

successful factor - key <마련이냐> success factors

money - 72  
luck - 47  
skills - 82  
people  
knowledge - 96  
attitude - 100

마련이냐 ~ or horse? or ~

Turing machines

→ simple but powerful

→ stack은 없다, 그냥 tape만 있다.

↔ 읽기만 할 수 있다, 쓰기만 할 수 있는 machine 있다.

⇒ algorithmic computation

Closure properties of context-free languages

CFL is closed under union,  
concat, star

Prove?  $\rightarrow$  CFG or PDA

Union of two language  $L_1(G_1)$   
and  $L_2(G_2)$

$G_{\text{union}} = (V_1 \cup V_2 \cup \{s\}, T_1 \cup T_2, P_1 \cup P_2, \cup \{s \rightarrow s_1 | s_2\})$

Union:  $\left\{ \begin{array}{l} s \rightarrow s_1 | s_2 \\ s_1 \rightarrow \dots \\ s_2 \rightarrow \dots \end{array} \right.$

Concat:  $\left\{ \begin{array}{l} s \rightarrow s_1 s_2 \\ s_1 \rightarrow \dots \\ s_2 \rightarrow \dots \end{array} \right.$

$$\text{star: } S \rightarrow S_1 S_1^* | \lambda$$

$$\text{ex) } L_1: S \rightarrow a S b | \lambda$$

$$L_2: S \rightarrow a a S b | \lambda$$


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$$L_1 \cup L_2: \begin{cases} S \rightarrow S_1 | S_2 \end{cases}$$

$$S_1 \rightarrow a S_1 b | \lambda$$

$$S_2 \rightarrow a a S_2 b | \lambda$$


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$$L_1 \cdot L_2: S \rightarrow S_1 S_2$$

$$S_1 \rightarrow a S_1 b | \lambda$$

$$S_2 \rightarrow a a S_2 b | \lambda$$


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$$L_1^*: S \rightarrow S_1 S_1^* | \lambda$$

$$S_1 \rightarrow a S_1 b | \lambda$$

CFL is not closed under  
intersection

Proof)

$$L_1 = \{0^n 1^n 0^m, n \geq 0, m \geq 0\}$$

$$L_2 = \{0^n 1^m 0^m, n \geq 0, m \geq 0\}$$

$$L_1 \cap L_2 = \{0^n 1^n 0^n, n \geq 0\} \rightarrow \text{not CFL}$$

$$\left. \begin{array}{l} L_1: S \rightarrow S_1 S_2 \\ S_1 \rightarrow 0 S_1 1 / \lambda \\ S_2 \rightarrow 0 S_2 1 / \lambda \end{array} \right\} \rightarrow \text{CFG}$$

$$\left. \begin{array}{l} L_2: S \rightarrow S_1 S_2 \\ S_1 \rightarrow 0 S_1 1 \\ S_2 \rightarrow 1 S_2 0 / \lambda \end{array} \right\} \rightarrow \text{CFG}$$

$$L_1 \cap L_2 = \{0^n 1^n 0^n, n \geq 0\} \rightarrow \text{not CFL}$$

$$\underline{\text{CFL} \cap \text{CFL} = \text{Not CFL}}$$

$$\overline{L_1 \cap L_2} = \overline{\overline{L_1} \cup \overline{L_2}}$$

한글 complement & closure 이(가),  
 $L_1 \cap L_2$  이 closure 가 된다.

	RL	CFL
Union	O	O
Concat	O	O
Star	O	O
Intersection	O	X
Complement	O	X

한글 operation 은 정의 할 때,  
 Union, concat, star 는 항상 closure,  
 intersection, complement, not CFL  
 한글 주 는 것 다.

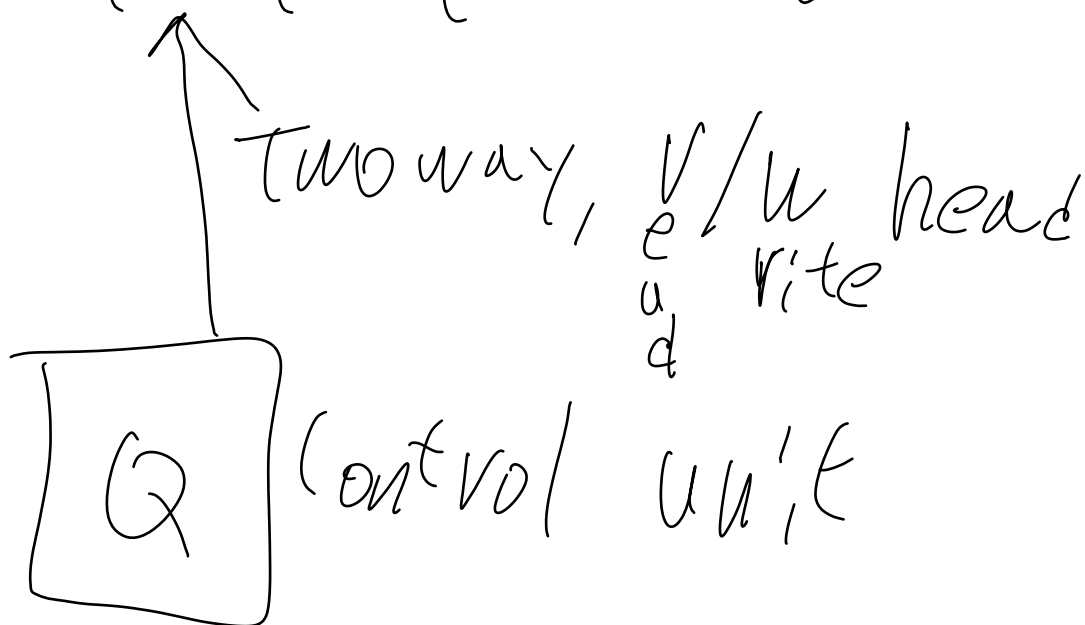
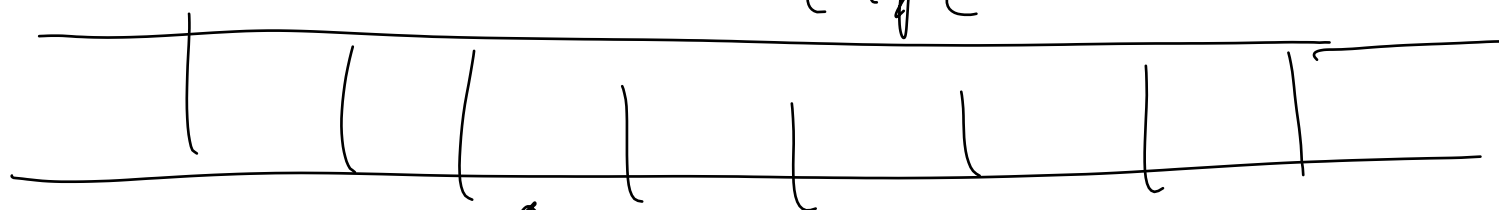
TM:

One Turing machine  $\leftrightarrow$  one

push down automata  $\leftrightarrow$  computation  
unlimited memory  $\leftrightarrow$  solvable

Turing machine: limited memory

tape



Tape is infinite

Basic operation?

1. Check current state.  
(If halt state  $\rightarrow$  terminate)

2. read symbol

3.  $M = (Q, \Sigma, \Gamma, \sqcup, q_0, F)$

$\Sigma$ : alphabet,  $\sqcup \notin \Sigma$   
 $\sqcup \in \Gamma$

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

$\delta$ : powerful in principle,  
small instructions are  
adequate for doing  
complicated tasks

$\sqcup \in \Gamma$ ,  $\sqcup$  is a blank

ex)  $M_1$ ,  $abbb$ ,  $\overline{a^2 b^2 b^2 c}$ .

input:  $abbb$   
output:  $bbbb$

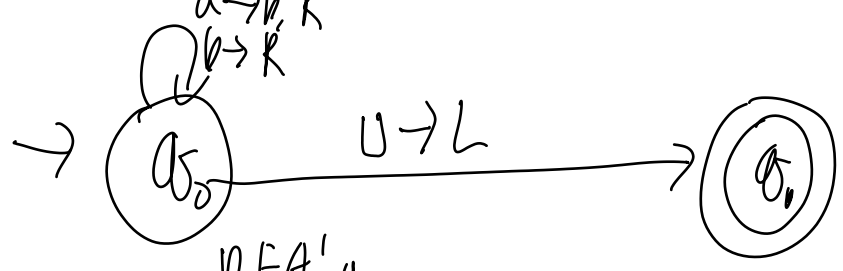
$q_0$   
 $\downarrow$

$a$	$b$	$\sqcup$
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$Q = \{q_0, q_1\}$   
 $\Sigma = \{a, b\}$   
 $\Gamma = \{a, b, \sqcup\}$   
 $q_0 = q_0$   
 $q_1 \in F$

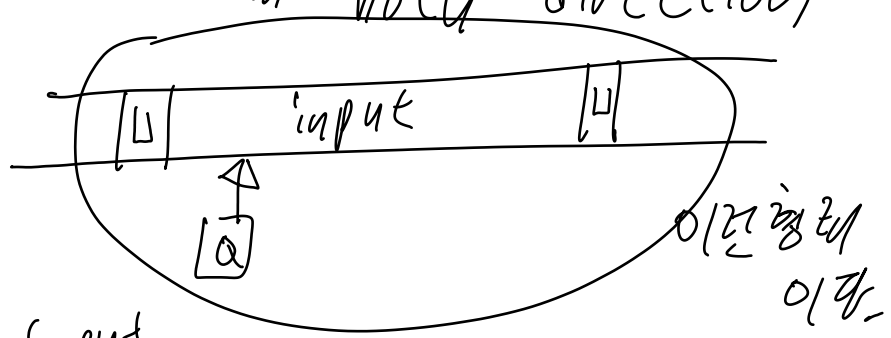
$\delta(q_0, a) = (q_0, b, R)$   
 $\delta(q_0, b) = (q_0, b, R)$   
 $\delta(q_0, \sqcup) = (q_1, \sqcup, L)$

$\delta(q_0, a) = (q_0, b, R)$   
 $\delta(q_0, b) = (q_0, b, R)$   
 $\delta(q_0, \epsilon) = (q_1, \epsilon, L)$



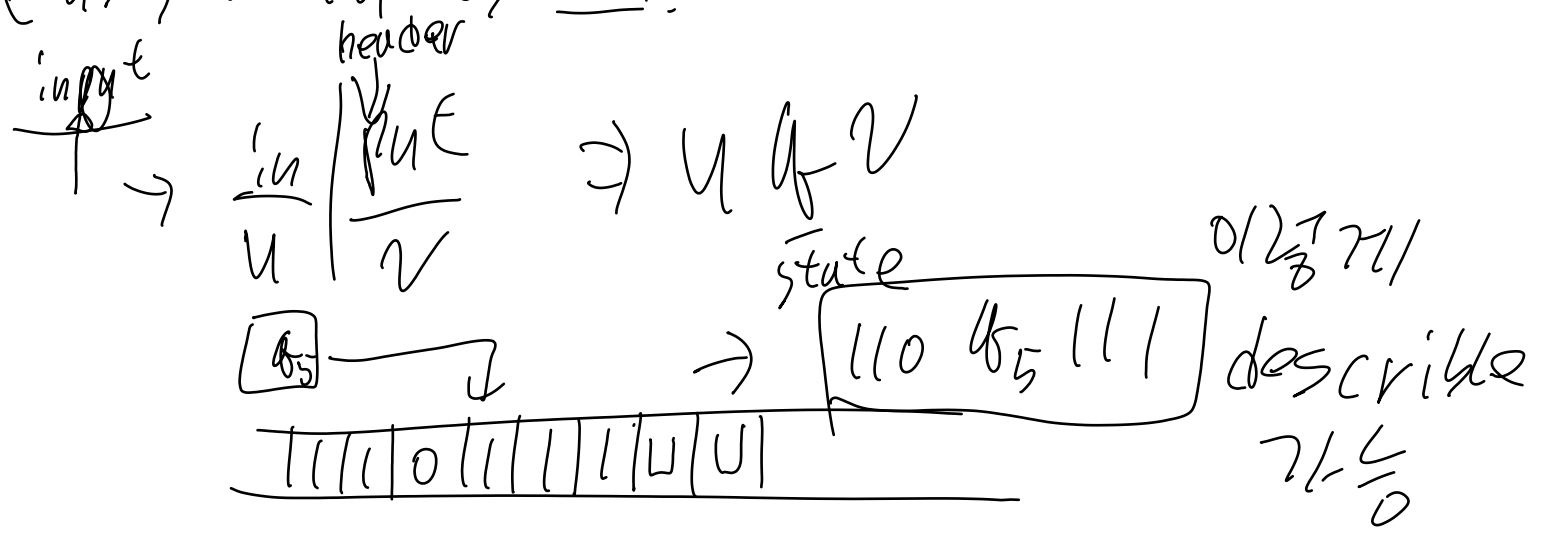
label  $\leftarrow$  DFA, a  
 PDA, a, b  $\rightarrow$  L  $\Rightarrow$  Design  
 TM, a  $\rightarrow$  b, R  
           a  $\rightarrow$  a, R  
            $\Rightarrow$  a  $\rightarrow$  R  
           TM!

standard TM.  
 $\rightarrow$  has tape that is unbounded  
 in both direction



$\Rightarrow$  same input, same state  $\rightarrow$  same action.

(configuration) of TM.





$\cup a q_i, b \nu \vdash \cup a q_i, a \nu \quad \text{if } (q_i, \nu) = (q_j, L, L);$   
 $\cup a q_i, b \nu \vdash \cup a q_i, \nu \quad \text{if } (q_i, \nu) = (q_j, L, R)$

ex)  $q_0$

$$\begin{array}{c|c|c|c} & a & b & \\ \hline & a & b & \end{array}$$

$q_0, a b \vdash b q_0, b \vdash b b q_0 \vdash b q_1, b$

이런 패턴을  
인식할 수 있는가?

각각 input 이 0 이거나  
 1 이거나  
 0 이거나 1 이거나  
 0 이거나 1 이거나