

## CHAPTER 1

# Vector-valued Functions

In the last chapter, you calculated the flight of the shell. For any time  $t$ , you could find a vector [distance, height]. This can be thought of as a function  $f$  that takes a number and returns a 2-dimensional vector. We call this a *vector-valued* function from  $\mathbb{R} \rightarrow \mathbb{R}^2 \rightarrow \mathbb{R}^3$ <sup>1</sup>.

### 1.1 Vector-valued functions: position

We often make a vector-valued function by defining several real-valued functions. For example, if you threw a hammer with an initial upward speed of 12 m/s and a horizontal speed of 4 m/s along the  $x$  axis from the point  $(1, 6, 2)$ , its position at time  $t$  (during its flight) would be given by:

$$f(t) = [4t + 1, 6, -4.8t^2 + 12t + 2]$$

In other words,  $x$  is increasing with  $t$ ,  $y$  is constant, and  $z$  is a parabola.

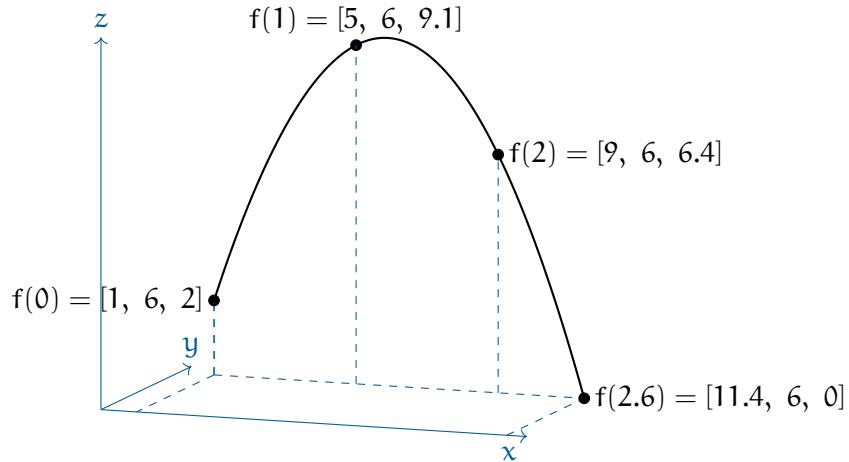


Figure 1.1: An example of a vector-valued function.

<sup>1</sup>the  $\mathbb{R}$  symbol represents the set of all real numbers; the  $\mathbb{R}^2$  symbol represents the set of all 2-dimensional vectors, and  $\mathbb{R}^3$  represents the set of all 3-dimensional vectors

## 1.2 Finding the velocity vector

Now that we have its position vector, we can differentiate each component separately to get its velocity as a vector-valued function:

$$\mathbf{f}'(t) = [4, 0, -9.8t + 12]$$

In other words, the velocity is constant along the  $x$ -axis, zero along the  $y$ -axis, and decreasing with time along the  $z$  axis.

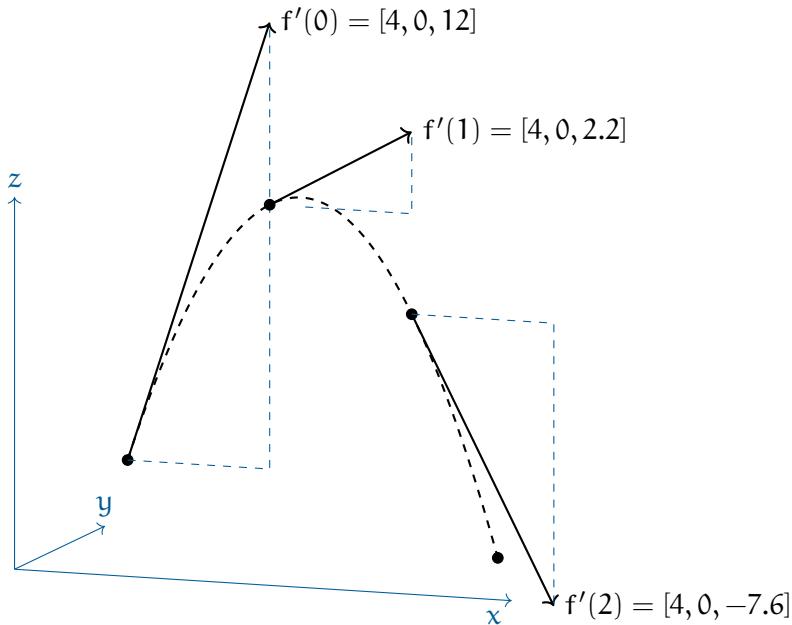


Figure 1.2: The derivatives of the position function (velocity) with respect to time.

## 1.3 Finding the acceleration vector

Now that we have its velocity, we can get its acceleration as a vector-valued function:

$$\mathbf{f}''(t) = [0, 0, -9.8]$$

There is no acceleration along the  $x$  or  $y$  axes. It is accelerating down at a constant  $9.8\text{m/s}^2$ .

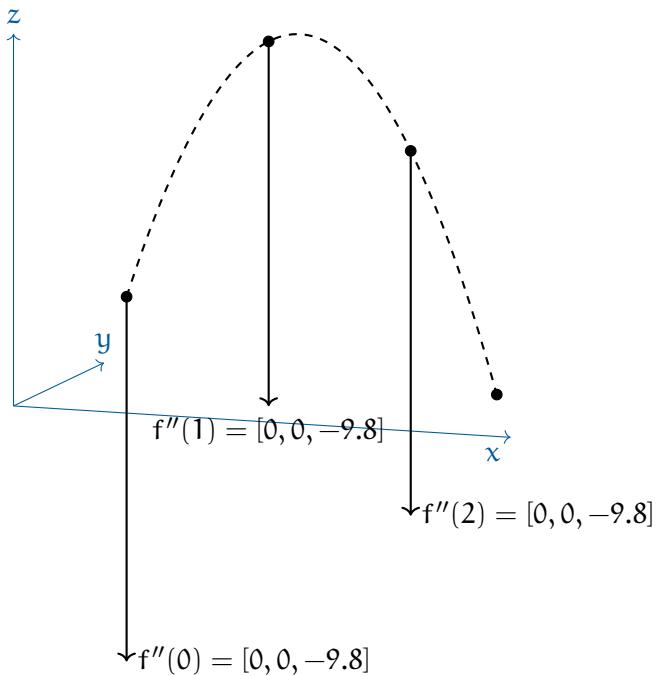


Figure 1.3: The acceleration vector is constant and points downward.

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*This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.*



## APPENDIX A

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# Answers to Exercises

