Chapter 1

Introduction to Polynomials

Watch Khan Academy's **Polynomials intro** video at `https://youtu.be/Vm7H0VTlIco`

A *monomial* is the product of a number and a variable raised to a non-negative (but possibly zero) integer power. Here are some examples of monomials:

$$3x^2 \qquad \pi x^2 \qquad 7x \qquad -\frac{2}{3}x^{12}$$

$$-2x^{15} \qquad (3.33)x^{100} \qquad 3 \qquad 0$$

The exponent is called the *degree* of the monomial. For example, $3x^{17}$ has degree 17, $-7x$ has degree 1, and 3.2 has degree 0 (because you can think of it as $(3.2)x^0$).. The exponent cannot be x (or any non-constant variable), as that becomes an exponential function rather than polynomial.[1]

The number in the product is called the *coefficient*. Example: $3x^{17}$ has a coefficient of 3, $-2x$ has a coefficient of -2, and $(3.4)x^{1000}$ has a coefficient of 3.4.

A *polynomial* is the sum of one or more monomials. Here are some polynomials:

$$4x^2 + 9x + 3.9 \qquad \pi x^2 + \pi x + \pi \qquad 7x + 2$$

$$-2x^{10} + (3.4)x - 45x^{900} - 1 \qquad 3.3 \qquad 3x^{20}$$

We say that each monomial is a *term* of the polynomial.

$x^{-5} + 12$ is *not* a polynomial because the first term has a negative exponent.

$x^2 - 32x^{\frac{1}{2}} + x$ is *not* a polynomial because the second term has a non-integer exponent.

$\frac{x+2}{x^2+x+5}$ is *not* a polynomial because it is not just a sum of monomials.

[1] A quadratic is a polynomial with a maximum degree of 2

Exercise 1 Identifying Polynomials

Circle only the polynomials.

Working Space

$-2x^3+\frac{1}{2}x+3.9$ $(4.5)x^2 + \pi x$ 7

$2x^{-10}+4x-1$ $x^{\frac{2}{3}}$ $3x^{20}+2x^{19}-5x^{18}$

Answer on Page 5

We typically write a polynomial starting at the term with the highest degree and proceed in decreasing order to the term with the lowest degree:

$$2x^9 - 3x^7 + \frac{3}{4}x^3 + x^2 + \pi x - 9.3$$

This is known as *the standard form*. The first term of the standard form is called *the leading term*, and we often call the coefficient of the leading term *the leading coefficient*. We sometimes speak of the degree of the polynomial, which is just the degree of the leading term.

Exercise 2 Standard of a Polynomial

Write $21x^2 - x^3 + \pi - 1000x$ in standard form. What is the degree of this polynomial? What is its leading coefficient?

Working Space

Answer on Page 5

Exercise 3 Evaluate a Polynomial

Let $y = x^3 - 3x^2 + 10x - 12$. What is y when x is 4?

Working Space

Answer on Page 5

We wouild be remiss in our duties if we didn't mention one more thing about polynomials: Mathematicians have defined a polynomial to be a sum of a *finite* number of monomials.

It is certainly possible to have a sum of an infinite number of monomials like this:

$$1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \dots$$

This is an example of an *infinite series*, which we don't consider polynomials. Infinite series are interesting and useful, but we will not discuss them in detail until later in the course.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

$\boxed{-2x^3 + \frac{1}{2}x + 3.9}$ $\boxed{(4.5)x^2 + \pi x}$ $\boxed{7}$

$2x^{-10} + 4x - 1$ $x^{\frac{2}{3}}$ $\boxed{3x^{20} + 2x^{19} - 5x^{18}}$

Answer to Exercise 2 (on page 2)

Standard form would be $-x^3 + 21x^2 - 1000x + \pi$. The degree is 3. The leading coefficient is -1

Answer to Exercise 3 (on page 3)

$4^3 - (3)(4^2) + (10)(4) - 12 = 64 - 48 + 40 - 12$. So $y = 44$

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