Repetitorium matematiky

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

$D = b^2 - 4ac$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$a = \log_b x \Rrightarrow b^a = x$
$\log a + \log b = \log a \cdot b$
$\log a - \log b = \log \frac{a}{b}$
$\log_a b^n = n \cdot \log_a b$
$a^{\log_a b} = b$
$\frac{\log a}{\log b} = \log_b a$
$\log(3+x) = 0 \rightarrow 3 + x = 1$

Test repetitorium

Cv 0.1. *Upravte do základního tvaru:* $\left(\frac{-16}{3}\right)\cdot\sqrt{\frac{2}{3}}+\frac{\frac{64}{9}}{2\cdot\sqrt{\frac{2}{3}}}$

$$\left(\frac{-16}{3}\right)\cdot\sqrt{\frac{2}{3}}+\frac{\frac{64}{9}}{2\cdot\sqrt{\frac{2}{3}}}=-\frac{2^4}{3}\cdot\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}}+\frac{\frac{2^6}{3^2}}{2\cdot\frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}}}=-\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}}+\left(\frac{2^6}{3^2}\cdot\frac{3^{\frac{1}{2}}}{2^{\frac{3}{2}}}\right)=-\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}}+\frac{2^{\frac{9}{2}}}{3^{\frac{3}{2}}}=\underline{\underline{0}}$$

Cv 0.2. *Zjednodušte* $\left(x-\frac{3x}{x+1}\right)\cdot\left(\frac{x-1}{x-2}-\frac{x}{x-1}\right)$

$$\begin{aligned}
\left(x-\frac{3x}{x+1}\right)\cdot\left(\frac{x-1}{x-2}-\frac{x}{x-1}\right)&=\left(x-\frac{3x}{x+1}\right)\cdot\left(\frac{x-1}{x-1}\cdot\frac{x-1}{x-2}-\frac{x}{x-1}\right)=\\
=\left(x-\frac{3x}{x+1}\right)\cdot\left(\frac{(x-1)^2}{(x-2)(x-1)}-\frac{x}{x-1}\right)&=\left(x-\frac{3x}{x+1}\right)\cdot\left(\frac{(x-1)^2-\big(x\cdot(x-2)\big)}{(x-2)(x-1)}\right)=\\
=\left(\frac{(x+1)x}{x+1}-\frac{3x}{x+1}\right)\cdot\left(\frac{(x-1)^2-\big(x^2-2x\big)}{(x-2)(x-1)}\right)&=\\
=\left(\frac{x^2-2x}{x+1}\right)\cdot\left(\frac{x^2-2x+1-x^2+2x}{(x-2)(x-1)}\right)=\left(\frac{x^2-2x}{x+1}\right)\cdot\left(\frac{1}{(x-2)(x-1)}\right)&=\\
=\frac{x(x-2)}{x+1}\cdot\frac{1}{(x-2)(x-1)}=\frac{x}{x+1}\cdot\frac{1}{x-1}=\underline{\underline{\frac{x}{x^2-1}}}&
\end{aligned}$$

Cv 0.3. *Zjednodušte* $\frac{3y+2}{y^2-2y+1}-\frac{6}{y^2-1}-\frac{3y-2}{y^2+2y+1}$

$$\begin{aligned}
\frac{3y+2}{y^2-2y+1}-\frac{6}{y^2-1}-\frac{3y-2}{y^2+2y+1}&=\frac{3y+2}{(y-1)(y-1)}-\frac{6}{(y+1)(y-1)}-\frac{3y+2}{(y+1)(y+1)}\\
&=\frac{((3y+2)\cdot(y+1))-((6)\cdot(y-1))}{(y-1)^2(y+1)}-\frac{3y+2}{(y+1)^2}\\
&=\frac{3y^2+3y+2y+2-6y+6}{(y-1)^2(y+1)}-\frac{3y+2}{(y+1)^2}\\
&=\frac{3y^2-y+8}{(y-1)^2(y+1)}-\frac{3y+2}{(y+1)^2}\\
&=\frac{((3y^2-y+8)\cdot(y+1))-((3y+2)(y^2-2y+1)}{(y^2-1)^2}\\
&=\frac{(3y^3+3y^2-y^2-y+8y+8)-(3y^3-6y^2+3y+2y^2-4y+2)}{(y^2-1)^2}\\
&=\frac{3y^3+3y^2-y^2-y+8y+8-3y^3+6y^2-3y-2y^2+4y-2}{(y^2-1)^2}\\
&=\frac{6y^2+8y+6}{(y^2-1)^2}=\underline{\underline{\frac{2(3y^2+4y+3)}{(y^2-1)^2}}}
\end{aligned}$$

Cv 0.4. *Řešte rovnici s neznámou* $x\in\mathbb{R}$

$$\begin{aligned}
\frac{x+5}{10}-\frac{x-4}{8}&=1\\
\frac{((x+5)\cdot(8))-((x-4)\cdot(10))}{80}&=1\\
\frac{8x+40-10x+40}{80}&=1\\
-2x+80&=80\\
-2x&=0\\
x&=\underline{\underline{0}}
\end{aligned}$$

Cv 0.5. *Řešte soustavu dvou rovnic o dvou neznámých x,$y \in \mathbb{R}$*

$$\begin{aligned} 3x &= -4y + 1 \\ 3y &= 4x \\ y &= \frac{4x}{3} \\ 3x &= -\frac{16x}{3} + 3 \\ 9x &= -16x + 3 \\ 25x &= 3 \\ x &= \underline{\underline{\frac{3}{25}}} \\ y &= \frac{\frac{4}{1} \cdot \frac{3}{25}}{\frac{3}{1}} = \frac{12}{25} \cdot \frac{1}{3} = \frac{12}{75} = \underline{\underline{\frac{4}{25}}} \end{aligned}$$

Cv 0.6. *Řešte nerovnici s neznámou $x \in \mathbb{R}$*

$$\begin{aligned} x^2 + 10 &> 7x \\ x^2 - 7x + 10 &> 0 \\ D = (-7)^2 - 4 \cdot 1 \cdot 10 &= 49 - 40 = 9 \\ x_{1,2} = \frac{7 \pm 3}{2} &\Rrightarrow x_1 = 5, x_2 = 2 \end{aligned}$$

$$x \in (-\infty, 2) \cup (5, \infty)$$

Cv 0.7. *Řešte rovnici s neznámou $x \in \mathbb{R}$*

$$\begin{aligned} 2^x - \frac{1}{2} &= \frac{1}{2} - 2^x \\ 2 \cdot 2^x &= 1 \\ 2^{x+1} &= 2^0 \\ x + 1 &= 0 \\ x &= \underline{\underline{-1}} \end{aligned}$$

Cv 0.8. *Určete definiční obor funkce $f(x)$*

$$f(x) = \frac{\log(2 - x)}{\log(x + 3)}$$

$$\log(x + 3) = 0$$

$$x \neq \{-3, 2\}$$

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$$\left(\frac{1}{x+1}-\frac{2x}{x^2-1}\right)\left(\frac{1}{x}-1\right)=\left(\frac{1}{x+1}-\frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x}-1\right)=$$
$$=\left(\frac{1}{x+1}\frac{x-1}{x-1}-\frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x}-1\right)=\left(\frac{x-1}{(x+1)(x-1)}-\frac{2x}{(x+1)(x-1)}\right)\left(\frac{1}{x}-1\right)=$$
$$=\left(\frac{-x-1}{(x+1)(x-1)}\right)\left(\frac{1}{x}-\frac{x}{x}\right)=\frac{-x-1}{(x+1)(x-1)}\cdot\frac{1-x}{x}=\frac{-(x+1)}{(x+1)(x-1)}\cdot\frac{1-x}{x}=$$
$$=\frac{-1}{x-1}*\frac{1-x}{x}=\frac{-(1-x)}{(x-1)(x)}=\frac{-(-1)(x-1)}{(x-1)x}=\underline{\underline{\frac{1}{x}}}$$

$$\left(\frac{3}{(x-3)^2}+\frac{1}{x+3}-\frac{6}{x^2-9}\right)\cdot\frac{x^2-6x+9}{2}=\left(\frac{3}{(x-3)(x-3)}+\frac{1}{x+3}-\frac{6}{(x+3)(x-3)}\right)\cdot\frac{(x-3)(x-3)}{2}=$$
$$=\left(\frac{3}{(x-3)(x-3)}+\frac{x-3}{(x+3)(x-3)}-\frac{6}{(x+3)(x-3)}\right)\cdot\frac{(x-3)(x-3)}{2}=$$
$$=\left(\frac{3}{(x-3)(x-3)}+\frac{x-9}{(x+3)(x-3)}\right)\cdot\frac{(x-3)(x-3)}{2}=$$
$$=\left(\frac{3}{(x-3)(x-3)}+\frac{x-9}{(x+3)(x-3)}\cdot\frac{(x-3)}{(x-3)}\right)\cdot\frac{(x-3)(x-3)}{2}=$$
$$=\left(\frac{3}{(x-3)^2}+\frac{(x-9)(x-3)}{(x+3)(x-3)^2}\right)\cdot\frac{(x-3)^2}{2}=\frac{3x+9+\big((x-9)(x-3)\big)}{(x+3)(x-3)^2}\cdot\frac{(x-3)^2}{2}=$$
$$=\frac{3x+9+\big((x-9)(x-3)\big)}{2(x+3)}=\frac{3x+9+(x^2-3x-9x+27)}{2(x+3)}=$$
$$=\underline{\underline{\frac{x^2-9x+36}{2(x+3)}}}$$

$$\frac{x^2+7x}{9-x^2}:\frac{x^2-49}{x+3}=\frac{x(x+7)}{(x+3)(-x+3)}\cdot\frac{x+3}{(x+7)(x-7)}=\underline{\underline{\frac{x}{(x-7)(3-x)}}}$$

$$f(a)=\frac{2x+3}{3x+4}$$
$$P_y=\left[0,\frac{3}{4}\right]$$
$$P_x=\left[-\frac{3}{2},0\right]$$
$$2x+3=0$$
$$x=-\frac{3}{2}$$

$$\frac{\frac{2}{3}\cdot\left(x+\frac{4}{3}\right)-\left(\frac{4}{3}-\frac{3}{2}\right)\cdot\frac{2}{3}}{x+\frac{4}{3}}$$