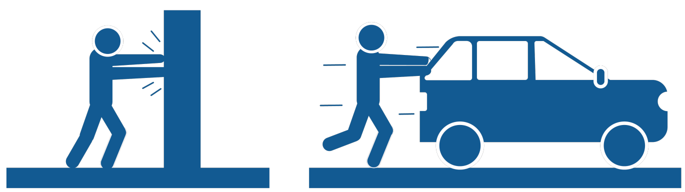
Chapter 1

Work and Energy

In this chapter, we are going to talk about how engineers define work and energy. It frequently takes force to get work done. Let's start with thinking about the relationship between force and energy. As we learned earlier, Force is measured in newtons, and one newton is equal to the force necessary to accelerate one kilogram at a rate of $1m/s^2$.

When you lean on a wall, you are exerting a force on the wall, but you aren't doing any work. On the other hand, if you push a car for a mile, you are clearly doing work. Work, to an engineer, is the force you apply to something, as well as the distance that something moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force applied over one meter.



For example, if you push a car with a force of 10 newtons for 12 meters, you have done 120 joules of work.

Formula for Work

$$W = F \cdot d$$

where W is the work in joules, F is the *force* in newtons, and d is the distance in meters.

If the force is not in the same direction as the distance, we can use the cosine of the angle between the force and the distance:

$$W = F \cdot d \cdot \cos(\theta)$$

where θ is the angle between the force and the distance.

The work-energy theorem (or work-energy principle) states that the net work done on an object is equal to the **change in its kinetic energy**. In other words, if you do work on an object, you are changing its kinetic energy. This is derived from Newton's second law of motion, covered in the previous chapter.

$$W = \Delta E = \Delta KE (\text{with units of Joules (J) or Newton-meters (Nm)})$$

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs (British Thermal Unit), and calories.

Let's go over a few different forms that energy can take.

1.1 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat
- Electricity
- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

1.1.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

1.1.2 Electricity

Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let's say your lightbulb consumes 60 watts of electricity, and you leave it on for 24 hours. We would say that you have consumed 1.44 kilowatt hours, or 3,600,000 joules.

1.1.3 Chemical Energy

As mentioned earlier, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosynthesis is a process that consumes energy. Therefore, the sugar molecule has more chemical energy than the carbon dioxide and water molecules that were used in its creation.

In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

Here is where things get tricky: People who work with food got tired of saying "kilocalories", so they just started using "Calorie" to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, "Calorie" probably means kilocalorie.

1.1.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the breaks, the energy from the motion of the car will be converted into heat in the breaks and under the tires.

How much energy does the car have?

$$E = \frac{1}{2}mv^2$$

Formula for Kinetic Energy

$$E = \frac{1}{2}mv^2$$

where E is the energy in joules, m is the mass in kilograms, and v is the speed in meters per second.

1.1.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

$$E = mgh$$

a rate of $9.8m/s^2$.

Formula for Gravitational Potential Energy

The formula for gravitational potentional energy is

$$E = mgh$$

where E is the energy in joules, m is the mass of the object you lifted, g is acceleration due to gravity, and h is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects in free-fall near Earth's surface accelerate at $9.8m/s^2$.

There are other kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy.

1.2 Conservation of Energy

The first law of thermodynamics says "Energy is neither created nor destroyed."

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. However, the total amount of energy in a closed system stays constant.

Exercise 1 The Energy of Falling

Working Space

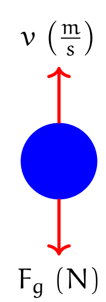
A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

Answer on Page 11

1.3 Work and Kinetic Energy

As stated above, the work-energy theorem tells us that the change in an object's kinetic energy is equal to the work done on that object. For now, we will only consider examples where the force and the direction of motion are parallel or perpendicular. When you learn about vectors, we will expand this to include forces that are skew to the direction of motion.

Consider what happens when you toss a ball in the air: once the ball leaves your hand, the only force acting on it is gravity. Initially, the ball is moving upwards while gravity points downwards:



Intuitively, we know that the ball will slow down (lose kinetic energy) as it moves upwards:

$$\Delta KE < 0$$

Since $W = \Delta KE$, we also know that gravity must be doing *negative work*. Whenever the direction of the force is opposite the direction of the motion, the work done by that force is negative.

Example: if the ball has a mass of 0.5 kg, how much kinetic energy does it lose as it moves upwards by 1 m?

Solution: The force acting on the ball is its weight, $F_w = mg$, and we will designate this as negative since weight points downwards. Using the work-energy theorem,

$$\Delta KE = F \cdot d = (mg) \cdot d = (0.5kg)\left(-9.8\frac{m}{s^2}\right)(1m) = -4.9J$$

Therefore, the ball loses 4.9 joules of kinetic energy for every 1 meter it moves upwards (the fact it is *losing kinetic energy* is represented by the result being negative).

Exercise 2 How far will you slide?

You are playing softball and have to slide into home. If you sprint at a maximum of 10 m/s and the force of friction between you and the ground is 0.3 times your weight, how far from the base can you start your slide and still reach home?

Working Space

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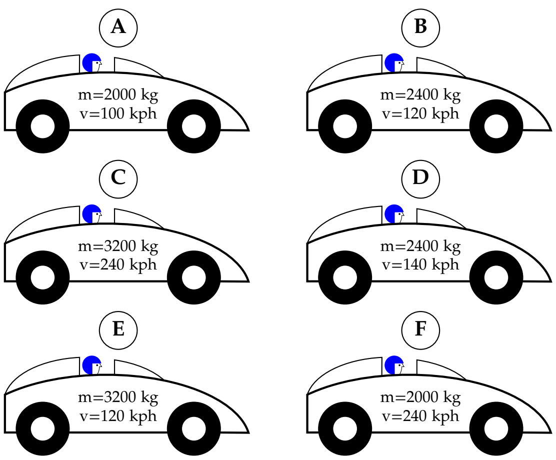
Exercise 3 Ranking Stopping Force

In drag racing, cars can reach speeds of 150 miles per hour (approximately 240 kilometers per hour). In order to be able to stop quickly and safely, drag racing cars are built with parachutes that deploy at the end of the race. Consider a drag race where cars of different masses reach different maximum speeds. There is 100 meters between the finish line and the fence surrounding the race track. If all the race cars deploy their parachutes at the finish line while going their maximum speed, rank the force needed from the parachute to stop each car in the required distance from least to greatest:



Working Space

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1.3.1 Forces that do no work

If the object you are pushing doesn't move. or the applied force is perpendicular to the direction of motion, that force does no work. Let's look at a few examples:

Pushing Against an Immobile Object

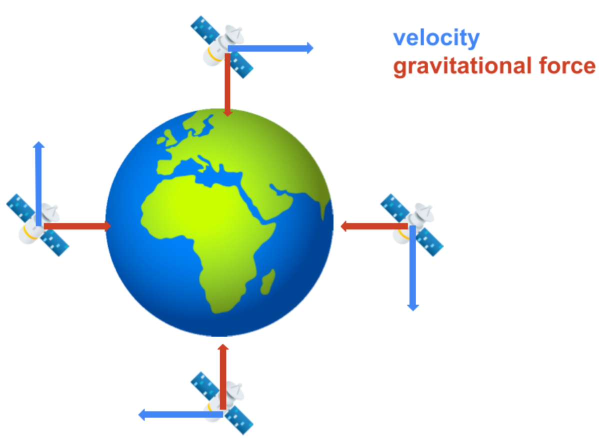
At the beginning of the chapter, we said that when you push on a wall, you don't do any work. Why is this? Well, if the wall is a good wall (that is, strong enough to not be pushed over by a person), the wall won't move while you push on it. This means the distance over which your push is applied is zero, and therefore the work done ($F \cdot d = F \cdot 0 = 0$) is zero joules.

Walking Across a Room with a Book

Imagine holding a book flat on your hands and walking at a constant velocity. Your hand is applying an upwards force to the book, but the book is moving horizontally. This means the force and direction of motion are *perpendicular*. Recall from the beginning of the chapter than if the force and distance are not parallel, then the work is given by $W = F \cdot d\cos(\theta)$. (When the vectors are parallel, $\theta = 0$ and $\cos(\theta) = 1$, while when the vectors point in opposite directions, $\theta = 180°$ and $\cos(\theta) = -1$.) When the vectors are perpendicular, then $\theta = 90°$ and $\cos(\theta) = 0$. Therefore, $W = 0$ as well and the upward force of your hands does no net work.

Circular Motion

We will discuss circular motion further in a subsequent chapter. For now, know that constant-speed circular motion is caused by a constant-magnitude force that always points to the center of the circle the object is moving in. For example, you can take a weight on the end of a string and spin it. The tension in the string spins the weight, and the string always points from the object to your hand (the center of the weight's circular path). For a satellite, that force is gravitational attraction to the Earth.



As a result, the force changes the *direction*, but not the *magnitude* of the satellite's velocity. Let's re-examine the equation for kinetic energy:

$$\mathrm{KE} = \frac{1}{2}mv^2$$

Since the velocity is squared, the direction of motion doesn't affect the kinetic energy (a ball moving at 5 m/s upwards has the same kinetic energy as if the ball were moving at 5 m/s downwards). So, a force that causes circular motion doesn't change a circling object's kinetic energy, and therefore does no work (as expected when force and direction of motion are perpendicular)!

1.4 Efficiency

Although energy is always conserved as it moves through different forms, scientists aren't always that good at controlling it.

In terms of an equation, efficiency is the ratio of the useful energy output to the total energy input. It is usually expressed as a percentage.

Formula for Efficiency

$$\text{Efficiency} = \frac{\text{Useful Energy Output}}{\text{Total Energy Input}} \times 100\%$$

where the useful energy output is the energy that is actually used to do work or complete a task, and the total energy input is the total energy consumed by the system.

A machine is considered 100% efficient only if all the input work is converted into useful output work, with no energy lost to heat, friction, or sound. 100% efficient process don't exist in real-life: every process loses some useful energy to heat.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

In general, if you are trying to increase efficiency in any system, the solution is usually easy to identify by the heat that is produced. Reduce the heat, increase the efficiency.

Light bulbs are an interesting case. To get the same amount of light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just “inefficient at making light” — it is actually counteracting the air conditioner!

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 5)

At the top of the ladder, the cannonball has $(9.8)(5)(3) = 147$ joules of potential energy.

At the bottom, the kinetic energy $\frac{1}{2}(5)v^2$ must be equal to 147 joules. So $v^2 = \frac{294}{5}$. This means it is going about 7.7 meters per second.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is negligible.)

Answer to Exercise 2 (on page 6)

$$F_f \cdot d = \Delta KE = KE_f - KE_i$$

You'll reach the maximum distance you can slide when you stop moving, so we will use a final velocity of zero, which means a final kinetic energy of zero:

$$F_f \cdot d = -KE_i = \frac{1}{2}mv^2$$

Since the force of friction is 0.3 times your weight, we know that:

$$F_f = 0.3F_w = 0.3mg$$

Substituting and canceling the mass:

$$(0.3mg) \cdot d = \frac{1}{2}mv^2$$

$$0.3g \cdot d = \frac{1}{2}v^2$$

Since we know g and v, we can solve for d:

$$d = \frac{v^2}{0.6g} = \frac{\left(10\frac{m}{s}\right)^2}{0.6\left(9.8\frac{m}{s^2}\right)} \approx 1.7m$$

So, if you want to reach home base, you should start your slide no more than 1.7 m from home.

Answer to Exercise 3 (on page 7)

Since all the cars need to stop in the same distance, the cars with the greatest kinetic energy will take the most force to stop. Calculating the kinetic energies (we won't change the units from kilometers per hour to meters per second, since we're just comparing the values):

Car	Mass [kg]	Max speed [kph]	KE [kg (kph)^2]
A	2000	100	1×10^7
B	2400	120	1.728×10^7
C	3200	240	9.216×10^7
D	2400	140	2.352×10^7
E	3200	120	2.304×10^7
F	2000	240	5.67×10^7

The correct ranking is A, B, E, D, F, C.

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