# Understanding Arboricity of Graphs

A LaTeX Manual with Intuition, Definitions, and Examples

## Introduction

Arboricity is a fundamental concept measuring how densely the edges of a graph can be decomposed into forests. Intuitively, it captures the minimum number of tree-like layers needed to cover all edges.

## Intuitive Definition

Two graphs, subgraphs, or paths are called edge-disjoint if they do not share any common edges. However, they may still share common vertices.

**Definition 2.1** (Arboricity). *For a graph $G = (V, E)$, the* arboricity $\gamma(G)$ *is the minimum number of edge-disjoint forests whose union is $G$.*

Think of each forest as a layer of non-cycling edges: arboricity tells us the fewest such layers to reconstruct the graph.

## Mathematical Characterization

Nash-Williams provided an equivalent formula:

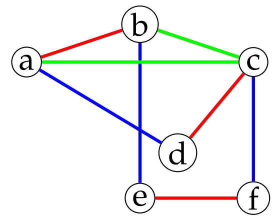
**Theorem 3.1** (Nash-Williams, 1961).

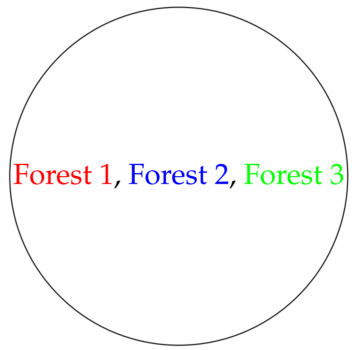
$$\gamma(G) = \max_{H \subseteq G,\, |V(H)| \geq 2} \left\lceil \frac{|E(H)|}{|V(H)| - 1} \right\rceil .$$

This considers every subgraph $H$: ratio of edges to vertices minus one, rounded up.

## Illustration

Below is a hand-positioned illustration of a sample graph decomposed into three forests. Nodes are placed manually to avoid graphdrawing requirements.




In this decomposition, edges are colored by forest: red, blue, and green, implying $\gamma(G) = 3$.

## Worked Example

Consider the complete graph $K_5$ with 5 vertices and 10 edges. For any subgraph $H$ with $n$ vertices,

$$\frac{|E(H)|}{|V(H)| - 1} \leq \frac{n(n-1)/2}{n-1} = \frac{n}{2}.$$

Maximizing at $n = 5$ gives $5/2 = 2.5 \Rightarrow \lceil 2.5 \rceil = 3$. Thus $\gamma(K_5) = 3$.

## Conclusion

Arboricity elegantly links combinatorial structure to decomposition into forests. Its computation via Nash–Williams provides a powerful tool in graph theory and its applications.

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