CHAPTER 1

Methods of Integration

1.1 Partial Fractions

1.2 u-substitution

1.3 Integration by Parts

1.4 Practice

Exercise 1

Using the substitution $u = x^2 - 3$, rewrite $\int_{-1}^{4} x(x^2-3)^5 dx$ in terms of u.

Working Space

Answer on Page 3

Exercise 2

Evaluate $\int_0^1 \frac{5x+8}{x^2+3x+2} dx$ without a calculator.

Working Space

Answer on Page 3

Exercise 3

Let f be a function such that $\int f(x) \sin x \, dx = -f(x) \cos x + \int 4x^3 \sin x \, dx$. Give a possible expression for $f(x)$.

Working Space

Answer on Page 3

Exercise 4

Evaluate $\int_1^\infty x e^{-x^2} \, dx$.

Working Space

Answer on Page 3

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 1)

If $u = x^2 - 3$, then $du = 2x dx$ and $x(x^2-3)^5 dx = \frac{1}{2}u^5 du$. When $x = -1$, $u = -2$ and when $x = 4$, $u = 13$. Putting it all together, we find an equivalent integral is $\frac{1}{2}\int_{-2}^{13} u^5\, du$.

Answer to Exercise 2 (on page 1)

We cannot use u-substitution because $\frac{d}{dx}(x^2 + 3x + 2) \neq n(5x + 8)$. We will use partial fractions to simplify the integrand. Settig up: $\frac{5x+8}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$. Rearranging, we find $5x + 8 = A(x + 2) + B(x + 1)$. Letting $x = -2$, we find that $B = 2$. And taking $x = -1$, we find $A = 3$. Therefore, $\int_0^1 \frac{5x+8}{x^2+3x+2}\, dx = \int_0^1 \frac{3}{x+1}\, dx + \int_0^1 \frac{2}{x+2}\, dx$. Evaluating the integrals, we get $3\ln(x+1)|_0^1 + 2\ln(x+2)|_0^1 = 3(\ln 2 - \ln 1) + 2(\ln 3 - \ln 2) = 3\ln 2 + 2\ln\frac{3}{2} = \ln 8 + \ln\frac{9}{4} = \ln\frac{8 \cdot 9}{4} = \ln 18$.

Answer to Exercise 3 (on page 2)

This question takes the form of integration by parts. That is, $\int f(x)g'(x)\, dx = f(x)g(x) - \int g(x)f'(x)\, dx$. If we let $g(x) = -\cos x$, then $g'(x) = \sin x$. The structure of the equation implies that $f'(x) = 4x^3$ and therefore that f could be $f(x) = x^4$.

Answer to Exercise 4 (on page 2)

Letting $u = -x^2$, then $du = -2x dx$ and $x dx = \frac{-1}{2} du$. Substituting u and du into the integral, we have $\int_{x=1}^{x=\infty} \frac{-1}{2} e^u\, du$, which equals $\frac{-1}{2}e^u = \frac{-1}{2}e^{-x^2}|_1^\infty$. Evaluating the statement, we get $\frac{-1}{2}(e^{-\infty} - e^{-1}) = \frac{-1}{2}(0 - \frac{1}{e}) = \frac{1}{2e}$