

CHAPTER 1

The Physics of Gases

Now, let's say you start to heat the helium inside the balloon. As the temperature goes up, the molecules inside will start to move faster.

Remember that the kinetic energy of an object with mass m and velocity v is given by

$$k = \frac{1}{2}mv^2 \quad (1.1)$$

So, you could say "As the temperature of the gas increases, the kinetic energy of the molecules increases." But a physicist would say "The temperature of the gas is how we measure its kinetic energy."

1.0.1 A Statistical Look At Temperature

If you say "This jar of argon gas is 25 degrees Celsius," you have told me about the *average* kinetic energy of the molecules in the jar. However, some molecules are moving very slowly. Others are moving really, really fast.

We could plot the probability distribution of the speeds of the molecules. For argon at 25 degrees Celsius, it would look like this:

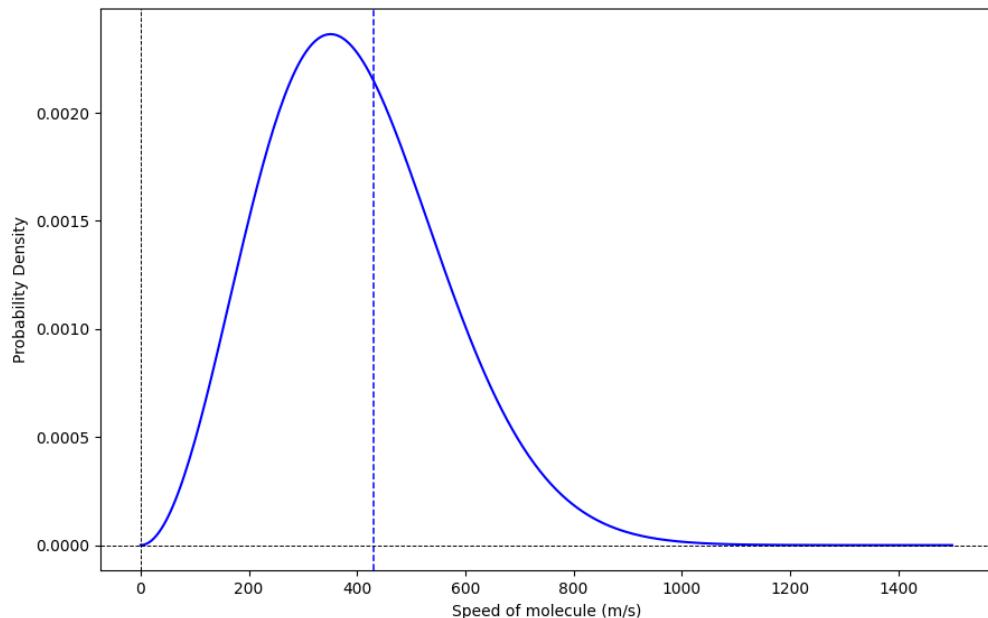


Figure 1.1: Probability distribution of molecular speeds for argon gas at 25 degrees Celsius.

The temperature, remember, is determined by the average kinetic energy of the molecules. Some molecules are moving slowly, and have less kinetic energy than the average. Some molecules are moving very quickly, and have more kinetic energy. The dotted line is the divider between the two groups: molecules moving at speeds to the left of the line have less kinetic energy than average; those on the right have more kinetic energy than average.

Where is that line? That is the RMS of the speeds of the molecules. That is, if we measured all the speeds of all the molecules $s_1, s_2, s_3, \dots, s_n$, that line would be given by the root of the mean of the squares:

$$v_{\text{rms}} = \sqrt{\frac{1}{n} (s_1^2 + s_2^2 + s_3^2 + \dots + s_n^2)}$$

If you have the same gas at a lower temperature, the distribution shifts toward zero: the peak becomes taller and narrower, and the tail of very fast molecules becomes much shorter. In other words, fewer molecules have high speeds, and more molecules cluster near lower speeds. Let's look at an example.

Here is probability distribution of molecular speeds for argon gas at 25 degrees and -100 degrees Celsius.

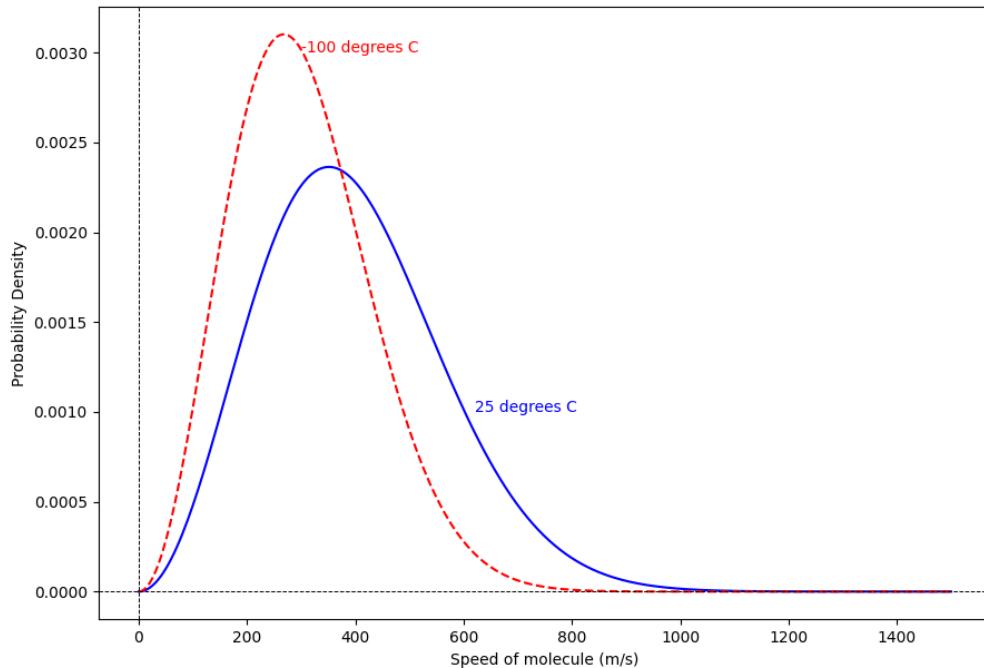


Figure 1.2: Probability distribution of molecular speeds for argon gas at 25 degrees and -100 degrees Celsius.

1.0.2 Absolute Zero and Degrees Kelvin

If you keep lowering the temperature, eventually, all the molecules stop moving. This is known as *absolute zero* — you can't make anything colder than absolute zero. Absolute zero is -273.15° Celsius or -459.67° Fahrenheit.

In addition to Celsius and Fahrenheit, there is a third temperature system: Kelvin. Kelvin has the same scale as Celsius, but it starts at absolute zero (such that every value differs by 273.15). So, 0 degrees Celsius is 273.15 degrees Kelvin, and 100 degrees Celsius is 373.15 degrees Kelvin.

Any time you are working with the physics of temperature, you will use Kelvin.

Sometimes, when reading about gases, you will see "STP" which stands for "Standard Temperature and Pressure." STP is defined to be 0° Celsius and 100 kPa.

1.1 Temperature and Volume

Let's say it's dawn, and a person is lying on a half-full air mattress in a field. The weight of the person will keep the pressure of the air inside constant (or pretty close).

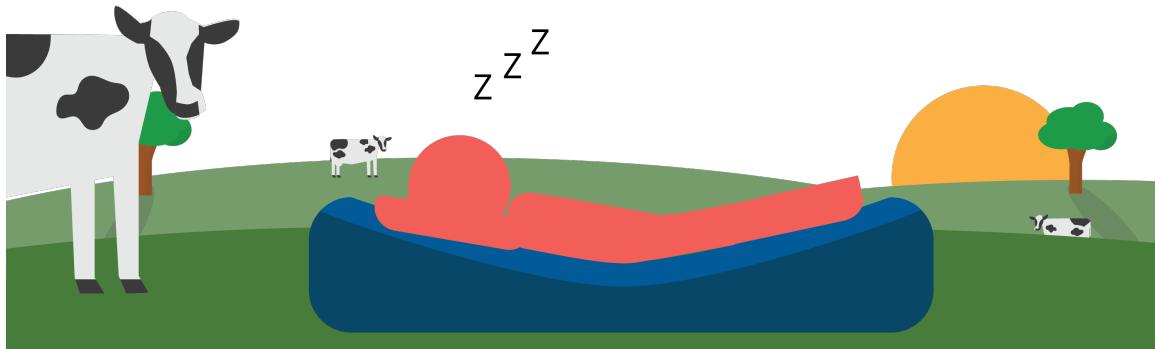


Figure 1.3: A man sleeping on an air mattress as the sun rises.

The molecules in the mattress are not entering or leaving that mattress. However, as the sun rises, the air inside will get warmer and expand. The person will be gently lifted by the expanding air. You might wonder: how much will the air expand?

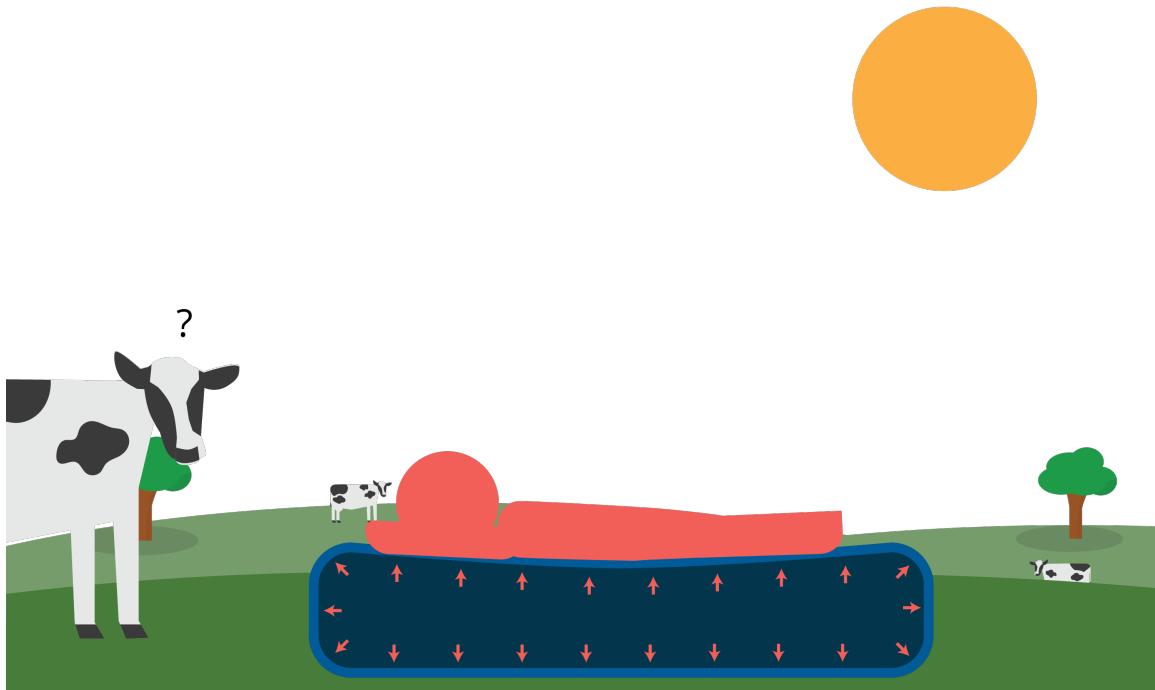


Figure 1.4: The air mattress inflates as the sun reaches its peak height in the sky.

If you have constant pressure and a constant number of molecules, the volume of the gas is proportional to the temperature in Kelvin:

$$V \propto T$$

This is known as Charles's Law.

Exercise 1 Temperature and Volume

Working Space

At dawn, the air inside mattress at dawn has a volume of 1000 liters and a temperature of 12 degrees Celsius.

At noon, that same air has a temperature of 28 degrees Celsius. The pressure on the gas has not changed at all.

What is the volume of the gas at noon?

Answer on Page 11

Note: Volume and temperature are only proportional as long as the substance is a gas. We will talk about liquids and solids soon.

1.2 Pressure and Volume

As you increase the pressure on a gas, the molecules will get pushed closer together, and the volume will decrease.

For example, if you put the cap on an empty plastic bottle and squeeze it. As you put the gas inside the bottle under pressure, its volume will decrease.

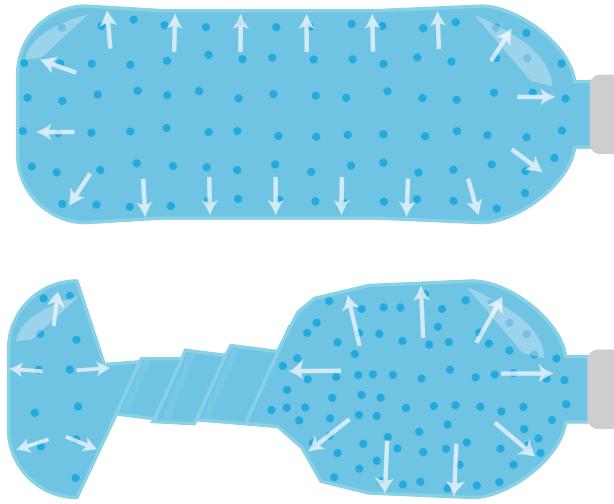


Figure 1.5: The water bottle shrinks as it is put under pressure.

If you keep the number of molecules and the temperature constant, the pressure of the gas and its volume are inversely proportional:

$$P \propto \frac{1}{V}$$

This is known as Boyle's Law. "But," you say with disbelief, "if I increase the pressure on my empty water bottle from 5 kPa to 10 kPa, the volume inside won't decrease by half!"

Don't forget that the air inside the bottle is under 101 kPa of atmospheric pressure before you even start to squeeze it.

Exercise 2 Temperature and Volume**Working Space**

At an altitude where the atmospheric pressure is 100 kPa, you seal air in a 1 liter water bottle.

Squeezing the water bottle, you raise the internal pressure by 20 kPa. What is the volume inside the bottle now?

Answer on Page 11**1.3 The Ideal Gas Law**

You are gradually getting an intuition for the relationship between the number of molecules, the volume, the pressure, and the temperature of a gas. We can actually bring these together in one handy equation.

Ideal Gas Law

$$PV = nRT$$

where:

- P is the pressure in pascals
- V is the volume in cubic meters
- n is the number of molecules in moles
- R is the molar gas constant: 8.31446
- T is the temperature in Kelvin

(You can remember this as the "Pivnert.")

From the name, you might predict the following: The Ideal Gas Law is not 100% accurate. However, for most purposes, it works remarkably well.

Notice that the ideal gas law says nothing about what kind of gas it is; it works regardless.

Exercise 3 Ideal Gas Law

Working Space

You have a cylinder containing O₂. The chamber inside has a radius of 12 cm and a length of 50 cm. The temperature inside the cylinder is 20 degrees Celsius. The pressure inside the tank is 600 kPa.

How many moles of O₂ are inside?

Answer on Page 11

1.4 Molecules Like To Stay Close to Each Other

When two molecules get close to each other a few things can happen:

- They can undergo a chemical reaction: electrons are exchanged or shared and a different molecule or molecules come into existence. This is the realm of chemistry, and we won't go into it in this course.
- One or both of them have so much kinetic energy that they just pass each other or bounce off each other. This is what happens in a gas.
- The two molecules can "stick" together. This is what happens in a liquid or a solid.

Why do they stick together if they aren't combined in a chemical reaction?

First, they don't get *too close*. If they get too close, their electron clouds repel each other with a strong force. This is what happens in a gas when two molecules bounce off of each other.

However, if the molecules are quite close to each other, there are forces that will attract them toward each other. These intermolecular forces are beyond the scope of this course, but they called Van der Waals forces and hydrogen bonds. The strength of these forces vary based on the two molecules involved.

Which is why some of the matter around you is in gas form (molecules that don't stick

together at the temperature and pressure you are living in because they have weak attractive forces) and some is non-gas (gangs of molecules with stronger attraction that makes them clump together as a liquid or a solid at that same temperature and pressure).

However, what if we change the temperature and pressure? We can change if and how the molecules clump together. This is known a *phase change*; We will cover it soon.

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APPENDIX A

Answers to Exercises

Answer to Exercise 2 (on page 7)

First, we convert the temperatures into Kelvin:

- Dawn: $12 + 273.15 = 285.15$
- Noon: $28 + 273.15 = 301.15$

So, the temperature T has increased by a factor of $\frac{301.15}{285.15} \approx 1.056$.

Thus, the volume of the air mattress has also increased by a factor of 1.056.

This means that the air mattress that had a volume of 1000 liters at dawn will have a volume 1056 liters at noon.

Answer to Exercise 2 (on page 7)

What is the pressure in kPa?

- Before squeezing: 100 kPa
- While squeezing: 120 kPa

So, the pressure P has increased by a factor of $\frac{120}{100} = 1.2$

$$1/1.2 \approx 0.833$$

The air in the bottle had a volume of 1 liter before squeezing, so it has a volume of 833 milliliters while being squeezed.

Answer to Exercise 3 (on page 8)

First, let's convert the known values to the right unit:

- Radius = 0.12 m
- Length = 0.5 m
- $T = 20 + 273.15 = 293.15$ degrees Kelvin
- $P = 600 \text{ kPa} = 600,000 \text{ Pa}$

The volume of the cylindrical chamber is $V = \pi r^2 h = \pi(0.12)^2 0(0.5) \approx 0.0226$.

The Ideal Gas Law tell us that $PV = nRT$. We are solving for n .

$$n = \frac{PV}{RT} = \frac{(600,000)(0.0226)}{(8.31446)(293.15)} \approx 5.68 \text{ moles of O}_2$$



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