

$\underline{T.M}$ design [

Acceptor
 Transducer

]

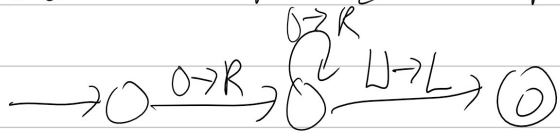
most powerful

$T.M \rightarrow \text{Acceptor?}$

Acceptor: after a seq of moves enter an accepting state and halt if we L
 otherwise, do not halt or halt in a non-accepting state

Transducer: get input. produce output
 s.t., transform input into output.

$00^* \stackrel{?}{\in} \text{Accept} \stackrel{?}{\in} \text{Acceptor}(T.M)?$



input: $x \rightarrow$ halt on initial state

$\emptyset \rightarrow \text{accept}$

$00 \rightarrow \text{accept}$

\vdots

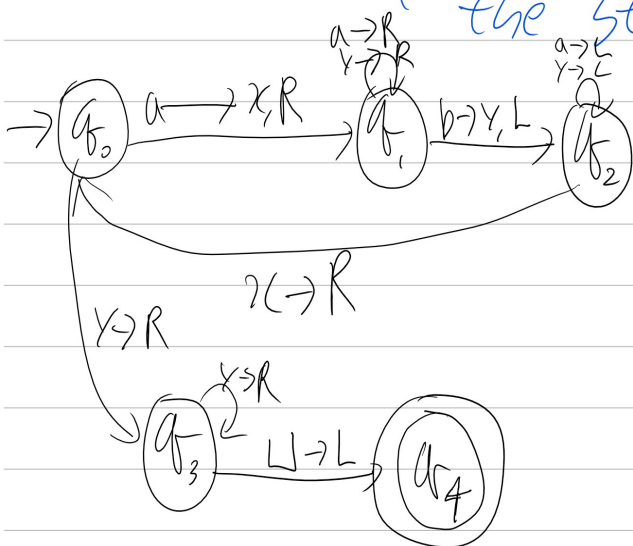
Transducer: $q_0 w \vdash_M q_5 \in (w)$

ex) design a TM that accepts $L = \{a^n b^n, n \geq 1\}$

→ count operations of a and b .

no more a and b → accept
 $a b b$?
 $a a b$?

→ replace the leftmost a w/ x
 • find the leftmost b and replace w/ y
 • no more a & b remaining; the string is in L .



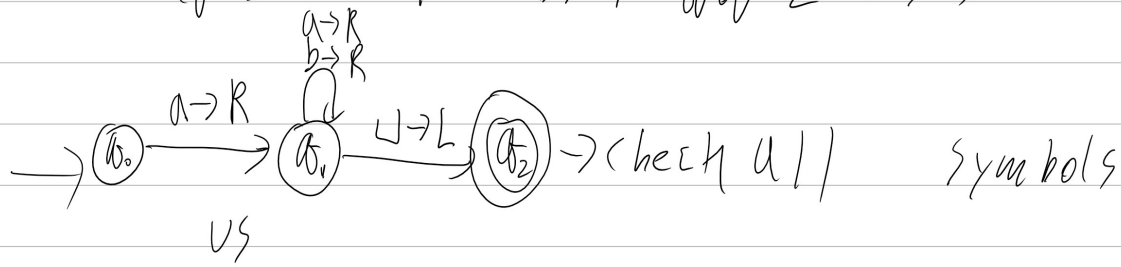
$x y$
 $x x b \rightarrow q_3$
 $x x b \rightarrow q_1$
 $x x y$
 rejected

$a a b b$ is acceptable?

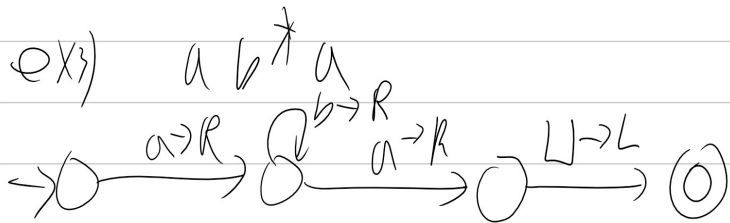
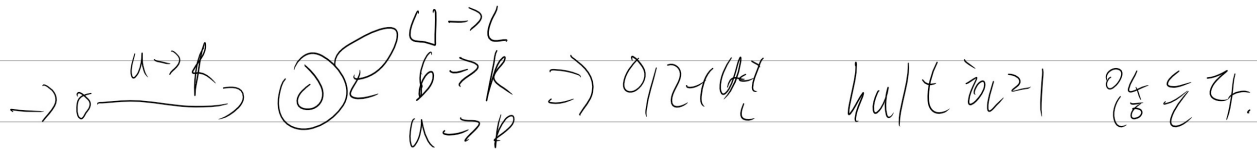
$q_0 a a b b \vdash x q_1 a b b \vdash x a q_1 b b \vdash x q_2 a y b \vdash q_2 x a y b$
 $\vdash x q_0 a y b \vdash x x q_1 y b \vdash x x y q_1 b \vdash x x q_2 x y \vdash x q_2 x x y$
 $\vdash x x q_0 x y \vdash x x x q_3 y \vdash x x x y q_3 \vdash x x x y q_4 y$
 accept states

$\Sigma = \{a, b\}$
 $\Gamma = \{a, b, x, y, \sqcup\}$

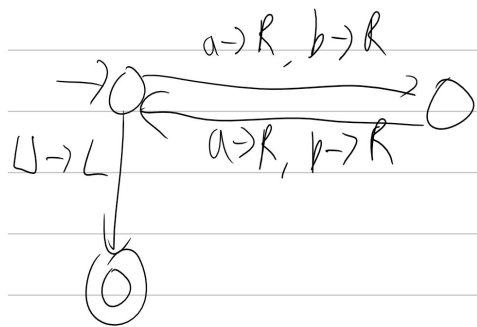
ex2) Design a TM w/ three states that accepts $L = \{a^n (ab)^* \mid n \geq 1\}$ over $\Sigma = \{a, b\}$



$\rightarrow q_0 \xrightarrow{a \rightarrow R} q_1 \rightarrow \text{check only the first character}$

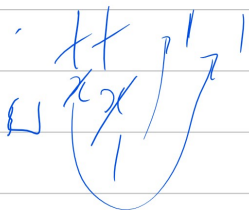


ex4) $L = \{w \mid |w| \text{ is even} \}$



ex5) Design a TM that copies the string of 1's. More precisely, $q_0 w 1^* q_f w w$, for any $w \in \{1\}^*$

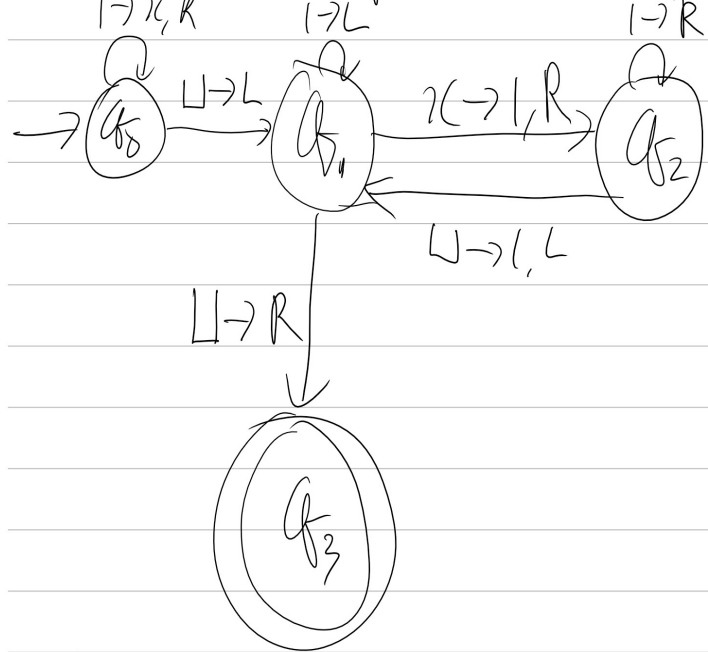
input: 11 \rightarrow output: 1111: transducer.



ex5)

Process:

1. Replace every 1 w/ π
2. Find the rightmost π and replace w/ 1
3. Travel to the right end, and creat a 1
4. repeat step 2,3 until no more π exist



$$q_0 \vdash \pi q_0 \cup \vdash q_1 \pi \vdash 1 q_2 \cup \vdash q_1 \cup \vdash q_1 \cup \vdash q_3 ()$$