6.6 Sampling from a finite population

$$X_i = \begin{cases} 1, & \text{if } i\text{-th member of the sample has the characteristic.} \\ 0, & o.w \end{cases}$$

- $P(X_i = 1) = p$
- $P(X_{k+1} = 1 \mid X_1, X_2, \ldots, X_k) = \left(Np - \sum_{i=1}^{k} X_i \right) / (N-k) \approx p,$

  That is, $\{X_i\}$ is approximately i.i.d. if N is large in relation to n.
- $X_1 + X_2 + \ldots + X_n \sim B(n,p) \sim N(np, npq)$
- $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n} \sim N\left(p, \dfrac{pq}{n} \right)$

Ex6.6a $n = 200, p = 0.45.$

$X = X_1 + X_2 + \ldots + X_n$

a) $E[X] = 200 \cdot 0.45 = 90, \qquad SD(X) = \sqrt{200 \cdot 0.45 \cdot 0.55} = 7.0356$

b) $P(X \geq 101) = P(X \geq 100.5)$ (the continuity corrrection)

$\approx P(Z \geq 1.4924) \approx 0.0678$

Ex6.6b Mean : 147, SD : 62, n=25.

$$\overline{X} \sim N\left(147, \left(\frac{62}{5} \right)^2 \right)$$

$$\mathbb{P}\{\overline{X} > 150\} = \mathbb{P}\left\{ \frac{\overline{X} - 147}{12.4} > \frac{150 - 147}{12.4} \right\}$$

$$\approx \mathbb{P}\{Z > 0.242\}$$