

<b>Commutativity:</b>	$x + y = y + x$	(Comm1)
	$x \cdot y = y \cdot x$	(Comm2)
<b>Associativity:</b>	$(x + y) + z = x + (y + z)$	(Assoc1)
	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	(Assoc2)
<b>Distributivity:</b>	$x + (y \cdot z) = (x + y) \cdot (x + z)$	(Distr1)
	$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	(Distr2)
<b>Identity:</b>	$x + 0 = x$	(Ident1)
	$x \cdot 1 = x$	(Ident2)
<b>Complement:</b>	$x + x' = 1$	(Compl1)
	$x \cdot x' = 0$	(Compl2)

Figure 3.1: The Laws of Boolean Algebra.

Boolean algebras provide an abstract representation of familiar ideas in various areas of study. Indeed we have already met concrete examples of Boolean algebras in the form of sets and propositions.

### Example 3.1 The Boolean Algebra of Sets

The power set  $\mathcal{P}(U)$  of a set  $U$  gives rise to a Boolean algebra, with the roles of 0, 1, +,  $\cdot$  and ' taken by  $\emptyset$ ,  $U$ ,  $\cup$ ,  $\cap$  and  $\bar{\phantom{x}}$ , respectively.

In this case, the laws give rise to the following set identities, which we confirmed in Section 2.7:

<b>Commutativity:</b>	$A \cup B = B \cup A$	(Comm1)
	$A \cap B = B \cap A$	(Comm2)
<b>Associativity:</b>	$(A \cup B) \cup C = A \cup (B \cup C)$	(Assoc1)
	$(A \cap B) \cap C = A \cap (B \cap C)$	(Assoc2)
<b>Distributivity:</b>	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(Distr1)
	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	(Distr2)
<b>Identity:</b>	$A \cup \emptyset = A$	(Ident1)
	$A \cap U = A$	(Ident2)
<b>Complement:</b>	$A \cup \bar{A} = U$	(Compl1)
	$A \cap \bar{A} = \emptyset$	(Compl2)