Discrete Mathematics (COSE211-03)

Consider a language L in which certain expressions are called *predicates* and certain expressions are called *sentences*, and we are given a rule that assigns to each predicate H and any expression X a sentence denoted $H(X)$.

Suppose that certain sentences of the language L are *proved.* To each sentence X is assigned $\sim X$ called the *negation* of X, and a sentence X is called *refutable* if its negation $\sim X$ is provable.

A sentence X is called *decidable* if it is either provable or refutable; otherwise *undecidable.*

Suppose now we are given the following three conditions.

G_1: Every predicate H has a sentence X that is provable if and only if $H(X)$ is provable. (X is called a *fixed point.*)

G_2: To each predicate H is assigned a predicate H' such that for every expression X, $H'(X)$ is provable if and only if $H(X)$ is refutable.

G_3: There is a predicate whose name is P such that for every sentence X, $P(X)$ is provable if and only if X is provable.

We assume that no sentence is both provable and refutable. Show that if the three conditions G_1, G_2, and G_3 are satisfied, $P(X)$ is undecidable, where X is a fixed point of the predicate P'. (Note that P is the predicate mentioned in G_3 and P' is the one that is assigned as is mentioned in G_2.)