Chapter 1

Common Polynomial Products

In math and physics, you will run into certain kinds of polynomials over and over again. In this chapter, We are going to cover some patterns that you will want to be able to recognize.

1.1 Difference of squares

Watch **Polynomial special products: difference of squares** from Khan Academy at `https://youtu.be/uNweU6I4Icw`.

If you are asked what $(3x-7)(3x+7)$ is, you would use the distributive property to expand that to $(3x)(3x) + (3x)(7) + (-7)(3x) + (-7)(7)$. Two of the terms cancel each other, so this is $(3x)^2 - (7)^2$. This would simplify to $9x^2 - 49$

You will see this pattern often. Anytime you see $(a + b)(a - b)$, you should immediately recognize it equals a^2-b^2. (Note that the order doesn't matter: $(a-b)(a+b)$ also a^2-b^2.)

Working the other way is important too. Any time you see $a^2 - b^2$, that you should recognize that you can change that into the product $(a + b)(a - b)$. Making something into a product like this is known as *factoring*. You probably have done prime factorization of numbers like $42 = 2 \times 3 \times 7$. In the next couple of chapters, you will learn to factorize polynomials.

Exercise 1 Difference of Squares

Simplify the following products:

1. $(2x-3)(2x+3)$
2. $(7+5x^3)(7-5x^3)$
3. $(x-a)(x+a)$
4. $(3-\pi)(3+\pi)$
5. $(-4x^3+10)(-4x^3-10)$
6. $(x+\sqrt{7})(x-\sqrt{7})$ Factor the following polynomials:
7. x^2-9
8. $49-16x^6$
9. π^2-25x^8
10. x^2-5

Working Space

Answer on Page 7

We are often interested in the roots of a polynomial. That is, we want to know "For what values of x does the polynomial evaluate to zer?" For example, when you deal with falling bodies, the first question you might ask would be "How many seconds before the hammer hits the ground?" Once you have factored a polynomial into binomials, you can easily find the roots.

For example, what are the roots of x^2-5? You just factored it into $(x+\sqrt{5})(x-\sqrt{5})$ This product is zero if and only if one of the factors is zero. The first factor is only zero when x is $-\sqrt{5}$. The second factor is zero only when x is $\sqrt{5}$. Those are the only two roots of this polynomial.

Let's check that result. $\sqrt{5}$ is a little more than 2.2. Using your Python code, you can graph the polynomial:

```
import poly.py
import matplotlib.pyplot as plt

# x**2 - 5
pn = [-5.0, 0.0, 1.0]
```

```
# These lists will hold our x and y values
x_list = []
y_list = []

# Start at x=-3
current_x =-3.0

# End at x=3.0
while current_x < 3.0:
    current_y = poly.evaluate_polynomial(pn, current_x)

    # Add x and y to respective lists
    x_list.append(current_x)
    y_list.append(current_y)

    # Move x forward
    current_x += 0.1

# Plot the curve
plt.plot(x_list, y_list)
plt.grid(True)
plt.show()
```

You should get a plot like this:

It does, indeed, seem to cross the x-axis near -2.2 and 2.2.

1.2 Powers of binomials

You can raise whole polynomials to exponents. For example,

$$(3x^3 + 5)^2 = (3x^3 + 5)(3x^3 + 5)$$
$$= 9x^6 + 15x^3 + 15x^3 + 25 = 9x^6 + 30x^3 + 25$$

A polynomial with two terms is called a *binomial*. $5x^9 - 2x^4$, for example, is a binomial. In this section, we are going to develop some handy techniques for raising a binomial to some power.

Looking at the previous example, you can see that for any monomials a and b, $(a + b)^2 = a^2 + 2ab + b^2$. This is referred to as a perfect square binomial. So, for example, $(7x^3 + \pi)^2 = 49x^6 + 14\pi x^3 + \pi^2$

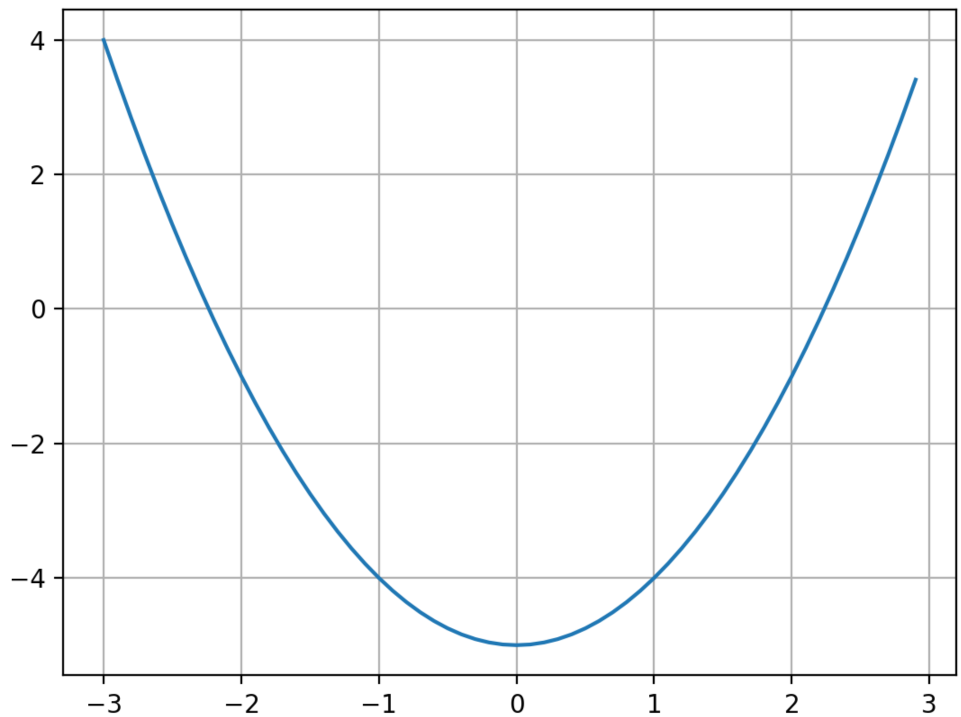

Figure 1.1: A graph of $x^2 - 5$ showing the roots on the x-axis.

Exercise 2 Squaring binomials

Simply the following

1. $(x+1)^2$
2. $(3x^5+5)^2$
3. $(x^3-1)^2$
4. $(x-\sqrt{7})^2$

Working Space

Answer on Page 7

What about $(x+2)^3$? You can do it as two separate multiplications:

$$\begin{aligned}(x+2)^3 &= (x+2)(x+2)(x+2)\\ &= (x+2)(x^2+4x+4) = x^3+4x^2+4x+2x^2+8x+8\\ &= x^3+6x^2+12x+8\end{aligned}$$

In general, we can say that for any monomials a and b, $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$.

What about higher powers? $(a+b)^4$, for example? You could use the distributive property four times, but it starts to get pretty tedious.

Here is a trick. This is known as *Pascal's triangle*

$$\begin{array}{ccccccccccccccc}
&&&&&&&1&&&&&&&\\
&&&&&&1&&1&&&&&&\\
&&&&&1&&2&&1&&&&&\\
&&&&1&&3&&3&&1&&&&\\
&&&1&&4&&6&&4&&1&&&\\
&&1&&5&&10&&10&&5&&1&&\\
&1&&6&&15&&20&&15&&6&&1&\\
1&&7&&21&&35&&35&&21&&7&&1\\
&&&&&&&\dots&&&&&&&
\end{array}$$

Each entry is the sum of the two above it.

The coefficients of each term are given by the entries in Pascal's triangle:

$$(a+b)^4 = 1a^4+4a^3b+6a^2b^2+4ab^3+1b^4$$

Exercise 3 Using Pascal's Triangle

Working Space

1. What is $(x + \pi)^5$?

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This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 2)

$(2x-3)(2x+3) = 4x^2 - 9$

$(7+5x^3)(7-5x^3) = 49 - 25x^6$

$(x-a)(x+a) = x^2 - a^2$

$(3-\pi)(3+\pi) = 9 - \pi^2$

$(-4x^3+10)(-4x^3-10) = 16x^6 - 100$

$(x+\sqrt{7})(x-\sqrt{7}) = x^2 - 7$

$x^2 - 9 = (x+3)(x-3)$

$49 - 16x^6 = (7+4x^3)(7+4^3)$

$\pi^2 - 25x^8 = (\pi + 5x^4)(\pi - 5x^4)$

$x^2 - 5 = (x+\sqrt{5})(x-\sqrt{5})$

Answer to Exercise 2 (on page 5)

$(x+1)^2 = x^2 + 2x + 1$

$(3x^5+5)^2 = 9x^1 0 + 30x^5 + 25$

$(x^3-1)^2 = x^6 - 2x^3 + 1$

$(x-\sqrt{7})^2 = x^2 - 2x\sqrt{7} + 7$

Answer to Exercise 3 (on page 6)

$(x+\pi)^5 = x^5 + 5\pi x^4 + 10\pi^2 x^3 + 10\pi^3 + x^2 + 5\pi^2 x + \pi^5$

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