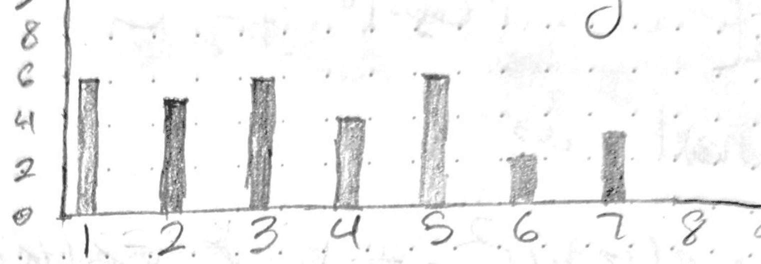
Eleg 4011 Homework 2 3/2/22

Problem 2

$$I = \begin{bmatrix} 1 & 3 & 4 & 2 & 2 & 5 \\ 1 & 1 & 7 & 5 & 1 & 2 \\ 6 & 5 & 0 & 5 & 2 & 4 \\ 3 & 5 & 1 & 0 & 4 & 3 \\ 1 & 3 & 2 & 3 & 2 & 3 \\ 0 & 6 & 7 & 7 & 4 & 5 \end{bmatrix}$$

a) Determine the Histogram



b) Determine the Cumulative distribution

$P_r(r_k) = n_k / MN$

$MN = 36$

use table on slide 14

28	18	9	24	24	7
28	28	2	7	28	24
3	7	22	7	24	9
18	7	28	22	9	18
28	18	24	18	24	18
22	3	2	2	9	7

c) Find point transform T[I]

$S_0 = T(r_0) = 7\,pr(r_0) = 7\left(\frac{790}{36}\right) = 153$

$S_1 = 7\left(\frac{1023}{36}\right) = 198$

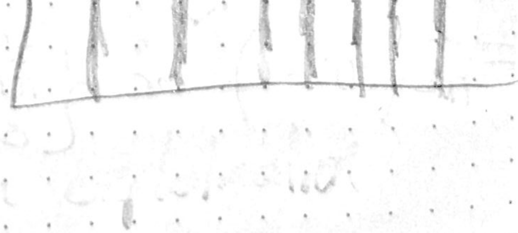
$S_2 = 165$ $\quad S_4 = 64$ $\quad S_6 = 24$

$S_3 = 128$ $\quad S_5 = 48$ $\quad S_7 = 16$

d) transformed image

198	128	64	165	165	48
198	198	16	48	198	165
24	48	153	48	165	64
128	48	18	153	64	128
198	128	165	128	165	128
153	24	16	16	64	48

e) Histogram equalized



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Problem 4

Prove $g\left(\frac{t}{T}\right) \xrightarrow{F} TG(uT)$

Definition

$$F\{g(ct)\} = \int_{-\infty}^{\infty} g(ct)\, e^{-i2\pi f t}\, dt \qquad u = ct \quad du = c\,dt$$

$$F\{g(ct)\} = \int_{-c\infty}^{c\infty} \frac{g(u)}{c} e^{-i2\pi f \frac{u}{c}}\, du$$

if $c \geq 0$

$$F\{g(ct)\} = \boxed{\frac{G\left(\frac{f}{c}\right)}{|c|}}$$

if $c < 0$

$$F(g(ct)) = \int_{\infty}^{-\infty} \frac{g(u)}{c} e^{-2\pi f \frac{u}{c}}\, du$$

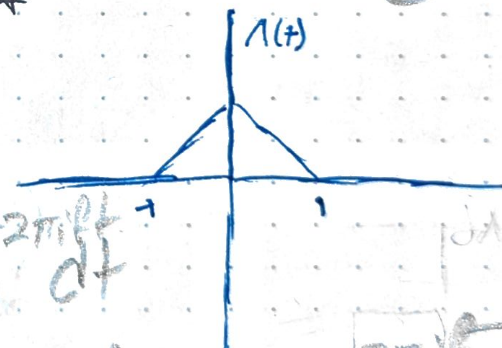
$$= -\int_{-\infty}^{\infty} \frac{g(u)}{c} e^{-2\pi i f \frac{u}{c}}\, du$$

$$= \frac{G\left(\frac{f}{c}\right)}{-c} = \boxed{\frac{G\left(\frac{f}{c}\right)}{|c|}}$$

Problem 5

Find fourier transform of

$$\Lambda(t) = \begin{cases} 1-|t|, & \text{if } |t| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



$$= \int_{-1}^{0} (1+t) e^{-2\pi i f t}\, dt + \int_{0}^{1} (1-t) e^{-2\pi i f t}\, dt$$

$$= \left[\frac{1+2\pi i f}{4\pi^2 f^2} - \frac{e^{2\pi i f}}{4\pi^2 f^2}\right] - \left[\frac{2\pi i f - 1}{4\pi^2 f^2} + \frac{e^{-2\pi i f}}{4\pi^2 f^2}\right]$$

$$= \frac{e^{-2\pi i f}\left(e^{2\pi i f} - 1\right)^2}{4\pi^2 f^2} = \frac{e^{-2\pi i f}\left(e^{\pi i f}\{e^{\pi i f} - e^{-\pi i f}\}\right)^2}{4\pi^2 f^2}$$

$$= \frac{e^{-2\pi i f} e^{2\pi i f} (2i)^2 \sin^2(\pi f)}{4\pi^2 f^2} = \left(\frac{\sin(\pi f)}{\pi f}\right)^2 = \boxed{\text{sinc}^2 f}$$