

① Age

$$\text{Info}(D) = I(9,5) = -\frac{9}{14} \log_2 \left(\frac{9}{14}\right) - \frac{5}{14} \log_2 \left(\frac{5}{14}\right) \approx 0.940$$

$$\text{Info}_{\text{age}} = 0.694$$

$$\text{Gain}_{\text{age}} = 0.940 - 0.694 = 0.246$$

② Income

age	income	student	credit rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

root node (income)

	L.	M.	H.
Yes	3	4	2
No.	1	2	2

$$\text{Info}_{\text{income}}(D) = \frac{4}{14} I(3,1) + \frac{6}{14} I(4,2) + \frac{4}{14} I(2,2) \\ = 0.911$$

$$\text{Gain}_{\text{income}} = 0.940 - 0.911 = 0.029$$

root node (student)

	std.	Not std.
Yes	6	3
No.	1	4

$$\text{Info}_{\text{std.}}(D) = \frac{7}{14} I(6,1) + \frac{7}{14} I(3,4) \\ = 0.789$$

$$\text{Gain}_{\text{income}} = 0.940 - 0.789 = 0.151$$

③ Student

age	income	student	credit rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

root node (credit)

	fair	excellent
Yes	6	3
No.	2	3

$$\text{Info}_{\text{Credit}}(D) = \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3) \\ = 0.892$$

$$\text{Gain}_{\text{income}} = 0.940 - 0.892 = 0.048$$

$$\text{Info}_{\text{income}}(D) = \frac{1}{5} I(1,0) + \frac{2}{5} I(1,1) + \frac{2}{5} I(0,2) = 0.8$$

$$\text{Info}_{\text{std.}}(D) = \frac{2}{5} I(2,0) + \frac{3}{5} I(2,0) = 0$$

$$\text{Info}_{\text{credit}}(D) = \frac{3}{5} I(1,2) + \frac{2}{5} I(1) = 0.951$$

$$\text{Gain}_{\text{income}} = 0.940 - 0.8 = 0.140$$

$$\text{Gain}_{\text{std.}} = 0.940 - 0.970 = 0.070$$

$$\text{Gain}_{\text{credit}} = 0.940 - 0.951 = 0.019$$

∴ study > income > credit_rating

1 Credit or Income

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

1.1) Income

	Low	Medium	High		fair	excellent
Yes	1	2	0	Yes	3	0
No	1	1	0	No	0	2

$$Info(D) = I(3,2) = -\frac{3}{5} \log_2(\frac{3}{5}) - \frac{2}{5} \log_2(\frac{2}{5}) = 0.971$$

$$Info_{credit}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} (0,2) = 0$$

$$Info_{income}(D) = \frac{3}{5} I(1,1) + \frac{2}{5} I(2,1) + \frac{0}{5} I(0,0) = 0.951$$

$$Gain_{income} = \frac{Info(D)}{Info_{income}(D)} = 0.971 - 0.951 = 0.020$$

$$Gain_{credit} = \frac{Info(D)}{Info_{credit}(D)} = 0.971 - 0 = 0.971 \checkmark$$

∴ Credit > income when Credit

1.2) Credit

