

A formal system (Alagar and Periyasamy, 2011) is a structure that consists of the following components: (1) A decidable set (i.e., there is an algorithm that can tell whether an arbitrary element is a member of the set or not) of expressions called well-formed formulas (wffs) (2) A decidable set of axioms which are wffs that are assumed to be true (3) A set of truth-preserving transformations, called inference rules. A theorem is a wff that is either an axiom or recursively constructed by applying inference rules to axioms or previously constructed theorems. A proof is a finite sequence of wffs such that each wff is either an axiom or derived from previous wffs according to inference rules. The set of proofs in a formal system is decidable, but the set of theorems in a formal system is recursively enumerable (Nelson, 1967).

MIU system (Hofstadter, 1999) is defined as follows. A wff is any positive length string over $\{M, I, U\}$. MI is the only axiom and there are four inference rules: (1) If a wff ends with I, U can be added at the end of the wff. (2) If the form of a wff is Mx , where x is any wff, then Mxx can be created as well. (3) A triple of I (i.e., III) in a wff can be replaced by U (4) UU in a wff can be deleted.

Proving a theorem can be cast as a search problem where we are to find a proof in the space defined by axioms and inference rules while verification involves mechanically checking whether each component of the proof in sequence (i.e., a wff) follows from previous wffs in the sequence. This will lead to discussions about what can be describable in a formal system, what can be provable in a formal system, etc.

Deriving a theorem T is an example of computation in the sense that it is a finite sequence from a starting point (i.e., a theorem) and an ending point (i.e., T). In fact, finding a proof for a theorem is an example of problem solving (Goldreich, 2012) and we can smoothly introduce situations where we cannot have a finite sequence that leads to an ending point from a starting point even though both starting and ending points are clearly defined.

A relevant computational problem (i.e., Theoremhood testing problem) can be defined as follows. Given a set of axioms, inference rules, and an arbitrary proposition P , determine whether P can be proved as a theorem or not. For some formal systems, this problem is decidable, but for others, it is undecidable. This means that there does not exist an algorithm that decides whether an arbitrary proposition is a theorem in a formal system or not.

pq- system is defined as follows (Hofstadter, 1999). A wff is any positive length string over $\{p, q, -\}$, and there is one axiom schema that generates