Explanation and Intuition of Graph Terms

1. Degeneracy of a Graph

Definition: Degeneracy (κ) of a graph is the smallest number such that every subgraph has a vertex with degree at most κ.

Intuition: It measures the "sparsity" of a graph in a smart way. Instead of average degree, it looks at the worst case in all subgraphs.

Examples:

- A tree has degeneracy 1.
- A cycle has degeneracy 2.
- A complete graph (clique) with n nodes has degeneracy $n-1$.

2. Constant Degeneracy Graphs

Definition: A graph has **constant degeneracy** if κ is a fixed constant, no matter how large the graph gets.

Why it's useful: These graphs stay sparse even as they grow, making them much easier to handle in streaming algorithms.

Examples of constant degeneracy graphs:

- Planar graphs (have $\kappa \leq 5$)
- Minor-closed families (e.g., no K_5 as a minor)
- Preferential attachment graphs (like many real-world networks)

Difference from general graphs:

- General graphs can have high degeneracy (e.g., cliques).
- Constant degeneracy graphs are always low-density and easier to process.

3. $\widetilde{O}$ vs O Notation

Big-O Notation ($O(f(n))$):

- Describes the upper bound on time/space.
- Hides constant factors.
- Example: $O(n^2)$ means the growth is at most proportional to n^2.

Soft-O Notation ($\widetilde{O}(f(n))$):

- Like O, but also hides polylogarithmic factors like $\log n$, $(\log n)^2$, etc.
- Think of it as "almost $O(f(n))$" but with extra logarithmic terms.
- Example:

$$\widetilde{O}(n) = O(n \cdot \log^k n) \quad \text{for some constant } k$$