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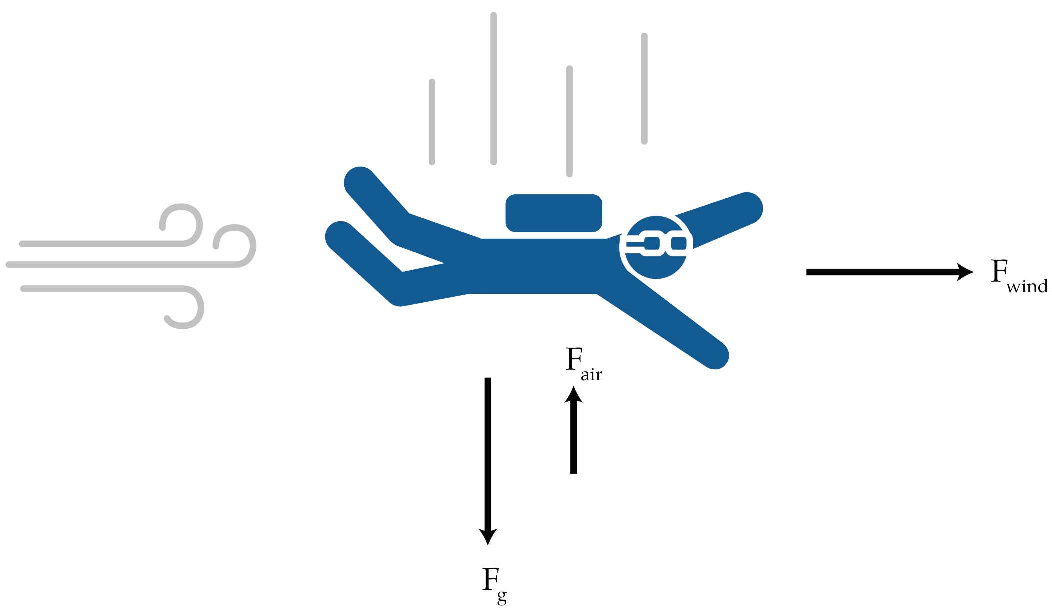
Chapter 1

Vectors

We have talked a some about forces, but in the calculations that we have done, we have only talked about the magnitude of a force. It is equally important to talk about its direction. To do the math on things with a magnitude and a direction (like forces), we need vectors.

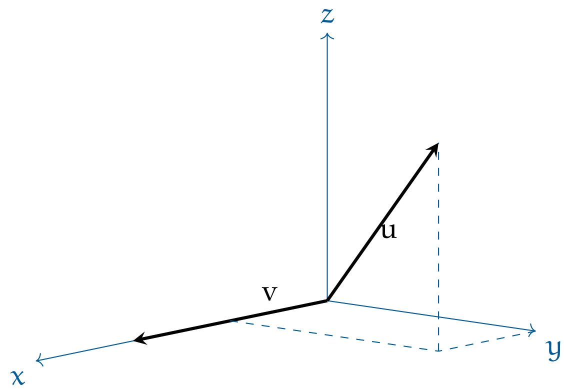
For example, if you jump out of a plane (hopefully with a parachute), several forces with different magnitudes and directions will be acting upon you. Gravity will push you straight down. That force will be proportional to your weight. If there were a wind from the west, it would push you toward the east. That force will be proportional to the square of the speed of the wind and approximately proportional to your size. Once you are falling, there will be resistance from the air that you are pushing through – that force will point in the opposite direction from the direction you are moving and will be proportional to the square of your speed.



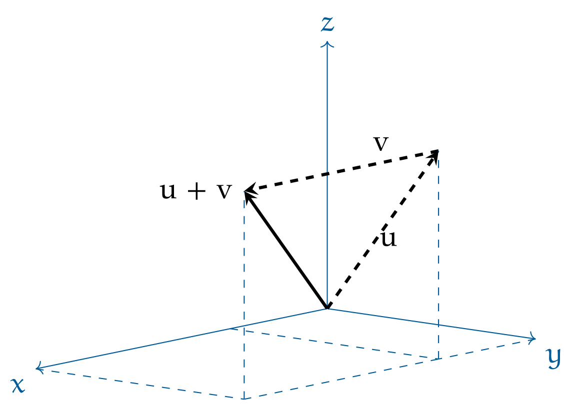
To figure out the net force (which will tell us how we will accelerate), we will need to add these forces together. So we need to learn to do math with vectors.

1.1 Adding Vectors

A vector is typically represented as a list of numbers, with each number representing a particular dimension. For example, if I am creating a 3-dimensional vector representing a force, it will have three numbers representing the amount of force in each of the three axes. For example, if a force of one newton is in the direction of the x-axis, I might represent the vector as $v = [1, 0, 0]$. Another vector might be $u = [0.5, 0.9, 0.7]$



Thinking visually, when we add to vectors, we put the starting point second vector at the ending point of the first vector.



If you know the vectors, you will just add them element-wise:

$$u + v = [0.5, 0.9, 0.7] + [1.0, 0.0, 0.0] = [1.5, 0.9.0.7]$$

These vectors have 3 components, so we say they are *3-dimensional*. Vectors can have any number of components. For example, the vector $[-12.2, 3, \pi, 10000]$ is 4-dimensional.

You can only add two vectors if they have the same dimension.

$$[12, -4] + [-1, 5] = [11, 1]$$

Addition is commutative: If you have two vectors a and b, then $a + b$ is the same as $b + a$.

Addition is also associative: If you have three vectors a, b, and c, it doesn't matter which order you add them in. That is, $a + (b + c) = (a + b) + c$.

A 1-dimensional vector is just a number. We say it is a *scalar*, not a vector.

Exercise 1 Adding vectors

Add the following vectors:

- $[1, 2, 3] + [4, 5, 6]$
- $[-1, -2, -3, -4] + [4, 5, 6, 7]$
- $[\pi, 0, 0] + [0, \pi, 0] + [0, 0, \pi]$

Working Space

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Exercise 2 Adding Forces

You are adrift in space. You are near two different stars. The gravity of one star is pulling you towards it with a force of $[4.2, 5.6, 9.0]$ newtons. The gravity of the other star is pulling you towards it with a force of $[-100.2, 30.2, -9.0]$ newtons. What is the net force?

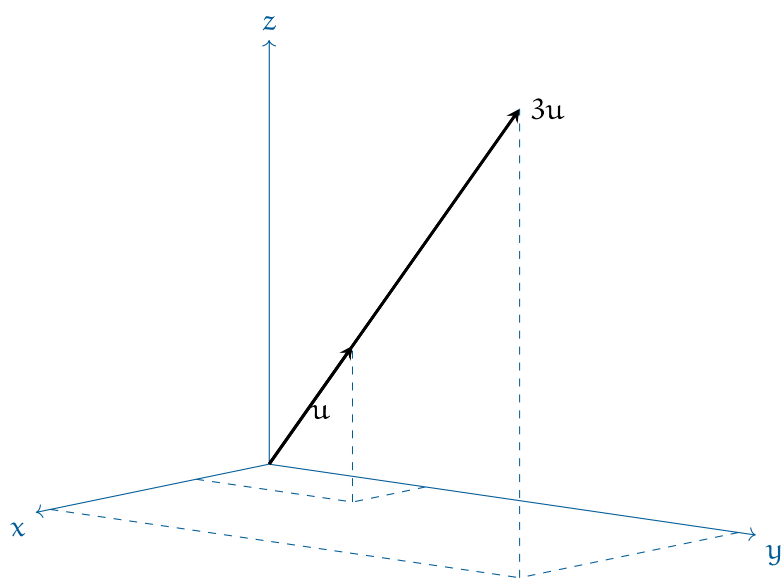
Working Space

Answer on Page 51

1.2 Multiplying a vector with a scalar

It is not uncommon to multiply a vector by a scalar. For example, a rocket engine might have a force vector v. If you fire 9 engines in the exact same direction, the resulting force vector would be $9v$.

Visually, when we multiply a vector u by a scalar a, we get a new vector that goes in the same direction as u but has a magnitude a times as long as u.



When you multiply a vector by a scalar, you just multiply each of the components by the scalar:

$$3 \times [0.5, 0.9, 0.7] = [1.5, 2.7, 3.6]$$

Exercise 3 Multiplying a vector and a scalar

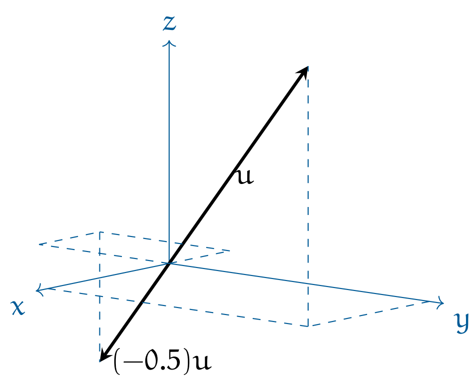
Simplify the following expressions:

- $2 \times [1, 2, 3]$
- $[-1, -2, -3, -4] \times -2$
- $\pi[\pi, 2\pi, 3\pi]$

Working Space

Answer on Page 51

Note that when you multiply a vector times a negative number, the new vector points in the opposite direction.



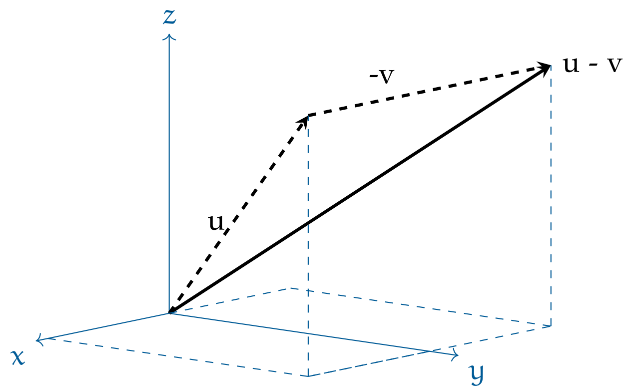
1.3 Vector Subtraction

As you might guess, when you subtract one vector from another, you just do element-wise subtraction:

$$[4, 2, 0] - [3, -2, 9] = [1, 4, -9]$$

So, $u - v = u + (-1v)$.

So visually, you reverse the one that is being subtracted:



1.4 Magnitude of a Vector

The *magnitude* of a vector is just its length. We write the magnitude of a vector v as $|v|$.

We compute the magnitude using the pythagorean theorem. If $v = [3, 4, 5]$, then

$$|v| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} \approx 7.07$$

(You might notice that the notation for the magnitude is exactly like the notation for absolute value. If you think of a scalar as a 1-dimensional vector, the absolute value and the magnitude are the same. For example, the absolute value of -5 is 5. If you take the magnitude of the one-dimenional vector $[-5]$, you get $\sqrt{25} = 5$.)

Notice that if you scale up a vector, its magnitude scales by the same amount. For example:

$$|7[3, 4, 5]| = 7\sqrt{50} \approx 7 \times 7.07$$

The rule then is: If you have any vector v and any scalar a:

$$|av| = |a||v|$$

Exercise 4 Magnitude of a Vector

Find the magnitude of the following vectors:

- $[1, 1, 1]$
- $[-5, -5, -5]$ (that is the same as $-5 \times [1, 1, 1]$)
- $[3, 4, -4] + [-2, -3, 5]$

Working Space

Answer on Page 51

1.5 Vectors in Python

NumPy is a library that allows you to work with vectors in Python. You might need to install it on your computer. This is done with `pip`. `pip3` installs things specifically for Python 3.

```
pip3 install NumPy
```

We can think of a vector as a list of numbers. There are also grids of numbers known as *matrices*. NumPy deals with both in the same way, so it refers to both of them as arrays.

The study of vectors and matrices is known as *Linear Algebra*. Some of the functions we need are in a sublibrary of NumPy called `linalg`.

As a convention, everyone who uses NumPy, imports it as *np*.

Create a file called `first_vectors.py`:

```
import NumPy as np

# Create two vectors
v = np.array([2,3,4])
u = np.array([-1,-2,3])
print(f"u = {u}, v = {v}")
```

```
# Add them
w = v + u
print(f"u + v = {w}")

# Multiply by a scalar
w = v * 3
print(f"v * 3 = {w}")

# Get the magnitude
# Get the magnitude
mv = np.linalg.norm(v)
mu = np.linalg.norm(u)
print(f"|v| = {mv}, |u| = {mu}")
```

When you run it, you should see:

```
> python3 first_vectors.py
u = [-1 -2  3], v = [2 3 4]
u + v = [1 1 7]
v * 3 = [ 6  9 12]
|v| = 5.385164807134504, |u| = 3.7416573867739413
```

1.5.1 Formatting Floats

The numbers 5.385164807134504 and 3.7416573867739413 are pretty long. You probably want it rounded off after a couple of decimal places.

Numbers with decimal places are called *floats*. In the placeholder for your float, you can specify how you want it formatted, including the number of decimal places.

Change the last line to look like this:

```
    print(f"|v| = {mv:.2f}, |u| = {mu:.2f}")
```

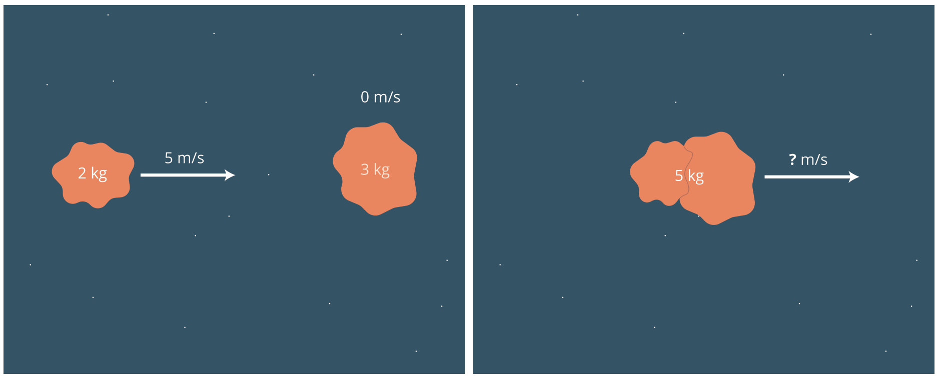
When you run the code, it will be neatly rounded off to two decimal places:

```
|v| = 5.39, |u| = 3.74
```

CHAPTER 2

Momentum

Let's say a 2 kg block of putty is flying through space at 5 meters per second, and it collides with a larger 3 kg block of putty that is not moving at all. When the two blocks deform and stick to each other, how fast will the resulting big block be moving?



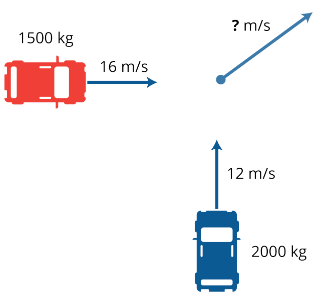
Every object has *momentum*. The momentum is a vector quantity: It points in the direction that the object is moving and has a magnitude equal to its mass times its speed.

Given a set of objects that are interacting, we can sum all their momentum vectors to get the total momentum. In such a set, the total momentum will stay constant.

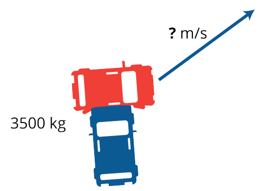
So, in our example, one object has a momentum vector of magnitude of 10 kg m/s, the other has a momentum of magnitude 0. Once they have merged, they have a combined mass of 5 kg. Thus, the velocity vector must have magnitude 2 m/s and pointing in the same direction that the first mass was moving.

Exercise 5 Cars on Ice

A car weighing 1000 kg is going north at 12 m/s. Another car weighing 1500 kg is going east at 16 m/s. They both hit a patch of ice (with zero friction) and collide. Steel is bent and the two objects become one. How what is the velocity vector (direction and magnitude) of the new object sliding across the ice?



Working Space



Answer on Page 52

Notice that kinetic energy ($1/2mv^2$) is *not* conserved here. Before the collision, the moving putty block has $(1/2)(2)(5^2) = 25$ joules of kinetic energy. Afterward, the big block has $(1/2)(5)(2^2) = 10$ joules of kinetic energy. What happened to the energy that was lost? It was used up deforming the putty.

What if the blocks were marble instead of putty? Then there would be very little deforming, so kinetic energy *and* momentum would be conserved. The two blocks would end up having different velocity vectors.

Let's assume for a moment that they strike each other straight on, so there is motion in only one direction, both before and after the collision. Can we solve for the speeds of the first block (v_1) and the second block (v_2)?

We end up with two equations. Conservation of momentum says:

$$2v_1 + 3v_2 = 10$$

Conservation of kinetic energy says:

$$(1/2)(2)(v_1^2) + (1/2)(3)(v_2^2) = 25$$

Using the first equation, we can solve for v_1 in terms of v_2:

$$v_1 = \frac{10 - 3v_2}{2}$$

Substituting this into the second equation, we get:

$$\left(\frac{10 - 3v_2}{2}\right)^2 + \frac{3v_2^2}{2} = 25$$

Simplifying, we get:

$$v_2^2 - 4v_2 + 0 = 0$$

This quadratic has two solutions: $v_2 = 0$ and $v_2 = 4$. $v_2 = 0$ represents the situation before the collision. Substituting in $v_2 = 4$:

$$v_1 = \frac{10 - 3(4)}{2} = -1$$

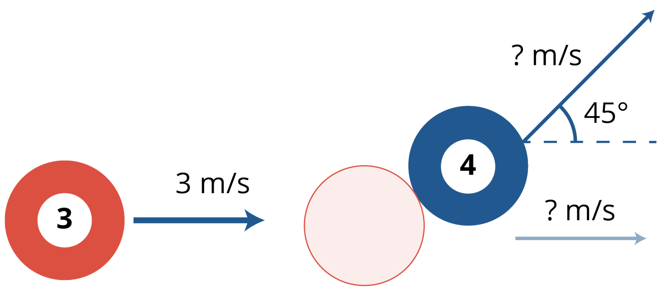
Thus, if the blocks are hard enough that kinetic energy is conserved, after the collision, the smaller block will be heading in the opposite direction at 1 m/s. The larger block will be moving at 4 m/s in the direction of the original motion.

Exercise 6 Billiard Balls

Working Space

A billiard ball weighing 0.4 kg and traveling at 3 m/s hits a billiard ball (same weight) at rest. It strikes obliquely so that the ball at rest starts to move at a 45 degree angle from the path of the ball that hit it.

Assuming all kinetic energy is conserved. How what is the velocity vector of each ball after the collision?



Answer on Page 52

Chapter 3

The Dot Product

If you have two vectors $u = [u_1, u_2, \ldots, w_n]$ and $v = [v_1, v_2, \ldots, v_n]$, we define the *dot product* $u \cdot v$ as

$$u \cdot v = (u_1 \times v_1) + (u_2 \times v_2) + \cdots + (u_n \times v_n)$$

So, for example,

$$[2, 4, -3] \cdot [5, -1, 1] = 2 \times 5 + 4 \times -1 + -3 \times 1 = 3$$

This may not seem like a very powerful idea, but dot products are *incredibly* useful. The enormous GPUs(Graphics Processing Unit) that let video games render scenes so quickly? They primarily function by computing huge numbers of dot products at mind-boggling speeds.

Exercise 7 Basic dot products

Compute the dot product of each pair of vectors:

- $[1, 2, 3]$, $[4, 5, -6]$
- $[\pi, 2\pi]$, $[2, -1]$
- $[0, 0, 0, 0]$, $[10, 10, 10, 10]$

Working Space

Answer on Page 53

3.1 Properties of the dot product

Sometimes we need an easy way to say "The vector of appropriate length is filled with zeros." We use the notation $\vec{0}$ to represent this. Then, for any vector v, this is true:

$$v \cdot \vec{0} = 0$$

The dot product is commutative:

$$v \cdot u = u \cdot v$$

The dot product of a vector with itself is its magnitude squared:

$$v \cdot v = |v|^2$$

If you have a scalar a then:

$$(v) \cdot (au) = a(v \cdot u)$$

So, if v and w are vectors that go in the same direction,

$$v \cdot w = |v||w|$$

If v and w are vectors that go in opposite directions,

$$v \cdot w = -|v||w|$$

if v and w are vectors that are perpendicular to each other, their dot product is zero:

$$v \cdot w = 0$$

3.2 Cosines and dot products

Furthermore, dot products' interaction with cosine makes them even more useful is what makes them so useful: If you have two vectors v and u,

$$v \cdot u = |v||u| \cos \theta$$

where θ is the angle between them.

So, for example, if two vectors v and u are perpendicular, the angle between them is $\pi/2$. The cosine of $\pi/2$ is 0: The dot product of any two perpendicular vectors is always 0. In fact, if the dot product of two non-zero vectors is 0, the vectors *must be* perpendicular.

Exercise 8 Using dot products

What is the angle between these each pair of vectors:

- $[1, 0]$, $[0, 1]$
- $[3, 4]$, $[4, 3]$

Working Space

Answer on Page 54

If you have two non-zero vectors v and u, you can always compute the angle between them:

$$\theta = \arccos(\frac{v \cdot u}{|v||u|})$$

3.3 Dot products in Python

NumPy will let you do dot products using the the symbol @. Open `first_vectors.py` and add the following to the end of the script:

```
# Take the dot product
d = v @ u
print("v @ u =", d)

# Get the angle between the vectors
a = np.arccos(d / (mv * mu))
print(f"The angle between u and v is {a * 180 / np.pi:.2f} degrees")
```

When you run it you should get:

```
v @ u = 4
The angle between u and v is 78.55 degrees
```

3.4 Work and Power

Earlier, we mentioned that mechanical work is the product of the force you apply to something and the amount it moves. For example, if you push a train with a force of 10 newtons as it moves 5 meters, you have done 50 joules of work.

What if you try to push the train sideways? That is, it moves down the track 5 meters, but you push it as if you were trying to derail it – perpendicular to its motion. You have done no work because the train didn't move at all in the direction you were pushing.

Now that you know about dot products: The work you do is the dot product of the force vector you apply and the displacement vector of the train. (The displacement vector is the vector that tells how the train moved while you pushed it.)

Similarly, we mentioned that power is the product of the force you apply and the velocity of the mass you are applying it to. It is actually the dot product of the force vector and the velocity vector.

For example, if you are pushing a sled with a force of 10 newtons and it is moving 2 meters per second, but your push is 20 degrees off, you aren't transferring 20 watts of power to the sled. You are transferring $10 \times 2 \times \cos(20 \text{ degrees}) \approx 18.8$ watts of power.

Chapter 4

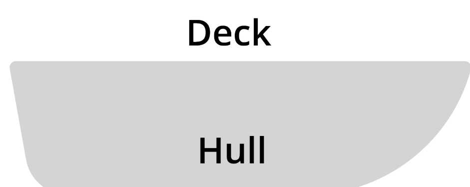
Boats

For centuries, engineers have been building boats. It is through boat design that humanity learned the lessons that made airplanes and rockets possible. You should know something about boats before we go any further.

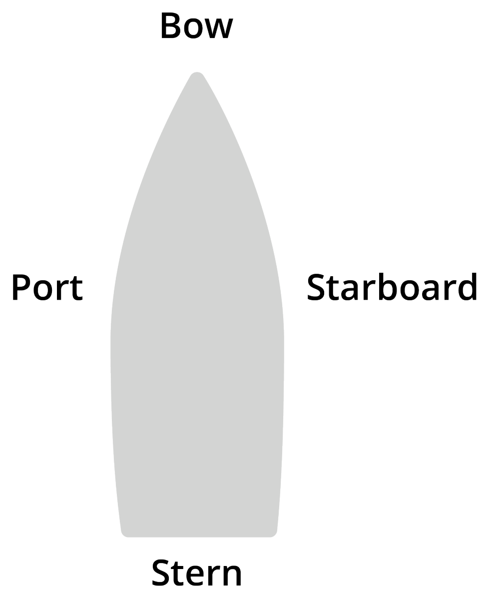
4.1 Basic Terminology

The front of a boat is called *the bow*. (It is pronounced exactly the same as "bough.") The back of the boat is called *the stern*.

The underside of the boat is called *the hull*. The top of the boat is called *the deck*.

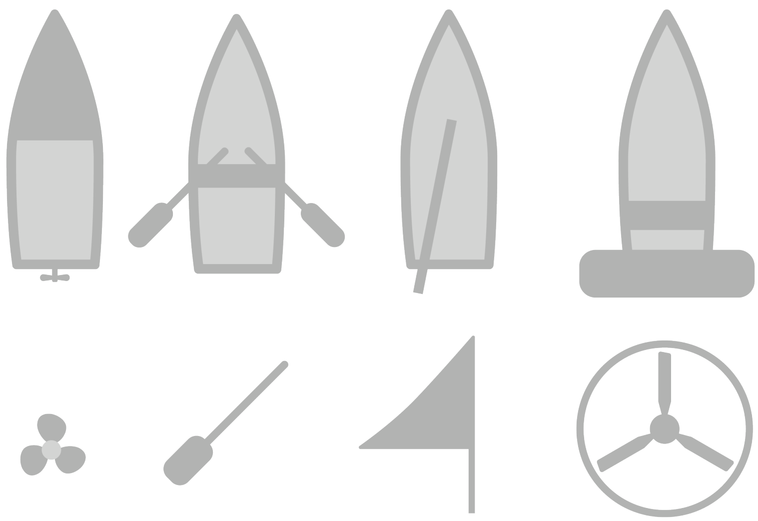


If you are standing at the stern and looking toward the bow, everything on your left is the *port* side. Everything on your right is the *starboard* side.



There are several different ways that boats are propelled:

- A motor turns a screw in the water, as in a motorboat. The screw is known as a *propeller*.
- A human pushes the water with a stick. If the stick is attached to the boat with a pivot (as in a rowboat) it is an *oar*. If the blade is not attached to the boat (as in a canoe), it is a *paddle*.
- The wind pushes the boat, as in a sailboat. The sails are held up by a *mast*.
- Some boats have a big fan that pushes the boat. These are called *airboats*. Airboats are not the most efficient boats, but they can travel on waterways with water just a couple of inches deep.



In the terms of physics, each of these method provide a *thrust vector* which is applied to the boat at a particular place and in a particular direction.

The speed of a boat usually measured in *knots*. 1 knot is 1 nautical mile per hour or 1.852 km per hour.

4.2 Why Boats Float Upright

Early in this sequence, we discussed buoyancy as a quantity: The magnitude of the buoyant force is equivalent to the weight of the liquid displaced.

We can also talk about the direction of the buoyant force: buoyancy pushes in the opposite direction as gravity.

How do we design boats so that they don't flip over?

4.2.1 Center of Buoyancy

Let's say you have a rowboat. If you push down on point on the floor near the front, the front of the boat will go down and the back of the boat will rise – that is, besides sinking in the water a little, the boat will rotate in that direction. If you push on the floor near the back of the boat, the back will sink a little lower and the front will rise. But there is a place, near the center of the boat, where if you push down, the boat will not rotate at all, it will just sink a little lower in the water. That point is known as the *center of buoyancy*.

How can we calculate the center of buoyancy? Imagine the shape of the water that was displaced by the boat. Now imagine that shape filled with water. The center of mass of that water is the center of buoyancy of the boat.

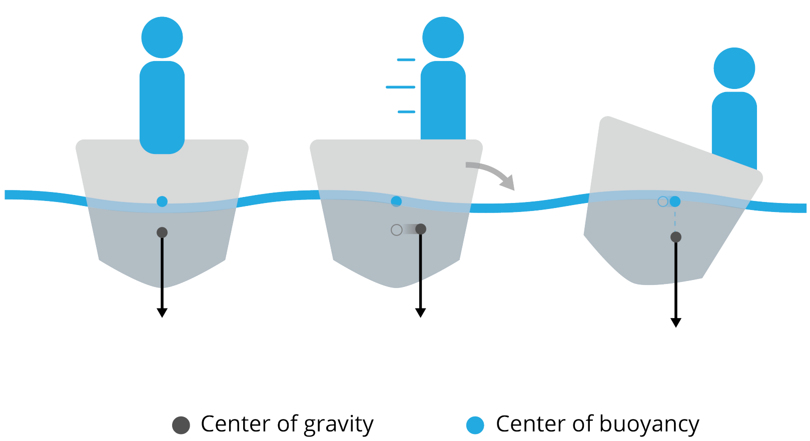
4.2.2 Center of Mass

Your boat and everything in it can be thought of as one object. That object has a center of mass. If you found the center of mass, you could balance the whole boat on it.

In a boat, if you move your body from the center of the boat to once side, you will have moved the center of mass. The boat will lean in that direction, which will change the center of buoyancy.

If you imagine a line is parallel to the force of gravity that passes through the center of mass of your boat, the boat will continue to increase its lean until the center of buoyancy is on that line.

If water comes over the sides of the boat before the center of gravity and center of buoyancy align, your boat will sink.



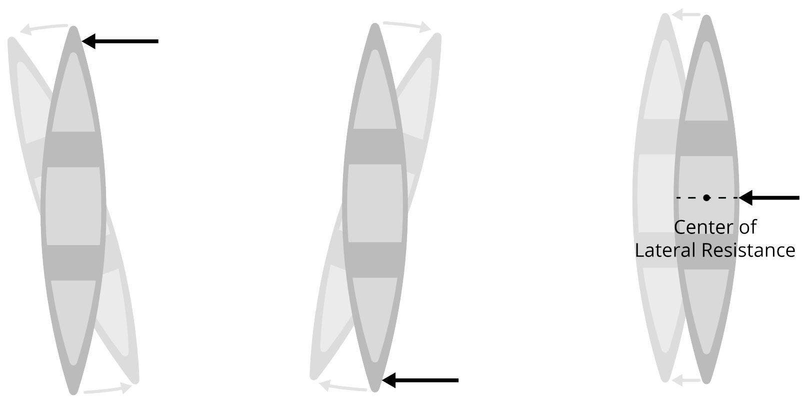
4.3 Center of Lateral Resistance

It isn't enough for a boat to float – for a boat to be useful, it must also be able to travel in a straight line.

Imagine that you are standing knee-deep in a lake next to a canoe. If you push the front of the canoe away from you, it will rotate – the back end will actually swing toward you. There is a point near the middle of the canoe where if you push it will not rotate in either direction – the boat will just slide sideways. This point is known as the *center of lateral resistance*.

The trick to making a boat travel in a straight line is to make sure that the line that contains the thrust vector passes through the center of lateral resistance.

An outboard motor allows you to direct the thrust vector: when the line of thrust passes the center of lateral resistance on the starboard side of the center of lateral resistance, the boat turns toward the port side.

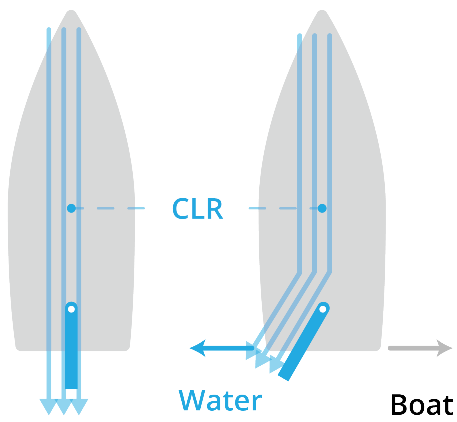


4.4 Steering with a Rudder

While outboard motors and airboats let you direct the thrust vector, most boats have a *rudder*. The rudder is a blade on a pivot near the back of the boat. The angle of the rudder can be adjusted so that water rushing past it gets pushed to one side or the other.

According to Newton's third law, when the water gets pushed to the left, the back of the boat gets pushed (with the same force) to the right. This causes the boat to rotate around its center of lateral resistance.

Note that a rudder only works when the boat passing through the water.



4.5 Boat Length and Resistance

FIXME: Write about wave length, boat length, and Froude number.

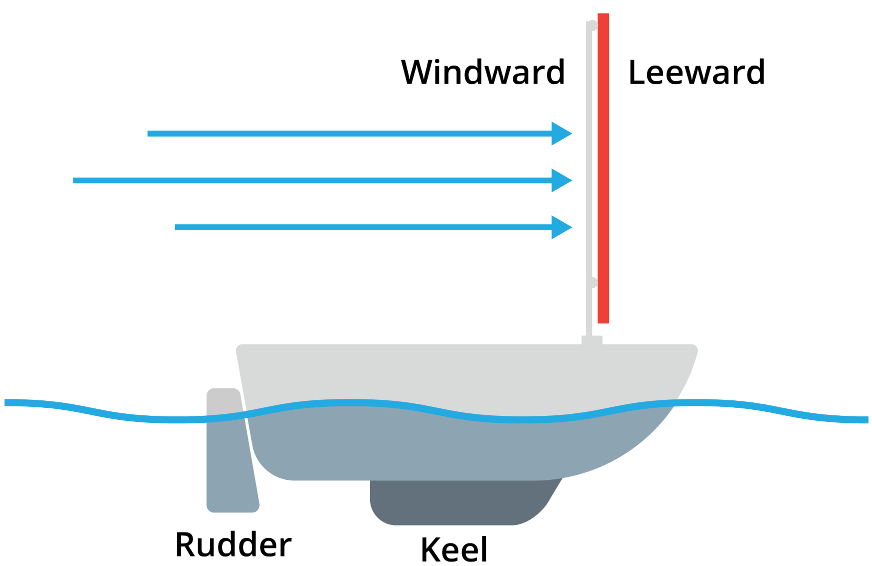
Chapter 5

Sailboats

Imagine that you have a canoe, and you are about to paddle from one island to another that is directly east of where you are standing. Imagine, also, that there is a steady wind coming from the west, and you have a big piece of plywood. You might be inspired to use it as a sail.

This situation is the most simple form of sailing: Wind comes from behind the boat and hits the sail which generates a force that pushes the boat in the direction of the wind.

The sail has two sides: The *windward* side is the one that is getting hit with the wind. The *leeward* side is the side away from the wind.



5.1 Magnitude of the Wind Force

The first natural question is: How much force will I have pushing my canoe through the water?

Wind Force

When the sail is perpendicular to the wind, the force of the wind on the sail in newtons (F_w) will be given by:

$$F_w = A\frac{dv^2}{2}$$

where A is the area of the sail in square meters d is the density of the gas in kg per cubic meter, and v is the wind speed in meters per second.

For air at STP, d is about 1.225 kg per cubic meter.

We call $\frac{dv^2}{2}$ the *wind pressure*. It is the amount of pressure that the windward side of plywood is experiencing that is above the pressure that the leeward side of the plywood is experiencing. (The leeward side might experience some turbulence, but the pressure it is experiencing is approximately 1 atmosphere.)

Let's your canoe is standing still and the wind is 0.5 m/s. Then the wind pressure is

$$P = \frac{1.225(0.5^2)}{2} = 0.153125 \text{ newtons per square meter}$$

Let's say your plywood sail is 2 meters tall and 1.5 meters wide. What will be the force of the wind?

$$F_w = AP = (3)(0.153125) \approx 0.46 \text{ newtons}$$

This is a very intuitive idea: There is a difference between the pressure on the windward side and the pressure on the leeward side, and the plywood experiences a force that pushes the boat through the water.

5.2 Direction and Location of the Wind Force

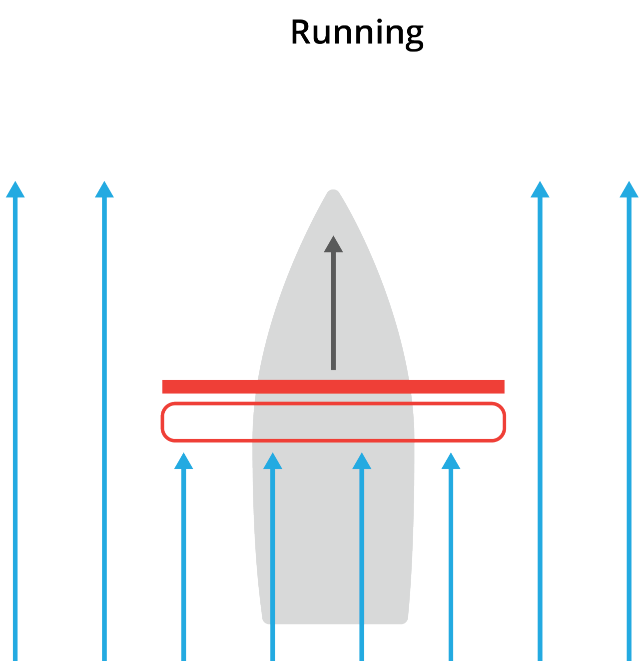
If there is low pressure on one side of the sail and high pressure on the other, the force vector will be perpendicular to the sail.

Where is this force vector applied? We can think of the force as being applied at the geometric center of the sail. This is called *the center of effort*. In this case, the center of effort is the exact center of the rectangular plywood.

The mast on a windsurfing board can be tilted from side to side. When the center of effort is over the center of lateral resistance, the board goes straight. To steer, the sailor moves the mast to one side of the center of lateral resistance which rotates the board.

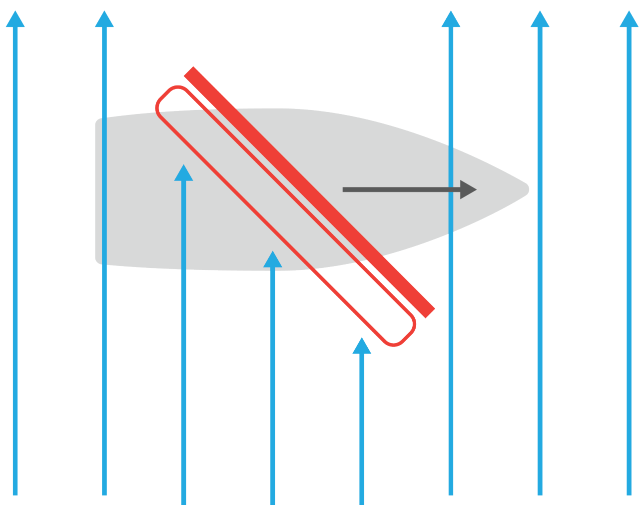
5.3 Beam Reach

When you are sailing in the same direction as the wind, sailors say you are *running*.



What if you want to go east and now the wind (still 0.5 m per second) is coming from the south? Sailing perpendicular to the wind is known as a *beam reach*.

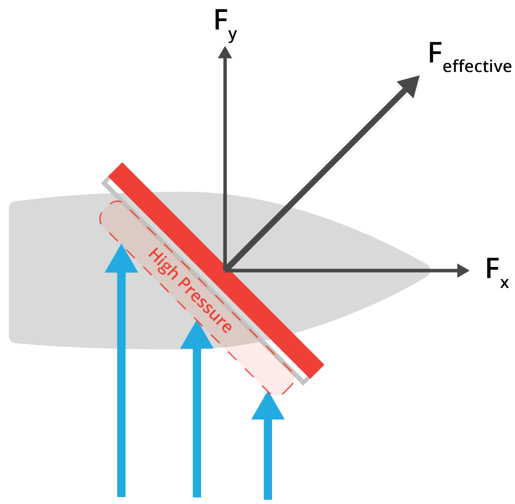
Beam Reach



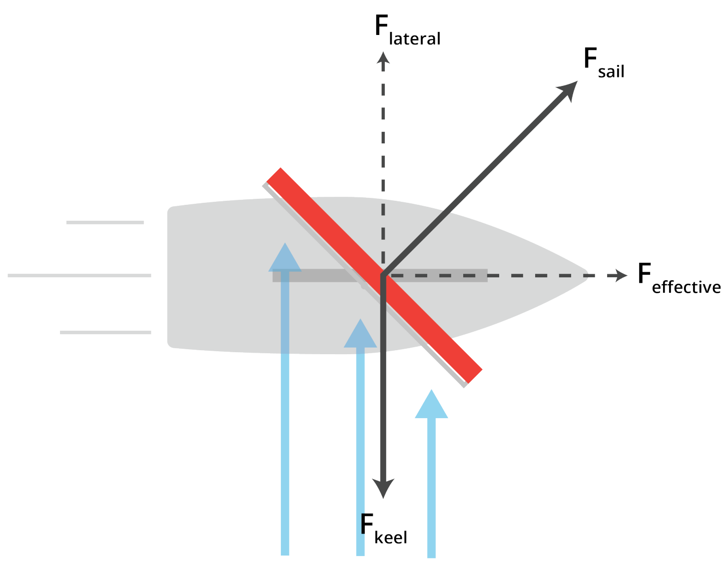
To do a beam reach, instead of mounting the plywood perpendicular to the boats direction of travel, you would mount it at a 45 degree angle. The wind pressure will build on the windward side of the plywood, and the plywood will experience a force pushing it at a 45° angle to the boat.

We can think of this force as having two components:

- One component pushes the boat forward (Yay!)
- One component pushes the boat sideways (Ugh.)



To minimize the effect of the sideways force, sailboats typically have a keel – a long fin on its underside that slows its sideways sliding.



Notice also that the "wind shadow" of the plywood is smaller when it is at a 45° angle to the wind. How much smaller? The effective area of your plywood has gone from 3 square meters to $\frac{3}{\sqrt{2}} \approx 2.12$ square meters. So the force generated will be smaller, and some of it will be wasted pushing the boat sideways.

If we assume that the wind pressure is still 0.153125 newtons per square meter. The force on the plywood will be about

$$F_w = AP = \frac{3}{\sqrt{2}}(0.154125) \approx 0.325 \text{ newtons}$$

However, the direction of that force is not all in the direction you want to go, so the effective force is:

$$F = F_w \frac{1}{\sqrt{2}} = \frac{3}{2}(0.154125) = 0.2311875 \text{ newtons}$$

Notice that we got twice as much effective force when we were running with the wind as when we are on a beam reach. However, any sailor will tell you that you can go much faster on a beam reach than you can running. Why?

5.4 Apparent Wind

When you are running, you can never go faster than the wind. As you go faster and faster, the wind that the boat experience decreases. For example, if you are going 0.2 m/s in a wind of 0.5 m/s, you (and your sail) will only experience wind at 0.3 m/s.We call the wind as experienced by the boat the *apparent wind*. The wind as observed by a stationary observer is called the *actual wind*.

If you are running with the wind, as you approach the speed of the wind, the force of that wind will decrease towards zero.

On a beam reach, as you go faster, the direction of the wind seems to change. If you are going 0.2 m/s east and the actual wind is 0.5 m/s from the south, the direction of the apparent wind will seem to come from about 22 degrees east of true south. The speed of the apparent wind will be about 0.54 m/s.

5.5 Close Reach

What if you want to go east, and the wind is coming from 40 degrees east of south? This would mean that you were sailing just 50 degrees away from straight into the wind. Is this possible?

If you put your sail at a 25 degree angle, you will still catch some wind and create some pressure on one side of the sail. Most of the resulting force would be trying to push your boat sideways, but some of it would be in the direction you were trying to travel.

Picking an angle for your sail that creates high pressure that makes a desirable force is known as the ”angle of attack”.

This is a non-intuitive result: A boat can sail into the wind!? The boat can’t sail directly into the wind – with each degree that the boat gets closer to straight into the wind, the force pushing it forward decreases and the force pushing it back increases. However, most boats can get within 45% if they have a well-shaped sail.

5.6 Shaping the Sail

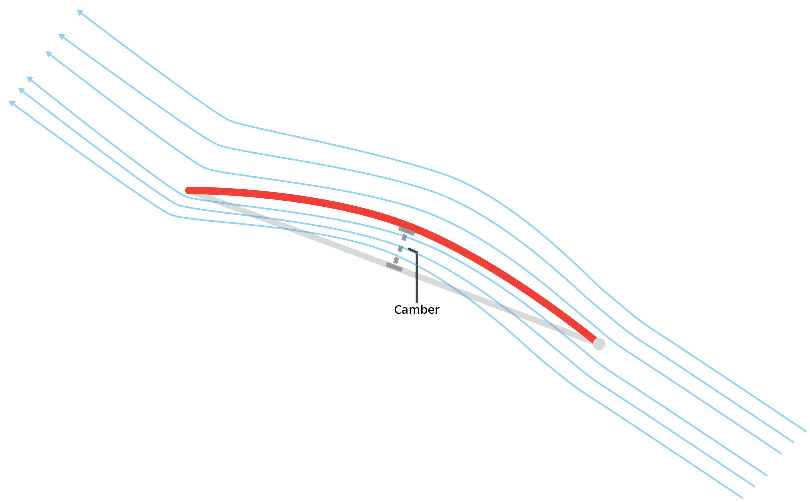
Most of the power of the wind can be captured with a piece of flat plywood. The wind hits it and creates a high pressure on the windward side. What about the other side of the plywood?

It turns out that if we can get the wind to travel smoothly over the back side of the plywood, the pressure on that side will be a little lower than if we had turbulence there. (I’m not going to go into this too deeply into why. If you want to know more, look up the Coanda effect.)

For example, if we were on a close reach, the very best sail we could have would gently pull the wind along its backside. It would look like this:

Of course, for the sail to work on either side of the boat, this asymmetrical design would not work. (Although, we should note that this design works great for airplane wings.)

When we make a sail out of cloth, we give it some curve known as *camber*. Slow winds require just a little camber, fast winds require more.



Some newer sailboats have wing sails that have two pieces that can be arranged to redirect the most air possible.

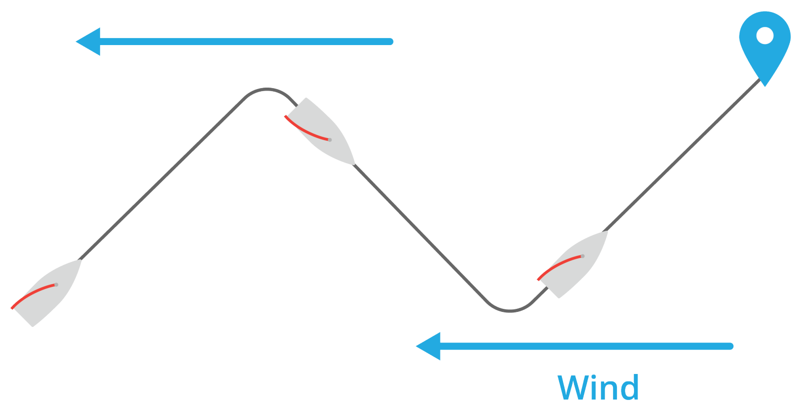
Note: when running with the wind, the turbulence on the leeward side of the sail is unavoidable. But when traveling perpendicular to the wind or on a close reach, the air should move smoothly over the leeward side of the sail. Many sailors have a piece of yarn taped on each side of the sail so they can see if the air is moving smoothly.

Many sailboats also have multiple sails. Besides the increase in the sail area, each sail also redirects the wind to pass smoothly over the leeward surface of the sail behind it.

5.7 Tacking into the Wind

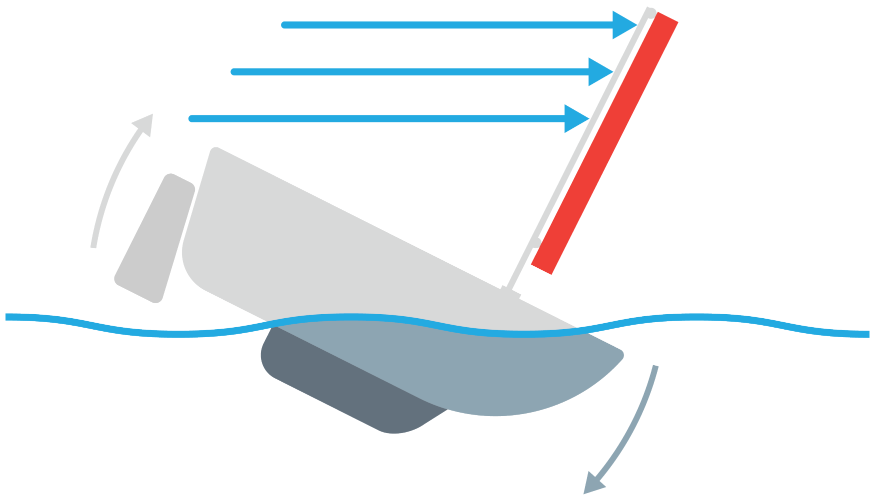
What if the wind is coming from the east and you really need to go directly into the wind?

The boat will not travel straight into the wind, instead you will travel on a close reach with the wind coming from one side of the boat. Then you will turn into the wind and continue turning until you are on a close reach with the wind coming from the other side. This is known as *tacking*.

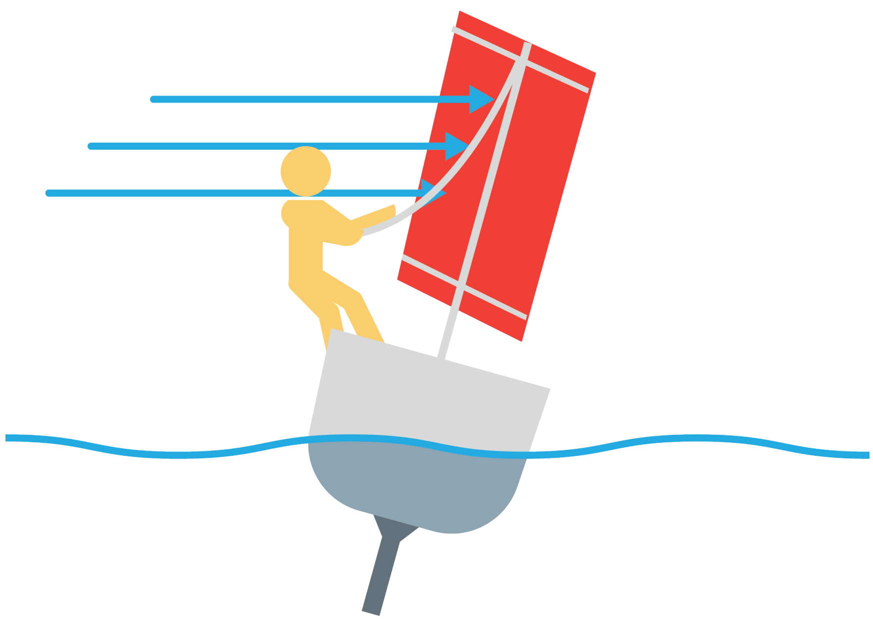


5.8 Heeling

The center of effort is near the center of your sail, which is usually pretty high in the air. Yes, the force generated will push your boat, but it will also rotate your boat. Sailors call the resulting lean *heeling*. Heeling too far is problematic – the rudder gets pulled out of the water, which makes it hard to steer the boat.



If a boat is heeling too far, sailors will move their weight to the windward side of the boat – some even wear harnesses that them push their weight out beyond the edge of the hull. If that doesn't work, they will reduce the sideways force on the sail.



Can the boat flip-over? (The word sailors use is *capsize*.) Most larger boats should not be able to capsize – as the sail gets pushed down toward the water, it loses power. So putting some weight in the keel will ensure that the boat doesn't turn upside-down. Small boats (think 3 or 4 meters long) can capsize, but they are small enough that the sailor can usually get them upright again without assistance.

There are boats with multiple hulls. A catamaran has two identical hulls that are side-by-side. As a result, the catamaran will not heel much at all. However, if a big gust of wind comes up, one hull can be pulled completely out of the water. Catamarans can be capsized.

If a boat is running when a big gust comes up, that rotating force will try to push the bow underwater. If the boat is going very fast, this can result in a somersault as the front of the boat slows and dips suddenly and the back of the boat it pitched up over it.

Chapter 6

Airplanes

Now that you understand how a sail works, you are ready to understand how an airplane works.

6.1 Drag and Thrust

As airplane flies, the air around it creates a big force that is trying to slow the plane down. Pilots call this *drag*.

To overcome drag, an airplane must have a source of *thrust*. This is usually done by pushing air toward the back of plane.

There are basically three types of propulsion on airplanes:

- Propeller planes have fan blades that are spun by an internal combustion engine and push the air toward the back of the plane.
- Jet planes suck air into a tube, mix it with fuel, and ignite the mixture. There are fan blades inside the tube that ensure the power from the burn is efficiently converted into thrust.
- Turboprop planes use both propeller and jet technology: they have propellers turned by jet engines.

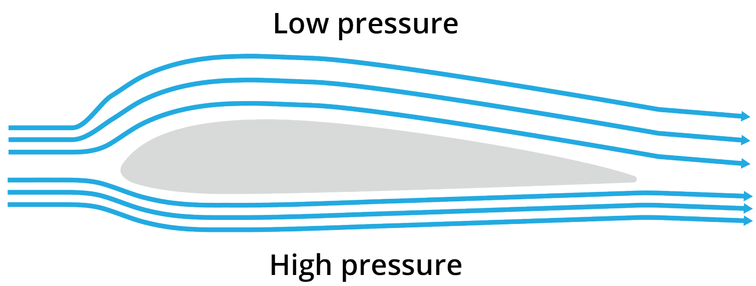
Drag is proportional to the square of the velocity. So, for example, if you fly twice as fast, the drag force will increase by a factor of 4.

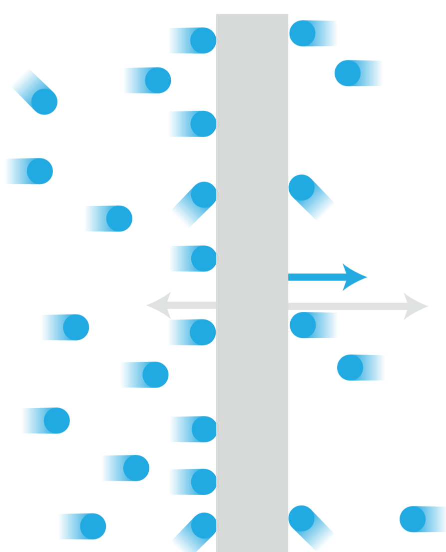
Passenger airplane fly fast – they have no trouble reaching 1000 km per hour. A Boeing 747 burns about 4 liters of fuel per second; the flight from London to New York requires about 70,000 kg of fuel. When the plane takes off, the fuel for the journey often weighs twice as much as the passengers.

6.2 Lift

Unlike a hot air balloon, an airplane is heavier than the air it displaces, so the force of gravity will pull it from the sky unless there is a counteracting force. We call the counteracting force *lift*.

Lift works just like a sailing into the wind: By picking an angle of attack, we create high pressure under the wing. By creating a nice smooth path for the air to travel over the top of the wing, we create low pressure over the wing. This difference creates the lift on the wings.





(There are some very bad explanations of this idea that overstate the importance of the rounded top of the wing. Most airplanes can fly upside down – if the most important part were the shape of the top, this would be impossible. The most important part is the angle of attack.)

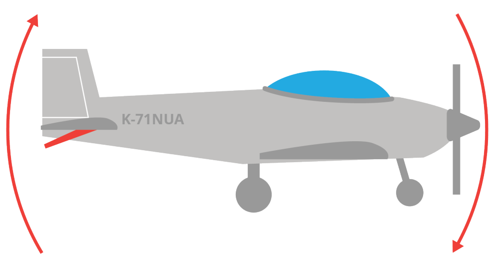
At high altitudes, the air is less dense. If two identical planes are flying at the same speed, but at different altitudes,:

- There is less drag on the higher airplane.
- The wings provide less left on the higher airplane.
- The air around the higher plane has less oxygen per liter, which can affect how the fuel burns.

6.3 Control

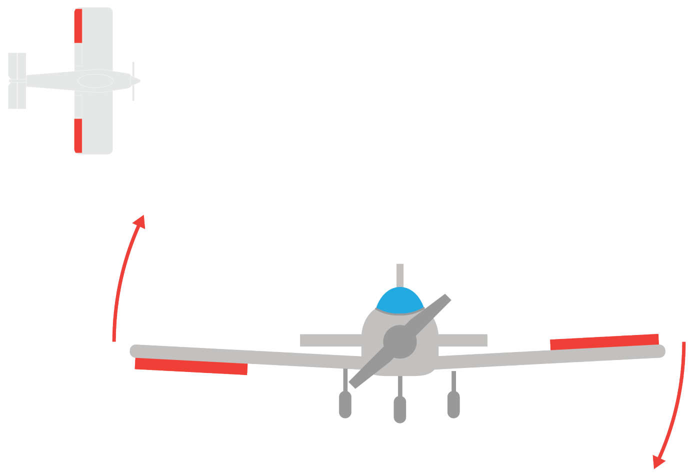
The first control that a pilot has is the throttle. By increasing the throttle, the pilot increases the plane's thrust.

The pilot has a stick. Pulling back on the stick causes the *elevator* on the tail of the plane to go up. Air hitting the elevator pushes the tail down and the nose up.

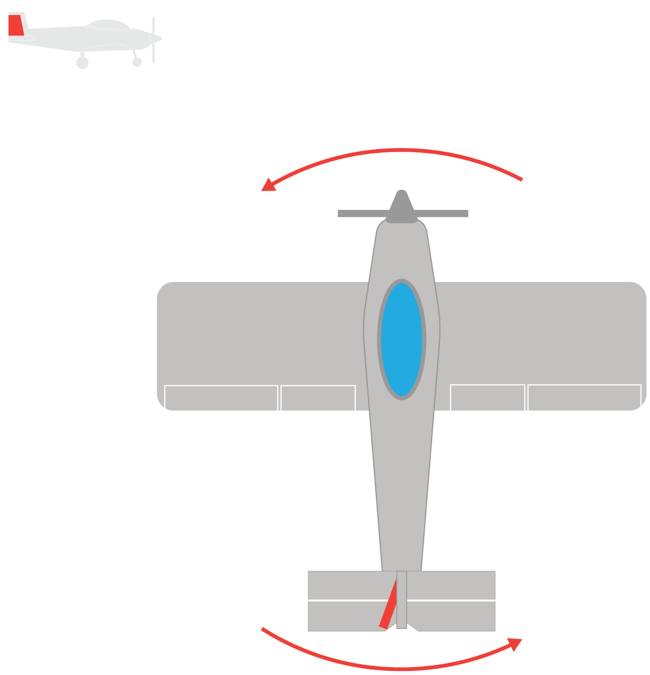


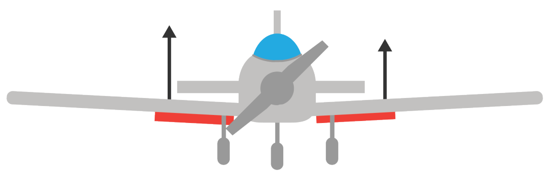
Pushing the stick forward will push the tail up and the nose down. We say "The elevator controls the *pitch* of the airplane."

The stick also goes left and right. Pushing the stick to the left, lifts the *aileron* on the left wing and lowers the aileron on the right wing. This pushes the left wing down and pushes the right wing up. We say "The ailerons control the *roll* of the airplane."



The pilot controls the *rudder* on the tail with his feet. Pushing the right side down will push the rudder to the right. This will push the tail of the plane to the left and the nose to the right. We say "The rudder controls the *yaw* of the airplane."

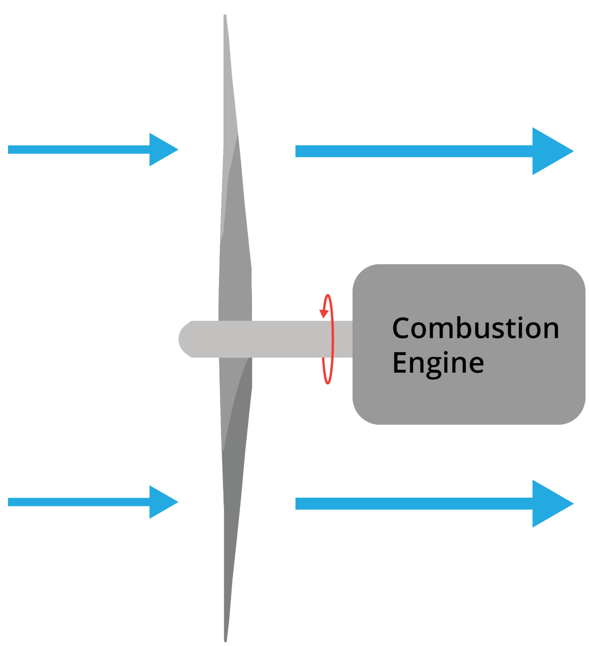




6.4 Thrust

There are several common ways that airplanes produce thrust.

The first is a propeller powered by a standard piston engine.

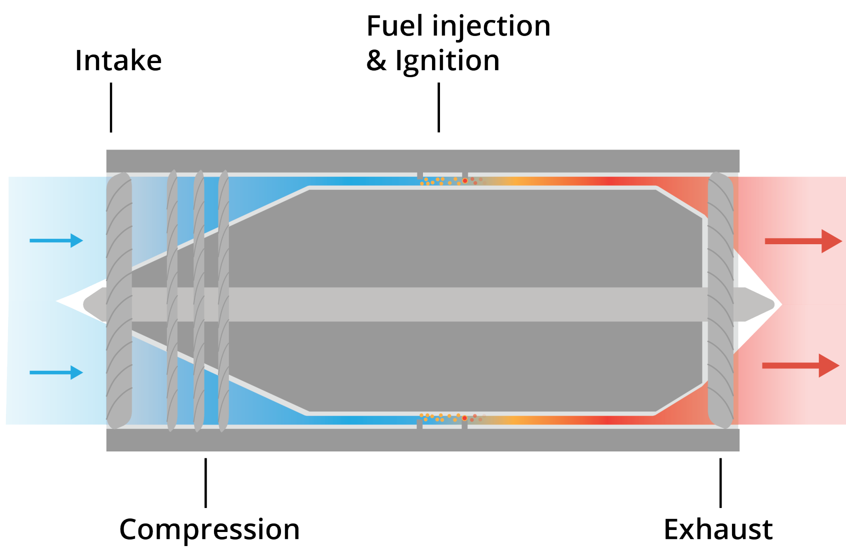


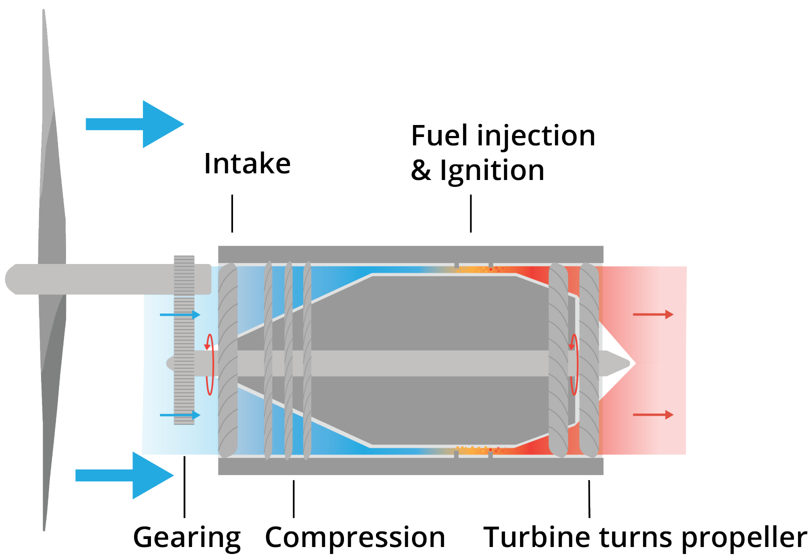
The propeller [HOW PROPELLER USES PRESSURE]

Another way to produce thrust is by using a *jet engine*. Jet engines take in air, and increase its pressure and temperature, and shoots it out of the back of the engine. The combination of making air move much faster while also

and XXX can make a jet engine more powerful than standard pistol-driven propellers.

Jet engines suck in air, compress it, add a fuel mixture, ignite the fuel, and shoot it out the back. Once the air exits the engine, it expands at a high speed, which further helps create thrust.



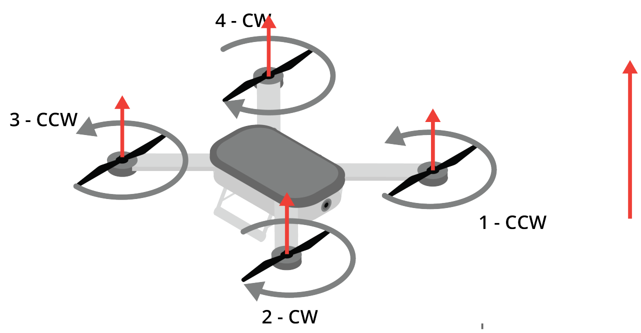


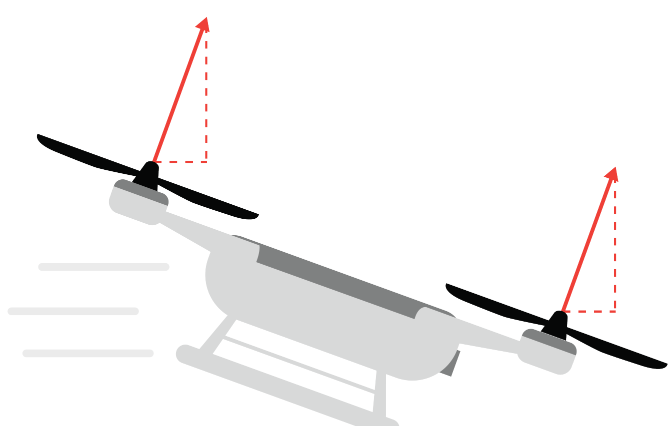
6.5 Gliders

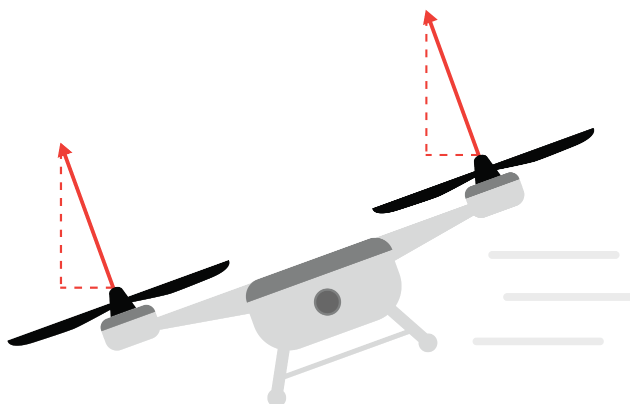
It is interesting to note that gliders fly with no engines. Gliding usually starts high in the air, where the glider has lots of potential energy. To stay aloft for a long time, glider pilots will look for places where air is rising – so they can ride those updrafts and regain that potential energy.

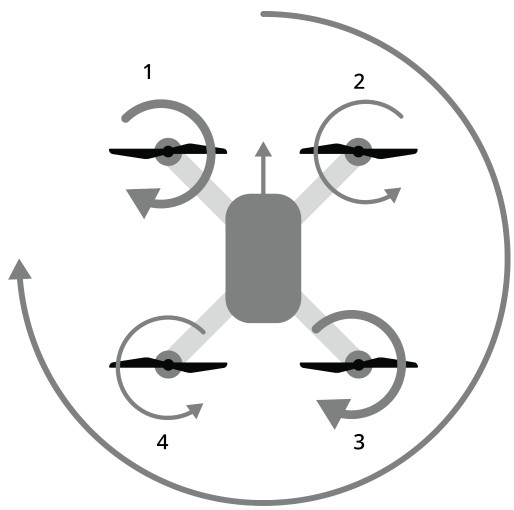
Chapter 7

Quadcopters





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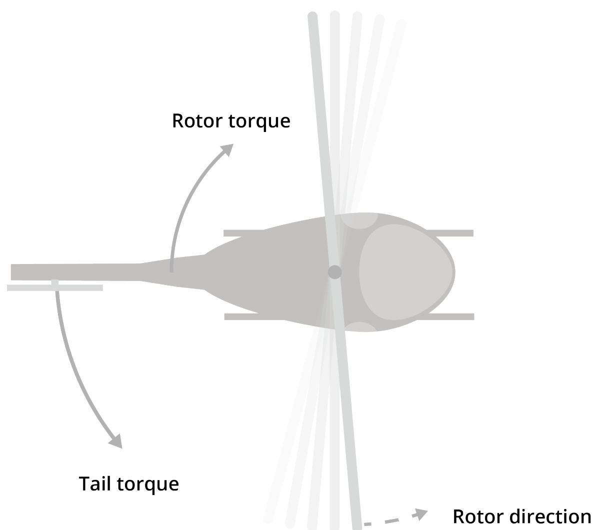
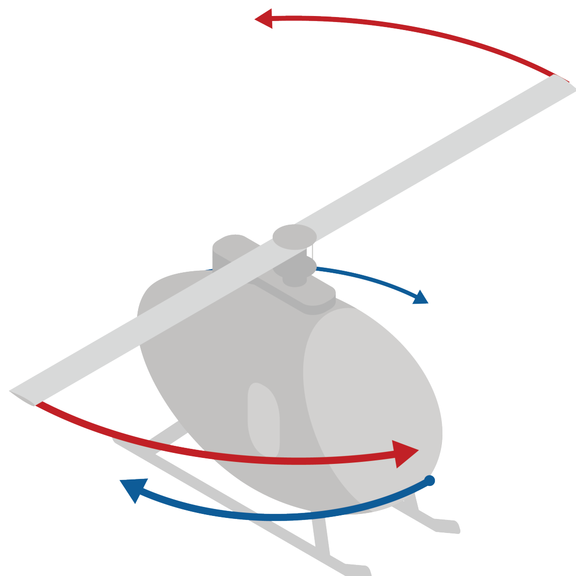
Chapter 8

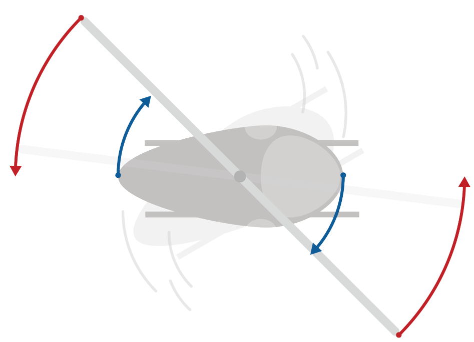
Helicopters

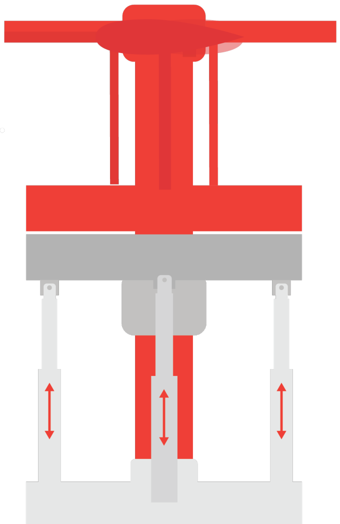
Instead of a fixed wing, the ”wing” rotates around to produce lift.

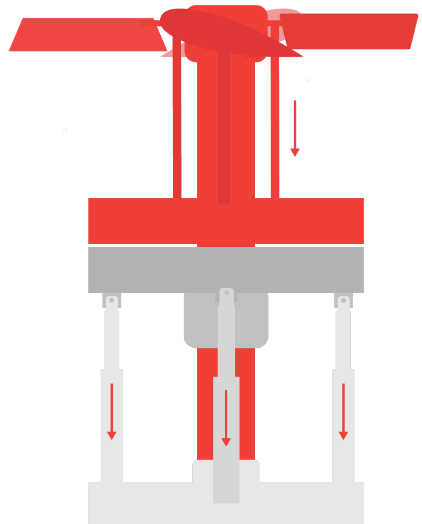


Sequence: Rotor/Tail, Show how the tail rotor works, then show how the control works for the main rotor









Still hard to see
what's going on in
the red hubs

Collective

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 5)

- $[1, 2, 3] + [4, 5, 6] = [5, 7, 9]$
- $[-1, -2, -3, -4] + [4, 5, 6, 7] = [3, 3, 3, 3]$
- $[\pi, 0, 0] + [0, \pi, 0] + [0, 0, \pi] = [\pi, \pi, \pi]$

Answer to Exercise 2 (on page 5)

To get the net force, you add the two forces:

$$F = [4.2, 5.6, 9.0] + [-100.2, 30.2, -9.0] = [-96, 35.8, 0.0] \text{ newtons}$$

Answer to Exercise 3 (on page 7)

- $2 \times [1, 2, 3] = [2, 4, 6]$
- $[-1, -2, -3, -4] \times -3 = [3, 6, 9, 12]$
- $\pi[\pi, 2\pi, 3\pi] = \pi^2, 2\pi^2, 3\pi^2]$

Answer to Exercise 4 (on page 9)

- $|[1, 1, 1]| = \sqrt{3} \approx 1.73$
- $|[-5, -5, -5]| = |-5 \times [1, 1, 1]| = 5\sqrt{3} \approx 8.66$
- $|[3, 4, 5] + [-2, -3, -4]| = |[1, 1, 1]| = \sqrt{3} \approx 1.73$

Answer to Exercise 5 (on page 12)

The momentum of the first car is 12,000 kg m/s in the north direction.

The momentum of the second car is 24,000 kg m/s in the east direction.

The new object will be moving northeast. What angle is the angle compared with the east?

$$\theta = \arctan \frac{12,000}{24,000} \approx 0.4636 \text{ radians } \approx 26.565 \text{ degrees north of east}$$

The magnitude of the momentum of the new object is $\sqrt{12,000^2 + 24,000^2} \approx 26,833$ kg m/s

Its new mass is 2,5000 kg. So the speed will be $26,833/2,500 = 10.73$ m/s.

Answer to Exercise 6 (on page 14)

The original forward momentum was 1.2 kg m/s. The original kinetic energy is $(1/2)(0.4)(3^2)$ $= 1.8$ joules.

Let s be the post-collision speed of the ball that had been at rest. Let x and y be the forward and sideways speeds (post-collision) of the other ball. Conservation of kinetic energy says

$$(1/2)(0.4)(s^2) + (1/2)(0.4)(x^2 + y^2) = 1.8$$

Forward momentum is conserved:

$$0.4\frac{s}{\sqrt{2}} + 0.4x = 1.2$$

Which can be rewritten:

$$x = 3 - \frac{s}{\sqrt{2}}$$

Sideways momentum stays zero:

$$(0.4)\frac{s}{\sqrt{2}} - 0.4y = 0.0$$

Which can be rewritten:

$$y = \frac{s}{\sqrt{2}}$$

Substituting into to the conservation of kinetic energy equation above:

$$(1/2)(0.4)(s^2) + (1/2)(0.4)(\left(3 - \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = 1.8$$

Which can be rewritten:

$$s^2 - \frac{3}{\sqrt{2}}s + 0 = 0$$

There are two solutions to this quadratic: $s = 0$ (before collision) and $s = \frac{3}{\sqrt{2}}$. Thus,

$$y = \frac{3}{2}$$

and

$$x = 3 - \frac{3}{2} = \frac{3}{2}$$

So both balls careen off at 45° angles at the exact same speed.

Answer to Exercise 7 (on page 15)

- $[1, 2, 3] \cdot [4, 5, -6] = 4 + 10 - 18 = -4$
- $[\pi, 2\pi] \cdot [2, -1] = 2\pi - 2\pi = 0$
- $[0, 0, 0, 0] \cdot [10, 10, 10, 10] = 0 + 0 + 0 + 0 = 0$

Answer to Exercise 8 (on page 17)

- $[1, 0] \cdot [0, 1] = 0$. The angle must be $\pi/2$.
- $[3, 4] \cdot [4, 3] = 24$. $|[3, 4]||[4, 3]| \cos(\theta) = 24$. $\cos(\theta) = \frac{24}{(5)(5)}$. $\theta = \arccos(\frac{24}{25}) \approx 0.284$ radians.

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