task1

October 18, 2022

1 Cohort 6 Screening

1.1 Task 1

I'm going to work on the first task in this cohort, as the papers featured as references as well as the problem itself seemed really interesting to me.

1.1.1 Task

To make a multiplier, for this we design the input of two positive integers to a function and this function will process a quantum algorithm that makes the multiplier (see Draper adder) and returns the result in an integer.

1.1.2 Contents

I start with short preliminaries where I introduce some helpers. Then I will build solution based on the multiplication circuit proposed by Ruiz-Perez et al. in [1]. Finally, I'll test and benchmark the solution. An alternative approach may be found in [3].

References:

[1] Ruiz-Perez et al., Quantum arithmetic with the Quantum Fourier Transform, 2017. (arXiv:1411.5949)

[2] T. G. Draper, Addition on a quantum computer, 2000. (arXiv:quant-ph/0008033v1)

[3] Muñoz-Coreas et al., T-count Optimized Design of Quantum Integer Multiplication, 2017. (arXiv:1706.05113)

1.1.3 Preliminaries

Below I define some useful helpers and import all dependencies, so as not to disrupt the process of the solution design afterwards.

Imports

[1]:
```
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit, Aer,␣
  ↪transpile, assemble
from qiskit.circuit.library import QFT

import matplotlib.pyplot as plt
import seaborn as sns
```

```
import numpy as np

import time

aer_simulator = Aer.get_backend('aer_simulator')
unitary_simulator = Aer.get_backend('unitary_simulator')
```

De/Encoding helpers.

[2]:
```
def dec_to_bin(t):
    """Return integer t in a form of binary string."""
    return "{0:b}".format(t)

def dec_to_bin_padded(t, n):
    """Return integer t in a form of binary string ensuring that it has exactly␣
 ↪n digits."""
    return "{0:b}".format(t).zfill(n)

def bin_to_dec(t):
    """Parse integer t encoded in binary string."""
    return int(t, 2)
```

The circuit is a concatenation of the following registers: * `n`-bit `a` encoding first operand * `n`-bit `b` encoding second operand * `2n`-bit `result` encoding the result of multiplication (when te circuit is ready)

[3]:
```
class QuantumMultiplication:
    """Class used as a struct holding references to all logical parts of the␣
 ↪circuit."""

    def __init__(self, n):
        self.n = n
        self.number_b = QuantumRegister(n, 'b')
        self.number_a = QuantumRegister(n, 'a')
        self.result = QuantumRegister(2*n, 'r')
        self.measurement = ClassicalRegister(2*n, 'c')
        self.circuit = QuantumCircuit(self.number_b, self.number_a, self.
 ↪result, self.measurement)
```

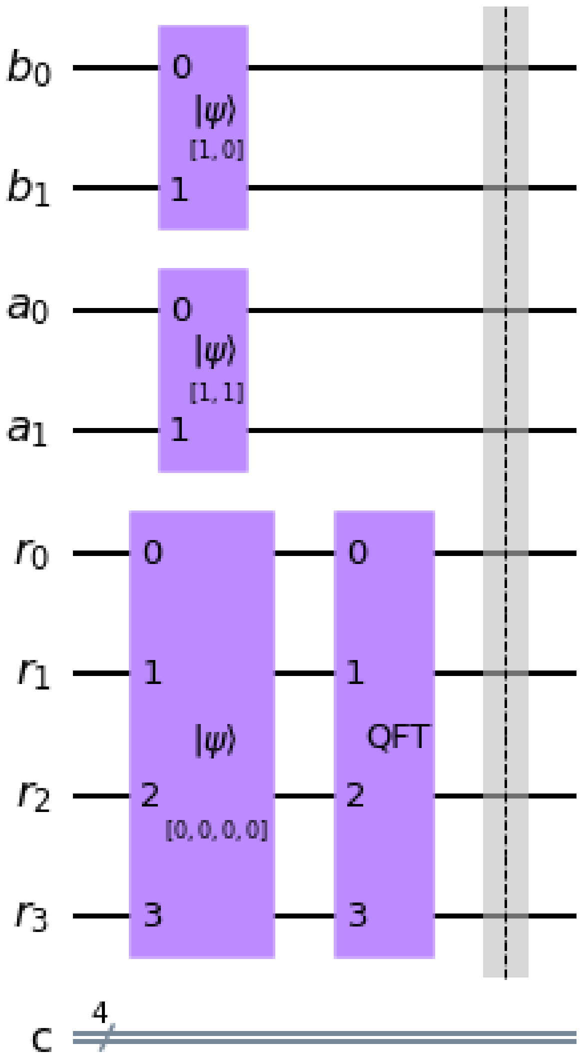
1.1.4 Implementation

First block of the composition is the preparation of the state, so encoding numbers on registers `a` and `b` and preparing state $|\phi(0)\rangle$ on the `result` register.

[4]:
```
n = 2
a = 2
b = 3
```

```
qm = QuantumMultiplication(n)

qm.circuit.initialize(dec_to_bin_padded(a, n), qm.number_b)
qm.circuit.initialize(dec_to_bin_padded(b, n), qm.number_a)
qm.circuit.initialize('0'*(2*n), qm.result)

qm.circuit.compose(QFT(num_qubits=2*qm.n, do_swaps=False, inverse=False), qm.
 ↪result, inplace=True)
qm.circuit.barrier()


qm.circuit.draw(output='mpl')
```

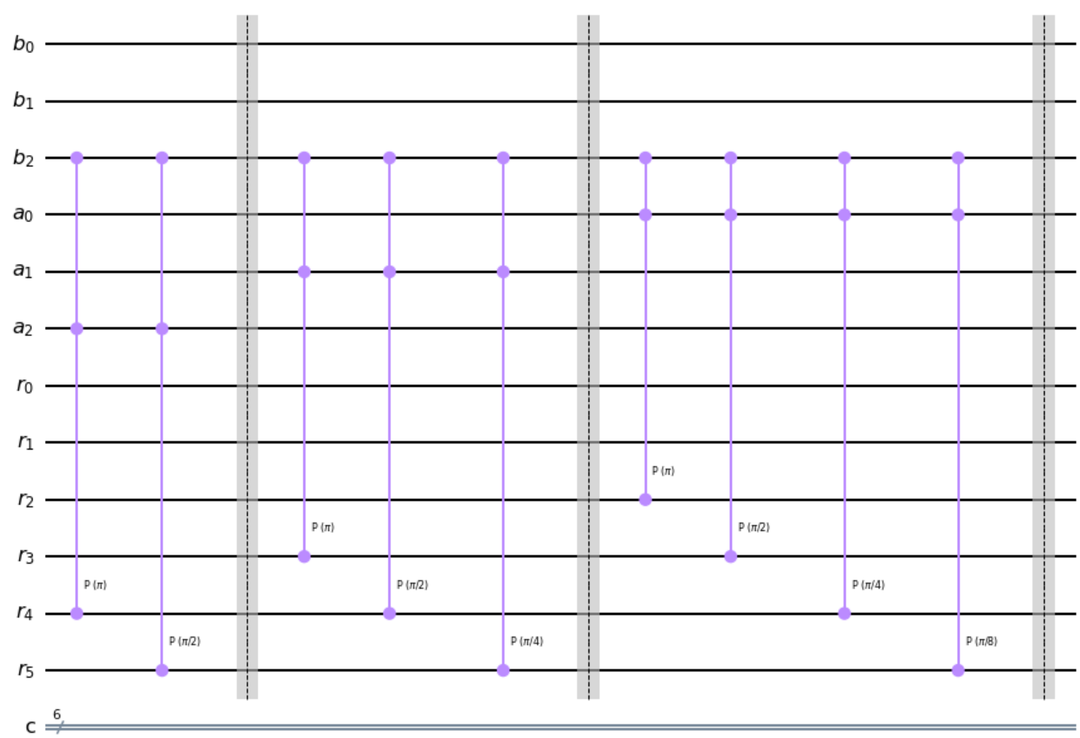
[4]:



Main block of the circuit is $2^i\Sigma$, being a controlled Draper adder (see [2]). Such a block consists of controlled phase rotation gates R_l (see eq. (15) in [1]), where $l = i + j + s - 2n$. Note that we skip R_l for $l \leq 0$, as these are rotations of multiples of 2π (and wouldn't change anything). Below I construct $2^0\Sigma$ on multiplication of 3-qbit numbers.

[5]:
```
n = 3
j = 1

qm = QuantumMultiplication(n)

for i in range(1, n+1):
    for s in range(1,2*n+1):
        l = i + j + s - 2*n
        if l <= 0:
            continue

        lam = (2 * np.pi) / (2**l)
        qm.circuit.mcp(lam, [qm.number_a[n-i], qm.number_b[n-j]], qm.
 ↪result[s-1])
    qm.circuit.barrier()

qm.circuit.draw(output='mpl')
```

[5]:



Now composition of whole multiplier seems easy. If we compose all adders, we need to finalize cirucit with `IQFT` and measurements of result qbits.

[6]:
```
def build_multiplication_circuit(a, b):
    n = max(len(dec_to_bin(x)) for x in (a,b))

    qm = QuantumMultiplication(n)

    # Initialize - initialize qbits and prepare QFT(0)
    qm.circuit.initialize("{0:b}".format(a).zfill(n), qm.number_b)
    qm.circuit.initialize("{0:b}".format(b).zfill(n), qm.number_a)
    qm.circuit.initialize('0'*(2*n), qm.result)

    qm.circuit.compose(QFT(num_qubits=2*n, do_swaps=False, inverse=False), qm.
 ↪result, inplace=True)
    qm.circuit.barrier()

    # Compose adders controlled by qbits of B
    for j in range(1, n+1):
        for i in range(1, n+1):
            for s in range(1,2*n+1):
                l = i + j + s - 2*n
                if l <= 0:
                    continue

                lam = (2 * np.pi) / (2**l)
                qm.circuit.mcp(lam, [qm.number_a[n-i], qm.number_b[n-j]], qm.
 ↪result[s-1])
            qm.circuit.barrier()

    # Finialize - add inverse QFT and measurements
    qm.circuit.compose(QFT(2*qm.n,  do_swaps=False, inverse=True), qm.result,␣
 ↪inplace=True)
    qm.circuit.measure(qm.result, qm.measurement)

    return qm.circuit
```

As for the sake of the convenience I used `.initialize` and `QFT()` Aer won't run my circuit without transpilation. Fortunately, both are easy to transpile, even "manually".

[7]:
```
EXPERIMENT_SHOTS = 100

def run_experiment(circuit):
    """
    Run experiment with circuit. Number of runs is conrolled by global const.
```

```
    Returned value is a dictionary with keys being parsed integers measured on␣
↪result register
    and values being normalised frequencies (0.0-1.0).
    """
    transpiled = transpile(circuit, backend=unitary_simulator)
    qobj = assemble(transpiled)
    result = aer_simulator.run(qobj, shots=EXPERIMENT_SHOTS).result()
    counts = result.get_counts()
    transformed_counts = {bin_to_dec(k): (v * 1.0) / EXPERIMENT_SHOTS for (k,v)␣
↪in counts.items()}
    return transformed_counts
```

[8]:
```
def multiplier(number_1, number_2):
    """
        number_1 : integer positive value that is the first parameter to the␣
↪multiplier function,
        number_2 : integer positive value that is the second parameter to the␣
↪multiplier function.
        Return the positive integer value of the multiplication between␣
↪number_1 and number_2
    """

    circuit = build_multiplication_circuit(number_1, number_2)
    normalised_measurements = run_experiment(circuit)
    result = max(normalised_measurements, key=normalised_measurements.get)

    return result

a = 2
b = 3

print(multiplier(a, b))
```

```
6
```

The function above is the function as described in the task. As it works at first glance, I may proceed with some benchmarks. To this end I define similar function returning `accuracy`, being the normalised frequency of correctly obtained result (i.e. `1.0` means that all simulations resulted with correct number)

[9]:
```
def benchmark_multiplier(number_1, number_2):
    circuit = build_multiplication_circuit(number_1, number_2)
    unified_measurements = run_experiment(circuit)

    expected_value = number_1 * number_2
    accuracy = unified_measurements.get(expected_value) or 0
```

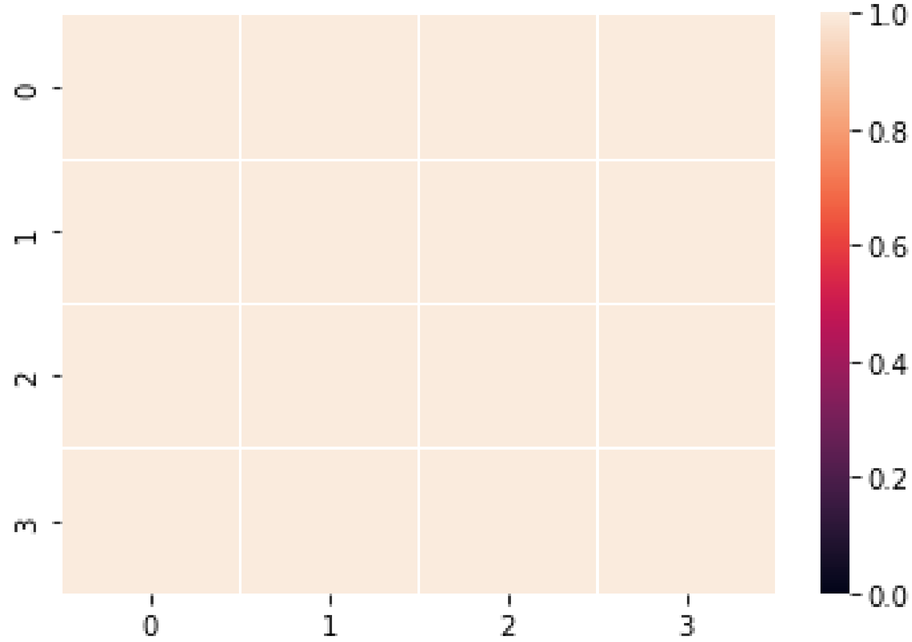
```
    return accuracy

a = 2
b = 3

print(benchmark_multiplier(a, b))
```

```
1.0
```

If we assume that number of bits of multiplied numbers n is relatively small, traversing all $N^2, N = 2^n$ pairs (a, b) is a reasonable goal. Finding minimum in array of results let us detect if there were any errors (any freq. other than `1.0` indicates error). The heatmap will serve as a convenient form of visualization.

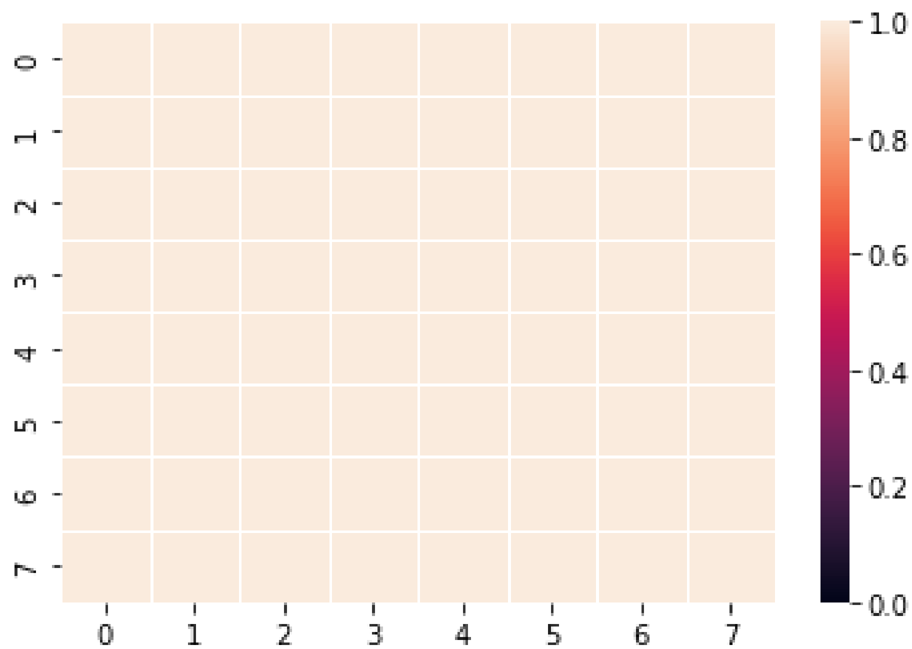
[10]:
```
def benchmark_accuracy(bits_number):
    max_value = 2**bits_number - 1
    possible_operands = list(range(max_value+1))

    results = np.array(list(list((benchmark_multiplier(a, b) for a in␣
 ↪possible_operands)) for b in possible_operands), dtype=float)
    return results

def draw_heatmap(array):
    sns.heatmap(array, linewidth=0.5, vmin=0.0, vmax=1.0)
    return plt.show()

results = benchmark_accuracy(2)
print("Min accuracy: {}".format(np.amin(results)))

draw_heatmap(results)
```

```
Min accuracy: 1.0
```



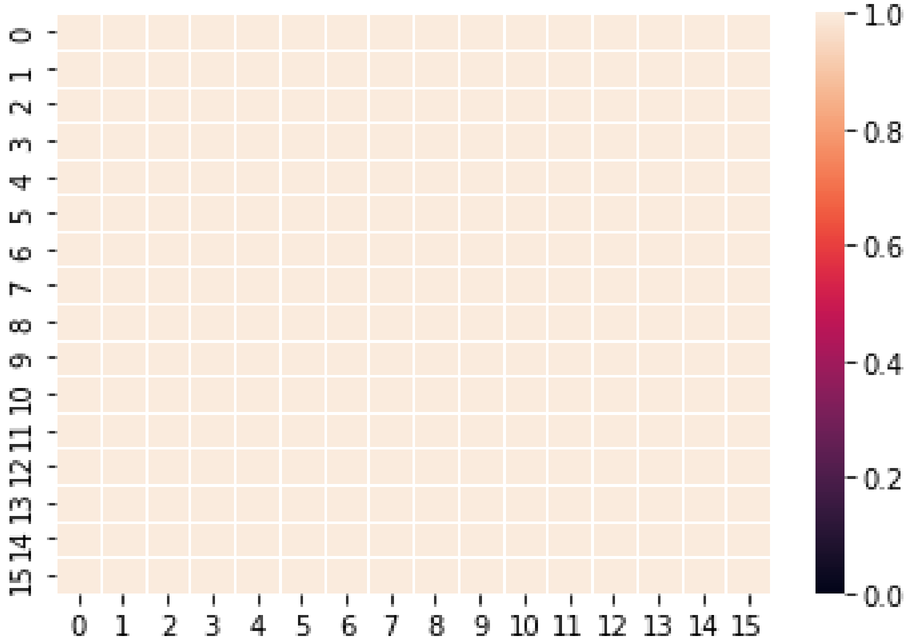
[11]:
```
results = benchmark_accuracy(3)
print("Min accuracy: {}".format(np.amin(results)))

draw_heatmap(results)
```

```
Min accuracy: 1.0
```



```
[12]: results = benchmark_accuracy(4)
print("Min accuracy: {}".format(np.amin(results)))

draw_heatmap(results)
```

Min accuracy: 1.0



Coloring whole space takes longer and longer. Signle multiplication lasts longer, but the main factor is number of multiplications performed. Complexity of single multiplication is $O(n^3)$, while complexity of performing all possible multiplications is $O((2^n)^2)$.

The other way of benchmarking, which allow to benchmark larger values of n (with upper bound of 6, due to simulation limitations) is to draw some random samples for given n.

```
[13]: REPEATS = 10

def benchmark_for(n, accuracies, times):
    accuracies.append([])
    times.append([])

    for r in range(REPEATS):
        a, b = np.random.randint(2**n, size=2)

        start = time.time()
```

```
        result = benchmark_multiplier(a,b)
        end = time.time()

        accuracies[-1].append(result)
        times[-1].append(end-start)
```

[14]:
```
n_values = range(2, 7)
accuracy_samples = []
time_samples = []

for n in n_values:
    benchmark_for(n, accuracy_samples, time_samples)
    log_row = "n = {}\t accuracy: {} +- {}; time: {:.3f} +- {:.3f}s".format(n,␣
 ↪np.mean(accuracy_samples[-1]), np.std(accuracy_samples[-1]), np.
 ↪mean(time_samples[-1]), np.std(time_samples[-1]))
    print(log_row)
```

```
n = 2    accuracy: 1.0 +- 0.0; time: 0.112 +- 0.033s
n = 3    accuracy: 1.0 +- 0.0; time: 0.215 +- 0.051s
n = 4    accuracy: 1.0 +- 0.0; time: 0.377 +- 0.068s
n = 5    accuracy: 1.0 +- 0.0; time: 0.696 +- 0.725s
n = 6    accuracy: 1.0 +- 0.0; time: 25.166 +- 15.054s
```

1.1.5 Conclusions

Referenced papers give examples of robust implementations of arithmetic using `QFT`. This shows the potential that phase encoding brings.