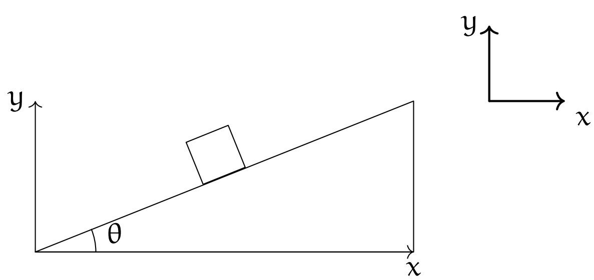
Chapter 1

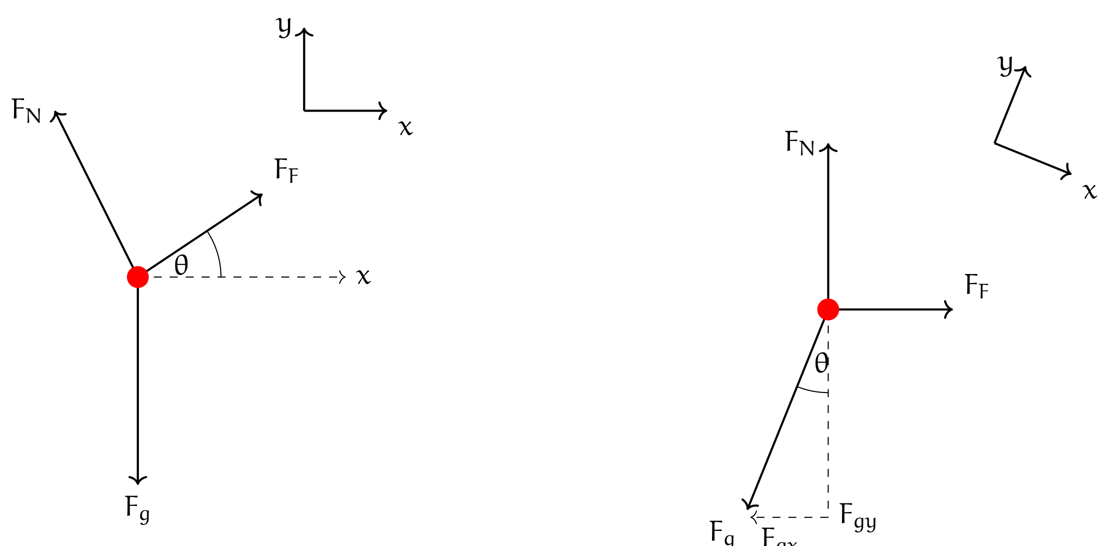
Inclined planes

Take a look at this box resting on an incline plane due to static friction and gravity.

Figure 1.1: A box on an inclined plane with $a = 0$ due to static friction and gravity.



Now for objects on an incline, such as a box on a ramp, we will want to tip our coordinate system. Why you ask? Because we are not restricted to having our y-axis 90° vertically and our x-axis flat horizontally. In this case we can tip our axes so that the x-axis is parallel to the incline and the y-axis is perpendicular to the incline, aligning with the normal force. This will make our calculations much easier.



(a) Free body diagram in standard orientation.

(b) Axes rotated by θ.

Figure 1.2: Free body diagrams on an incline.

From this new perspective, we can see that the gravitational force F_g can be broken down into two components: one parallel to the incline (F_{gx})[1] and one perpendicular to the incline (F_{gy}). The normal force F_N acts perpendicular to the surface of the incline, while the frictional force F_F acts parallel to the surface, opposing motion. The coordinate planes in the corner are included to show the original coordinate system relative to our rotated FBD.[2]

We can set up the following equations based on Newton's second law:

$$\sum F_x = F_{gx} - F_F = ma_x = 0, \qquad \sum F_y = F_N - F_{gy} = ma_y = 0$$

Using the trigonometric components, we can express the forces in terms of the angle θ:

$$\sum F_x = mg\sin(\theta) - F_F = ma_x = 0, \qquad \sum F_y = F_N - mg\cos(\theta) = ma_y = 0$$

Solving for the normal force and frictional force, we find:

$$F_N = mg\cos(\theta), \qquad F_F = mg\sin(\theta)$$

The following relationships are true for acceleration on an incline:

$$F_F = \mu F_N, \qquad a_x = g(\sin\theta - \mu\cos\theta)$$

Where μ is the coefficient of friction between the box and the incline.

Let's apply all of this to an example problem:

[1]Note that some professors will use down the incline as positive. This book will consistently use up the incline as positive as rotating coordinate plane aligns well with this.

[2]Note also that the dashed line components are not actual forces, they are drawn in for illustration of how forces at angles effect the standard forces.

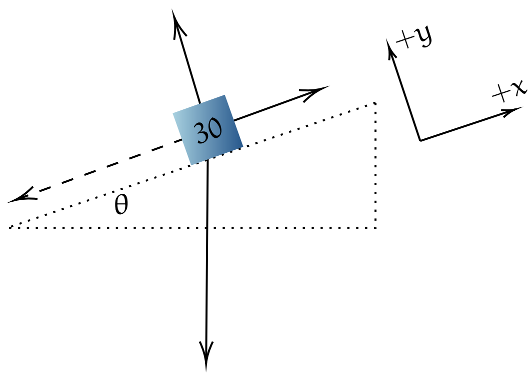
Exercise 1 Box sliding down an incline

A 30 kg block is sliding down an incline plane that makes a 30° with the horizontal. Given that the coefficient of kinetic friction is 0.3, find the following. Note that up the inclined plane is considered the positive x-direction.

(a) The normal force acting on the block.

(b) The frictional force acting on the block.

(c) The acceleration of the block down the incline.



Working Space

Answer on Page ??

Exercise 2 Solving for angle θ

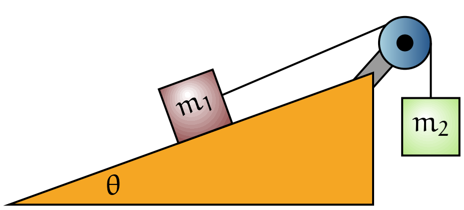
A 40 kg box is sitting on a rough inclined plane, held in place by static friction. The coefficient of static friction μ_s between the box and the incline is found to be 0.464. What is the maximum angle θ that the incline can be tilted to before the box starts to slide down?

Working Space

Answer on Page ??

Exercise 3 Pulleys and Inclines

A block at rest of mass m_1 is resting on a rough inclined plane making an angle θ with the horizontal. The coefficient of kinetic friction between the block and the incline is μ_k. The block is attached by a light string over a frictionless pulley to a hanging block of mass m_2. Once they are connected by a string, the hanging block begins to descend and the block of m_1 moves upwards along the incline. Draw the free body diagrams of each mass. Derive an equation for the acceleration of m_2, only in terms of $m_1, m_2, g, \theta,$ and μ_k.



Working Space

Answer on Page ??

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

APPENDIX A

Answers to Exercises

Answer to Exercise ?? (on page ??)

(a) The normal force must be equal to the perpendicular component of the gravitational force, since there is no acceleration in the y-direction of our rotated y-axis:

$$F_N = mg\cos(\theta) = (30\,\text{kg})(9.8\,\text{m/s}^2)\cos(30^\circ) \approx 254.6\,\text{N}$$

(b) The frictional force can be found using the coefficient of kinetic friction and the normal force:

$$F_F = \mu_k F_N = (0.3)(254.6\,\text{N}) \approx 76.4\,\text{N}$$

(c) The acceleration can be found by setting up the equation for the sum of forces in the x-direction:

$$\sum F_x = F_F - mg\sin(\theta) = ma_x$$

Solving for a_x:

$$\begin{aligned} a_x &= \frac{F_F - mg\sin(\theta)}{m} \\ &= \frac{76.4\,\text{N} - (30\,\text{kg})(9.8\,\text{m/s}^2)\sin(30^\circ) -}{30\,\text{kg}} \\ &= \frac{76.4 - 147}{30} \\ &\approx \frac{-70.6}{30} \\ &\approx -2.35\,\text{m/s}^2 \end{aligned}$$

Answer to Exercise ?? (on page ??)

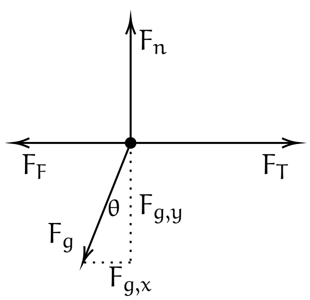
For a box to be on the verge of sliding, the frictional force (static) must equal the component of gravitational force parallel to the incline. Thus, we can set up the following equation:

$$
\begin{aligned}
\sum F_x &= mg\sin(\theta) - F_F = 0 \\
mg\sin(\theta) &= F_F \\
mg\sin(\theta) &= \mu_s F_N \\
mg\sin(\theta) &= \mu_s mg\cos(\theta) \\
\tan(\theta) &= \mu_s
\end{aligned}
$$

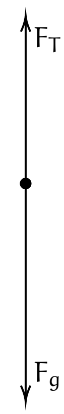
Solving for θ, we get $\theta = \tan^{-1}(\mu_s)$. Plugging in the value of μ_s, we get $\theta = 24.89°$

Answer to Exercise ?? (on page ??)

Let's create free body diagrams for both objects.



FBD Block m_1



FBD Block m_2

Note that the dashed lines are components of forces, not forces themselves. We can see that the common force uniting the two objects is F_T. We can solve for the acceleration in the x-direction of the block of mass m_1 and the acceleration in the y-direction of m_2.

$$
\begin{aligned}
F_{net,m_1,x} &= F_T - m_1 g\sin\theta - \mu_k m_1 g\cos\theta \\
&= m_1 a_x \\
F_{net,m_2,y} &= m_2 g - F_T \\
&= m_2 a_x \\
F_{net,m_1,x} + F_{net,m_2,y} &= m_2 g - m_1 g\sin\theta - \mu_k m_1 g\cos\theta = (m_1 + m_2)a \\
a &= \frac{m_2 g - m_1 g\sin\theta - \mu_k m_1 g\cos\theta}{m_1 + m_2}
\end{aligned}
$$

Notice that both masses share the same magnitude of acceleration. m_2 accelerates vertically, m_1 accelerates horizontally, but their magnitude is identical. It is *not* the same as the net force on m_2 alone.