

Repetitorium matematiky

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$D = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$a = \log_b x \Rightarrow b^a = x$$

$$\log a + \log b = \log a \cdot b$$

$$\log a - \log b = \log \frac{a}{b}$$

$$\log_a b^n = n \cdot \log_a b$$

$$a^{\log_a b} = b$$

$$\frac{\log a}{\log b} = \log_b a$$

$$\log(3+x) = 0 \rightarrow 3+x = 1$$

Test repetitorium

Cv 0.1. Upravte do základního tvaru: $\left(\frac{-16}{3}\right) \cdot \sqrt{\frac{2}{3}} + \frac{\frac{64}{9}}{2 \cdot \sqrt{\frac{2}{3}}}$

$$\left(\frac{-16}{3}\right) \cdot \sqrt{\frac{2}{3}} + \frac{\frac{64}{9}}{2 \cdot \sqrt{\frac{2}{3}}} = -\frac{2^4}{3} \cdot \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}} + \frac{\frac{2^6}{3^2}}{2 \cdot \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}}} = -\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} + \left(\frac{2^6}{3^2} \cdot \frac{3^{\frac{1}{2}}}{2^{\frac{3}{2}}}\right) = -\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} + \frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} = \underline{\underline{0}}$$

Cv 0.2. Zjednodušte $\left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-2} - \frac{x}{x-1}\right)$

$$\begin{aligned} &\left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-2} - \frac{x}{x-1}\right) = \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-1} \cdot \frac{x-1}{x-2} - \frac{x}{x-1}\right) = \\ &= \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2}{(x-2)(x-1)} - \frac{x}{x-1}\right) = \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2 - (x \cdot (x-2))}{(x-2)(x-1)}\right) = \\ &= \left(\frac{(x+1)x}{x+1} - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2 - (x^2 - 2x)}{(x-2)(x-1)}\right) = \\ &= \left(\frac{x^2 - 2x}{x+1}\right) \cdot \left(\frac{x^2 - 2x + 1 - x^2 + 2x}{(x-2)(x-1)}\right) = \left(\frac{x^2 - 2x}{x+1}\right) \cdot \left(\frac{1}{(x-2)(x-1)}\right) = \\ &= \frac{x(x-2)}{x+1} \cdot \frac{1}{(x-2)(x-1)} = \frac{x}{x+1} \cdot \frac{1}{x-1} = \underline{\underline{\frac{x}{x^2-1}}} \end{aligned}$$

Cv 0.3. Zjednodušte $\frac{3y+2}{y^2-2y+1} - \frac{6}{y^2-1} - \frac{3y-2}{y^2+2y+1}$

$$\begin{aligned} \frac{3y+2}{y^2-2y+1} - \frac{6}{y^2-1} - \frac{3y-2}{y^2+2y+1} &= \frac{3y+2}{(y-1)(y-1)} - \frac{6}{(y+1)(y-1)} - \frac{3y+2}{(y+1)(y+1)} \\ &= \frac{((3y+2) \cdot (y+1)) - ((6) \cdot (y-1))}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2} \\ &= \frac{3y^2+3y+2y+2-6y+6}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2} \\ &= \frac{3y^2-y+8}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2} \\ &= \frac{((3y^2-y+8) \cdot (y+1)) - ((3y+2)(y^2-2y+1))}{(y^2-1)^2} \\ &= \frac{(3y^3+3y^2-y^2-y+8y+8) - (3y^3-6y^2+3y+2y^2-4y+2)}{(y^2-1)^2} \\ &= \frac{3y^3+3y^2-y^2-y+8y+8-3y^3+6y^2-3y-2y^2+4y-2}{(y^2-1)^2} \\ &= \frac{6y^2+8y+6}{(y^2-1)^2} = \underline{\underline{\frac{2(3y^2+4y+3)}{(y^2-1)^2}}} \end{aligned}$$

Cv 0.4. Řešte rovnici s neznámou $x \in \mathbb{R}$

$$\begin{aligned} \frac{x+5}{10} - \frac{x-4}{8} &= 1 \\ \frac{((x+5) \cdot (8)) - ((x-4) \cdot (10))}{80} &= 1 \\ \frac{8x+40-10x+40}{80} &= 1 \\ -2x+80 &= 80 \\ -2x &= 0 \\ x &= \underline{\underline{0}} \end{aligned}$$

Cv 0.5. Řešte soustavu dvou rovnic o dvou neznámých $x, y \in \mathbb{R}$

$$3x = -4y + 1$$

$$3y = 4x$$

$$y = \frac{4x}{3}$$

$$3x = -\frac{16x}{3} + 3$$

$$9x = -16x + 3$$

$$25x = 3$$

$$x = \frac{3}{\underline{\underline{25}}}$$

$$y = \frac{\frac{4}{1} \cdot \frac{3}{\underline{\underline{25}}}}{\frac{3}{1}} = \frac{12}{25} \cdot \frac{1}{3} = \frac{12}{75} = \frac{4}{\underline{\underline{25}}}$$

Cv 0.6. Řešte nerovnici s neznámou $x \in \mathbb{R}$

$$x^2 + 10 > 7x$$

$$x^2 - 7x + 10 > 0$$

$$D = (-7)^2 - 4 \cdot 1 \cdot 10 = 49 - 40 = 9$$

$$x_{1,2} = \frac{7 \pm 3}{2} \Rightarrow x_1 = 5, x_2 = 2$$

$$x \in (-\infty, 2) \cup (5, \infty)$$

Cv 0.7. Řešte rovnici s neznámou $x \in \mathbb{R}$

$$2^x - \frac{1}{2} = \frac{1}{2} - 2^x$$

$$2 \cdot 2^x = 1$$

$$2^{x+1} = 2^0$$

$$x + 1 = 0$$

$$x = \underline{\underline{-1}}$$

Cv 0.8. Určete definiční obor funkce $f(x)$

$$f(x) = \frac{\log(2-x)}{\log(x+3)}$$

$$\log(x+3) = 0$$

$$x \neq \{-3, 2\}$$

$$\begin{aligned}
 & \left(\frac{1}{x+1} - \frac{2x}{x^2-1} \right) \left(\frac{1}{x} - 1 \right) = \left(\frac{1}{x+1} - \frac{2x}{(x+1)(x-1)} \right) \left(\frac{1}{x} - 1 \right) = \\
 & = \left(\frac{1}{x+1} \cdot \frac{x-1}{x-1} - \frac{2x}{(x+1)(x-1)} \right) \left(\frac{1}{x} - 1 \right) = \left(\frac{x-1}{(x+1)(x-1)} - \frac{2x}{(x+1)(x-1)} \right) \left(\frac{1}{x} - 1 \right) = \\
 & = \left(\frac{-x-1}{(x+1)(x-1)} \right) \left(\frac{1}{x} - \frac{x}{x} \right) = \frac{-x-1}{(x+1)(x-1)} \cdot \frac{1-x}{x} = \frac{-(x+1)}{(x+1)(x-1)} \cdot \frac{1-x}{x} = \\
 & = \frac{-1}{x-1} \cdot \frac{1-x}{x} = \frac{-(1-x)}{(x-1)(x)} = \frac{-(-1)(x-1)}{(x-1)x} = \frac{1}{\underline{\underline{x}}}
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{3}{(x-3)^2} + \frac{1}{x+3} - \frac{6}{x^2-9} \right) \cdot \frac{x^2-6x+9}{2} = \left(\frac{3}{(x-3)(x-3)} + \frac{1}{x+3} - \frac{6}{(x+3)(x-3)} \right) \cdot \frac{(x-3)(x-3)}{2} = \\
 & = \left(\frac{3}{(x-3)(x-3)} + \frac{x-3}{(x+3)(x-3)} - \frac{6}{(x+3)(x-3)} \right) \cdot \frac{(x-3)(x-3)}{2} = \\
 & = \left(\frac{3}{(x-3)(x-3)} + \frac{x-9}{(x+3)(x-3)} \right) \cdot \frac{(x-3)(x-3)}{2} = \\
 & = \left(\frac{3}{(x-3)(x-3)} + \frac{x-9}{(x+3)(x-3)} \cdot \frac{(x-3)}{(x-3)} \right) \cdot \frac{(x-3)(x-3)}{2} = \\
 & = \left(\frac{3}{(x-3)^2} + \frac{(x-9)(x-3)}{(x+3)(x-3)^2} \right) \cdot \frac{(x-3)^2}{2} = \frac{3x+9+((x-9)(x-3))}{(x+3)(x-3)^2} \cdot \frac{(x-3)^2}{2} = \\
 & = \frac{3x+9+((x-9)(x-3))}{2(x+3)} = \frac{3x+9+(x^2-3x-9x+27)}{2(x+3)} = \\
 & = \frac{x^2-9x+36}{\underline{\underline{2(x+3)}}}
 \end{aligned}$$

$$\frac{x^2+7x}{9-x^2} : \frac{x^2-49}{x+3} = \frac{x(x+7)}{(x+3)(-x+3)} \cdot \frac{x+3}{(x+7)(x-7)} = \frac{x}{\underline{\underline{(x-7)(3-x)}}}$$

$$f(a) = \frac{2x+3}{3x+4}$$

$$P_y = \left[0, \frac{3}{4} \right]$$

$$P_x = \left[-\frac{3}{2}, 0 \right]$$

$$2x+3=0$$

$$x = -\frac{3}{2}$$

$$\frac{\frac{2}{3} \cdot \left(x + \frac{4}{3}\right) - \left(\frac{4}{3} - \frac{3}{2}\right) \cdot \frac{2}{3}}{x + \frac{4}{3}}$$