Chebyshev's inequality

In probability theory, Chebyshev's inequality (also called the Bienaymé–Chebyshev inequality) provides an upper bound on the probability of deviation of a random variable (with finite variance) from its mean. More specifically, the probability that a random variable deviates from its mean by more than $k\sigma$ is at most $1/k^2$, where k is any positive constant and σ is the standard deviation (the square root of the variance).

The rule is often called Chebyshev's theorem, about the range of standard deviations around the mean, in statistics. The inequality has great utility because it can be applied to any probability distribution in which the mean and variance are defined. For example, it can be used to prove the weak law of large numbers.

The term Chebyshev's inequality may also refer to Markov's inequality, especially in the context of analysis. They are closely related, and some authors refer to Markov's inequality as "Chebyshev's First Inequality," and the similar one referred to on this page as "Chebyshev's Second Inequality."

Chebyshev's inequality is tight in the sense that for each chosen positive constant, there exists a random variable such that the inequality is in fact an equality.[3]

Probabilistic statement [edit]

Let X (integrable) be a random variable with finite non-zero variance σ^2 (and thus finite expected value μ).[9] Then for any real number $k > 0$,

$$\Pr(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

Only the case $k > 1$ is useful. When $k \leq 1$ the right-hand side $\frac{1}{k^2} \geq 1$ and the inequality is trivial as all probabilities are ≤ 1.

As an example, using $k = \sqrt{2}$ shows that the probability values lie outside the interval $(\mu - \sqrt{2}\sigma, \mu + \sqrt{2}\sigma)$ does not exceed $\frac{1}{2}$.

Equivalently, it implies that the probability of values lying within the interval (i.e. its "coverage") is *at least* $\frac{1}{2}$.

k	Min. % within *k* standard deviations of mean	Max. % beyond *k* standard deviations from mean
1	0%	100%
$\sqrt{2}$	50%	50%
1.5	55.55%	44.44%
2	75%	25%
$2\sqrt{2}$	87.5%	12.5%
3	88.8888%	11.1111%
4	93.75%	6.25%
5	96%	4%
6	97.2222%	2.7778%
7	97.9592%	2.0408%
8	98.4375%	1.5625%
9	98.7654%	1.2346%
10	99%	1%