① $f: \mathbb{R}^2 \mapsto \mathbb{R}, (x_1, x_2) \mapsto -\cos(x_1^2 + x_2^2 + x_1 x_2)$

b) $\frac{\partial f}{\partial x_1} = (2x_1 + x_2) \cdot \sin(x_1^2 + x_2^2 + x_1 x_2)$

$\frac{\partial f}{\partial x_2} = (2x_2 + x_1) \cdot \sin(x_1^2 + x_2^2 + x_1 x_2)$

$\hookrightarrow \nabla f = \begin{pmatrix} (2x_1 + x_2) \cdot \sin(x_1^2 + x_2^2 + x_1 x_2) \\ (2x_2 + x_1) \cdot \sin(x_1^2 + x_2^2 + \cancel{x_1 x_2)} \end{pmatrix}$

c) $\frac{\partial^2 f}{\partial x_1 \partial x_1} = 2\sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_1 \cdot x_2) \cdot (2x_1 \cdot x_2) \cdot (\cos(x_1^2 + x_2^2 + x_1 x_2)$

$\Rightarrow 2\sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_1 \cdot x_2)^2 \cdot \cos(x_1^2 + x_2^2 + x_1 x_2)$

$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_1 + x_2) \cdot (2x_2 + x_1) \cdot \cos(x_1^2 + x_2^2 + x_1 x_2)$

$\frac{\partial^2 f}{\partial x_2 \partial x_2} = 2\sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_2 + x_1)^2 \cdot \cos(x_1^2 + x_2^2 + x_1 x_2)$

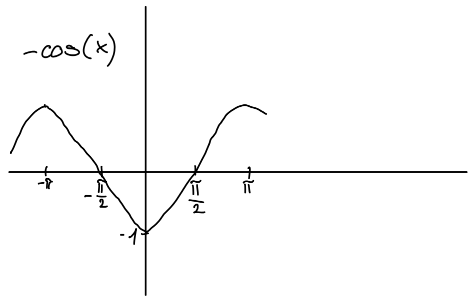
$\frac{\partial^2 f}{\partial x_2 \partial x_1} = \sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_2 + x_1) \cdot (2x_1 + x_2) \cdot \cos(x_1^2 + x_2^2 + x_1 x_2)$

$\hookrightarrow \nabla^2 f = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2 \partial x_2} \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2\sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_1 + x_2)^2 \cdot \cos(x_1^2 + x_2^2 + x_1 x_2) & \sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_1 - x_2)(2x_2 + x_1) \cdot \cos(x_1^2 + x_2^2 + x_1 x_2) \\ \sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_2 + x_1)(2x_1 + x_2) \cdot \cos(x_1^2 + x_2^2 + x_1 x_2) & 2\sin(x_1^2 + x_2^2 + x_1 x_2) + (2x_2 + x_1)^2 \cdot \cos(x_1^2 + x_2^2 + x_1 x_2) \end{pmatrix}$

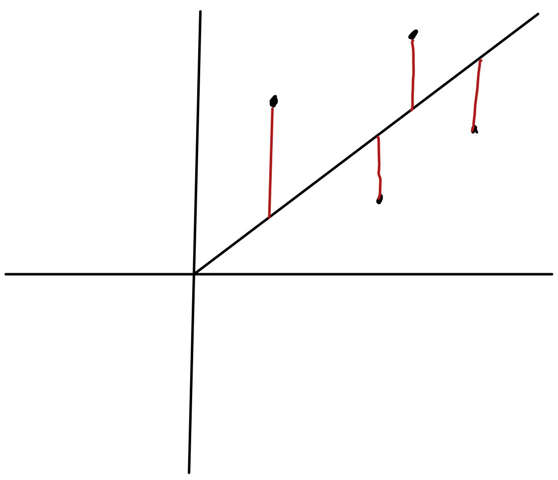
d) $S_r = \{ (x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 + x_1 x_2 < r \}$

$\rightarrow r \in \mathbb{R}, r > 0$



② $x_k = (0{,}7; 1{,}0), (0{,}8; 0{,}2), (1{,}5; 1{,}4)$
$(1{,}6; 1{,}5), (2{,}0; 1{,}8)$

a) $\min\limits_{\theta \in \mathbb{R}^2} \sum\limits_{i=1}^{n} \left(\theta^T (x^i - y^i)^2\right)$



$\Rightarrow mx^i + t = y^i$

$\theta_0 x^i + \theta_1 = y^i$

$\theta = \begin{pmatrix} \theta_0 \\ \theta_1 \end{pmatrix}$ $\quad x^i = \begin{pmatrix} x^i \\ 1 \end{pmatrix}$

$\Rightarrow \min\limits_{\theta \in \mathbb{R}^2} \sum\limits_{i=1}^{5} \left(\theta^T x^i - y^i\right)^2$

b)

$X = (x^1, \dots, x^k)^T$ , $y = \begin{pmatrix} y^1 \\ \vdots \\ y^k \end{pmatrix}$

$\hookrightarrow X = \begin{pmatrix} 0{,}7 & 0{,}8 & 1{,}5 & 1{,}6 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}^T$ , $y = \begin{pmatrix} 1 \\ 0{,}2 \\ 1{,}4 \\ 1{,}5 \\ 1{,}8 \end{pmatrix}$

$\min\limits_{\theta \in \mathbb{R}^2} \sum\limits_{i=1}^{5} \left(\theta^T x^i - y^i\right)^2 \Rightarrow (X\theta - y)^T (X\theta - y)$

Ableiten nach $\theta$:

$$2X^T (X\theta - y)^T = 0$$
$$X^T X \theta - X^T y = 0$$
$$\theta = (X^T X)^{-1} X^T y$$

$X\theta - \vec{y}$

$\begin{pmatrix} 0{,}7 ; 1 \\ 0{,}8 ; 1 \\ 1{,}5 ; 1 \\ 1{,}6 ; 1 \\ 2{,}0 ; 1 \end{pmatrix} \cdot \begin{pmatrix} \theta_0 \\ \theta_1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0{,}2 \\ 1{,}4 \\ 1{,}5 \\ 1{,}8 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} \quad \Rightarrow (X\theta - y)^T (X\theta - y) = (a_1, a_2, a_3, a_4, a_5) \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix}$

$= a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2$

3) $c = \begin{pmatrix} 2,0 \\ 1,0 \end{pmatrix}$

$\min_{x \in \mathbb{R}^2} f(x) = x^T c$

$mx + t = y$

rot) $x_2 \geq -1x_1 + 1$ $\quad \rightarrow x_2 \geq -1x_1 + 1 \quad | +x_1$

$\Rightarrow \underset{1}{x_1} + \underset{1}{x_2} \geq 1$

blau) $x_2 \geq 8x_1 - 12$ $\quad \rightarrow x_2 \geq 8x_1 - 12 \quad | -8x_1$

$\Rightarrow \underset{-8}{-8x_1} + \underset{1}{x_2} \geq -12$

grün) $x_2 \leq 2x_1 + 2$ $\quad \rightarrow x_2 \leq 2x_1 + 2 \quad | -2x_1$

$\underset{-2}{-2x_1} + \underset{-1}{x_2} \leq 2$ $\quad \rightarrow ?\ 2x - x_2 \geq -2$

gelb) $x_2 \leq \frac{2}{3}x_1 + \frac{8}{3}$ $\quad \rightarrow x_2 \leq \frac{2}{3}x_1 + \frac{8}{3} \quad | -\frac{2}{3}x_1$

$\underset{-\frac{2}{3}}{-\frac{2}{3}x_1} + \underset{-1}{x_2} \leq \frac{8}{3}$ $\quad \rightarrow ?\ \frac{2}{3}x_1 - x_2 \geq -\frac{8}{3}$

$A = \begin{pmatrix} 1 & 1 \\ -8 & 1 \\ 2 & -1 \\ \frac{2}{3} & -1 \end{pmatrix}$ $\quad b = \begin{pmatrix} 1 \\ -12 \\ -2 \\ -\frac{8}{3} \end{pmatrix}$