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Chapter 1

Definite Integrals

Integrals are a fundamental concept in calculus, which are used to calculate areas, volumes, and many other things. A definite integral calculates the net area between the function and the x-axis over a given interval.

1.1 Definition

The definite integral of a function $f(x)$ over an interval $[a, b]$ is defined as the limit of a Riemann sum:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \tag{1.1}$$

where x_i^* is a sample point in the i^{th} subinterval of a partition of $[a, b]$, $\Delta x = \frac{b-a}{n}$ is the width of each subinterval, and the limit is taken as the number of subintervals n approaches infinity.

Chapter 2

Antiderivatives

In your study of calculus, you have learned about derivatives, which allow us to find the rate of change of a function at any given point. Derivatives are powerful tools that help us analyze the behavior of functions. Now, we will explore another concept called antiderivatives, which are closely related to derivatives.

An antiderivative, also known as an integral or primitive, is the reverse process of differentiation. It involves finding a function whose derivative is equal to a given function. In simple terms, if you have a function and you want to find another function that, when differentiated, gives you the original function back, you are looking for its antiderivative.

The symbol used to represent an antiderivative is $\int$. It is called the integral sign. For example, if $f(x)$ is a function, then the antiderivative of $f(x)$ with respect to x is denoted as $\int f(x), dx$. The dx at the end indicates that we are integrating with respect to x.

Finding antiderivatives requires using specific techniques and rules. Some common antiderivative rules include:

- The power rule: If $f(x) = x^n$, where n is any real number except -1, then the antiderivative of $f(x)$ is given by $\int f(x), dx = \frac{1}{n+1}x^{n+1} + C$, where C is the constant of integration.

- The constant rule: The antiderivative of a constant function is equal to the constant times x. For example, if $f(x) = 5$, then $\int f(x), dx = 5x + C$.
- The sum and difference rule: If $f(x)$ and $g(x)$ are functions, then $\int (f(x) + g(x)), dx = \int f(x), dx + \int g(x), dx$. Similarly, $\int (f(x) - g(x)), dx = \int f(x), dx - \int g(x), dx$.

Antiderivatives have various applications in mathematics and science. They allow us to calculate the total accumulation of a quantity over a given interval, compute areas under curves, and solve differential equations, among other things.

It is important to note that an antiderivative is not a unique function. Since the derivative of a constant is zero, any constant added to an antiderivative will still be an antiderivative of the original function. This is why we include the constant of integration, denoted by C, in the antiderivative expression.

In summary, antiderivatives are the reverse process of differentiation. They help us find functions whose derivatives match a given function. Understanding antiderivatives is crucial for various advanced calculus concepts and real-world applications.

Now, let's explore different techniques and methods for finding antiderivatives and discover how they can be applied in solving problems.

Chapter 3

The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus is a theorem that connects the concept of differentiating a function with the concept of integrating a function. This theorem is divided into two parts:

3.1 First Part

The first part of the Fundamental Theorem of Calculus states that if f is a continuous real-valued function defined on a closed interval $[a, b]$ and F is the function defined, for all x in $[a, b]$, by:

$$F(x) = \int_a^x f(t)\, dt \tag{3.1}$$

Then, F is uniformly continuous and differentiable on the open interval (a, b), and $F'(x) =$

$f(x)$ for all x in (a, b).

3.2 Second Part

The second part of the Fundamental Theorem of Calculus states that if f is a real-valued function defined on a closed interval $[a, b]$ that admits an antiderivative F on $[a, b]$, and f is integrable on $[a, b]$ (it need not be continuous), then

$$\int_a^b f(t)\,dt = F(b) - F(a). \tag{3.2}$$

Appendix A

Answers to Exercises

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