

the logical apparatus of a theory has been assumed to be formulated wholly in terms of rules of reasoning.

First-Order Logical Consequence

To illustrate the notion of logical consequence, let us introduce a simple first-order predicate language. In such a language, we can formulate statements about the *individuals* in some *domain* of individuals, using a collection of *predicates* to express properties of and relations between individuals. In our example of a first-order language we can say that an individual has the property of being a *fnoffle*, and that one individual *glorfs* another. We say that “fnoffle” is a *unary predicate* and “glorfs” is a *binary predicate*. To form complex sentences we have at hand the logical constructions “if-then,” “not,” “or,” “and,” “if and only if,” and we also use *variables* ranging over the domain of individuals to express statements of the form “for every (individual) x it holds that...” and “there is at least one (individual) x such that...”

Let us consider a few examples of sentence in this language, and how they might be formulated in ordinary language.

- There is at least one fnoffle: there is an x such that x is a fnoffle.
- No fnoffle glorfs itself: for every x , if x is a fnoffle it is not the case that x glorfs x .
- Every fnoffle is a fnoffle: for every x , if x is a fnoffle then x is a fnoffle.
- Every fnoffle is glorfed by at least one fnoffle: for every x , if x is a fnoffle then there is a y such that y is a fnoffle and y glorfs x .
- There is a fnoffle which glorfs every fnoffle: there is an x such that x is a fnoffle and for every y , if y is a fnoffle then x glorfs y .

In specifying the language, nothing is said about just what domain of individuals we are talking about, or what “ x is a fnoffle” or “ x glorfs y ” is supposed to mean. The specification of the language is just a matter of syntax, and does not involve any questions of meaning. (See Section 2.4.) We can, however, make the following observation: no matter what domain of individuals we are talking about, no matter what subset of that domain is singled out by “is a fnoffle,” and no matter what relation between individuals is specified by “glorfs,” if the fifth sentence above is true, then