파생금융상품론 11주차 과제

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14.2)

$\sigma = 0.3$
거래일 기준 표준편차 : $\sigma/\sqrt{252} = 0.0189$

14.4)

$S_0 = 50, K = 50, T = 3/12, r = 0.1, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.2417$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.0917$$

$$p = -S_0 N(-d_1) + Ke^{-rT}N(-d_2)$$

$$p = -50 \times N(-0.2417) + 50e^{-0.1\times 3/12}N(-0.0917)$$

$$p = -50 \times 0.4045 + 50e^{-0.1\times 3/12} \times 0.4635$$

$$p \approx 2.3759$$

14.5)

$S_0 = 50, K = 50, T = 3/12, t = 2/12, r = 0.1, \sigma = 0.3$

$$D = 1.5e^{-0.1*2/12}$$

$$D = 1.4752$$

$$S_0^* = S_0 - D = 48.5248$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.042$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.108$$

$$p = -S_0^* N(-d_1) + Ke^{-rT} N(-d_2)$$
$$p = -48.5248 \times N(-0.042) + 50e^{-0.1\times 3/12} N(0.108)$$
$$p = -48.5248 \times 0.4832 + 50e^{-0.1\times 3/12} \times 0.543$$
$$p \approx 3.0302$$

14.13)

$S_0 = 52, K = 50, T = 3/12, r = 0.12, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.5365$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.3865$$
$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$
$$c = 52 \times N(0.5365) - 50e^{-0.12\times 3/12} N(0.3865)$$
$$c \approx 52 \times 0.7042 - 50e^{-0.12\times 3/12} \times 0.6504$$
$$c \approx 5.0574$$

14.14)

$S_0 = 69, K = 70, T = 6/12, r = 0.05, \sigma = 0.35$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.1666$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.0809$$
$$p = -S_0 N(-d_1) + Ke^{-rT} N(-d_2)$$
$$p = -69 \times N(-0.1666) + 70e^{-0.05\times 6/12} N(0.0809)$$
$$p = -69 \times 0.4338 + 70e^{-0.05\times 6/12} \times 0.5322$$
$$p \approx 6.4018$$

special)

$$c + Ke^{-rT} = p + S_0$$
$$c + Ke^{-rT} - p - S_0 = 0$$
$$where, p = -S_0 N(-d_1) + Ke^{-rT} N(-d_2), c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$
$$c + Ke^{-rT} - p - S_0$$
$$= S_0 N(d_1) - Ke^{-rT} N(d_2) + Ke^{-rT} + S_0 N(-d_1) - Ke^{-rT} N(-d_2) - S_0$$

$$= S_0(N(d_1) + N(-d_1) - 1) + Ke^{-rT}(-N(d_2) - N(-d_2) + 1)$$
$$= S_0(1 - 1) + Ke^{-rT}(-1 + 1)$$
$$= 0$$
$$c + Ke^{-rT} - p - S_0 = 0, c + Ke^{-rT} - p - S_0 = 0$$

풋콜패리티가 성립한다.

lecture 1)

$\mu = 0.16, T = 6/12, \sigma = 0.2, S_0 = 40, \alpha = 0.95$

$$lnS_T \sim \phi((ln(S_0) + \mu - \frac{\sigma^2}{2})T, \sigma^2 T)$$
$$ln(S_T) \sim \phi(3.7589, 0.02)$$
$$3.7589 - 1.96 \times 0.1414 < ln(S_T) < 3.7589 + 1.96 \times 0.1414$$
$$3.4817 < ln(S_T) < 4.0361$$
$$32.5147 < S_T < 56.6032$$

lecture 2)

$\mu = 0.2, T = 12/12, \sigma = 0.4, S_0 = 20, \alpha = 0.95$

$$lnS_T \sim \phi((ln(S_0) + \mu - \frac{\sigma^2}{2})T, \sigma^2 T)$$
$$ln(S_T) \sim \phi(3.1157, 0.16)$$
$$3.1157 - 1.96 \times 0.4 < ln(S_T) < 3.1157 + 1.96 \times 0.4$$
$$2.3317 < ln(S_T) < 3.8997$$
$$10.2958 < S_T < 49.3892$$

lecture 2-1)

$\mu = 0.2, T = 12/12, \sigma = 0.4, S_0 = 20, \alpha = 0.95$

$$E(S_T) = S_0 e^{\mu T} = 24.4281$$
$$var(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1) = 103.5391$$
$$std(S_T) = \sqrt{S_T} = 10.1754$$
$$24.4281 - 1.96 \times 10.1754 < S_T < 24.4281 + 1.96 \times 10.1754$$
$$4.4842 < S_T < 44.3719$$

lecture 3)

$\mu = 0.17, T = 36/12, \sigma = 0.2, \alpha = 0.95$

$$x = \frac{1}{T} ln(\frac{S_T}{S_0}) \sim \phi((\mu - \frac{\sigma^2}{2}), \frac{\sigma^2}{T})$$

$$x \sim= \phi(0.15, 0.0133)$$

$$0.15 - 1.96 \times 0.1155 < x < 0.15 + 1.96 \times 0.1155$$

$$-0.0763 < x < 0.3763$$

lecture 4)

$S_0 = 50, r = 0.25$
거래일 기준 하루 변화량 : $r \times S_0/252 = 0.0496$
lecture 5)

$S_0 = 50, K = 50, T = 3/12, t = 2/12, r = 0.1, \sigma = 0.3$

$$D = 1.5e^{-0.1*2/12}$$

$$D = 1.4752$$

$$S_0^* = S_0 - D = 48.5248$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.042$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.108$$

$$c = S_0^* N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 48.5248 \times N(0.042) - 50e^{-0.1\times 3/12} N(-0.108)$$

$$c \approx 48.5248 \times 0.5168 - 50e^{-0.1\times 3/12} \times 0.457$$

$$c \approx 2.7895$$

lecture 5)

$S_0 = 50, K = 50, T = 3/12, t = 2/12, r = 0.1, \sigma = 0.3$

$$D = 1.5e^{-0.1*2/12}$$

$$D = 1.4752$$

$$S_0^* = S_0 - D = 48.5248$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.042$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.108$$

$$p = -S_0^* N(-d_1) + Ke^{-rT} N(-d_2)$$

$$p = -48.5248 \times N(-0.042) + 50e^{-0.1\times 3/12} N(0.108)$$

$$p = -48.5248 \times 0.4832 + 50e^{-0.1\times 3/12} \times 0.543$$

$$p \approx 3.0302$$

lecture 6)

$$S_0 = 40, K = 40, T = 6/12, t_1 = 2/12, t_2 = 5/12, r = 0.09, \sigma = 0.3$$

$$D = 0.5e^{-0.09*2/12} 0.5e^{-0.09*5/12}$$

$$D = 0.9742$$

$$S_0^* = S_0 - D = 39.0258$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.202$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.0102$$

$$c = S_0^* N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 39.0258 \times N(0.202) - 40e^{-0.09\times 6/12} N(-0.0102)$$

$$c \approx 39.0258 \times 0.58 - 40e^{-0.09\times 6/12} \times 0.4959$$

$$c \approx 3.6387$$

lecture 6)

$$S_0 = 42, K = 40, T = 6/12, t_1 = 2/12, t_2 = 5/12, r = 0.1, \sigma = 0.2$$

$$D = 0.5e^{-0.1*2/12} 0.5e^{-0.1*5/12}$$

$$D = 0.9713$$

$$S_0^* = S_0 - D = 41.0287$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.6038$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.4624$$

$$c = S_0^* N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 41.0287 \times N(0.6038) - 40e^{-0.1\times 6/12} N(0.4624)$$

$$c \approx 41.0287 \times 0.727 - 40e^{-0.1\times 6/12} \times 0.6781$$

$$c \approx 4.0079$$

lecture 7)

$S_0 = 52, K = 50, T = 3/12, r = 0.12, q = 0.05, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.4531$$

$$d_2 = \frac{ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.3031$$

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 52 \times e^{-0.05*3/12} \times N(0.4531) - 50e^{-0.12\times 3/12} N(0.3031)$$

$$c \approx 52 \times e^{-0.05*3/12} \times 0.6748 - 50e^{-0.12\times 3/12} \times 0.6191$$

$$c \approx 4.6119$$

lecture 7)

$S_0 = 52, K = 50, T = 3/12, r = 0.12, q = 0.05, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.4531$$

$$d_2 = \frac{ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.3031$$

$$p = -S_0 e^{-qT} N(-d_1) + Ke^{-rT} N(-d_2)$$

$$p = -52 \times e^{-0.05*3/12} \times N(-0.4531) + 50e^{-0.12\times 3/12} N(-0.3031)$$

$$p = -52 \times e^{-0.05*3/12} \times 0.3252 + 50e^{-0.12\times 3/12} \times 0.3809$$

$$p \approx 1.7801$$

14.2)

$\sigma = 0.3$
거래일 기준 표준편차 : $\sigma/\sqrt{252} = 0.0189$
14.4)

$S_0 = 50, K = 50, T = 3/12, r = 0.1, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.2417$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.0917$$

$$p = -S_0 N(-d_1) + Ke^{-rT} N(-d_2)$$

$$p = -50 \times N(-0.2417) + 50e^{-0.1\times 3/12} N(-0.0917)$$

$$p = -50 \times 0.4045 + 50e^{-0.1\times 3/12} \times 0.4635$$

$$p \approx 2.3759$$

14.5)

$S_0 = 50, K = 50, T = 3/12, t = 2/12, r = 0.1, \sigma = 0.3$

$$D = 1.5e^{-0.1*2/12}$$

$$D = 1.4752$$

$$S_0^* = S_0 - D = 48.5248$$

$$d_1 = \frac{ln(S_0^*/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.042$$

$$d_2 = \frac{ln(S_0^*/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.108$$

$$p = -S_0^* N(-d_1) + Ke^{-rT} N(-d_2)$$

$$p = -48.5248 \times N(-0.042) + 50e^{-0.1\times 3/12} N(0.108)$$

$$p = -48.5248 \times 0.4832 + 50e^{-0.1\times 3/12} \times 0.543$$

$$p \approx 3.0302$$

14.7)

$\mu = 0.15, T = 24/12, \sigma = 0.25, \alpha = 0.95$

$$x = \frac{1}{T} ln(\frac{S_T}{S_0}) \sim \phi((\mu - \frac{\sigma^2}{2}), \frac{\sigma^2}{T})$$

$$x \sim= \phi(0.1187, 0.0312)$$

$$0.1187 - 1.96 \times 0.1768 < x < 0.1187 + 1.96 \times 0.1768$$

$$-0.2277 < x < 0.4652$$

14.8-a)

$$\frac{\Delta S}{S} \sim \phi(\mu \times dt, \sigma^2 dt)$$

$$S_T = S + \Delta S \sim \phi(S + S \times \mu * dt, S^2\sigma^2 dt)$$

$$= \phi(41.04, \sqrt{9.4045})$$

유로피언 콜옵션이 행사될 확률은 다음과 같다.

$$P(S_T > K) = P(S_T > 40)$$

$$= P(Z > \frac{40 - 41.04}{9.4045})$$

$$= P(Z > -0.1106)$$

$$= P(Z < 0.1106)$$

$$= 0.544$$

14.8-b)

$$\frac{\Delta S}{S} \sim \phi(\mu \times dt, \sigma^2 dt)$$

$$S_T = S + \Delta S \sim \phi(S + S \times \mu * dt, S^2\sigma^2 dt)$$

$$= \phi(41.04, \sqrt{9.4045})$$

유로피언 풋옵션이 행사될 확률은 다음과 같다.

$$P(S_T < K) = P(S_T < 40)$$

$$= P(Z < \frac{40 - 41.04}{9.4045})$$

$$= P(Z < -0.1106)$$

$$= 0.456$$

14.13)

$S_0 = 52, K = 50, T = 3/12, r = 0.12, \sigma = 0.3$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.5365$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.3865$$

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 52 \times N(0.5365) - 50e^{-0.12\times 3/12}N(0.3865)$$
$$c \approx 52 \times 0.7042 - 50e^{-0.12\times 3/12} \times 0.6504$$
$$c \approx 5.0574$$

14.14)

$S_0 = 69, K = 70, T = 6/12, r = 0.05, \sigma = 0.35$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.1666$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.0809$$
$$p = -S_0N(-d_1) + Ke^{-rT}N(-d_2)$$
$$p = -69 \times N(-0.1666) + 70e^{-0.05\times 6/12}N(0.0809)$$
$$p = -69 \times 0.4338 + 70e^{-0.05\times 6/12} \times 0.5322$$
$$p \approx 6.4018$$

14.26)

$\mu = 0.18, T = 24/12, \sigma = 0.3, S_0 = 50, \alpha = 0.95$

$$lnS_T \sim \phi((ln(S_0) + \mu - \frac{\sigma^2}{2})T, \sigma^2T)$$
$$ln(S_T) \sim \phi(4.182, 0.18)$$
$$4.182 - 1.96 \times 0.4243 < ln(S_T) < 4.182 + 1.96 \times 0.4243$$
$$3.3505 < ln(S_T) < 5.0136$$
$$28.516 < S_T < 150.4424$$

14.29-(a))

$S_0 = 30, K = 29, T = 4/12, r = 0.05, \sigma = 0.25$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.4225$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.2782$$
$$c = S_0N(d_1) - Ke^{-rT}N(d_2)$$
$$c = 30 \times N(0.4225) - 29e^{-0.05\times 4/12}N(0.2782)$$
$$c \approx 30 \times 0.6637 - 29e^{-0.05\times 4/12} \times 0.6096$$

$$c \approx 2.5247$$

14.29-(c))

$S_0 = 30, K = 29, T = 4/12, r = 0.05, \sigma = 0.25$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.4225$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \approx 0.2782$$

$$p = -S_0 N(-d_1) + Ke^{-rT} N(-d_2)$$

$$p = -30 \times N(-0.4225) + 29e^{-0.05\times 4/12} N(-0.2782)$$

$$p = -30 \times 0.3363 + 29e^{-0.05\times 4/12} \times 0.3904$$

$$p \approx 1.0454$$