Ex6.3b $X_i \sim N(3, 0.09), \quad W \sim N(400, 1600)$

$$Y = X_1 + X_2 + ... + X_n - W \sim N(3n - 400, 0.09n + 1600)$$

$$P(Y \geq 0) = P\left(Z \geq \frac{-(3n-400)}{\sqrt{0.09n+1600}}\right) \approx 0.1$$

sol)

$$\frac{3n-400}{\sqrt{0.09n+1600}} \leq 1.28 \rightarrow n \geq 117$$

Application of Central Limit Theorem

X~B(n, p)

$X = X_1 + X_2 + ... + X_n$

$$X_i = \begin{cases} 1 & \text{if the } i\text{th trial is a sucess,} \\ 0 & \text{otherwise.} \end{cases}$$

as n→ ∞ , X→N(np, np(1-p))

$E[X_i] = p$, $Var[X_i] = p(1-p)$

$$\frac{X - np}{\sqrt{np(1-p)}} \rightarrow N(0,1) \quad \text{as } n \rightarrow \infty$$

$P(X=k) \sim P(k - 0.5 \leq X \leq k + 0.5)$

n이 커지면 정규분포로 근사된다.

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \longrightarrow N(0,1) \quad \text{in distribution, as } n \rightarrow \infty.$$

모집단이 무엇이든,

표본평균($\overline{X}$)의 값은 정규분포로 근사할 수 있다.

6.3.2 How large a sample is needed?

sample의 n이 30이상,

B(n, p)의 경우, min(np, n(1-p))가 5이상이여야 한다.

6.4 The sample variable

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

$$E[S^2] = \sigma^2$$

$$\sum_{i=1}^{n}(X_i - \overline{X})^2 = \left\{\sum_{i=1}^{n}(X_i - \mu)^2\right\} - n(\overline{X} - \mu)^2$$

$$\begin{aligned}
&\sum_{i=1}^{n}(X_i - \bar{X})^2 \\
&= \sum_{i=1}^{n}(X_i - \mu - (\bar{X} - X))^2 \\
&= \sum_{i=1}^{n}(X_i - \mu)^2 - 2(X_i - \mu)(\bar{X} - X) + (\bar{X} - \mu)^2 \\
&= \{\sum_{i=1}^{n}(X_i - \mu)^2\} - n(\bar{X} - \mu)^2
\end{aligned}$$

$$\sum_i (x_i - \overline{x})^2 = \left(\sum_i x_i^2\right) - n \cdot \overline{x}^2$$

6.5 Sampling distributions from a normal distribution

6.5.1 Distribution of the Sample Mean

Let $X_1, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$. Then

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1).$$

6.5.2 Joint Distribution of $\overline{X}$ and S^2

Let $X_1, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$.

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{and} \quad S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$$

(1) X and S are independent

(2) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$$\frac{(n-1)S^2}{\sigma^2} = \frac{X_1 - \bar{X}}{\sigma^2} + \frac{X_2 - \bar{X}}{\sigma^2} + ... + \frac{X_n - \bar{X}}{\sigma^2}$$

$$\chi^2_{n-1} = Z_1^2 + Z_2^2 + ... + Z_{n-1}^2$$