Contents

Chapter 1

Introduction to the Kontinua Sequence

The purpose of this book is to help you along the long and difficult trek to becoming a modern problem solver. As you explore this path, you will learn how to use the tools of math, computers, and science.

If this path is so arduous, it is only fair to ask why you should bother in the first place. There are big problems out there that will require expert problem solvers. Those people will make the world a better place, while also enjoying interesting and lucrative careers. We are talking about engineers, scientists, doctors, computer programmers, architects, actuaries, and mathematicians. Right now, those occupations represent about 6% of all the jobs in the United States. Soon, that number is expected to rise above 10%. On average, people in that 10% of the population are expected to have salaries twice that of their non-technical counterparts.

Solving problems is difficult. At some point on this journey, you will see people who are better at solving problems than you are. You, like every other person who has gone on this journey, may think "I have worked so hard on this, but that person is better at it than I am. I should quit." *Don't*.

Instead, remember these two important facts. First, solving problems is like a muscle. The more you do, the better you get at it. It is OK to say "I am not good at this yet." That just means you need more practice.

Second, you don't need to be the best in the world. 10 million people your age can be better at solving problems than you, *and you can still be in the top 10% of the world*. If you complete this journey, there will be problems for you to solve and a job where your problem-solving skills will be appreciated.

Where do we start?

The famous physicist Richard Feynman once asked, "If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence was passed on to the next generation of creatures, what statement would contain the most information in the fewest words?"

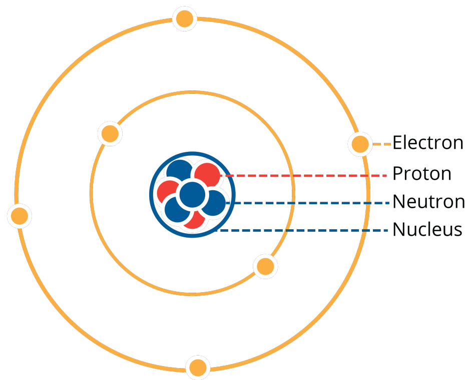
His answer was "All things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.”

That seems like a good place to start.

Chapter 2

Matter and Energy

All things (including the air around you) are made of atoms. Atoms are incredibly tiny — there are more atoms in a drop of water than there are drops of water in all the oceans.

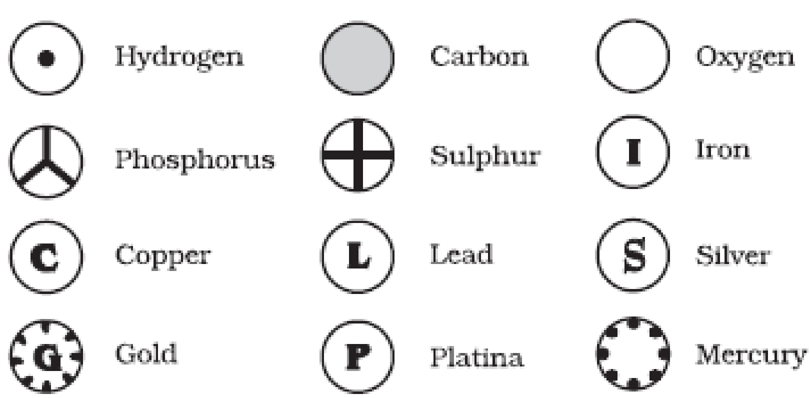


Every atom has a nucleus that contains protons and neutrons. Orbiting around the nucleus is a cloud of electrons. However, the mass of the atom comes mainly from the protons and neutrons, since they are about 2000 times as massive as an electron!

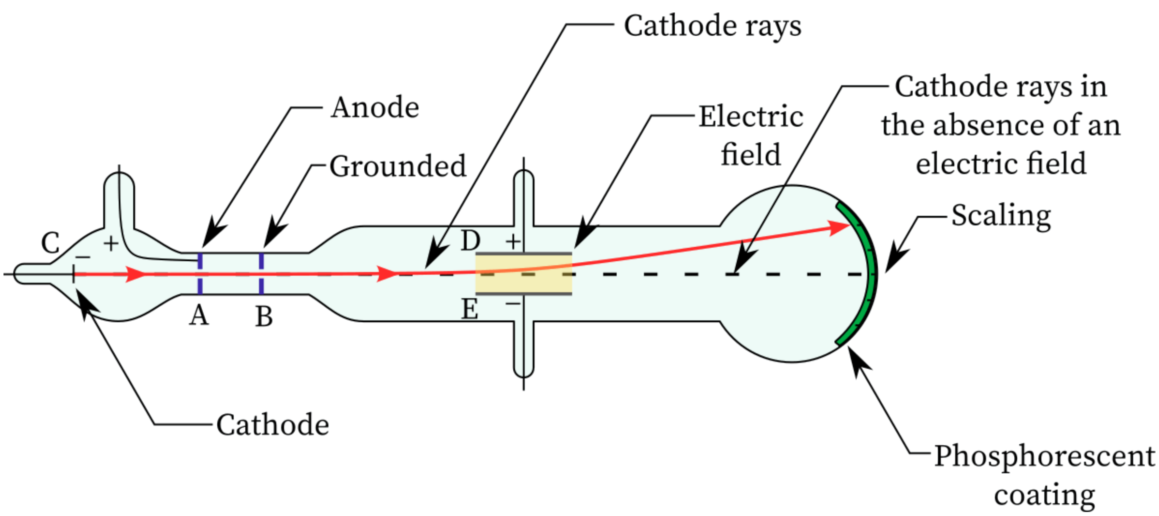
2.0.1 Models of the Atom

Over the history of science, there have been many ideas about the structure of atoms. This history is a good example of how science develops: unexpected results drive scientists to update their models, moving us closer and closer to a true model of the atom.

During his investigations into the behavior of gases, John Dalton (lived 1766-1844) noted that different elements combine in strict ratios. For example, he noted that nitrogen and oxygen combine in a 1:1 and 1:2 fashion, but no ratio in between.

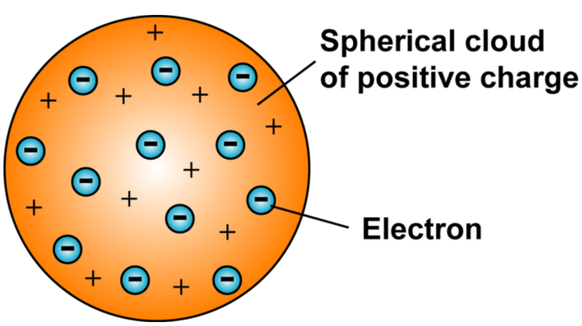


This first model of the atom is rudimentary; each element is a unique atom, and those atoms cannot be subdivided. The atom is modeled as one large, solid, uniform, and neutral object. Some scientists, including the British physicist J.J. Thomson (1856-1940) thought that larger atoms (like lead) might be able to be broken down into smaller atoms (like hydrogen). Thomson had been experimenting with cathode ray tubes and discovered that the these rays traveled much faster than thought possible for a particle the size of a hydrogen atom.



This, combined with the observation that cathode rays could be deflected by electrical charge, led him to postulate two things:

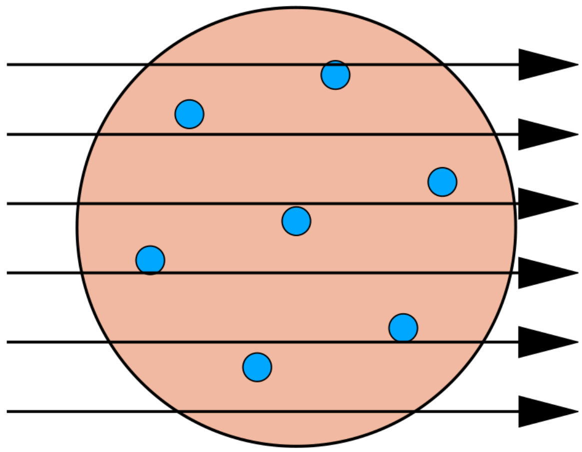
1. Atoms can be broken into parts smaller than a hydrogen atom
2. The part of atoms that composes cathode rays is negatively charged



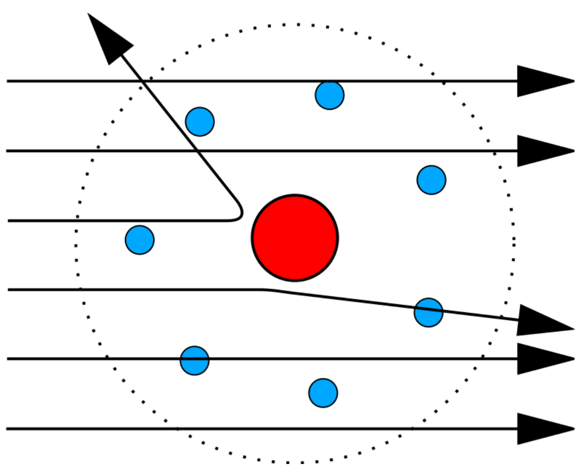
The presence of "corpuscules" (as Thomson called them) that were negatively charged and smaller than a hydrogen atom contradicted Dalton's theory. Thomson updated his model of the atom, adding small, negatively charged subatomic particles (now called electrons) that were embedded in a larger, uniform, positive sphere. Suddenly, the atom went from neutral and indivisible to made of different types of charged particles.

At the time, physicists were very interested in the mass-to-charge ratios of various particles (Thomson was able to determine the mass-to-charge ratio of the electron during his experiments), and Ernest Rutherford (1871-1937) was investigating the mass-to-charge ratio of alpha particles. (Alpha particles, we now know, are composed of two protons and two neutrons. They are emitted from certain radioactive elements, including uranium.)

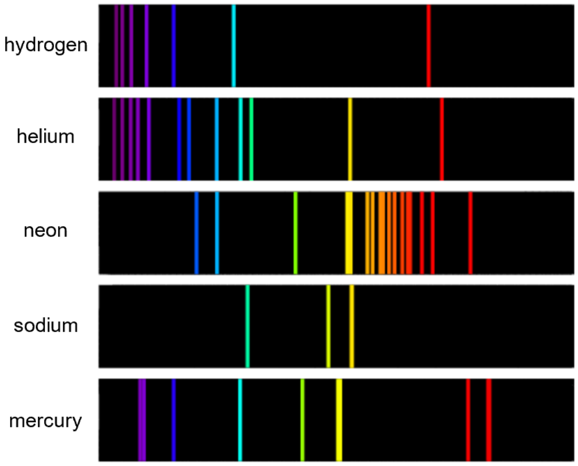
Rutherford needed consistent scattering of alpha particles in order to collect the data necessary to determine the particles' mass-to-charge ratio. He achieved this by bombarding extremely thin gold foil with alpha particles. The Thomson model of the atom would predict that particles would be slightly deflected, as illustrated below:



However, a small but significant portion of the alpha particles were deflected over 90 deg! To explain this, Rutherford modeled the atom as mostly empty space with a small, dense, positive center (we now call this the nucleus).



At the same time that Rutherford was conducting his gold foil experiments, Niels Bohr was investigating the hydrogen line series. FIXME insert figure of hydrogen lines. When hydrogen is electrically excited, it emits specific bands of color, not a complete spectrum. Every element has a unique emission spectrum.



Bohr, upon learning of Rutherford's experiments, embraced the Rutherford model over the Thomson model and postulated that electrons existed only at discrete distances from the nucleus. When electrified, a hydrogen atom's electrons would gain energy and "jump" up one or more levels. The electron would be unstable in this energized state, and eventually "fall" back to the lowest energy level, emitting the extra energy as light. different colors of light have different energies: violet being the most energetic and red being the least. The different levels had differing amounts of energy between them, resulting in only those colors corresponding to the exact energy step between levels being emitted. This model, called the Bohr model or the Rutherford-Bohr model, expands on the Rutherford model by limiting electrons to specific distances from the nucleus, and is often compared to a model of the solar system FIXME add image of Bohr model.

This is likely the model you are most familiar with seeing, and it is the one we will use most often in this text.

The previous graphic is slightly inaccurate. While it is a convenient model for thinking about atoms, in reality, electrons don't neatly orbit the nucleus. Scientists don't know exactly where an electron will be in relation to the nucleus, but they do know where it is most likely to be. They use a cloud that is thicker in the center but fades out at the edges to represent an electron's position.

We classify atoms by the numbers of protons they have. An atom with one proton is a hydrogen atom, an atom with two protons is a helium atom, and so forth (refer to periodic table on pg..). We say that hydrogen and helium are *elements* because the classification of elements is based on the proton number. And we give each element an atomic symbol. Hydrogen gets H, helium gets He, oxygen gets O, carbon gets C, and so on.

Often, two hydrogen atoms will attach to an oxygen atom. The result is a water molecule. Why do they cluster together? Because they share electrons in their clouds.

A molecule is described by the elements it contains. Water is H_2O because it has two hydrogen atoms and one oxygen atom.

There are many kinds of molecules. You know a few:

- Table salt is crystals made of $NaCl$ molecules: a sodium atom attached to a chlorine atom.
- Baking soda, or sodium bicarbonate, is $NaHCO_3$.
- Vinegar is a solution including acetic acid (CH_3COOH).
- O_2 is the oxygen molecules that you breathe out of the air (air, a blend of gases, is mostly N_2.).

2.0.2 Reading the Periodic Table

The Periodic Table organizes what we know about the structure of different elements. Each element has its own block or tile on the Periodic Table, and the information on the tile tells us about the structure of that atom. Take a look at the tile for carbon: (FIXME add carbon tile)

There are two key numbers: the atomic number and the average atomic mass. The atomic number tells us how many protons there are in the nucleus of any atom of carbon. All carbon atoms have 6 protons. The other number is the average atomic mass.

Have you heard of carbon-14 dating? The phrase "carbon-14" refers to a rare type of carbon that decays radioactively. By seeing how much carbon-14 has decayed, scientists can estimate the age of organic materials, such as bone or ash. Carbon-14 is a radioactive isotope (or version) of carbon. The 14 refers to the mass number - the total amount of protons AND neutrons in the nucleus. The most common isotope of carbon is carbon-12, with 6 protons and 6 neutrons in its nucleus. Carbon-14, on the other hand, has 8 neutrons, which makes the nucleus unstable, leading to radioactive decay. FIXME tow models comparing the structure of C-12 and C-14. FIXME resource: atom builder PhET. The average atomic mass is the weighted average of all the carbon atoms in existence. Since the vast majority of carbon is carbon-12, the average atomic mass is very close to 12. You cannot determine the mass number of an individual atom from the periodic table; it only tells you the average of all the isotopes. However, especially for light atoms (atoms in the first two rows of the periodic table), you can usually determine the mass number of the most common isotope by rounding the average atomic mass to the nearest whole number.

2.1 Chemical Reaction

Sometimes two hydrogen atoms form a molecule (H_2). Sometimes two oxygen atoms form a molecule (O_2). If you mix these together and light a match, they will rearrange themselves into water molecules. This is called a *chemical reaction*. In any chemical reaction, the atoms are rearranged into new molecules.

Some chemical reactions (like the burning of hydrogen gas described above) are *exothermic* — that is, they give off energy. Burning hydrogen gas happens quickly and gives off a lot of energy. If you have enough, it will make quite an explosion!

Other chemical reactions are *endothermic* — they consume energy. Photosynthesis, the process by which plants consume energy from the sun to make sugar from CO_2 and H_2O requires an endothermic chemical reaction.

2.2 Mass and Acceleration

Each atom has a mass, which means everything made up of those atoms has mass as well (and that's pretty much everything!). We measure mass in grams. A paper clip is about 1 gram of steel. An adult human can weigh 70,000 grams, so for larger things, we often talk about kilograms, which is 1000 grams.

The first interesting thing about mass is that objects with more mass require more force to accelerate. For example, pushing a bicycle so that it accelerates from a standstill to jogging speed in 2 seconds requires much less force than pushing a train so that it accelerates at the same rate.

Newton's Second Law of Motion

The force necessary to accelerate an object of mass m is given by:

$$F = ma$$

This means the force is equal to the mass times the acceleration.

What are the units here? We already know that mass is measured in kilograms. We can measure velocity in meters per second, but that is different from acceleration. Acceleration is the rate of change in velocity. So if we want to go from 0 to 5 meters per second (that's jogging speed) in two seconds, that is a change in velocity of 2.5 meters per second every second. We would say this acceleration is $2.5m/s^2$.

What about measuring force? Newton decided to name the unit after himself: The force necessary to accelerate one kilogram at $1m/s^2$ is known as *a newton*. It is often denoted by the symbol N.

Exercise 1 Acceleration

Working Space

While driving a bulldozer, you come across a train car (with no brakes and no locomotive) sitting on a track in the middle of a city. The train car has a label telling you that it weighs 2,400 kg. There is a timebomb welded to the interior of the train car, and the timer tells you that you can safely push the train car for 120 seconds. To get the train car to where it can explode safely, you need to accelerate it to 20 meters per second. Fortunately, the track is level and the train car's wheels have almost no rolling resistance.

With what force, in newtons, do you need to push the train for those 120 seconds?

Answer on Page 47

2.3 Mass and Gravity

The second interesting thing about mass is that masses are attracted to each other by the force we call *gravity*. The force of attraction between two objects is proportional to the product of their masses, and inversely proportional to their distance squared. This means that as objects get farther away, the force decreases. This is why you are more attracted to the earth than you are to distant stars, even though they have much more mass than the earth.

Newton's Law of Universal Gravitation

Two masses (m_1 and m_2) that are a distance of r from each other are attracted toward each other with a force of magnitude:

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant. If you measure the mass in kilograms and the distance in meters, G is about 6.674×10^{-11}. That will get you the force of the attraction in newtons.

Exercise 2 Gravity

Working Space

The earth's mass is about 6×10^{24} kilograms.

Your spacecraft's mass is 6,800 kilograms.

Your spacecraft is also about 100,000 km from the center of the earth. (For reference, the moon is about 400,000 km from the center of the earth.)

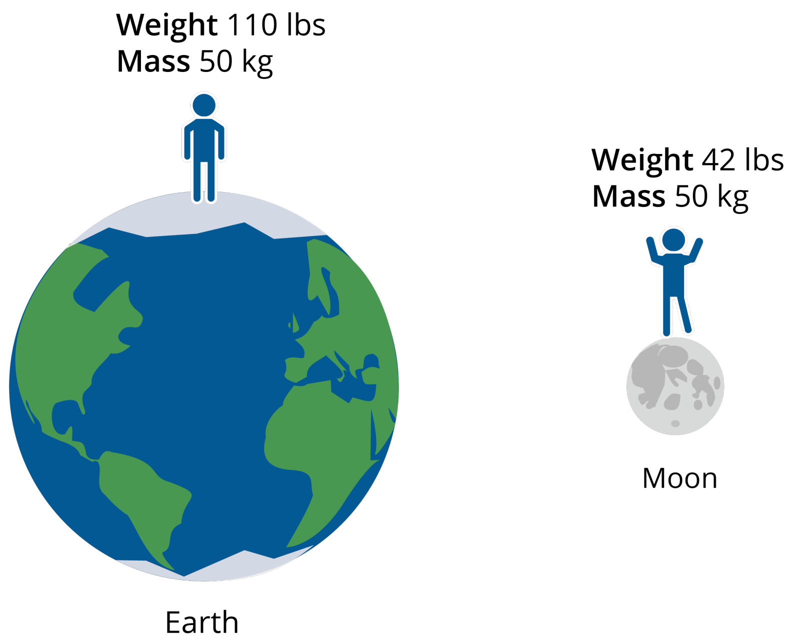
What is the force of gravity that is pulling your spacecraft and the earth toward each other?

Answer on Page 47

2.4 Mass and Weight

Gravity pulls on things proportional to their mass, so we often ignore the difference between mass and weight.

The weight of an object is the force due to the object's mass and gravity. When we say, "This potato weighs 1 pound," we actually mean "This potato weighs 1 pound on earth." That same potato would weigh about one-fifth of a pound on the moon.



However, that potato has a mass of 0.45 kg no matter where it is in the universe.

FIXME Global layout note: Let's discuss adding Title's and Captions to all graphics.

For example:
TITLE: Mass versus Weight
CAPTION: Human Earth weight: 150lbs / Moon weight:??lbs
Potato Earth weight: .25lbs / Moon weight: ??lbs

FIXME: Allison thinks it would be funny if the person in the graphic were holding a potato and we also added the weight and mass of the potato to the caption. No worries if this type of edit isn't in the budget!

FIXME: What are your thoughts about using the metric system consistently – in which case we'll replace pounds here with kilos. Max notes: we should explicitly use kilos for mass and pounds or newtons for weight. Kilos are a scalar measure of the amount of matter and pounds are a vector force of gravity on a particular piece of matter. Many students struggle to differentiate between mass and weight at a theoretical level due to casual comparison between pounds and kilos.

Chapter 3

Atomic and Molecular Mass

A proton and a neutron have about the same mass. An electron, on the other hand, has much less mass: One neutron weighs about the same amount as 2000 electrons. This means that the mass of any object comes mostly from the protons and neutrons in the nucleus of its atoms.

We know how many protons an atom has by what element it is, but how do we know the number of neutrons?

If you fill a balloon with helium, it will have two different kinds of helium atoms. Most of the helium atoms will have 2 neutrons, but a few will have only 1 neutron. We say that these are two different *isotopes* of helium. We call them helium-4 (or ^{4}He) and helium-3 (or ^{3}He). Isotopes are named for the sum of protons and neutrons the atom has: helium-3 has 2 protons and 1 neutron.

A hydrogen atom nearly always has just 1 proton and no neutrons. A helium atom nearly always has 2 protons and 2 neutrons. So, if you have a 100 hydrogen atoms and 100 helium atoms, the helium will have about 4 times more mass than the hydrogen. We say "Hydrogen is about 1 atomic mass unit (amu), and helium-4 is about 4 atomic mass units."

What, precisely, is an atomic mass unit? It is defined as 1/12 of the mass of a carbon-12 atom. Scientists have measured the mass of helium-4, and it is about 4.0026 atomic mass units. (By the way, an atomic mass unit is also called a *dalton*.)

Now you are ready to take a good look at the periodic table of elements. Here is the version from Wikipedia:

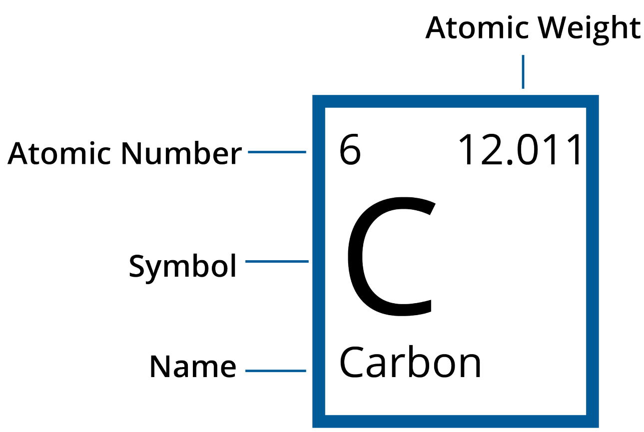
Periodic Table of Elements

IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIIIB	VIIIB	VIIIB	IB	IIB	IIIA	IVA	VA	VIA	VIIA	VIIIA
1 **H** Hydrogen 1.01																	2 **He** Helium 4.00
3 **Li** Lithium 6.94	4 **Be** Beryllium 9.01											5 **B** Boron 10.81	6 **C** Carbon 12.01	7 **N** Nitrogen 14.01	8 **O** Oxygen 16.00	9 **F** Fluorine 19.00	10 **Ne** Neon 20.18
11 **Na** Sodium 22.99	12 **Mg** Magnesium 24.31											13 **Al** Aluminum 26.98	14 **Si** Silicon 28.09	15 **P** Phosphorus 30.97	16 **S** Sulfur 32.06	17 **Cl** Chlorine 35.45	18 **Ar** Argon 39.95
19 **K** Potassium 39.10	20 **Ca** Calcium 40.08	21 **Sc** Scandium 44.96	22 **Ti** Titanium 47.87	23 **V** Vanadium 50.94	24 **Cr** Chromium 52.00	25 **Mn** Manganese 54.94	26 **Fe** Iron 55.85	27 **Co** Cobalt 58.93	28 **Ni** Nickel 58.69	29 **Cu** Copper 63.55	30 **Zn** Zinc 65.38	31 **Ga** Gallium 69.72	32 **Ge** Germanium 72.63	33 **As** Arsenic 74.92	34 **Se** Selenium 78.97	35 **Br** Bromine 79.90	36 **Kr** Krypton 83.80
37 **Rb** Rubidium 85.47	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.91	40 **Zr** Zirconium 91.22	41 **Nb** Niobium 92.91	42 **Mo** Molybdenum 95.95	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.91	46 **Pd** Paladium 106.42	47 **Ag** Silver 107.87	48 **Cd** Cadmium 112.41	49 **In** Indium 114.82	50 **Sn** Tin 118.71	51 **Sb** Antimony 121.76	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90	54 **Xe** Xenon 131.29
55 **Cs** Cesium 132.91	56 **Ba** Barium 137.33	57 - 71 **Lanthanides**	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.95	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.21	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.22	78 **Pt** Platinum 195.08	79 **Au** Gold 196.97	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.38	82 **Pb** Lead 207.20	83 **Bi** Bismuth 208.98	84 **Po** Polonium (209)	85 **At** Astatine (210)	86 **Rn** Radon (222)
87 **Fr** Francium (223)	88 **Ra** Radium (226)	89 - 103 **Actinides**	104 **Rf** Rutherfordium (265)	105 **Db** Dubnium (268)	106 **Sg** Seaborgium (271)	107 **Bh** Bohrium (270)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (276)	110 **Ds** Darmstadtium (281)	111 **Rg** Roentgenium (280)	112 **Cn** Copernicium (285)	113 **Nh** Nihonium (284)	114 **Fl** Flerovium 289	115 **Mc** Moscovium (288)	116 **Lv** Livermorium (293)	117 **Ts** Tennessine (294)	118 **Og** Oganesson (294)

57 **La** Lanthanum 138.91	58 **Ce** Cerium 140.12	59 **Pr** Praseodymium 140.91	60 **Nd** Neodymium 144.24	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.96	64 **Gd** Gadolinium 157.25	65 **Tb** Terbium 158.93	66 **Dy** Dysprosium 162.50	67 **Ho** Holmium 164.93	68 **Er** Erbium 167.26	69 **Tm** Thulium 168.93	70 **Yb** Ytterbium 173.05	71 **Lu** Lutetium 174.97
89 **Ac** Actinium (227)	90 **Th** Thorium 232.04	91 **Pa** Protactinium 231.04	92 **U** Uranium 238.03	93 **Np** Neptunium (237)	94 **Pu** Plutonium (244)	95 **Am** Americium (243)	96 **Cm** Curium (247)	97 **Bk** Berkelium (247)	98 **Cf** Californium (251)	99 **Es** Einsteinium (252)	100 **Fm** Fermium (257)	101 **Md** Mendelevium (258)	102 **No** Nobelium (259)	103 **Lr** Lawrencium (262)

There is a square for each element. In the middle, you can see the atomic symbol and the name of the element. In the upper-right corner is the atomic number — the number of protons in the atom.

In the upper-left corner is the atomic mass in atomic mass units.



Look at the atomic mass of boron. About 80% of all boron atoms have six neutrons. The other 20% have only 5 neutrons. This difference is why most boron atoms have a mass of about 11 atomic mass units, but some have a mass of about 10 atomic mass units. The atomic mass of boron is equivalent to the average mass of a boron atom: 10.811.

Exercise 3 Mass of a Water Molecule

Working Space

Using the periodic table, what is the average mass of one water molecule in atomic mass units?

Answer on Page 47

3.1 Molar Mass

An atomic mass unit is a very, very, very small unit; we would much rather work in grams. It turns out that $6.02214076 \times 10^{23}$ atoms equal 1 mole (a standard measure for chemistry). Scientists use this number so often that they gave it a name: *the Avogadro constant* or *Avogadro's number*.

If you have 12 doughnuts, that's a dozen doughnuts. If you have $6.02214076 \times 10^{23}$ doughnuts, you have a *mole* of doughnuts. This isn't really a practical measurement, as a mole of doughnuts would be about the size of the earth. We use moles for small things like molecules.

Let's say you want to know how much a mole of $NaCl$ weighs. From the periodic table, you see that Na has an atomic mass of 22.98976 atomic mass units, and Cl has 35.453 atomic mass units. One atom of $NaCl$ has a mass of $22.98976 + 35.453 = 58.44276$ atomic mass units. This means a mole of $NaCl$ has a mass of 58.44276 grams. Handy, right?

Exercise 4 Burning Methane

Working Space

Natural gas is mostly methane (CH_4). When one molecule of methane burns, two oxygen molecules (O_2) are consumed. One molecule of H_2O and one molecule of CO_2 are produced.

If I need 200 grams of water, how many grams of methane do I need to burn?

(This is how the hero in "The Martian" made water for his garden.)

Answer on Page 47

3.2 Heavy atoms aren't stable

When you look at the periodic table, there are a surprisingly large number of elements. You might be told to "Drink milk so that you can get the calcium you need." However, no one has told you "You should eat kale so that you get enough copernicium in your diet."

Copernicium, with 112 protons and 173 neutrons, has only been observed in a lab. It is highly radioactive and unstable (meaning it decays). A copernicium atom usually lives for less than a minute before decaying.

The largest stable element is lead, which has 82 protons and between 122 and 126 neutrons. Elements with lower atomic numbers than lead, have at least one stable isotope, while elements with higher atomic numbers than lead don't.

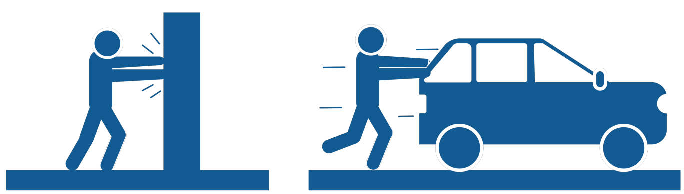
Bismuth, with an atomic number of 83, is *almost* stable. In fact, most bismuth atoms will live for billions of years before decaying!

Chapter 4

Work and Energy

In this chapter, we are going to talk about how engineers define work and energy. It frequently takes force to get work done. Let's start with thinking about the relationship between force and energy. As we learned earlier, Force is measured in newtons, and one newton is equal to the force necessary to accelerate one kilogram at a rate of $1\text{m}/\text{s}^2$.

When you lean on a wall, you are exerting a force on the wall, but you aren't doing any work. On the other hand, if you push a car for a mile, you are clearly doing work. Work, to an engineer, is the force you apply to something, as well as the distance that it moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force over one meter.



For example, if you push a car uphill with a force of 10 newtons for 12 meters, you have done 120 joules of work.

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs(British Thermal Unit), and calories.

Let's go over a few different forms that energy can take.

4.1 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat

- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

4.1.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

4.1.2 Electricity

Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let's say your lightbulb consumes 60 watts of electricity, and you leave it on for 24 hours. We would say that you have consumed 1.44 kilowatt hours, or 3,600,000 joules.

4.1.3 Chemical Energy

As mentioned early, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosythesis is a process that releases energy. Therefore, the sugar molecule has more chemical energy than the carbon dioxide and water molecules that were used in its creation.

In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

Here is where things get tricky: People who work with food got tired of saying "kilocalories", so they just started using "Calorie" to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, "Calorie" probably means kilocalorie.

4.1.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the breaks, the energy from the motion of the car will be converted into heat in the breaks and under the tires.

How much energy does the car have?

Formula for Kinetic Energy

$$E = \frac{1}{2}mv^2$$

where E is the energy in joules, m is the mass in kilograms, and v is the speed in meters per second.

4.1.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

On the surface of the earth, gravity will accelerate a heavy object downward at a rate of $9.8 m/s^2$.

Formula for Gravitational Potential Energy

The formula for gravitational potentional energy is

$$E = mgh$$

where E is the energy in joules, m is the mass of the object you lifted, g is acceleration due to gravity, and h is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects accelerate at $9.8 m/s^2$.

There are other kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy.

4.2 Conservation of Energy

The first law of thermodynamics says "Energy is neither created nor destroyed."

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. However, the total amount of energy in a closed system stays constant.

Exercise 5 The Energy of Falling

Working Space

A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

Answer on Page 48

4.3 Efficiency

Although energy is always conserved as it moves through different forms, scientists aren't always that good at controlling it.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

In general, if you are trying to increase efficiency in any system, the solution is usually easy to identify by the heat that is produced. Reduce the heat, increase the efficiency.

Light bulbs are an interesting case. To get the light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just "inefficient at making light" — it is actually counteracting the air conditioner!

Chapter 5

Units and Conversions

Accurate measurements are at the heart of good data and good problem solving. Engineers need to be able to describe many different types of phenomena, such as distance, sound, light, force, and more.

At this point, you are working with a lot of units: grams for weight, joules for energy, newtons for force, meters for distance, seconds for time, and so on. For each type of measurement, there are several different units. For example, distance can be measured in feet, miles, and light-years.

Some Equalencies

Distance	
1 mile	1.6093 kilometers
1 foot	0.3048 meters
1 inch	2.54 centimeters
1 light-year	9.461×10^{12} kilometers
Volume	
1 milliliter	1 cubic centimeter
1 quart	0.9461 liters
1 gallon	3.7854 liters
1 fluid ounce	29.6 milliliters
Mass	
1 pound	0.4535924 kilograms
1 ounce	0.4535924 grams
1 metric ton	1000 kilograms
Force	
1 newton	1 kilogram meter per sec^2
Pressure	
1 pascal	1 newton per square meter
1 bar	0.98692 atmosphere
1 pound per square inch	6897 pascals
Energy	
1 joule	1 newton meter
1 calorie	4.184 joules
1 kilowatt-hour	3.6×10^6 joules

(You don't need to memorize these! Just remember that this page is here.)

In the metric system, prefixes are often used to express a multiple. Here are the common prefixes:

Common Prefixes for Metric Units

giga	$\times 10^9$
mega	$\times 10^6$
kilo	$\times 10^3$
milli	$\div 10^3$
micro	$\div 10^6$
nano	$\div 10^9$

(These are worth memorizing. Here's a mnemonic: "King Henry Doesn't Usually Drink Chocolate Milk." Or Kilo, Hecto, Deca, Unit (for example: gram), Deci, Centi, Mili.

5.1 Conversion Factors

Here is a really handy trick to remembering how to do conversions between units.

Often, you will be given a table like the one above, and someone will ask you "How many miles are in 0.23 light-years?" You know that 1 mile = 1.6093 kilometers and that 1 light-year is 9.461×10^{12} kilometers. How do you do the conversion?

The trick is to treat the two parts of the equality as a fraction that equals 1. In other words, you think:

$$\frac{1 \text{ miles}}{1.6093 \text{ km}} = \frac{1.6093 \text{ km}}{1 \text{ miles}} = 1$$

and

$$\frac{1 \text{ light-years}}{9.461 \times 10^{12} \text{ km}} = \frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}} = 1$$

We call these fractions *conversion factors*.

Now, your problem is

$$0.23 \text{ light-years} \times \textit{ Some conversion factors} = ?\text{ miles}$$

Note that when you multiply fractions together, things in the numerators can cancel with things in the denominator:

$$\left(\frac{31\pi}{47}\right)\left(\frac{11}{37\pi}\right) = \left(\frac{31\cancel{\pi}}{47}\right)\left(\frac{11}{37\cancel{\pi}}\right) = \left(\frac{31}{47}\right)\left(\frac{11}{37}\right)$$

When working with conversion factors, you will do the same with the units:

$$0.23 \text{ light-years} \left(\frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}}\right)\left(\frac{1 \text{ miles}}{1.6093 \text{ km}}\right) =$$
$$0.23 \cancel{\text{light-years}} \left(\times \frac{9.461 \times 10^{12} \cancel{\text{km}}}{1 \cancel{\text{light-years}}}\right)\left(\frac{1 \text{ miles}}{1.6093 \cancel{\text{km}}}\right) = \frac{(0.23)(9.461 \times 10^{12})}{1.6093} \text{ miles}$$

Exercise 6 Simple Conversion Factors

Working Space

How many calories are in 4.5 kilowatt-hours?

Answer on Page 48

5.2 Conversion Factors and Ratios

Conversion factors also work on ratios. For example, if you are told that a bug is moving 0.5 feet every 120 milliseconds, what is that in meters per second?

The problem then is

$$\frac{0.5 \text{ feet}}{120 \text{ milliseconds}} = \frac{? \text{ m}}{\text{second}}$$

So you will need conversion factors to replace the “feet” with “meters” and to replace “milliseconds” with “seconds”:

$$\left(\frac{0.5\ \cancel{\text{feet}}}{120\ \cancel{\text{milliseconds}}}\right)\left(\frac{0.3048\ \text{meters}}{1\ \cancel{\text{feet}}}\right)\left(\frac{1000\ \cancel{\text{milliseconds}}}{1\ \text{second}}\right) = \frac{(0.5)(0.3048)(1000)}{120}\ \text{m/second}$$

Exercise 7 Conversion Factors

Working Space

The hole in the bottom of the boat lets in 0.1 gallons every 2 minutes. How many milliliters per second is that?

Answer on Page 48

5.3 When Conversion Factors Don't Work

Conversion factors only work when the units being converted are proportional to each other. Gallons and liters, for example, are proportional to each other: If you have n gallons, you have $n \times 3.7854$ liters.

Degrees celsius and degrees farenheit are *not* proportional to each other. If your food is n degrees celsius, it is $n \times \frac{9}{5} + 32$ degrees farenheit. You can't use conversion factors to convert celsius to farenheit.

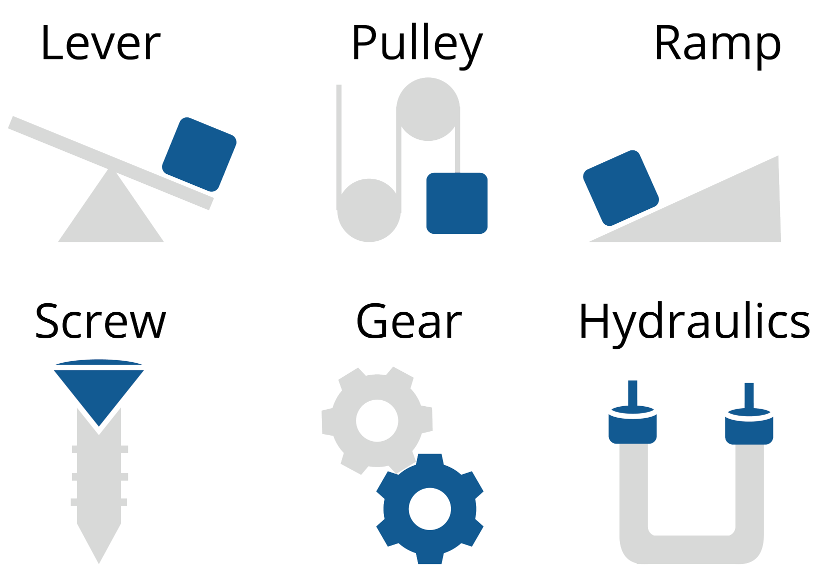
Chapter 6

Simple Machines

As mentioned earlier, physicists define work as the force applied times the distance over which it is applied. For example, if you push your car 100 meters with a force of 17 newtons, you have done 1700 joules of work.

Humans have long needed to move heavy objects, so many centuries ago, we developed simple machines to reduce the amount of force necessary to perform such tasks. These include:

- Levers
- Pulleys
- Inclined planes
- Gears
- Hydraulics
- Screws



While these machines can reduce the force needed, they do not change the total amount of work that must be done. For instance, if the force is reduced by a factor of three, the distance over which the force must be applied increases by the same factor.

The term *mechanical advantage* refers to the increase in force achieved by using these machines.

6.1 Levers

A lever pivots on a fulcrum. To decrease the necessary force, the load is placed closer to the fulcrum than where the force is applied.

Physicists also discuss the concept of *torque* created by a force. When you apply force to a lever, the torque is the product of the force you exert and the distance from the point of rotation.

Torque is typically measured in newton-meters.

To balance two torques, the products of force and distance must be equal. Thus, assuming the forces are applied in the correct direction, the equation becomes:

$$R_L F_L = R_A F_A$$

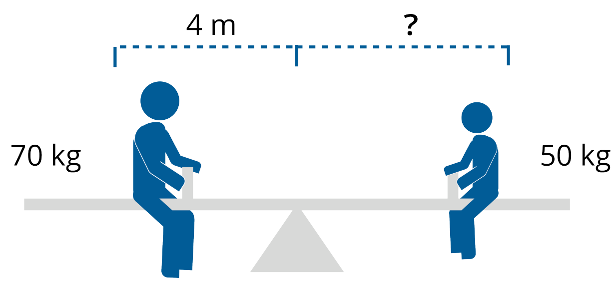
where R_L and R_A represent the distances from the fulcrum to where the load's force and the applied force are exerted, respectively, and F_L and F_A are the magnitudes of the forces.

Exercise 8 Lever

Paul, who weighs 70 kilograms, sits on a see-saw 4 meters from the fulcrum. Jan, who weighs 50 kilograms, wishes to balance the see-saw. How far should Jan sit from the fulcrum?

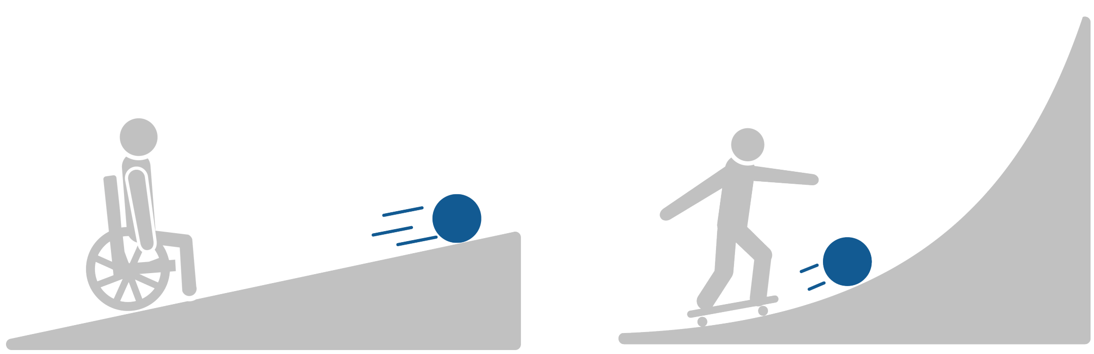
Working Space

Answer on Page 48



6.2 Inclined Planes

Inclined planes, or ramps, allow you to roll or slide objects to a higher level. Steeper ramps require less mechanical advantage. For instance, it is much easier to roll a ball up a wheelchair ramp than a skateboard ramp.



Assuming the incline has a constant steepness, the mechanical advantage is equal to the ratio of the length of the inclined plane to the height it rises.

If friction is neglected, the force required to push a weight up the inclined plane is given by:

$$F_A = \frac{V}{L} F_G$$

where F_A is the applied force, L is the length of the inclined plane, V is the vertical rise, and F_G is the gravitational force acting on the mass.

(We will discuss sine function later, but in case you're familiar with it, note that:

$$\frac{V}{L} = \sin\theta$$

where θ is the angle between the inclined plane and the horizontal surface.)

Exercise 9 Ramp

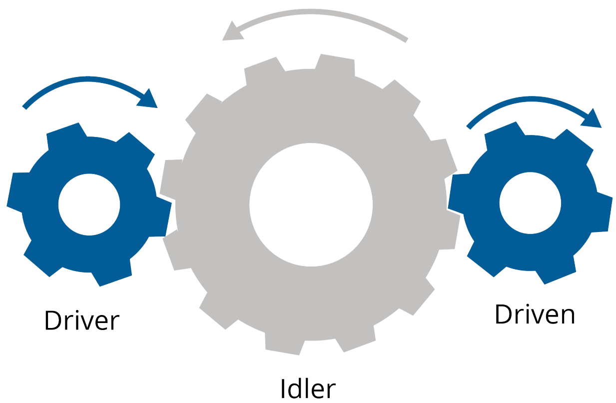
A barrel of oil weighs 136 kilograms. You can apply a force of up to 300 newtons. You need to get the barrel onto a platform that is 2 meters high. What is the shortest length of inclined plane you can use?

Working Space

Answer on Page 49

6.3 Gears

Gears have teeth that mesh with each other. When you apply torque to one gear, it transfers torque to the other. The resulting torque is increased or decreased depending on the ratio of the number of teeth on the gears.



If N_A is the number of teeth on the gear you are turning with a torque of T_A, and N_L is the number of teeth on the gear it is turning, the resulting torque is:

$$T_L = \frac{N_A}{N_L} T_A$$

Exercise 10 Gears

In a bicycle, the goal is not always to gain mechanical advantage, but to spin the pedals slower while applying more force.

You like to pedal your bike at 70 revolutions per minute. The chainring connected to your pedals has 53 teeth. The circumference of your tire is 2.2 meters. You want to ride at 583 meters per minute.

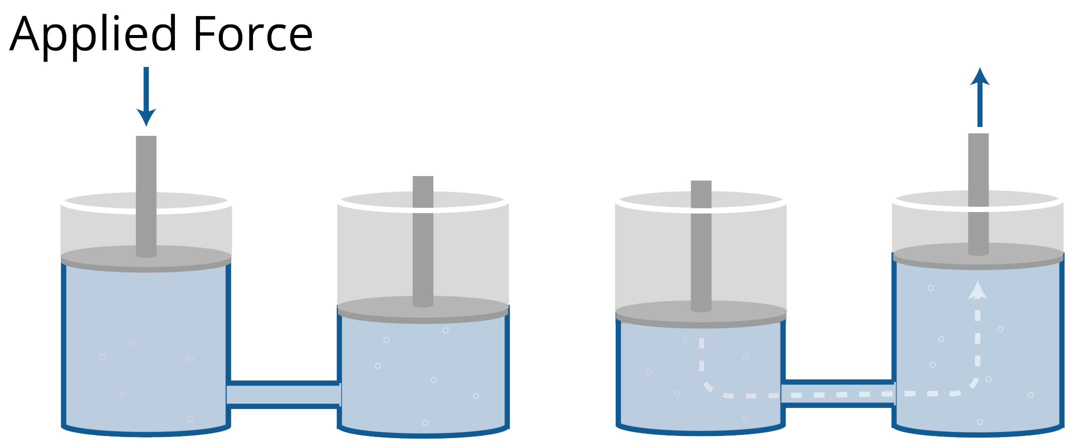
How many teeth should the rear sprocket have?

Working Space

Answer on Page 49

6.4 Hydraulics

In a hydraulic system, such as a car's braking system, you exert force on a piston filled with fluid. The fluid transmits this pressure into another cylinder, where it pushes yet another piston that moves the load.



The pressure in the fluid is typically measured in pascals (Pa), which is equivalent to N/m^2. We will use pascals for this calculation.

To calculate the pressure you create, divide the force applied by the area of the piston head. To determine the force on the other piston, multiply the pressure by the area of the second piston.

Exercise 11 Hydraulics

Your car has disc brakes. When you apply 2,500,000 pascals of pressure to the brake fluid, the car stops quickly. As the car designer, you want this to require only 12 newtons of force from the driver's foot.

What should the radius of the master cylinder (the piston the driver pushes) be?

Working Space

Answer on Page 49

Chapter 7

Buoyancy

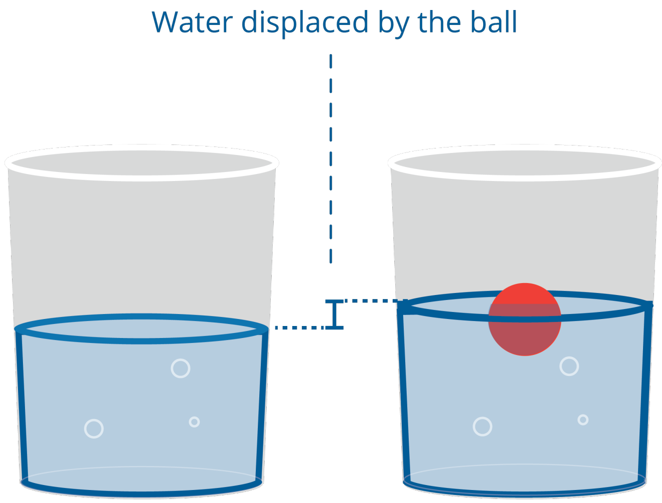
The word buoyancy probably brings to mind images of floating in water. Before we dive in, let's zoom out for a better understanding of everything buoyancy entails. You may be thinking: I want to be a computer programmer, why do I need to know about buoyancy? You might be surprised! This topic is much bigger than it seems at first glance.

Buoyancy concerns the ways in which liquids and gasses interact with gravity. The concept of buoyancy is connected to fundamental concepts about how the universe works. The *buoyant force*, as it is known in engineering, is an important concept that has wide ranging applications. A big part of engineering is moving stuff around, and understanding buoyancy helps us solve problems where we need to move things in and through fluids. Even if you don't have plans to build a robotic submarine, these are incredibly useful ideas to be familiar with. We will start exploring the topic with familiar scenarios around boats and water.

When you put a boat into water, it will sink into the water until the mass of the water it displaces is equal to the mass of the boat. We think of this in terms of forces. Gravity pulls the mass of the boat down; the *buoyant force* pushes the boat up. A boat dropped into the water will bob up and down at first before reaching an *equilibrium* where the two forces are equal.

The buoyant force pushes things up, fighting against the force of gravity. The force is equal to the weight of the fluid being replaced. For example, a cubic meter of freshwater has a mass of about 1000kg. If you submerge anything with a volume of one meter in freshwater on earth, the buoyant force will be about 9800 newtons.

For some things, like a block of styrofoam, this buoyant force will be sufficient to carry it to the surface. Once it reaches the surface, it will continue to rise (displacing less water) until the mass of the water it displaces is equal to its mass. And then we say "It floats!"



For some things, like a block of lead, the buoyant force is not sufficient to lift it to the surface, and then we say "It sinks!"

This is why a helium balloon floats through the air. The air that it displaces weighs more than the balloon and the helium itself. (It is easy to forget that air has a mass, but it does.)

Exercise 12 Buoyancy

You have an aluminum box that has a heavy base, so it will always float upright. The box and its contents weigh 10 kg. Its base is 0.3 m x 0.4 m. It is 1m tall.

When you drop it into freshwater ($1000 \text{kg}/\text{m}^3$), how far will it sink before it reaches equilibrium?

Working Space

Answer on Page 50

7.1 The Mechanism of Buoyancy: Pressure

As you dive down in the ocean, you will experience greater and greater pressure from the water. And if you take a balloon with you, you will gradually see it get smaller as the water pressure compresses the air in the balloon.

Let's say you are 3 meters below the surface of the water. What is the pressure in Pascals (newtons per square meter)? You can think of the water as a column of water crushing down upon you. The pressure over a square meter is the weight of 3 cubic meters of water pressing down.

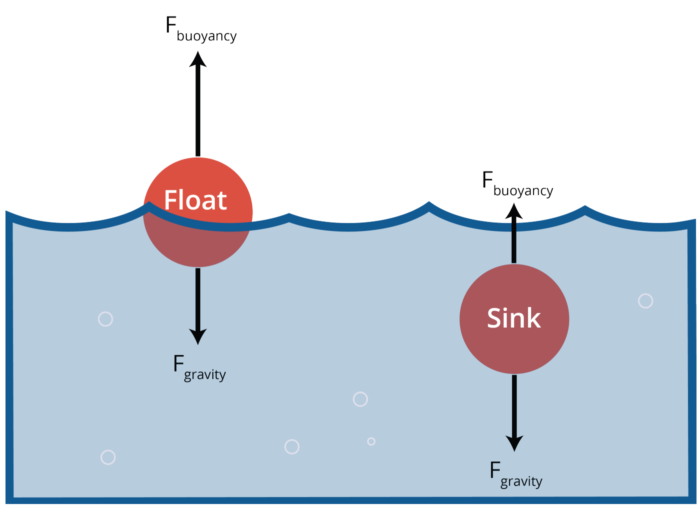
$$p = (3)(1000)(9.8) = 29,400 \text{ Pa}$$

This is called *hydrostatic pressure*. The general rule for hydrostatic pressure in Pascals p is

$p = dgh$

where d is the density of the fluid in kg per cubic meter, g is the acceleration due to gravity in m/s^2, and h is the height of the column of fluid above you.

So where does buoyant force come from? Basically, the pressure pushing up on the deepest part of the object is higher than the pressure pushing down on the shallowest part of the object. That is where bouyancy comes from.



Exercise 13 Hydrostatic Pressure

Working Space

You dive into a tank of olive oil on Mars. How much more hydrostatic pressure does your body experience at 5 meters deep than it did at the surface?

The density of olive oil is about 900 kg per square meter. The acceleration due to gravity on Mars is 3.721 m/s^2.

Answer on Page 50

7.2 The Mechanism of Buoyancy: Density

Keep in mind that although the pressure is increasing as you go deeper, the buoyant force will *not increase*, because the buoyant force is always equal to the weight of the fluid that is displaced, regardless whether that is 1 meter or 100 meters underwater.

Due to the added minerals, saltwater is denser than freshwater. This causes objects to float better in the sea than they do in a river. Lipids, like fats and oils, are less dense than water, allowing them to float on top of a glass of water. When you're facing a grease fire, you're told not to put water on it. That's because the water sinks below the grease, then boils, throwing burning grease everywhere.

Chapter 8

Heat

All mass in the universe has heat, whether you're looking at a block of dry ice (frozen CO_2, $-78.5°$ C) or the surface of the sun ($5,600°$ C). As long as the mass is above absolute zero — the coldest possible temperature in the universe — there is some amount of heat in it.

8.1 How Heat Works

As you heat up an object, you are imparting energy into it. Where does this energy go? The atoms take this energy and they begin to move, vibrating and bumping into each other, causing the heat to spread throughout. Everytime the atoms collide and bounce off of each other, they emit a tiny amount of energy as light. In most cases, that light is in the infrared spectrum, but in extreme cases can be visible, such as with molten lava or hot metal.

As objects interact, they either put heat into colder objects or take heat from warmer objects. That's what allows you to heat up anything in the first place. The hot air from a stove or bunsen burner interacts with the pan or test tube you're heating, passing the air's heat on. How could you model this?

8.2 Specific Heat Capacity

If you are heating something, the amount of energy you need to transfer to it depends on three things: the mass of the thing you are heating, the amount of temperature change you want, and the *specific heat capacity* of that substance.

Energy in Heat Transfer

The energy moved in a heat transfer is given by

$$E = mc\Delta_T$$

where m is the mass, Δ_T is the change in temperature, and c is the specific heat capacity of the substance.

(Note that this assumes there isn't a phase change. For example, this formula works nicely on warming liquid water, but it gets more complicated if you warm the water past its boiling point.)

Can we guess the specific heat capacity of a substance? It is very, very difficult to guess the specific heat of a substance, so we determine it by experimentation.

For example, it takes 0.9 joules to raise the temperature of solid aluminum one degree Celsius. So we say "The specific heat capacity of aluminum is 0.9 J/g °C."

The specific heat capacity of liquid water is about 4.2 J/g °C.

Let's say you put a 1 kg aluminum pan that is 80° C into 3 liters of water that is 20° C. Energy, in the form of heat, will be transferred from the pan to the water until they are at the same temperature. (We call this "thermal equilibrium.")

What will the temperature of the water be?

To answer this question, the amount of energy given off by the pan must equal the amount of energy absorbed by the water. They also need to be the same temperature at the end. Let T be the final temperature of both.

3 liters of water weighs 3,000 grams, so the change in energy in the water will be:

$$E_W = mc\Delta_T = (3000)(4.2)(T - 20) = 12600T - 252000 \text{ joules}$$

The pan weighs 1000 grams, so the change in energy in the pan will be::

$$E_P = mc\Delta_T = (1000)(0.9)(T - 80) = 900T - 72000 \text{ joules}$$

The total energy stays the same, so $E_W + E_P = 0$. This means you need to solve

$$(12600T - 252000) + (900T - 72000) = 0$$

And find that the temperature at equilibrium will be

$$T = 24°C$$

Exercise 14 Thermal Equilibrium

Working Space

Just as you put the aluminium pan in the water as described above, someone also puts a 1.2 kg block of copper cooled to 10 ° C. The specific heat of solid copper is about 0.4 J/g °C.

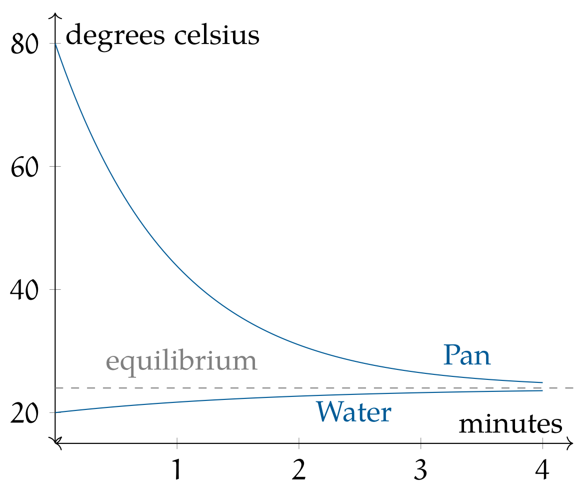
What is the new temperature at equilibrium?

Answer on Page 50

8.3 Getting to Equilibrium

When two objects with different temperatures are touching, the speed at which they exchange heat is proportional to the differences in their temperatures. As their temperatures get closer together, the heat exchange slows down.

In our example, the pan and the water will get close to equilibrium quickly, but they may never actually reach equilibrium.



Exercise 15 Cooling Your Coffee

You have been given a ridiculously hot cup of coffee and a small pitcher of chilled milk.

You need to start chugging your coffee in three minutes, and you want it as cool as possible at that time. When should you add the milk to the coffee?

Working Space

Answer on Page 50

8.4 Specific Heat Capacity Details

For any given substance, the specific heat capacity often changes a great deal when the substance changes state. For example, ice is 2.1 J/g °C, whereas liquid water is 4.2 J/g°C.

Even within a given state, the specific heat capacity varies a bit based on the temperature and pressure. If you are trying to do these sorts of calculations with great accuracy, you will want to find the specific heat capacity that matches your situation. For example, I might look for the specific heat capacity for water at 22°C at 1 atmosphere of pressure (atm).

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 11)

To get the train to 20 meters per second in 120 seconds, you must accelerate it with a constant rate of $\frac{1}{6}m/s^2$. You remember that $F = ma$, so $F = 2400 \times \frac{1}{6}$. This means you will need to push the train with a force of 400 newtons for the 120 seconds before the bomb goes off.

Answer to Exercise 2 (on page 12)

$$F = G\frac{m_1 m_2}{r^2} = (6.674 \times 10^{-11})\frac{(6.8^3)(6 \times 10^{24})}{(10^5)^2} = 6.1 \times 10^6$$

About 6 million newtons.

Answer to Exercise 3 (on page 17)

The average hydrogen atom has a mass of 1.00794 atomic mass units.

The average oxygen atom has a mass of 15.9994.

$2 \times 1.00794 + 15.9994 = 18.01528$ atomic mass units.

Answer to Exercise 4 (on page 18)

From the last exercise, you know that 1 mole of water weighs 18.01528 grams, meaning 200 grams of water is about 11.1 moles. So you need to burn 11.1 moles of methane.

What does one mole of methane weigh? Using the periodic table: $12.0107 + 4 \times 1.00794 = 16.04246$ grams.

$16.0424 \times 11.10 = 178.1$ grams of methane.

Answer to Exercise 5 (on page 24)

At the top of the ladder, the cannonball has $(9.8)(5)(3) = 147$ joules of potential energy.

At the bottom, the kinetic energy $\frac{1}{2}(5)v^2$ must be equal to 147 joules. So $v^2 = \frac{294}{5}$. This means it is going about 7.7 meters per second.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is neglible.)

Answer to Exercise 6 (on page 29)

$$4.5 \cancel{\text{kWh}} \left(\frac{3.6 \times 10^6 \cancel{\text{joules}}}{1 \cancel{\text{kWh}}}\right) \left(\frac{1 \text{ calories}}{4.184 \cancel{\text{joules}}}\right) = \frac{(4.5)(3.6 \times 10^6)}{4.184} = 1.08 \times 10^6 \text{calories}$$

Answer to Exercise 7 (on page 30)

$$\frac{0.1 \cancel{\text{gallons}}}{2 \cancel{\text{minutes}}} \left(\frac{3.7854 \cancel{\text{liters}}}{1 \cancel{\text{gallons}}}\right) \left(\frac{1000 \text{ milliliters}}{1 \cancel{\text{liters}}}\right) \left(\frac{1 \cancel{\text{minutes}}}{60 \text{ seconds}}\right) = \frac{(0.1)(3.7854)(1000)}{(2)(60)} \text{ ml/second} = 3.1545 \text{ ml/second}$$

Answer to Exercise 8 (on page 32)

Paul exerts a force of $70 \times 9.8 = 686$ newtons at a distance of 4 meters from the fulcrum, creating a torque of $686 \times 4 = 2744$ newton-meters. Jan exerts a force of $50 \times 9.8 = 490$ newtons.

Let r be the distance from the fulcrum to Jan's seat. To balance the torques:

$$490 \times r = 2744$$

Solving for r, we find $r = \frac{2744}{490} \approx 5.6$ meters.

Answer to Exercise 9 (on page 34)

The weight of the barrel is $136 \times 9.8 = 1332.8$ newtons.

Let L be the length of the inclined plane. The force needed to push the barrel up is related by:

$$300 = \frac{2}{L} \times 1332.8$$

Solving for L, we find $L = \frac{2 \times 1332.8}{300} \approx 8.885$ meters.

Answer to Exercise 10 (on page 35)

The equation relating these quantities is:

$$583 = 70 \times 2.2 \times \frac{53}{n}$$

Solving for n, we find $n = 14$ teeth.

Answer to Exercise 11 (on page 37)

We are solving for the radius r of the piston. The area of the piston is πr^2, so the pressure is:

$$\text{Pressure} = \frac{12}{\pi r^2}$$

Setting the pressure equal to 2,500,000 pascals:

$$2,500,000 = \frac{12}{\pi r^2}$$

Solving for r, we find:

$$r = \sqrt{\frac{12}{\pi \times 2.5 \times 10^6}} \approx 0.00124 \text{ meters.}$$

Answer to Exercise 12 (on page 40)

Equilibrium will be achieved when the box has displaced 10 kg of water. In other words, when it has displaced 0.01 cubic meters.

The area of the base of the box is 0.12 square meters. So if the box sinks x meters into the water it will displace $0.12x$ cubic meters.

Thus at equilibrium $x = \frac{0.01}{0.12} \approx 0.083$ m. So the box will sink 8.3 cm into the water before reaching equilibrium.

Answer to Exercise 13 (on page 42)

$$p = dgh = (900)(3.721)(5) = 16,744.5 \text{ Pa}$$

Answer to Exercise 14 (on page 45)

$$E_C = (1200)(0.4)(T - 10) = 480T - 4800$$

Total energy stays constant:

$$0 = (12600T - 252000) + (900T - 72000) + (480T - 4800)$$

Solving for T gets you $T = 23.52°$ C.

Answer to Exercise 15 (on page 46)

During the 3 minutes, you want the coffee to give off as much of its heat as possible, so you want to maximize the difference between the temperature of the coffee and the temperature of the room around it.

You wait until the last moment to put the milk in.

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