1. Consider the problem of minimizing a quadratic function:

$$\texttt{minimize} \;\; f(x) = x^T P x + q^T x$$

for symmetric P. Show that if $P \not\succeq 0$, that is not positive semidefinite and f is not convex, show that the problem is unbounded.

2. Consider minimizing

$$f(x) = \frac{\|Ax - b\|_2^2}{c^T x + d}, \quad \operatorname{dom} f = \left\{x | c^T x + d > 0\right\}$$

Show that the minimizer x^* is given by

$$x^* = x_1 + t x_2$$

wherer $x_1 = (A^T A)^{-1} A^b$ and $x_2 = (A^T A)^{-1} c$ and $t \in \mathbb{R}$ can be solved using a quadratic equation.

3. Suppose we minimize the following objective

$$f(x) = \frac{1}{2}\left(x_1^2 + \gamma x_2^2\right)$$

for $\gamma > 0$. With $x^{(0)} = (\gamma, 1)$, Show that the expressions for the iterates $x^{(k)}$ is given by

$$x_1^{(k)} = \gamma \left(\frac{\gamma - 1}{\gamma + 1}\right)^k, \quad x_2^{(k)} = \left(-\frac{\gamma - 1}{\gamma + 1}\right)^k$$

4. A random variable $X \in \{0, 1\}$ satisfies

$$\operatorname{prob}(X = 1) = p = \frac{\exp\left(a^T x + b\right)}{1 + \exp\left(a^T x + b\right)}$$

where $x \in \mathbb{R}^n$ is a vector of variables that affect the probability, and a and b are known parameters. We can think of $X = 1$ as the event that a consumer buys a product, and x as a vector of variables that affect the probability, e.g., advertising effort, retail price, discounted price, packaging expense, and other factors. The variable x, which we are to optimize over, is subject to a set of linear constraints, $Fx \succeq g$. Formulate the following problems as convex optimization problems.

   (a) *Maximizing buying probability*. The goal is to choose x to maximize p.

   (b) *Maximizing expected profit*. Let $c^T x + d$ be the profit derived from selling the product, which we assume is positive for all feasible x. The goal is to maximize the expected profit, which is $p(c^T x + d)$.

5. Consider the robust linear discrimination problem given by

$$\begin{array}{ll} \text{maximize} & t \\ \text{subject to} & a^T x_i - b \geq t, \quad i = 1, \ldots, N \\ & a^T y_i - b \leq -t, \quad i = 1, \ldots, M \\ & \|a\|_2 \leq 1 \end{array}$$

(a) Show that the optimal value t is positive if and only if the two sets of points can be linearly separated. When the two sets of points can be linearly separated, show that the inequality $\|a\|_2 \leq 1$ is tight, i.e., we have $\|a^*\|_2 = 1$, for the optimal a^*.

(b) Using the change of variables $\tilde{a} = a/t$, $\tilde{b} = b/t$, prove that the problem (8.23) is equivalent to the QP

$$\begin{array}{rl} \texttt{minimize} & \|\tilde{a}\|_2 \\ \texttt{subject to} & \tilde{a}^T x_i - b \geq 1, \; i = 1, \ldots, N \\ & \tilde{a}^T y_i - b \leq -1, \; i = 1, \ldots, M \end{array}$$