Contents

Chapter 1

Electromagnetic Waves

Sound is a compression wave — to travel, it needs a medium to compress: air, water, etc. (Regardless of what you have seen in movies, sound does not travel through a vacuum!)

Light is an electromagnetic wave — it causes fluctuations in the electric and magnetic fields that are everywhere. It can cross a vacuum, as it does to reach us from the sun.

Electromagnetic waves travel at about 300 million meters per second in a vacuum. The waves travel slower through things. For example, an electromagnetic wave travels at 225 million meters in water.

Electromagnetic waves come in different frequencies. For example, the light coming out of a red laser pointer is usually about 4.75×10^{14} Hz. The wifi data sent by your computer is carried on an electromagnetic wave too. It is usually close to 2.4×10^{6} Hz or 5×10^{6} Hz.

Because we know how fast the waves are moving, we sometimes talk about their wavelengths instead of their frequencies. The light coming out of a laser pointer is $300 \times 10^{6} / 4.76 \times 10^{14} = 630 \times 10^{-9}$ m, or 630 nm.

Exercise 1 Wavelengths

A green laser pointer emits light at 5.66×10^{14} Hz. What is its wavelength in a vacuum?

Working Space

Answer on Page 51

We have given names to different ranges of the electromagnetic spectrum:

Name	Hertz	Meters
Gamma rays	$\times 10$	$\times 10$
X-rays	$\times 10$	$\times 10$
Ultraviolet	$\times 10$	$\times 10$
Blue	$\times 10$	$\times 10$
Red	$\times 10$	$\times 10$
Infrared	$\times 10$	$\times 10$
Microwaves	$\times 10$	$\times 10$
Radio waves	$\times 10$	$\times 10$

(You may have heard of "cosmic rays" and wonder why they are not listed in this table. Cosmic rays are actually the nuclei of atoms that have been stripped of their electron cloud. These particles come flying out of the sun at very high speeds. They were originally thought to be electromagnetic waves, and were mistakenly named "rays".)

In general, the lower frequency the wave is, the better it passes through a mass. A radio wave, for example, can pass through the walls of your house, but visible light cannot. The people who designed the microwave oven chose the frequency of 2.45 GHz because the energy from those waves tended to get absorbed in the first few inches of food that it passed through.

1.1 The greenhouse effect

Humans have dug up a bunch of long carbon-based molecules (like oil and coal) and burned them, releasing large amounts of CO_2 into the atmosphere. It may not be obvious why that has made the planet warmer, but the answer is electromagnetic waves.

A warm object gives off infrared electromagnetic waves. That's why, for example, motion detectors in security systems are actually infrared detectors: even in a dark room, your body gives off a lot of infrared radiation.

You may have heard of "heat-seeking missiles." These are more accurately called "Infrared homing missiles" because they follow objects giving off infrared radiation – hot things like jet engines.

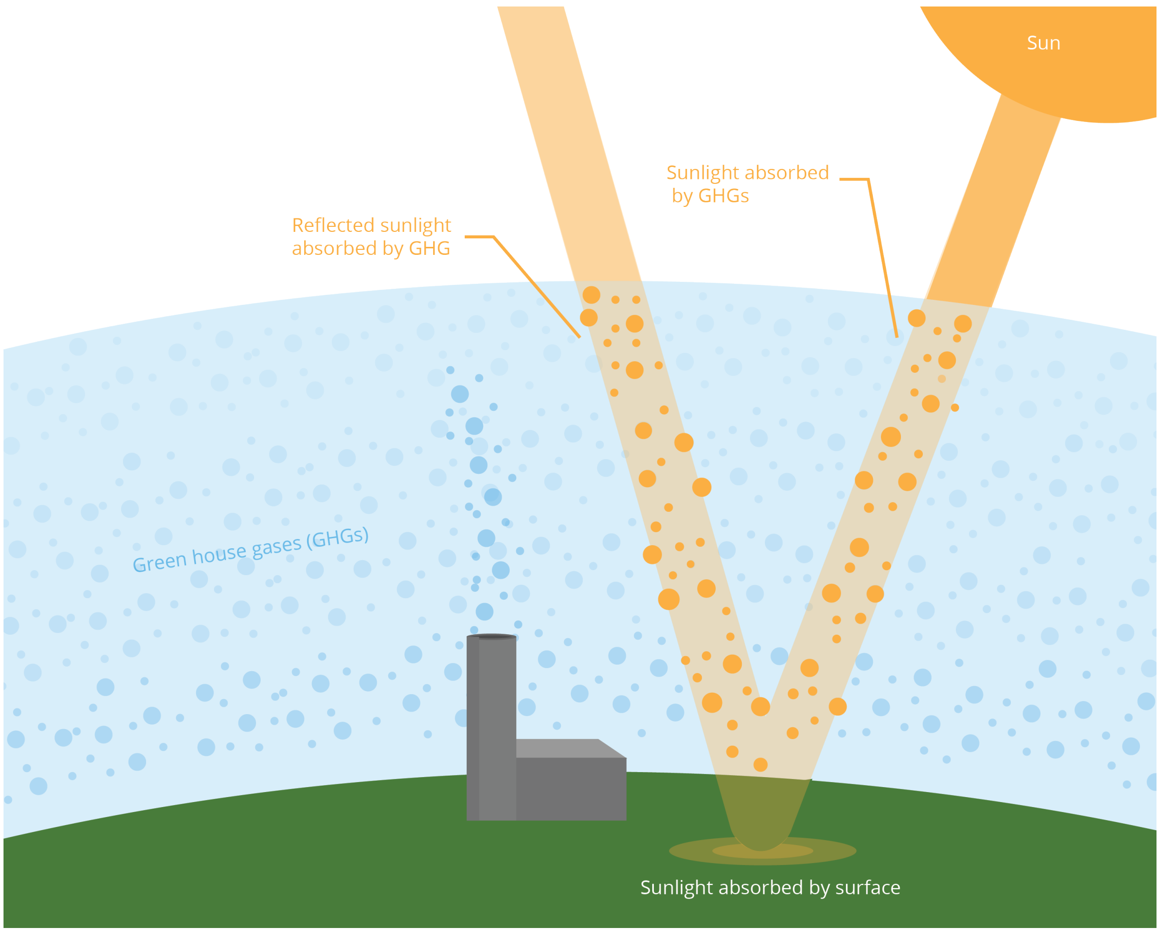
The sun beams a lot of energy to our planet in the form of electromagnetic radiation: visible light, infrared, ultraviolet. (How much? At the top of the atmosphere directly facing the sun, we get 1,360 watts of radiation per square meter. That is a lot of power!)

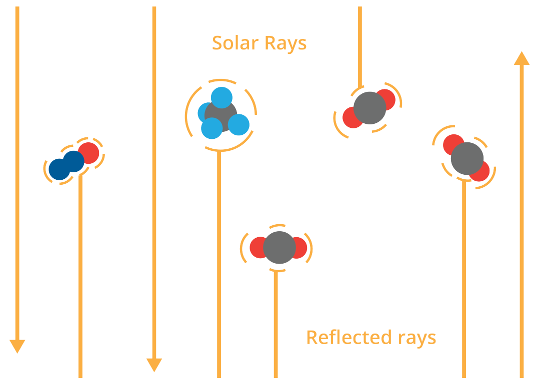
Some of that radiation just reflects back into space. 23% is reflected by the clouds and the atmosphere, while 7% makes it all the way to the surface of the planet and is reflected back into space.

The other 71% is absorbed. 48% is absorbed by the surface and 23% is absorbed by the atmosphere. All of that energy warms the planet and the atmosphere so that it gives off infrared radiation. The planet lives in equilibrium; the infrared radiation leaving our atmosphere is exactly the same amount of energy as that 71% of the radiation that it absorbs.

(If the planet absorbs more energy than it releases, the planet gets hotter. Hotter things release more infrared. When the planet is in equilibrium again, it stops getting hotter.)

So, what is the problem with CO_2 and other large molecules in the atmosphere? They absorb the infrared radiation instead of letting it escape into space. This means the planet must be hotter to maintain equilibrium.





The planet is getting hotter, and it is creating a multitude of problems:

- Weather patterns are changing, which leads to extreme floods and droughts.
- Ice and snow in places like Greenland are melting and flowing into the oceans. This is raising sea levels.
- Biomes with biodiversity are resilient. Rapidly changing climate is destroying biodiversity everywhere, which is making these ecosystems very fragile.
- In many places, permafrost, which has trapped large amounts of methane in the ground for millenia, is melting.

That last item is particularly scary, because methane is a large gas molecule — it absorbs even more infrared radiation than CO_2. As it escapes the permafrost, the problem will get worse.

Scientists are working on four kinds of solutions:

- **Stop increasing the amount of greenhouse gases in our atmosphere.** It is hoped that non-carbon based energy systems like solar, wind, hydroelectric, and nuclear could let us stop burning carbon. Given the methane already being released, it maybe too late for this solution to work on its own.
- **Take some of greenhouse gases out of our atmosphere and sequester them somewhere.** The trunk of a tree is largely made up of carbon molecules. When you grow a tree where there had not been one before, you are sequestering carbon inside the tree. There are also scientists that are trying to develop a process that pulls greenhouse gases out of the air and turn them into solids.
- **Decrease the amount of solar radiation that is absorbed by our planet and its atmosphere.** Clouds reflect a lot of radiation back into space. Could we increase the cloudiness of our atmosphere? Or maybe launch mirrors into orbit around our planet?
- **Adapt to the changing climate.** These scientists are assuming that global warming

> will continue, and are working to minimize future human suffering. How will we relocate a billion people as the oceans claim their homes? When massive heat waves occur, how will we keep people from dying? As biodiversity decreases, how can we make sure that species that are important to human existence survive?

What are the greenhouse gases and how much does each contribute to keeping the heat from exiting to space? These numbers are still being debated, but this will give you a feel:

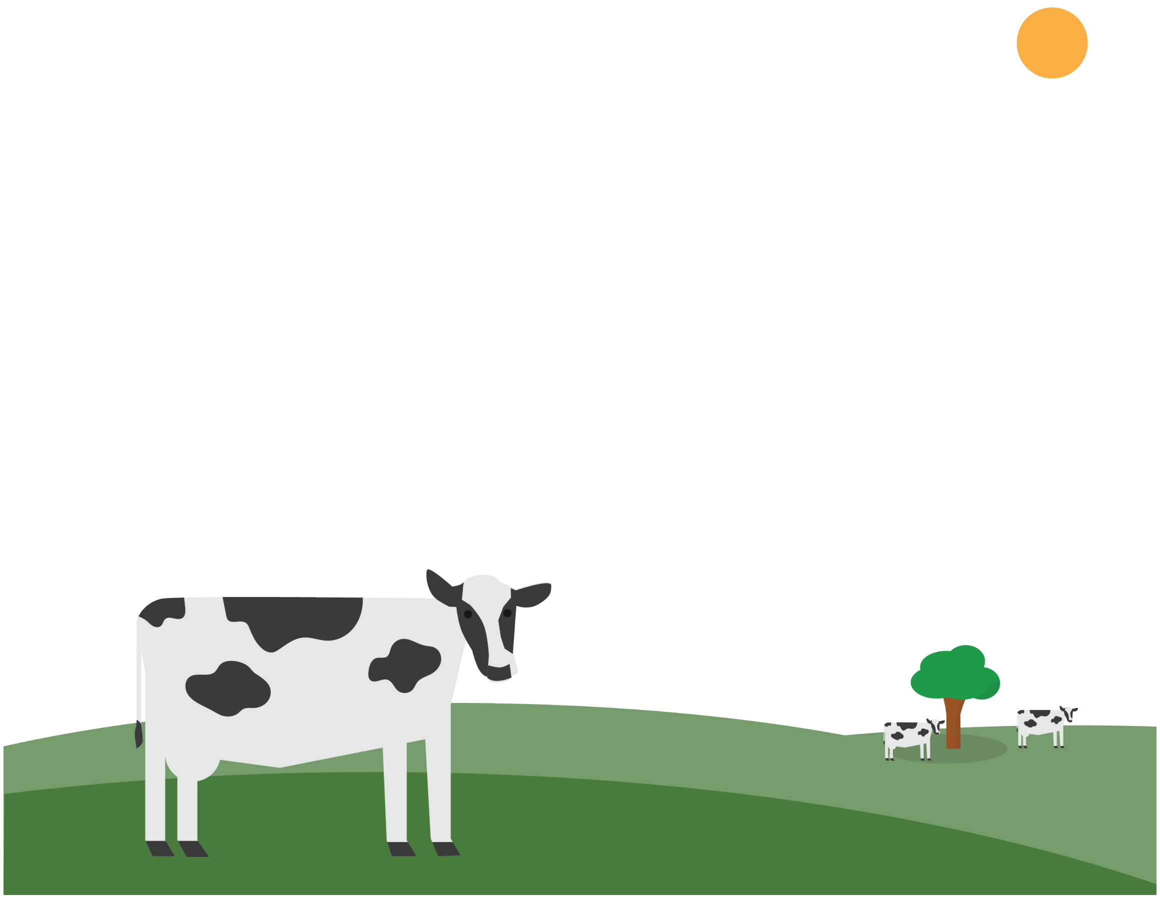
Water vapor	H_2O	36 - 72 %
Carbon dioxide	CO_2	9 - 26 %
Methane	CH_4	4 - 9 %
Ozone	O_3	3 - 7 %

Notice that while we talk a lot about carbon dioxide, the most important greenhouse gas is actually water. Why don't we talk about it? Given the enormous surfaces of the oceans, it is difficult to imagine any way to permanently decrease the amount of water in the air. Additionaly, a great deal of water in the air is in the form of clouds, which help reflect radiation before it is absorbed.

Chapter 2

How Cameras Work

Let's say it is a sunny day and you are standing in a field a few meters from a cow. You use the camera on your phone to take a picture of the cow. How does that whole process work?

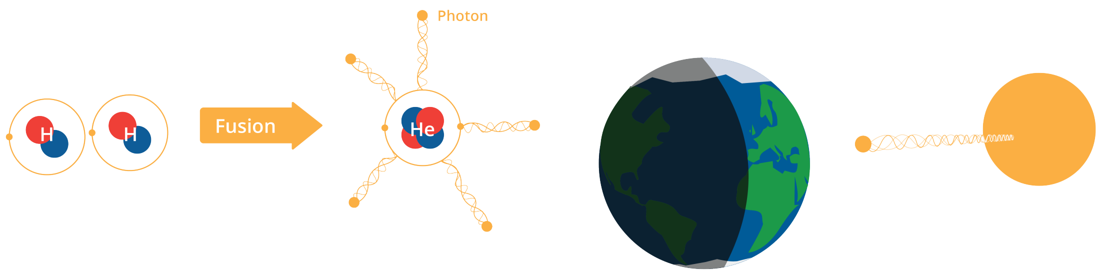


2.1 The Light That Shines On the Cow

The sun is a sphere of hot gas. About 70% of the gas is hydrogen. About 28% is helium. There's also a little carbon, nitrogen, and oxygen.

Gradually, the sun is converting hydrogen into helium through a process known as "nuclear fusion" (which we will be discussing more in a future chatper). A large amount of heat is created in this process. This heat makes the gases glow.

How does heat make things glow? The heat pushes the electrons into higher orbitals. When they come back down to a lower orbital, they release a photon of energy, which travels away from the atom as an electromagnetic wave.



Heat is not the only way to push the electrons into a higher orbital. For example, a fluorescent lightbulb is filled with gas. When we pass electricity through the gas, its electrons are moved to a higher orbital. When they fall, light is created.

What is the frequency of the wave that the photon travels on? Depending on what orbital it falls from and how far it falls, the photon created has different amounts of energy. The amount of energy determines the frequency of the electromagnetic wave.

Formula for enegy of a photon

If you want to know the amount of energy E in a photon, here is the formula:

$$E = \frac{hc}{\lambda}$$

where c is the speed of light, λ is the wavelength of the electromagnetic wave, and h Planck's constant: $6.63 \times 10^{-34} m^2 kg/s$

For example, a red laser light has a wavelength of about 630 nm. So, the energy in each photon is:

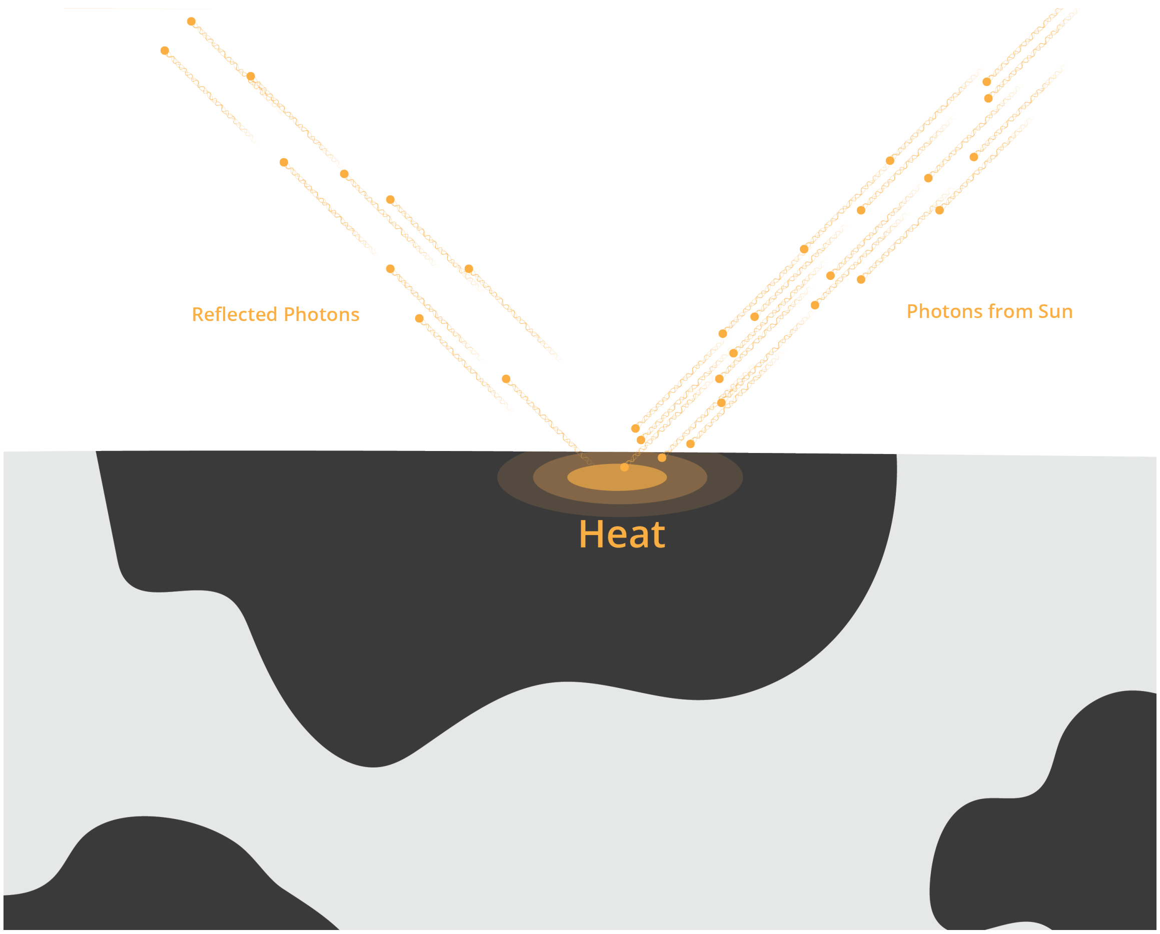
$$\frac{(300 \times 10^6)(6.63 \times 10^{-34})}{630 \times 10^{-9}} = 3.1 \times 10^{-19} \text{ joules}$$

In the sun, there are several kinds of molecules and each has a few different orbitals that the electrons can live in. Thus, the light coming from the sun is made up of electromagnetic waves of many different frequencies.

We can see some of these frequencies as different colors, but some are invisible to humans, such as ultraviolet and infrared.

2.2 Light Hits the Cow

When these photons from the sun hit the cow, the hide and hairs of the cow will absorb some of the photons. These photons will become heat and make the cow feel warm. Some of the photons will not be absorbed – they will leave the cow. When you say "I see the cow," what you are really saying is "I see some photons that were not absorbed by the cow."



Different materials absorb different amounts of each wavelength. A plant, for example, absorbs a large percentage of all blue and red photons that hit it, but it absorbs only a small percentage of the green photons that hit it. Thus, we say "That plant is green."

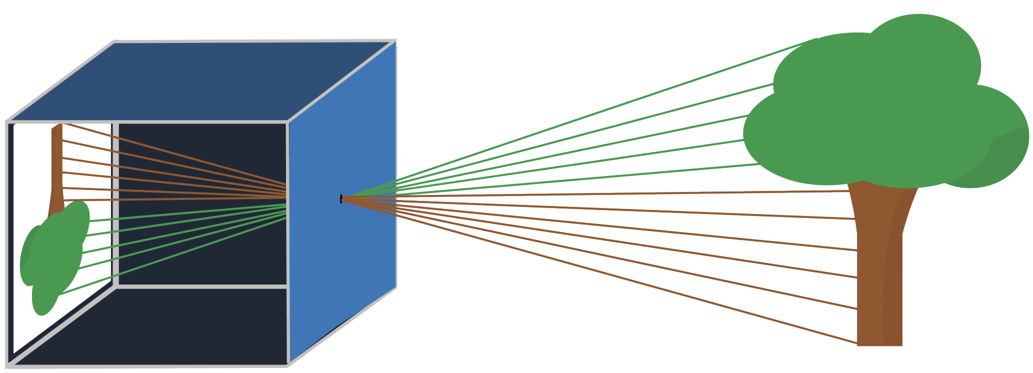
White things absorb very small percentages of photons of any visible wavelength. Black things absorb very *large* percentages of photons of any visible wavelength.

Before we go on, let's review: The sun creates photons that travel as electromagnetic waves of assorted wavelengths to the cow. Many of those photons are absorbed, but some are not. Some of those photons that are not absorbed go into the lens of our camera.

2.3 Pinhole camera

The simplest cameras have no lenses. They are just a box. The box has a tiny hole that allows photons to enter. The side of the box opposite the hole is flat and covered with film or some other photo-sensitive material.

The photons entering the box continue in the same direction they were going when they passed through the hole. Thus, the photons that entered from high hit the back wall at a low point. The photons that came from the left hit the back wall on the right. This is how the image is projected onto the back wall, rotated 180 degrees; what was up is down, what was on the left is on the right.



Exercise 2 Height of the image

Working Space

FIXME: cow swap

Let's say that that the pinhole is exactly the same height as the shoulder of the cow, and that the shoulder is directly above one hoof. This means the pinhole, the shoulder, and the hoof form a right triangle.

Now, let's say that the camera is being held perpendicular to the ground. The pinhole, the image of the shoulder, and the image of the hoof on the back wall of the camera now also form a right triangle.

These two triangles are similar.

The shoulder is 2 meters from the hoof. The cow is standing 3 meters from the camera. The distance from the pinhole to the back wall of the camera is 3 cm. How tall is the image of the cow on the back wall of the camera?

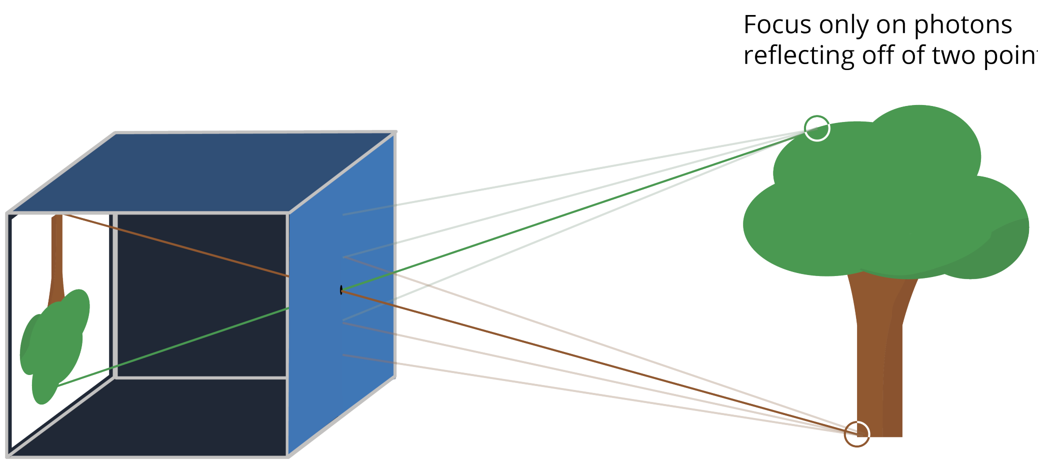
Answer on Page 51

2.4 Lenses

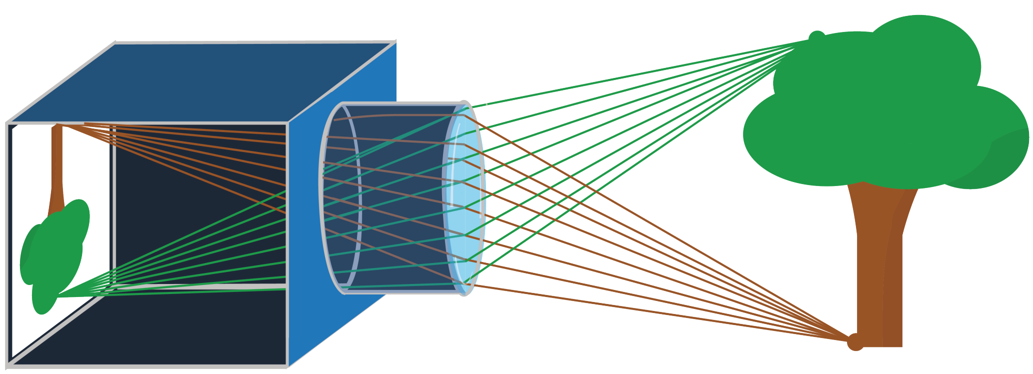
Now, a quick review: A photon leaves the sun in some random direction. It travels 150 million km from the sun and hits a cow. It is not absorbed by the cow, and heads off in a new direction. It passes through the pinhole and hits the back wall of the camera. That seems incredibly improbable, right?

It actually is relatively improbable, especially if there isn't a lot of light — like you are taking the picture at dusk. To increase the odds, we added a *lens* to the camera.

If you focus a lens on a wall and you draw a dot on that wall, the lens is designed such that all the photons from the dot that hit the lens get redirected to the same spot on the back wall of the camera — regardless of which path it took to get to the lens.



Note that the image still gets flipped. There is a *focal point* that all the photons pass through.



The distance from the lens to its focal point is called the lens's *focal length*. Telephoto lenses, that let you take big pictures of things that are far away, have long focal lengths. Wide-angle lenses have short focal lengths.

2.5 Sensors

The camera on your phone has a sensor on the back wall of the camera. The sensor is broken up into tiny rectangular regions called pixels. When you say a sensor is 6000 by 4000 pixels, we are saying the sensor is a grid of 24,000,000 pixels: 6000 pixels wide and 4000 pixels tall.

Each pixel has three types of cavities that take in photos. One of the cavities measures the amount of short wavelength light, like blues and violets. One of the cavities measures the long wavelength light, like reds and oranges. One of the cavities measures the intensity of wavelengths in the middle, like greens.

Thus, if your camera has a resolution of 6000×4000, the image is 72,000,000 numbers: Every one of the 24,000,000 pixels yeilds three numbers: intensity of long wavelength, mid wavelength, and long wavelength light. We call these numbers “RGB” for Red, Green, and Blue.

CHAPTER 3

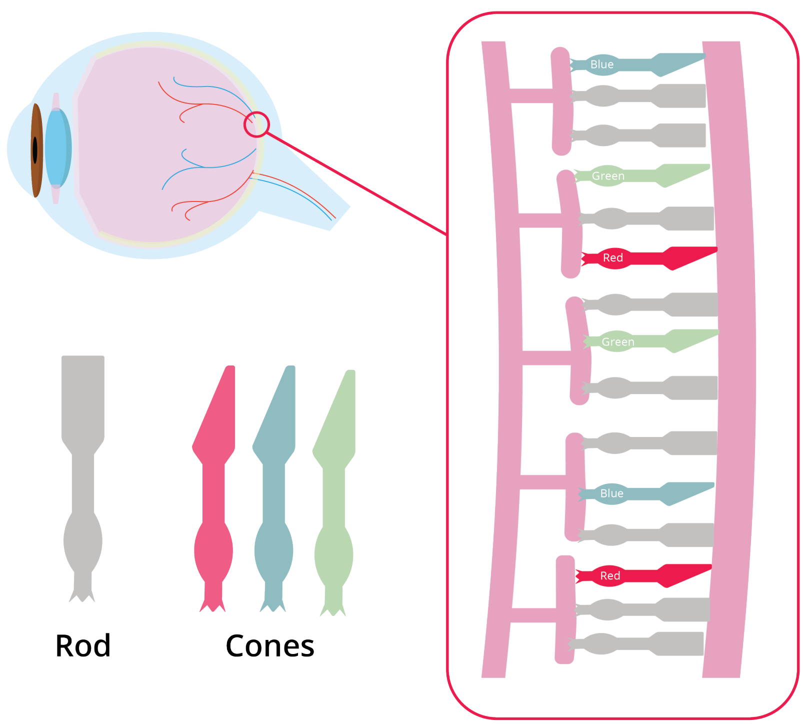
How Eyes Work

Dr. Craig Blackwell has made a great video on the mechanics of the eye. You should watch it: https://youtu.be/Z8asc2SfFHM

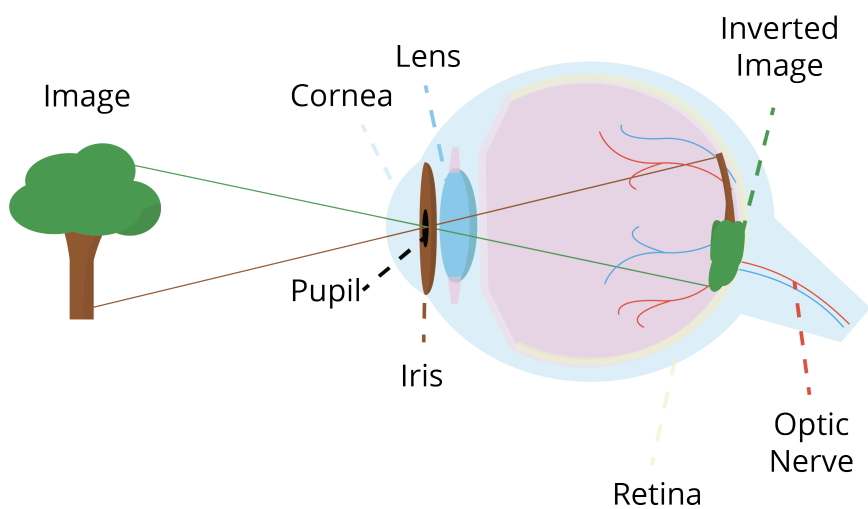
Mechanically, your eye works a lot like a camera. The eye is a sphere with two lenses on the front: The outer lens is called the *cornea*, while the second lens is simply called "the lens."

Between the two lenses is an aperture that opens wide when there is very little light, and closes very small when there is bright light. The opening is called the *pupil* and the tissue that forms the pupil is called the *iris*. When people talk about the color of your eyes, they are talking about the color of your iris. The blackness at the center of your iris is your pupil.

There are two types of photoreceptor cells in your retina: rods and cones. The rods are more sensitive; in very dark conditions, most of our vision is provided by the rods. The cones are used when there is plenty of light, and they let us see colors.



The white part around the outside of the eyeball? That is called the *sclera*.



The walls of the eye are lined inside with the *retina*, which has sensors that pick up the light and send impulses down the optic nerve to your brain.

Just like a camera, the images are flipped when they get projected on the back of the eye.

3.1 Eye problems

Now that you know the mechanics of the eye, let's go over a few things that commonly go wrong with the eye.

3.1.1 Glaucoma

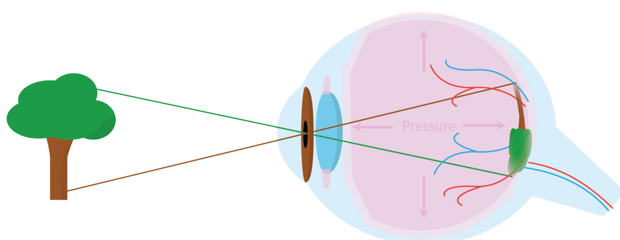
The space between your cornea and lens is filled with a fluid called *aqueous humor*. To feed the cells of the cornea and lens, the aqueous humor carries oxygen and nutrients like blood would, but unlike blood, it is transparent so you can see. Aqueous humor is constantly being pumped into and out of that chamber. If aqueous humor has trouble exiting, the pressure builds up and can damage the eye. This is known as *glaucoma*.

3.1.2 Cataracts

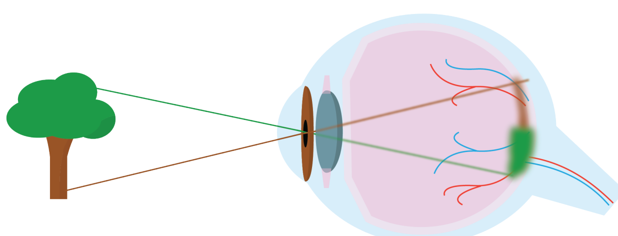
The lens should be clear. As a person ages, the proteins in the lens break down and clump together, becoming opaque. This can also be accelerated by diabetes, too much exposure to sunlight, obesity, and high blood pressure. From the outside, the eye will look cloudy. This is called a *cataract*, and it makes it difficult for the person to see.

This problem can be corrected, however. The person's cloudy lens is removed and replaced with a clear, manufactured lens.

Glaucoma



Cataracts



3.1.3 Nearsightedness, farsightedness, and astigmatism

If you are in a dark room and a tiny LED is turned on, the photons from that LED can pass through your cornea in many different places. If your eye is focusing on that light correctly, all the photons should meet up at the same place on the retina.

FIXME: Diagram here

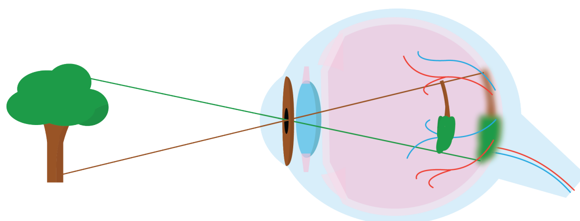
If the lenses are bending the light too much, the photons meet up before they hit the retina and get smeared a bit across it. To the person, the LED would appear blurry. The eye is said to be *nearsighted* or *myopic*.

If the lenses are not bending it enough, the photons would meet up behind the retina. Once again, they get smeared a bit across the retina and the LED looks blurry to the person. The eye is said to be *farsighted* or *hyperoptic*.

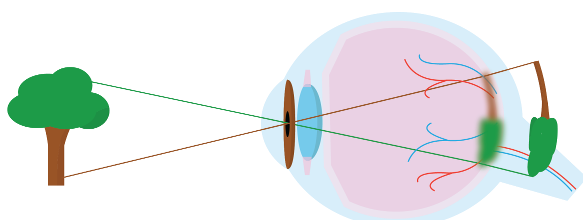
Your lenses are supposed to bend the photons the same amount vertically and horizontally. If one dimension is focused, but the other is myopic or hyperoptic, the eye is said to have *astigmatism*.

Myopia, hyperoptia, and astigmatism can be corrected with glasses or contact lenses. Doctors can also do surgical corrections, usually by changing the shape of the cornea.

Near Sighted



Far Sighted



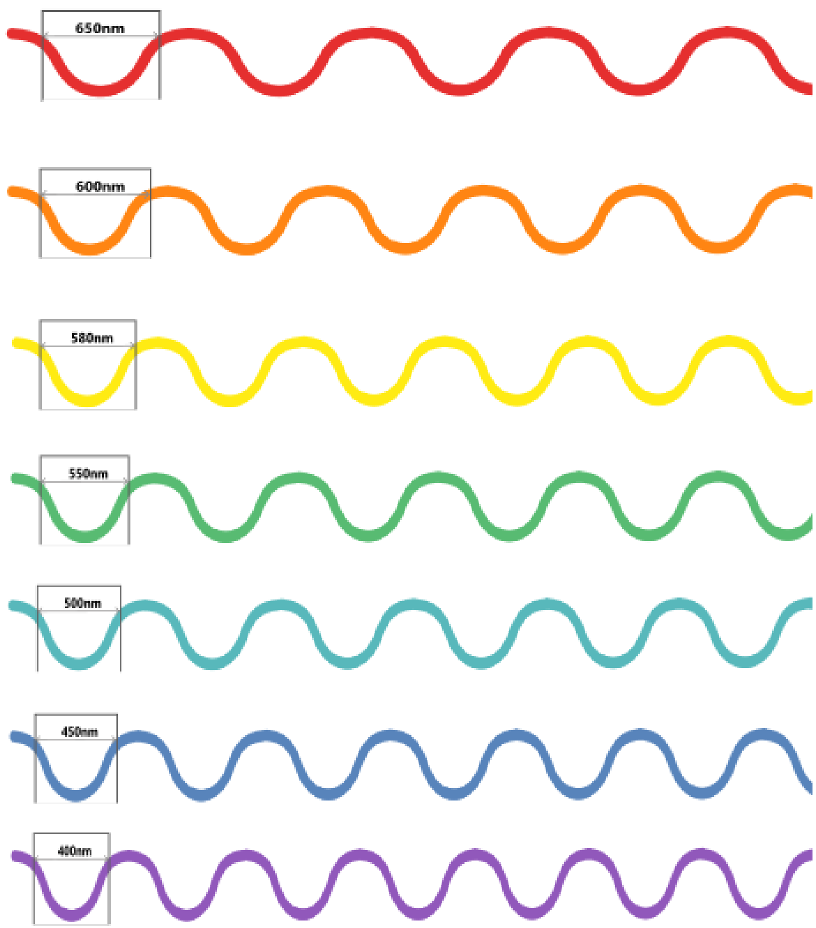
3.2 Seeing colors

TED-Ed has made a good video on how we see color. Watch it here: `https://youtu.be/18_fZPHasdo`

When a rainbow forms, you are seeing different wavelengths separating from each other. In the rainbow:

- Red is about 650 nm.
- Orange is about 600 nm.
- Yellow is about 580 nm.

- Green is about 550 nm.
- Cyan is about 500 nm.
- Blue is about 450 nm.
- Violet is about 400 nm.



If you shine a light with a wavelength of 580 nm on a white piece of paper, you will see yellow.

However, if you shine two lights with wavelengths of 650 nm (red) and 550 nm (green), you will also see yellow.

Why? Our ears can hear two different frequencies at the same time. Why can't our eyes see two colors in the same place?

As mentioned above, the cone photoreceptors in our eyes let us see colors. There are three kinds of cones:

- Blue: Cones that are most sensitive to frequencies near 450nm.
- Green: Cones that are most sensitive to frequencies near 550nm.
- Red: Cones that let us see the frequencies up to about 700nm.

When a wavelength of 580 nm hits your retina, it excites the red and green receptors, and your brain interprets that mix as yellow.

Similarly, when light that contains both 650 nm and 550 nm waves hits your retina, it excites the red and green receptors, and your brain interprets that mix as yellow.

You can't tell the difference!

Now we know why the sensors on the camera are RGB. The camera is recording the scene as closely as necessary to fool your eye.

A TV or a color computer monitor only has three colors of pixels: red, green, and blue. By controlling the mix of them, it creates the sensation of thousands of colors to your eye.

3.3 Pigments

A color printer works in the opposite fashion. Instead of radiating colors, it puts pigments on the paper that absorb certain frequencies. A pigment that absorbs only frequencies near 650 nm (red) will appear to your eye as cyan. This makes sense, because the sensation of cyan is created when your blue and green receptors are activated.

Thus, pigment colors come in:

- Cyan: absorbs frequencies around red
- Magenta: absorbs freqencies around green
- Yellow: absorbs frequencies around blue

If you buy ink for a color printer, you know there is typically a fourth ink: black. If you put cyan, magenta, and yellow pigments on paper, the mix won't absorb all the visible spectrum in a consistent manner. Our eyes are pretty sensitive to this, so we would see brown. This is why we add black ink to get pretty grays and blacks.

We call this approach to color CMYK (as opposed to RGB). If an artist is creating an image to be viewed on a screen, they will typically make an RGB image. If they are creating an image to be printed using pigments, they typically create a CMYK image. (Most of us don't care so much — we just let the computer do conversions between the two color spaces for us.)

Chapter 4

Reflection

What happens when light hits a mass?

In a previous chapter, we talked about light as a wave, and we mentioned that each color in the rainbow is a different wavelength. You can also think of light as particles of energy called *photons*. Every photon comes with an amount of energy that determines what color it is.

When we are talking about light interacting with objects, your intuition will be right more often if you think of light as a beam of photons.

When a photon comes from the sun and hits an object, one of several things can happen:

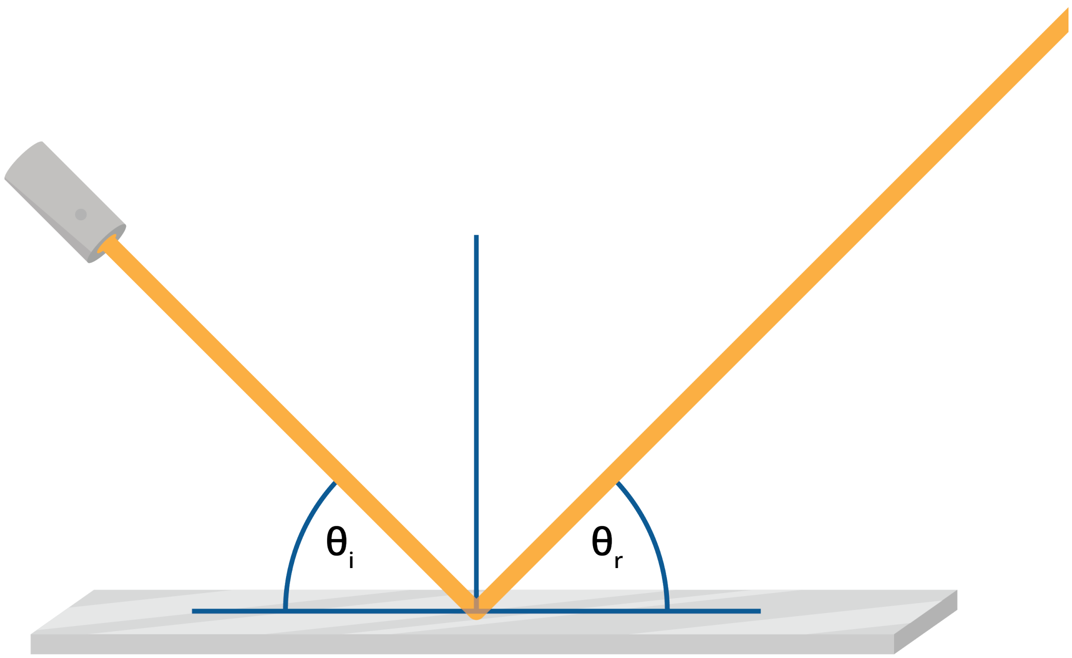
- The energy of the photon is absorbed by the object. It makes the object a little warmer. If a large proportion of photons hitting the mass are absorbed like this, we say the object is "black".
- The photon bounces off the object. If the surface is very smooth, the photons bounce in a predictable manner, and we call this *reflection* and we say the object is "shiny".
- If the surface is rough and the photons are not absorbed, the photons are scattered in random directions. We call this *diffusion*. If most of the photons hitting an object are bounced in random directions, we say that the object is "white".
- The photon passes through the mass. If the mass has smooth surfaces and a consistent composition, the photons will pass throught the mass in a predictable manner. We say that the mass is "transparent".
- If the photons pass through, but in an unpredictable, scattering manner, we say the mass is "translucent".

No object absorbs every photon, but chemists are always coming up with "blacker" materials. Vantablack, for example, is a super-black paint that absorbs 99.965% of all photons in the visible spectrum.

No object reflects every photon, but a mirror is pretty close. Let's talk about reflections in a mirror.

4.1 Reflection

When a beam of light hits mirror, it bounces of the mirror at the same angle it approached from. That is, if it approaches nearly perpendicular to the mirror, it departs nearly perpendicular to the mirror. If it hits the mirror at a glancing angle, it departs at an angle close to the mirrors surface.

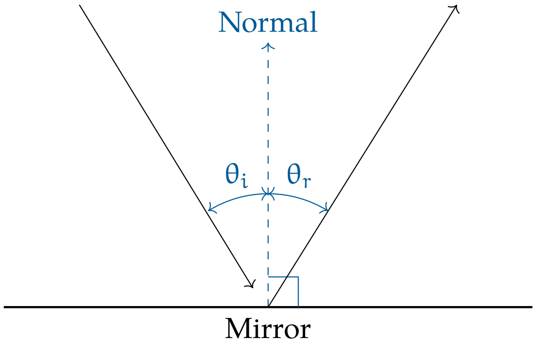


Law of Reflection

The angle of incidence, denoted as θ_i, is equal to the angle of reflection, denoted as θ_r. This law can be mathematically expressed as:

$$\theta_i = \theta_r$$

where θ_i is the angle between the incident light ray and the normal to the surface, and θ_r is the angle between the reflected light ray and the normal.



Exercise 3 Law of Reflection

Working Space

You are standing 4 meters from a mirror hung on a wall. The bottom of the mirror is the same height as your chin, so you can't see your whole body. You stick a piece of masking tape to your body.

You walk forward until you are only 3 meters from the mirror, then put a piece of masking tape on your body at the new cut-off point. Is the new masking tape higher or lower on your body?

Answer on Page 51

Exercise 4 Photons and Color

Working Space

There are red photons.

Are there black photons?

Are there white photons?

Are there yellow photons?

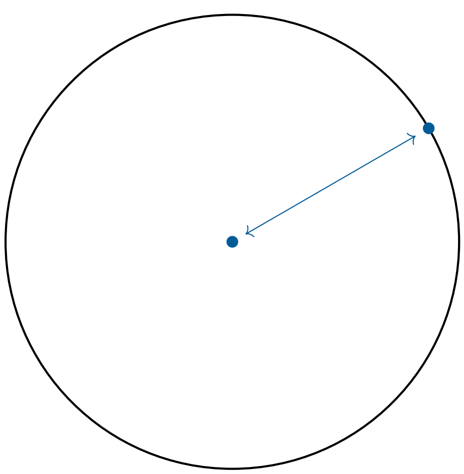
Answer on Page 51

4.2 Curved Mirrors

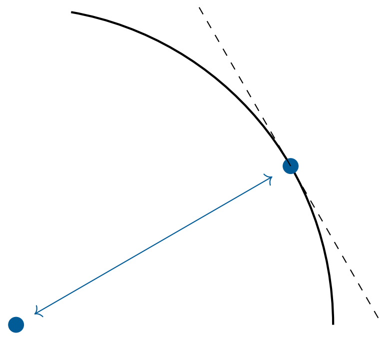
Flat mirrors are common and useful, but things get more interesting once you bend the mirrors. In this section, we are going to talk about a few different kinds of curved mirrors.

4.2.1 The Reflective Properties of Circles and Spheres

For example, if you were inside a circular room (a cylinder, actually), you could imagine standing in the center and pointing a flash light in any horizontal direction. The beam of light would bounce right back to you.



How do you know this? Because the tangent line is always perpendicular to the radius to the point of tangency:



You could create a spherical room with mirror walls. You would create a platform in the center where you could stand, and if you pointed your flashlight in any direction, its beam of light would shine back at you.

4.2.2 Ellipses and Ellipsoids

Intuitively, you know what an ellipse is: an oval. However, the ellipse is actually an oval with some special properties. This is a good time to talk about those properties.

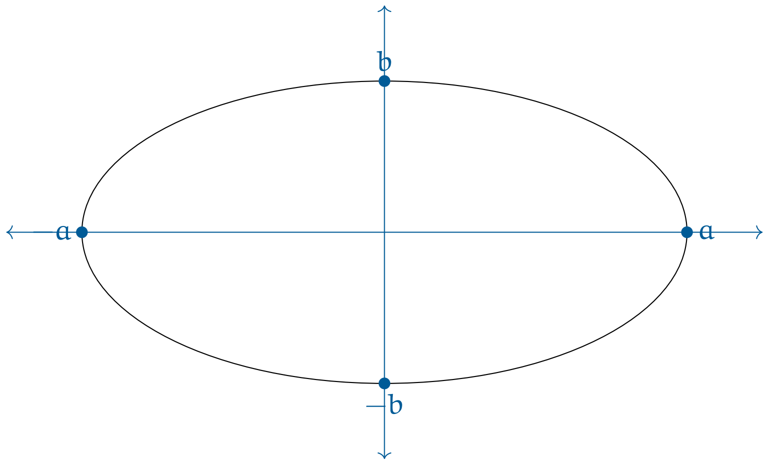
Mathematicians talk about a *standard* ellipse. A standard ellipse is centered on the origin $(0, 0)$ and its long axis is parallel with the x-axis or the y-axis.

Equation for a Standard Ellipse

To be precise, a standard ellipse is the set of points (x, y) that are solutions to the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Note that $(a, 0), (-a, 0), (0, b), (0, -b)$ are all part of the set. The complete set looks like this:

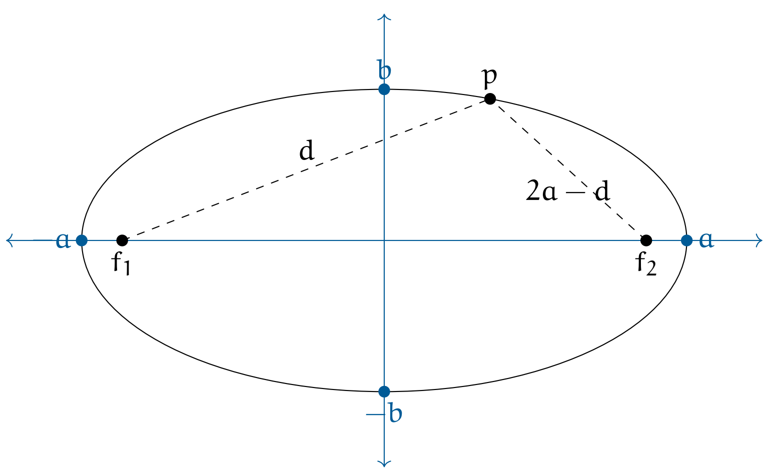


The area contained inside this ellipse is given by

$$A = \pi a b$$

We can now talk about two special points: the *foci*. Each focal point is on the long axis of the ellipse. Let's assume for a moment that $a > b$. (Everything works the same if $b > a$, but it gets confusing if we try to deal with both cases simultaneously.)

If p is a point on the ellipse, the distance from p to focal point 1 plus the distance from p to focal point 2 is always $2a$.



How do we find the foci? We know they are on the long axis and that they are symmetrical across the short axis. All we need to know is how far are they from the short axis.

Distance from Center to the Foci

If you have an ellipse with a long axis that extends a from the center, and a short axis that extends b from the center, the foci lie on the long axis and are c from the center. Where

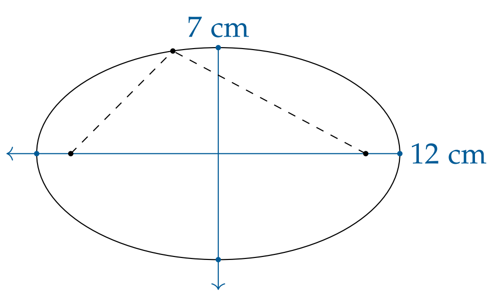
$$c = \sqrt{a^2 - b^2}$$

Exercise 5 Foci of an ellipse

Working Space

You need to draw an ellipse that is 12 cm long and 7 cm wide. You have a string, two pushpins, a ruler, and a pencil. Using the ruler, you draw two perpendicular axes.

You will stick one pin at each focal point. Each end of the string will be tied to a push pin. Using the pencil to keep the string taut, you will draw an ellipse.



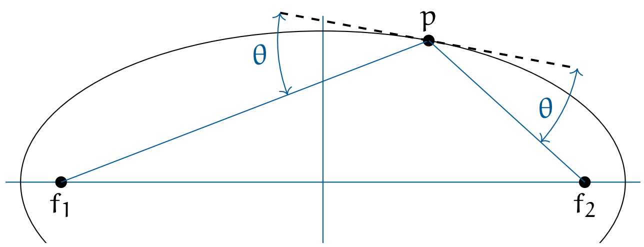
How far from the short axis are the pushpins placed?

How long is the string between them?

Answer on Page 52

The Reflective Property of Ellipses

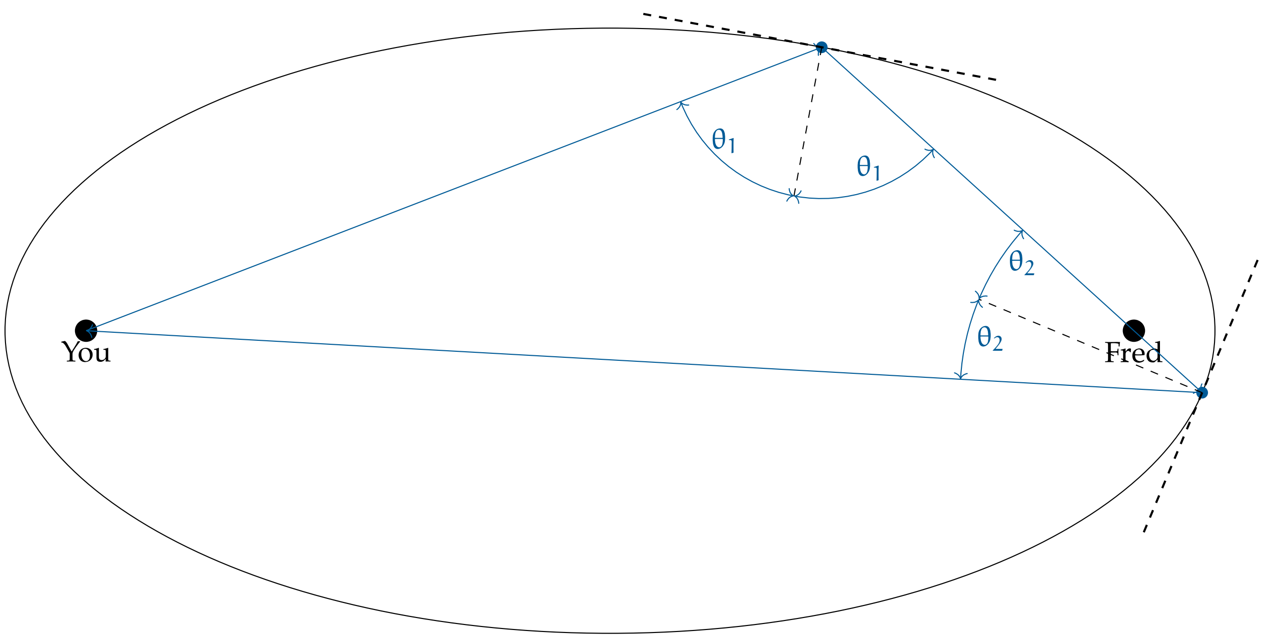
Here is something else that is wonderful about an ellipse: Pick any point p on the ellipse. Draw a line from p to each focal point. Draw the line tangent to the ellipse at p. You will see that the angle between the tangent and the line to focal point 1 is equal to the angle between the tangent and the line to focal point 2.



This is known as "The Reflective Property of Ellipses".

Imagine you and your friend Fred are at an ellipse-shaped skating rink, and the edge of the rink is mirrored. You sit at one focal point and your friend sits at the other. If you point a flashlight at the mirror (in any direction!), the beam will bounce off the wall and head directly for Fred.

If Fred ducks out of the way, the beam will bounce again and head back to you.



This will work for sound as well. If you whisper while on the focal point, Fred (at the other focal point) will hear you surprisingly well, because all the soundwaves that hit the wall will bounce (just like the light) straight at Fred.

4.2.3 Elliptical Orbits

One more fun fact about ellipses: We often imagine the planets traveling in circular orbits with the sun at the center — they actually travel in elliptical orbits, with the sun as one of the focal points.

The earth is closest to the sun around January 3rd: 147 million km.

The earth is farthest from the sun around July 3rd: 152 million km.

(Note that these dates are not the same as the solstices: The southern hemisphere is tilted the most toward the sun around December 21 and tilted most away around June 21.)

4.2.4 Ellipsoids

Just as we can pull the ideas of a circle into three dimensions to make a sphere, we can extend the ideas of the ellipse into three dimensions to talk about ellipsoids. Ellipsoids are like blimps.

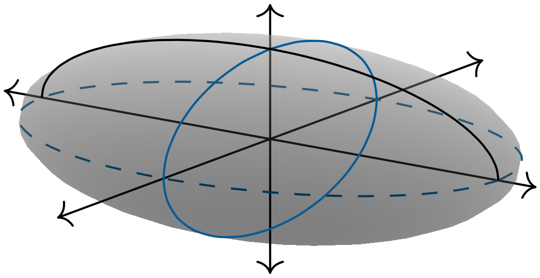
The standard ellipsoids are centered at the origin and aligned with the three axes.

Equation for a Standard Ellipsoid

To be precise, a standard ellipse is the set of points (x, y, z) that satify the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Note that $(a, 0, 0), (0, b, 0), (0, 0, c)$ are all part of the set. The complete set looks like this:



The volume bounded by this ellipsoid is

$$V = \frac{4}{3}\pi abc$$

Of course, a, b, and c can be any positive number, but in the real world, we find ourselves working regularly with ellipsoids where two of the numbers are the same.

Oblate Spheroid

If two axes have the same length and one is shorter, you get something that looks like a sphere compressed in one direction — like a pumpkin. These are called *oblate spheroids*.

The earth is actually an oblate spheroid; the axis that goes through the north and south pole is shorter than the axes that pass through the equator. How much shorter? Just a little, relatively speaking. The equator is 6,378 km from the center of the earth; the north pole is 21 km closer.

Prolate Spheroid

If two axes have the same length and one is longer, you get something that looks like a sphere stretched in one direction — like a rugby ball. It is called a *prolate spheroid*.

Like an ellipse, prolate spheroids have two focal points.

Focal Points of a Prolate Spheroid

If the long axis has a radial length of a and the two shorter axes have radial length b, then the focal points are on the long axis. The distance from the center to the focal point is given by

$$c = \sqrt{a^2 - b^2}$$

For any point p on the prolate spheroid, the sum of the distances from p to the focus points will always be $2a$.

It has the reflective property: A photon shot in any any direction from one focal point will bounce off the wall and head directly at the other.

Exercise 6 Volume of Ruby Ball

Working Space

Some jokesters once thought it would be fun to make something that looked like a rugby ball, but made out of lead.

A rugby ball is about 30 cm long and has a circumference of 60 cm at its midpoint. A cubic centimeter of lead has a mass of 11.34 grams.

How much would a solid (not hollow) lead ruby ball weigh?

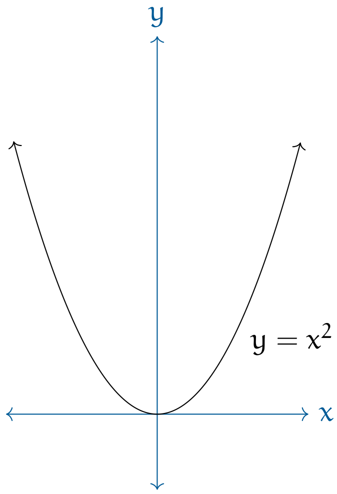
Answer on Page 52

4.2.5 Parabolas and Parabolic Reflectors

You are familiar with quadratic functions:

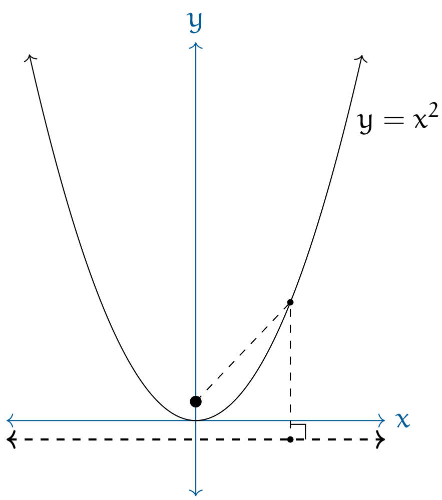
$$y = ax^2 + bx + c$$

If a is not zero, the graph of a quadratic is a curved line called a *parabola*. The first parabola that most mathematicians think of is the graph of $y = x^2$:



Every parabola has a *focus* and a *directix*. The focus is a point on the parabola's axis of symmetry. The directrix is a line perpendicular to the axis of symmetry. Every point on the parabola is equal distance from the focus and the directrix.

For the graph of $y = x^2$, the focus is the point $(0, \frac{1}{4})$. The directrix is the line $y = -\frac{1}{4}$:



For example, the point $(1, 1)$ is on this parabola. It is $5/4$ from the directorix. How far is it from the focus? 1 horizontally and $3/4$ vertically.

$$\sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}$$

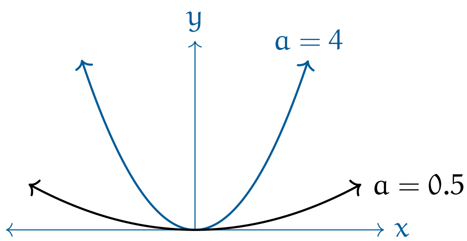
Thus, we have confirmed that $(1, 1)$ is equal distances from the focus and the directrix.

When we think about a parabola and its properties, we usually rotate and translate it to be symmetric around the y-axis, flip it so that it is low in the middle and rising on both sides, and push it up or down until the low point is is on the x-axis.

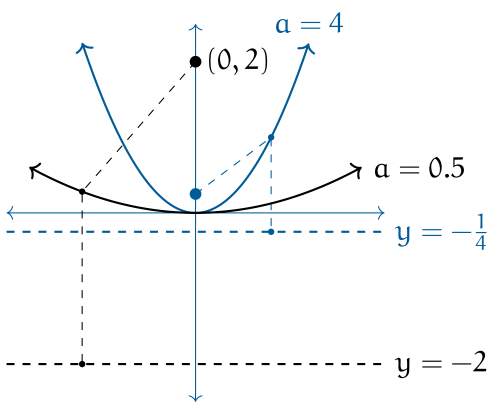
Then, they can all be written:

$$y = \frac{a}{4}x^2$$

where $a > 0$. If a is small, the parabola opens wider.

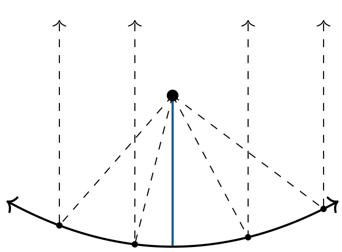


The focus is at $(0, \frac{1}{a})$ and the directorix is the line $y = -\frac{1}{a}$.



Reflective Property of a Parabola

Assume you have a parabola-shaped mirror. A beam of light shot from the focus will bounce off the mirror in the direction of the axis of symmetry:



This is why your flashlight has a parabolic mirror. The lightbulb is at the focus, so any photons that hit the mirror are redirected straight forward.

(Note that in the real world, we use parabolic dishes: a rotated around its axis of symmetry.)

The reflection works exactly the same in reverse. There are solar cookers that are big parabolic mirrors. They let you put a pot on the focus point. You move the dish until its axis of symmetry is pointed at the sun.

You will also see a lot of antennas have parabolic dishes. Note that photons that come in parallel to the axis of symmetry are redirected to a single point: where the receiver is.

Sometimes in a science museum, you will see two parabolic dishes far apart and pointed at each other. One person speaks with their mouth at the focus of one. The other person listens with their ear at the focus of the other. Even though you are very far apart, it sounds like they are really, really close.

This is because the speaker's parabolic wall focuses the sound energy in a nice beam the size of the wall pointed straight at the listener's parabolic wall. The listener's wall focuses the energy of that beam at the listener's ear.

Chapter 5

Refraction

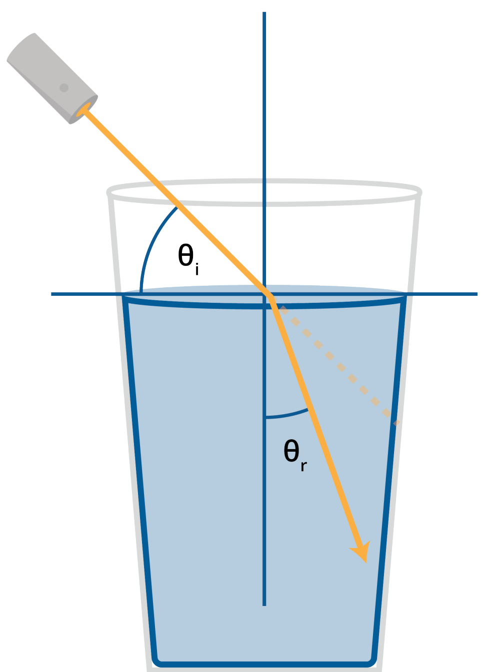
The refraction of light is a phenomenon where light changes its direction when it passes from one medium to another. The change in direction is due to a change in the speed of light as it moves from one medium to another.

This phenomenon is explained by Snell's law, which states:

$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2) \tag{5.1}$$

where:

- n_1 and n_2 are the indices of refraction for the first and second media, respectively. The index of refraction is the ratio of the speed of light in a vacuum to the speed of light in the medium. It is a dimensionless quantity.
- θ_1 and θ_2 are the angles of incidence and refraction, respectively. These angles are measured from the normal (perpendicular line) to the surface at the point where light hits the boundary.



The angle of incidence (θ_1) is the angle between the incident ray and the normal to the interface at the point of incidence. Similarly, the angle of refraction (θ_2) is the angle between the refracted ray and the normal.

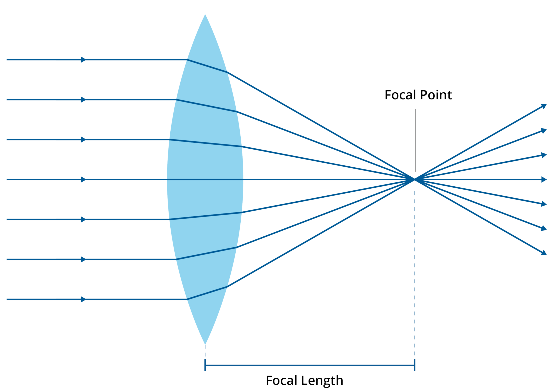
When light travels from a medium with a lower refractive index to a medium with a higher refractive index, it bends towards the normal. Conversely, when light travels from a medium with a higher refractive index to one with a lower refractive index, it bends away from the normal.

Chapter 6

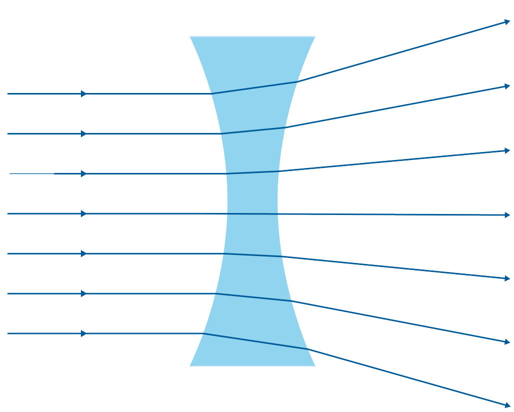
Lenses

Lenses are optical devices with perfect or approximate axial symmetry that transmit and refract light, converging or diverging the beam. There are two main types of lenses, distinguished by their shape and the way they refract light:

- **Converging (or Convex) Lenses:** These are thicker at the center than at the edges. When parallel light rays enter a convex lens, they converge to a point called the focal point. Examples of converging lenses include magnifying glasses and camera lenses.



- **Diverging (or Concave) Lenses:** These are thinner at the center than at the edges. When parallel light rays enter a concave lens, they diverge or spread out. These lenses are often used in glasses to correct nearsightedness.



6.1 Focal Length

The focal length of a lens is the distance between the center of the lens and the focal point. It is determined by the lens shape and the refractive index of the lens material. For a converging lens, the focal length is positive; for a diverging lens, the focal length is negative.

6.2 Refractive Index

The refractive index of a material is a measure of how much the speed of light is reduced inside the material. The refractive index n of a material is given by the ratio of the speed of light in a vacuum c to the speed of light v in the material:

$$n = \frac{c}{v}$$

The refractive index affects how much a light ray changes direction, or refracts, when entering the material at an angle. A higher refractive index indicates that light travels slower in that medium, and the light ray will bend more towards the normal.

Lenses work by refracting light at their two surfaces. By choosing the right lens shape and material, lenses can be designed to bring light to a focus, spread it out, or perform more complex transformations.

Chapter 7

Images in Python

An image is usually represented as a three-dimensional array of 8-bit integers. NumPy arrays are the most commonly used library for this sort of data structure.

If you have an RGB image that is 480 pixels tall and 640 pixels wide, you will need a $480 \times 640 \times 3$ NumPy array.

There is a separate library (imageio) that:

- Reads an image file (like JPEG files) and creates a NumPy array.
- Writes a NumPy array to a file in standard image formats

Let's create a simply python program that creates a file containing an all-black image that is 640 pixels wide and 480 pixels tall. Create a file called `create_image.py`:

```
import NumPy as np
import imageio
import sys

# Check command-line arguments
if len(sys.argv) < 2:
    print(f"Usage {sys.argv[0]} <outfile>")
    sys.exit(1)

# Constants
IMAGE_WIDTH = 640
IMAGE_HEIGHT = 480

# Create an array of zeros
image = np.zeros((IMAGE_HEIGHT, IMAGE_WIDTH, 3), dtype=np.uint8)

# Write the array to the file
imageio.imwrite(sys.argv[1], image)
```

To run this, you will need to supply the name of the file you are trying to create. The extension (like .png or .jpeg) will tell imageio what format you want written. Run it now:

```
python3 create_image.py blackness.png
```

Open the image to confirm that it is 640 pixels wide, 480 pixels tall, and completely black.

7.1 Adding color

Now, let's walk through through the image, pixel-by-pixel, adding some red. We will gradually increase the red from 0 on the left to 255 on the right.

```
import NumPy as np
import imageio
import sys

# Check command-line arguments
if len(sys.argv) < 2:
    print(f"Usage sys.argv[0] <outfile>")
    sys.exit(1)

# Constants
IMAGE_WIDTH = 640
IMAGE_HEIGHT = 480

# Create an array of zeros
image = np.zeros((IMAGE_HEIGHT, IMAGE_WIDTH, 3), dtype=np.uint8)

for col in range(IMAGE_WIDTH):

    # Red goes from 0 to 255 (left to right)
    r = int(col * 255.0 / IMAGE_WIDTH)

    # Update all the pixels in that column
    for row in range(IMAGE_HEIGHT):
        # Set the red pixel
        image[row, col, 0] = r

# Write the array to the file
imageio.imwrite(sys.argv[1], image)
```

When you run the function to create a new image, it will be a fade from black to red as you move from left to right:

Now, inside the inner loop, update the blue channel so that it goes from zero at the top to 255 at the bottom:

```
    # Update all the pixels in that column
    for row in range(IMAGE_HEIGHT):

        # Update the red channel
        image[row,col,0] = r

        # Blue goes from 0 to 255 (top to bottom)
        b = int(row * 255.0 / IMAGE_HEIGHT)
        image[row,col,2] = b

imageio.imwrite(sys.argv[1], image)
```

When you run the program again, you will see the color fades from black to blue as you go down the left side. As you go down the right side, the color fades from red to magenta.

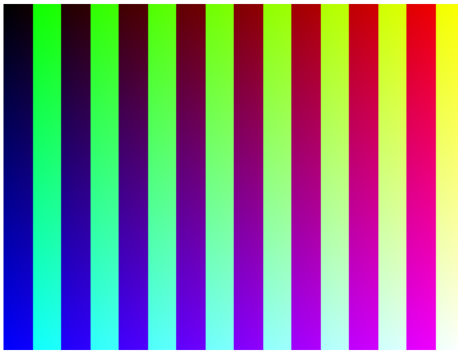
Notice that red and blue with no green looks magenta to your eye.

Next, let's add some stripes of green:

```
import NumPy as np
```

```
import imageio
import sys

# Check command line arguments
if len(sys.argv) < 2:
    print(f"Usage sys.argv[0] <outfile>")
    sys.exit(1)

# Constants
IMAGE_WIDTH = 640
IMAGE_HEIGHT = 480
STRIPE_WIDTH = 40
pattern_width = STRIPE_WIDTH * 2

# Create an image of all zeros
image = np.zeros((IMAGE_HEIGHT, IMAGE_WIDTH, 3), dtype=np.uint8)

# Step from left to right
for col in range(IMAGE_WIDTH):

    # Red goes from 0 to 255 (left to right)
    r = int(col * 255.0 / IMAGE_WIDTH)

    # Should I add green to this column?
    should_green = col % pattern_width > STRIPE_WIDTH

    # Update all the pixels in that column
    for row in range(IMAGE_HEIGHT):

        # Update the red channel
        image[row,col,0] = r

        # Should I add green to this pixel?
        if should_green:
            image[row,col,1] = 255

        # Blue goes from 0 to 255 (top to bottom)
        b = int(row * 255.0 / IMAGE_HEIGHT)
        image[row,col,2] = b

imageio.imwrite(sys.argv[1], image)
```

When you run this version, you will see the previous image in half the stripes. In the other half, you will see that green fades to cyan down the left side, and yellow fades to white down the right side.



7.2 Using an existing image

imageio can also be used to read in any common image file format. Let's read in an image and save each of the red, green, and blue channels out as its own image.

Create a new file called `separate_image.py`:

```
import imageio
import sys
import os

# Check command line arguments
if len(sys.argv) < 2:
    print(f"Usage {sys.argv[0]} <infile>")
    sys.exit(1)

# Read the image
path = sys.argv[1]
image = imageio.imread(path)

# What is the filename?
filename = os.path.basename(path)

# What is the shape of the array?
original_shape = image.shape

# Log it
print(f"Shape of {filename}:{original_shape}")

# Names of the colors for the filenames
colors = ['red','green','blue']

# Step through each of the colors
```

```
for i in range(3):

    # Create a new image
    newimage = np.zeros(original_shape, dtype=np.uint8)

    # Copy one channel
    newimage[:,:,i] = image[:,:,i]

    # Save to a file
    new_filename = f"{colors[i]}_{filename}"
    print(f"Writing {new_filename}")
    imageio.imwrite(new_filename, newimage)
```

Now, you can run the program with any common RGB image type:

```
python3 separate_image.py dog.jpg
```

This will create three images: `red_dog.jpg`, `green_dog.jpg`, and `blue_dog.jpg`.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 3)

$$\frac{300 \times 10^6}{5.66 \times 10^{14}} = 530 \times 10^{-9} = 530 \text{ nm}$$

Answer to Exercise 2 (on page 13)

The two triangles are similar; one is 2 m and 3m, the other is x cm and 3 cm.

The image of the cow is 2 cm tall.

Answer to Exercise 3 (on page 27)

Assuming the mirror is truly vertical and the floor is truly horizontal, the new cut off should be exactly the same as the old one: It should be below your chin the same amount that your eyes are above your chin.

Illustration Here

Answer to Exercise 4 (on page 28)

Are there white photons? No. What we call "white" is a blend of photons that are several different colors.

Some people like to say white light is the combination of all visible colors of photons in equal amounts. That seems oddly specific and unusual.

Maybe it is better to imagine it from the human experience of white light. In our eyes, we have three different types of color-sensing cones, which generally correspond to the the red, green, and blue regions of the spectrum. When all three are excited about equal amounts, humans experience that as white. On your computer screen, for example, what you see as white is just a blend of three colors of photons: a red, a green, and a blue.

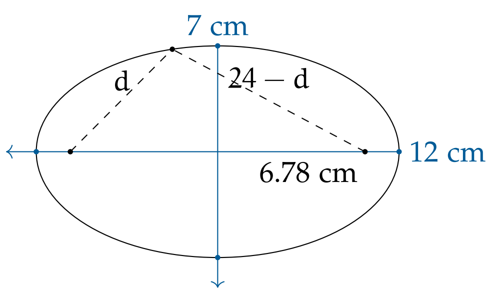
Are there black photons? No. What we call "black" is an absence of photons in the visible range.

Are there yellow photons? Yes! There is a region of the color spectrum that is yellow. It has a wavelength of about 527 nm. Photons at this energy level excite both our green-sensitive and red-sensitive cones. Your computer monitor does not actually create light with a 527 nm wavelength. Instead, it creates red light and green light, which our eyes interpret as yellow.

Answer to Exercise 5 (on page 31)

The length of the string is easy: $2 \times 12 = 24$ cm.

The distance from the center to the focal point is $\sqrt{12^2 - 7^2} approx 6.78$ cm.



Answer to Exercise 6 (on page 35)

We need the distance from the center out to each of the three axes. We know that $a = \left(\frac{1}{2}\right) 30 = 15$ cm.

We can calculate the b and c (which are equal) using the circumference given: $2b\pi = 60$, so $c = b \approx 9.55$ cm.

The volume, then is

$$V = \frac{4}{3}\pi(15)(9.55)(9.55) \approx 5,730 \text{ cubic centimeters}$$

The mass would be $5,730 \times 11.34 = 64,973$ grams or about 65 kg.

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