Chapter 1

Circular Motion

We can recall the motion of 2D objects, such as a projectile launched or a car driving along a road. They have components of linear motion, such as position, velocity, acceleration, and force.

But that only works for objects moving in a straight line. What about objects moving in a circle?

1.1 Introduction to Uniform Circular Motion

Recall that in linear motion, You may be surprised to know that we can use very similar equations to describe circular motion, provided you use the correct variables.

The angular displacement, θ, is the angle in *radians* that the object has traveled. The angular velocity, ω, is the rate of change of the angular displacement, and the angular acceleration, α, is the rate of change of the angular velocity.

Linear motion	Rotational motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Table 1.1: Kinematic equations for linear and rotational motion.

Linear velocity, v, is related to angular velocity by the equation:

$$v = r\omega$$

The *Period*, T, is the time it takes to complete one full rotation. The frequency, f, is the number of rotations per second. The two are related by:

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

$$f = \frac{1}{T}$$

The *centripetal force*, usually labelled differently (such as tension or gravity), is given by the equation:

$$F = mr\omega^2 = \frac{mv^2}{r}$$

The *angular velocity*, ω is rotations with respect to time. It can be defined in the following ways:

$$\omega_{\text{inst}} = \frac{d\theta}{dt}, \qquad \omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Angular acceleration:

$$\alpha = \frac{d\omega}{dt}, \qquad \alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$$

Note that all of the kinematic formulas still work as long as you stay in the same relative frame of reference (rotational or linear). You cannot "mix and match" the two sets of equations.

You may be asking, "If the object is moving at a constant speed, why is there acceleration?" Recall that acceleration is a vector quantity, meaning it has both magnitude and direction. Even though the speed (the magnitude of the velocity) is constant, the direction of the velocity vector is always changing. Since acceleration is the rate of change of velocity, any change in direction means there is acceleration. This acceleration is always directed toward the center of the circle, which is why it is called centripetal (center-seeking) acceleration.

Now that we've established the basic relationships for circular motion, let's apply them to a practical example.

1.2 The Flying Billiard Ball

Let's say you tie a 0.16 kg billiard ball to a long string and begin to swing it around in a circle above your head. The string, perpendicular, is 3 meters long, and the ball returns to where it started every 4 seconds. We will assume the ball moves at a constant speed. If you start your stopwatch as the ball crosses the x-axis, the coordinates of the position (x, y, z) of the ball at any time t given by:

$$p(t) = \left[3\cos\left(\frac{2\pi}{4}t\right), 3\sin\left(\frac{2\pi}{4}t\right), 2\right]$$

(This assumes that the ball would be going counter-clockwise if viewed from above. The spot you are standing on is considered the origin $[0, 0, 0]$.)

Notice that the height is a constant — 2 meters in this case. That isn't very interesting, so we will talk just about the first two components. Figure 1.1 shows a visual of the situation:

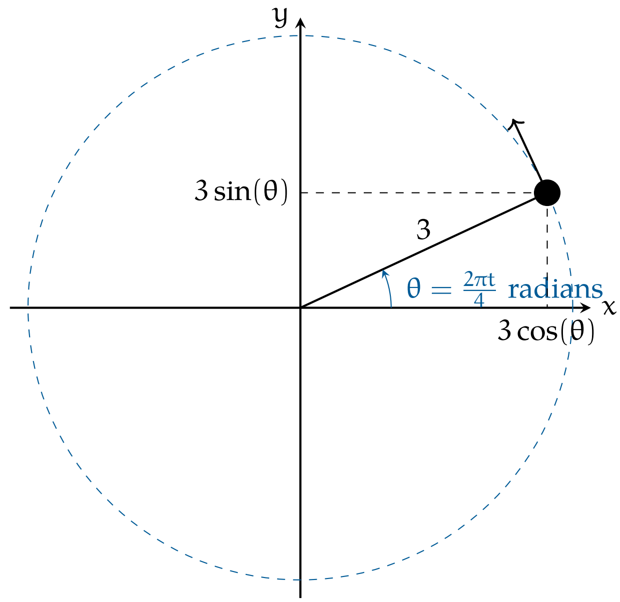


Figure 1.1: A diagram of our ball and string scenario.

In this case, the radius, r, is 3 meters. The period, T, is 4 seconds. In general, we say that circular motion is given by:

$$p(t) = \left[r\cos\frac{2\pi t}{T}, r\cos\frac{2\pi t}{T} \right]$$

A common question is "How fast is it turning right now?" If you divide the 2π radians of a circle by the 4 seconds it takes, you get the answer "About 1.57 radians per second." This is known as *angular velocity* and we typically represent it with the lowercase Omega: ω. (Yes, it looks a lot like a "w".) To be precise, in our example, the angular velocity is $\omega = \frac{\pi}{2}$. Note that in this scenario, the angular velocity and linear *speed* are constant. However, the *velocity* vector is not constant; as we will see in the next section, the direction of the velocity vector is always changing.

Notice that this is different from the question "How fast is it going? (referring to *linear velocity*)" This ball is traveling the circumference of $6\pi \approx 18.85$ meters every 4 seconds. This means the speed of the ball is about 4.71 meters per second.

It is very important to distinguish between angular velocity and linear velocity. Angular velocity is how fast the angle is changing and is referred to by ω, while linear velocity, v, is how fast the object is moving along its path. While linear velocity has a constant speed, it has always changing direction. The angular velocity is constant, but the linear velocity is not.

1.3 Velocity

The velocity of the ball is a vector, and we can find that vector by differentiating each component of the position vector.

For any constants a and b:

Expression $f(x)$	Derivative $f'(x)$
$a \sin bt$	$ab \cos bt$
$a \cos bt$	$-ab \sin bt$

Thus, in our example, the velocity of the ball at any time t is given by:

$$v(t) = \left[-\frac{3(2\pi)}{4} \sin \frac{2\pi t}{4}, \frac{3(2\pi)}{4} \cos \frac{2\pi t}{4}, 0\right]$$

Notice that the velocity vector is perpendicular to the position vector. It has a constant magnitude.

In general, an object traveling in a circle at a constant speed has the velocity vector:

$$v(t) = [-r\omega \sin \omega t, r\omega \cos \omega t]$$

where $t = 0$ is the time that it crosses the x axis. If ω is negative, that means the motion would be clockwise when viewed from above.

The magnitude of the velocity vector is $r\omega$. See Figure 1.4a for a visual of the velocity vector. Note that the z-coordinate is constant, so its derivative is zero, and it is eliminated from our vector for simplification.

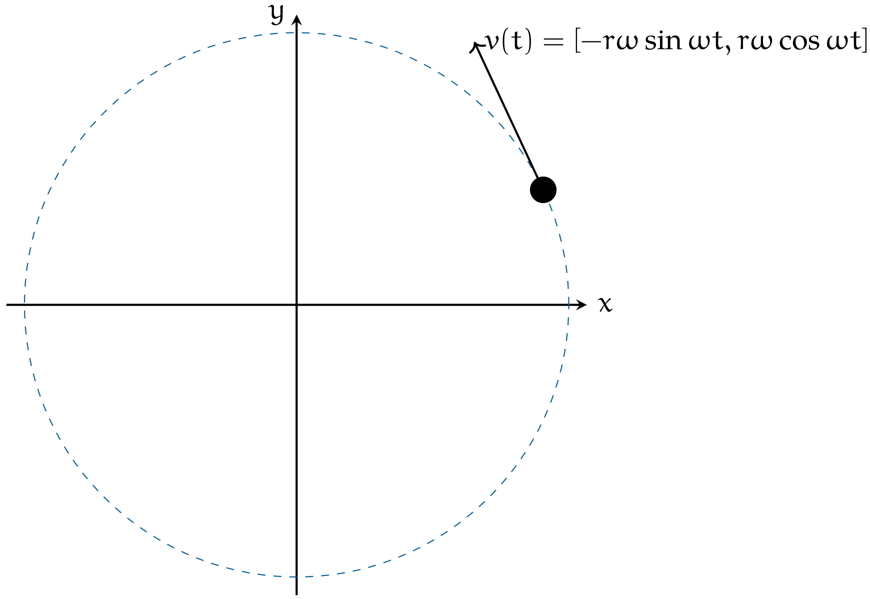


Figure 1.2: Velocity vector of the ball in circular motion.

Exercise 1 A Particle in Motion

Working Space

[This question was originally presented as two multiple-choice problems on the 2012 AP Physics C exam.] The x and y coordinates of a particle as it moves in a circle are given by:

$$x = 5\cos(3t) y = 5\sin(3t)$$

What is the radius of the particle's circular path? What is the particle's *speed*? Based on your answers, how long does it take the particle to complete the circular path?

Answer on Page 23

1.4 Acceleration

We can get the (linear) acceleration by differentiating the components of the velocity vector.

$$a(t) = \left[-r\omega^2\cos\omega t, -r\omega^2\sin\omega t\right] = -r\omega^2\left[\cos\omega t, \sin\omega t\right]$$

Notice that the acceleration vector points **toward the center** of the circle it is traveling on. That is, when an object is traveling on a circle at a constant (uniform) speed (notice that coefficients r and ω are constant), its only acceleration is toward the center of the circle. The acceleration does not come from the motion itself, it comes from the constantly changing direction. This is known as *centripetal acceleration*.

The magnitude of the acceleration vector is $a_c = r\omega^2$, or more commonly, $a_c = \frac{v^2}{r}$. This gives us the common calculation for whatever force acts as the centripetal force: $F_c = ma_c = \frac{mv^2}{r}$.[1]

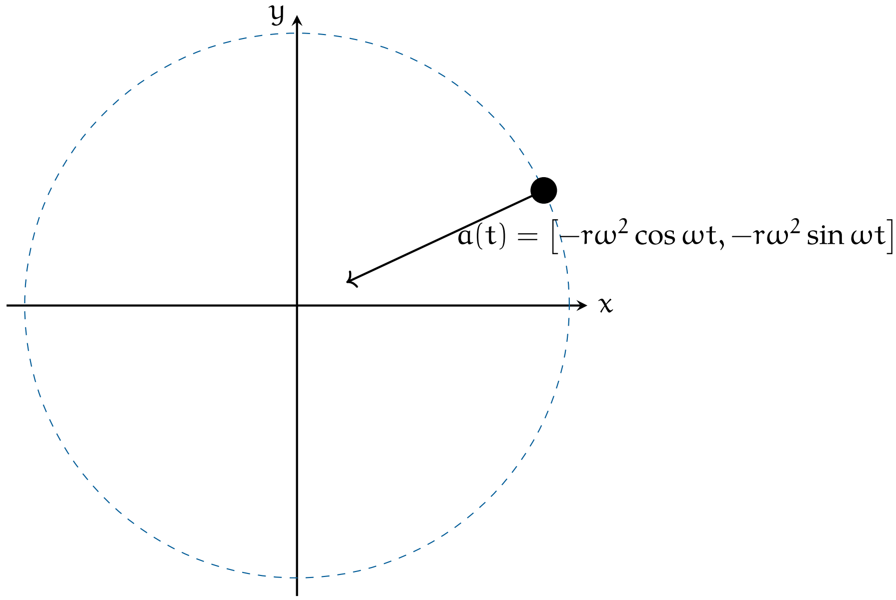
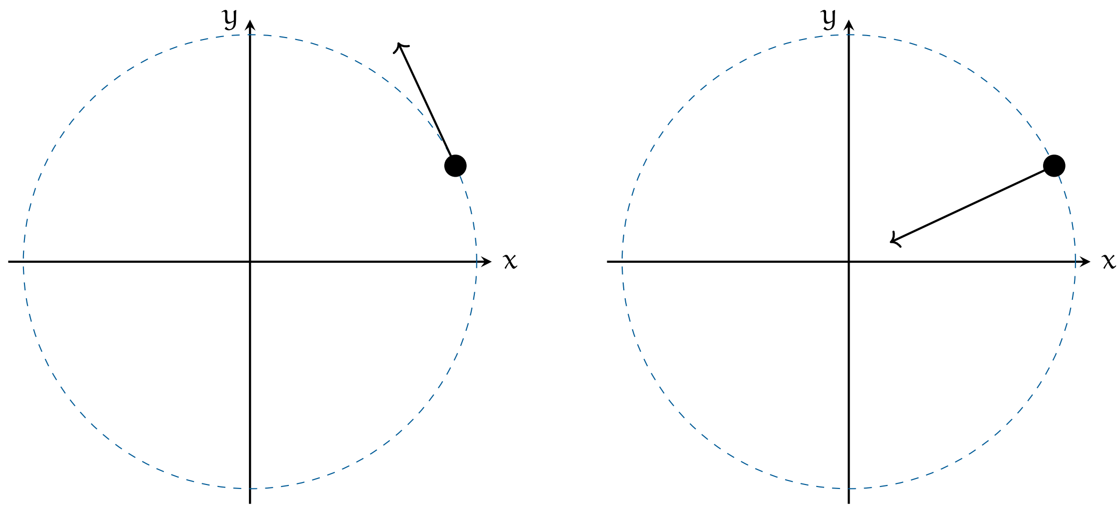


Figure 1.3: Acceleration vector of the particle in circular motion.

Let's compare the velocity and acceleration paths from figures 1.4a and 1.4b. Notice that the velocity vector is always tangent to the circle, while the acceleration vector is always pointing toward the center of the circle.

[1]You will see centripetal acceleration noted as a_c or a_r, standing for centripetal or radial, respectively. Note that both refer to the same acceleration.



(a) Velocity vector. (b) Acceleration vector.

Figure 1.4: Uniform circular motion: velocity (tangent) and acceleration (centripetal).

1.5 Centripetal force

What would happen if there were no force pulling the ball toward the center of the circle? It would fly off in a straight line, according to Newton's First Law.

The force you are exerting on the string is what causes it to accelerate toward the center of the circle. We call this the *centripetal force*. It makes sense that the centripetal force and centripetal acceleration are in the same direction, as they are proportional.

Recall that $F = ma$. The magnitude of the acceleration is $r\omega^2 = 3\left(\frac{2\pi}{4}\right)^2 \approx 7.4\text{m/s}^2$. The mass of the ball is 0.16 kg. So, the force pulling against your hand is about 1.18 newtons.

The general rule is that when something is traveling in a circle at a constant speed, the centripetal force needed to keep it traveling in a circle is:

$$F = mr\omega^2$$

If you know the radius r and the speed v of the object, here is the rule:

$$F = \frac{mv^2}{r}$$

We didn't introduce centripetal force as a type of force because it isn't its own force, like friction or gravity. Rather, calling a force "centripetal" tells us what the force is doing. A centripetal force is any force that causes circular motion, a force which acts in the *radial* direction. For a satellite, the centripetal force is gravity. In the opening example with the billiard ball, the centripetal force is the tension in the string.

Example: A child is sitting on a merry-go-round as it spins. What provides the centripetal force?

Solution: In this case, the child is rotating horizontally, so the centripetal force must also be horizontal. Therefore, the centripetal force isn't the child's weight or the normal force. It is the *friction* between the child and the merry-go-round that keeps the child turning. Here is a free body diagram:

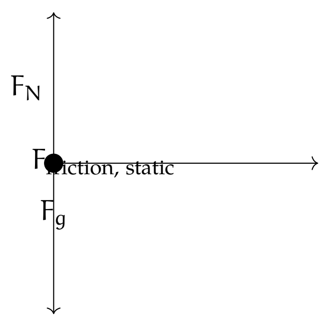


Figure 1.5: The static friction is what causes the centripetal force.

Example: If the child is 1.2 meters from the center of the merry-go-round and it spins at 0.25 rotations/second, what is the coefficient of friction between the child and the merry-go-round?

Solution: Using our FBD and applying Newton's Second Law, we see that:

$$F_N - F_g = 0 \rightarrow F_N = m_{child} g$$

$$F_f = mr\omega^2 = \mu F_N = \mu mg$$

Since the child isn't accelerating vertically, we know that the normal force equals the child's weight. The horizontal force, friction, must equal the mass of the child times the acceleration. Additionally, from the vertical component, we know the friction is equal to μmg, since the normal force is equal to the child's weight. Looking at the second equation, we see that:

$$mr\omega^2 = \mu mg$$

We were given ω in rotations per second, but we need radians per second:

$$\frac{0.25\text{rotation}}{1\text{second}} = \frac{1\text{rotation}}{4\text{seconds}} = \frac{2\pi\text{radians}}{4\text{seconds}} = \frac{\pi}{2}\frac{rad}{s}$$

We can divide m from both sides of $mr\omega^2 = \mu mg$ (notice: we don't need to know the child's mass to determine the coefficient of friction!):

$$r\omega^2 = \mu g \rightarrow \mu = \frac{r\omega^2}{g} = \frac{(1.2\text{m})\left(\frac{\pi}{2\text{s}}\right)^2}{9.8\frac{\text{m}}{\text{s}^2}} = 0.302$$

Therefore, the coefficient of friction between the child and the merry-go-round is 0.302.

1.5.1 Banked Turns

Have you ever been driving on the highway and taken a turn where the road is at an angle? Or maybe you've seen a sign like Figure 1.6 on the road as you take a turn.

A banked curve is designed so that a car can safely turn without slipping, even in the rain, if the driver does not exceed the indicated speed. Engineers choose an angle such that the bank provides sufficient force to turn the car without friction. Let's look at a free body diagram for a car taking a banked turn (see figure 1.7).

Figure 1.6: A banked turn sign on the highway.

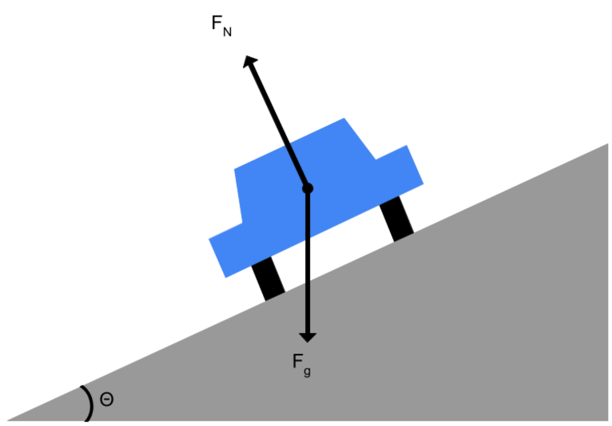


Figure 1.7: If there is no friction due to rain, then the only two forces acting on the car are gravity and the normal force.

In the past, we've split the vector for the force of gravity into components that are parallel and perpendicular to the ramp. This time, we will split the normal force into x and y components (see figure 1.8).

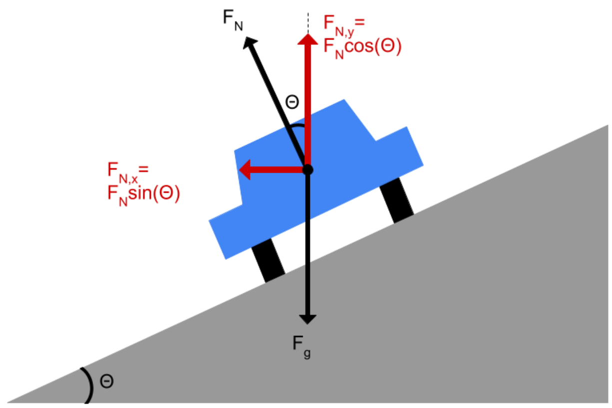


Figure 1.8: Geometrically, the x component of the normal force is given by $F_N \cos(\theta)$ and the y component by $F_N \sin(\theta)$.

Assuming the radius of the turn is 20 m, at what angle should the engineers build the banked turn? (Let's also assume the maximum speed is 25 mph, like the traffic sign above.) First, let's apply Newton's Second Law in the x and y directions:

$$(1)\ F_N \cos(\theta) - F_g = 0$$

$$(2)\ F_N \sin(\theta) = \frac{mv^2}{r}$$

We know that $F_g = mg$. From this and equation (1) we see that:

$$F_N = \frac{mg}{\cos(\theta)}$$

Substituting for F_N into equation (2):

$$\left(\frac{mg}{\cos(\theta)}\right)\sin(\theta) = \frac{mv^2}{r}$$

The mass can be divided from both sides, and $\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$:

$$g\tan(\theta) = \frac{v^2}{r}$$

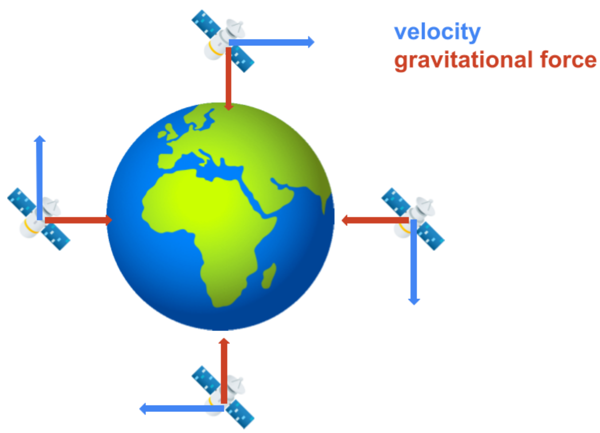
Solving for θ and substituting for the speed ($25\ \text{mph} \approx 11.18\frac{m}{s}$) and radius:

$$\theta = \arctan\left(\frac{v^2}{gr}\right) = \arctan\left(\frac{\left(11.18\frac{m}{s}\right)^2}{\left(9.8\frac{m}{s^2}\right)(20m)}\right)$$

$$\theta = \arctan(0.6377) \approx 32.53^\circ$$

1.6 Modeling Circular Motion

What causes circular motion is a constant force perpendicular to the motion. Consider a satellite circling the Earth. The only force acting on the satellite is gravity, yet the satellite does not fall to the Earth. Why? Let's look at the relative direction of motion and gravity for some different positions of the satellite (we'll assume this satellite is moving clockwise from our point of view):



No matter what position the satellite is in, the velocity and gravity vectors are perpendicular.

Exercise 2 Circular Motion

Just as your car rolls onto a circular track with a radius of 200 m, you realize your 0.4 kg cup of coffee is on the slippery dashboard of your car. While driving 120 km/hour, you hold the cup to keep it from sliding.

What is the maximum amount of force you would need to use? (The friction of the dashboard helps you, but the max is when the friction is zero.)

Working Space

Answer on Page 23

Exercise 3 Twirling a Whistle

The lifeguard at a local pool is twirling their whistle horizontally. You wonder if the lifeguard could spin the whistle fast enough to break the string. The string the whistle is attached to can hold a maximum mass of 20 kg before breaking. If the lifeguard's whistle string is 0.35 m long and the average whistle has a mass of 165 grams, what is the maximum tangential speed the lifeguard can spin the whistle? How many rotations per second would the whistle be spinning at? Based on this, do you think the lifeguard is capable of spinning the whistle fast enough to break the string?

Working Space

Answer on Page 24

Exercise 4 The Gravitron

The Gravitron is a carnival ride where riders "stick" to the wall of a spinning cylinder as the floor beneath them drops away. A video explanation is given here: `https://www.youtube.com/watch?v=ifAY5tbYDmQ`. Draw a free body diagram of a rider. If the coefficient of friction between a rider and the wall is 0.32 and the ride is 10 meters across, what angular velocity must the ride reach before the floor drops away?

Working Space

Answer on Page 25

1.7 Flying Pigs and Tension

Take a look at the following two videos that involve flying pigs!

1. Flying Pig: Uniform Circular Motion and Centripetal Force
2. flying pig - circular motion [workthrough demo]

This involves a combination of circular motion and tension. Let's say we have a pig at making angle θ with the vertical and the string of length L, causing the pig to move in a circular path with radius r. We can draw the following two diagrams for our pig, as shown in Figure 1.9.

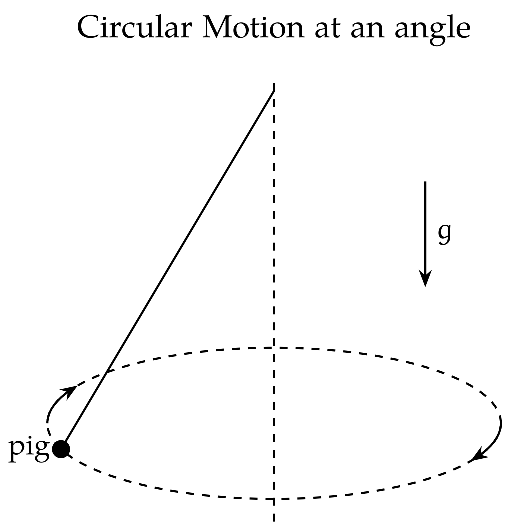


Figure 1.9: The uniform motion of a flying pig attached to a string.

If this is confusing, think of a tetherball from your local playground. It follows the same mechanics!

At any point, we can draw a free body diagram for the pig. Figure 1.10 shows the forces acting on the pig at all points.

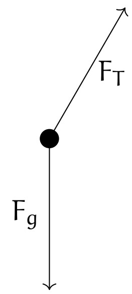


Figure 1.10: The FBD of the flying pig at any point.

To analyze the net forces, the tension force needs to be split into its x and y components. The y component of the tension force must balance out the weight of the pig (as it is not moving vertically), while the x component of the tension force provides the centripetal force to keep the pig moving in a circle.

$$\sum F_y = 0 \rightarrow F_{T_y} - F_g = 0 \rightarrow F_{T_y} = F_g = F_T \cos\theta = mg$$

The centripetal force is found by:

$$F_{T_x} = F_c = F_T \sin\theta = ma_r = \frac{mv^2}{r}$$

Dividing the second equation by the first equation gives:

$$\frac{F_T \sin(\theta)}{F_T \cos(\theta)} = \frac{\frac{mv^2}{r}}{mg} \rightarrow \tan(\theta) = \frac{v^2}{rg}$$

Solving for θ gives us $\theta = \arctan\left(\frac{v^2}{rg}\right)$. We can also solve for v:

$$v = \sqrt{rg\tan\theta} = \sqrt{Lg\sin\theta\tan\theta}$$

Since r can be solved for θ using $r = L\sin\theta$.

Explicitly, we can express our tension force as $F_T = \frac{mg}{\cos(\theta)}$ (coming from our net force in the y-direction).

We can also explicitly solve for ω:

$$
\begin{aligned}
\tan(\theta) &= \frac{v^2}{rg} \\
&= \frac{(r\omega)^2}{rg} = \frac{r\omega^2}{g} \\
\omega^2 &= \frac{g\tan(\theta)}{r} \\
\omega &= \sqrt{\frac{g\tan(\theta)}{r}}
\end{aligned}
$$

What does all of this tell us?

- The vertical component of tension balances the weight of the pig, so there is no vertical motion.
- The horizontal component of tension provides the centripetal force that keeps the pig moving in a circle.
- The faster the pig moves, the larger the angle θ and the greater the tension in the string.
- The tension is always greater than the pig's weight: $F_T = \frac{mg}{\cos\theta}$.
- The angular speed required for a given angle is $\omega = \sqrt{\frac{g\tan\theta}{r}}$. This increases with larger angles, and decreases with a shorter radius.

Exercise 5 Playground Tetherball

A tetherball of mass .5kg is attached to a string of length 2.5m. When the ball is swung around in a horizontal circle, the string makes an angle of 38° with the vertical. If the ball is in uniform circlular motion, what is the speed of the ball?

Working Space

Answer on Page 26

1.8 Non-uniform circular motion

We have talked enough about uniform circular motion, where the speed is constant. However, in many real-world scenarios, the speed is not constant. This is known as *non-uniform circular motion*. In this case, there are two components of acceleration: the centripetal acceleration $a_\perp$ (toward the center of the circle) and the tangential acceleration $a_\parallel$ (along the direction of motion). The tangential acceleration is responsible for changes in speed, while the centripetal acceleration is responsible for changes in direction. We will expand on this in the oscillations chapter, but this acceleration is caused by a *restoring force*, a force that tries to pull the object back toward equilibrium whenever it's displaced. We will introduce this concept here, but go into more detail in the oscillations chapter.

Let's look at a few examples of these concepts in action.

1.8.1 Roller Coaster Loop

Now we can talk about a roller coaster. A roller coaster loop is a great example of circular motion. As the roller coaster cart goes through the loop, it experiences both centripetal and tangential acceleration.

We will analyze the cart on a roller coaster at positions at the top, bottom, and sides of the loop, as shown in Figure 1.11

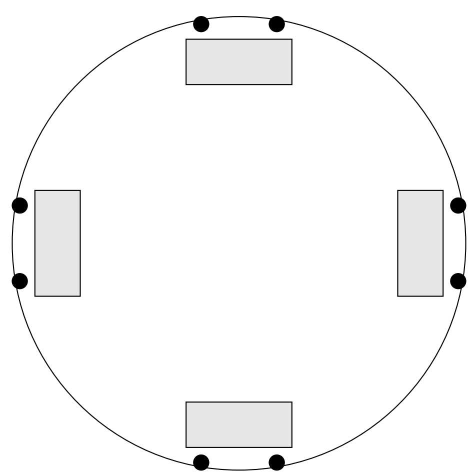


Figure 1.11: Carts at four positions around the loop with wheels on the track.

At the bottom

At the very bottom of the loop, the forces acting on the cart are gravity and the normal force from the track. Let's imagine the cart goes around the circular track infinitely without "exiting" the loop. There must, then, be a force keeping the cart moving in a circle. This force is the centripetal force, which points toward the center of the loop. The only two forces acting on the cart are gravity (downward) and the normal force from the track (upward). We know that a centripetal force must satisfy the formula $F_{net} = F_N - F_g = \frac{mv^2}{r}$. Therefore, the normal force must be greater than the weight of the cart at the bottom of the loop: $F_N = \frac{mv^2}{r} + mg$. This is represented in Figure 1.12.

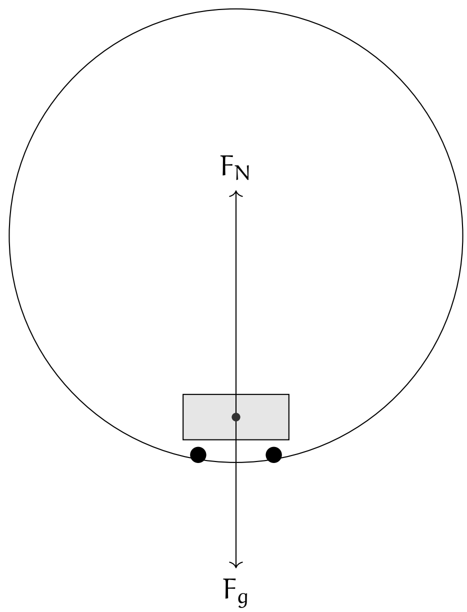


Figure 1.12: The FBD of the roller coaster at the bottom.

At the top

At the very top of the loop, the forces acting on the cart are gravity and the normal force from the track. Both forces point *downward* this time, toward the center of the loop. We can say that $F_{net} = F_N + F_g = \frac{mv^2}{r}$. Solving for the normal force gives: $F_N = \frac{mv^2}{r} - mg$.

Notice that the normal force is less than the weight of the cart at the top of the loop. This means that the centripetal acceleration would have to be greater than or equal mg. This is represented in Figure 1.13.

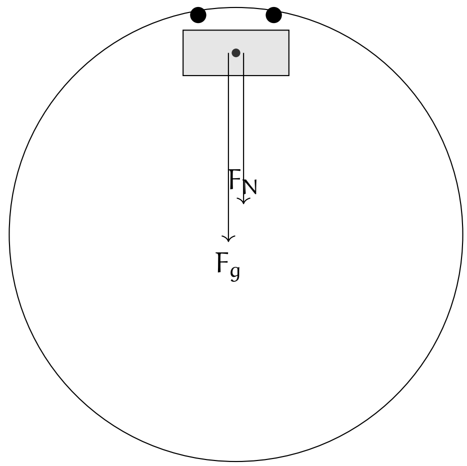


Figure 1.13: The FBD of the roller coaster at the top.

Note that for both the top and bottom of the loop, the minimum speed required to maintain contact with the track is given by $v = \sqrt{rg}$. If the cart goes any slower such that $v < \sqrt{rg}$, it will lose contact with the track, as there will not be enough normal force to provide the centripetal force needed to keep the cart moving in a circle. Think about where this observation comes from!

At the sides

Now at the sides of the loop (say 0 and π if we view the roller coaster as a unit circle), the net force is directed at an angle. The normal force from the track acts horizontally, pointing toward the center of the circle, while the gravitational force acts vertically downward. Since these two forces are perpendicular to each other, the net force is the vector sum of the normal and gravitational forces.

At the right side ($\theta = 0$), the normal force points leftward (toward the center), and gravity points downward, so the net force points diagonally down and to the left, toward the lower-left quadrant of the circle.

At the left side ($\theta = \pi$), the normal force points rightward (toward the center), and gravity still points downward, so the net force points diagonally down and to the right, toward the lower-left quadrant of the circle.

While the normal force provides the centripetal component of the net force, gravity introduces a tangential component, causing the cart to accelerate (speed up) along the track. Since the centripetal force is not directly towards the center of the circle, the cart must be speeding up. This is modeled in Figure 1.14.

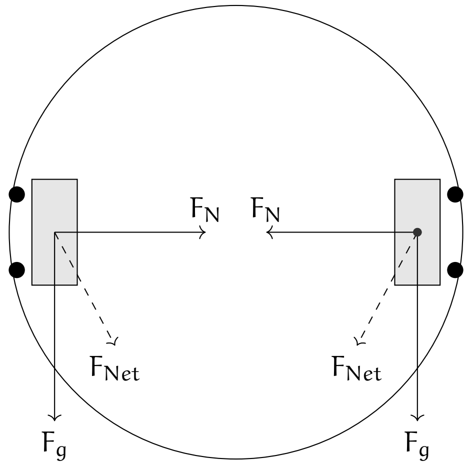


Figure 1.14: The FBD of the roller coaster at the sides.

Exercise 6 Roller Coaster Loop

This question is taken from the AP Physics C Review Book, 19th edition.

A roller-coaster car enters the circular loop portion of the ride. At the very top of the circle (where people are upside down), the car has a speed of 25 m/s, and the acceleration points straight down. If the diameter of the loop is 50 m and the total mass of the car plus passengers is 1,200 kg, find the magnitude of the normal force exerted by the track on the car at this point. Also find the normal force exerted by the track on the car when it is at the very bottom of the loop, given it is still traveling at 25 m/s. You may use $g = 10\ \text{m/s}^2$ for simpler calculations.

Working Space

Answer on Page 26

Our next chapter will explore oscillations, restoring forces, simple harmonic motion, and more concepts of harmonic motion.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 6)

The radius is 5 m, because the coefficients of both the x and y functions is 5.

Recall that the velocity in each direction is given by the derivative of the position functions:

$$v_x(t) = \frac{d}{dt}\left[5\cos(3t)\right] = -15\sin(3t)$$
$$v_y(t) = \frac{d}{dt}\left[5\sin(3t)\right] = 15\cos(3t)$$

The overall *speed* can be found from the x and y components of the velocity:

$$\begin{aligned}|v| &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{[-15\sin(3t)]^2 + [15\cos(3t)]^2} \\ &= \sqrt{15^2\left[\sin(3t)^2 + \cos(3t)^2\right]} \\ &= \sqrt{15^2} = 15\frac{m}{s}\end{aligned}$$

Notice that this is the coefficient for both components of the velocity.

To complete the path, the particle must travel 10π m. If the speed is $15\frac{m}{s}$, then the time it takes is:

$$t = \frac{d}{v} = \frac{10\pi \text{ m}}{15\frac{m}{s}} = \frac{2\pi}{3}s \approx 2.09 \text{ s}$$

Answer to Exercise 2 (on page 12)

$$\frac{120 \text{ km}}{1 \text{hour}} = \frac{1000 \text{ m}}{1 \text{ km}}\frac{120 \text{ km}}{1 \text{hour}}\frac{1 \text{ hour}}{3600 \text{ seconds}} = 33.3 \text{ m/s}$$

$$F = \frac{mv^2}{r} = \frac{0.4(33.3)^2}{200} = 2.2 \text{ newtons}$$

Answer to Exercise 3 (on page 13)

Givens:

$$T_{max} = (20\text{kg}) \cdot \left(9.8\frac{m}{s^2}\right) = 196\text{N}$$

$$r = 0.35\text{m}$$

$$m_{whistle} = 165\text{g} = 0.165\text{kg}$$

Unknown:

$$v = ?$$

$$f = ?$$

Equation(s):

$$F = ma$$

$$a = \frac{v^2}{r}$$

$$v = rf$$

Solution: First, we find the tangential speed if the tension in the string is the maximum tension:

$$T_{max} = m_{whistle} a = m_{whistle} \frac{v^2}{r}$$

$$v = \sqrt{\frac{T_{max} r}{m_{whistle}}}$$

$$v = \sqrt{\frac{196\text{N}\,(0.35\text{m})}{0.165\text{kg}}} \approx 0.645\frac{m}{s}$$

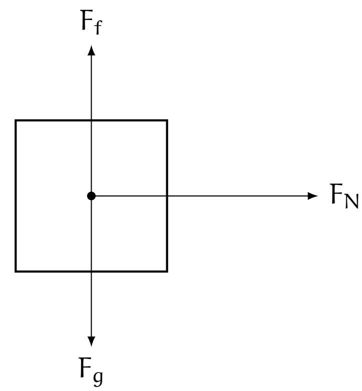
Therefore, the maximum tangential speed of the whistle before the string breaks is $0.645\frac{m}{s}$. Now finding the equivalent frequency (rotations per second):

$$f = \frac{v}{r} = \frac{0.645\frac{m}{s}}{0.35\text{m}} \approx 1.842\text{Hz}$$

So, to break the string, the lifeguard would have to spin it at nearly 2 rotations per second, which is achievable. The lifeguard could possibly break the string.

Answer to Exercise 4 (on page 13)

There are three forces acting on the rider: gravity, friction with the wall, and the normal force with the wall:



The FBD lets us write equations for Newton's Second Law in each dimension:

$$(1)\ \ ma_x = F_N$$

$$(2)\ \ ma_y = F_f - F_g = \mu F_N - mg$$

Because the rider doesn't fall down, we know that $a_y = 0\frac{m}{s^2}$ and therefore equation (2) becomes:

$$\mu F_N - mg = 0 \rightarrow F_N = \frac{mg}{\mu}$$

Having solved for F_N, we substitute for it into equation (1):

$$ma_x = \frac{mg}{\mu} \rightarrow a_x = \frac{g}{\mu}$$

Since we know g and μ, we can calculate a_x:

$$a_x = \frac{9.8\frac{m}{s^2}}{0.32} = 30.625\frac{m}{s^2}$$

This is the minimum acceleration needed to keep the rider from slipping down. We can now use the relationship between centripetal acceleration, tangential velocity, and the radius to find the angular velocity:

$$a = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{30.625\frac{m}{s^2}}{10\ m}} \approx 1.75\frac{rad}{s}$$

Answer to Exercise 5 (on page 16)

Givens:

- Mass of tetherball: $m = 0.5$ kg
- Length of string: $L = 2.5$ m
- Angle with vertical: $\theta = 38°$

The components of the tension force are seperated as follows:

$$T\cos\theta = mg, \qquad T\sin\theta = \frac{mv^2}{r}$$

Dividing the second equation by the first equation gives:

$$\tan\theta = \frac{v^2}{rg} \rightarrow v = \sqrt{rg\tan\theta}$$

But we are not given r. We can find r using trigonometry:

$$r = L\sin\theta$$

Now we can find v:

$$v = \sqrt{(L\sin\theta)g\tan\theta}$$

Solving this gives us:

$$v = \sqrt{(2.5\ \mathrm{m}\sin 38°)(9.8\frac{\mathrm{m}}{\mathrm{s}^2})\tan 38°} \approx 3.4\frac{\mathrm{m}}{\mathrm{s}}$$

The speed of the tetherball is approximately $3.4\frac{\mathrm{m}}{\mathrm{s}}$.

Answer to Exercise 6 (on page 20)

- At the top of the loop: At the top of the loop, both the gravitational force and the normal force point downward. Therefore, we can write:

$$F_{net} = F_N + F_g = \frac{mv^2}{r}$$

Solving for the normal force gives:

$$F_N = \frac{mv^2}{r} - F_g$$

Substituting for $F_g = mg$ and $r = \frac{d}{2} = 25$ m:

$$F_N = \frac{(1200 \text{ kg})(25 \frac{\text{m}}{\text{s}})^2}{25 \text{ m}} - (1200 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})$$

$$F_N = 30000 \text{ N} - 12000 \text{ N} = 18000 \text{ N}$$

- At the bottom of the loop: At the bottom of the loop, the gravitational force points downward while the normal force points upward. Therefore, we can write:

$$F_{net} = F_N - F_g = \frac{mv^2}{r}$$

Solving for the normal force gives:

$$F_N = \frac{mv^2}{r} + F_g$$

Substituting for $F_g = mg$ and $r = \frac{d}{2} = 25$ m:

$$F_N = \frac{(1200 \text{ kg})(25 \frac{\text{m}}{\text{s}})^2}{25 \text{ m}} + (1200 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})$$

$$F_N = 30000 \text{ N} + 12000 \text{ N} = 42000 \text{ N}$$

INDEX