Contents

Chapter 1

Introduction to Polynomials

Watch Khan Academy's **Polynomials intro** video at https://youtu.be/Vm7H0VTlIco

A *monomial* is the product of a number and a variable raised to a non-negative (but possibly zero) integer power. Here are some monomials:

$3x^2$	πx^2	$7x$	$-\frac{2}{3}x^{12}$
$-2x^{15}$	$(3.33)x^{100}$	3	0

The exponent is called the *degree* of the monomial. Examples: $3x^{17}$ has degree 17, $-7x$ has degree 1, and 3.2 has degree 0 (because you can think of it as $(3.2)x^0$).

The number in the product is called the *coefficient*. Example: $3x^{17}$ has a coefficient of 3, $-2x$ has a coefficient of -2, and $(3.4)x^{1000}$ has a coefficient of 3.4.

A *polynomial* is the sum of one or more monomials. Here are some polynomials:

$4x^2 + 9x + 3.9$ $\pi x^2 + \pi x + \pi$ $7x + 2$

$-2x^{10} + (3.4)x - 45x^{900} - 1$ 3.3 $3x^{20}$

We say that each monomial is a *term* of the polynomial.

$x^{-5} + 12$ is *not* a polynomial because the first term has a negative exponent.

$x^2 - 32x^{\frac{1}{2}} + x$ is *not* a polynomial because the second term has a non-integer exponent.

$\frac{x+2}{x^2+x+5}$ is *not* a polynomial because it is not just a sum of monomials.

Exercise 1 Identifying Polynomials

Circle only the polynomials.

Working Space

$-2x^3 + \frac{1}{2}x + 3.9(4.5)x^2 + \pi x$ 7

$2x^{-10} + 4x - 1$ $x^{\frac{2}{3}}$ $3x^{20} + 2x^{19} - 5x^{18}$

Answer on Page 31

We typically write a polynomial starting at the term with the highest degree and proceed in decreasing order to the term with the lowest degree:

$$2x^9 - 3x^7 + \frac{3}{4}x^3 + x^2 + \pi x - 9.3$$

This is known as *the standard form*. The first term of the standard form is called *the leading term*, and we often call the coefficient of the leading term *the leading coefficient*. We sometimes speak of the degree of the polynomial, which is just the degree of the leading term.

Exercise 2 Standard of a Polynomial

Write $21x^2 - x^3 + \pi - 1000x$ in standard form. What is the degree of this polynomial? What is its leading coefficient?

Working Space

Answer on Page 31

Exercise 3 Evaluate a Polynomial

Let $y = x^3 - 3x^2 + 10x - 12$. What is y when x is 4?

Working Space

Answer on Page 32

I would be remiss in my duties if I didn't mention one more thing about polynomials: mathematicians have defined a polynomial to be a sum of a *finite* number of monomials.

It is certainly possible to have a sum of an infinite number of monomials like this:

$$1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \dots$$

This is an example of an *infinite series*; we don't consider them polynomials. Infinite series are interesting and useful, but I will not discuss them much until later in the course.

Chapter 2

Python Lists

Watch CS Dojo's **Introduction to Lists in Python** video at https://www.youtube.com/watch?v=tw7ror9x32s

To review, Python list is an indexed collection. The indices start at zero. You can create a list using square brackets.

Now you are going to write a program that makes an array of strings. Type this code into a file called `faves.py`:

```
favorites = ["Raindrops", "Whiskers", "Kettles", "Mittens"]
favorites.append("Packages")
print("Here are all my favorites:", favorites)
print("My most favorite thing is", favorites[0])
print("My second most favorite is", favorites[1])
number_of_faves = len(favorites)
print("Number of things I like:", number_of_faves)

for i in range(number_of_faves):
```

```
    print(i, ": I like", favorites[i])
```

Run it:

```
$ python3 faves.py
Here are all my favorites: ['Raindrops', 'Whiskers', 'Kettles', 'Mittens', 'Packages']
My most favorite thing is Raindrops
My second most favorite is Whiskers
Number of things I like: 5
0 : I like Raindrops
1 : I like Whiskers
2 : I like Kettles
3 : I like Mittens
4 : I like Packages
```

After you have run the code, study it until the output makes sense.

Exercise 4 Assign into list

Before you list the items, replace "Mittens" with "Gloves".

Working Space

Answer on Page 32

2.1 Evaluating Polynomials in Python

First, before you go any further, you need to know that raising a number to a power is done with ** in Python. So for example, to get 5^2, you would write `5**2`.

Back to polynomials: if you had a polynomial like $2x^3 - 9x + 12$, you could write it like this: $12x^0 + (-9)x^1 + 0x^2 + 2x^3$. We could use this representation to keep a polynomial in a Python list. We would simply store all the coefficients in order:

```
pn1 = [12,-9,0,2]
```

In the list, the index of each coefficient would correspond to the degree of that monomial. For example, in the list 2 is at index 3, so that entry represents $2x^3$.

In the last chapter, you evaluated the polynomial $x^3 - 3x^2 + 10x - 12$ at $x = 4$. Now you will write code that does that evalution. Create a file called `polynomials.py` and type in the following:

```
def evaluate_polynomial(pn, x):
    sum = 0.0
    for degree in range(len(pn)):
        coefficient = pn[degree]
        term_value = coefficient * x ** degree
        sum = sum + term_value
    return sum

pn1 = [-12.0, 10.0, -3.0, 1.0]
y = evaluate_polynomial(pn1, 4.0)
print("Polynomial 1: When x is 4.0, y is", y)
```

Run it. It should evaluate to 44.0.

2.2 Walking the list backwards

Now you are going to make a function that makes a pretty string to represent your polynomial. Here is how it will be used:

```
def polynomial_to_string(pn):
    ...Your Code Here...

pn_test = [-12.0, 10.0, 0.0, 1.0]
print(polynomial_to_string(pn1))
```

This would output:

```
1.0x**3 + 10.0x + -12.0
```

This is not as simple as you might hope. In particular:

- You should skip the terms with a coefficient of zero
- The term of degree 1 has an x, but no exponent
- The term of degree 0 has neither an x nor an exponent

- Standard form demands that you list the terms in the reverse order from that of your coefficients list. You will need to walk the list from last to first.

Add this function to your `polynomials.py` file after your `evaluate_polynomial` function:

```
def polynomial_to_string(pn):

    # Make a list of the monomial strings
    monomial_strings = []

    # Start at the term with the largest degree
    degree = len(pn) - 1

    # Go through the list backwards stop after constant term
    while degree >= 0:
        coefficient = pn[degree]

        # Skip any term with a zero coefficient
        if coefficient != 0.0:

            # Describe the monomial
            if degree == 0:
                monomial_string = "{}".format(coefficient)
            elif degree == 1:
                monomial_string = "{}x".format(coefficient)
            else:
                monomial_string = "{}x^{}".format(coefficient, degree)

            # Add it to the list
            monomial_strings.append(monomial_string)

        # Move to the previous term
        degree = degree - 1

    # Deal with the zero polynomial
    if len(monomial_strings) == 0:
        monomial_strings.append("0.0")

    # Make a string that joins the terms with a plus sign
    return " + ".join(monomial_strings)
```

Note that in a list n items, the indices go from 0 to $n-1$. So when we are walking the list backwards, we start at `len(pn) - 1` and stop at zero.

Look over the code and google the functions you aren't familar with. For example, if you

want to know about the (join) function, google for "python join".

Now change your code to use the new function:

```
pn1 = [-12.0, 10.0, -3.0, 1.0]
y = evaluate_polynomial(pn1, 4.0)
print("y =", polynomial_to_string(pn1))
print("    When x is 4.0, y is", y)
```

Run the program. Does the function work?

Exercise 5 Evaluate Polynomials

Using the function that you just wrote, add a few lines of code to `polynomials.py` to evaluate the following polynomials:

- Find $4x^4 - 7x^3 - 2x^2 + 5x + 2.5$ at $x = 8.5$. It should be 16481.875
- Find $5x^5 - 9$ at $x = 2.0$. It should be 151.0

Working Space

Answer on Page 32

2.3 Plot the polynomial

We can evaluate a polynomial at many points and plot them on a graph. You are going to write the code to do this. Create a new file called `plot_polynomial.py`. Copy your `evaluate_polynomial` function into the new file.

Add a line at the beginning of the program that imports the plotting library matplotlib:

```
import matplotlib.pyplot as plt
```

After the `evaluate_polynomial` function:

- Create a list with polynomial coefficients.

- Create two empty arrays, one for x values and one for y values.

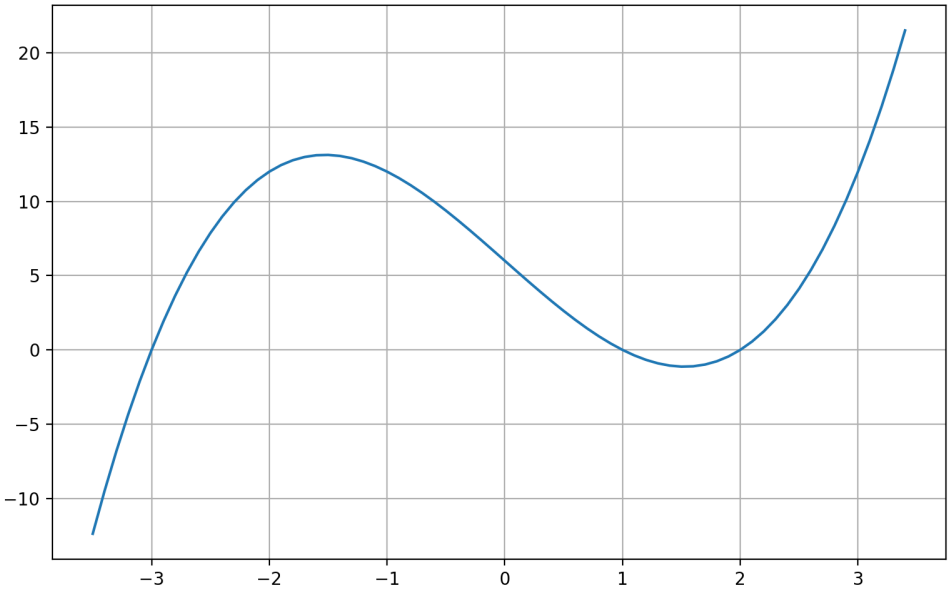
- Fill the x array with values from -3.5 to 3.5. Evaluate the polynomial at each of these points; put those values in the y array.

- Plot them

Like this:

```
# x**3 - 7x + 6
pn = [6.0, -7.0, 0.0, 1.0]

# These lists will hold our x and y values
x_list = []
y_list = []

# Start at x=-3.5
current_x =-3.5

# End at x=3.5
while current_x <= 3.5:
    current_y = evaluate_polynomial(pn, current_x)

    # Add x and y to respective lists
    x_list.append(current_x)
    y_list.append(current_y)

    # Move x forward
    current_x += 0.1

# Plot the curve
plt.plot(x_list, y_list)
plt.grid(True)
plt.show()
```

You should get a beautiful plot like this:



If you received an error that the matplotlib was not found, use pip to install it:

```
$ pip3 install matplotlib
```

Exercise 6 Observations

Where does your polynomial cross the y-axis? Looking at the polynomial $x^3 - 7x+6$, could you have guessed that value?

Working Space

Where does your polynomial cross the x-axis? The places where a polynomial crosses the x-axis is called *its roots*. Later in the course, you will learn techniques for finding the roots of a polynomial.

Answer on Page 32

Chapter 3

Adding and Subtracting Polynomials

Watch Khan Academy's **Adding polynomias** video at https://youtu.be/ahdKdxsTj8E

When adding two monomials of the same degree, you sum their coefficients:

$$7x^3 + 4x^3 = 11x^3$$

Using this idea, when adding two polynomials, you convert it into one long polynomial and then simplify by combining terms with the same degree. For example:

$$\begin{aligned}(10x^3 - 2x + 13) + (-5x^2 + 7x - 12) \\ &= 10x^3 + (-2)x + 13 + (-5)x^2 + 7x + (-12) \\ &= 10x^3 + (-5)x^2 + (-2+7)x + (13-12) \\ &= 10x^3 - 5x^2 + 5x + 1\end{aligned}$$

Exercise 7 Adding Polynomials Practice

Working Space

Add the following polynomials:

1. $2x^3 - 5x^2 + 3x - 9$ and $x^3 - 2x^2 - 2x - 9$

2. $3x^5 - 5x^3 + 3x^2 - x - 3$ and $2x^4 - 2x^3 - 2x^2 + x - 9$

Answer on Page 32

Notice that in the second question, the degree 1 term disappears completely: $(-x) + x = 0$

One more tricky thing that can happen: Sometimes the coefficients don't add nicely. For example:

$$\pi x^2 - 3x^2 = (\pi - 3)x^2$$

That is as far as you can simplify it.

3.1 Subtraction

Now watch Khan Academy's **Subtracting polynomials** at https://youtu.be/5ZdxnFspyP8.

When subtracting one polynomial from the other, it is a lot like adding two polynomials. The difference: when make the two polynomials into one long polynomial, we multiply each monomial that is being subtracted by -1. For example:

$$\begin{aligned}(2x^2 - 3x + 9) - (5x^2 - 7x + 4) &= 2x^2 + (-3)x + 9 + (-5)x^2 + 7x + (-4) \\ &= (2-5)x^2 + (-3+7)x + (9-4) \\ &= -3x^2 + 4x + 5\end{aligned}$$

Exercise 8 Subtracting Polynomials Practice

Working Space

Add the following polynomials:

1. $(2x^3 - 5x^2 + 3x - 9) - (x^3 - 2x^2 - 2x - 9)$

2. $(3x^5 - 5x^3 + 3x^2 - x - 3) - (2x^4 - 2x^3 - 2x^2 + x - 9)$

Answer on Page 32

3.2 Adding Polynomials in Python

As a reminder, in our Python code, we are representing a polynomial with a list of coefficients. The first coefficient is the constant term. The last coefficient is the leading coefficient. So, we can imagine $-5x^3 + 3x^2 - 4x + 9$ and $2x^3 + 4x^2 - 9$ would look like this: *FIXME: Diagram here*

To add the two polynomials then, we sum the coefficients for each degree. *FIXME: Diagram here*

Create a file called `add_polynomials.py`, and type in the following:

```
def add_polynomials(a, b):
    degree_of_result = len(a)
    result = []
    for i in range(degree_of_result):
        coefficient_a = a[i]
        coefficient_b = b[i]
        result.append(coefficient_a + coefficient_b)
    return result
```

```
polynomial1 = [9.0, -4.0, 3.0, -5.0]
polynomial2 = [-9.0, 0.0, 4.0, 2.0]
polynomial3 = add_polynomials(polynomial1, polynomial2)

print('Sum =', polynomial3)
```

Run the program.

Unfortunately, this code only works if the polynomails are the same length. For example, try making `polynomial1` have a larger degree than `polynomial2`:

```
# x**4 - 5x**3 + 3x**2 - 4x + 9
polynomial1 = [9.0, -4.0, 3.0, -5.0, 1.0]

# 2x**3 + 4x**2 - 9
polynomial2 = [-9.0, 0.0, 4.0, 2.0]
polynomial3 = add_polynomials(polynomial1, polynomial2)
print('Sum =', polynomial3)
```

See the problem?

Exercise 9 Dealing with polynomials of different degrees

Working Space

Can you fix the function `add_polynomials` to handle polynomials of different degrees?
Here is a hint: In Python, there is a `max` function that returns the largest of the numbers it is passed.

```
biggest = max(5,7)
```

Here `biggest` would be set to 7.
Here is another hint: If you have an array `mylist`, `i`, a non-negative integer, is only a legit index if `i < len(mylist)`.

Answer on Page 33

3.3 Scalar multiplication of polynomials

If you multiply a polynomial with a number, the distributive property applies:

$$(3.1)(2x^2 + 3x + 1) = (6.2)x^2 + (9.3)x + 3.1$$

(When we are talking about things that are more complicated than a number, we use the word *scalar* to mean "Just a number". So this is the product of a scalar and a polynomial.)

In `add_polynomials.py`, add a function to that multiplies a scalar and a polynomial:

```
def scalar_polynomial_multiply(s, pn):
    result = []
    for coefficient in pn:
        result.append(s * coefficient)
    return result
```

Somewhere near the end of the program, test this function:

```
polynomial4 = scalar_polynomial_multiply(5.0, polynomial1)
print('Scalar product =', polynomial_to_string(polynomial4))
```

Exercise 10 Subtract polynomials in Python

Working Space

Now implement a function that does subtraction using `scalar_polynomial_multiply` and `add_polynomials`.
It should look like this:

```
def subtract_polynomial(a, b):
    ...Your code here...

polynomial5 = [9.0, -4.0, 3.0, -5.0]
polynomial6 = [-9.0, 0.0, 4.0, 2.0, 1.0]
polynomial7 = subtract_polynomial(polynomial5, polynomial6)
print('Difference =', polynomial_to_string(polynomial7))
```

Answer on Page 33

Chapter 4

Multiplying Polynomials

Watch Khan Academy's **Multiplying monomials** at https://youtu.be/Vm7H0VTlIco.

To review, when you multiply two monomials, you take the product of their coefficients and the sum of their degrees:

$$(2x^6)(5x^3) = (2)(5)(x^6)(x^3) = 10x^9$$

If you have a product of more than two monomials, multiply *all* the coefficients and sum *all* the exponents:

$$(3x^2)(2x^3)(4x) = (3)(2)(4)(x^2)(x^3)(x^1) = 24x^6$$

Exercise 11 Multiplying monomials

Multiply these monomials

Working Space

1. $(3x^2)(5x^3)$

2. $(2x)(4x^9)$

3. $(-5.5x^2)(2x^3)$

4. $(\pi)(-2x^5)$

5. $(2x)(3x^2)(5x^7)$

Answer on Page 33

4.1 Multiplying a monomial and a polynomial

Watch Khan Academy's **Multiplying monomials by polynomials** at https://youtu.be/pD2-H15ucNE.

When multiplying a monomial and a polynomial, you use the the distributive property.

Then it is just multiplying several pairs of monomials:

$$\begin{aligned}(3x^2)(4x^3 - 2x^2 + 3x - 7) &= (3x^2)(4x^3) + (3x^2)(-2x^2) + (3x^2)(3x) + (3x^2)(-7) \\ &= 12x^5 - 6x^4 + 9x^3 - 21x^2\end{aligned}$$

Exercise 12 Multiplying a monomial and a polynomial

Working Space

Multiply these monomials

1. $(3x^2)(5x^3 - 2x + 3)$

2. $(2x)(4x^9 - 1)$

3. $(-5.5x^2)(2x^3 + 4x^2 + 6)$

4. $(\pi)(-2x^5 + 3x^4 + x)$

5. $(2x)(3x^2)(5x^7 + 2x)$

Answer on Page 34

4.2 Multiplying polynomials

Watch Khan Academy's **Multiplying binomials by polynomials** video at https://youtu.be/D6mivA_8L8U

When you are multiplying two polynomials, you will use the distributive property several times to make it one long polynomial. Then you will combine the terms with the same degree. For example,

$$\begin{aligned}(2x^2-3)(5x^2+2x-7)&\\ &= (2x^2)(5x^2+2x-7)+(-3)(5x^2+2x-7)\\ &= (2x^2)(5x^2)+(2x^2)(2x)+(2x^2)(-7)+(-3)(5x^2)+(-3)(2x)+(-3)(-7)\\ &= 10^4+4x^3+-14x^2+-15x^2+-6x+21 = 10^4+4x^3+-29x^2+-6x+21\end{aligned}$$

One common form that you will see is multiplying two binomials together:

$$(2x+7)(5x+3) = (2x)(5x+3)+(7)(5x+3) = (2x)(5x)+(7)(5x)+(2x)(3)+(7)(3)$$

Notice the product has become the sum of four parts: the firsts, the inners, the outers, and the lasts. People sometimes use the mnemonic FOIL to remember this pattern, but there is a general rule that works for all product of polynomials, not just binomials. Here it is: Every term in the first will be multiplied by every term in the second, and then just add them together.

So, for example, if you have a polynomial s with three terms and you multiply it by a polynomial t with five terms, you will get a sum of 15 terms – each term is a product of two monomials, one from s and one from t. (Of course, several of those terms might have the same degree, so they will be combined together when you simplify. Thus you typically end up with a polynomial with less than 15 terms.)

Using this rule, here is how I would multiply $2x^2-3x+1$ and $5x^2+2x-7$:

$$\begin{aligned}(2x^2-3x+1)(5x^2+2x-7) &= \begin{array}{rcrcrc}(2x^2)(5x^2) & + & (2x^2)(2x) & + & (2x^2)(-7) & +\\ (-3x)(5x^2) & + & (-3x)(2x) & + & (-3x)(-7) & +\\ (1)(5x^2) & + & (1)(2x) & + & (1)(-7) & \end{array}\\ &= 10x^4+4x^3+(-14)x^2+(-15)x^3+(-6)x^2+21x+5x^2+2x+(-7)\\ &= 10x^4+(4-15)x^3+(-14-6+5)x^2+(21+2)x+(-7)\\ &= 10x^4-11x^3-15x^2+23x-7\end{aligned}$$

Note that the product (before combining terms with the same degree) has $3\times 3 = 9$ terms – every possible combination of a term from the first polynomial and a term from the second polynomial.

One common source of error: losing track of the negative signs. You will need to be really careful. I have found that it helps to use + between all terms, and use negative coefficients to express subtraction. For example, if the problem says $4x^2-5x-3$, you should work with that as $4x^2+(-5)x+(-3)$

Exercise 13 Multiplying polynomials

Multiply the following pairs of polynomials:

1. $2x + 1$ and $3x - 2$

2. $-3x^2 + 5$ and $4x - 2$

3. $-2x - 1$ and $-3x - \pi$

4. $-2x^5 + 5x$ and $3x^5 + 2x$

Working Space

Answer on Page 34

Exercise 14 Observations

Let's say I have two polynomials, p_1 and p_2. p_1 has degree 23. p_2 has degree 12. What is the degree of their product?

Working Space

Answer on Page 34

Chapter 5

Multiplying Polynomials in Python

At this point, you have created a nice toolbox of functions for dealing with lists of coefficients as polynomials. Create a file called `poly.py` and copy the folowing functions into it:

- `evaluate_polynomial`
- `polynomial_to_string`
- `add_polynomials`
- `scalar_polynomial_multiply`
- `subtract_polynomial`

Now create another file in the same directory called `test.py`. Type this into that file:

```
import poly

polynomial_a = [9.0, -4.0, 3.0, -5.0]
print('Polynomial A =', poly.polynomial_to_string(polynomial_a))

polynomial_b = [-9.0, 0.0, 4.0, 2.0, 1.0]
print('Polynomial B =', poly.polynomial_to_string(polynomial_b))

# Evaluation
value_of_b = poly.evaluate_polynomial(polynomial_b, 3)
print('Polynomial B at 3 =', value_of_b)

# Adding
a_plus_b = poly.add_polynomials(polynomial_a, polynomial_b)
print('A + B =', poly.polynomial_to_string(a_plus_b))

# Scalar multiplication
b_scalar = poly.scalar_polynomial_multiply(-3.2, polynomial_b)
print('-3.2 * Polynomial B =', poly.polynomial_to_string(b_scalar))

# Subtraction
a_minus_b = poly.subtract_polynomial(polynomial_a, polynomial_b)
print('A - B =', poly.polynomial_to_string(a_minus_b))
```

When you run it, you should get the following:

```
Polynomial A = -5.0x^3 + 3.0x^2 + -4.0x + 9.0
Polynomial B = 1.0x^4 + 2.0x^3 + 4.0x^2 + -9.0
Polynomial B at 3 = 162.0
A + B = 1.0x^4 + -3.0x^3 + 7.0x^2 + -4.0x
-3.2 * Polynomial B = -3.2x^4 + -6.4x^3 + -12.8x^2 + 28.8
A - B = -1.0x^4 + -7.0x^3 + -1.0x^2 + -4.0x + 18.0
```

Now you are ready to implement multiplication of polynomials. The function will look like this:

```
def multiply_polynomials(a, b):
  ...Your code here...
```

It will return a list of coefficients.

In an exercise in the last chapter, you were asked " Let's say I have two polynomials, p_1 and p_2. p_1 has degree 23. p_2 has degree 12. What is the degree of their product?" The answer was $23 + 12 = 35$.

In our implementation, a polynomial of degree 23 is held in a list of length 24.

In Python we wil be trying to multiply a polynomial `a` and a polynomial `b` represented as lists. What is the degree of that product?

```
result_degree = (len(a) - 1) + (len(b) - 1)
```

Now, we need to create an array of zeros that is one longer than that. Here is a cute Python trick: if you have a list, you can replicate it using the * operator.

```
a = [5,7]
b = a * 4
print(b)
# [5, 7, 5, 7, 5, 7, 5, 7]
```

Here's how you will get a list of zeros:

```
result = [0.0] * (result_degree + 1)
```

We will step through `a` getting the index and value of each entry. You can do this in one line using `enumerate`:

```
for a_degree, a_coefficient in enumerate(a):
```

For each of those, we will step through the entire `b` polynomial. As you multiply together each term, you will add it to appropriate coefficient of the result.

Here is the whole function:

```
def multiply_polynomials(a, b): # What is the degree of the resulting
polynomial?  result_degree = (len(a) - 1) + (len(b) - 1)

    # Make a list of zeros to hold the coefficents result = [0.0] *
    (result_degree + 1)

    # Iterate over the indices and values of a for a_degree,
    a_coefficient in enumerate(a):

        # Iterate over the indices and values of b for b_degree,
        b_coefficient in enumerate(b):
```

```
            # Calculate the resulting monomial coefficient =
            a_coefficient * b_coefficient degree = a_degree + b_degree

            # Add it to the right bucket
            result[degree] = result[degree] + coefficient

    return result
```

Take a long look at that function. When you understand it, type it into `poly.py`.

In `test.py`, try out the new function:

```
# Multiplication
a_times_b = poly.multiply_polynomials(polynomial_a, polynomial_b)
print('A x B =', poly.polynomial_to_string(a_times_b))
```

This is an example of a *nested loop*. The outer loop steps through the polynomial `a`. For each step it takes, the inner loop steps through the entire polynomial `b`.

5.1 Something surprising about lists

You can imagine that you might want to create two very similar polynomials. Let's say polynomial `c` is $x^2 + 2x + 1$ and polynomial `d` is $x^2 - 2x + 1$. You might think you are very clever to just alter that degree 1 coefficient like this:

```
c = [1.0, 2.0, 1.0]
d = c
d[1] = -2.0
```

If you printed out `c`, you would get $[1.0, -2.0, 1.0]$. Why? You assigned two variables (`c` and `d`) to the *the same list*. So when you use one reference (`d`) to change the list, you see the change if you look at the list from either reference. *FIXME: Diagram of two references to the same list here.*

To create two separate lists, you would need to explicitly make a copy:

```
c = [1.0, 2.0, 1.0]
d = c.copy()
d[1] = -2.0
```

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 4)

$\boxed{-2x^3 + \frac{1}{2}x + 3.9}$

$\boxed{(4.5)x^2 + \pi x}$

$\boxed{7}$

$2x^{-10} + 4x - 1$

$x^{\frac{2}{3}}$

$\boxed{3x^{20} + 2x^{19} - 5x^{18}}$

Answer to Exercise 2 (on page 5)

Standard form would be $-x^3 + 21x^2 - 1000x + \pi$. The degree is 3. The leading coefficient is -1

Answer to Exercise 3 (on page 5)

$4^3 - (3)(4^2) + (10)(4) - 12 = 64 - 48 + 40 - 12$. So $y = 44$

Answer to Exercise 4 (on page 8)

```
favorites[3] = "Gloves"
```

Answer to Exercise 5 (on page 11)

```
pn2 = [2.5, 5.0, -2.0, -7.0, 4.0]
y = evaluate_polynomial(pn2, 8.5)
print("Polynomia 2: When x is 8.5, y is", y)

pn3 = [-9.0, 0.0, 0.0, 0.0, 0.0, 5.0]
y = evaluate_polynomial(pn3, 2.0)
print("Polynomial 3: When x is 2.0, y is", y)
```

Answer to Exercise 6 (on page 13)

The polynomial crosses the y-axis at 6. When x is zero, all the terms are zero except the last one. Thus you can easily tell that $x^3 - 7x + 6$ will cross the y-axis at $y = 6$.

Looking at the graph, you tell that the curve crosses the y-axes near -3, 1 and 2. If you plug those numbers into the polynomial, you would find that it evalutes to zero at each one. Thus, $x = -3$, $x = 1$, and $x = 2$ are roots.

Answer to Exercise 7 (on page 16)

$3x^3 - 7x^2 + x - 18$ and $3x^5 - 7x^3 + x^2 - 12$

Answer to Exercise 8 (on page 17)

$x^3 - 3x^2 + 5x$ and $x^5 - 3x^3 + 5x^2 - 2x + 6$

Answer to Exercise 9 (on page 18)

```
def add_polynomials(a, b):
    degree_of_result = max(len(a), len(b))
    result = []
    for i in range(degree_of_result):
        if i < len(a):
            coefficient_a = a[i]
        else:
            coefficient_a = 0.0

        if i < len(b):
            coefficient_b = b[i]
        else:
            coefficient_b = 0.0

        result.append(coefficient_a + coefficient_b)
    return result
```

Answer to Exercise 10 (on page 19)

```
def subtract_polynomial(a, b):
    neg_b = scalar_polynomial_multiply(-1.0, b)
    return add_polynomials(a, neg_b)
```

Answer to Exercise 11 (on page 22)

$(3x^2)(5x^3) = 15x^5$

$(2x)(4x^9) = 8x^{10}$

$(-5.5x^2)(2x^3) = -11x^5$

$(\pi)(-2x^5) = -2\pi x^5$

$(2x)(3x^2)(5x^7) = 30x^{10}$

Answer to Exercise 12 (on page 23)

$(3x^2)(5x^3 - 2x + 3) = 15x^6 - 6x^3 + 6x^2$

$(2x)(4x^9 - 1) = 8x^{10} - 2x$

$(-5.5x^2)(2x^3 + 4x^2 + 6) = 11x^5 - 22x^4 + 33x^2$

$(\pi)(-2x^5 + 3x^4 + x) = -2\pi x^5 + 3\pi x^4 + \pi x$

$(2x)(3x^2)(5x^7 + 2x) = 30x^{10} + 12x^4$

Answer to Exercise 13 (on page 25)

$(2x + 1)(3x - 2) = 6x^2 - x - 2$

$(-3x^2 + 5)(4x - 2) = -12x^3 + 6x^2 + 20x - 10$

$(-2x - 1)(-3x - \pi) = 6x^2 + (4 + 2\pi)x + \pi$

$(-2x^5 + 5x)(3x^5 + 2x) = -6x^{10} + 12x^6 + 10x^2$

Answer to Exercise 14 (on page 25)

The degree of the product is determined by the term that is the product of the highest degree term in p_1 and the highest degree term in p_2. Thus, the product of a degree 23 polynomial and a degree 12 polynomial has degree 35.

Index