Chapter 1

Exponential Decay

In a previous chapter, we saw that an investment of P getting compound interest with an annual interest rate of r, grows exponentially. At the end of year t, your balance would be

$$P(1+r)^t$$

Because r is positive, this number grows as time passes. You get a nice exponential growth curve that looks something like this:

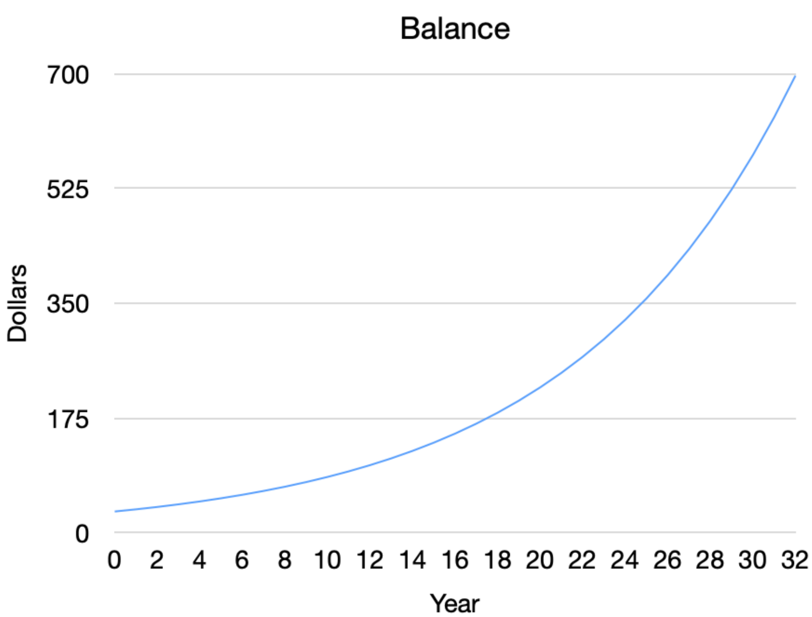

Figure 1.1: A diagram showing (exponential) compound interest.

This is $30 invested with a 10% annual interest rate. So, the formula for the balance after t years would be

$$(30)(1.1)^t$$

What if r were negative? This would be *exponential decay*.

1.1 Radioactive Decay

Until around 1970, there were companies making watches whose faces and hands were coated with radioactive paint. The paint usually contained radium. When a radium atom decays, it gives off some energy, loses two protons and two neutrons, and becomes becomes a different element (radon). Some of the energy given off is visible light. Thus, these watches glow in the dark.

How many of the radium atoms in the paint decay each century? About 4.24%.

Notice the quantity of atoms lost is proportional to the number of atoms you have. This is exponential decay. If we assume that we start with a million radium atoms, the number of atoms decreases over time like this:

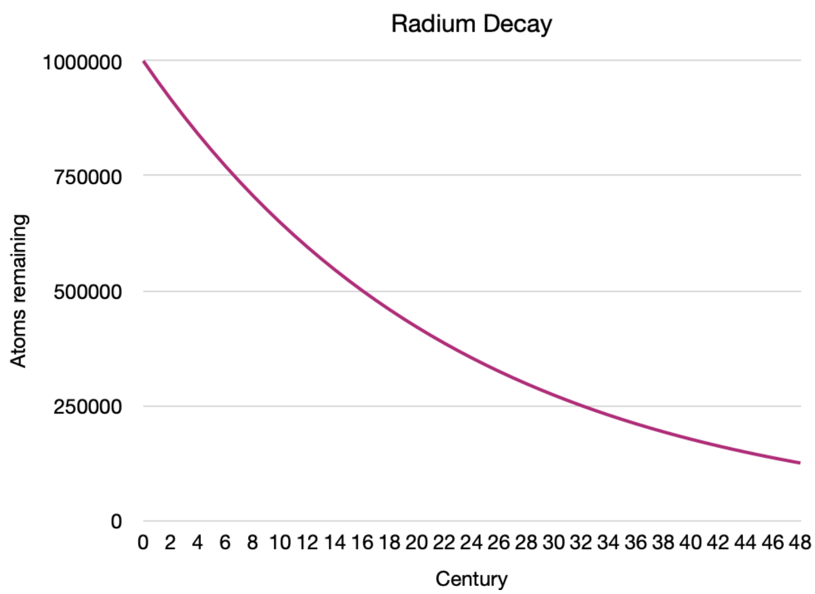

Figure 1.2: A diagram showing the decay and half life of radium.

- We start with 1,000,000 atoms.
- At 16 centuries, we have only 500,000 (half as many) left.
- 16 centuries after that, we have only 250,000 (half again) left.
- 16 centuries after that, we have only 125,000 (half again) left.

A nuclear chemist would say that radium has a *half-life* of 1,600 years; its lifespan decreases by half its original amount every 1,600 years. Note that this means that if you bought a watch with glowing hands in 1960, it will be glowing half as brightly in the year 3560.

How do we calculate the amount of radium left at the end of century t? If you start with P atoms, at the end of the t-th century, you will have

$$P(1 - 0.0424)^t$$

This is exponential decay.

1.2 Model Exponential Decay

Let's say you get hired to run a company with 480,000 employees. Each year, 1/8 of your employees leave the company for one reason or another (retirement, quitting, etc.). For some reason, you never hire any new employees.

Make a spreadsheet that indicates how many of the original 480,000 employees will still be around at the end of each year for the next 12. Next, make a bar graph from that data.

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

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