Chapter 1

Optimization

Optimization is a branch of mathematics that involves finding the best solution from all feasible solutions. In the field of operations research, optimization plays a crucial role. Whether it is minimizing costs, maximizing profits, or reducing the time taken to perform a task, optimization techniques are employed to make decisions effectively and efficiently.

1.1 Optimization Problems

An optimization problem consists of maximizing or minimizing a real function by systematically choosing the values of real or integer variables from within an allowed set. This function is known as the objective function.

A standard form of an optimization problem is:

$$\underset{x}{\text{minimize}} \quad f(x) \quad \text{subject to} \quad g_i(x) \leq 0, ; i = 1, \ldots, m \quad h_j(x) = 0, ; j = 1, \ldots, p$$

where

- $f(x)$ is the objective function,
- $g_i(x) \leq 0$ are the inequality constraints,
- $h_j(x) = 0$ are the equality constraints.

1.2 Types of Optimization Problems

There are several different types of optimization problems, such as:

- **Linear Programming:** The objective function and the constraints are all linear.
- **Integer Programming:** The solution space is restricted to integer values.
- **Nonlinear Programming:** The objective function and/or the constraints are nonlinear.
- **Stochastic Programming:** The objective function and/or constraints involve random variables.

These problems are solved using different techniques and algorithms, many of which are a subject of active research.

1.3 Applications

Optimization techniques have a wide variety of applications in many fields, such as economics, engineering, transportation, and scheduling problems.

FIXME expand this chapter with examples of calculus optimization using min max from previous chapter

1.4 Calculus and Optimization

Calculus plays a key role in optimization because maximum and minimum values of a function often occur at critical points. A critical point occurs when the derivative of a function is equal to zero or does not exist. For a function defined on an interval, the maximum or minimum value may occur at a critical point or at an endpoint of the interval. This idea provides a systematic method for solving optimization problems.

In order to solve optimization problems, two key components are important:

- **The objective function**, which is the quantity that we want to maximize or minimize. Common objective functions include volume, area, and cost.
- **The constraint**, which is the condition that limits the possible values of the objective function. Fixed values of volume, perimeter, time, or weight are common constraints. When solving examples, a constraint may be worded like "The weight must be 50 kg" or "We have 30 years to maximize total profit." Whatever value is fixed in the problem statement is the constraint.

1.5 General Strategy for Solving Optimization Problems

Most calculus optimization problems follow these steps:

1. Identify the quantity to be optimized.
2. Write an equation for the objective function.
3. Use the given constraints to rewrite the function using one variable.
4. Determine the appropriate domain.
5. Find the derivative of the function.

6. Find critical points by setting the derivative equal to zero.
7. Interpret the result in the context of the problem.

When solving optimization problems, some common mistakes include forgetting to define variables, not using the constraints of the problem appropriately, misinterpreting or failing to interpret the final answer, and giving answers without appropriate units.

1.6 Using Python to Visualize Your Optimization Problem

You may be familiar with the use of Python to find the derivative of functions. We are going to use Python to supplement your knowledge of Optimization problems! Below is a script that can be used to visualize what you are solving for including maximums, minimums, and

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This is a draft chapter from the Kontinua Project. Please see our website (`https://kontinua.org/`) for more details.

Appendix A

Answers to Exercises

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