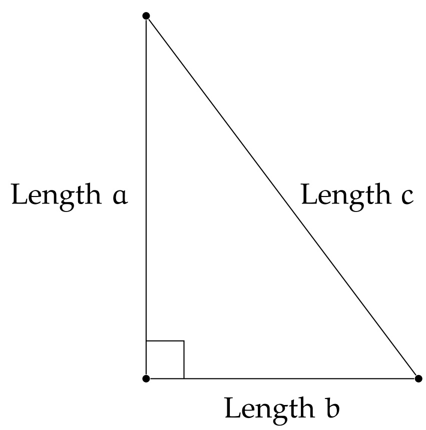
Chapter 1

Pythagorean Theorem

Watch's Khan Academy's Intro to the Pythagorean Theorem video at https://youtu.be/AA6RfgP-AHU.

If you have a right triangle, the edges that touch the right angle are called *the legs*. The third edge, which is always the longest and opposite the right angle, is known as *the hypotenuse*. The Pythagorean Theorem gives us the relationship between the length of the legs and the length of the hypotenuse.



The Pythagorean Theorem tells us that $a^2 + b^2 = c^2$, given that c is the hypotenuse.

For example, if one leg has a length of 3 and the other has a length of 4, then $a^2 + b^2 = 3^2 + 4^2 = 25$. Thus, c^2 must equal 25. This means you know the hypotenuse must be of length 5. This works for any right triangle

In reality, it rarely works out to be such a tidy number. For example, what is the length of the hypotenuse if the two legs are 3 and 6? $a^2 + b^2 = 3^2 + 6^2 = 45$. The length of the hypotenuse is the square root of that: $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$, which is approximately 6.708203932499369.

Common side lengths for these triangles are referred to as *Pythagorean triples*, meaning they evaluate to a whole number. Some common examples are $(3, 4, 5)$, $(5, 12, 13)$, and $(8, 15, 17)$. Multiples of right triangles are also triangles ie. $(3, 4, 5) \implies (6, 8, 10)$, which we will touch on next chapter.

There are also angle-based right triangles, consisting of ratios of the angles of the triangles. The most common ones are 45°-45°-90° and the 30°-60°-90°. We will discuss these further in depth, but know for now that they are vital in trigonometry, and consist of Pythagorean triples as side lengths.

Exercise 1 Find the Missing Length

What is the missing measure?

Leg 1 = 6, Leg 2 = 8, Hypotenuse = ?
(It should be a whole number.)

Leg 1 = 5, Leg 2 = ?, Hypotenuse = 13
(It should be a whole number.)

Leg 1 = ?, Leg 2 = 15, Hypotenuse = 17
(It should be a whole number.)

Leg 1 = 3, Leg 2 = 3, Hypotenuse = ?
(It is an irrational number. Give the exact answer and then use a calculator to get an approximation.)

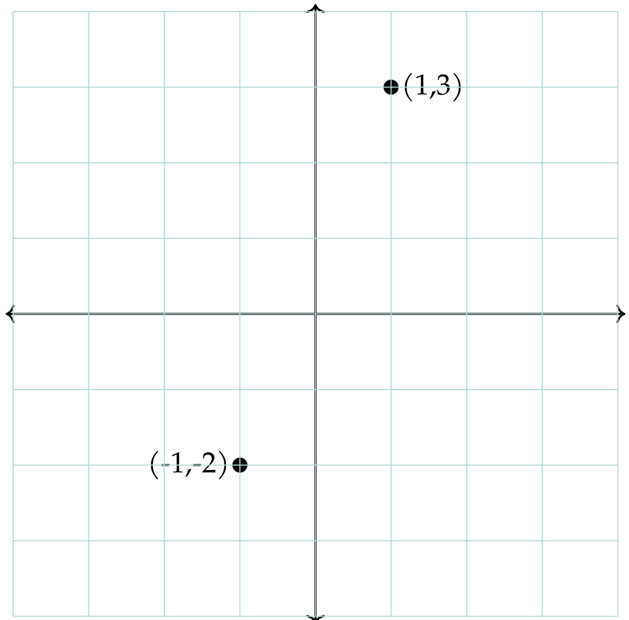
Working Space

Answer on Page 5

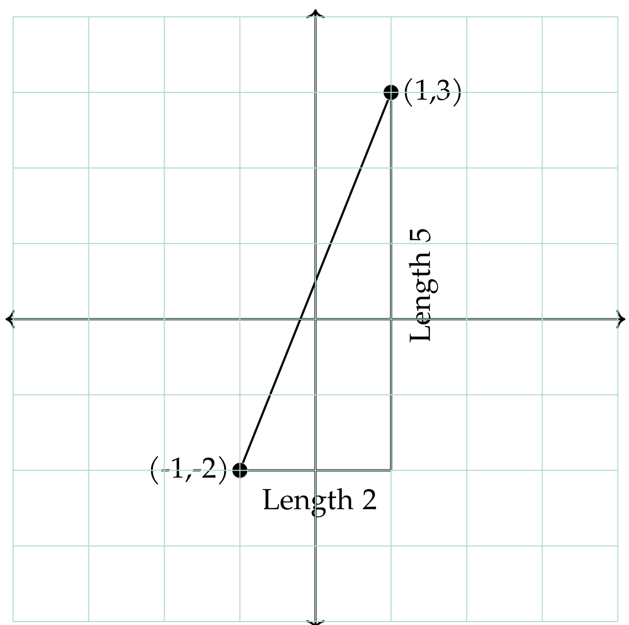
A square's diagonal is a special case of the Pythagorean Theorem such that $c = \sqrt{a^2 + b^2} = \sqrt{2s^2}$. FIXME Diagram here

1.1 Distance between Points

What is the distance between these two points?



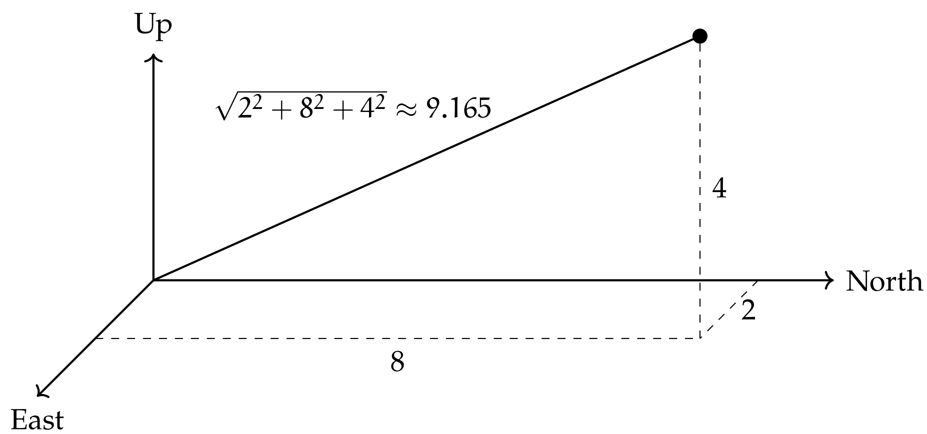
We can draw a right triangle and use the Pythagorean Theorem:



The distance between the two points is $\sqrt{2^2+5^2} = \sqrt{29} \approx 5.385165$. In other words, you square the change in x and add it to the square of the change in y. The distance is the square root of that sum.

1.2 Distance in 3 Dimensions

What if the point is in three-dimensional space? For example, you move 2 meters East, 8 meters North, and 4 meters up in the air. How far are you from where you started? You just square each, sum them, and take the square root: $\sqrt{2^2+8^2+4^2} = \sqrt{84} = 2\sqrt{21} \approx 9.165$ meters.



This leads us to a formal definition of the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Or in 3D space:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

10 because $6^2 + 8^2 = 10^2$

12 because $5^2 + 12^2 = 13^2$

8 because $8^2 + 15^2 = 17^2$

$3\sqrt{2} \approx 4.24$ because $3^2 + 3^2 = \left(3\sqrt{2}\right)^2$

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