This shows that $h$ is representable by $f$, a contradiction. Therefore, $\phi$ has a fixed point. Call this fixed point $S$. Then $S = \phi(S) = P(\ulcorner S \urcorner)$. Since equality in $F_1$ is simply the equivalence of formulas, $S \equiv P(\ulcorner S \urcorner)$. □

The following result uses the Gödel numbering of proofs.

**Theorem 5.2 (Gödel's First Incompleteness Theorem.)** There is a statement $S \in F_0$ about arithmetic such that $S$ is true iff $S$ is not provable.

*Proof* Let $P(y, x)$ be the formula denoting "$y$ is the Gödel number of a proof of a statement whose Gödel number is $x$". This formula is an arithmetical predicate having two free variables; the variables are to take values from $\mathbb{N}$. Let $Q(x) = \forall y \neg P(y, x)$. Now, $Q(x) \in F_1$. By Löb's Theorem, there is a statement $S \in F_0$ such that $S \equiv Q(\ulcorner S \urcorner)$. That is, $S \equiv \forall y \neg P(y, \ulcorner S \urcorner)$ or that $S \equiv \neg \exists y P(y, \ulcorner S \urcorner)$. This says that $S$ is true iff it is not the case that there is a natural number $y$ which is the Gödel number of a proof of $S$. □

**6. Conclusion**

No mathematical discourse is complete without raising further problems. To tackle the problems one must master the techniques and for this, exercises are of help. Let us have some exercises in the form of applications of CLT/CDT. Your interest may take you to the materials in [2,3,4].

Our first exercise is Russell's paradox. It says that the set of all sets which are not members of themselves is a member of itself and at the same time, it cannot be a member of itself. It is, of course, not a contradiction since the phrase 'set of all sets which are not members of themselves' may not be meaningful. The exercise is: show that the collection of all sets which are not members of themselves is not a set.