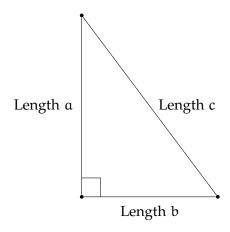
Pythagorean Theorem

Watch's Khan Academy's Intro to the Pythagorean Theorem video at https://youtu.be/AA6RfgP-AHU.

If you have a right triangle, the edges that touch the right angle are called *the legs*. The third edge, which is always the longest and opposite the right angle, is known as *the hypotenuse*. The Pythagorean Theorem gives us the relationship between the length of the legs and the length of the hypotenuse.



The Pythagorean Theorem tells us that $a^2 + b^2 = c^2$.

For example, if one leg has a length of 3 and the other has a length of 4, then $a^2 + b^2 = 3^2 + 4^2 = 25$. Thus, c^2 must equal 25. This means you know the hypotenuse must be of length 5. This works for any right triangle

In reality, it rarely works out to be such a tidy number. For example, what is the length of the hypotenuse if the two legs are 3 and 6? $a^2 + b^2 = 3^2 + 6^2 = 45$. The length of the hypotenuse is the square root of that: $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$, which is approximately 6.708203932499369.

Common side lengths for these triangles are referred to as *Pythagorean triples*, meaning they evaluate to a whole number. Some common examples are (3,4,5), (5,12,13), and (8,15,17). Multiples of right triangles are also triangles ie. $(3,4,5) \Longrightarrow (6,8,10)$, which we will touch on next chapter.

There are also angle-based right triangles, consisting of ratios of the angles of the triangles. The most common ones are $45^{\circ}-45^{\circ}-90^{\circ}$ and the $30^{\circ}-60^{\circ}-90^{\circ}$. We will discuss these further

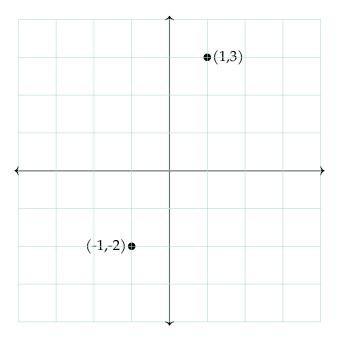
in depth, but know for now that they are vital in trigonometry, and consist of Pythagorean triples as side lengths.

Exercise 1 Find the Missing Length

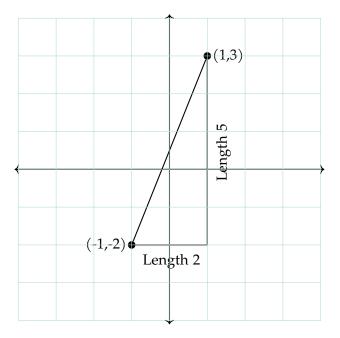
```
Working Space
What is the missing measure?
Leg 1 = 6, Leg 2 = 17
8, Hypotenuse = ?
                    (It should be a
(It should be a whole number.)
whole number.)
                    \text{Leg } 1 = 3, \text{Leg } 2 =
Leg 1 = 5, Leg 2 3, Hypotenuse = ?
= ?, Hypotenuse = (It is an irrational
                    number. Give the
13
(It should be a exact answer and
whole number.)
                    then use a calcu-
                    lator to get an ap-
\text{Leg } 1 = ?, \text{Leg } 2 =
                    proximation.)
15, Hypotenuse =
                                                              Answer on Page 5
```

1.1 Distance between Points

What is the distance between these two points?



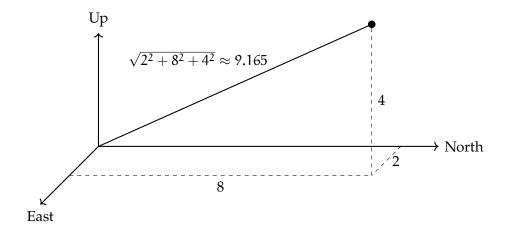
We can draw a right triangle and use the Pythagorean Theorem:



The distance between the two points is $\sqrt{2^2+5^2}=\sqrt{29}\approx 5.385165$. In other words, you square the change in x and add it to the square of the change in y. The distance is the square root of that sum.

1.2 Distance in 3 Dimensions

What if the point is in three-dimensional space? For example, you move 2 meters East, 8 meters North, and 4 meters up in the air. How far are you from where you started? You just square each, sum them, and take the square root: $\sqrt{2^2 + 8^2 + 4^2} = \sqrt{84} = 2\sqrt{21} \approx 9.165$ meters.



This leads us to a formal definition of the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Or in 3D space:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

APPENDIX A

Answers to Exercises

Answer to Exercise 1 (on page 2)

10 because
$$6^2 + 8^2 = 10^2$$

12 because
$$5^2 + 12^2 = 13^2$$

8 because
$$8^2 + 15^2 = 17^2$$

$$3\sqrt{2} \approx 4.24 \text{ because } 3^2 + 3^2 = \left(3\sqrt{2}\right)^2$$



INDEX

distance in 3 dimensions, 4 distance using Pythagorean theorem, 2

Pythagorean theorem, 1 Pythagorean triples, 1