Remark: Let $f, g \in C(\mathbb{R}/\mathbb{Z}; \mathbb{C})$. Define the operator

$$T_f : C(\mathbb{R}/\mathbb{Z}; \mathbb{C}) \longrightarrow C(\mathbb{R}/\mathbb{Z}; \mathbb{C}), \qquad (T_f g)(x) = (f * g)(x).$$

By the basic properties of convolution, T_f is a linear operator on $C(\mathbb{R}/\mathbb{Z}; \mathbb{C})$.

Moreover, for each Fourier basis function $e_n(x) = e^{2\pi i n x}$ we have

$$(f * e_n)(x) = \widehat{f}(n)\, e_n(x),$$

which shows that

$$T_f(e_n) = \widehat{f}(n)\, e_n.$$

Thus each e_n is an eigenfunction of T_f with eigenvalue $\widehat{f}(n)$.