① $f: \mathbb{R} \mapsto \mathbb{R}, \; x \mapsto -\frac{1}{2}\cos(x)\exp\left(\frac{1}{4}(x+2)^2\right)+3$

$x \in [-2, 2]$, $\varphi = 0{,}618$

b)

$x_{left} = (-2 ; 3{,}21)$ $\quad x_{right} = (2 ; 14{,}30)$

$x_{best} = x_{right} - \varphi(x_{right} - x_{left})$ $\quad = x_{left} + (1-\varphi)(x_{right} - x_{left})$

$x_{new} = x_{right} - (1-\varphi)(x_{right} - x_{left})$

1. Iteration

$x_{best} = -2 + (1-\varphi)\cdot(2-(-2))$

$\approx -0{,}472$

$f(x_{best}) \approx 2{,}20$

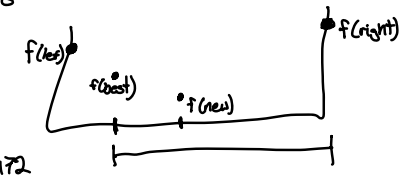
$x_{new} = 2 - (1-\varphi)\cdot(2-(-2))$

$= 0{,}472$

$f(x_{new}) \approx 0{,}948$

$F(X_{best}) > f(x_{new})$

$\rightarrow$ new $x_{best} = x_{new} = 0{,}472$



$\hookrightarrow [x_{best} ; x_{right}]$

$\hookrightarrow [-0{,}472 ; 2]$

2. Iteration

$x_{best} = -0{,}472 + (1-\varphi)(2-(-0{,}472))$

$\approx 0{,}472$

$f(x_{best}) \approx 0{,}948$

$x_{new} = 2 - (1-\varphi)\cdot(2-(-0{,}472))$

$\approx 1{,}06$

$f(x_{new}) = 0{,}460$

$\hookrightarrow f(x_{best}) > f(x_{new}) \rightarrow [0{,}472 , 2]$

$\hookrightarrow$ new $x_{best} = x_{new} = 1{,}06$

c) $f(x) = ax^2 + bx + c$

$P_1(-2, 3{,}21)$, $P_2(2, 14{,}36)$, $P_3(-0{,}47, 2{,}2)$

P_1:

$3{,}21 = 4a - 2b + c$

$\Rightarrow c = -4a + 2b + 3{,}21$

P_2:

$2{,}2 = 0{,}2209a - 0{,}47b - 4a + 2b + 3{,}21 \quad | -2{,}2$

$0 = -3{,}78a + 1{,}53b + 1{,}01 \quad | +3{,}78a$

$3{,}78a = 1{,}53b + 1{,}01 \quad | :3{,}78$

$a = \frac{1{,}53b}{3{,}78} + \frac{1{,}01}{3{,}78} = 0{,}4b + 0{,}26$

P_3:

$14{,}36 = 4a + 2b + c$

$\Rightarrow 14{,}36 = 4\cdot(0{,}4b + 0{,}28) + 2b - 4\cdot(0{,}4b + 0{,}28) + 2b + 3{,}21$

$\Rightarrow 14{,}36 = 4b + 3{,}21 \quad | -14{,}36$

$\Rightarrow 0 = 4b - 11{,}15 \quad | -4b$

$\Rightarrow -4b = -11{,}15 \quad | :-4$

$\Rightarrow b = 2{,}79$

$c = -4\cdot 1{,}38 + 2\cdot 2{,}79 + 3{,}21 = 3{,}27$

$0{,}4\cdot 2{,}79 + 0{,}26 = 1{,}38 = a$

$f(x) = 1{,}38x^2 + 2{,}79x + 3{,}27$

1. Iteration $f(x) = 1{,}38x^2 + 2{,}79x + 3{,}27$

$f'(x) = 2{,}76x + 2{,}79$

$f''(x) = 2{,}76$

$f'(x) = 0$

$\Rightarrow 2{,}76x + 2{,}79 = 0 \quad | -2{,}79 \quad | :2{,}76$

$\Rightarrow \quad x_{new} = -1,01$

$f(x_{best}) < f(x_{new})$ -> keine änderung von x_{best}

$x_{new} < x_{best}$

↳ $x_{left} = x_{new}$ neues Intervall $[-1,01; 2]$

$x_{best} = -0,47$

2. Iteration

$x_{left} \Rightarrow 1,02a - 1,01b + c = 2,65$

$\Rightarrow -1,02a + 1,01b + 2,65 = c \quad -> -1,02 \cdot 1,9 + 1,01 \cdot 2 + 2,65 = 2,7 = c$

$x_{best} \Rightarrow 0,22a - 0,47b - 1,02a + 1,01b + 2,65 = 2,2 \quad | -2,2$

$\Rightarrow -0,8a + 0,54b + 0,45 = 0 \quad | + 0,8a \; | : 0,8$

$\Rightarrow 0,67b + 0,56 = a \quad -> 0,67 \cdot 2 + 0,56 = a = 1,9$

$x_{right} \Rightarrow 4 \cdot (0,67b + 0,56) + 2b - 1,02 \cdot (0,67b + 0,56) + 1,01b + 2,65$

$\Rightarrow 2,68b + 2,24 + 2b - 0,68b - 0,57 + 1,01b + 2,65$

$\Rightarrow 5,01b + 4,32 = 14,36 \quad | -4,32 \; | : 5,01$

$\Rightarrow b = 2$

↳ $f(x) = 1,9x^2 + 2x + 2,7$

$f'(x) = 3,8x + 2$

$f'(x) = 0$

$\Rightarrow 3,8x + 2 = 0 \quad | -2 \; | : 3,8$

$x_{new} = -0,52$

$f(x_{new}) = 2,24 > f(x_{best}) = 2,2$ -> keine änderung

$x_{new} < x_{best}$ -> $x_{new} = x_{left}$

↳ neues Intervall = $[-0,52; 2]$ mit $x_{best} = -0,47$