

Projektpraktikum Series 3 Documentation

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1 Introduction

Welcome to our documentation of Projektpraktikum Series 3. Our task was to write a program which creates certain matrices and runs experiments to test the accuracy of matrix operations. Due to physical limitations, computers can often cause problems when executing complex algorithms involving large matrices, especially when the sizes of individual matrix entries vary wildly.

2 Implementation

2.1 Matrix class

Our Matrix class holds the methods we use to create matrices and execute operations.

2.1.1 `create_matrix_and_inv`

The method `create_matrix_and_inv` generates a matrix as well as its inverse, and

2.1.2 `condition`

`condition` calculates the condition of a given matrix by multiplying the infinity norm of the matrix with the infinity norm of the inverse matrix and returns the result.

2.1.3 `lu`

`lu` carries out a LU (Lower-Upper) decomposition on a given matrix and returns both resulting matrices.

2.1.4 `solve`

`solve` accepts a matrix A and a vector b and solves the equation $Ax = b$ by first applying LU decomposition and then using Gaussian elimination to return the vector x.

2.2 Execution of experiments

2.2.1 `main3.1B`

`main3.1B` calculates and prints the analytic property

$$||I - MM^1||$$

as well as the condition of the matrix with the infinity norm

$$\text{cond}_\infty(M) = ||M||_\infty ||M^{-1}||_\infty$$

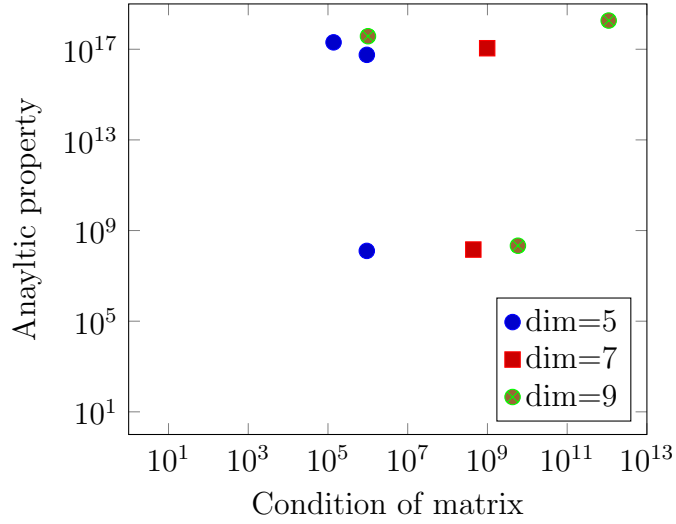
for Hilbert Matrices and tridiagonal matrices with the dimensions 5, 7 and 9 with the data types `float16`, `float32` and `float64`.

3 Experimental results

3.1 Analytical Property

3.1.1 Hilbert Matrix

Analytic property as a function of matrix condition

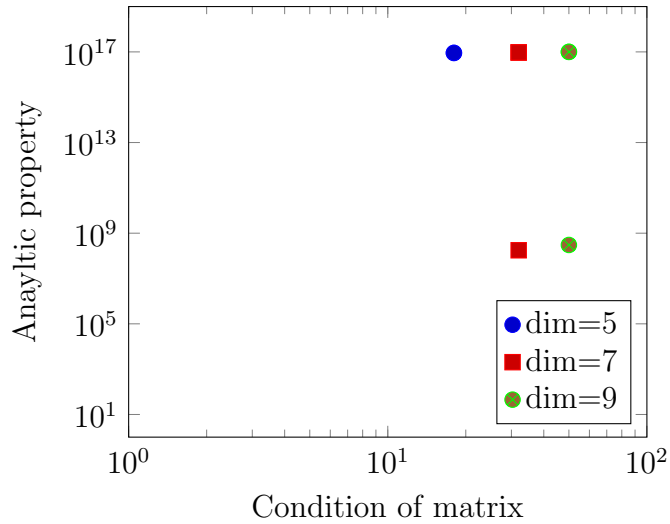


Note: for $\text{dim} = 7$ with `float16`, our result was 'Cannot calculate inverse of M: singular matrix'. This is likely due to a rounding error causing the rank of the matrix to fall below 7.

The three marks at approximately 10^9 are all for `float16`.

3.1.2 Tridiagonal 'String' matrix

Analytic property as a function of matrix condition



Note: for $\text{dim} = 5$ with `float32`, our result was 'Cannot calculate inverse of M: singular matrix'. It is unclear why this is the case only for `float32`; it functions for both `float16` and `float64`

We see a similar pattern here. For both matrices, we see that the condition and size of the matrices have very little effect on the analytic property, while

4 Conclusion