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# Constructing a Neuron using Column Generation

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## Abstract

We consider the problem of describing neurons in single neuron images. To do this we map the problem to a problem of constructing a lineage to one

## 1 Formulation

### 1.1 Tracks

- We describe the set of detections as  $\mathcal{D}$  which we index with  $d$ .
- We describe the set of tracks as  $\mathcal{P}$  which we index with  $p$ .
- We use  $X \in \{0, 1\}^{|\mathcal{D} \times \mathcal{P}|}$  to denote a mapping of detections to tracks where  $X_{dp} = 1$  indicates that track  $d$  is associated with track  $p$ .
- A track consists of a sequence of sub-tracks each of  $k$  detections where  $k$  is a user defined hyper-parameter that trades off model complexity and efficiency of inference.
- We use  $\bar{X} \in \{0, 1\}^{|\mathcal{D} \times \mathcal{P}|}$  to denote a mapping of detections to tracks where  $X_{dp} = 1$  indicates that track  $d$  is associated with track  $p$  and detection  $d$  is not in the first  $k - 1$  detections on the track. .
- We define the set of subtracks as  $\mathcal{S}$  which we index with  $s$ . A given subtrack has elements  $\{s_1, s_2, s_3 \dots s_k\}$  ordered in time from earliest to latest.
- We define a mapping of tracks to subtracks using a matrix  $S^0 \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$ . We use  $S_{sp}^0 = 1$  to indicate that track  $p$  contains subtrack  $s$ .
- In order to describe the first subtrack on a track we use matrix  $S^+ \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$  which we index with  $s, p$  where  $S_{sp}^+ = 1$  if and only if subtrack  $s$  is the first subtrack on track  $p$ .
- In order to describe the first subtrack on a track we use matrix  $S^- \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$  which we index with  $s, p$  where  $S_{sp}^- = 1$  if and only if subtrack  $s$  is the first subtrack on track  $p$ .
- The subtracks in the sequence that describes a track overlap eachother. Hence if a sub-tracks  $s_1$  is succeeded by another subtrack  $s_2$  on a given track then the final  $k - 1$  elements on  $s_1$  are the same as the earliest  $k - 1$  elements in  $s_2$ .
- We use  $Q \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{S}|}$  which we index by  $s_1, s_2$  respectively. We set  $Q_{s_1 s_2} = 1$  if and only if subtrack  $s_1$  can succeed  $s_2$ .

### 1.2 Costs

We associate tracks with costs with costs using the following notation. We use  $\Theta \in \mathbb{R}^{\mathcal{P}}$  which we index by  $p$  to associate tracks with costs. We use  $\Theta_p$  to associate track  $p$  with a cost.

- We use  $\theta \in \mathbb{R}^{|\mathcal{S}|, 3}$  which we index by  $s/[+, -, 0]$  respectively.
- We use  $\theta_{s+}$  to denote the cost of starting a track at subtrack  $s$ .

- We use  $\theta_{s-}$  to denote the cost of terminating a track at subtrack  $s$ .
- We use  $\theta_{s0}$  to denote the cost of including a subtrack  $s$  in a track .

We associate a track with cost with cost  $\Theta_p$  as follows.

$$\Theta_p = \sum_{s \in \mathcal{S}} S_{sp}^+ \theta_{s+} + S_{sp}^- \theta_{s-} + S_{sp}^0 \theta_{s0} \quad (1)$$

### 1.3 Collection of Tracks

We describe a collection of tracks that describe a neuron using a vector  $\gamma \in \{0, 1\}^{|\mathcal{P}|}$  which we index with  $p$ . We set  $\gamma_p = 1$  if and only if track  $p$  is included in the neuron.

We use  $\Gamma$  to describe the set of all possible neurons. This is a subset of  $\gamma \in \{0, 1\}^{|\mathcal{P}|}$ . The cost associated with a neuron described by  $\gamma$  is defined by the sum of the tracks that compose it. The selection of the lowest cost neuron is thus written below

$$\min_{\gamma \in \Gamma} \sum_p \gamma_p \Theta_p \quad (2)$$

### 1.4 Feasibility

We now define what makes a neuron feasible. A neuron consists of a sequence of tracks.