

Signs of S_0 : φ , $*$, N .

Preliminary definitions: (1) By an *expression* (of S_0) we mean any string built from the three signs of S_0 . (2) By the (formal) quotation of an expression, we mean the expression surrounded by stars. (3) By the *norm* of an expression, we mean the expression followed by its own (formal) quotation.

Formation rules for (individual) designators

- (1) The quotation of any expression is a designator.
- (2) If E is a designator, so is ' NE ' (i.e., ' N ' followed by E).

Alternative definition

'(1)' A designator is an expression which is either a quotation (of some other expression) or a quotation preceded by one or more ' N 's.

Rules of designation in S_0

- R1. The quotation of an expression E designates E .
- R2. If E_1 designates E_2 , then ' NE_1 ' designates the *norm* of E_2 .

Definition of a sentence of S_0

- (1) A sentence of S_0 is an expression consisting of ' φ ' followed by a designator.

The semantical system S_P

For any property P , we define the semantical system S_P as follows:

- (1) The rules for designators, designation and sentence formation in S_P are the same as in S_0 .
 - (2) The rule of truth for S_P is the following:
- R3. For any designator E , ' φE ' is true in S_P iff the expression designated by E (in S_P) has the property P .

THEOREM 2.1. There exists an expression of S_0 , which designates itself.

PROOF. ' $*N*$ ' designates ' N ' [By Rule 1].

Hence ' $N*N*$ ' designates the norm of ' N ' [By Rule 2] which is ' $N*N*$ '.

Thus ' $N*N*$ ' designates itself.

THEOREM 2.2. There exists a sentence G of S_0 , such that for any property P , G is true in $S_P \iff G$ has the property P .

PROOF. ' $N*\varphi N*$ ' designates ' $\varphi N*\varphi N*$ ' [By R1 and R2].

Thus G , viz., ' $\varphi N*\varphi N*$ ' is our desired sentence.

REMARK. G is, of course, the formalized version of ' W contains the norm of ' W contains the norm of.' ' φ ' is but an abbreviation of ' W contains,' and ' N ' abbreviates 'the norm of.'

⁷ If we wished to construct a miniature system L_P which formalizes the diagonal function in the same way as S_P does the norm function, we take four signs, viz., ' φ ', ' $*$ ', ' D ', ' x ', and the rules R_1 , (same as S_P), R_2 : If E_1 designates E_2 , then ' DE_1 ' designates the diagonalization of E_2 (i.e., the result of replacing each occurrence of ' x ' in E_2 by the quotation of E_2), R_3 : If E_1 designates E_2 , then ' φE_1 ' is a sentence and is true in L_P if and only if E_2 has the property P . Then the expression of Theorem 2.1, which designates itself, is ' $D*Dx*$ ', and the Tarski sentence (of Theorem 2.2) for P is ' $\varphi D*\varphi Dx*$ '.