Contents

Chapter 1

Sound

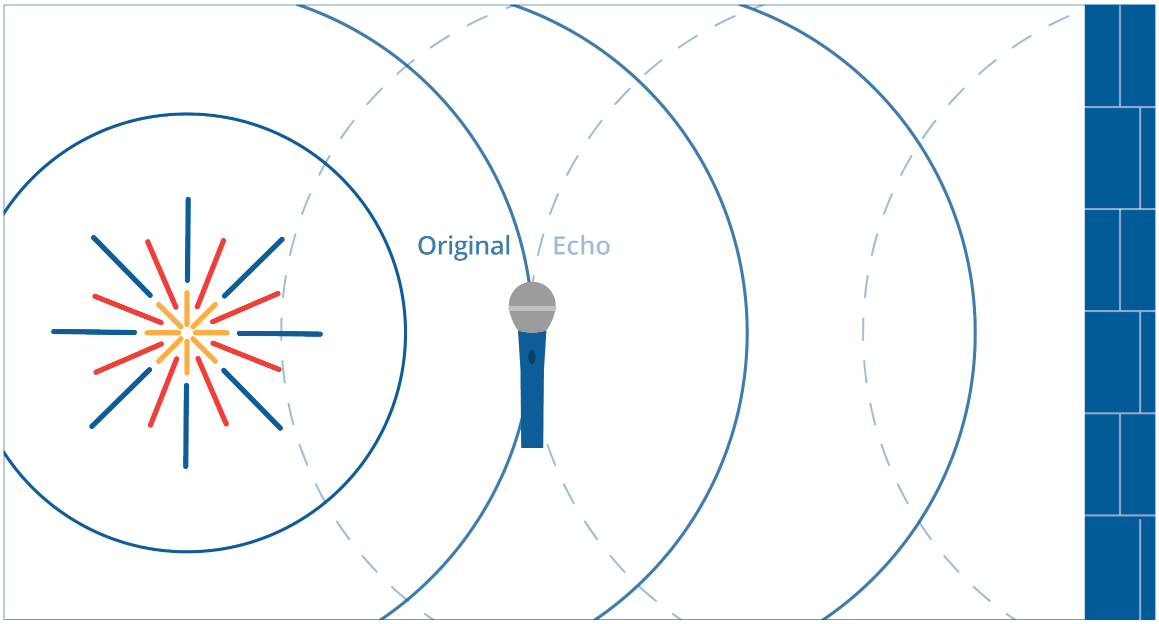
When you set off a firecracker, it makes a sound.

Let's break that down a little more: Inside the cardboard wrapper of the firecracker, there is potassium nitrate (KNO_3), sulfur (S), and carbon(C). These are all solids. When you trigger the chemical reactions with a little heat, these atoms rearrange themselves to be potassium carbonate (K_2CO_3), potassium sulfate (K_2SO_4), carbon dioxide (CO_2), and nitrogen (N_2). Note that the last two are gasses.

The molecules of a solid are much more tightly packed than the molecules of a gas. So after the chemical reaction, the molecules expand to fill a much bigger volume. The air molecules nearby get pushed away from the firecracker. They compress the molecules beyond them, and those compress the molecules beyond them.

This compression wave radiates out as a sphere; its radius growing at about 343 meters per second ("The speed of sound").

The energy of the explosion is distributed around the surface of this sphere. As the radius increases, the energy is spread more and more thinly around. This is why the firecracker seems louder when you are closer to it. (If you set off a firecracker in a sewer pipe, the sound will travel much, much farther.)



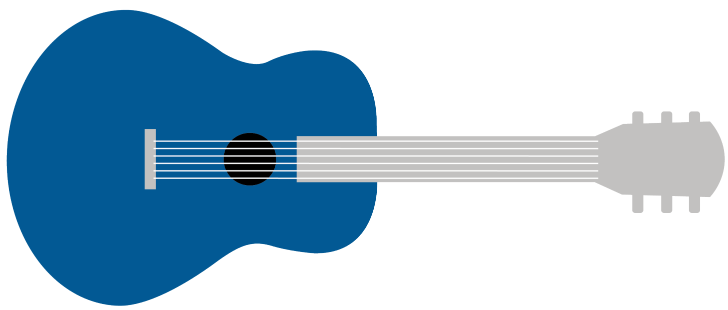
This compression wave will bounce off of hard surfaces. If you set off a firecracker 50 meters from a big wall, you will hear the explosion twice. We call the second one "an echo."

The compression wave will be absorbed by soft surfaces. If you covered that wall with pillows, there would be almost no echo.

The study of how these compression waves move and bounce is called *acoustics*. Before you build a concert hall, you hire an acoustician to look at your plans and tell you how to make it sound better.

1.1 Pitch and frequency

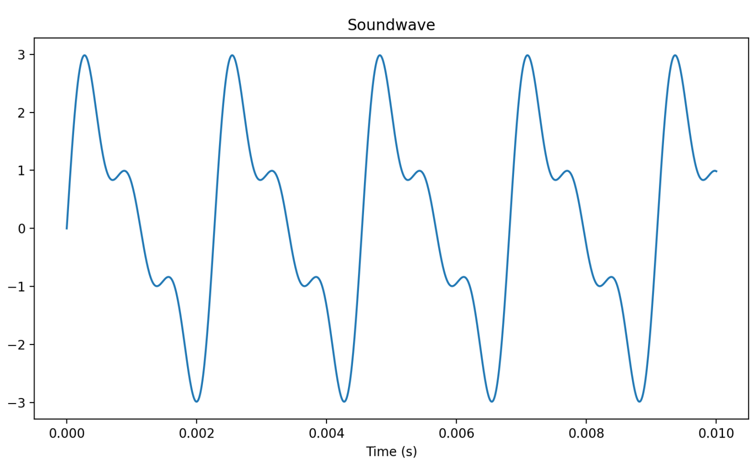
The string on a guitar is very similar to the weighted spring example. The farther the string is displaced, the more force it feels pushing it back to equilibrium. Thus, it moves back and forth in a sine wave. (OK, it isn't a pure sine wave, but we will get to that later.)



The string is connected to the center of the boxy part of the guitar, which is pushed and pulled by the string. That creates compression waves in the air around it.

If you are in the room with the guitar, those compression waves enter your ear, push and pull your eardrum, which is attached to bones that move a fluid that tickles tiny hairs, called *cilia* in your inner ear. That is how you hear.

We sometimes see plots of sound waveforms. The x-axis represents time. The y-axis represents the amount the air is compressed at the microphone that converted the air pressure into an electrical signal.



If the guitar string is made tighter (by the tuning pegs) or shorter (by the guitarist's fingers on the strings), the string vibrates more times per second. We measure the number of waves per second and we call it the *frequency* of the tone. The unit for frequency is *Hertz*: cycles per second.

Musicians have given the different frequencies names. If the guitarist plucks the lowest note on his guitar, it will vibrate at 82.4 Hertz. The guitarist will say "That pitch is low E." If the string is made half as long (by a finger on the 12th fret), the frequency will be twice as fast (164.8 Hertz), and the guitarist will say "That is E an octave up."

For any note, the note that has twice the frequency is one octave up. The note that has half the frequency is one octave down.

The octave is a very big jump in pitch, so musicians break it up into 12 smaller steps. If the guitarist shortens the E string by one fret, the frequency will be $82.4 \times 1.059463 \approx 87.3$ Hertz.

Shortening the string one fret always increases the frequency by a factor of 1.059463. Why?

Because $1.059463^1 2 = 2$. That is, if you take 12 of these hops, you end up an octave higher.

This, the smallest hop in western music, is referred to as *half step*.

Exercise 1 Notes and frequencies

The note A near the middle of the piano, is 440Hz. The note E is 7 half steps above A. What is its frequency?

Working Space

Answer on Page 37

1.2 Chords and harmonics

Of course, a guitarist seldom plays only one string at a time. Instead, he uses the frets to pick a pitch for each string and strums all six strings.

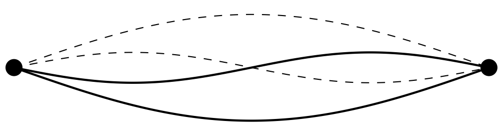
Some combinations of frequencies sound better than others. We have already talked about the octave: if one string vibrates twice for each vibration of another, they sound sweet together.

Musicians speak of “the fifth”. If one string vibrates three times and the other vibrates twice in the same amount of time, they sound sweet together.

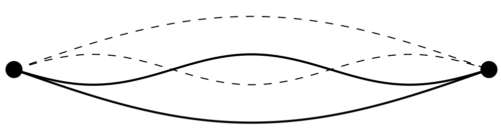
If one string vibrates 4 times while the other vibrates 3 times, they sound sweet together. Musicians call this “the third.”

Each of these different frequencies tickle different cilia in the inner ear, so you can hear all six notes at the same time when the guitarist strums his guitar.

When a string vibrates, it doesn’t create a single sine wave. Yes, the string vibrates from end-to-end and this generates a sine wave at what we call *the fundamental frequency*. However, there are also “standing waves” on the string. One of these standing waves is still at the centerpoint of the string, but everything to the left of the centerpoint is going up while everything to the right is going down. This creates *an overtone* that is twice the frequency of the fundamental.



The next overtone has two still points – it divides the string into three parts. The outer parts are up while the inner part is down. Its frequency is three times the fundamental frequency.



And so on: 4 times the fundamental, 5 times the fundamental, etc.

In general, tones with a lot of overtones tend to sound bright. Tones with just the fundamental sound thin.

Humans can generally hear frequencies from 20Hz to 20,000Hz (or 20kHz). Young people tend to be able to hear very high sounds better than older people.

Dogs can generally hear sounds in the 65Hz to 45kHz range.

1.3 Making waves in Python

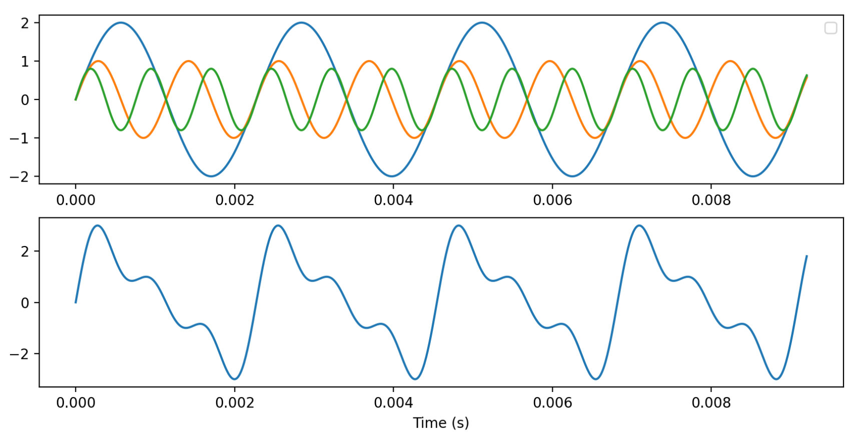
Let’s make a sine wave and add some overtones to it. Create a file `harmonics.py`

```
import matplotlib.pyplot as plt
import math
```

```
# Constants: frequency and amplitude
fundamental_freq = 440.0 # A = 440 Hz
fundamental_amp = 2.0

# Up an octave
first_freq = fundamental_freq * 2.0 # Hz
first_amp = fundamental_amp * 0.5

# Up a fifth more
second_freq = fundamental_freq * 3.0 # Hz
second_amp = fundamental_amp * 0.4

# How much time to show
max_time = 0.0092 # seconds

# Calculate the values 10,000 times per second
time_step = 0.00001 # seconds

# Initialize
time = 0.0
times = []
totals = []
fundamentals = []
firsts = []
seconds = []

while time <= max_time:
    # Store the time
    times.append(time)

    # Compute value each harmonic
    fundamental = fundamental_amp * math.sin(2.0 * math.pi * fundamental_freq * time)
    first = first_amp * math.sin(2.0 * math.pi * first_freq * time)
    second = second_amp * math.sin(2.0 * math.pi * second_freq * time)

    # Sum them up
    total = fundamental + first + second

    # Store the values
    fundamentals.append(fundamental)
    firsts.append(first)
    seconds.append(second)
    totals.append(total)

    # Increment time
```

```
    time += time_step

# Plot the data
fig, ax = plt.subplots(2, 1)

# Show each component
ax[0].plot(times, fundamentals)
ax[0].plot(times, firsts)
ax[0].plot(times, seconds)
ax[0].legend()

# Show the totals
ax[1].plot(times, totals)
ax[1].set_xlabel("Time (s)")

plt.show()
```

When you run it, you should see a plot of all three sine waves and another plot of their sum:



1.3.1 Making a sound file

The graph is pretty to look at, but make a file that we can listen to.

The WAV audio file format is supported on pretty much any device, and a library for writing WAV files comes with Python. Let's write some sine waves and some noise into a WAV file.

Create a file called `soundmaker.py`

```
import wave
import math
import random

# Constants
frame_rate = 16000 # samples per second
duration_per = 0.3 # seconds per sound
frequencies = [220, 440, 880, 392] # Hz
amplitudes = [20, 125]
baseline = 127 # Values will be between 0 and 255, so 127 is the baseline
samples_per = int(frame_rate * duration_per) # number of samples per sound

# Open a file
wave_writer = wave.open('sound.wav', 'wb')

# Not stereo, just one channel
wave_writer.setnchannels(1)

# 1 byte audio means everything is in the range 0 to 255
wave_writer.setsampwidth(1)

# Set the frame rate
wave_writer.setframerate(frame_rate)

# Loop over the amplitudes and frequencies
for amplitude in amplitudes:
    for frequency in frequencies:
        time = 0.0
        # Write a sine wave
        for sample in range(samples_per):
            s = baseline + int(amplitude * math.sin(2.0 * math.pi * frequency * time))
            wave_writer.writeframes(bytes([s]))
            time += 1.0 / frame_rate

        # Write some noise after each sine wave
        for sample in range(samples_per):
            s = baseline + random.randint(0, 15)
            wave_writer.writeframes(bytes([s]))

# Close the file
wave_writer.close()
```

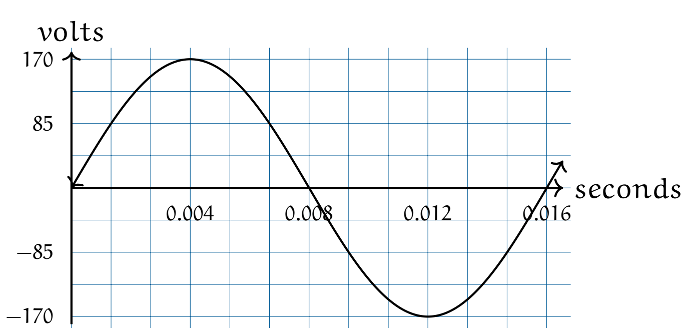
When you run it, it should create a sound file with several tones of different frequencies and volumes. Each tone should be followed by some noise.

CHAPTER 2

Alternating Current

We have discussed the voltage and current created by a battery. A battery pushes the electrons in one direction at a constant voltage; this is known as *Direct Current* or DC. A battery typically provides between 1.5 and 9 volts.

The electrical power that comes into your home on wires is different. If you plotted the voltage over time, it would look like this:



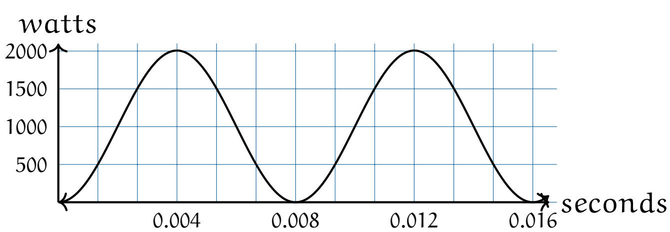
The x axis here represents ground. When you insert a two-prong plug into an outlet, one is "hot" and the other is "ground". Ground represents 0 volts and should be the same voltage as the dirt under the building.

The voltage is a sine wave at 60Hz. Your voltage fluctuates between -170v and 170v. Think for a second what that means: The power company pushes electrons at 170v and then pulls electrons at 170v. It alternates back and forth this way 60 times per second.

2.1 Power of AC

Let's say you turn on your toaster which has a resistance of 14.4 ohms. How much energy (in watts) does it change from electrical energy to heat? We know that $I = V/R$ and we know that watts of power are IV. So given a voltage of V, the toaster is consuming V^2/R watts.

However, V is fluctuating. Let's plot the power the toaster is consuming:



Another sine wave! Here is a lesser-known trig identity: $(\sin(x))^2 = \frac{1}{2} - \frac{1}{2}\cos(2x)$

So this is actually a cosine wave flipped upside down, scaled down by half the peak power, and translated up so that it is never negative. Note that it is also twice the frequency of the voltage sine wave.

If we say the peak voltage is V_p and the resistance of the toaster is R, the power is given by

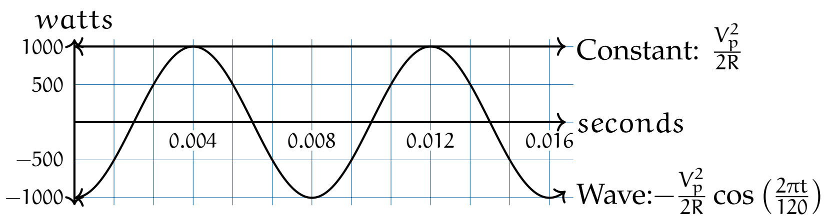
$$\frac{V_p^2}{2R} - \frac{V_p^2}{2R}\cos\left(\frac{2\pi t}{120}\right)$$

As a toaster user and as someone who pays a power bill, you are mostly interested in the average power. To get the average power, you take the area under the power graph and divide it by the amount of time.

We can think of the area under the curve as two easy-to-integrate quantities summed:

- A constant function of $y = fracV_p^2 2R$

- A wave $y = -\frac{V_p^2}{2R}\cos\left(\frac{2\pi t}{120}\right)$



When we integrate that constant function we get $\frac{tV_p^2}{2R}$

When we integrate that wave for a complete cycle we get...zero! The positive side of the wave is canceled out by the negative side.

So, the average power is $\frac{V_p^2}{2R}$ watts.

Someone at some point said "I'm used to power being V^2/R. Can we define a voltage measure for AC power such that this is always true?"

So we started using V_{rms} which is just $\frac{V_p}{\sqrt{2}}$. If you look on the back of anything that plugs into a standard US power outlet, it will say something like "For 120v". What they mean is "For 120v RMS, so we expect the voltage to fluctuate back and forth from 170v to -170v."

Notice that this is the same Root-Mean-Squared that we defined earlier, but now we know that if $y = \sin(x)$, the RMS of y is $1/\sqrt{2} \approx 0.707$.

For current, we do the same thing: If the current is AC, the power consumed by a resistor is $I_{RMS}^2 R$, where I_{RMS} is the peak current divided by $sqrt2$.

2.2 Power Line Losses

A wire has some resistance. Thinner wires tend to have more resistance than thicker ones. Aluminum wires tend to have more resistance than copper wires.

Let's say that the power that comes to your house has to travel 20 km from the generator in a cable that has about 1Ω of resistance per km. Let's say that your home is consuming 12 kilowatts of power.

If that power is 120v RMS from the generator to your home, what percentage of the power is lost heating the power line? 10 amps RMS flow through your home. When that current goes through the wire, $I^2R = (100)(20) = 2000watts$ is lost to heat.

So the power company would need to supply 14 kilowatts of power, knowing that 2 kilowatts would be lost on the wires.

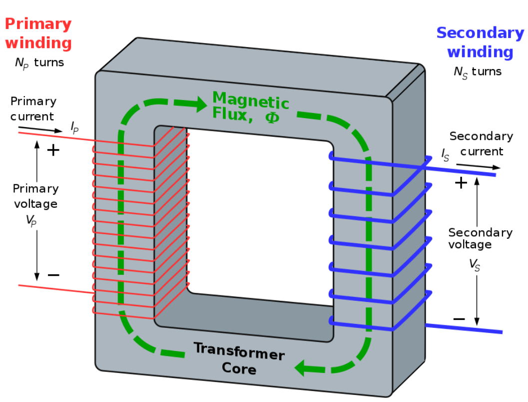
What if the power company moved the power at 120,000 volts RMS? Now only 0.01 amps RMS flow through your home. When that current goes through the wire $I^2R = (0.0001)(20) = .002$ watts of power are lost on the power lines.

It is much, much more efficient. The only problem is that 120,000 volts would be incredibly dangerous. So the power company moves power long distances at very high voltages, like 765 kV. Before the power is brought into your home, it is converted into a lower voltage using a *transformer*.

2.3 Transformers

A transformer is a device that converts electrical power from one voltage to another. A good transformer is more than 95% efficient. The details of magnetic fields, flux, and inductance are beyond the scope of this chapter, so I am going to give a relatively simple explanation and admit that it is incomplete.

A transformer is a ring with two sets of coils wrapped around it.



(Diagram from Wikipedia)

When alternating current is run through the primary winding, it creates magnetic flux in the ring. The magnetic flux induces current in the secondary winding.

If V_P is the voltage across the primary winding and V_S is the voltage across the secondary winding, they are related by the following equation:

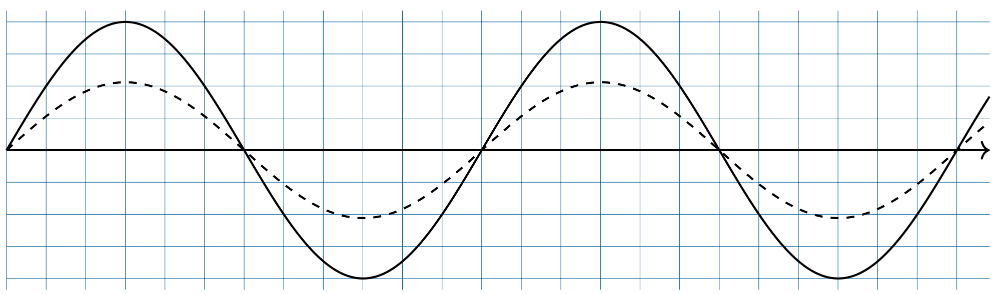
$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

where N_P and N_S are the number of turns in the primary and secondary windings.

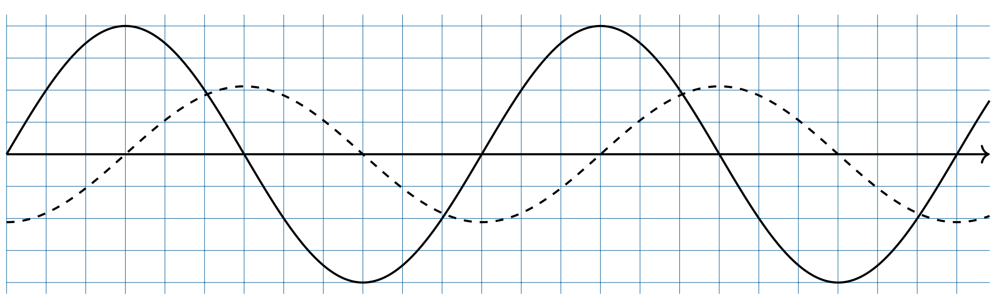
There are usually at least two transformers between you and the very high voltage lines. There are transformers at the substation that make the voltage low enough to travel on regular utility poles. On the utility poles, you will see cans that contain smaller transformers. Those step the voltage down to make the power safe to enter your home.

2.4 Phase and 3-phase power

If two waves are "in sync" we say they have the same *phase*.

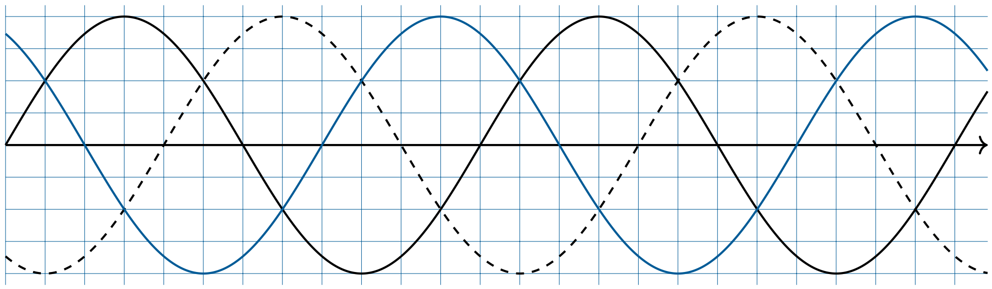


If they are the same frequency, but are not in-sync, we can talk about the difference in their phase.



Here we see that the smaller wave is lagging by $\pi/2$ or $90°$.

In most power grids, there are usually 3 wires carrying the power. The voltage on each is $2\pi/3$ out of phase with the other two:



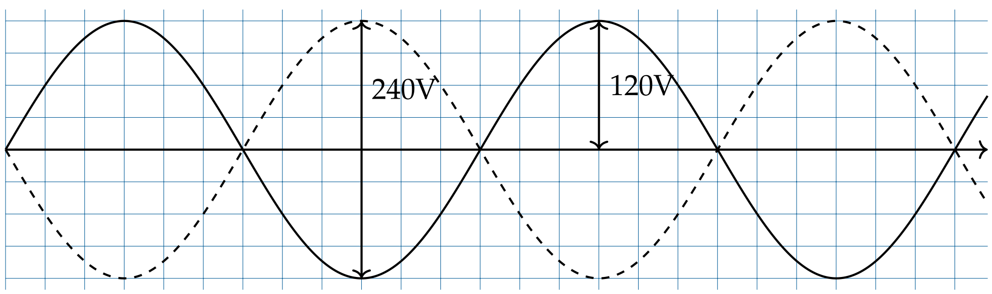
This is nice in two ways:

- While the power in each wire is fluctuating, the total power is not fluctuating at all.
- While the power plant is pushing and pulling electrons on each wire, the total number number of electrons leaving the load is zero.

(Both these assume that there each wire is attached to a load with the same constant resistance.)

In big industrial factories, you will see all three wires enter the building. Large amounts of smooth power delivery means a lot to an industrial user.

In residential settings, each home gets its power from one of the three wires. However, two wires typically carry power into the home. Each one carries 120V RMS, but they are out of phase by 180 degrees. Lights and small appliances are connected to one of the wires and ground, so they get 120V RMS. Large appliances, like air conditioners and washing machines, are connected across the two wires so they get 240V RMS.



How do you get two circuits, 180 degrees out of phase, from one circuit? Using a center-tap transformer.

FIXME: Diagram here

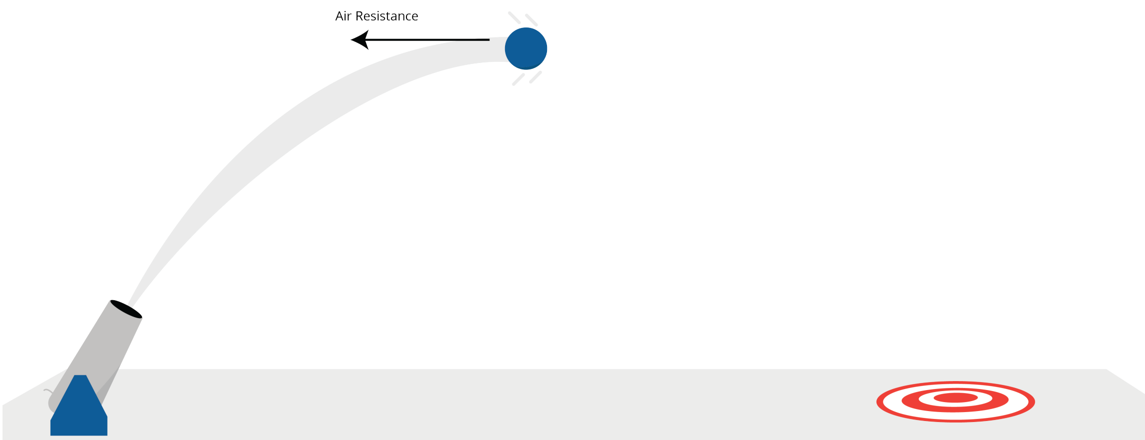
Chapter 3

Drag

The very first computers were created to do calculations of how artillery would fly when shot at different angles. The calculations were similar to the ones you just did for the flying hammer with two important differences:

- They were interested in two dimensions: the height and the distance across the ground.

- However, artillery flies a lot faster than a hammer, so they had to worry about drag from the air.



3.1 Wind resistance

The first thing they did was put one of the shells in a wind tunnel. They measured how much force was created when they pushed 1 m/s of wind over the shell. Let's say it was 0.1 newtons.

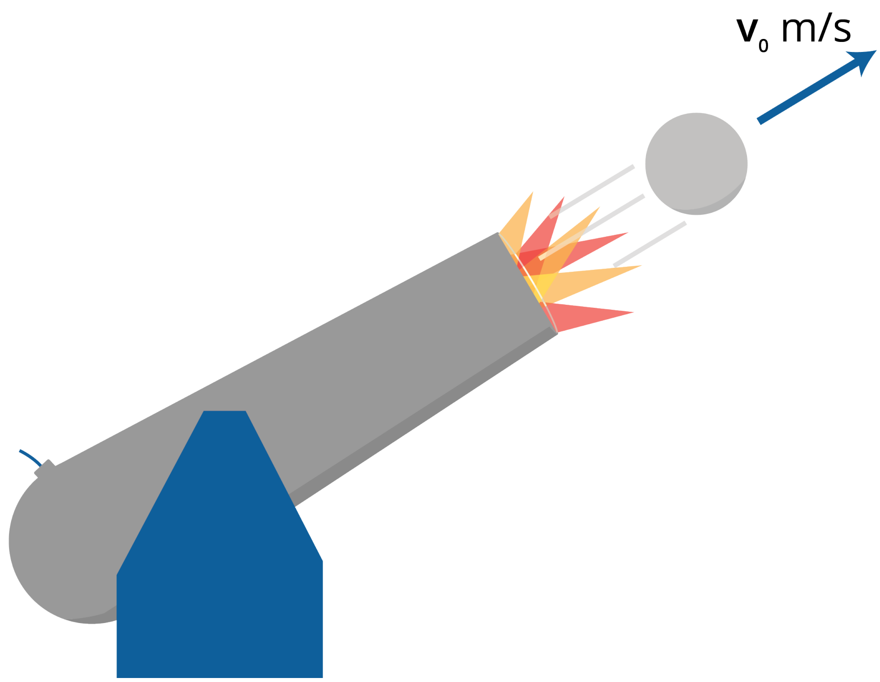
One of the interesting things about the drag from the air (often called *wind resistance*) is that it increases with the *square* of the speed. Thus, if the wind pushing on the shell is 3 m/s, instead of 1 m/s, the resistance is $3^2 \times 0.1 = 0.9$ newtons.

(Why? Intuitively, three times as many air molecules are hitting the shell and each molecule is hitting it three times harder.)

So, if a shell is moving with the velocity vector v, the force vector of the drag points in the exact opposite direction. If μ is the force of wind resistance of the shell at 1 m/s, then the magnitude of the drag vector is $\mu|v|^2$.

3.2 Initial velocity and acceleration due to gravity

Let's say a shell is shot out of a tube at s m/s, and let's say the tube is tilted θ radians above level. Then, the initial velocity will be given by the vector $[s\cos(\theta), s\sin(\theta)]$



(The velocity of the shell is actually a 3-dimensional vector, but we are only going to worry about height and horizontal distance; we are assuming that the operator pointed it in the right direction.)

To figure out the path of the shell, we need to compute its acceleration. We remember that

$$F = ma$$

(Note that F and a are vectors.) Dividing both sides by m we get:

$$a = \frac{F}{m}$$

So let's figure out the net force on the shell so that we can calculate the acceleration vector.

If the shell has a mass of b, the force due to gravity will be in the downward direction with a magnitude of $9.8b$ newtons.

To get the net force, we will need to add the force due to gravity with the force due to wind resistance.

3.3 Simulating artillery in Python

Create a file called artillery.py.

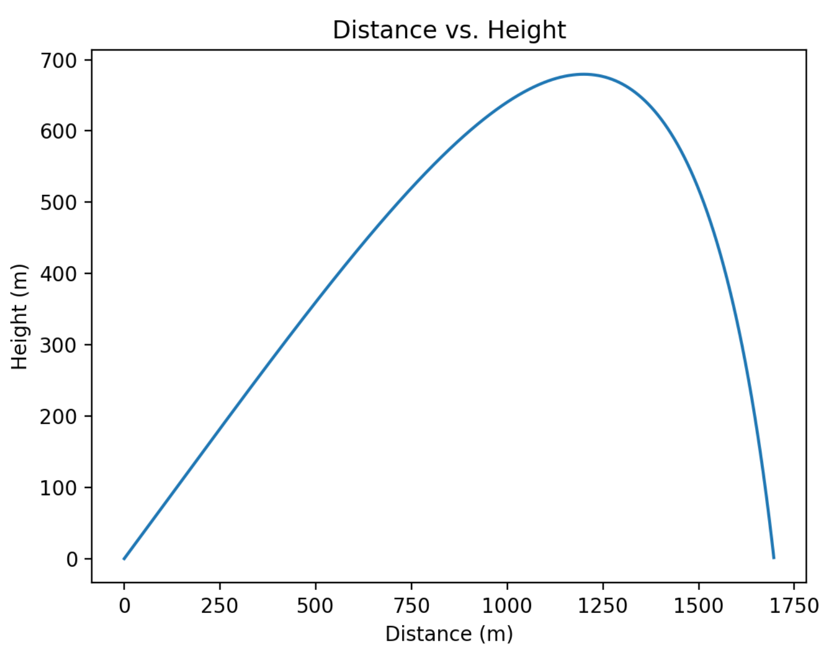
```
import numpy as np
import matplotlib.pyplot as plt

# Constants
mass = 45 # kg
start_speed = 300.0 # m/s
theta = np.pi/5 # radians (36 degrees above level)
time_step = 0.01 # s
wind_resistance = 0.05 # newtons in 1 m/s wind
force_of_gravity = np.array([0.0, -9.8 * mass]) # newtons

# Initial state
position = np.array([0.0, 0.0]) # [distance, height] in meters
velocity = np.array([start_speed * np.cos(theta), start_speed * np.sin(theta)])
time = 0.0 # seconds

# Lists to gather data
distances = []
heights = []
times = []

# While shell is aloft
while position[1] >= 0:
    # Record data
    distances.append(position[0])
    heights.append(position[1])
    times.append(time)

    # Calculate the next state
    time += time_step
    position += time_step * velocity

    # Calculate the net force vector
    force = force_of_gravity - wind_resistance * velocity**2

    # Calculate the current acceleration vector
    acceleration = force / mass
```

```
        # Update the velocity vector
        velocity += time_step * acceleration

    print(f"Hit the ground {position[0]:.2f} meters away at {time:.2f} seconds.")

    # Plot the data
    fig, ax = plt.subplots()
    ax.plot(distances, heights)
    ax.set_title("Distance vs. Height")
    ax.set_xlabel("Distance (m)")
    ax.set_ylabel("Height (m)")
    plt.show()
```

When you run it, you should get a message like:

```
Hit the ground 1696.70 meters away at 20.73 seconds.
```

You should also see a plot of the shell's path:



3.4 Terminal velocity

If you shot the shell very, very high in the sky, it would keep accelerating toward the ground until the force of gravity and the force of the wind resistance were equal. The speed at which this happens is called the *terminal velocity*. The terminal velocity of a falling human is about 53 m/s.

Exercise 2 Terminal velocity

What is the terminal velocity of shell described in our example?

Working Space

Answer on Page 37

Chapter 4

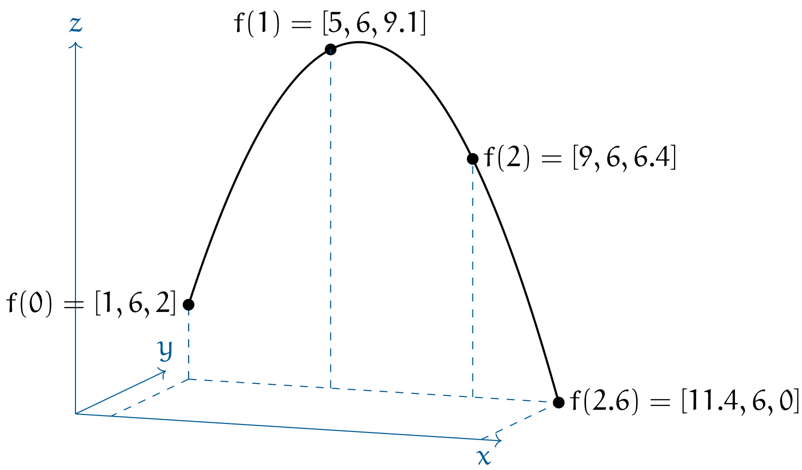
Vector-valued Functions

In the last chapter, you calculated the flight of the shell. For any time t, you could find a vector $[\mathtt{distance}, \mathtt{height}]$. This can be thought of as a function f that takes a number and returns a 2-dimensional vector. We call this a *vector-valued* function from $\mathbb{R} \to \mathbb{R}^2$.

We often make a vector-valued function by defining several real-valued functions. For example, if you threw a hammer with an initial upward speed of 12 $m/2$ and a horizontal speed of 4 m/s along the x axis from the point $(1, 6, 2)$, its position at time t (during its flight) would be given by:

$$f(t) = [4t + 1, 6, -4.8t^2 + 12t + 2]$$

That is, x is increasing with t, y is constant, and z is a parabola.

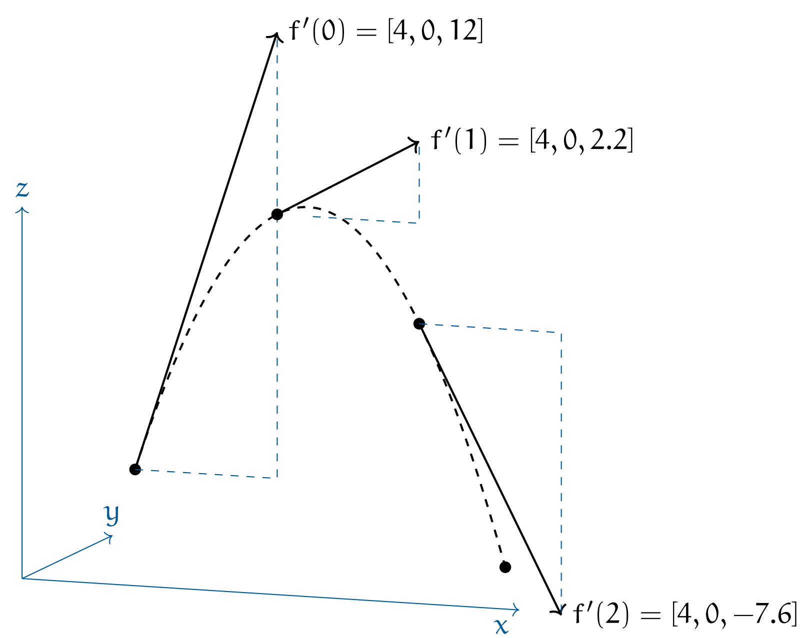


4.1 Finding the velocity vector

Now that we have its position vector, we can differentiate each component separately to get its velocity as a vector-valued function:

$$f'(t) = [4, 0, -9.8t + 12]$$

That is, the velocity is constant along the x-axis, zero along the y-axis, and decreasing with time along the z axis.

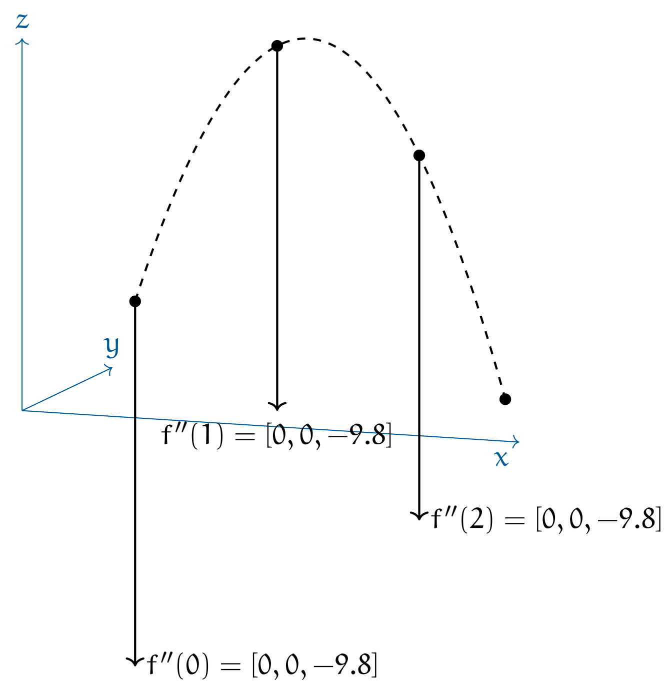


4.2 Finding the acceleration vector

Now that we have its velocity, we can get its acceleration as a vector-valued function:

$$f''(t) = [0, 0, -9.8]$$

There is no acceleration along the x or y axes. It is accelerating down at a constant $9.8m/s^2$.



Chapter 5

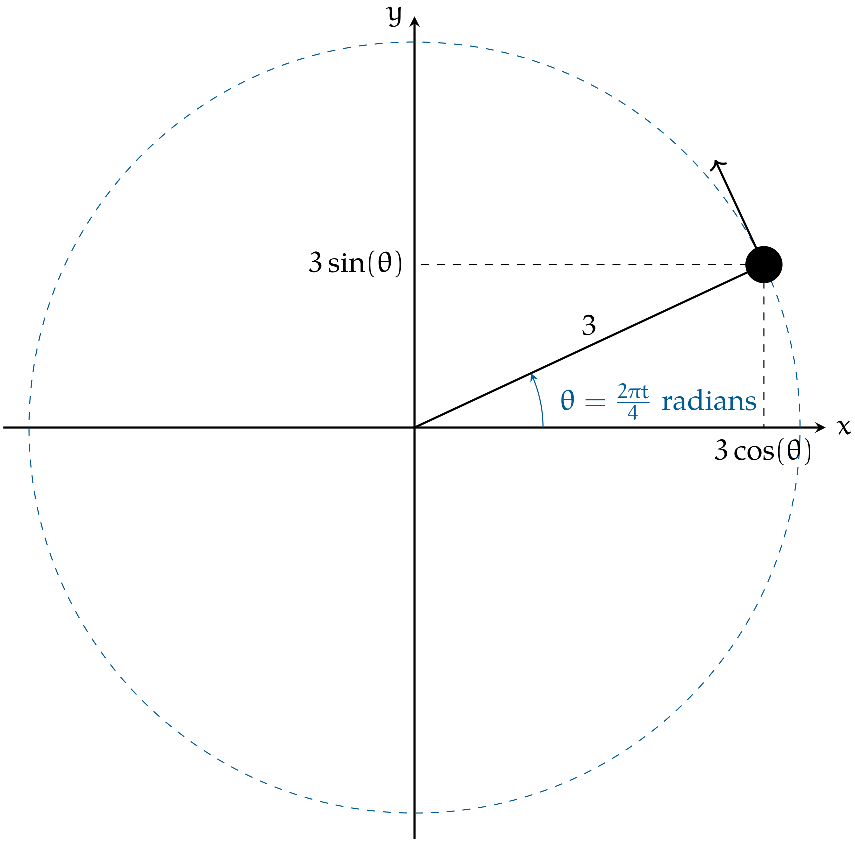
Circular Motion

Let's say you tie a 0.16 kg billard ball to a long string and begin to swing it around in a circle above your head. Let's say the string is 3 meters long, and the ball returns to where it started every 4 seconds. If you start your stopwatch as the ball crosses the x-axis, the position of the ball at any time t given by:

$$p(t) = [3\cos\left(\frac{2\pi}{4}t\right), 3\sin\left(\frac{2\pi}{4}t\right), 2]$$

(This assumes that the ball would be going counter-clockwise if viewed from above. The spot you are standing on is considered the origin $[0, 0, 0]$.)

Notice that the height is a constant – 2 meters in this case. That isn't very interesting, so we will talk just about the first two components. Here is what it would look like from above:



In this case, the radius, r, is 3 meters. The period, T is 4 seconds. In general, we say that circular motion is given by:

$$p(t) = \left[r\cos\frac{2\pi t}{T}, r\cos\frac{2\pi t}{T}\right]$$

A common question is "How fast is it turning right now?" If you divide the 2π radians of a circle by the 4 seconds it takes, you get the answer "About 1.57 radians per second." This is known as *angular velocity* and we typically represent it with the lowercase Omega: ω. (Yes, it looks a lot like a "w".) To be precise, in our example, the angular velocity is $\omega = \frac{\pi}{2}$.

Notice that this is different from the question "How fast is it going?" This ball is traveling the circumference of $6\pi \approx 18.85$ meters every 4 seconds. So the speed of the ball is about 4.71 meters per second.

5.1 Velocity

The velocity of the ball is a vector, and we can find that vector by differentiating each component of the position vector.

For any constants a and b:

Expression	Derivative
$a \sin bt$	$ab \cos bt$
$a \cos bt$	$-ab \sin bt$

Thus, in our example, the velocity of the ball at any time t is given by:

$$v(t) = \left[-\frac{3(2\pi)}{4} \sin \frac{2\pi t}{4}, \frac{3(2\pi)}{4} \cos \frac{2\pi t}{4}, 0\right]$$

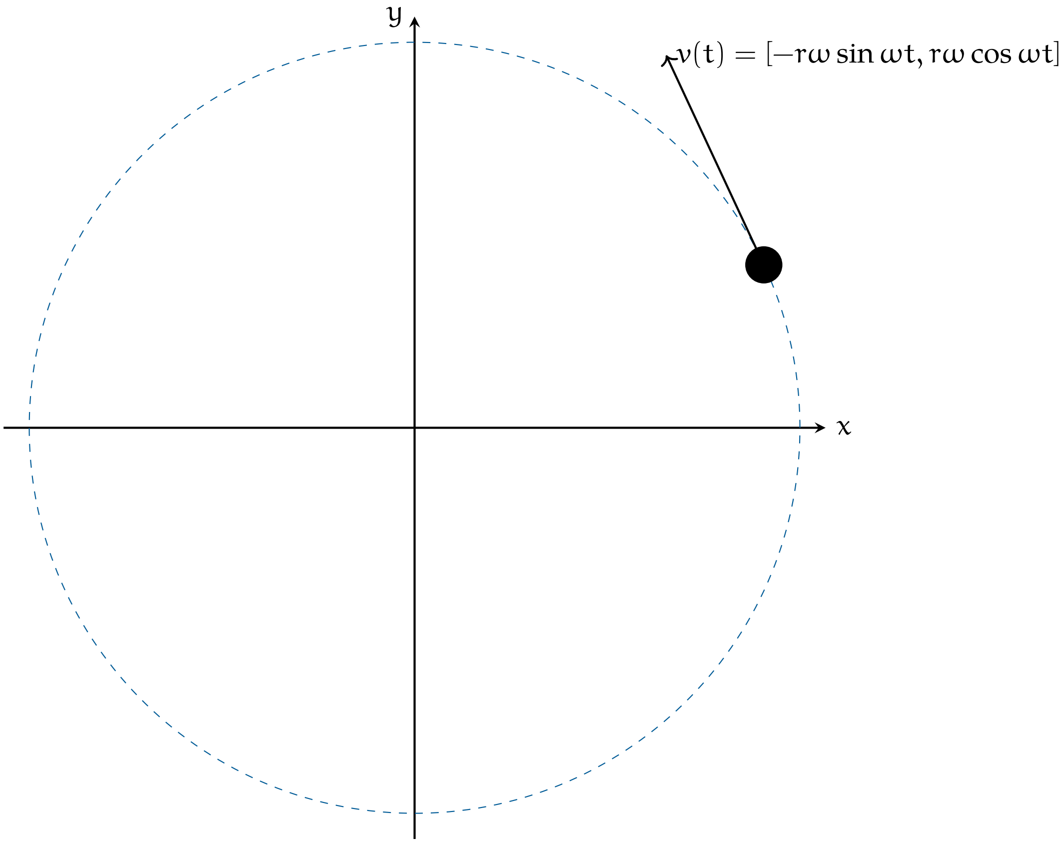
Notice that the velocity vector is perpendicular to the position vector. It has a constant magnitude.

In general, an object traveling in a circle at a constant speed has the velocity vector:

$$v(t) = [-r\omega \sin \omega t, r\omega \cos \omega t]$$

where $t = 0$ is the time that it crosses the x axis. If $\omega$is negative, that means the motion would be clockwise when viewed from above.

The magnitude of the velocity vector is $r\omega$.

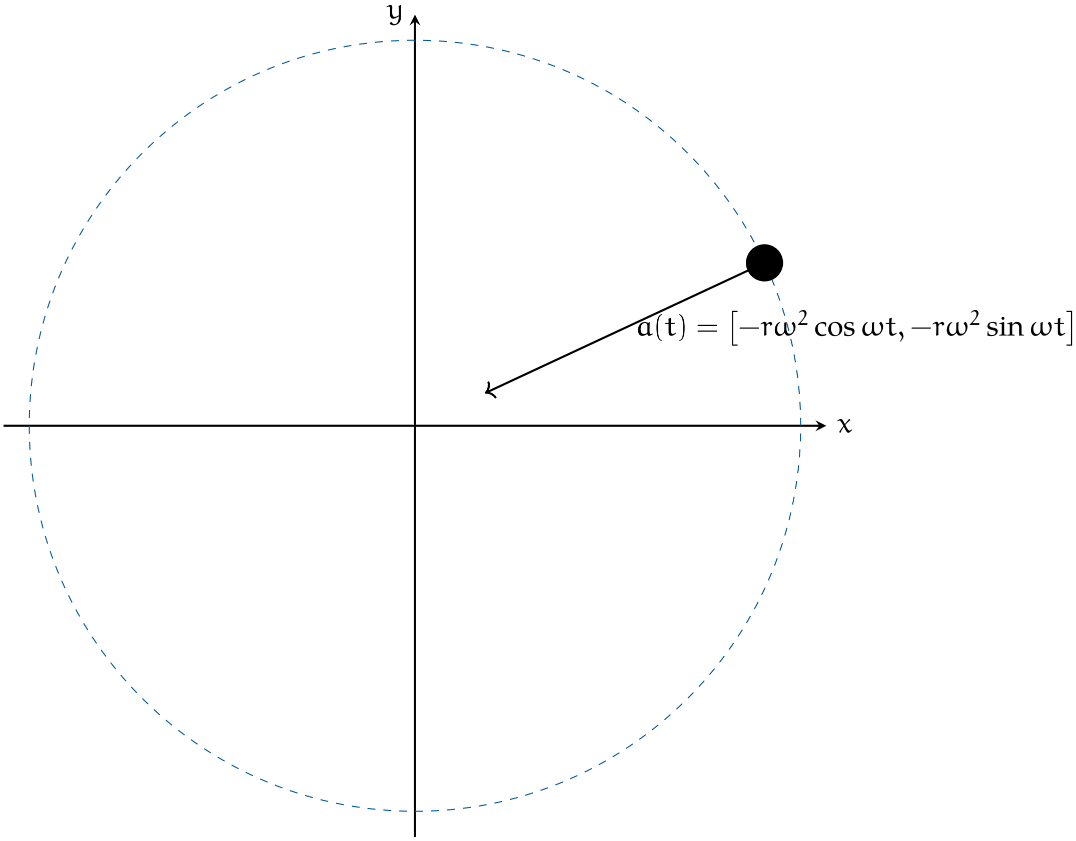


5.2 Acceleration

We can get the acceleration by differentiating the components of the velocity vector.

$$a(t) = \left[-r\omega^2 \cos \omega t, -r\omega^2 \sin \omega t\right]$$

Notice that the acceleration vector points toward the center of the circle it is traveling on. That is, when an object is traveling on a circle at a constant speed, its only acceleration is toward the center of the circle.



The magnitude of the acceleration vector is $r\omega^2$.

5.3 Centripetal force

How hard is the ball pulling against your hand? That is, if you let go, the ball would fly in a straight line. The force you are exerting on the string is what causes it to accelerate toward the center of the circle. We call this the *centripetal force*.

Recall that $F = ma$. The magnitude of the acceleration is $r\omega^2 = 3\left(\frac{2pi}{4}\right)^2 \approx 7.4$ m/s. The mass of the ball is 0.16 kg. So the force pulling against your hand is about 1.18 newtons.

The general rule is that when something is traveling in a circle at a constant speed, the centripetal force needed to keep it traveling in a circle is:

$$F = mr\omega^2$$

If you know the radius r and the speed v of the object, here is the rule:

$$F = \frac{mv^2}{r}$$

Exercise 3 Circular Motion

Just as your car rolls onto a circular track with a radius of 200 m, you realize your 0.4 kg cup of coffee is on the slippery dashboard of your car. While driving 120 km/hour, you hold the cup to keep it from sliding.
What is the maximum amount of force you would need to use (The friction of the dashboard helps you, but the max is when the friction is zero.)

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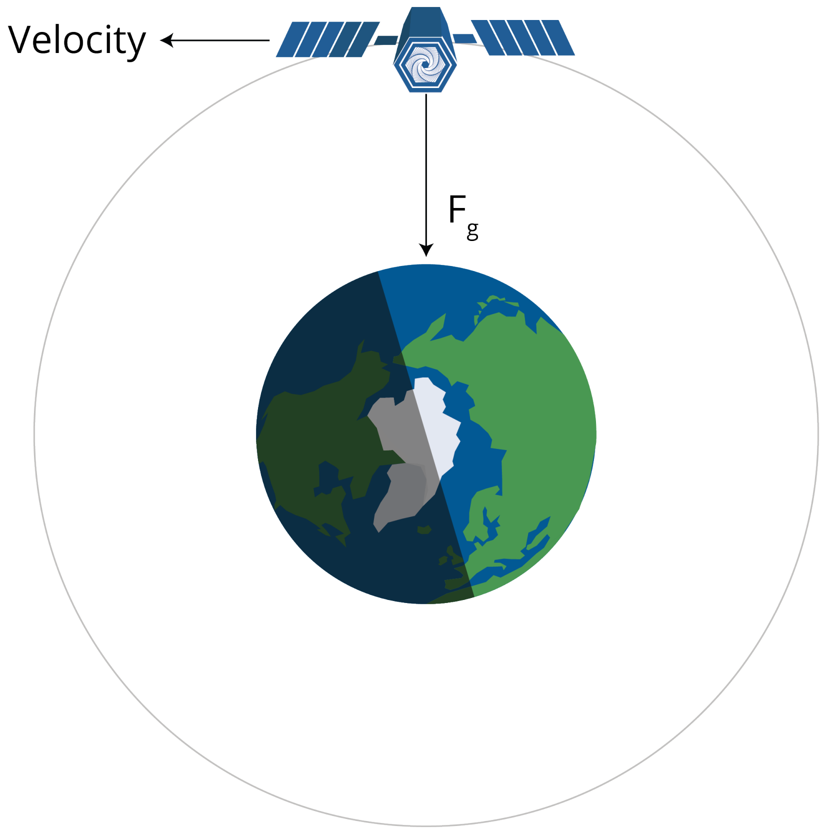
Chapter 6

Orbits

A satellite stays in orbit around the planet because the pull of the planet's gravity causes it to accelerate toward the center of the planet. The satellite must be moving at a very particular speed to keep a constant distance from the planet – to travel in a circular orbit. If it is moving too slowly, it will get closer to the planet. If it is going too fast, it will get farther



from the planet.

The radius of the earth is about 6.37 million meters. A satellite that is in a low orbit is typically about 2 million meters above the ground. At that distance, the acceleration due to gravity is more like $6.8m/s^2$, instead of the $9.8m/s^2$ that we experience on the surface of the planet.

How fast does the satellite need to be moving in a circle with a radius of 8.37 million meters to have an acceleration of $6.8m/s^2$? Real fast.

Recall that the acceleration vector is

$$a = \frac{v^2}{r}$$

Thus the velocity v needs to be:

$$v = \sqrt{ar} = \sqrt{6.8(8.37x10^6} = 7,544 \text{ m/s}$$

(That's 16,875 miles per hour.)

When a satellite falls out of orbit, it enters the atmosphere at that 7,544 m/s. The air rushing by generates so much friction that the satellite gets very, very hot and usually disintegrates.

6.1 Astronauts are not weightless

Some people see astronauts floating inside an orbiting spacecraft and think there is no gravity: that the astronauts are so far away that the gravity of the planet doesn't affect them. This is incorrect. The gravity might be slightly less (Maybe 6 newtons per kg instead of 9.8 newtons per kg), but the weightless they experience is because they and the spacecraft is in free fall. They are just moving so fast (in a direction perpendicular to gravity) that they don't collide with the planet.

Exercise 4 Mars Orbit

The radius of Mars is 3.39 million meters. The atmosphere goes up another 11 km. Let's say you want to put a satellite in a circular orbit around Mars with a radius of 3.4 million meters.
The acceleration due to gravity on the surface of Mars is $3.721 \mathrm{m/s}^2$. We can safely assume that it is approximately the same 11 km above the surface.
How fast does the satellite need to be traveling in its orbit? How long will each orbit take?

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6.2 Geosynchronous Orbits

The planet earth rotates once a day. Satellites in low orbits circle the earth many times a day. Satellites in very high orbits circle less than once per day. There is a radius at which a satellite orbits exactly once per day. Satellites at this radius are known as "geosynchronous" or "geostationary" because they are always directly over a place on the planet.

The radius of a circular geosynchronous orbit is 42.164 million meters. (About 36 km above the surface of the earth.)

A geosynchronous satellite travels at a speed of 3,070 m/s.

Geosynchronous satellites are used for the Global Positioning Satellite system, weather monitoring system, and communications system.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 6)

A is 440 Hz. Each half-step is a multiplication by $\sqrt[12]{2} = 1.059463094359295$ So the frequency of E is $(440)(2^{7/12}) = 659.255113825739859$

Answer to Exercise 2 (on page 22)

The force of gravity is $9.8 \times 45 = 441$ newtons.

At any speed s, the force of wind resistance is $0.05 \times s^2 = 0.05s^2$ newtons.

At terminal velocity, $0.05s^2 = 441$.

Solving for s, we get $s = \sqrt{\frac{441}{0.05}}$

Thus, terminal velocity should be about 94 m/s.

Answer to Exercise 3 (on page 32)

$$\frac{120 \text{ km}}{1 \text{hour}} = \frac{1000 \text{ m}}{1 \text{ km}} \frac{120 \text{ km}}{1 \text{hour}} \frac{1 \text{ hour}}{3600 \text{ seconds}} = 33.3 \text{ m/s}$$

$$F = \frac{mv^2}{r} = \frac{0.4(33.3)^2}{200} = 2.2 \text{ newtons}$$

Answer to Exercise 3 (on page 35)

$$v = \sqrt{3.721(3.4 \times 10^6)} = 3,557 \text{ m/s}$$

The circular orbit is $2\pi(3.4 \times 10^6) = 21.4 \times 10^6$ meters in circumference.

The period of the orbit is $(21.4 \times 10^6)/3,557 \approx 6,000$ seconds.