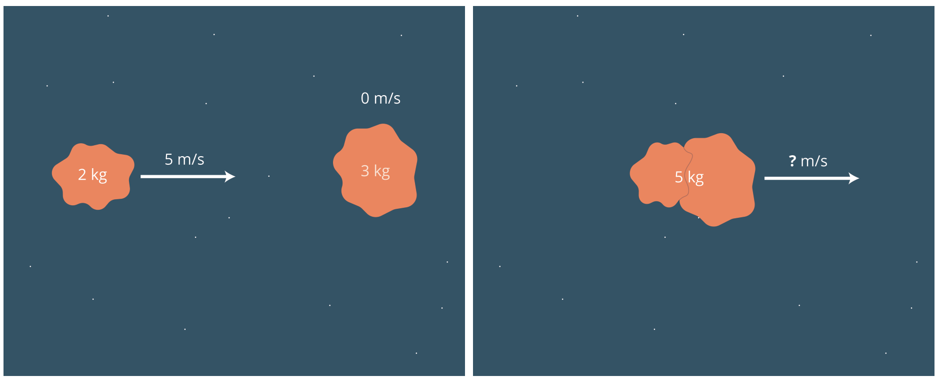
Chapter 1

Momentum

Let's say a 2 kg block of putty is flying through space at 5 meters per second, and it collides with a larger 3 kg block of putty that is not moving at all. When the two blocks deform and stick to each other, how fast will the resultant big block be moving?



Every object has *momentum*. The momentum is a vector quantity — it points in the direction that the object is moving and has a magnitude equal to its mass times its speed.

$$p = mv$$

Given a set of objects that are interacting, we can sum all their momentum vectors to get the total momentum. In such a set, the total momentum will stay constant.

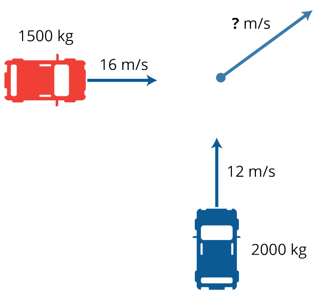
$$p = p_1 + p_2 = m_1 v_1 + m_2 v_2$$

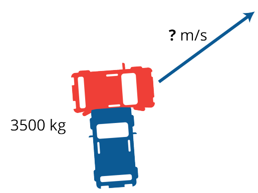
In our example, one object has a momentum vector of magnitude of 10 kg m/s, the other has a momentum of magnitude 0. Once they have merged, they have a combined mass of 5 kg. This means the velocity vector must have magnitude 2 m/s and pointing in the same direction that the first mass was moving.

Exercise 1 Cars on Ice

A car weighing 1000 kg is going north at 12 m/s. Another car weighing 1500 kg is going east at 16 m/s. They both hit a patch of ice (with zero friction) and collide. Steel is bent, and the two objects become one. How what is the velocity vector (direction and magnitude) of the new object sliding across the ice?

Working Space





Answer on Page 5

Note that kinetic energy ($1/2mv^2$) is *not* conserved here. Before the collision, the moving putty block has $(1/2)(2)(5^2) = 25$ joules of kinetic energy. Afterward, the big block has $(1/2)(5)(2^2) = 10$ joules of kinetic energy. What happened to the energy that was lost? It was used up deforming the putty.

What if the blocks were marble instead of putty? Then there would be very little deforming, so kinetic energy *and* momentum would be conserved. The two blocks would end up having different velocity vectors.

Let's assume for a moment that they strike each other straight on, so there is motion in only one direction, both before and after the collision. Can we solve for the speeds of the first block (v_1) and the second block (v_2)?

We end up with two equations. Conservation of momentum says:

$$2v_1 + 3v_2 = 10$$

Conservation of kinetic energy says:

$$(1/2)(2)(v_1^2) + (1/2)(3)(v_2^2) = 25$$

Using the first equation, we can solve for v_1 in terms of v_2:

$$v_1 = \frac{10 - 3v_2}{2}$$

Substituting this into the second equation, we get:

$$\left(\frac{10 - 3v_2}{2}\right)^2 + \frac{3v_2^2}{2} = 25$$

Simplifying, we get:

$$v_2^2 - 4v_2 + 0 = 0$$

This quadratic has two solutions: $v_2 = 0$ and $v_2 = 4$. $v_2 = 0$ represents the situation before the collision. Substituting in $v_2 = 4$:

$$v_1 = \frac{10 - 3(4)}{2} = -1$$

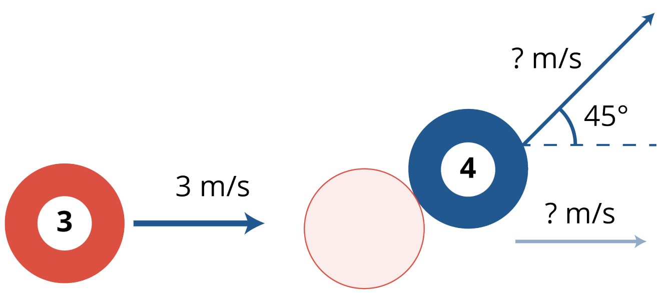
Thus, if the blocks are hard enough that kinetic energy is conserved, after the collision, the smaller block will be heading in the opposite direction at 1 m/s. The larger block will be moving at 4 m/s in the direction of the original motion.

Exercise 2 Billiard Balls

Working Space

A billiard ball weighing 0.4 kg and traveling at 3 m/s hits a billiard ball (same weight) at rest. It strikes obliquely (neither perpendicular nor parallel), so that the ball at rest starts to move at a 45 degree angle from the path of the ball that hit it.

Assuming all kinetic energy is conserved, what is the velocity vector of each ball after the collision?



Answer on Page 5

APPENDIX A

Answers to Exercises

Answer to Exercise 1 (on page 2)

The momentum of the first car is 12,000 kg m/s in the north direction.

The momentum of the second car is 24,000 kg m/s in the east direction.

The new object will be moving northeast. What is the angle compared with the east?

$$\theta = \arctan \frac{12,000}{24,000} \approx 0.4636 \text{ radians } \approx 26.565 \text{ degrees north of east}$$

The magnitude of the momentum of the new object is $\sqrt{12,000^2 + 24,000^2} \approx 26,833$ kg m/s

Its new mass is 2,5000 kg. So the speed will be $26,833/2,500 = 10.73$ m/s.

Answer to Exercise 2 (on page 4)

The original forward momentum was 1.2 kg m/s. The original kinetic energy is $(1/2)(0.4)(3^2)$ = 1.8 joules.

Let s be the post-collision speed of the ball that had been at rest. Let x and y be the forward and sideways speeds (post-collision) of the other ball. Conservation of kinetic energy says

$$(1/2)(0.4)(s^2) + (1/2)(0.4)(x^2 + y^2) = 1.8$$

Forward momentum is conserved:

$$0.4\frac{s}{\sqrt{2}} + 0.4x = 1.2$$

Which can be rewritten:

$$x = 3 - \frac{s}{\sqrt{2}}$$

Sideways momentum stays zero:

$$(0.4)\frac{s}{\sqrt{2}} - 0.4y = 0.0$$

Which can be rewritten:

$$y = \frac{s}{\sqrt{2}}$$

Substituting into to the conservation of kinetic energy equation above:

$$(1/2)(0.4)(s^2) + (1/2)(0.4)(\left(3 - \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = 1.8$$

Which can be rewritten:

$$s^2 - \frac{3}{\sqrt{2}}s + 0 = 0$$

There are two solutions to this quadratic: $s = 0$ (before collision) and $s = \frac{3}{\sqrt{2}}$. Thus,

$$y = \frac{3}{2}$$

and

$$x = 3 - \frac{3}{2} = \frac{3}{2}$$

So, both balls careen off at 45° angles at the exact same speed.

Index