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a) $\dfrac{\partial x^T x}{\partial x}$

$\Rightarrow x^T x = x_1^2 + x_2^2 + x_3^2$

$\Rightarrow = 2x_1 + 2x_2 + 2x_3 = 2x$

b) $\dfrac{\partial \|x-c\|_2^2}{\partial x} \rightarrow$

$u = x - c$

$x = [x_1, x_2, x_2]^T$

$c = [1, 2, 3]^T$

$\| u \|_2 = \sqrt{\sum_{i=1}^{n} (u)^2}$

$\| u \|_2^2 = \sum_{i=1}^{n} (u)^2$

$\dfrac{\partial \|u\|_2^2}{\partial x} = u_1^2 + u_2^2 + u_3^2 = u^T u = (x-c)^T (x-c)$

c) $\frac{\partial \|x-c\|_2}{\partial x} = \nearrow \sqrt{\sum_{i=1}^{n} (x_i - c_i)^2}$

$$\sqrt{\sum_{i=1}^{n} (x_i-c_i)^2} = \|x-c\|_2 = \sqrt{(x-c)^T(x-c)} = \sqrt{(x-c)^2}$$

$$\Rightarrow \frac{\partial \sqrt{(x-c)^2}}{\partial x} \Rightarrow f(x) = \left((x-c)^2\right)^{\frac{1}{2}}$$

$$\left.\begin{aligned} \hookrightarrow g(u) &= u^{\frac{1}{2}} \\ h(x) &= (x-c)^2 \end{aligned}\right\} f(x) = g(h(x))$$

-> Kettenregel

$$\frac{\partial g}{\partial u} \cdot \frac{\partial h}{\partial x} = \frac{1}{2} u^{\frac{1}{2}} \cdot (2(x-c))$$

$$= \frac{1}{2} \frac{1}{u} \cdot (2(x-c))$$

$$= \frac{1}{2} \frac{1}{\sqrt{(x-c)^2}} \cdot 2(x-c)$$

$$= \frac{x-c}{\sqrt{(x-c)^2}} = = \frac{x-c}{\|x-c\|_2}$$

3 $f: [-1,2] \mapsto \mathbb{R}; \quad x \mapsto \exp(x^3 - 2x^2)$

a)
$$f'(x) = (3x^2 - 4x) \cdot \exp(x^3 - 2x^2)$$

c) $f'(x) = 0$

$$\hookrightarrow \underbrace{(3x^2 - 4x)}_{0} \cdot \underbrace{\exp(x^3 - 2x^2)}_{>0} = 0$$

$$\Rightarrow 3x^2 - 4x = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(-4)^2 + 4 \cdot 3 \cdot 0}}{6} \rightarrow x_1 = \frac{4}{3}$$

$$x_2 = 0$$

d) $f''(x) = (6x-4) \cdot \exp(x^3-2x^2)$

$+ (3x^2-4x) \cdot (3x^2-4x) \cdot \exp(x^3-2x^2)$

$\Rightarrow (6x-4) \cdot \exp(x^3-2x^2)$

$+ (3x^2-4x)^2 \cdot \exp(x^3-2x^2)$

$f''(x) = \exp(x^3-2x^2) \cdot \left((3x^2-4x)^2 + (6x-4)\right)$

e) $f''(\frac{4}{3}) \approx 1{,}223 > 0 \rightarrow$ konkav (nach oben gekrümmt
↳ Lokales Minimum

$f''(0) = -4 < 0 \rightarrow$ konvex (nach unten gekrümmt)
↳ Lokales Maximum

f) $x_1 = \frac{4}{3}$, $x_2 = 0$, boundry Points: $x_3 = -1$, $x_4 = 2$

$f(x_1) = \exp(\frac{4}{3}^3 - 2 \cdot \frac{4}{3}^2) = 0{,}3056$

$f(x_2) = \exp(0) = 1$

$f(x_3) = \exp((-1)^3 - 2 \cdot (-1)^2) = 0{,}04978$

$$f(x_4) = \exp(2^3 - 2 \cdot 2^2) = 1$$

global max : x_2 & x_4

global min : x_3