

3 Way Merge Sort

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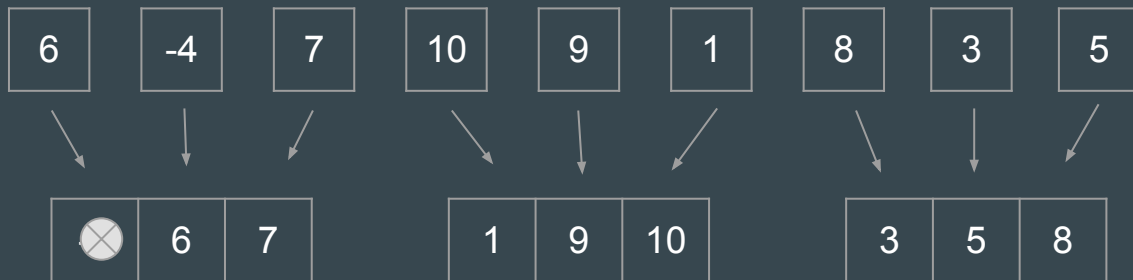
3 Way Merge Sort

- Ideia básica: Dividir para conquistar.
- Divide recursivamente o conjunto de dados até que cada subconjunto possua apenas 1 elemento.
- Combina 3 subconjuntos de forma a obter 1 conjunto maior e ordenado.
- O processo se repete até que exista apenas 1 conjunto.

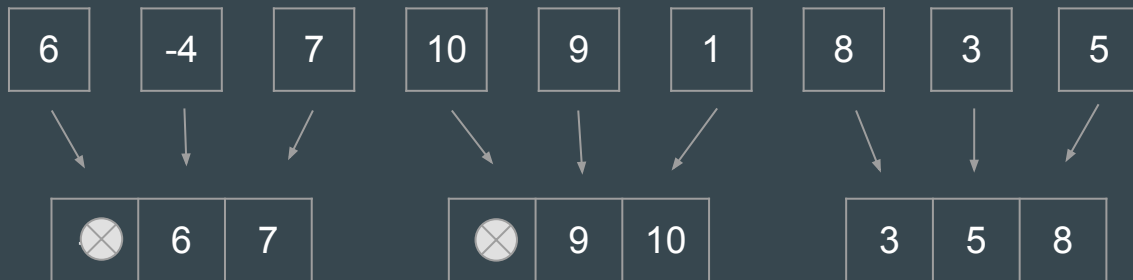
Funcionamento

6	-4	7	10	9	1	8	3	5
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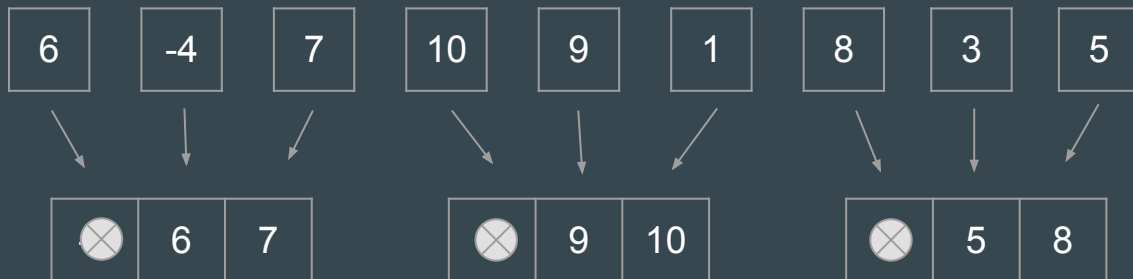




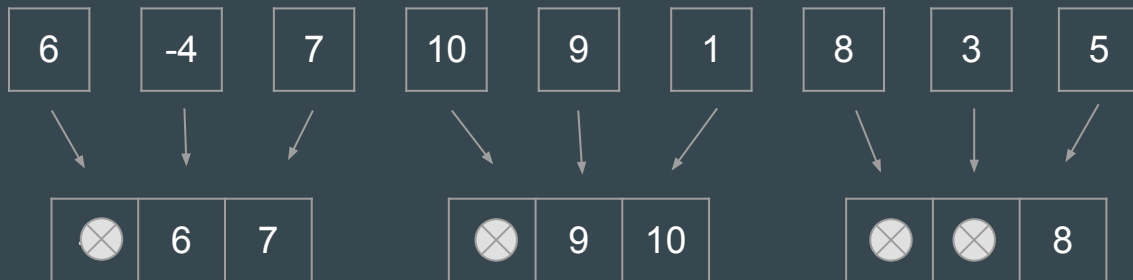
-4								
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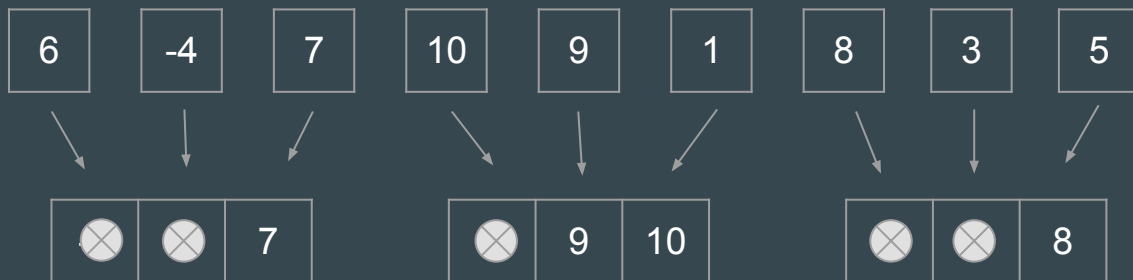
-4	1							
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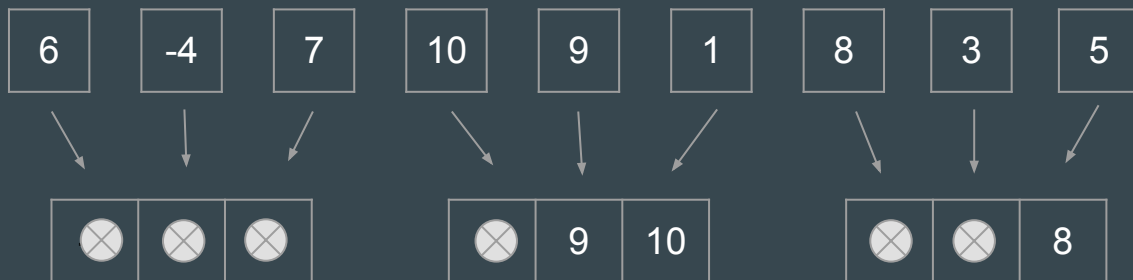
-4	1	3						
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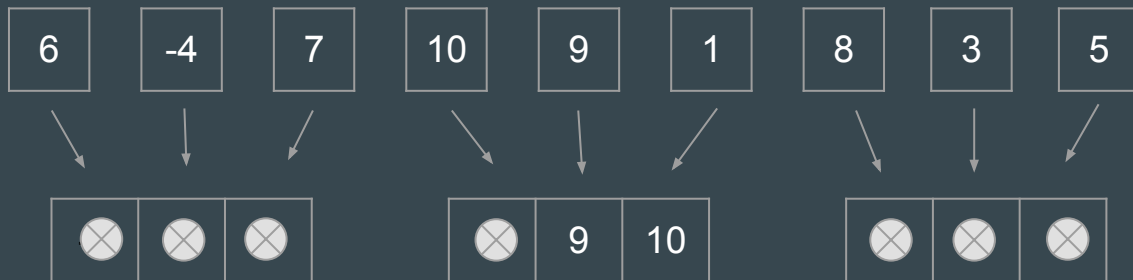
-4	1	3	5					
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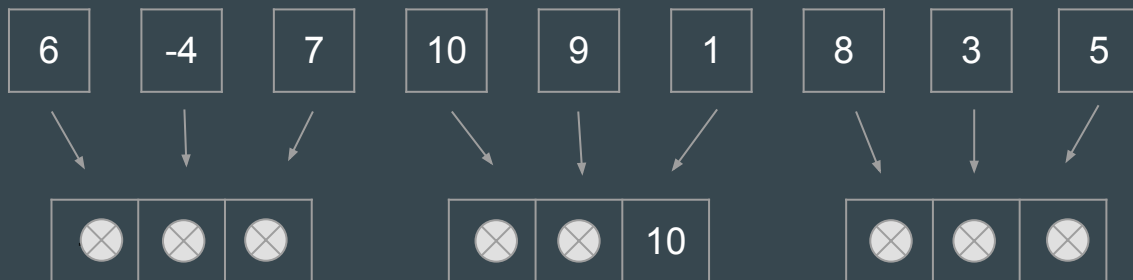
-4	1	3	5	6				
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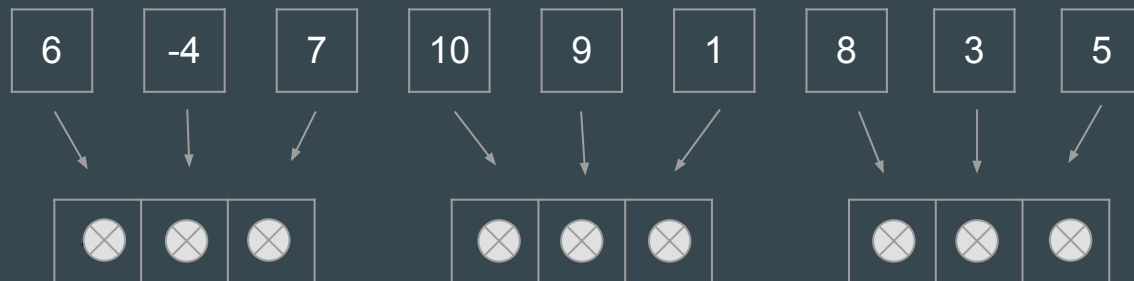
-4	1	3	5	6	7			
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-4	1	3	5	6	7	8		
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-4	1	3	5	6	7	8	9	
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-4	1	3	5	6	7	8	9	10
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Conjunto ordenado

Relação de Recorrência

$$T(n) = \begin{cases} 1 & , \text{ se } n = 1 \\ 3T(n/3) + n + 1 & , \text{ se } n > 1 \end{cases}$$

$$T(n) = 3T\left(\frac{n}{3}\right) + n + 1$$

$$T\left(\frac{n}{3}\right) = 3T\left[3T\left(\frac{n}{9}\right) + \frac{n}{3} + 1\right] + n + 1$$

$$T\left(\frac{n}{3}\right) = 9T\left(\frac{n}{9}\right) + 2n + 3 + 1$$

$$T\left(\frac{n}{9}\right) = 9T\left[3T\left(\frac{n}{27}\right) + \frac{n}{9} + 1\right] + 2n + 3 + 1$$

$$T\left(\frac{n}{9}\right) = 27T\left(\frac{n}{27}\right) + 3n + 9 + 3 + 1$$

$$T\left(\frac{n}{27}\right) = 27T\left[3T\left(\frac{n}{81}\right) + \frac{n}{27} + 1\right] + 3n + 9 + 3 + 1$$

$$T\left(\frac{n}{27}\right) = 81T\left(\frac{n}{81}\right) + 4n + 27 + 9 + 3 + 1$$

$$T(n) = 3^4 T\left(\frac{n}{3^4}\right) + 4n + 3^3 + 3^2 + 3^1 + 3^0$$

$$T(n) = 3^k T\left(\frac{n}{3^k}\right) + kn + \sum_{i=0}^{k-1} 3^i$$

$$T(n) = 3^k T\left(\frac{n}{3^k}\right) + kn + \frac{1 - 3^{k-1}}{1 - 3}$$

$$T(n) = 3^k T\left(\frac{n}{3^k}\right) + kn + \frac{3^{k-1} - 1}{2}$$

FAZENDO $n = 3^k$ e $\log_3 n = k$:

$$T(n) = 3^k T(1) + kn + \frac{3^{k-1} - 1}{2}$$

$$T(n) = 3^k + kn + \frac{3^{k-1} - 1}{2}$$

$$T(n) = n + n \log_3 n + \frac{3^{\log_3 n - 1} - 1}{2}$$

$$T(n) = n + n \log_3 n + \frac{n - 3}{6} \quad \Rightarrow \quad O(n \log_3 n)$$