SOLUTION If we let X_i be the amount consumed by the ith member of the sample, $i = 1, \ldots, 25$, then the desired probability is

$$P\left\{\frac{X_1 + \cdots + X_{25}}{25} > 150\right\} = P\{\overline{X} > 150\}$$

where $\overline{X}$ is the sample mean of the 25 sample values. Since we can regard the X_i as being independent random variables with mean 147 and standard deviation 62, it follows from the central limit theorem that their sample mean will be approximately normal with mean 147 and standard deviation 62/5. Thus, with Z being a standard normal random variable, we have

$$\begin{aligned} P\{\overline{X} > 150\} &= P\left\{\frac{\overline{X} - 147}{12.4} > \frac{150 - 147}{12.4}\right\} \\ &\approx P\{Z > .242\} \\ &\approx .404 \quad \blacksquare \end{aligned}$$

Chapter 6 Problems

1. Suppose that X_1, X_2, X_3 are independent with the common probability mass function

$$P\{X_i = 0\} = .2, \quad P\{X_i = 1\} = .3, \quad P\{X_i = 3\} = .5, \quad i = 1, 2, 3$$

   (a) Plot the probability mass function of $\overline{X}_2 = \dfrac{X_1 + X_2}{2}$.

   (b) Determine $E[\overline{X}_2]$ and $\text{Var}(\overline{X}_2)$.

   (c) Plot the probability mass function of $\overline{X}_3 = \dfrac{X_1 + X_2 + X_3}{3}$.

   (d) Determine $E[\overline{X}_3]$ and $\text{Var}(\overline{X}_3)$.

2. If 10 fair dice are rolled, approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.

3. Approximate the probability that the sum of 16 independent uniform (0, 1) random variables exceeds 10.

4. A roulette wheel has 38 slots, numbered 0, 00, and 1 through 36. If you bet 1 on a specified number, you either win 35 if the roulette ball lands on that number or lose 1 if it does not. If you continually make such bets, approximate the

probability that

(a) you are winning after 34 bets;
(b) you are winning after 1,000 bets;
(c) you are winning after 100,000 bets.

Assume that each roll of the roulette ball is equally likely to land on any of the 38 numbers.

5. A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of .3 inches.

(a) Approximate the probability that the salt on hand will suffice for the next 50 days.
(b) What assumption did you make in solving part (a)?
(c) Do you think this assumption is justified? Explain briefly.

6. Fifty numbers are rounded off to the nearest integer and then summed. If the individual roundoff errors are uniformly distributed between $-.5$ and $.5$, what is the approximate probability that the resultant sum differs from the exact sum by more than 3?

7. A six-sided die, in which each side is equally likely to appear, is repeatedly rolled until the total of all rolls exceeds 400. Approximate the probability that this will require more than 140 rolls.

8. The amount of time that a certain type of battery functions is a random variable with mean 5 weeks and standard deviation 1.5 weeks. Upon failure, it is immediately replaced by a new battery. Approximate the probability that 13 or more batteries will be needed in a year.

9. The lifetime of a certain electrical part is a random variable with mean 100 hours and standard deviation 20 hours. If 16 such parts are tested, find the probability that the sample mean is

(a) less than 104;
(b) between 98 and 104 hours.

10. A tobacco company claims that the amount of nicotine in its cigarettes is a random variable with mean 2.2 mg and standard deviation .3 mg. However, the sample mean nicotine content of 100 randomly chosen cigarettes was 3.1 mg. What is the approximate probability that the sample mean would have been as high or higher than 3.1 if the company's claims were true?

11. The lifetime (in hours) of a type of electric bulb has expected value 500 and standard deviation 80. Approximate the probability that the sample mean of n such bulbs is greater than 525 when

(a) $n = 4$;
(b) $n = 16$;

(c) $n = 36$;
(d) $n = 64$.

12. An instructor knows from past experience that student exam scores have mean 77 and standard deviation 15. At present the instructor is teaching two separate classes — one of size 25 and the other of size 64.

(a) Approximate the probability that the average test score in the class of size 25 lies between 72 and 82.
(b) Repeat part (a) for a class of size 64.
(c) What is the approximate probability that the average test score in the class of size 25 is higher than that of the class of size 64?
(d) Suppose the average scores in the two classes are 76 and 83. Which class, the one of size 25 or the one of size 64, do you think was more likely to have averaged 83?

13. If X is binomial with parameters $n = 150$, $p = .6$, compute the exact value of $P\{X \leq 80\}$ and compare with its normal approximation both **(a)** making use of and **(b)** not making use of the continuity correction.

14. Each computer chip made in a certain plant will, independently, be defective with probability .25. If a sample of 1,000 chips is tested, what is the approximate probability that fewer than 200 chips will be defective?

15. A club basketball team will play a 60-game season. Thirty-two of these games are against class A teams and 28 are against class B teams. The outcomes of all the games are independent. The team will win each game against a class A opponent with probability .5, and it will win each game against a class B opponent with probability .7. Let X denote its total number of victories in the season.

(a) Is X a binomial random variable?
(b) Let X_A and X_B denote, respectively, the number of victories against class A and class B teams. What are the distributions of X_A and X_B?
(c) What is the relationship between X_A, X_B, and X?
(d) Approximate the probability that the team wins 40 or more games.

16. Argue, based on the central limit theorem, that a Poisson random variable having mean λ will approximately have a normal distribution with mean and variance both equal to λ when λ is large. If X is Poisson with mean 100, compute the exact probability that X is less than or equal to 116 and compare it with its normal approximation both when a continuity correction is utilized and when it is not. The convergence of the Poisson to the normal is indicated in Figure 6.5.

17. Use the text disk to compute $P\{X \leq 10\}$ when X is a binomial random variable with parameters $n = 100, p = .1$. Now compare this with its **(a)** Poisson and

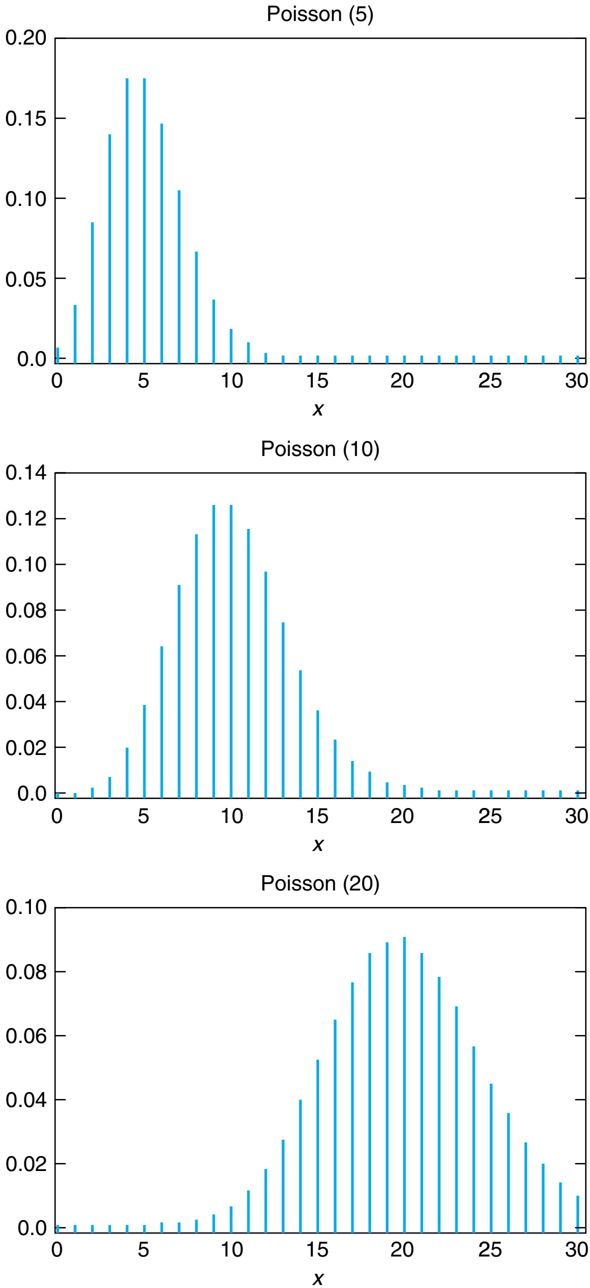

FIGURE 6.5 *Poisson probability mass functions.*

(b) normal approximation. In using the normal approximation, write the desired probability as $P\{X < 10.5\}$ so as to utilize the continuity correction.

18. The temperature at which a thermostat goes off is normally distributed with variance σ^2. If the thermostat is to be tested five times, find

(a) $P\{S^2/\sigma^2 \leq 1.8\}$
(b) $P\{.85 \leq S^2/\sigma^2 \leq 1.15\}$

where S^2 is the sample variance of the five data values.

19. In Problem 18, how large a sample would be necessary to ensure that the probability in part (a) is at least .95?

20. Consider two independent samples — the first of size 10 from a normal population having variance 4 and the second of size 5 from a normal population having variance 2. Compute the probability that the sample variance from the second sample exceeds the one from the first. (*Hint*: Relate it to the F-distribution.)

21. Twelve percent of the population is left-handed. Find the probability that there are between 10 and 14 left-handers in a random sample of 100 members of this population. That is, find $P\{10 \leq X \leq 14\}$, where X is the number of left-handers in the sample.

22. Fifty-two percent of the residents of a certain city are in favor of teaching evolution in high school. Find or approximate the probability that at least 50 percent of a random sample of size n is in favor of teaching evolution, when

(a) $n = 10$;
(b) $n = 100$;
(c) $n = 1{,}000$;
(d) $n = 10{,}000$.

23. The following table gives the percentages of individuals of a given city, categorized by gender, that follow certain negative health practices. Suppose a random sample of 300 men is chosen. Approximate the probability that

(a) at least 150 of them rarely eat breakfast;
(b) fewer than 100 of them smoke.

	Sleeps 6 Hours or Less per Night	Smoker	Rarely Eats Breakfast	Is 20 Percent or More Overweight
Men	22.7	28.4	45.4	29.6
Women	21.4	22.8	42.0	25.6

Source: U.S. National Center for Health Statistics, Health Promotion and Disease Prevention.

24. (Use the table from Problem 23.) Suppose a random sample of 300 women is chosen. Approximate the probability that

(a) at least 60 of them are overweight by 20 percent or more;
(b) fewer than 50 of them sleep 6 hours or less nightly.

25. (Use the table from Problem 23.) Suppose random samples of 300 women and of 300 men are chosen. Approximate the probability that more women than men rarely eat breakfast.

26. The following table uses data concerning the percentages of teenage male and female full-time workers whose annual salaries fall in different salary groupings. Suppose random samples of 1,000 men and 1,000 women were chosen. Use the table to approximate the probability that

(a) at least half of the women earned less than \$20,000;
(b) more than half of the men earned \$20,000 or more;
(c) more than half of the women and more than half of the men earned \$20,000 or more;
(d) 250 or fewer of the women earned at least \$25,000;
(e) at least 200 of the men earned \$50,000 or more;
(f) more women than men earned between \$20,000 and \$24,999.

Earnings Range	Percentage of Women	Percentage of Men
\$4,999 or less	2.8	1.8
\$5,000 to \$9,999	10.4	4.7
\$10,000 to \$19,999	41.0	23.1
\$20,000 to \$24,999	16.5	13.4
\$25,000 to \$49,999	26.3	42.1
\$50,000 and over	3.0	14.9

Source: U.S. Department of Commerce, Bureau of the Census.

27. In 1995 the percentage of the labor force that belonged to a union was 14.9. If five workers had been randomly chosen in that year, what is the probability that none of them would have belonged to a union? Compare your answer to what it would be for the year 1945, when an all-time high of 35.5 percent of the labor force belonged to a union.

28. The sample mean and sample standard deviation of all San Francisco student scores on the most recent Scholastic Aptitude Test examination in mathematics were 517 and 120. Approximate the probability that a random sample of 144 students would have an average score exceeding

(a) 507;
(b) 517;
(c) 537;
(d) 550.

29. The average salary of newly graduated students with bachelor's degrees in chemical engineering is $53,600, with a standard deviation of $3,200. Approximate the probability that the average salary of a sample of 12 recently graduated chemical engineers exceeds $55,000.

30. A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of the components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least .95?

Chapter 7 Problems

1. Let $X_1, \ldots, X_n$ be a sample from the distribution whose density function is

$$f(x) = \begin{cases} e^{-(x-\theta)} & x \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Determine the maximum likelihood estimator of θ.

2. Determine the maximum likelihood estimator of θ when $X_1, \ldots, X_n$ is a sample with density function

$$f(x) = \tfrac{1}{2} e^{-|x-\theta|}, \qquad -\infty < x < \infty$$

3. Let $X_1, \ldots, X_n$ be a sample from a normal μ, σ^2 population. Determine the maximum likelihood estimator of σ^2 when μ is known. What is the expected value of this estimator?

4. Determine the maximum likelihood estimates of a and λ when $X_1, \ldots, X_n$ is a sample from the Pareto density function

$$f(x) = \begin{cases} \lambda a^{\lambda} x^{-(\lambda+1)}, & \text{if } x \geq a \\ 0, & \text{if } x < a \end{cases}$$

5. Suppose that $X_1, \ldots, X_n$ are normal with mean μ_1; $Y_1, \ldots, Y_n$ are normal with mean μ_2; and $W_1, \ldots, W_n$ are normal with mean $\mu_1 + \mu_2$. Assuming that all $3n$ random variables are independent with a common variance, find the maximum likelihood estimators of μ_1 and μ_2.

6. River floods are often measured by their discharges (in units of feet cubed per second). The value v is said to be the value of a 100-year flood if

$$P\{D \geq v\} = .01$$

where D is the discharge of the largest flood in a randomly chosen year. The following table gives the flood discharges of the largest floods of the Blackstone River in Woonsocket, Rhode Island, in each of the years from 1929 to 1965. Assuming that these discharges follow a lognormal distribution, estimate the value of a 100-year flood.

Annual Floods of the Blackstone River (1929–1965)

Year	Flood Discharge (ft^3/s)
1929	4,570
1930	1,970
1931	8,220
1932	4,530
1933	5,780
1934	6,560
1935	7,500
1936	15,000
1937	6,340
1938	15,100
1939	3,840
1940	5,860
1941	4,480
1942	5,330
1943	5,310
1944	3,830
1945	3,410
1946	3,830
1947	3,150
1948	5,810
1949	2,030
1950	3,620
1951	4,920
1952	4,090
1953	5,570
1954	9,400
1955	32,900
1956	8,710
1957	3,850
1958	4,970
1959	5,398
1960	4,780
1961	4,020
1962	5,790
1963	4,510
1964	5,520
1965	5,300

7. Recall that X is said to have a lognormal distribution with parameters μ and σ^2 if $\log(X)$ is normal with mean μ and variance σ^2. Suppose X is such a lognormal random variable.
(a) Find $E[X]$.
(b) Find $\text{Var}(X)$.

Hint: Make use of the formula for the moment generating function of a normal random variable.

(c) The following are, in minutes, travel times to work over a sequence of 10 days.

$$42, 28, 53, 57, 67, 39, 35, 50, 44, 39$$

Assuming an underlying lognormal distribution, use the data to estimate the mean travel time.

8. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = .1$ mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.

- **(a)** Determine a 95 percent confidence interval estimate of the true weight.
- **(b)** Determine a 99 percent confidence interval estimate of the true weight.

9. The PCB concentration of a fish caught in Lake Michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of .08 ppm (parts per million). Suppose the results of 10 independent measurements of this fish are

$$11.2,\ 12.4,\ 10.8,\ 11.6,\ 12.5,\ 10.1,\ 11.0,\ 12.2,\ 12.4,\ 10.6$$

- **(a)** Give a 95 percent confidence interval for the PCB level of this fish.
- **(b)** Give a 95 percent lower confidence interval.
- **(c)** Give a 95 percent upper confidence interval.

10. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students.

11. Let $X_1, \ldots, X_n, X_{n+1}$ be a sample from a normal population having an unknown mean μ and variance 1. Let $\bar{X}_n = \sum_{i=1}^{n} X_i/n$ be the average of the first n of them.

- **(a)** What is the distribution of $X_{n+1} - \bar{X}_n$?
- **(b)** If $\bar{X}_n = 4$, give an interval that, with 90 percent confidence, will contain the value of X_{n+1}.

12. If $X_1, \ldots, X_n$ is a sample from a normal population whose mean μ is unknown but whose variance σ^2 is known, show that $(-\infty, \overline{X} + z_\alpha \sigma/\sqrt{n})$ is a $100(1 - \alpha)$ percent lower confidence interval for μ.

13. A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2 mg. Compute a 99 percent two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarette's nicotine content is $\sigma = .2$ mg.

14. In Problem 13, suppose that the population variance is not known in advance of the experiment. If the sample variance is .04, compute a 99 percent two-sided confidence interval for the mean nicotine content.

15. In Problem 14, compute a value c for which we can assert "with 99 percent confidence" that c is larger than the mean nicotine content of a cigarette.

16. Suppose that when sampling from a normal population having an unknown mean μ and unknown variance σ^2, we wish to determine a sample size n so as to guarantee that the resulting $100(1 - \alpha)$ percent confidence interval for μ will be of size no greater than A, for given values α and A. Explain how we can approximately do this by a double sampling scheme that first takes a subsample of size 30 and then chooses the total sample size by using the results of the first subsample.

17. The following data resulted from 24 independent measurements of the melting point of lead.

330°C	322°C	345°C
328.6°C	331°C	342°C
342.4°C	340.4°C	329.7°C
334°C	326.5°C	325.8°C
337.5°C	327.3°C	322.6°C
341°C	340°C	333°C
343.3°C	331°C	341°C
329.5°C	332.3°C	340°C

Assuming that the measurements can be regarded as constituting a normal sample whose mean is the true melting point of lead, determine a 95 percent two-sided confidence interval for this value. Also determine a 99 percent two-sided confidence interval.

18. The following are scores on IQ tests of a random sample of 18 students at a large eastern university.

130, 122, 119, 142, 136, 127, 120, 152, 141,
132, 127, 118, 150, 141, 133, 137, 129, 142

(a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university.
(b) Construct a 95 percent lower confidence interval estimate.
(c) Construct a 95 percent upper confidence interval estimate.

19. Suppose that a random sample of nine recently sold houses in a certain city resulted in a sample mean price of $222,000, with a sample standard deviation of $22,000. Give a 95 percent upper confidence interval for the mean price of all recently sold houses in this city.

20. A company self-insures its large fleet of cars against collisions. To determine its mean repair cost per collision, it has randomly chosen a sample of 16 accidents. If the average repair cost in these accidents is \$2,200 with a sample standard deviation of \$800, find a 90 percent confidence interval estimate of the mean cost per collision.

21. A standardized test is given annually to all sixth-grade students in the state of Washington. To determine the average score of students in her district, a school supervisor selects a random sample of 100 students. If the sample mean of these students' scores is 320 and the sample standard deviation is 16, give a 95 percent confidence interval estimate of the average score of students in that supervisor's district.

22. Each of 20 science students independently measured the melting point of lead. The sample mean and sample standard deviation of these measurements were (in degrees centigrade) 330.2 and 15.4, respectively. Construct **(a)** a 95 percent and **(b)** a 99 percent confidence interval estimate of the true melting point of lead.

23. A random sample of 300 CitiBank VISA cardholder accounts indicated a sample mean debt of \$1,220 with a sample standard deviation of \$840. Construct a 95 percent confidence interval estimate of the average debt of all cardholders.

24. In Problem 23, find the smallest value v that "with 90 percent confidence," exceeds the average debt per cardholder.

25. Verify the formula given in Table 7.1 for the $100(1-\alpha)$ percent lower confidence interval for μ when σ is unknown.

26. The following are the daily number of steps taken by a certain individual in 20 weekdays.

2,100	1,984	2,072	1,898
1,950	1,992	2,096	2,103
2,043	2,218	2,244	2,206
2,210	2,152	1,962	2,007
2,018	2,106	1,938	1,956

Assuming that the daily number of steps is normally distributed, construct **(a)** a 95 percent and **(b)** a 99 percent two-sided confidence interval for the mean number of steps. **(c)** Determine the largest value v that, "with 95 percent confidence," will be less than the mean range.

27. Studies were conducted in Los Angeles to determine the carbon monoxide concentration near freeways. The basic technique used was to capture air samples in special bags and to then determine the carbon monoxide concentration by using a spectrophotometer. The measurements in ppm (parts per million) over a sampled period during the year were 102.2, 98.4, 104.1, 101, 102.2, 100.4, 98.6, 88.2, 78.8, 83, 84.7, 94.8, 105.1, 106.2, 111.2, 108.3, 105.2, 103.2, 99, 98.8. Compute a 95 percent two-sided confidence interval for the mean carbon monoxide concentration.

28. A set of 10 determinations, by a method devised by the chemist Karl Fischer, of the percentage of water in a methanol solution yielded the following data.

$$.50, .55, .53, .56, .54,$$
$$.57, .52, .60, .55, .58$$

Assuming normality, use these data to give a 95 percent confidence interval for the actual percentage.

29. Suppose that $U_1, U_2, \ldots$ is a sequence of independent uniform (0,1) random variables, and define N by

$$N = \min\{n : U_1 + \cdots + U_n > 1\}$$

That is, N is the number of uniform (0, 1) random variables that need to be summed to exceed 1. Use random numbers to determine the value of 36 random variables having the same distribution as N, and then use these data to obtain a 95 percent confidence interval estimate of $E[N]$. Based on this interval, guess the exact value of $E[N]$.

30. An important issue for a retailer is to decide when to reorder stock from a supplier. A common policy used to make the decision is of a type called s, S: The retailer orders at the end of a period if the on-hand stock is less than s, and orders enough to bring the stock up to S. The appropriate values of s and S depend on different cost parameters, such as inventory holding costs and the profit per item sold, as well as the distribution of the demand during a period. Consequently, it is important for the retailer to collect data relating to the parameters of the demand distribution. Suppose that the following data give the numbers of a certain type of item sold in each of 30 weeks.

$$14, 8, 12, 9, 5, 22, 15, 12, 16, 7, 10, 9, 15, 15, 12,$$
$$9, 11, 16, 8, 7, 15, 13, 9, 5, 18, 14, 10, 13, 7, 11$$

Assuming that the numbers sold each week are independent random variables from a common distribution, use the data to obtain a 95 percent confidence interval for the mean number sold in a week.

31. A random sample of 16 professors at a large private university yielded a sample mean annual salary of \$90,450 with a sample standard deviation of \$9,400. Determine a 95 percent confidence interval of the average salary of all professors at that university.

32. Let $X_1, \ldots, X_{n+1}$ be a sample from a population with mean μ and variance σ^2. As noted in the text, the natural predictor of X_{n+1} based on the data values $X_1, \ldots, X_n$ is $\bar{X}_n = \sum_{i=1}^{n} X_i/n$. Determine the mean square error of this predictor. That is, find $E[(X_{n+1} - \bar{X}_n)^2]$.

33. National Safety Council data show that the number of accidental deaths due to drowning in the United States in the years from 1990 to 1993 were (in units of one thousand) 5.2, 4.6, 4.3, 4.8. Use these data to give an interval that will, with 95 percent confidence, contain the number of such deaths in 1994.

34. The daily dissolved oxygen concentration for a water stream has been recorded over 30 days. If the sample average of the 30 values is 2.5 mg/liter and the sample standard deviation is 2.12 mg/liter, determine a value which, with 90 percent confidence, exceeds the mean daily concentration.

35. Verify the formulas given in Table 7.1 for the $100(1 - \alpha)$ percent lower and upper confidence intervals for σ^2.

36. The capacities (in ampere-hours) of 10 batteries were recorded as follows:

$$140, 136, 150, 144, 148, 152, 138, 141, 143, 151$$

(a) Estimate the population variance σ^2.
(b) Compute a 99 percent two-sided confidence interval for σ^2.
(c) Compute a value v that enables us to state, with 90 percent confidence, that σ^2 is less than v.

37. Find a 95 percent two-sided confidence interval for the variance of the diameter of a rivet based on the data given here.

6.68	6.66	6.62	6.72
6.76	6.67	6.70	6.72
6.78	6.66	6.76	6.72
6.76	6.70	6.76	6.76
6.74	6.74	6.81	6.66
6.64	6.79	6.72	6.82
6.81	6.77	6.60	6.72
6.74	6.70	6.64	6.78
6.70	6.70	6.75	6.79

Assume a normal population.

38. The following are independent samples from two normal populations, both of which have the same standard deviation σ.

$$16, 17, 19, 20, 18 \qquad \text{and} \qquad 3, 4, 8$$

Use them to estimate σ.

39. The amount of beryllium in a substance is often determined by the use of a photometric filtration method. If the weight of the beryllium is μ, then the value given by the photometric filtration method is normally distributed with mean μ and standard deviation σ. A total of eight independent measurements of 3.180 mg of beryllium gave the following results.

$$3.166, 3.192, 3.175, 3.180, 3.182, 3.171, 3.184, 3.177$$

Use the preceding data to

(a) estimate σ;

(b) find a 90 percent confidence interval estimate of σ.

40. If $X_1, \ldots, X_n$ is a sample from a normal population, explain how to obtain a $100(1-\alpha)$ percent confidence interval for the population variance σ^2 when the population mean μ is known. Explain in what sense knowledge of μ improves the interval estimator compared with when it is unknown.
Repeat Problem 38 if it is known that the mean burning time is 53.6 seconds.

41. A civil engineer wishes to measure the compressive strength of two different types of concrete. A random sample of 10 specimens of the first type yielded the following data (in psi)

Type 1:	3,250,	3,268,	4,302,	3,184,	3,266
	3,297,	3,332,	3,502,	3,064,	3,116

whereas a sample of 10 specimens of the second yielded the data

Type 2:	3,094,	3,106,	3,004,	3,066,	2,984,
	3,124,	3,316,	3,212,	3,380,	3,018

If we assume that the samples are normal with a common variance, determine

(a) a 95 percent two-sided confidence interval for $\mu_1 - \mu_2$, the difference in means;

(b) a 95 percent one-sided upper confidence interval for $\mu_1 - \mu_2$;

(c) a 95 percent one-sided lower confidence interval for $\mu_1 - \mu_2$.

42. Independent random samples are taken from the output of two machines on a production line. The weight of each item is of interest. From the first machine, a sample of size 36 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 64 is taken, with a sample mean weight of 130 grams and a sample variance of 5. It is assumed that the weights of items from the first machine are normally distributed with mean μ_1 and variance σ^2 and that the weights of items from the second machine are normally distributed with mean μ_2 and variance σ^2 (that is, the variances are assumed to be equal). Find a 99 percent confidence interval for $\mu_1 - \mu_2$, the difference in population means.

43. Do Problem 42 when it is known in advance that the population variances are 4 and 5.

44. The following are the daily numbers of company website visits resulting from advertisements on two different types of media.

Type I		*Type II*	
481	572	526	537
506	561	511	582
527	501	556	605
661	487	542	558
501	524	491	578

Find a 99 percent confidence interval for the mean difference in daily visits assuming normality with unknown but equal variances.

45. If $X_1, \ldots, X_n$ is a sample from a normal population having known mean μ_1 and unknown variance σ_1^2, and $Y_1, \ldots, Y_m$ is an independent sample from a normal population having known mean μ_2 and unknown variance σ_2^2, determine a $100(1 - \alpha)$ percent confidence interval for σ_1^2/σ_2^2.

46. Two analysts took repeated readings on the hardness of city water. Assuming that the readings of analyst i constitute a sample from a normal population having variance σ_i^2, $i = 1, 2$, compute a 95 percent two-sided confidence interval for σ_1^2/σ_2^2 when the data are as follows:

Coded Measures of Hardness

Analyst 1	**Analyst 2**
.46	.82
.62	.61
.37	.89
.40	.51
.44	.33
.58	.48
.48	.23
.53	.25
	.67
	.88

47. A problem of interest in baseball is whether a sacrifice bunt is a good strategy when there is a man on first base and no outs. Assuming that the bunter will be out but will be successful in advancing the man on base, we could compare the probability of scoring a run with a player on first base and no outs with the probability of scoring a run with a player on second base and one out.

The following data resulted from a study of randomly chosen major league baseball games played in 1959 and 1960.

(a) Give a 95 percent confidence interval estimate for the probability of scoring at least one run when there is a man on first and no outs.
(b) Give a 95 percent confidence interval estimate for the probability of scoring at least one run when there is a man on second and one out.

Base Occupied	Number of Outs	Number of Cases in Which 0 Runs Are Scored	Total Number of Cases
First	0	1,044	1,728
Second	1	401	657

48. A random sample of 1,200 engineers included 48 Hispanic Americans, 80 African Americans, and 204 females. Determine 90 percent confidence intervals for the proportion of all engineers who are

(a) female;
(b) Hispanic Americans or African Americans.

49. To estimate p, the proportion of all newborn babies that are male, the gender of 10,000 newborn babies was noted. If 5,106 of them were male, determine **(a)** a 90 percent and **(b)** a 99 percent confidence interval estimate of p.

50. An airline is interested in determining the proportion of its customers who are flying for reasons of business. If they want to be 90 percent certain that their estimate will be correct to within 2 percent, how large a random sample should they select?

51. A recent newspaper poll indicated that Candidate A is favored over Candidate B by a 53 to 47 percentage, with a margin of error of ± 4 percent. The newspaper then stated that since the 6-point gap is larger than the margin of error, its readers can be certain that Candidate A is the current choice. Is this reasoning correct?

52. A market research firm is interested in determining the proportion of households that are watching a particular sporting event. To accomplish this task, they plan on using a telephone poll of randomly chosen households. How large a sample is needed if they want to be 90 percent certain that their estimate is correct to within $\pm .02$?

53. In a recent study, 79 of 140 meteorites were observed to enter the atmosphere with a velocity of less than 25 miles per second. If we take $\hat{p} = 79/140$ as an estimate of the probability that an arbitrary meteorite that enters the atmosphere will have a speed less than 25 miles per second, what can we say, with 99 percent confidence, about the maximum error of our estimate?

54. A random sample of 100 items from a production line revealed 17 of them to be defective. Compute a 95 percent two-sided confidence interval for the probability that an item produced is defective. Determine also a 99 percent upper confidence interval for this value. What assumptions are you making?

55. Of 100 randomly detected cases of individuals having lung cancer, 67 died within 5 years of detection.

- **(a)** Estimate the probability that a person contracting lung cancer will die within 5 years.
- **(b)** How large an additional sample would be required to be 95 percent confident that the error in estimating the probability in part (a) is less than .02?

56. Derive $100(1-\alpha)$ percent lower and upper confidence intervals for p, when the data consist of the values of n independent Bernoulli random variables with parameter p.

57. Suppose the lifetimes of batteries are exponentially distributed with mean θ. If the average of a sample of 10 batteries is 36 hours, determine a 95 percent two-sided confidence interval for θ.

58. Determine $100(1-\alpha)$ percent one-sided upper and lower confidence intervals for θ in Problem 57.

59. Let $X_1, X_2, \ldots, X_n$ denote a sample from a population whose mean value θ is unknown. Use the results of Example 7.7b to argue that among all unbiased estimators of θ of the form $\sum_{i=1}^{n} \lambda_i X_i$, $\sum_{i=1}^{n} \lambda_i = 1$, the one with minimal mean square error has $\lambda_i \equiv 1/n$, $i = 1, \ldots, n$.

60. Consider two independent samples from normal populations having the same variance σ^2, of respective sizes n and m. That is, $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are independent samples from normal populations each having variance σ^2. Let S_x^2 and S_y^2 denote the respective sample variances. Thus both S_x^2 and S_y^2 are unbiased estimators of σ^2. Show by using the results of Example 7.7b along with the fact that

$$\mathrm{Var}(\chi_k^2) = 2k$$

where χ_k^2 is chi-square with k degrees of freedom, that the minimum mean square estimator of σ^2 of the form $\lambda S_x^2 + (1-\lambda)S_y^2$ is

$$S_p^2 = \frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}$$

This is called the *pooled estimator* of σ^2.

61. Consider two estimators d_1 and d_2 of a parameter θ. If $E[d_1] = \theta$, $\text{Var}(d_1) = 6$ and $E[d_2] = \theta + 2$, $\text{Var}(d_2) = 2$, which estimator should be preferred?

62. Suppose that the number of accidents occurring daily in a certain plant has a Poisson distribution with an unknown mean λ. Based on previous experience in similar industrial plants, suppose that a statistician's initial feelings about the possible value of λ can be expressed by an exponential distribution with parameter 1. That is, the prior density is

$$p(\lambda) = e^{-\lambda}, \qquad 0 < \lambda < \infty$$

Determine the Bayes estimate of λ if there are a total of 83 accidents over the next 10 days. What is the maximum likelihood estimate?

63. The functional lifetimes in hours of computer chips produced by a certain semiconductor firm are exponentially distributed with mean $1/\lambda$. Suppose that the prior distribution on λ is the gamma distribution with density function

$$g(x) = \frac{e^{-x}x^2}{2}, \qquad 0 < x < \infty$$

If the average life of the first 20 chips tested is 4.6 hours, compute the Bayes estimate of λ.

64. Each item produced will, independently, be defective with probability p. If the prior distribution on p is uniform on (0, 1), compute the posterior probability that p is less than .2 given

(a) a total of 2 defectives out of a sample of size 10;
(b) a total of 1 defective out of a sample of size 10;
(c) a total of 10 defectives out of a sample of size 10.

65. The breaking strength of a certain type of cloth is to be measured for 10 specimens. The underlying distribution is normal with unknown mean θ but with a standard deviation equal to 3 psi. Suppose also that based on previous experience we feel that the unknown mean has a prior distribution that is normally distributed with mean 200 and standard deviation 2. If the average breaking strength of a sample of 20 specimens is 182 psi, determine a region that contains θ with probability .95.

SOLUTION Since there is a small probability of an industrial accident in any given minute, it would seem that the weekly number of such accidents should have approximately a Poisson distribution. If we let X_1 denote the total number of accidents during an 8-week period at plant 1, and let X_2 be the number during a 6-week period at plant 2, then if the safety conditions did not differ at the two plants we would have that

$$\lambda_2 = \tfrac{3}{4}\lambda_1$$

where $\lambda_i \equiv E[X_i]$, $i = 1, 2$. Hence, as $X_1 = 133, X_2 = 149$ it follows that the p-value of the test of

$$H_0 : \lambda_2 = \tfrac{3}{4}\lambda_1 \quad \text{versus} \quad H_1 : \lambda_2 \neq \tfrac{3}{4}\lambda_1$$

is given by

$$\begin{aligned} p\text{-value} &= 2\min\left(P\left\{\text{Bin}\left(282, \tfrac{4}{7}\right) \geq 133\right\}, P\left\{\text{Bin}\left(282, \tfrac{4}{7}\right) \leq 133\right\}\right) \\ &= 9.408 \times 10^{-4} \end{aligned}$$

Thus, the hypothesis that the safety conditions at the two plants are equivalent is rejected. ■

Chapter 8 Problems

1. Consider a trial in which a jury must decide between the hypothesis that the defendant is guilty and the hypothesis that he or she is innocent.
   (a) In the framework of hypothesis testing and the U.S. legal system, which of the hypotheses should be the null hypothesis?
   (b) What do you think would be an appropriate significance level in this situation?

2. A colony of laboratory mice consists of several thousand mice. The average weight of all the mice is 32 grams with a standard deviation of 4 grams. A laboratory assistant was asked by a scientist to select 25 mice for an experiment. However, before performing the experiment the scientist decided to weigh the mice as an indicator of whether the assistant's selection constituted a random sample or whether it was made with some unconscious bias (perhaps the mice selected were the ones that were slowest in avoiding the assistant, which might indicate some inferiority about this group). If the sample mean of the

25 mice was 30.4, would this be significant evidence, at the 5 percent level of significance, against the hypothesis that the selection constituted a random sample?

3. A population distribution is known to have standard deviation 20. Determine the p-value of a test of the hypothesis that the population mean is equal to 50, if the average of a sample of 64 observations is
(a) 52.5; **(b)** 55.0; **(c)** 57.5.

4. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of .02. Suppose 10 independent measurements yielded the following pH values:

8.18	8.17
8.16	8.15
8.17	8.21
8.22	8.16
8.19	8.18

(a) What conclusion can be drawn at the $\alpha = .10$ level of significance?
(b) What about at the $\alpha = .05$ level of significance?

5. The mean breaking strength of a certain type of fiber is required to be at least 200 psi. Past experience indicates that the standard deviation of breaking strength is 5 psi. If a sample of 8 pieces of fiber yielded breakage at the following pressures,

210	198
195	202
197.4	196
199	195.5

would you conclude, at the 5 percent level of significance, that the fiber is unacceptable? What about at the 10 percent level of significance?

6. It is known that the average height of a man residing in the United States is 5 feet 10 inches and the standard deviation is 3 inches. To test the hypothesis that men in your city are "average," a sample of 20 men have been chosen. The heights of the men in the sample follow:

Man	Height in	Inches	Man
1	72	70.4	11
2	68.1	76	12
3	69.2	72.5	13
4	72.8	74	14
5	71.2	71.8	15
6	72.2	69.6	16
7	70.8	75.6	17
8	74	70.6	18
9	66	76.2	19
10	70.3	77	20

What do you conclude? Explain what assumptions you are making.

7. Suppose in Problem 4 that we wished to design a test so that if the pH were really equal to 8.20, then this conclusion will be reached with probability equal to .95. On the other hand, if the pH differs from 8.20 by .03 (in either direction), we want the probability of picking up such a difference to exceed .95.
   (a) What test procedure should be used?
   (b) What is the required sample size?
   (c) If $\overline{x} = 8.31$, what is your conclusion?
   (d) If the actual pH is 8.32, what is the probability of concluding that the pH is not 8.20, using the foregoing procedure?

8. Verify that the approximation in Equation 8.3.7 remains valid even when $\mu_1 < \mu_0$.

9. A British pharmaceutical company, Glaxo Holdings, has recently developed a new drug for migraine headaches. Among the claims Glaxo made for its drug, called somatriptan, was that the mean time it takes for it to enter the bloodstream is less than 10 minutes. To convince the Food and Drug Administration of the validity of this claim, Glaxo conducted an experiment on a randomly chosen set of migraine sufferers. To prove its claim, what should they have taken as the null and what as the alternative hypothesis?

10. The weights of salmon grown at a commercial hatchery are normally distributed with a standard deviation of 1.2 pounds. The hatchery claims that the mean weight of this year's crop is at least 7.6 pounds. Suppose a random sample of 16 fish yielded an average weight of 7.2 pounds. Is this strong enough evidence to reject the hatchery's claims at the
    (a) 5 percent level of significance;
    (b) 1 percent level of significance?
    (c) What is the p-value?

11. Consider a test of $H_0 : \mu \leq 100$ versus $H_1 : \mu > 100$. Suppose that a sample of size 20 has a sample mean of $\overline{X} = 105$. Determine the p-value of this outcome if the population standard deviation is known to equal
(a) 5; **(b)** 10; **(c)** 15.

12. An advertisement for a new toothpaste claims that it reduces cavities of children in their cavity-prone years. Cavities per year for this age group are normal with mean 3 and standard deviation 1. A study of 2,500 children who used this toothpaste found an average of 2.95 cavities per child. Assume that the standard deviation of the number of cavities of a child using this new toothpaste remains equal to 1.

(a) Are these data strong enough, at the 5 percent level of significance, to establish the claim of the toothpaste advertisement?

(b) Do the data convince you to switch to this new toothpaste?

13. There is some variability in the amount of phenobarbital in each capsule sold by a manufacturer. However, the manufacturer claims that the mean value is 20.0 mg. To test this, a sample of 25 pills yielded a sample mean of 19.7 with a sample standard deviation of 1.3. What inference would you draw from these data? In particular, are the data strong enough evidence to discredit the claim of the manufacturer? Use the 5 percent level of significance.

14. Twenty years ago, entering male high school students of Central High could do an average of 24 pushups in 60 seconds. To see whether this remains true today, a random sample of 36 freshmen was chosen. If their average was 22.5 with a sample standard deviation of 3.1, can we conclude that the mean is no longer equal to 24? Use the 5 percent level of significance.

15. The mean response time of a species of pigs to a stimulus is .8 seconds. Twenty-eight pigs were given 2 oz of alcohol and then tested. If their average response time was 1.0 seconds with a standard deviation of .3 seconds, can we conclude that alcohol affects the mean response time? Use the 5 percent level of significance.

16. Suppose that team A and team B are to play a National Football League game and that team A is favored by f points. Let $S(A)$ and $S(B)$ denote the scores of teams A and B, and let $X = S(A) - S(B) - f$. That is, X is the amount by which team A beats the point spread. It has been claimed that the distribution of X is normal with mean 0 and standard deviation 14. Use data from randomly chosen football games to test this hypothesis.

17. A medical scientist believes that the average basal temperature of (outwardly) healthy individuals has increased over time and is now greater than 98.6 degrees Fahrenheit (37 degrees Celsius). To prove this, she has randomly selected 100 healthy individuals. If their mean temperature is 98.74 with a sample standard deviation of 1.1 degrees, does this prove her claim at the 5 percent level? What about at the 1 percent level?

18. Use the results of a Sunday's worth of NFL professional football games to test the hypothesis that the average number of points scored by winning teams is less than or equal to 28. Use the 5 percent level of significance.

19. Use the results of a Sunday's worth of major league baseball scores to test the hypothesis that the average number of runs scored by winning teams is at least 5.6. Use the 5 percent level of significance.

20. A car is advertised as having a gas mileage rating of at least 30 miles/gallon in highway driving. If the miles per gallon obtained in 10 independent experiments are 26, 24, 20, 25, 27, 25, 28, 30, 26, 33, should you believe the advertisement? What assumptions are you making?

21. A producer specifies that the mean lifetime of a certain type of battery is at least 240 hours. A sample of 18 such batteries yielded the following data.

237	242	232
242	248	230
244	243	254
262	234	220
225	236	232
218	228	240

Assuming that the life of the batteries is approximately normally distributed, do the data indicate that the specifications are not being met?

22. Use the data of Example 2.3i of Chapter 2 to test the null hypothesis that the average noise level directly outside of Grand Central Station is less than or equal to 80 decibels.

23. An oil company claims that the sulfur content of its diesel fuel is at most .15 percent. To check this claim, the sulfur contents of 40 randomly chosen samples were determined; the resulting sample mean and sample standard deviation were .162 and .040. Using the 5 percent level of significance, can we conclude that the company's claims are invalid?

24. A company supplies plastic sheets for industrial use. A new type of plastic has been produced and the company would like to claim that the average stress resistance of this new product is at least 30.0, where stress resistance is measured in pounds per square inch (psi) necessary to crack the sheet. The following random sample was drawn off the production line. Based on this sample, would the claim clearly be unjustified?

30.1	32.7	22.5	27.5
27.7	29.8	28.9	31.4
31.2	24.3	26.4	22.8
29.1	33.4	32.5	21.7

Assume normality and use the 5 percent level of significance.

25. It is claimed that a certain type of bipolar transistor has a mean value of current gain that is at least 210. A sample of these transistors is tested. If the sample mean value of current gain is 200 with a sample standard deviation of 35, would the claim be rejected at the 5 percent level of significance if

(a) the sample size is 25;
(b) the sample size is 64?

26. A manufacturer of capacitors claims that the breakdown voltage of these capacitors has a mean value of at least 100 V. A test of 12 of these capacitors yielded the following breakdown voltages:

$$96, 98, 105, 92, 111, 114, 99, 103, 95, 101, 106, 97$$

Do these results prove the manufacturer's claim? Do they disprove them?

27. A sample of 10 fish were caught at lake A and their PCB concentrations were measured using a certain technique. The resulting data in parts per million were

Lake A: 11.5, 10.8, 11.6, 9.4, 12.4, 11.4, 12.2, 11, 10.6, 10.8

In addition, a sample of 8 fish were caught at lake B and their levels of PCB were measured by a different technique than that used at lake A. The resultant data were

Lake B: 11.8, 12.6, 12.2, 12.5, 11.7, 12.1, 10.4, 12.6

If it is known that the measuring technique used at lake A has a variance of .09 whereas the one used at lake B has a variance of .16, could you reject (at the 5 percent level of significance) a claim that the two lakes are equally contaminated?

28. A method for measuring the pH level of a solution yields a measurement value that is normally distributed with a mean equal to the actual pH of the solution and with a standard deviation equal to .05. An environmental pollution scientist claims that two different solutions come from the same source. If this were so, then the pH level of the solutions would be equal. To test the plausibility of this claim, 10 independent measurements were made of the pH level for both solutions, with the following data resulting.

Measurements of Solution A	Measurements of Solution B
6.24	6.27
6.31	6.25
6.28	6.33
6.30	6.27
6.25	6.24
6.26	6.31
6.24	6.28
6.29	6.29
6.22	6.34
6.28	6.27

(a) Do the data disprove the scientist's claim? Use the 5 percent level of significance.

(b) What is the p-value?

29. The following are the values of independent samples from two different populations.

Sample 1	122, 114, 130, 165, 144, 133, 139, 142, 150
Sample 2	108, 125, 122, 140, 132, 120, 137, 128, 138

Let μ_1 and μ_2 be the respective means of the two populations. Find the p-value of the test of the null hypothesis

$$H_0 : \mu_1 \leq \mu_2$$

versus the alternative

$$H_1 : \mu_1 > \mu_2$$

when the population standard deviations are $\sigma_1 = 10$ and
(a) $\sigma_2 = 5$; **(b)** $\sigma_2 = 10$; **(c)** $\sigma_2 = 20$.

30. The data below give the lifetimes in hundreds of hours of samples of two types of electronic tubes. Past lifetime data of such tubes have shown that they can often be modeled as arising from a lognormal distribution. That is, the logarithms of the data are normally distributed. Assuming that variance of the logarithms is equal for the two populations, test, at the 5 percent level of significance, the hypothesis that the two population distributions are identical.

Type 1	32, 84, 37, 42, 78, 62, 59, 74
Type 2	39, 111, 55, 106, 90, 87, 85

31. The viscosity of two different brands of car oil is measured and the following data resulted:

Brand 1	10.62, 10.58, 10.33, 10.72, 10.44, 10.74
Brand 2	10.50, 10.52, 10.58, 10.62, 10.55, 10.51, 10.53

Test the hypothesis that the mean viscosity of the two brands is equal, assuming that the populations have normal distributions with equal variances.

32. It is argued that the resistance of wire A is greater than the resistance of wire B. You make tests on each wire with the following results.

Wire A	**Wire B**
.140 ohm	.135 ohm
.138	.140
.143	.136
.142	.142
.144	.138
.137	.140

What conclusion can you draw at the 10 percent significance level? Explain what assumptions you are making.

In Problems 33 through 40, assume that the population distributions are normal and have equal variances.

33. Twenty-five men between the ages of 25 and 30, who were participating in a well-known heart study carried out in Framingham, Massachusetts, were randomly selected. Of these, 11 were smokers and 14 were not. The following data refer to readings of their systolic blood pressure.

Smokers	Nonsmokers
124	130
134	122
136	128
125	129
133	118
127	122
135	116
131	127
133	135
125	120
118	122
	120
	115
	123

Use these data to test the hypothesis that the mean blood pressures of smokers and nonsmokers are the same.

34. In a 1943 experiment (Whitlock and Bliss, "A Bioassay Technique for Antihelminthics," *Journal of Parasitology*, **29**, pp. 48–58) 10 albino rats were used to study the effectiveness of carbon tetrachloride as a treatment for worms. Each rat received an injection of worm larvae. After 8 days, the rats were randomly divided into two groups of 5 each; each rat in the first group received a dose of .032 cc of carbon tetrachloride, whereas the dosage for each rat in the second group was .063 cc. Two days later the rats were killed, and the number of adult worms in each rat was determined. The numbers detected in the group receiving the .032 dosage were

$$421, 462, 400, 378, 413$$

whereas they were

$$207, 17, 412, 74, 116$$

for those receiving the .063 dosage. Do the data prove that the larger dosage is more effective than the smaller?

35. A professor claims that the average starting salary of industrial engineering graduating seniors is greater than that of civil engineering graduates. To study this claim, samples of 16 industrial engineers and 16 civil engineers, all of whom graduated in 2006, were chosen and sample members were queried about their starting salaries. If the industrial engineers had a sample mean salary of \$72,700 and a sample standard deviation of \$2,400, and the civil engineers had a sample mean salary of \$71,400 and a sample standard deviation of \$2,200, has the professor's claim been verified? Find the appropriate p-value.

36. In a certain experimental laboratory, a method A for producing gasoline from crude oil is being investigated. Before completing experimentation, a new method B is proposed. All other things being equal, it was decided to abandon A in favor of B only if the average yield of the latter was clearly greater. The yield of both processes is assumed to be normally distributed. However, there has been insufficient time to ascertain their true standard deviations, although there appears to be no reason why they cannot be assumed equal. Cost considerations impose size limits on the size of samples that can be obtained. If a 1 percent significance level is all that is allowed, what would be your recommendation based on the following random samples? The numbers represent percent yield of crude oil.

A	23.2, 26.6, 24.4, 23.5, 22.6, 25.7, 25.5
B	25.7, 27.7, 26.2, 27.9, 25.0, 21.4, 26.1

37. A study was instituted to learn how the diets of women changed during the winter and the summer. A random group of 12 women were observed during the month of July and the percentage of each woman's calories that came from fat was determined. Similar observations were made on a different randomly selected group of size 12 during the month of January. The results were as follows:

July	32.2, 27.4, 28.6, 32.4, 40.5, 26.2, 29.4, 25.8, 36.6, 30.3, 28.5, 32.0
January	30.5, 28.4, 40.2, 37.6, 36.5, 38.8, 34.7, 29.5, 29.7, 37.2, 41.5, 37.0

Test the hypothesis that the mean fat percentage intake is the same for both months. Use the **(a)** 5 percent level of significance and **(b)** 1 percent level of significance.

38. To learn about the feeding habits of bats, 22 bats were tagged and tracked by radio. Of these 22 bats, 12 were female and 10 were male. The distances flown (in meters) between feedings were noted for each of the 22 bats, and the following summary statistics were obtained.

Female Bats	Male Bats
$n = 12$	$m = 10$
$\overline{X} = 180$	$\overline{Y} = 136$
$S_x = 92$	$S_y = 86$

Test the hypothesis that the mean distance flown between feedings is the same for the populations of both male and of female bats. Use the 5 percent level of significance.

39. The following data summary was obtained from a comparison of the lead content of human hair removed from adult individuals that had died between 1880 and 1920 with the lead content of present-day adults. The data are in units of micrograms, equal to one-millionth of a gram.

	1880–1920	**Today**
Sample size:	30	100
Sample mean:	48.5	26.6
Sample standard deviation:	14.5	12.3

(a) Do the above data establish, at the 1 percent level of significance, that the mean lead content of human hair is less today than it was in the years between 1880 and 1920? Clearly state what the null and alternative hypotheses are.

(b) What is the p-value for the hypothesis test in part (a)?

40. Sample weights (in pounds) of newborn babies born in two adjacent counties in western Pennsylvania yielded the following data.

$$n = 53, \qquad m = 44$$

$$\overline{X} = 6.8, \qquad \overline{Y} = 7.2$$

$$S^2 = 5.2, \qquad S^2 = 4.9$$

Consider a test of the hypothesis that the mean weight of newborns is the same in both counties. What is the resulting p-value?

41. To verify the hypothesis that blood lead levels tend to be higher for children whose parents work in a factory that uses lead in the manufacturing process, researchers examined lead levels in the blood of 33 children whose parents worked in a battery manufacturing factory (Morton, D., Saah, A., Silberg, S., Owens, W., Roberts, M., and Saah, M., "Lead Absorption in Children of Employees in a Lead-Related Industry," *American Journal of Epidemiology*, **115**, 549–555, 1982). Each of these children was then *matched* by another child who was of similar age, lived in a similar neighborhood, had a similar exposure to traffic, but whose parent did not work with lead. The blood levels of the 33 cases (sample 1) as well as those of the 33 controls (sample 2) were then used to test the hypothesis that the average blood levels of these groups are the same. If the resulting sample means and sample standard deviations were

$$\bar{x}_1 = .015, \quad s_1 = .004, \quad \bar{x}_2 = .006, \quad s_2 = .006$$

find the resulting p-value. Assume a common variance.

42. Ten pregnant women were given an injection of pitocin to induce labor. Their systolic blood pressures immediately before and after the injection were:

Patient	Before	After	Patient	Before	After
1	134	140	6	140	138
2	122	130	7	118	124
3	132	135	8	127	126
4	130	126	9	125	132
5	128	134	10	142	144

Do the data indicate that injection of this drug changes blood pressure?

43. A question of medical importance is whether jogging leads to a reduction in one's pulse rate. To test this hypothesis, 8 nonjogging volunteers agreed to begin a 1-month jogging program. After the month their pulse rates were determined and compared with their earlier values. If the data are as follows, can we conclude that jogging has had an effect on the pulse rates?

Subject	1	2	3	4	5	6	7	8
Pulse Rate Before	74	86	98	102	78	84	79	70
Pulse Rate After	70	85	90	110	71	80	69	74

44. If $X_1, \ldots, X_n$ is a sample from a normal population having unknown parameters μ and σ^2, devise a significance level α test of

$$H_0 = \sigma^2 \leq \sigma_0^2$$

versus the alternative

$$H_1 = \sigma^2 > \sigma_0^2$$

for a given positive value σ_0^2.

45. In Problem 44, explain how the test would be modified if the population mean μ were known in advance.

46. A gun-like apparatus has recently been designed to replace needles in administering vaccines. The apparatus can be set to inject different amounts of the serum, but because of random fluctuations the actual amount injected is normally distributed with a mean equal to the setting and with an unknown variance σ^2. It has been decided that the apparatus would be too dangerous to use if σ exceeds .10. If a random sample of 50 injections resulted in a sample standard deviation of .08, should use of the new apparatus be discontinued? Suppose the level of significance is $\alpha = .10$. Comment on the appropriate choice of a significance level for this problem, as well as the appropriate choice of the null hypothesis.

47. A pharmaceutical house produces a certain drug item whose weight has a standard deviation of .5 milligrams. The company's research team has proposed a new method of producing the drug. However, this entails some costs and will be adopted only if there is strong evidence that the standard deviation of the weight of the items will drop to below .4 milligrams. If a sample of 10 items is produced and has the following weights, should the new method be adopted?

5.728	5.731
5.722	5.719
5.727	5.724
5.718	5.726
5.723	5.722

48. The production of large electrical transformers and capacitators requires the use of polychlorinated biphenyls (PCBs), which are extremely hazardous when released into the environment. Two methods have been suggested to monitor the levels of PCB in fish near a large plant. It is believed that each method will result in a normal random variable that depends on the method. Test the hypothesis at the $\alpha = .10$ level of significance that both methods have the same variance, if a given fish is checked 8 times by each method with the following data (in parts per million) recorded.

Method 1	6.2, 5.8, 5.7, 6.3, 5.9, 6.1, 6.2, 5.7
Method 2	6.3, 5.7, 5.9, 6.4, 5.8, 6.2, 6.3, 5.5

49. In Problem 31, test the hypothesis that the populations have the same variances.

50. If $X_1, \ldots, X_n$ is a sample from a normal population with variance σ_x^2, and $Y_1, \ldots, Y_n$ is an independent sample from normal population with variance σ_y^2, develop a significance level α test of

$$H_0 : \sigma_x^2 < \sigma_y^2 \qquad \text{versus} \qquad H_1 : \sigma_x^2 > \sigma_y^2$$

51. The amount of surface wax on each side of waxed paper bags is believed to be normally distributed. However, there is reason to believe that there is greater variation in the amount on the inner side of the paper than on the outside. A sample of 75 observations of the amount of wax on each side of these bags is obtained and the following data recorded.

Wax in Pounds per Unit Area of Sample	
Outside Surface	**Inside Surface**
$\bar{x} = .948$	$\bar{y} = .652$
$\sum x_i^2 = 91$	$\sum y_i^2 = 82$

Conduct a test to determine whether or not the variability of the amount of wax on the inner surface is greater than the variability of the amount on the outer surface ($\alpha = .05$).

52. In a famous experiment to determine the efficacy of aspirin in preventing heart attacks, 22,000 healthy middle-aged men were randomly divided into two equal groups, one of which was given a daily dose of aspirin and the other a placebo that looked and tasted identical to the aspirin. The experiment was halted at a time when 104 men in the aspirin group and 189 in the control group had had heart attacks. Use these data to test the hypothesis that the taking of aspirin does not change the probability of having a heart attack.

53. In the study of Problem 52, it also resulted that 119 from the aspirin group and 98 from the control group suffered strokes. Are these numbers significant to show that taking aspirin changes the probability of having a stroke?

54. A standard drug is known to be effective in 72 percent of the cases in which it is used to treat a certain infection. A new drug has been developed and testing has found it to be effective in 42 cases out of 50. Is this strong enough evidence to prove that the new drug is more effective than the old one? Find the relevant p-value.

55. Three independent news services are running a poll to determine if over half the population supports an initiative concerning limitations on driving automobiles in the downtown area. Each wants to see if the evidence indicates that over half the population is in favor. As a result, all three services will be testing

$$H_0 : p \leq .5 \quad \text{versus} \quad H_1 : p > .5$$

where p is the proportion of the population in favor of the initiative.

(a) Suppose the first news organization samples 100 people, of which 56 are in favor of the initiative. Is this strong enough evidence, at the 5 percent level of significance, to reject the null hypothesis and so establish that over half the population favors the initiative?

(b) Suppose the second news organization samples 120 people, of which 68 are in favor of the initiative. Is this strong enough evidence, at the 5 percent level of significance, to reject the null hypothesis?

(c) Suppose the third news organization samples 110 people, of which 62 are in favor of the initiative. Is this strong enough evidence, at the 5 percent level of significance, to reject the null hypothesis?

(d) Suppose the news organizations combine their samples, to come up with a sample of 330 people, of which 186 support the initiative. Is this strong enough evidence, at the 5 percent level of significance, to reject the null hypothesis?

56. It has been a long held belief that the proportion of California births of African America mothers that result in twins is about 1.32 percent. (The twinning rate appears to be influenced by the ethnicity of the mother; claims are that it is 1.05 for Caucasian Americans, and 0.72 percent for Asian Americans.) A public health scientist believes that this number is no longer correct and has decided to test the null hypothesis that the proportion is 1.32 percent by gathering data on the next 1, 000 recorded birthing events, where twin birds are regarded as a single birthing event, in California.

(a) What is the minimal number of twin births that would have to be observed in order to reject the null hypothesis at the 5 percent level of significance?

(b) What is the probability the null hypothesis will be rejected if the actual twinning rate is 1.80?

57. An ambulance service claims that at least 45 percent of its calls involve life-threatening emergencies. To check this claim, a random sample of 200 calls was selected from the service's files. If 70 of these calls involved life-threatening emergencies, is the service's claim believable at the

(a) 5 percent level of significance;

(b) 1 percent level of significance?

58. A standard drug is known to be effective in 75 percent of the cases in which it is used to treat a certain infection. A new drug has been developed and has been found to be effective in 42 cases out of 50. Based on this, would you accept, at the 5 percent level of significance, the hypothesis that the two drugs are of equal effectiveness? What is the p-value?

59. Do Problem 58 by using a test based on the normal approximation to the binomial.

60. In a study of the effect of two chemotherapy treatments on the survival of patients with multiple myeloma, each of 156 patients was equally likely to be given either one of the two treatments. As reported by Lipsitz, Dear, Laird, and Molenberghs in a 1998 paper in *Biometrics*, the result of this was that 39 of the 72 patients given the first treatment and 44 of the 84 patients given the second treatment survived for over 5 years.

(a) Use these data to test the null hypothesis that the two treatments are equally effective.

(b) Is the fact that 72 of the patients received one of the treatments while 84 received the other consistent with the claim that the determination of the treatment to be given to each patient was made in a totally random fashion?

61. Let X_1 denote a binomial random variable with parameters (n_1, p_1) and X_2 an independent binomial random variable with parameters (n_2, p_2). Develop a test, using the same approach as in the Fisher-Irwin test, of

$$H_0 : p_1 \le p_2$$

versus the alternative

$$H_1 : p_1 > p_2$$

62. Verify that Equation 8.6.5 follows from Equation 8.6.4.

63. Let X_1 and X_2 be binomial random variables with respective parameters n_1, p_1 and n_2, p_2. Show that when n_1 and n_2 are large, an approximate level α test of $H_0 : p_1 = p_2$ versus $H_1 : p_1 \ne p_2$ is as follows:

$$\text{reject} \quad H_0 \quad \text{if} \quad \frac{|X_1/n_1 - X_2/n_2|}{\sqrt{\dfrac{X_1 + X_2}{n_1 + n_2}\left(1 - \dfrac{X_1 + X_2}{n_1 + n_2}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} > z_{\alpha/2}$$

Hint: **(a)** Argue first that when n_1 and n_2 are large

$$\frac{\dfrac{X_1}{n_1} - \dfrac{X_2}{n_2} - (p_1 - p_2)}{\sqrt{\dfrac{p_1(1 - p_1)}{n_1} + \dfrac{p_2(1 - p_2)}{n_2}}} \mathrel{\dot\sim} \mathcal{N}(0, 1)$$

where $\dot\sim$ means "approximately has the distribution."

(b) Now argue that when H_0 is true and so $p_1 = p_2$, their common value can be best estimated by $(X_1 + X_2)/(n_1 + n_2)$.

64. Use the approximate test given in Problem 63 on the data of Problem 60.

65. Patients suffering from cancer must often decide whether to have their tumors treated with surgery or with radiation. A factor in their decision is the 5-year survival rates for these treatments. Surprisingly, it has been found that patients' decisions often seem to be affected by whether they are told the 5-year survival rates or the 5-year death rates (even though the information content is identical). For instance, in an experiment a group of 200 male prostate cancer patients were randomly divided into two groups of size 100 each. Each member of the first group was told that the 5-year survival rate for those electing surgery was 77 percent, whereas each member of the second group was told that the 5-year death rate for those electing surgery was 23 percent. Both groups were given the same information about radiation therapy. If it resulted that 24 members of the first group and 12 of the second group elected to have surgery, what conclusions would you draw?

66. The following data refer to Larry Bird's results when shooting a pair of free throws in basketball. During two consecutive seasons in the National Basketball Association, Bird shot a pair of free throws on 338 occasions. On 251 occasions he made both shots; on 34 occasions he made the first shot but missed the second one; on 48 occasions he missed the first shot but made the second one; on 5 occasions he missed both shots.

(a) Use these data to test the hypothesis that Bird's probability of making the first shot is equal to his probability of making the second shot.

(b) Use these data to test the hypothesis that Bird's probability of making the second shot is the same regardless of whether he made or missed the first one.

67. In the 1970s, the U.S. Veterans Administration (Murphy, 1977) conducted an experiment comparing coronary artery bypass surgery with medical drug therapy as treatments for coronary artery disease. The experiment involved 596 patients, of whom 286 were randomly assigned to receive surgery, with the remaining 310 assigned to drug therapy. A total of 252 of those receiving surgery, and a total of 270 of those receiving drug therapy were still alive 3 years after treatment. Use these data to test the hypothesis that the survival probabilities are equal.

68. Test the hypothesis, at the 5 percent level of significance, that the yearly number of earthquakes felt on a certain island has mean 52 if the readings for the past 8 years are 46, 62, 60, 58, 47, 50, 59, 49. Assume an underlying Poisson distribution and give an explanation to justify this assumption.

69. In 1995, the Fermi Laboratory announced the discovery of the top quark, the last of six quarks predicted by the "standard model of physics." The evidence for its existence was statistical in nature and involved signals created when antiprotons and protons were forced to collide. In a *Physical Review Letters* paper documenting the evidence, Abe, Akimoto, and Akopian (known in physics circle as the three A's) based their conclusion on a theoretical analysis that indicated that the number of decay events in a certain time interval would have a Poisson distribution with a mean equal to 6.7 if a top quark did not exist and with a larger mean if it did exist. In a careful analysis of the data the three A's showed that the actual count was 27. Is this strong enough evidence to prove the hypothesis that the mean of the Poisson distribution was greater than 6.7?

70. For the following data, sample 1 is from a Poisson distribution with mean λ_1 and sample 2 is from a Poisson distribution with mean λ_2. Test the hypothesis that $\lambda_1 = \lambda_2$.

Sample 1	24, 32, 29, 33, 40, 28, 34, 36
Sample 2	42, 36, 41

71. A scientist looking into the effect of smoking on heart disease has chosen a large random sample of smokers and of nonsmokers. She plans to study these two groups for 5 years to see if the number of heart attacks among the members of the smokers' group is significantly greater than the number among the nonsmokers. Such a result, the scientist feels, should be strong evidence of an association between smoking and heart attacks. Given that

(a) older people are at greater risk of heart disease than are younger people; and
(b) as a group, smokers tend to be somewhat older than nonsmokers,

would the scientist be justified in her conclusion? Explain how the experimental design can be improved so that meaningful conclusions can be drawn.

72. A researcher wants to analyze the average yearly increase in a stock over a 20-year period. To do so, she plans to randomly choose 100 stocks from the listing of current stocks, discarding any that were not in existence 20 years ago. She will then compare the current price of each stock with its price 20 years ago to determine its percentage increase. Do you think this is a valid method to study the average increase in the price of a stock?