.

Newton's Second Law of Motion

The force necessary to accelerate an object of mass m at an acceleration of a is given by:

$$F = ma$$

This is said as "the force is equal to the mass times the acceleration."

This is known as Newton's Second Law of Motion.[1]

What are the units here? We already know that mass is measured in kilograms. We can measure velocity in meters per second, but that is different from acceleration. So if we want to go from 0 to 5 meters per second (that's jogging speed) in two seconds, that is a change in velocity of 2.5 meters per second every second. We would say this acceleration is $2.5 m/s^2$.

1.2.1 Velocity versus Acceleration

Acceleration is the change in velocity. If an object is speeding up or slowing down, it is accelerating. In everyday language, we often use *decelerate* to indicate slowing down, but in physics you can use the word accelerate (slowing down is just negative acceleration). Since velocity is a vector (it has a magnitude and direction), changing direction is *also acceleration*. On the other hand, an object moving at a constant velocity (same speed, same direction) is *not accelerating*!

[1]This is a simplified version of Newton's Law of Gravitation. The formula $F = G\frac{m_1 m_2}{r^2}$ simplifies very close to $F = ma$ for any object within the Earth's atmosphere.

Exercise 1 Is it accelerating?

State whether the described object is accelerating or not.

1. A satellite orbiting the Earth at a constant speed.
2. A car moving due west at a constant 30 mile per hour.
3. A child coming to a stop on their bicycle.
4. A roller coaster going around a loop at a constant speed.
5. A roller coaster speeding up as it goes down the initial hill.
6. A book sitting on a table.

Working Space

Answer on Page 13

It's a common misconception that all objects in motion are accelerating. If an object is moving with a constant velocity (same speed, same direction), then it is not accelerating.

1.2.2 Calculating Acceleration

When an object is speeding up or slowing down, we can calculate the acceleration by dividing the change in velocity by the time it takes to make that change.

Calculating Acceleration

The acceleration of an object from an initial velocity, v_i, to a final velocity, v_f, over a period of time, t, is given by:

$$a = \frac{v_f - v_i}{t}$$

Notice that if the velocity does not change, then $v_f - v_i = 0$ and the acceleration is also zero.

Example: Your car can go from zero to 60 mph in 3 seconds. What is the acceleration in m/s^2?

Solution: First, let's convert from the imperial units of miles per hour to the SI units of meters per second. You can do this using a search engine, but we will show how to do it by hand below. (You will learn more about this method in the Units chapter).

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx \frac{26.82 \text{ m}}{\text{s}}$$

Now we have the starting velocity (0 m/s), the ending velocity (26.82 m/s), and the time (3 s), and we can find the acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{26.82\frac{m}{s} - 0\frac{m}{s}}{3s} \approx 8.94\frac{m}{s^2}$$

1.2.3 Determining Force

What about measuring force? Newton decided to name the unit after himself: The force necessary to accelerate one kilogram at $1m/s^2$ is known as *a newton*. It is often denoted by the symbol N.

$$1\text{N} = 1\frac{\text{kg} \cdot m}{s^2}$$

Example: If the car in the above example has a mass of 1500 kg, how much force does the engine use to accelerate the car?

Solution: We have already found the car's acceleration: 8.94 m/s^2. With the mass and acceleration, we can use Newton's Second Law to find the force needed to accelerate the car:

$$F = m \cdot a = 1500 \text{ kg} \cdot 8.94\frac{m}{s^2} = 13410 \text{ N}$$

Exercise 2 Acceleration

Working Space

While driving a bulldozer, you come across a train car (with no brakes and no locomotive) sitting on a track in the middle of a city. The train car has a label telling you that it has a mass of 2,400 kg. There is a time-bomb welded to the interior of the train car, and the timer tells you that you can safely push the train car for 120 seconds. To get the train car to where it can explode safely, you need to accelerate it to 20 meters per second. Fortunately, the track is level and the train car's wheels have almost no rolling resistance.

With what force, in newtons, do you need to push the train for those 120 seconds?

Answer on Page 13

1.3 Net Force

So far, we've looked at examples where only one force is acting on an object. In reality, there are usually multiple forces acting on an object. For example, the engine pushes your car forward while friction pulls it backwards. Or your chair is pushing up on you while gravity pulls you down. How then can we describe the motion of an object if more than one force is acting on it?

We can rearrange Newton's Law:

$$a = \frac{F_{net}}{m}$$

This means that an object's acceleration is directly proportional to the *net force* acting on the object and inversely proportional to the object's mass. The *net force* is the vector sum of all the forces acting on an object. The vector sum just means we have to take the direction of the force into account. Usually, up and right are positive while down and left are negative. You can see some examples in figure 1.1.

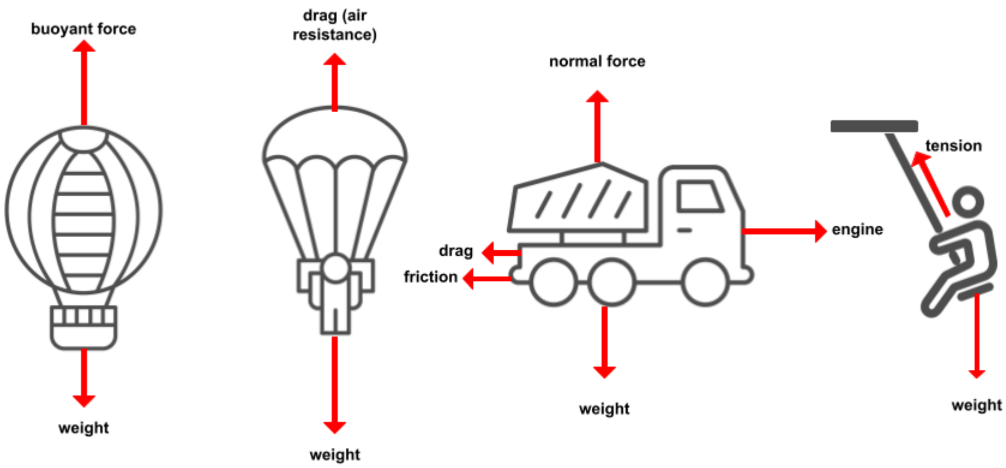
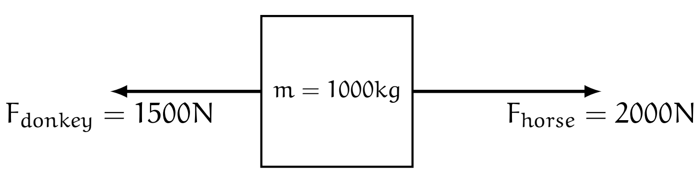


Figure 1.1: The hot air balloon has an upwards net force. The parachuter has a downwards net force. The truck has a rightward net force. The swinger has a leftward net force.

For now, we'll only look at parallel forces (up/down or left/right). You'll learn to use vector addition to combine orthogonal (at right angles) and skew (at angles other than parallel or right) forces in a later chapter.

Example: A donkey and a horse are each pulling a cart. The donkey pulls to the left with a force of 1500 N, while the horse pulls to the right with a force of 2000 N. What is the net force on the cart? If the cart has a mass of 1000 kg, in what direction and with what magnitude will the cart accelerate?

Solution: We begin by drawing a diagram (this is good practice - while a diagram may not be necessary for such a simple question, it will be very useful as we examine more complex scenarios in future chapters).



It is customary to take right as positive, so the horse applies a positive force while the donkey applies a negative force to the cart. Therefore, the net force is:

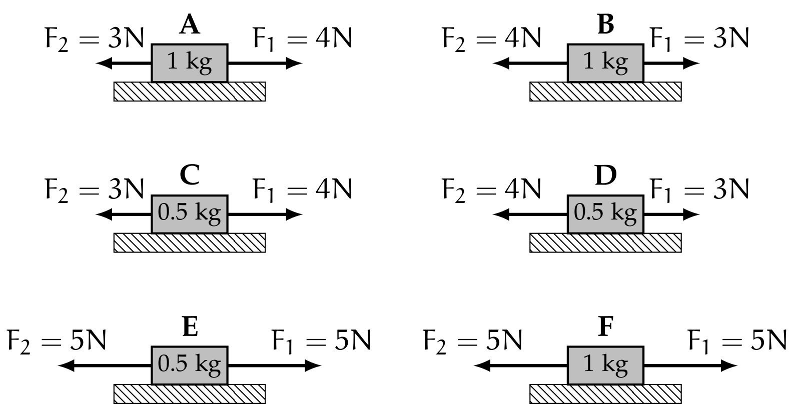
$$F_{net} = F_{horse} - F_{donkey} = 2000N - 1500N = 500N$$

Since the net force is positive, it points to the right and the cart will accelerate to the right at a rate of:

$$a = \frac{F_{net}}{m} = \frac{500\text{N}}{1000\text{kg}} = 0.5\frac{m}{s^2}$$

Exercise 3 Net Force and Acceleration

Rank the acceleration of the boxes shown below from greatest to least. All surfaces are frictionless and each box starts at rest. Take left as negative, and negative accelerations are less than a zero acceleration.



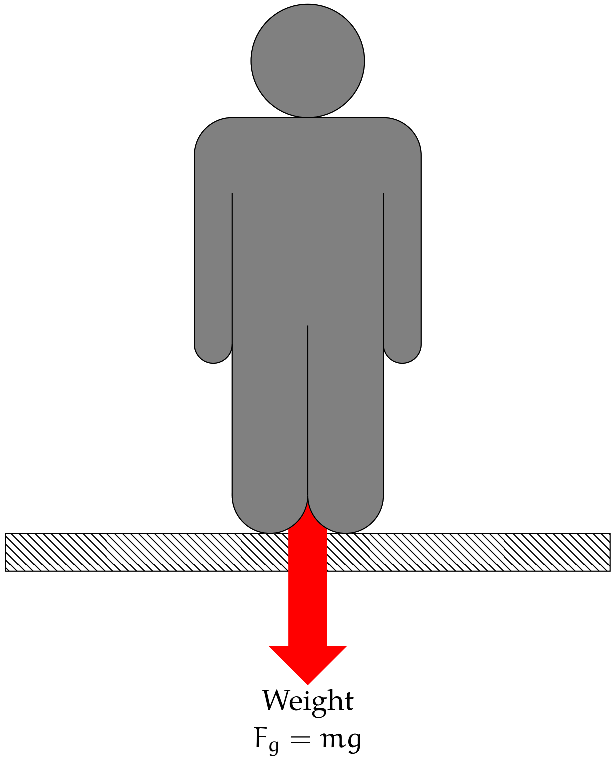
Working Space

Answer on Page 13

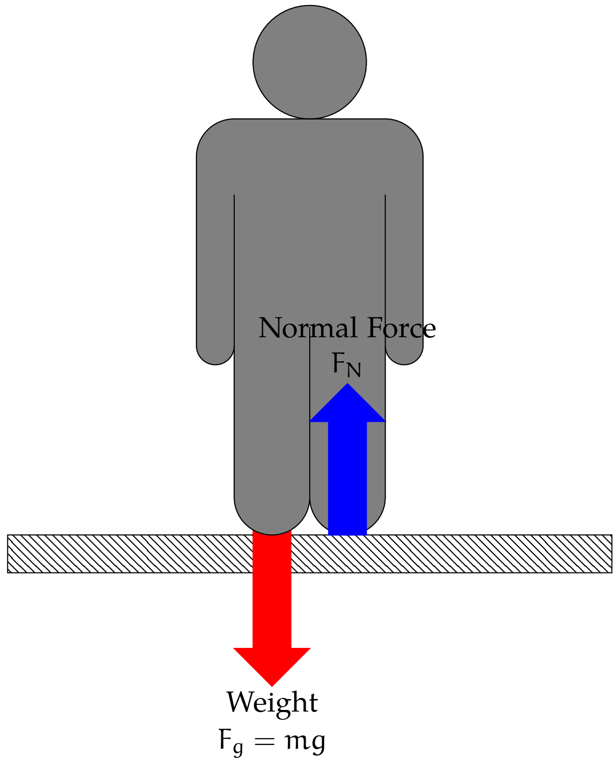
1.4 Normal Force and Apparent Weight

If you were to stand on a scale while riding an elevator, you would see the scale fluctuate as the elevator accelerates up and down (if you have a bathroom scale and live in a building with an elevator, you can try this yourself!). The reading on the scale is your *apparent weight* and is equal to the normal force between you and the floor.

What is a *normal force*? First, the word “normal” doesn’t have the colloquial meaning of average or usual. In mathematics, “normal” means perpendicular. A normal force is perpendicular to the *contact* surface between two objects. Let’s look at a person standing at rest. We know that gravity is pulling down on the person:



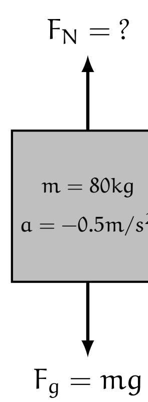
Since the person is not accelerating, there must be some other force acting on them in the upwards direction that balances out the person's weight (recall that if $a = 0\text{m}/\text{s}^2$, then it must be true that $F_{net} = 0\text{N}$). In this case, the balancing force is the *normal force* of the floor pushing up on the person's feet:



When you step on a bathroom scale, it is actually measuring the normal force! When you are not accelerating, your weight and the normal force between you and the scale are equal, so the scale gives you an accurate measure of your weight. Let's look at what happens if you are accelerating, as in an elevator.

Example: Maria has a mass of 80. kg. If the elevator in her apartment building initially accelerates at 0.50 m/s^2, what is her apparent weight as the elevator begins to move down?

Solution: We begin with a diagram:



Her apparent weight is the normal force. Seeing that the net force acting on Maria is given by $F_N - F_g$, we can use Newton's Second Law to find F_N:

$$F_{net} = F_N - F_g = ma$$
$$F_N - (80\text{kg})\left(9.8\frac{m}{s^2}\right) = (80\text{kg})\left(-0.5\frac{m}{s^2}\right)$$
$$F_N = (80\text{kg})\left(9.8\frac{m}{s^2} - 0.5\frac{m}{s^2}\right) = 744\text{N}$$

Maria's apparent weight is 744 newtons when the elevator is accelerating downwards.

Exercise 4 Moving Elevators

A 700-N person is standing on a scale in an elevator. Rank the reading on the scale from greatest to least for the following scenarios:

1. The elevator has an initial upward velocity of 2 m/s and an upward acceleration of 3 m/s^2
2. The elevator is initially at rest and has an upward acceleration of 3 m/s^2
3. The elevator has an initial downward velocity of 2 m/s and an upward acceleration of 5 m/s^2
4. The elevator is initially at rest and has a downward acceleration of 9.8 m/s^2
5. The elevator has an initial upward velocity of 2 m/s and a downward acceleration of 5 m/s^2
6. The elevator has an initial downward velocity of 2 m/s and is not accelerating

Working Space

Answer on Page 14

Let's talk about the effect of normal force on the net force. On standard flat ground, the Normal Force will be equal to the gravitational force (on Earth), assuming the object is stationary. This normal force also influences another contact force, friction, which we will talk about in the next chapter. If an object is on an incline such as a ramp, the normal force is *perpendicular* to the ramp.

1.5 Newtons and Joules

You now know the fundamental units for newtons and joules, which hints at the relationship between force and energy:

$$N = \frac{kg \cdot m}{s^2} \; J = \frac{kg \cdot m^2}{s^2} = N \cdot m$$

A joule is equivalent to a newton times a meter. So, multiplying a force (newtons) by a displacement (meters) tells you how the energy (joules) of the object changes. This relationship is described by the Work-Kinetic Energy Theorem, which we will explore in the next chapter.

1.6 Newton's Third Law

Now we can discuss Newton's Third Law, which states that every action has an equal but opposite reaction. In other words, if body A exerts a force on body B, then body B simultaneously exerts a force of equal magnitude and opposite direction on body A. Mathematically, we can express this as:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

Any time we use Newton's Second Law, we can combine this with the Third Law to state that *the sum of the forces equals zero* in some action-reaction pair, assuming that the system is isolated. For example, if two boxes (1 and 2) are applying a force on each other, we can say that (no matter their mass values), $F_{\text{Box 1 on 2}} = -F_{\text{Box 2 on 1}}$. This applies to physics concepts like air resistance and drag, tension forces of conjoined boxes or train cars, and gravitation of planets, all of which we will discuss in future Sequences.

Note the following:

- Action-reaction pairs do *not* cancel when applying Newton's Second Law to a single body.
- The forces are always of the same type (both contact, both gravitational, etc).
- They occur simultaneously; there is no time delay.

Like we said above, gravity and the normal force are usually action-reaction pairs. Similarly, a force applied on a box always has an equal but opposite force on the root cause of the force. Two objects connected by a massless rope have the same tension force on each other ($F_{1 \text{ on } 2} = -F_{2 \text{ on } 1}$) **Example:** Consider a swimmer pushing the wall of a pool. What forces are in play?

Solution: While the swimmer exerts a force $\mathbf{F}_{\text{wall}}$ on the wall, the wall exerts the reaction $-\mathbf{F}_{\text{wall}}$ on the swimmer, propelling the swimmer forward. Although the wall does not move due to mass and acceleration differences, the forces are equal and opposite.

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 3)

1. Acceleration: the satellite is moving in a circle, therefore changing direction and accelerating.
2. Not acceleration: the car isn't changing speed or direction.
3. Acceleration: the child is changing speed.
4. Acceleration: the roller coaster is changing direction.
5. Acceleration: the roller coaster is changing speed.
6. Not acceleration: the book isn't changing speed or direction.

Answer to Exercise 2 (on page 5)

If you accelerate to 20 m/s in 120 s, the acceleration is:

$$a = \frac{v_f - v_i}{t} = \frac{20 \text{ m/s} - 0 \text{ m/s}}{120 \text{ s}} = \frac{1}{6} \frac{\text{m}}{\text{s}^2}$$

To achieve this acceleration, you will need to apply a force of:

$$F = m \cdot a = 2400 \text{ kg} \cdot \frac{1}{6} \frac{\text{m}}{\text{s}^2} = 400 \text{ N}$$

Answer to Exercise 3 (on page 7)

C, A, E/F, B, D

$$a_A = \frac{F_{net,A}}{m_A} = \frac{4\text{N} - 3\text{N}}{1\text{kg}} = \frac{1\text{N}}{1\text{kg}} = 1 \frac{\text{m}}{\text{s}^2}$$

$$a_B = \frac{F_{net,B}}{m_B} = \frac{3\text{N} - 4\text{N}}{1\text{kg}} = \frac{-1\text{N}}{1\text{kg}} = -1 \frac{\text{m}}{\text{s}^2}$$

$$a_C = \frac{F_{net,C}}{m_C} = \frac{4N - 3N}{0.5kg} = \frac{1N}{0.5kg} = 2\frac{m}{s^2}$$
$$a_D = \frac{F_{net,D}}{m_D} = \frac{3N - 4N}{0.5kg} = \frac{-1N}{0.5kg} = -2\frac{m}{s^2}$$
$$a_E = \frac{F_{net,E}}{m_E} = \frac{5N - 5N}{0.5kg} = \frac{0N}{0.5kg} = 0\frac{m}{s^2}$$
$$a_F = \frac{F_{net,F}}{m_F} = \frac{5N - 5N}{1kg} = \frac{0N}{1kg} = 0\frac{m}{s^2}$$

Answer to Exercise 4 (on page 10)

3, 1/2, 6, 5, 4 The velocity does not affect the apparent weight, only the acceleration. If the elevator is accelerating upwards, the apparent weight increases. If the elevator is accelerating downwards, the apparent weight decreases. In fact, for scenario 4, the elevator is in free-fall and the person has no apparent weight. When the elevator is not accelerating, the person's apparent weight is their true weight.

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