Contents

1 Vectors and Matrices **3**

1.1 Applications of Matrix-Vector Multiplication *4*

1.1.1 Linear Transformations *4*

1.1.2 Systems of Linear Equations *4*

1.1.3 Computer Graphics *4*

2 Linear Combinations **5**

2.1 Weighted Averages of Vectors *5*

3 Vector Spans and Independence **7**

3.1 Vector Independence *7*

4 Matrices **9**

4.1 Defining a Matrix *9*

4.2 Types of Matrices *10*

A Answers to Exercises **11**

Index 13

Chapter 1

Vectors and Matrices

In linear algebra, one of the fundamental operations is the multiplication of a matrix by a vector. Given a matrix A of size $m \times n$ and a vector x of size $n \times 1$, the product Ax is a new vector of size $m \times 1$.

The i-th component of the product vector Ax is computed by taking the dot product of the i-th row of A and the vector x:

$$(Ax)_i = \sum_{j=1}^{n} a_{ij} x_j \tag{1.1}$$

where a_{ij} is the element in the i-th row and j-th column of A, and x_j is the j-th element of x.

1.1 Applications of Matrix-Vector Multiplication

Matrix-vector multiplication is a crucial operation in many areas of science and engineering:

1.1.1 Linear Transformations

Matrices can represent linear transformations, such as rotations, scaling, and shearing. Multiplying a vector by a matrix applies the transformation represented by the matrix to the vector.

1.1.2 Systems of Linear Equations

A system of linear equations can be written in matrix form as $Ax = b$. Solving this system involves operations on A and b, and the solution vector x is found by various methods such as Gaussian elimination or LU decomposition.

1.1.3 Computer Graphics

In computer graphics, transformations of objects in the scene (like rotation, scaling, and translation) are done using matrix operations. The vertices of objects are represented as vectors, and transformations are applied by multiplying these vectors by transformation matrices.

These are just a few examples of the uses of matrix-vector multiplication. The operation is also fundamental to many algorithms in machine learning, physics, economics, and other fields.

Chapter 2

Linear Combinations

A linear combination of vectors involves combining vectors using scalar multiplication and addition. Given vectors $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n \in \mathbb{R}^m$ and scalars $a_1, a_2, ..., a_n \in \mathbb{R}$, a linear combination of these vectors is any vector of the form

$$\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + ... + a_n\mathbf{v}_n$$

Each scalar a_i scales the corresponding vector $\mathbf{v}_i$, and the results are added together to produce a new vector $\mathbf{w}$.

2.1 Weighted Averages of Vectors

A weighted average of vectors is a specific type of linear combination where the coefficients (or weights) a_i are non-negative and sum to 1:

$$\sum_{i=1}^{n} a_i = 1, \quad a_i \geq 0$$

A weighted average of vectors $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n$ is then defined as

$$\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + ... + a_n\mathbf{v}_n$$

In this case, each a_i not only scales the corresponding vector $\mathbf{v}_i$, but also represents the proportion of that vector in the final average vector $\mathbf{w}$.

Chapter 3

Vector Spans and Independence

In linear algebra, the span of a set of vectors is the set of all possible linear combinations of those vectors. If the set $S = \{v_1, v_2, ..., v_n\}$ contains vectors from a vector space V, then the span of S is given by:

$$\mathrm{Span}(S) = \{a_1 v_1 + a_2 v_2 + ... + a_n v_n : a_1, a_2, ..., a_n \in \mathbb{R}\} \tag{3.1}$$

This means that any vector in the $\mathrm{Span}(S)$ can be written as a linear combination of the vectors in S.

3.1 Vector Independence

A set of vectors $S = \{v_1, v_2, ..., v_n\}$ is said to be linearly independent if the only solution to the equation

$$a_1 v_1 + a_2 v_2 + ... + a_n v_n = 0 \tag{3.2}$$

is $a_1 = a_2 = ... = a_n = 0$. This means that no vector in the set can be written as a linear combination of the other vectors.

If there exists a nontrivial solution (i.e., a solution where some $a_i \neq 0$), then the vectors are said to be linearly dependent. This means that at least one vector in the set can be written as a linear combination of the other vectors.

The concept of vector independence is crucial in many areas of linear algebra, including the study of vector spaces, bases, and rank.

CHAPTER 4

Matrices

In mathematics, a matrix is a rectangular array of numbers arranged in rows and columns. The individual numbers in the matrix are called its elements or entries. Matrices have a wide variety of applications in various fields, such as physics, computer science, engineering, and economics. They are a fundamental tool in linear algebra and help in solving systems of linear equations.

4.1 Defining a Matrix

A matrix with m rows and n columns is called an $m \times n$ matrix or simply an m-by-n matrix, and m and n are called its dimensions.

The general form of a 2×3 matrix A is given by:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \, a_{21} & a_{22} & a_{23} \end{bmatrix}$$

4.2 Types of Matrices

There are several specific types of matrices.

- **Row Matrix:** A matrix is said to be a row matrix if it has only one row.
- **Column Matrix:** A matrix is said to be a column matrix if it has only one column.
- **Square Matrix:** A square matrix has the same number of rows and columns.
- **Zero Matrix:** A zero matrix or null matrix is a matrix all of whose entries are zero.
- **Identity Matrix:** The identity matrix, or sometimes called the unit matrix, of size n is the $n \times n$ square matrix with 1 on the diagonals and zero everywhere else.

Appendix A

Answers to Exercises

INDEX

linear combonations, 5

weighted averages, 5