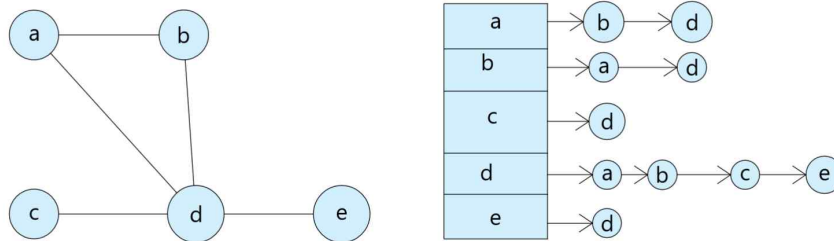
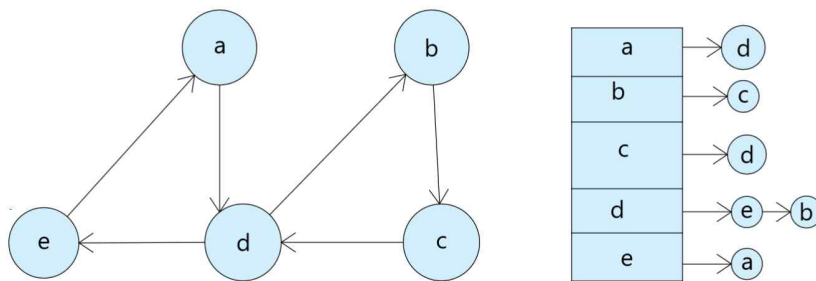


(1) an undirected graph



(2) an directed graph



The transitive closure  $\rightarrow tc()$

(1) a binary relation  $R$ :

$tc(R) =$  a binary relation  $R^*$

(2) a directed graph  $G$ :

$tc(G) =$  transitive closure of  $G$

$\rightarrow tc(R)$  and  $tc(G)$  are equivalent

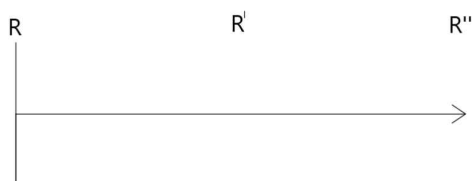
$R$  is a subset of  $A \times A$

with

1.  $R \subseteq R^*$ ,

2.  $R^*$  is a transitive relation such that  $\forall$ (transitive relations on  $A$  that contains  $R$ )

$R^* \subseteq R^{**}$



smallest  $R^*$ 이 무엇인지 아는 것이 가장

ex)  $A=\{a, b, c, d, e\} \rightarrow |A \times A| = 25$

$R=\{(a, b), (b, c)\}$ : not transitive

$\rightarrow tc(R)? R^*$

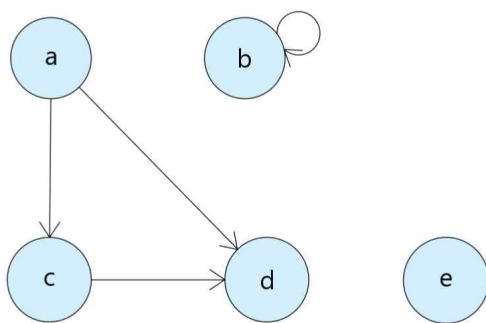
$R_1 = R \cup \{(a, c)\} \rightarrow$  transitive

$R_2 = R_1 \cup \{(d, d)\} \rightarrow$  transitive

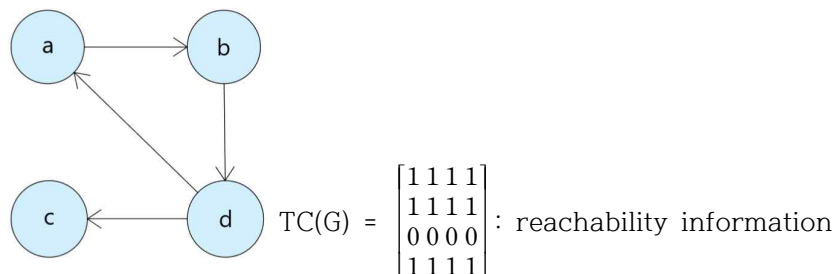
$R_3 = R_1 \cup \{(a, a), (d, d)\} \rightarrow$  transitive

ex)  $R=\{(a, c), (c, d), (a, d), (b, b)\} \rightarrow$  already transitive  $TC(R)=R$

$V=\{a, b, c, d, e\}$ 이면,



ex) 4 nodes  $E=\{(a, b), (b, d), (d, a), (d, c)\}$ ,  $V=\{a, b, c, d\}$



$$R^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}, R^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}, R^2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, R^3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, R^4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = TC(G)$$

1.  $R^0$

2.  $R^k[i, j] = \vee (R^{k-1}[i, j]), (\wedge R^{k-1}[i, k], R^{k-1}[k, j])$ 으로 구한다.

-a closed semiring-

$(S, +, \cdot, 0, 1)$

$+, \cdot \rightarrow$  binary operation

1.  $(S, +, 0) \rightarrow$  a monoid closed under  $+$   $\rightarrow a+0=0+a=a$

$(S, \cdot, 1) \rightarrow$  a monoid  $\rightarrow a \cdot 1 = 1 \cdot a = a$

$+$  is associative  $a+(b+c)=(a+b)+c$

0 is annihilator  $\rightarrow a \cdot 0 = 0 \cdot a = 0$

$$2. a+a=a, a+b=b+a$$

$$3. a \cdot (b+c)=a \cdot b+a \cdot c$$

$$(b+c) \cdot a=b \cdot a+c \cdot a$$

$$4. \sum_i^\infty a_i \text{의 값은 존재하며, unique하다. (countably many +)}$$

$$5. \cdot \text{은 countably many sum에 distribute 가능하다.}$$

$$\rightarrow \sum_i^\infty a_i \cdot \sum_i^\infty a_j = \sum_{i,j}^\infty a_i \cdot a_j = \sum_i^\infty \left( \sum_j^\infty a_i \cdot a_j \right)$$