Chapter 1

Exponents

Let's take quick look at exponents. Ancient scientists started coming up with a lot of formulas that involved multiplying the same number several times. For example, if they knew that a sphere was r centimeters in radius, its volume in milliliters was

$$V = \frac{4}{3} \times \pi \times r \times r \times r$$

They did two things to make the notation less messy. First, they decided that if two numbers were written next to each other, the reader would assume that meant "multiply them". Second, they came up with the exponent, a little number that was lifted off the baseline of the text, that meant "multiply it by itself". For example 5^3 was the same as $5 \times 5 \times 5$.

Now, the formula for the volume of a sphere is written

$$V = \frac{4}{3}\pi r^3$$

Tidy, right? In an exponent expression like this, we say that r is *the base* and 3 is *the exponent*.

1.1 Identities for Exponents

What about exponents of exponents? What is $\left(5^3\right)^2$?

$$\left(5^3\right)^2 = (5 \times 5 \times 5)^2 = (5 \times 5 \times 5)(5 \times 5 \times 5) = 5^6$$

In general, for any a, b, and c:

$$\left(a^b\right)^c = a^{(bc)}$$

If you have $\left(5^3\right)\left(5^4\right)$, that is just $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ or 5^7

The general rule is that for any a, b, and c

$$\left(a^{b}\right)\left(a^{c}\right) = a^{(b+c)}$$

Mathematicians *love* this rule, so we keep extending the idea of exponents to keep this rule true. For example, at some point, someone asked "What about 5^0?" According to the rule, 5^2 must equal $5^{(2+0)}$ which must equal $\left(5^2\right)\left(5^0\right)$. Thus, 5^2 must be 1. So mathematicians declared "Anything to the power of 0 is 1".

We don't typically assume that $0^0 = 1$. It is just too weird. So, we say that for any a not equal to zero,

$$a^0 = 1$$

What about $5^{(-2)}$? By our beloved rule, we know that $\left(5^{-2}\right)\left(5^5\right)$ must be equal to 5^3, right? So 5^{-2} must be equal to $\frac{1}{5^2}$.

We say that for any a not equal to zero and any b,

$$a^{-b} = \frac{1}{a^b}$$

This makes dividing one exponential expression by another (with the same base) easy:

$$\frac{a^b}{a^c} = a^{(b-c)}$$

We often say "cancel out" for this. Here, we can "cancel out" x^2:

$$\frac{x^5}{x^2} = x^3$$

What about $5^{\frac{1}{3}}$? By the beloved rule, we know that $5^{\frac{1}{3}}5^{\frac{1}{3}}5^{\frac{1}{3}}$ must equal 5^1. Thus, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

We say that for any a and b not equal to zero and any c greater than zero,

$$a^{\frac{b}{c}} = \left(\sqrt[c]{a}\right)^b = \left(\sqrt[c]{a^b}\right)$$

Before you go on to the exercises, note that the beloved rule demands a common base.

- We can combine these: $\left(5^2\right)\left(5^4\right) = 5^6$
- We cannot combine: $\left(5^2\right)\left(3^5\right)$

With that said, we note for any a,b, and c:

$$(ab)^c = (a^c)(b^c)$$

So, for example, if we were asked to simplify $\left(3^4\right)\left(6^2\right)$, we would note that $6 = 2 \times 3$, so

$$\left(3^4\right)\left(6^2\right) = \left(3^4\right)\left(3^2\right)\left(2^2\right) = \left(3^6\right)\left(2^2\right)$$

If these ideas are new to you (or maybe you are just having trouble remembering them), watch the Khan Academy's **Intro to rational exponents** video at `https://youtu.be/1ZfXc4nHooo`.

Appendix A

Answers to Exercises

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