
Constructing a Neuron using Column Generation

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Abstract

We consider the problem of describing neurons in single neuron images. To do this we map the problem to a problem of constructing a lineage to one

1 Formulation

1.1 Tracks

- We describe the set of detections as \mathcal{D} which we index with d .
- We describe the set of tracks as \mathcal{P} which we index with p .
- We use $X \in \{0, 1\}^{|\mathcal{D} \times \mathcal{P}|}$ to denote a mapping of detections to tracks where $X_{dp} = 1$ indicates that track d is associated with track p .
- A track consists of a sequence of sub-tracks each of k detections where k is a user defined hyper-parameter that trades off model complexity and efficiency of inference.
- We use $\bar{X} \in \{0, 1\}^{|\mathcal{D} \times \mathcal{P}|}$ to denote a mapping of detections to tracks where $X_{dp} = 1$ indicates that track d is associated with track p and detection d is not in the first $k - 1$ detections on the track. .

1.2 Sub-Tracks

- We define the set of subtracks as \mathcal{S} which we index with s . A given subtrack has elements $\{s_1, s_2, s_3 \dots s_k\}$ ordered in time from earliest to latest.
- We use $F \in \{0, 1\}^{|\mathcal{D}| \times |\mathcal{S}|}$ which we index by d, s respectively. We set $F_{ds} = 1$ if and only if detection d is in subtrack s .
- We use $F^- \in \{0, 1\}^{|\mathcal{D}| \times |\mathcal{S}|}$ which we index by d, s respectively. We set $F_{ds}^- = 1$ if and only if detection d is the final detection on subtrack s .
- We define a mapping of tracks to subtracks using a matrix $S^0 \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$. We use $S_{sp}^0 = 1$ to indicate that track p contains subtrack s as neither the start nor the end.
- In order to describe the first subtrack on a track we use matrix $S^+ \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$ which we index with s, p where $S_{sp}^+ = 1$ if and only if subtrack s is the first subtrack on track p .
- In order to describe the first subtrack on a track we use matrix $S^- \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{P}|}$ which we index with s, p where $S_{sp}^- = 1$ if and only if subtrack s is the first subtrack on track p .
- The subtracks in the sequence that describes a track overlap each other. Hence if a sub-track s_1 is succeeded by another subtrack s_2 on a given track then the final $k - 1$ elements on s_1 are the same as the earliest $k - 1$ elements in s_2 .
- We use $Q \in \{0, 1\}^{|\mathcal{S}| \times |\mathcal{S}|}$ which we index by s_1, s_2 respectively. We set $Q_{s_1 s_2} = 1$ if and only if subtrack s_1 can succeed s_2 .

1.3 Costs

We associate tracks with costs with costs using the following notation. We use $\Theta \in \mathbb{R}^{\mathcal{P}}$ which we index by p to associate tracks with costs. We use Θ_p to associate track p with a cost.

- We use $\theta \in \mathbb{R}^{|\mathcal{S}|,3}$ which we index by $s/[+, -, 0]$ respectively.
- We use θ_{s+} to denote the cost of starting a track at subtrack s .
- We use θ_{s-} to denote the cost of terminating a track at subtrack s .
- We use θ_{s0} to denote the cost of including a subtrack s in a track as neither the start nor the end.

We associate a track with cost with cost Θ_p as follows.

$$\Theta_p = \sum_{s \in \mathcal{S}} S_{sp}^+ \theta_{s+} + S_{sp}^- \theta_{s-} + S_{sp}^0 \theta_{s0} \quad (1)$$

1.4 Collection of Tracks

We describe a collection of tracks that describe a neuron using a vector $\gamma \in \{0, 1\}^{|\mathcal{P}|}$ which we index with p . We set $\gamma_p = 1$ if and only if track p is included in the neuron.

We use Γ to describe the set of all possible neurons. This is a subset of $\gamma \in \{0, 1\}^{|\mathcal{P}|}$. The cost associated with a neuron described by γ is defined by the sum of the tracks that compose it. The selection of the lowest cost neuron is thus written below

$$\min_{\gamma \in \Gamma} \sum_p \gamma_p \Theta_p \quad (2)$$

1.5 Feasibility

We assume that the soma is defined by a subtrack s_0 which is a special subtrack that initializes the neuron.

A track is included or not included : $\gamma_p \in \{0, 1\}$

No two tracks can continue through a given detection

$$\sum_p \gamma_p \hat{X}_{dp} \leq 1 \quad \forall d \quad (3)$$

A detection can not be part of more than two tracks. This blocks succession in close proximity.

$$\sum_p \gamma_p X_{dp} \leq 2 \quad \forall d \quad (4)$$

A track can not split off a subtrack unless that sub-track is already on a track.

$$\sum_p \sum_s Q_{ss_1} S_{s_1 p}^+ \gamma_p \leq \sum_p S_{sp}^0 \gamma_p \quad (5)$$

If a track terminates at a given detection then no detections can start off it. A strong penalty for ending a track early negates the need for this. Since this strong penalty has been described I ignore this constraint in the document.

$$\sum_p \gamma_p \sum_s (F_{ds} - F_{ds}^-) \leq (1 - \sum_p \gamma_p \sum_s S_p^- F_{ds}^-) \quad (6)$$

2 LP relaxation

$$\begin{aligned} \min_{\gamma \geq 0} \quad & \sum_p \gamma_p \Theta_p \\ \text{s.t.} \quad & \sum_p \gamma_p \hat{X}_{dp} \leq 1 \\ & \sum_p \gamma_p X_{dp} \leq 2 \\ & \sum_p \sum_s Q_{ss_1} S_{s_1 p}^+ \gamma_p \leq \sum_p S_{sp}^0 \gamma_p \end{aligned} \quad (7)$$

We now take the dual form of this. We use lagrange multipliers $\lambda^1 \in \mathbb{R}_{0+}^{|\mathcal{D}|}, \lambda^2 \in \mathbb{R}_{0+}^{|\mathcal{D}|}, \lambda^3 \in \mathbb{R}_{0+}^{|\mathcal{S}|}$ to respreset nteh constraints above in dual form.

$$\max_{\substack{\lambda^1 \geq 0 \\ \lambda^2 \geq 0 \\ \lambda^3 \geq 0}} - \sum_d (\lambda_d^1 + 2\lambda_d^2) \quad (8)$$

$$\Theta_p + \sum_d \hat{X}_{dp} \lambda_d^1 + \sum_d X_{dp} \lambda_d^2 - \sum_s (S_{sp}^0) \lambda_s^3 + \sum_{s_1} \lambda_{s_1}^3 \sum_{s_2} Q_{s_1 s_2} S_{s_2 p}^+ \geq 0 \quad (9)$$

Finding the most violated constraint is a dynamic program. Many constraints can be generated at once.