

CHAPTER 1

Introduction to Polynomials

Watch Khan Academy's **Polynomials intro** video at <https://youtu.be/Vm7H0VT1Ico>

A *monomial* is the product of a number and a variable raised to a non-negative (but possibly zero) integer power. Here are some examples of monomials:

$$3x^2$$

$$\pi x^2$$

$$7x$$

$$-\frac{2}{3}x^{12}$$

$$-2x^{15}$$

$$(3.33)x^{100}$$

$$3$$

$$0$$

The exponent is called the *degree* of the monomial. For example, $3x^{17}$ has degree 17, $-7x$ has degree 1, and 3.2 has degree 0 (because you can think of it as $(3.2)x^0$).¹.. The exponent cannot be x (or any non-constant variable), as that becomes an exponential function rather than polynomial.¹

The number in the product is called the *coefficient*. Example: $3x^{17}$ has a coefficient of 3, $-2x$ has a coefficient of -2, and $(3.4)x^{1000}$ has a coefficient of 3.4.

A *polynomial* is the sum of one or more monomials. Here are some polynomials:

$$4x^2 + 9x + 3.9$$

$$\pi x^2 + \pi x + \pi$$

$$7x + 2$$

$$-2x^{10} + (3.4)x - 45x^{900} - 1$$

$$3.3$$

$$3x^{20}$$

We say that each monomial is a *term* of the polynomial.

$x^{-5} + 12$ is *not* a polynomial because the first term has a negative exponent.

$x^2 - 32x^{\frac{1}{2}} + x$ is *not* a polynomial because the second term has a non-integer exponent.

$\frac{x+2}{x^2+x+5}$ is *not* a polynomial because it is not just a sum of monomials.

¹A quadratic is a polynomial with a maximum degree of 2

Exercise 1 Identifying Polynomials

Circle only the polynomials.

Working Space

$$-2x^3 + \frac{1}{2}x + 3.9(4.5)x^2 + \pi x^7$$

$$2x^{-10} + 4x - 1 \quad x^{\frac{2}{3}} \quad 3x^{20} + 2x^{19} - 5x^{18}$$

Answer on Page ??

We typically write a polynomial starting at the term with the highest degree and proceed in decreasing order to the term with the lowest degree:

$$2x^9 - 3x^7 + \frac{3}{4}x^3 + x^2 + \pi x - 9.3$$

This is known as *the standard form*. The first term of the standard form is called *the leading term*, and we often call the coefficient of the leading term *the leading coefficient*. We sometimes speak of the degree of the polynomial, which is just the degree of the leading term.

Exercise 2 Standard of a Polynomial

Write $21x^2 - x^3 + \pi - 1000x$ in standard form. What is the degree of this polynomial? What is its leading coefficient?

Working Space

Answer on Page ??

Exercise 3 Evaluate a Polynomial

Let $y = x^3 - 3x^2 + 10x - 12$. What is y when x is 4?

Working Space

Answer on Page ??

We would be remiss in our duties if we didn't mention one more thing about polynomials: Mathematicians have defined a polynomial to be a sum of a *finite* number of monomials.

It is certainly possible to have a sum of an infinite number of monomials like this:

$$1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \dots$$

This is an example of an *infinite series*, which we don't consider polynomials. Infinite series are interesting and useful, but we will not discuss them in detail until later in the course.

This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.

APPENDIX A

Answers to Exercises

Answer to Exercise ?? (on page ??)

$$-2x^3 + \frac{1}{2}x + 3.9$$

$$(4.5)x^2 + \pi x$$

7

$$2x^{-10} + 4x - 1$$

$$x^{\frac{2}{3}}$$

$$3x^{20} + 2x^{19} - 5x^{18}$$

Answer to Exercise ?? (on page ??)

Standard form would be $-x^3 + 21x^2 - 1000x + \pi$. The degree is 3. The leading coefficient is -1 .

Answer to Exercise ?? (on page ??)

$$4^3 - (3)(4^2) + (10)(4) - 12 = 64 - 48 + 40 - 12. \text{ So } y = 44$$



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