Example 3.2 The Boolean Algebra of Propositions

The set of propositions gives rise to a Boolean algebra, with the roles of 0, 1, $+$, $\cdot$ and $'$ taken by false, true, $\vee$, $\wedge$ and $\neg$, respectively. (Equality $p = q$ is interpreted by equivalence $p \Leftrightarrow q$.)

In this case, the laws give rise to the following equivalences, which we confirmed in Section 1.7:

Commutativity:	$p \vee q \Leftrightarrow q \vee p$	*(Comm1)*
	$p \wedge q \Leftrightarrow q \wedge p$	*(Comm2)*
Associativity:	$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$	*(Assoc1)*
	$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	*(Assoc2)*
Distributivity:	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	*(Distr1)*
	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	*(Distr2)*
Identity:	$p \vee \text{false} \Leftrightarrow p$	*(Ident1)*
	$p \wedge \text{true} \Leftrightarrow p$	*(Ident2)*
Complement:	$p \vee \neg p \Leftrightarrow \text{true}$	*(Compl1)*
	$p \wedge \neg p \Leftrightarrow \text{false}$	*(Compl2)*

Example 3.3 The two-valued Boolean Algebra

The two-element set $\mathbb{B} = \{0, 1\}$ itself gives rise to an important Boolean algebra, with the operations defined as follows:

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

x	x'
0	1
1	0

As we shall see, this particular algebra is of fundamental importance in the design of digital circuits.

Exercise 3.3 (Solution on page 421)

Verify that the laws of Boolean algebra hold for the two-valued Boolean algebra $\mathbb{B}$.

From now on we shall typically omit $\cdot$ and write xy rather than $x \cdot y$, and freely omit parentheses by allowing $\cdot$ to bind tighter than $+$ and $'$ to bind tighter than $\cdot$; thus for example, we shall write $x + (y \cdot (z'))$ simply as $x + yz'$.