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CHAPTER 1

# u-Substitution

U-Substitution, also known as the method of substitution, is a technique used to simplify the process of finding antiderivatives and integrals of complicated functions. The method is similar to the chain rule for differentiation in reverse.

Suppose we have an integral of the form:

$$\int f(g(x)) \cdot g'(x)\, dx \tag{1.1}$$

The u-substitution method suggests letting a new variable $u$ equal to the inside function $g(x)$, i.e.,

$$u = g(x) \tag{1.2}$$

Then, the differential of $u$, $du$, is given by:

$$du = g'(x)\, dx \tag{1.3}$$

Substituting $u$ and $du$ back into the integral gives us a simpler integral:

$$\int f(u)\, du \tag{1.4}$$

This new integral can often be simpler to evaluate. Once the antiderivative of $f(u)$ is found, we can substitute $u = g(x)$ back into the antiderivative to get the antiderivative of the original function in terms of $x$.

The method of u-substitution is a powerful tool for evaluating integrals, especially when combined with other techniques like integration by parts, partial fractions, and trigonometric substitutions.

# Chapter 2

# Differential Equations

Differential equations are equations involving an unknown function and its derivatives. They play a crucial role in mathematics, physics, engineering, economics, and other disciplines due to their ability to describe change over time or in response to changing conditions.

## 2.1 Ordinary Differential Equations

An ordinary differential equation (ODE) involves a function of a single independent variable and its derivatives. The order of an ODE is determined by the order of the highest derivative present in the equation. An example of a first-order ODE is:

$$\frac{dy}{dx} + y = x \tag{2.1}$$

Here, $y$ is the function of the independent variable $x$, and $\frac{dy}{dx}$ represents its first derivative.

## 2.2 Partial Differential Equations

Partial differential equations (PDEs), on the other hand, involve a function of multiple independent variables and their partial derivatives. An example of a PDE is the heat equation, a second-order PDE:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \tag{2.2}$$

In this equation, $u = u(x, t)$ is a function of the two independent variables $x$ and $t$, $\frac{\partial u}{\partial t}$ is the first partial derivative of $u$ with respect to $t$, and $\frac{\partial^2 u}{\partial x^2}$ is the second partial derivative of $u$ with respect to $x$.

Appendix A

# Answers to Exercises

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