1) $f: \mathbb{R}^2 \to \mathbb{R},\ (x_1, x_2) \mapsto x_1^2 + 0{,}5 x_2^2 + x_1 x_2$

a) $f'(x_1) = 2x_1 + x_2$

$f'(x_2) = x_2 + x_1$

$\hookrightarrow (2x_1 + x_2,\ x_2 + x_1) = \nabla f = C^1 \to$ smooth

b) $\nabla f(1,1) = (2 \cdot 1 + 1,\ 1+1) = \underline{\underline{(3,2)}}$

c) $-\nabla f(1,1) = -(3; 2) = (-3; -2)$

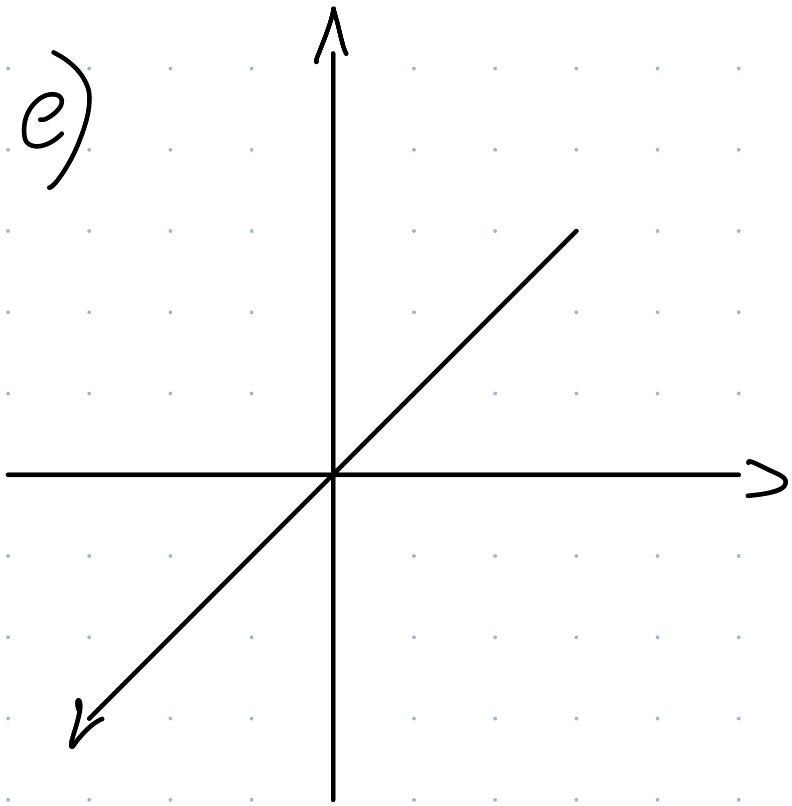
d) $\nabla f(x) \cdot v = 0$

$\Rightarrow (3, 2) \cdot (v_1, v_2) = 0$

$\Rightarrow (3, 2) \cdot (-2, 3) = 0$

$\Rightarrow \quad -6 + 6 = 0 \quad \rightarrow v = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

e)



$f(\tilde{x}(t)) = f(1,1)$

$$\frac{\partial f(\tilde{x}(t))}{\partial t} = \frac{\partial f(x)}{\partial t} = 0$$

Chain-Rule:

$$\frac{\partial f}{\partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial t} = 0$$

$\Rightarrow \nabla f(x)^T$

↳

2

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 0 & 3 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

a)

$$Ab \begin{pmatrix} 1 \cdot 3 + 2 \cdot 0 + 3 \cdot 1 \\ 2 \cdot 3 + 1 \cdot 2 + 1 \cdot 4 \\ 0 \cdot 3 + 2 \cdot 3 + 2 \cdot 1 \\ 1 \cdot 3 + 2 \cdot 1 + 0 \cdot 1 \end{pmatrix} \Rightarrow Ab = \begin{pmatrix} 6 \\ 12 \\ 8 \\ 5 \end{pmatrix}$$

b)

$A^T b$ hält nur wenn A von der form $n \times 3$ wäre

c) $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 0 & 3 & 2 \end{pmatrix}$

$$\left.\begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 4 & 2 & 1 \\ 0 & 3 & 2 & 0 & 3 \end{array}\right. = 2 + 18 - 12 = 8 \neq 0$$

↳ Invertierbar ✓

d) A^{-1}

$$\left(\begin{matrix}1 & 0 & 3\\ 2 & 1 & 4\\ 0 & 3 & 2\end{matrix}\right) \left(\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right) \begin{matrix}\\ -2\cdot \mathrm{I}\\ \\ \end{matrix}$$

$$\Rightarrow \left(\begin{matrix}1 & 0 & 3\\ 0 & 1 & -2\\ 0 & 3 & 2\end{matrix}\right) \left(\begin{matrix}1 & 0 & 0\\ -2 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right) \begin{matrix}\\ \\ -3\cdot \mathrm{II}\end{matrix}$$

$$= \left(\begin{matrix}1 & 0 & 3\\ 0 & 1 & -2\\ 0 & 0 & 8\end{matrix}\right) \left(\begin{matrix}1 & 0 & 0\\ -2 & 1 & 0\\ 6 & -3 & 1\end{matrix}\right) \begin{matrix}\\ \\ :8\end{matrix}$$

$$= \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ \frac{3}{4} & -\frac{3}{8} & \frac{1}{8} \end{pmatrix} \begin{array}{l} -3 \cdot \mathrm{III} \\ +2 \cdot \mathrm{III} \\ \end{array}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{5}{4} & \frac{9}{8} & -\frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} & \frac{2}{8} \\ \frac{3}{4} & -\frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

③

a) $f(x+t(y-x))$

$g(x+t(y-x))$

$$f(x+t(y-x)) + g(x+t(y-x))$$

$$\leq$$

$$\Big(f(x) + t(f(y) - f(x))\Big) + \Big(g(x) + t(g(y) - g(x))\Big)$$

$\Rightarrow (f+g)(x+t(y-x)) \leq (f+g)(x) + t((f+g)(y) - (f+g)(x))$

$\rightarrow$ konvex

b) proof: $g \circ f \rightarrow$ konvex, $h = g \circ f$

$$\Rightarrow h(x+t(y-x)) = g(f(x+t(y-x)))$$
$$\leq g(f(x) + t(f(y) - f(x)))$$
$$\leq g(f(x)) + t(g(f(y)) - g(f(x)))$$

$$= h(x) + t(h(y) - h(x))$$

4) $f: \mathbb{R}^2 \to \mathbb{R},\ (x_1, x_2) \mapsto \exp(\pi \cdot x_1) - \sin(\pi \cdot x_2) + \pi \cdot x_1 \cdot x_2$

a)

$$\frac{\partial f}{\partial x_1} = \pi \cdot \exp(\pi \cdot x_1) + \pi \cdot x_2 \Rightarrow \pi \cdot (\exp(\pi \cdot x_1) + x_2)$$

$$\frac{\partial f}{\partial x_2} = -\cos(\pi \cdot x_2) \cdot \pi + \pi \cdot x_1 = \pi \cdot (-\cos(\pi \cdot x_2) + x_1)$$

$$\nabla f = \pi \cdot \left(\exp(\pi \cdot x_1) + x_2,\ -\cos(\pi \cdot x_2) + x_1\right)$$

b) $$\frac{\partial^2 f}{\partial x_1} = \pi \cdot \left(\pi \cdot \exp(\pi \cdot x_1)\right) \Rightarrow \pi^2 \cdot \exp(\pi \cdot x_1)$$

$$\frac{\partial^2 f}{\partial x_2} = \tilde{\pi}^1 \cdot \sin(\pi \cdot x_2)$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \tilde{\pi}$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = \pi$$

$$\nabla^2 f(x) = \begin{pmatrix} \pi^2 \cdot \exp(\tilde{\pi} \cdot x_1) & \pi \\ \tilde{\pi} & \tilde{\pi}^2 \cdot \sin(\pi \cdot x_2) \end{pmatrix}$$

$$\Rightarrow \pi \cdot \begin{pmatrix} \pi \cdot \exp(\pi \cdot x_1) & 1 \\ 1 & \pi \cdot \sin(\pi \cdot x_2) \end{pmatrix}$$

c) $T_{1,a}(x) \qquad a = (0, 1)$

$$F(x) = f(a) + \nabla f(a)^T (x - a)$$

$$= 1 + (2\pi, \pi) \cdot (x_1, x_2 - 1)$$

$$= 1 + 2\pi x_1 + \pi x_2 - \pi$$

d) $T_2(x,a)$

$$= f(a) + \nabla f(a)^T (x-a) + \frac{1}{2}(x-a)^T H(a)(x-a)$$

$$\Rightarrow \frac{1}{2}(x_1, x_2 - 1) \cdot \pi \begin{pmatrix} \pi & 1 \\ 1 & 0 \end{pmatrix} (x_1, x_2 - 1)$$

$$= \left(\frac{1}{2}x_1, \frac{1}{2}x_2 - 0{,}5\right) \cdot \underbrace{\begin{pmatrix} \pi^2 & \pi \\ \pi & 0 \end{pmatrix} (x_1, x_2 - 1)}$$

$$\Rightarrow \left(\frac{1}{2}x_1, \frac{1}{2}x_2 - 0{,}5\right) \cdot \begin{pmatrix} \pi^2 x_1 & \pi x_2 - \pi \\ \pi x_1 & 0 \end{pmatrix}$$

$$\Rightarrow \frac{1}{2}x_1 \cdot \left(\pi^2 x_1 + \pi x_2 - \pi\right) + \left(\frac{1}{2}x_2 - 0{,}5\right) \cdot \left(\pi x_1\right)$$

$$\Rightarrow \frac{1}{2}\pi^2 x_1^2 + \frac{1}{2}x_1 \pi x_2 - \frac{1}{2}\pi x_1 + \frac{1}{2}\pi x_1 x_2 - \frac{1}{2}\pi x_1$$

$$= \frac{1}{2}\pi^2 x_1^2 + \pi x_1 x_2 - \pi x_1$$

$$\Rightarrow 1 + 2\pi x_1 + \pi x_2 - \pi + \frac{1}{2}\pi^2 x_1^2 + \pi x_1 x_2 - \pi x_1$$

$$= 1 + \pi x_1 + \pi x_2 - \pi + \frac{1}{2}\pi^2 x_1^2 + \pi x_1 x_2$$