① Explain the concept of homogeneous coordinates and their role in 2D transformations.

$$\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \stackrel{def}{=} \begin{bmatrix} ax \\ ay \\ a \end{bmatrix}$$

Standard image coordinates → homogeneous coordinates

how to convert them back

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} x_1/x_3 \\ x_2/x_3 \end{bmatrix}$$

homog. coord. → Standard coord.

→ They represend 2D points with 3D dimensions

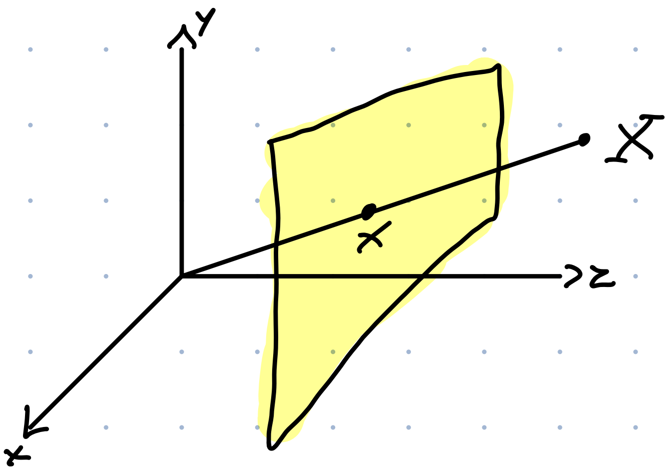
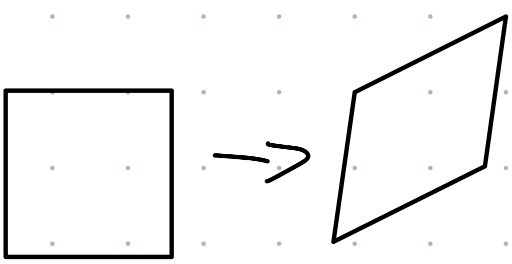


image point in standard coord. $x = \begin{bmatrix} x_1/x_3 \\ x_2/x_3 \end{bmatrix}$

image point in homogeneous coordinates $\underline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

x is a projection of a point $\underline{X}$ on the image plane

② Describe the properties of affine transformations. How do they affect lines, parallel lines, and ratios?



Properties:

- lines map to lines
- parallel lines map to parallel lines
- ratios of segments within a line are preserved
- compositions of affine transformations are affine transforms

③ Outline the steps involved in determining an unknown affine transformation given two sets of corresponding points.

Affine transform:

uniform scaling + shearing + rotation + translation

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 1 \end{bmatrix}$$

unknowns

$$x' = Mx$$

point correspondences

1. Find pairs of corresponding points

$$(x_1, x_1')$$
$$(x_2, x_2')$$
$$\vdots$$

2. Write down an Objective:

$$\min_M \sum_i \|x' - Mx\|^2$$

$$Ax = b$$

3. Least squares:

$$x = (A^T A)^{-1} A^T b$$

Affine transformation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$