Repetitorium matematiky

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

$D = b^2 - 4ac$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$a = \log_b x \Rightarrow b^a = x$
$\log a + \log b = \log a \cdot b$
$\log a - \log b = \log \frac{a}{b}$
$\log_a b^n = n \cdot \log_a b$
$a^{\log_a b} = b$
$\frac{\log a}{\log b} = \log_b a$

Test repetitorium

Cv 0.1. Upravte do základního tvaru: $\left(\frac{-16}{3}\right) \cdot \sqrt{\frac{2}{3}} + \frac{\frac{64}{9}}{2 \cdot \sqrt{\frac{2}{3}}}$

$$\left(\frac{-16}{3}\right) \cdot \sqrt{\frac{2}{3}} + \frac{\frac{64}{9}}{2 \cdot \sqrt{\frac{2}{3}}} = -\frac{2^4}{3} \cdot \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}} + \frac{\frac{2^6}{3^2}}{2 \cdot \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}}} = -\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} + \left(\frac{2^6}{3^2} \cdot \frac{3^{\frac{1}{2}}}{2^{\frac{3}{2}}}\right) = -\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} + \frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}} = \underline{0}$$

Cv 0.2. Zjednodušte $\left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-2} - \frac{x}{x-1}\right)$

$$\left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-2} - \frac{x}{x-1}\right) = \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{x-1}{x-1} \cdot \frac{x-1}{x-2} - \frac{x}{x-1}\right) =$$

$$= \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2}{(x-2)(x-1)} - \frac{x}{x-1}\right) = \left(x - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2 - \left(x \cdot (x-2)\right)}{(x-2)(x-1)}\right) =$$

$$= \left(\frac{(x+1)x}{x+1} - \frac{3x}{x+1}\right) \cdot \left(\frac{(x-1)^2 - \left(x^2 - 2x\right)}{(x-2)(x-1)}\right) =$$

$$= \left(\frac{x^2 - 2x}{x+1}\right) \cdot \left(\frac{x^2 - 2x + 1 - x^2 + 2x}{(x-2)(x-1)}\right) = \left(\frac{x^2 - 2x}{x+1}\right) \cdot \left(\frac{1}{(x-2)(x-1)}\right) =$$

$$= \frac{x(x-2)}{x+1} \cdot \frac{1}{(x-2)(x-1)} = \frac{x}{x+1} \cdot \frac{1}{x-1} = \underline{\underline{\frac{x}{x^2 - 1}}}$$

Cv 0.3. Zjednodušte $\frac{3y+2}{y^2-2y+1} - \frac{6}{y^2-1} - \frac{3y-2}{y^2+2y+1}$

$$\frac{3y+2}{y^2-2y+1} - \frac{6}{y^2-1} - \frac{3y-2}{y^2+2y+1}$$

$$= \frac{3y+2}{(y-1)(y-1)} - \frac{6}{(y+1)(y-1)} - \frac{3y+2}{(y+1)(y+1)}$$

$$= \frac{((3y+2) \cdot (y+1)) - ((6) \cdot (y-1))}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2}$$

$$= \frac{3y^2 + 3y + 2y + 2 - 6y + 6}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2}$$

$$= \frac{3y^2 - y + 8}{(y-1)^2(y+1)} - \frac{3y+2}{(y+1)^2}$$

$$= \frac{((3y^2 - y + 8) \cdot (y+1)) - ((3y+2)(y^2 - 2y + 1)}{(y^2-1)^2}$$

$$= \frac{(3y^3 + 3y^2 - y^2 - y + 8y + 8) - (3y^3 - 6y^2 + 3y + 2y^2 - 4y + 2)}{(y^2-1)^2}$$

$$= \frac{3y^3 + 3y^2 - y^2 - y + 8y + 8 - 3y^3 + 6y^2 - 3y - 2y^2 + 4y - 2}{(y^2-1)^2}$$

$$= \frac{6y^2 + 8y + 6}{(y^2-1)^2} = \underline{\underline{\frac{2(3y^2 + 4y + 3)}{(y^2-1)^2}}}$$

Cv 0.4. Řešte rovnici s neznámou $x \in \mathbb{R}$

$$\frac{x+5}{10} - \frac{x-4}{8} = 1$$

$$\frac{((x+5) \cdot (8)) - ((x-4) \cdot (10))}{80} = 1$$

$$\frac{8x + 40 - 10x + 40}{80} = 1$$

$$-2x + 80 = 80$$

$$-2x = 0$$

$$x = \underline{\underline{0}}$$

Cv 0.5. Řešte soustavu dvou rovnic o dvou neznámých $x,y \in \mathbb{R}$

$$3x = -4y + 1$$

$$3y = 4x$$

$$y = \frac{4x}{3}$$

$$3x = -\frac{16x}{3} + 3$$

$$9x = -16x + 3$$

$$25x = 3$$

$$x = \underline{\underline{\frac{3}{25}}}$$

$$y = \frac{\frac{4}{1} \cdot \frac{3}{25}}{\frac{3}{1}} = \frac{12}{25} \cdot \frac{1}{3} = \frac{12}{75} = \underline{\underline{\frac{4}{25}}}$$

Cv 0.6. Řešte nerovnici s neznámou $x \in \mathbb{R}$

$$x^2 + 10 > 7x$$

$$x^2 - 7x + 10 > 0$$

$$D = (-7)^2 - 4 \cdot 1 \cdot 10 = 49 - 40 = 9$$

$$x_{1,2} = \frac{7 \pm 3}{2} \Rrightarrow x_1 = 5, x_2 = 2$$

$$x \in (-\infty, 2) \cup (5, \infty)$$

Hodina 07.10.2022

$$\left(\frac{1}{x+1} - \frac{2x}{x^2-1}\right)\left(\frac{1}{x} - 1\right) = \left(\frac{1}{x+1} - \frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x} - 1\right) =$$

$$= \left(\frac{1}{x+1}\frac{x-1}{x-1} - \frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x} - 1\right) = \left(\frac{x-1}{(x+1)(x-1)} - \frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x} - 1\right) =$$

$$= \left(\frac{-x-1}{(x+1)(x-1)}\right)\left(\frac{1}{x} - \frac{x}{x}\right) = \frac{-x-1}{(x+1)(x-1)} \cdot \frac{1-x}{x} = \frac{-(x+1)}{(x+1)(x-1)} \cdot \frac{1-x}{x} =$$

$$= \frac{-1}{x-1} * \frac{1-x}{x} = \frac{-(1-x)}{(x-1)(x)} = \frac{-(-1)(x-1)}{(x-1)x} = \underline{\underline{\frac{1}{x}}}$$

$$\left(\frac{3}{(x-3)^2} + \frac{1}{x+3} - \frac{6}{x^2-9}\right) \cdot \frac{x^2-6x+9}{2} = \left(\frac{3}{(x-3)(x-3)} + \frac{1}{x+3} - \frac{6}{(x+3)(x-3)}\right) \cdot \frac{(x-3)(x-3)}{2} =$$

$$= \left(\frac{3}{(x-3)(x-3)} + \frac{x-3}{(x+3)(x-3)} - \frac{6}{(x+3)(x-3)}\right) \cdot \frac{(x-3)(x-3)}{2} =$$

$$= \left(\frac{3}{(x-3)(x-3)} + \frac{x-9}{(x+3)(x-3)}\right) \cdot \frac{(x-3)(x-3)}{2} =$$

$$= \left(\frac{3}{(x-3)(x-3)} + \frac{x-9}{(x+3)(x-3)} \cdot \frac{(x-3)}{(x-3)}\right) \cdot \frac{(x-3)(x-3)}{2} =$$

$$= \left(\frac{3}{(x-3)^2} + \frac{(x-9)(x-3)}{(x+3)(x-3)^2}\right) \cdot \frac{(x-3)^2}{2} = \frac{3x+9 + \big((x-9)(x-3)\big)}{(x+3)(x-3)^2} \cdot \frac{(x-3)^2}{2} =$$

$$= \frac{3x+9 + \big((x-9)(x-3)\big)}{2(x+3)} = \frac{3x+9 + (x^2-3x-9x+27)}{2(x+3)} =$$

$$= \underline{\underline{\frac{x^2-9x+36}{2(x+3)}}}$$

$$\frac{x^2+7x}{9-x^2} : \frac{x^2-49}{x+3} = \frac{x(x+7)}{(x+3)(-x+3)} \cdot \frac{x+3}{(x+7)(x-7)} = \underline{\underline{\frac{x}{(x-7)(3-x)}}}$$