In probability theory, Markov's inequality gives an upper bound on the probability that a non-negative random variable is greater than or equal to some positive constant. Markov's inequality is tight in the sense that for each chosen positive constant, there exists a random variable such that the inequality is in fact an equality.[1]

Markov's inequality (and other similar inequalities) relate probabilities to expectations, and provide (frequently loose but still useful) bounds for the cumulative distribution function of a random variable.

Markov's inequality can also be used to upper bound the expectation of a non-negative random variable in terms of its distribution function.

Markov inequality

The Markov inequality

- Use a bit of information about a distribution to learn something about probabilities of "extreme events"
- "If $X \geq 0$ and $\mathbf{E}[X]$ is small, then X is unlikely to be very large"

> **Markov inequality:** If $X \geq 0$ and $a > 0$, then $\mathbf{P}(X \geq a) \leq \dfrac{\mathbf{E}[X]}{a}$

$$E[X] = \int_0^\infty x f_X(x)\,dx \geq \int_a^\infty x f_X(x)\,dx$$

$$\geq \int_a^\infty a\, f_X(x)\,dx = a\,P(X \geq a)$$

$$E[X] \geq a * P(X \geq a) \rightarrow P(X \geq a) \leq \frac{E[x]}{a}$$