

Pumping Lemma

$$\forall L \in \mathcal{A} \exists m \in \mathbb{Z}^+ \forall w \in L (|w| \geq m) \exists w = xyz, |xy| \leq m, |y| \geq 1 \forall i \in \{0, 1, 2, \dots\} xy^iz \in L$$

( $\mathcal{A}$  is the set of all regular languages)

Prove  $L = \{ww^R \mid w \text{ is a positive length string over } \{a, b\}\}$  is not regular

1. Assume that  $L$  is regular
2.  $m \in \mathbb{Z}^+$
3.  $|w| \geq m, w = a^m b^m b^m a^m, |w| = 4m$
4. Let  $y = a^i$  then, contradiction occurs

1. recursive theorem
  2. self-copying programs
  3. abstract theorem
- 을 배운다.

1. recursion theorem:

if a computer program  $T$ , such as Java, C, C++, exists  
(Turing complete programming languages)  
then, a computer program  $R$  exists.

What is  $R$ ?

그전에  $t$ 가 뭔지 생각해 보자.

$t$ : a computable function ( $\Sigma^* X \Sigma^* \rightarrow \Sigma^*$ )

$\Sigma$ : a finite nonempty set of symbols

$\Sigma^*$ : the set of all strings over  $\Sigma$

ex)  $t(ab, bba) = aba$

$R$  computes function value of  $t$

$r: \Sigma^* \rightarrow \Sigma^*$

$R: \forall_{w \in \Sigma} \cdot r(w) = t(\langle R \rangle, w)$

$\langle R \rangle$ : a string, a text

ex) 프로그램 이름이 Test이면,  $\langle \text{Test} \rangle$ : all string code of Test이다.

2. self-copying programs

대부분의 컴퓨터는 Von Neumann Computer이다.

==Stored program computer

-> we can load "data" and "program" together on memory

그리고 그러한 컴퓨터에는 Instruction Pointer(Program Counter)가 있다.

-> 현재 실행되고 있는 프로그램의 줄 수를 지칭한다.

그렇다면 self-copying programs이 뭐냐?

self-copying programs은 다음과 같은 구조를 가진다.

```
1. program self copy
2.     L=ip-1
3.     loop until line[L]="end"
4.     {
5.         print(line[L])
6.         L=L+1
7.     }
8.     print("end")
9. end
```

-> 이 코드의 결과물은 정확히 이 코드와 일치하다.

What line has to do the capability of Von Nohiman architecture -> 5

print (line[L])

line[L]은 Data로, print는 Command로 해석가능하다.

->본 노이만 컴퓨터는 이렇게 두 가지로 해석이 가능하다.

3. abstract theorem

-> abstract-recursion theorem

a denumerable set of objects (countably infinite objects)

->  $E_1, E_2, E_3, \dots E_n, \dots$

$\Sigma$  : a set of functions ( $Z^+ \rightarrow Z^+$ )

The definition of  $\Sigma$  is that  $\Sigma$  is closed under "composition"

->  $\forall f, g \in \Sigma \exists h \in \Sigma \forall x \in Z^+ f(g(x)) = h(x)$

$F(x, y): Z^+ \times Z^+ \rightarrow Z^+$  (ordered pair of functions)

Theorem:

if:

1.  $\Sigma$  is closed under composition
2.  $F(x, x) \in \Sigma$  ( $Z^+ \rightarrow Z^+$ )
3.  $\forall f \in \Sigma \exists a \in Z^+ \forall x \in Z^+ E_{F(a, x)} = E_{f(x)}$

then:

$\forall f \in \Sigma \exists i \in Z^+ E_i = E_{f(x)}$