number from which the formula P can be constructed back. Moreover, each formula is interpreted in the set of natural numbers. Thus formulas in $F_0 \cup F_1$ are taken as properties of natural numbers, or as is commonly said, they are arithmetical predicates. The scheme of Gödel numbering is easily extended to proofs. That is, each proof also has a Gödel number; of course, you have to fix an axiomatic system for the first order logic here, when you talk about proofs. From the Gödel numbering it follows (see [3]) that there exists a function $\delta : \mathbb{N} \to \mathbb{N}$ which satisfies $\delta(\ulcorner P(x) \urcorner) = \ulcorner P(\ulcorner P(x) \urcorner) \urcorner$. In the proof of the following result we make use of this function (also called a diagonalization function) δ.

Theorem 5.1 (Löb's Theorem). For any formula $P(x) \in F_1$, there exists a statement $S \in F_0$ such that $S \equiv P(\ulcorner S \urcorner)$ holds.

Proof Define the function $h : F_1 \to F_0$ by $h = \phi \circ f \circ \psi$, where

$\phi : F_0 \to F_0$ with $\phi(C) = P(\ulcorner C \urcorner)$,

$f : F_1 \times F_1 \to F_0$ with $f(Q(x), R(y)) = R(\ulcorner Q(x) \urcorner)$, and

$\psi : F_1 \to F_1 \times F_1$ with $\psi(P(x)) = (P(x), P(x))$.

Let $\delta : \mathbb{N} \to \mathbb{N}$ be the diagonalization function with $\delta(\ulcorner Q(x) \urcorner) = \ulcorner Q(\ulcorner Q(x) \urcorner) \urcorner$

for any formula $Q(x) \in F_1$. Suppose that ϕ does not have a fixed point. Then by CDT, h is not representable by f. Now,

$$g(Q(x)) = \phi(f(\psi(Q(x)))) = \phi(f(Q(x), Q(x))) = \phi(Q(\ulcorner Q(x) \urcorner))$$

$$= P(\ulcorner Q(\ulcorner Q(x) \urcorner) \urcorner) = P(\delta(\ulcorner Q(x) \urcorner)) = f(Q(x), P(\delta(y))).$$