

CHAPTER 1

Mass, Weight, and Gravity

Mass is a measure of the amount of matter in an object. Weight is the force of gravity on that object. An object's mass is the same no matter where it is in the universe: the amount of "stuff" in an object does not depend on its location. However, an object's weight *does* change: the same object has a different weight on the Moon than it does on Earth or Venus. Since the force of gravity on the Moon is about 1/6th as on Earth, you would weigh 1/6th as much on the Moon, but your mass would be the same.

1.1 Kilograms versus Pounds

In elementary school, you probably learned that 1 kilogram is about 2.2 pounds. This is only true *on Earth*. In everyday language, we may use kilograms and pounds, or mass and weight, interchangeably; scientists do not.

Mass	Weight
measure of <i>amount</i>	measure of <i>force</i>
units include grams, kilograms, milligrams	units include pounds, Newtons, dynes
a <i>scalar</i> measurement	a <i>vector</i> measurement
does <i>not</i> depend on location	does depend on location

On Earth, a 1 kilogram mass weighs 2.2 pounds. That same mass weighs 0.37 pounds on the Moon. You learned to "convert" between pounds in kilograms in elementary school because everywhere on Earth, a 1-kg mass weighs 2.2 lbs. The conversion doesn't apply to other locations in the universe because mass and weight are different.

1.1.1 Scalars and Vectors

One of the fundamental differences between mass and weight is the type of measurement: scalar versus vector. *Scalar* measurements are just a counting number: they tell the amount of magnitude. Mass is a scalar measurement: it tells how much matter is in an object. Energy is also a scalar measurement, as well as temperature and density. *Vectors*, on the other hand, also have a *direction*. A complete measure of a force gives the magnitude and direction. So, your weight isn't just 185 pounds: it's 185 pounds *downward*. Velocity is a speed plus a direction, such as 35 mph east. We will expand on this later in the vectors chapter!

1.2 Mass and Gravity

When we discuss weight, we usually mean the gravitational force between an object and the largest nearby object. My weight on Earth, a spacecraft's weight on the Moon, or a rover's weight on Mars are all examples. But planets and moons don't have some special property that makes them emit gravity. Rather, *all objects with mass are gravitationally attracted to each other*. But, compared to the mass of the Earth, the mass of all the other objects around you (a table, your family, your house or apartment complex) is very, very small.

The force of the gravitational attraction between two objects is proportional to the product of their masses, and inversely proportional to their distance squared. This means that as objects get farther away, the force decreases. If you double the distance, the force quarters. Quadruple the distance, the force is $1/16^{\text{th}}$ as much. This is why you are more attracted to the Earth than you are to distant stars, even though they have much more mass than the Earth.

Newton's Law of Universal Gravitation

Two masses (m_1 and m_2) that are a distance of r from each other are attracted toward each other with a force of magnitude:

$$F = G \frac{m_1 m_2}{r^2} \quad (1.1)$$

where G is the universal gravitational constant. If you measure the mass in kilograms and the distance in meters, G is about 6.674×10^{-11} . That will get you the force of the attraction in newtons.

Exercise 1 Gravitation between two planets

A small moon orbits a planet at a distance of 4.0×10^6 m from the planet's center. The mass of the planet is 6.0×10^{24} kg, and the mass of the moon is 8.0×10^{22} kg.

Working Space

1. Using Newton's Law of Universal Gravitation, calculate the gravitational force between the planet and the moon.
2. If the moon were moved to a distance of 8.0×10^6 m from the planet's center (twice the original distance), determine the new gravitational force.

Answer on Page ??

Exercise 2 **Gravity***Working Space*

The Earth's mass is about 6×10^{24} kilograms.

Your spacecraft's mass is 6,800 kilograms.

Your spacecraft is also about 100,000 km from the center of the Earth. (For reference, the moon is about 400,000 km from the center of the Earth.)

What is the magnitude of the force of gravity that is pulling your spacecraft and the Earth toward each other?

*Answer on Page ??***1.3 Mass and Weight**

Gravity pulls on things proportional to their mass, so we often ignore the difference between mass and weight.

The weight of an object is the force due to the object's mass and gravity. When we say, "This potato weighs 1 pound," we actually mean "This potato weighs 1 pound on Earth." That same potato would weigh about one-fifth of a pound on the moon (see figure ??).

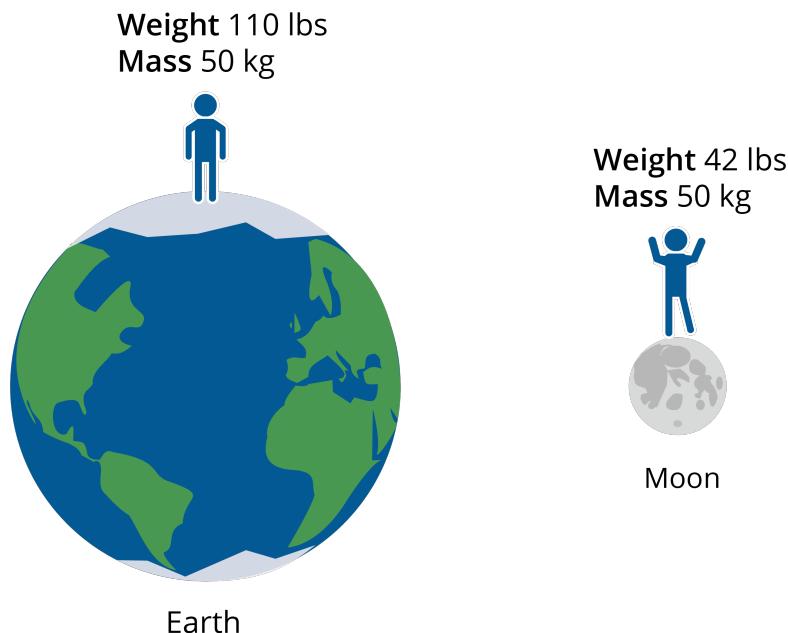


Figure 1.1: Mass is a measure of all the matter in an object. Weight is a measure of the force of gravity on that object. Mass is not location-dependent, while weight is.

However, that potato has a mass of 0.45 kg no matter where it is in the universe.

We talked about the Law of Universal Gravitation. We can simplify it for our calculations to derive the weight of objects near to the surface of the earth.

Starting with Equation ??, for objects near Earth's surface, the radius is essentially constant and equal to Earth's radius R_E . Substituting $r = R_E$ gives us:

$$F = G \frac{Mm}{R_E^2} \quad (1.2)$$

The only thing variable in this equation is m , so the object's mass can be factored out, leaving:

$$F = m \left(G \frac{M}{R_E^2} \right) \quad (1.3)$$

The quantity in parentheses involves only constants: the gravitational constant G , Earth's mass M , and Earth's radius R_E . Since this value stays the same for all objects near the surface, we define it as the gravitational field strength:

$$g = G \frac{M}{R_E^2} \quad (1.4)$$

Substituting this definition back into the force equation gives:

$$F = mg \quad (1.5)$$

where m is the mass of some object, and $g = 9.8 \text{ m/s}^2$ is the gravitational constant, a constant found by $G \frac{M}{R_E^2}$.

The steps in sequence are shown below:

$$\begin{aligned} F &= G \frac{Mm}{r^2} \\ &= G \frac{Mm}{R_E^2} \\ &= m \left(G \frac{M}{R_E^2} \right) \\ g &= G \frac{M}{R_E^2} \\ F &= mg \end{aligned}$$

Equation ?? is the equation we will use as a standard for the force of gravity, F_g , for most objects on Earth. Objects undergoing projectile motion, objects at rest, and objects hung by tension all have a force of gravity acting on them,¹

Exercise 3 May the Force be with You 1

A hammer is falling along the side of a building, close to the surface of the Earth. What is its acceleration?

Working Space

Answer on Page ??

¹You may occasionally use 10 for g to simplify expressions if the problem says to do so, but use 9.8 otherwise.

Exercise 4 May the Force be with You 2

A coin is dropped. It is measured that the force of gravity on the coin is .0198 N. What is the mass of the coin?

Working Space

Answer on Page ??

1.4 Center of Mass

The *center of mass* of a body is the point where its mass is evenly balanced in all directions, as if all the mass were concentrated at that single point. When we talk about *Free Body Diagrams* in a future chapter, we will assume the entire body's center of mass is located at a point that we use to analyze forces from. For an object with different densities at different points — for example, a hammer with a dense metal head and a lighter wooden handle — the center of mass is found by averaging the positions of all parts, giving more weight to the heavier regions. You will see the term *center of mass* throughout this sequence; we will assume you understand how center of mass is calculated, but typically will not fully determine it.

For a one-dimensional object on some line, the center of mass can be found using the formula:

1-Dimensional Center of Mass

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + m_n}$$

where x_{CM} and $x_1 \dots x_n$ are the distance from a *chosen origin* location. Notice how this is similar the formula for averages.

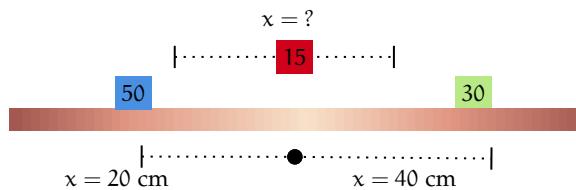
We can rewrite this using summation notation:

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i}$$

Exercise 5 Balancing a seesaw

Three masses are placed on a seesaw. M_1 has a mass of 30 lbs and is placed 40 cm to the right the equilibrium point of the seesaw. M_2 has a mass of 50 lbs and is placed 20 cm to the left of the equilibrium point. Where does M_3 , which weighs 15 lbs, need to go in order for the seesaw to balanced? Assume the seesaw plank itself is uniformly distributed.

Working Space



Answer on Page ??

In higher dimensions, it is easier to use integration to calculate the coordinates of the center of mass of some body, especially when it is not uniform (equally distributed). Although we haven't introduced integration formally, it is important to begin thinking about these topics:

2D and 3D Center of Mass

$$\mathbf{r} = x \hat{\mathbf{i}} + y \hat{\mathbf{j}} + z \hat{\mathbf{k}}$$

$$dm = \rho(x, y, z) dV$$

$$\text{2D : } M = \iint_V \rho(\mathbf{r}) dV, \quad \text{3D : } M = \iiint_V \rho(\mathbf{r}) dV$$

where M is the net mass, dm is some infinitesimal weight at point \mathbf{r} , the position vector within V . V is the dimensions of some the 3d boundary, and $\rho(x, y, z)$ is the density at the infinitesimal point (x, y, z)

The individual components can be found by:

$$\begin{aligned}x_{CM} &= \frac{1}{M} \iiint_V x \rho(x, y, z) dV \\y_{CM} &= \frac{1}{M} \iiint_V y \rho(x, y, z) dV \\z_{CM} &= \frac{1}{M} \iiint_V z \rho(x, y, z) dV\end{aligned}$$

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APPENDIX A

Answers to Exercises

Answer to Exercise ?? (on page ??)

(a)

$$\begin{aligned} F &= G \frac{m_1 m_2}{r^2} \\ &= (6.674 \times 10^{-11}) \frac{(6.0 \times 10^{24})(8.0 \times 10^{22})}{(4.0 \times 10^6)^2} \\ &= 2.0022 \times 10^{24} \end{aligned}$$

- (b) Now we have a new distance, $r' = 8.0 \times 10^6$ m, double the original. Notice that gravitational force is inversely proportional to the square of the distance:

$$F \propto \frac{1}{r^2}$$

So if distance doubles,

$$r' = 2r \quad \Rightarrow \quad F' = \frac{F}{4}$$

Which results in 5.0055×10^{23} N

Answer to Exercise ?? (on page ??)

$$F = G \frac{m_1 m_2}{r^2} = (6.674 \times 10^{-11}) \frac{(6.8 \times 10^3)(6 \times 10^{24})}{(10^5)^2} = 6.1 \times 10^6$$

About 6 million newtons.

Answer to Exercise ?? (on page ??)

The gravitational acceleration is 9.8 m/s^2 , no matter the mass of the object.

Answer to Exercise ?? (on page ??)

The mass of the coin is $\frac{.0198 \text{ N}}{9.8 \text{ m/s}^2} = 0.002 \text{ grams}$

Answer to Exercise ?? (on page ??)

We can set up a balanced center of mass equation for this problem. Our problem is only in one dimension, so there is only one distance to be checked. If we set our $x_{CM} = 0$ where the fulcrum or pivot of a seesaw would be by assuming the balance scenario, we can solve our problem:

$$\begin{aligned}x_{CM} &= \frac{\sum m_i x_i}{\sum m_i} \\0 &= \frac{(30)(40) + (50)(-20) + (15)d}{30 + 50 + 15} \\&= \frac{1200 - 1000 + (15)d}{85} \\&= \frac{200 + (15)d}{85} \\d &= -13\frac{1}{3} \text{ m}\end{aligned}$$

Since our answer is negative, it means to the left of the fulcrum (given we chose left to be negative). Thus, M_3 should be placed $13\frac{1}{3}$ m to the left.



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