7강

Linear Algebra review

- Vector space
- Basis/Dimension
- Nullspace
- Range
- Rank
- Determinant
- ...and more to cover as we move on

Vector space

→ 벡터 공간

- a vector space V consists of
  - A set of vectors

    벡터 공간은 벡터들로 이루어짐.

  - Addition operator

    → Vector space에 대해서 Addition operation을 진행 할 수 있다.

$$x = (x_1, x_2) \in R^2$$
$$y = (y_1, y_2) \in R^2$$
$$x + y = (x_1 + y_1, x_2 + y_2)$$

  - multiplication with scalar

$$cx = (cx_1, cx_2)$$

  - special element 0 vector

    → Origin(원점)

→ vector space는 linear combination에 대해 닫혀있다.

$$x, y \in V,\ cx + dy \in V$$

→ 이는 원점을 반드시 포함한다.

- Example :
  - V1 = R^n
  - V2 = {0}
  -

$$V_3 = span(v_1, ..., v_k)\ with\ v_1, ..., v_k \in R^n$$
$$where\ span(v_1, ..., v_k) = \{c_1 v_1 + ... + c_k v_k \mid c_1, ..., c_k \ \in \ R\}$$

span이라는 operator가 있는데,

이 operator는 k개의 벡터에 대해서 그 벡터들에 대한 임의의 combination을 의미한다.

Subspace

- Subspace of a vector space is
  - i) subset of a vector space and (어떤 vector space의 subspace여야하고,)
  - ii) itself is a vector space (그리고 그것이 vector space의 성질을 만족해야함)
- V_1, V_2, V_3 are subspaces

R^2은 R^3의 subspace이다.

자기 자신은 자기 자신의 subspace이다.

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independent set of vectors

- we say vectors v1,...,vk are linearly independent when

$$c_1v_1 + ... \ + c_kv_k = 0$$
$$\rightarrow c_1 = ... = c_k = 0$$

- The only way to make the linear combinations of linearly independent vectors to zero

  is to make all the coefficients zero

- No vector v_i, 1≤ i ≤k, can be expressed as linear combination of other vectors

$$v_3 \neq c_1v_1 + c_2v_2$$

independence는 2개 이상의 벡터가 있어야 성립이 가능하다.

- Not to be confused with orthogonality of vectors (독립과 수직은 다르다.)
  - If v_1, ..., v_k are mutually orthogonal, they are linearly independent
  - converse is not necessarily true

  orthogonal → linealy independent (O)

  linearly independent → orthogonal (X)

  수직 여부는 내적의 결과로 판단 가능하다.

Basis and Dimension

- set of vectors {v_1, ..., v_k} is a basis of vector space V if
  1. {v_1, ..., v_k} spans V, or

     $$V = span(v_1, ..., v_k)$$

  2. v_1, ... ,v_k are linearly independent

- Any point x ∈ V can be uniquely expressed as

$$c_1v_1 + \cdots + c_kv_k$$

for some c_1, ..., c_k

V안의 임의의 점을 선택해서 unique하게 그것을 basis의 linear combination으로 표현가능하다.

$$if\ c_1v_1 + ...\ + c_kv_k = x,\ c_1^{'}v_1^{'} + ...\ + c_k^{'}v_k^{'} = x$$
$$then\ (c_1 - c_1^{'})x_1 + ...\ + (c_k - c_k^{'})x_k = 0$$
$$x_1, ..., x_k\ are\ not\ independent \rightarrow contradiction$$

- For given vector space V and any of its basis, (basis 벡터는 다양해 질 수 있으나)

  the number of vectors in the basis is fixed (basis 벡터의 갯수는 일정하다.)

- The number of basis vectors is called dimension of V, denoted by dim(V)

  그 일정한 basis 벡터의 갯수를 dim(V)이라고 한다.

Basis and Dimension

- By default, we let dim({0}) = 0

  (in other mathematical definition of dimensions,

  a single point other than 0 is also defined to have 0 dimension)

  0차원은 점이다.

- Examples : consider V_1 = {αv| α ∈R} for some v ∈ R^n

  1차원은 원점을 지나는 직선이다.

  (원점을 안지나면 안된다. vectorspace가 되려면 반드시 원점을 지나야 한다.)

  - V_1 represents a line going through origin, and is parallel to v
  - V_1 is a subspace of R^n: it is a subset of R^n, and is vector space, and contains {0}
  - Dimension of V_1 is 1, although it contains a point from R^n!

dim(V_1) = 1

벡터가 1개이기에 independent 여부는 신경쓰지 않는다.

- Consider v_1,v_2 ∈ R^3 where v_1, v_2 are linearly independent.

  Plane V_2 = {α1v1 + α2v2 | α1,α2 ∈ R} goes through the origin and is a subspace with dimension 2

- But note the same plane can be expressed as v ∈ R^3 | c^T * v = 0}

  using some vector c ∈R^n orthogonal to vectors on the plane!

  3차원 내에서의 평면인 vectorspace를 2개의 basis로 나타낼 수도 있지만, 하나의 벡터에 대한 수직성으로 나타낼 수도 있는 것이다.

Matrix vector multiplication

- Useful things to know
- Let A ∈ R^{m×n} and

$$A = [a_1\ a_2\ \dots\ a_n]$$

where ai is the ith column of A and x = (x_1, ..., x_n) then

$$Ax = x_1a_1 + \cdots + x_na_n = \sum_{i=1}^{n} x_ia_i$$

That is, it is linear combination of columns

- Let A ∈R^{m×n} and

$$A = \begin{bmatrix} a_1^T \\ a_2^T \\ \dots \\ a_m^T \end{bmatrix}$$

where a^T_i is the ith row of A and

$$x^TA = x_1a_1^T + \cdots + x_ma_m^T = \sum_{i=1}^{m} x_ia_i^T$$

That is, it is linear combination of rows

Matrix matrix multiplication

- Let A ∈ R^{mxr} and B ∈ R^{rxn}

$$AB = A\begin{bmatrix} b_1 & b_2 & ... & b_n \end{bmatrix} = \begin{bmatrix} Ab_1 & Ab_2 & ... & Ab_n \end{bmatrix}$$

or

$$AB = \begin{bmatrix} a_1^T \\ a_2^T \\ ... \\ a_n^T \end{bmatrix} B = \begin{bmatrix} a_1^T B \\ a_2^T B \\ ... \\ a_n^T B \end{bmatrix}$$

Range

- Range of a matrix A ∈ R^{m×n}, denoted byR(A) is defined as

$$R(A) = \{Ax \mid x \in R^n\}$$

  - R(A) is equivalent to span(a_1, ..., a_n) where a_i ∈ R^m are columns of A
  - That is, R(A) is the subspace (subset of R^m) spanned by columns of A

    → m ≥ n

lineear system x → A → Ax인 것이다.

- set of vectors 'hit' by linear mapping y = Ax
- set of vectors such that, for given y in R(A), equation Ax −y = 0 w.r.t. x has solution

  → y가 range 안에 있어야 solution이 존재한다는 의미이다.

Range: interpretation

- let

$$v \in R(A) \; and \; \not\in R(A)$$

→ 하나는 range안에 들어가고, 아닌 것도 있다.

→ 그게 무슨 의미인가.

- let y = Ax output of a sensor to input x
  - y = v is possible/consistent output
  - y = w is impossible/inconsistent

- R(A) represents achievable outputs

  R(A)은 취할 수 있는 값들이다.

- R(A) is subspace

- suppose R(A) = R^m (special case)
  - any output y ∈ R^m is possible

    R(x)의 결과값, Ax는 m차원을 넘어갈 수가 없는 것이다.