the fourth sentence is also true. This is what is meant by saying that the fourth sentence is a *logical consequence* (in first-order logic) of the fifth sentence. We can also verify that the fifth sentence is *not* in this sense a logical consequence of the fourth sentence. For suppose we take the domain of individuals to contain only 0 and 1, interpret "is a fnoffle" as "is a member of the domain," and interpret "x glorfs y" as "x is identical with y." Then the fourth sentence is true, but the fifth sentence is false.

The third sentence above is an example of a *logically true* statement: it is true no matter what domain of individuals we choose, and no matter what "is a fnoffle" is taken to mean in that domain. In a formulation of predicate logic that includes logical axioms, those axioms are logically true statements.

Thus, informally speaking, to say that a sentence A in a first-order language is a logical consequence of a set M of sentences in that language means that for any domain of individuals, and for any specification of what the predicates used in the language mean when applied to individuals in that domain, if every sentence in M is true (when understood in accordance with this specification), then so is A. An *interpretation* of a first-order language is a structure consisting of a domain of individuals together with subsets of that domain and relations between elements in the domain corresponding to the predicates of the language. An interpretation is called a *model* of a first-order theory T if all the axioms of T are true when read using that interpretation. So another way of formulating the concept of logical consequence is that A is a logical consequence of the axioms of T if A is true in every model of T.

Soundness and Completeness of the Rules of Logic

The formal rules of logical reasoning used in a first-order theory have the property of being *sound* with respect to the notion of logical consequence. What this means is that anything that can be proved from a set of axioms using these rules of reasoning is also a logical consequence of the axioms in the sense defined. The *soundness theorem* for first-order logic establishes that this is the case. What the completeness theorem shows is that the converse holds: if A is in fact a logical consequence of a set of axioms, then there is a proof of A using those axioms and the logical rules of reasoning.

Recalling that a model of a theory is an interpretation in which all of the axioms of the theory are true, we can formulate the completeness theorem combined with the soundness theorem as the statement that for a