파생금융상품론 9주차 과제

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12.1)

$S_0 = 40, S_u = 42, S_d = 38, T = 1/12, r = 0.08, K = 39$

$$f_u = max(S_u - K, 0), f_u = 3$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{3 - 0}{42 - 38}$$
$$\Delta = 0.75$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 40 * 0.75 - (42 * 0.75 - 3)e^{-0.08*1/12}$$
$$f \approx 1.6894$$

따라서 유로피언 콜 옵션의 가격은 약 $1.6894이다.

12.4)

$S_0 = 50, S_u = 55, S_d = 45, T = 6/12, r = 0.1, K = 50$

$$f_u = max(K - S_u, 0), f_u = 0$$
$$f_d = max(K - S_d, 0), f_d = 5$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$

$$\Delta = \frac{0-5}{55-45}$$
$$\Delta = -0.5$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 50 * (-0.5) - (55 * (-0.5) - 0)e^{-0.1*6/12}$$
$$f \approx 1.1588$$

따라서 유로피언 풋 옵션의 가격은 약 $1.1588이다.
12.5)

$S_0 = 100, u = 1.1, d = 0.9, N = 2, \Delta t = 6/12, T = 12/12, r = 0.08, K = 100$

$$S_{uu} = S_0 * u * u, S_{uu} = 121.0$$
$$S_{ud} = S_0 * u * d, S_{ud} = 99.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 81.0$$

$$f_{uu} = max(S_{uu} - K, 0), f_{uu} = 21.0$$
$$f_{ud} = max(S_{ud} - K, 0), f_{ud} = 0$$
$$f_{dd} = max(S_{dd} - K, 0), f_{dd} = 0$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.08*6/12} - 0.9}{1.1 - 0.9}$$
$$p \approx 0.7041$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.08*12/12}(0.7041^2 21.0 + 2 * 0.7041(1 - 0.7041)0 + (1 - 0.7041)^2 0)$$
$$f \approx 9.6105$$

따라서 유로피언 콜 옵션의 가격은 약 $9.6105이다.
추가문제1)

$S_0 = 20, S_u = 22, S_d = 18, T = 3/12, r = 0.12, K = 21$

$$f_u = max(S_u - K, 0), f_u = 1$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{1 - 0}{22 - 18}$$
$$\Delta = 0.25$$

$$f_0 = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f_0 = 20 * 0.25 - (22 * 0.25 - 1)e^{-0.12*3/12}$$
$$f_0 \approx 0.633$$

따라서 유로피언 콜 옵션의 가격은 약 \$0.633이다.$f = 0.6 < f_0 = 0.633$따라서 차익거래 전략은
1. $(S_0\Delta - f_0)$만큼 채권 매수,
2. 콜옵션을 f에 매수,
3. 기초자산을 Δ주를 S_0에 공매도,
4. $(f_0 - f)$만큼 채권을 매수하는 전략이다.

	현재시점	만기 시점	
전략	현재시점	$S_T = S_u$	$S_T = S_d$
$(S_0\Delta - f_0)$만큼 채권 매수	$-(S_0\Delta - f_0)$	$S_u\Delta - f_u$	$S_d\Delta - f_d$
콜옵션을 f에 매수	-f	$S_u - K = f_u$	$0=\mathrm{f}_d$
기초자산 Δ주를 S_0에 공매도	$S_0\Delta$	$-S_u\Delta$	$-S_d\Delta$
$(f_0 - f)$만큼 채권 매수	$-(f_0 - f)$	$(f_0 - f)e^{rT}$	$(f_0 - f)e^{rT}$
합계	0	$(f_0 - f)e^{rT}$	$(f_0 - f)e^{rT}$

값들을 대입하면

	현재시점	만기 시점	
전략	현재시점	$S_T = 22$	$S_T = 18$
4.367만큼 채권 매수	-4.367	4.5	4.5
콜옵션을 0.6에 매수	-0.6	1	0
기초자산 0.25주를 20에 공매도	5.0	-5.5	-4.5
0.033만큼 채권 매수	-0.033	0.034	0.034
합계	0	0.034	0.034

추가문제2)

$S_0 = 50, K = 52, \sigma = 0.3, r = 0.05, \Delta t = 12/12, T = 24/12, q = 0.0$

$$u = e^{\sigma\sqrt{\Delta t}}, u = 1.3499$$
$$d = e^{-\sigma\sqrt{\Delta t}}, d = 0.7408$$

$$S_u = S_0 * u, S_u = 67.4929$$
$$S_d = S_0 * d, S_d = 37.0409$$

$$S_{uu} = S_0 * u * u, S_{uu} = 91.1059$$
$$S_{ud} = S_0 * u * d, S_{ud} = 50.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 27.4406$$

$$f_{uu} = max(K - S_{uu}, 0), f_{uu} = 0$$
$$f_{ud} = max(K - S_{ud}, 0), f_{ud} = 2.0$$
$$f_{dd} = max(K - S_{dd}, 0), f_{dd} = 24.5594$$

$$p = \frac{e^{(r-q)*\Delta t} - d}{u - d}$$
$$p = \frac{e^{(0.05-0.0)*12/12} - 0.7408}{1.3499 - 0.7408}$$
$$p \approx 0.5097$$

$$f_u = max(e^{-(r-q)\Delta t}(pf_{uu} + (1-p)f_{ud}), K - S_u)$$
$$f_u = max(0.9327, -15.4929)$$
$$f_u = 0.9327$$

$$f_d = max(e^{-(r-q)\Delta t}(pf_{ud} + (1-p)f_{dd}), K - S_d)$$
$$f_d = max(12.423, 14.9591)$$
$$f_d = 14.9591$$

$$f = max(e^{-(r-q)\Delta t}(pf_u + (1-p)f_d), K - S_0)$$
$$f = max(7.4284, 2)$$
$$f = 7.4284$$

따라서 아메리칸 풋 옵션의 가격은 약 $7.4284이다.
추가문제3)

$S_0 = 810, K = 800, \sigma = 0.2, r = 0.05, \Delta t = 3/12, T = 6/12, q = 0.02$

$$u = e^{\sigma\sqrt{\Delta t}}, u = 1.1052$$
$$d = e^{-\sigma\sqrt{\Delta t}}, d = 0.9048$$

$$S_{uu} = S_0 * u * u, S_{uu} = 989.3362$$
$$S_{ud} = S_0 * u * d, S_{ud} = 810.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 663.1719$$

$$f_{uu} = max(S_{uu} - K, 0), f_{uu} = 189.3362$$
$$f_{ud} = max(S_{ud} - K, 0), f_{ud} = 10.0$$
$$f_{dd} = max(S_{dd} - K, 0), f_{dd} = 0$$

$$p = \frac{e^{(r-q)*\Delta t} - d}{u - d}$$
$$p = \frac{e^{(0.05-0.02)*3/12} - 0.9048}{1.1052 - 0.9048}$$
$$p \approx 0.5126$$

$$f = e^{-(r-q)T}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-(0.05-0.02)*6/12}(0.5126^2 189.3362 + 2*0.5126(1-0.5126)10.0 + (1-0.5126)^2 0)$$
$$f \approx 53.9313$$

따라서 유로피언 콜 옵션의 가격은 약 $53.9313이다.
12.1)

$S_0 = 40, S_u = 42, S_d = 38, T = 1/12, r = 0.08, K = 39$

$$f_u = max(S_u - K, 0), f_u = 3$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{3 - 0}{42 - 38}$$
$$\Delta = 0.75$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 40 * 0.75 - (42 * 0.75 - 3)e^{-0.08*1/12}$$
$$f \approx 1.6894$$

따라서 유로피언 콜 옵션의 가격은 약 $1.6894이다.
12.4)

$S_0 = 50, S_u = 55, S_d = 45, T = 6/12, r = 0.1, K = 50$

$$f_u = max(K - S_u, 0), f_u = 0$$
$$f_d = max(K - S_d, 0), f_d = 5$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{0 - 5}{55 - 45}$$
$$\Delta = -0.5$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 50 * (-0.5) - (55 * (-0.5) - 0)e^{-0.1*6/12}$$
$$f \approx 1.1588$$

따라서 유로피언 풋 옵션의 가격은 약 $1.1588이다.

12.5)

$S_0 = 100, u = 1.1, d = 0.9, N = 2, \Delta t = 6/12, T = 12/12, r = 0.08, K = 100$

$$S_{uu} = S_0 * u * u, S_{uu} = 121.0$$
$$S_{ud} = S_0 * u * d, S_{ud} = 99.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 81.0$$

$$f_{uu} = max(S_{uu} - K, 0), f_{uu} = 21.0$$
$$f_{ud} = max(S_{ud} - K, 0), f_{ud} = 0$$
$$f_{dd} = max(S_{dd} - K, 0), f_{dd} = 0$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.08*6/12} - 0.9}{1.1 - 0.9}$$
$$p \approx 0.7041$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.08*12/12}(0.7041^2 21.0 + 2 * 0.7041(1 - 0.7041)0 + (1 - 0.7041)^2 0)$$
$$f \approx 9.6105$$

따라서 유로피언 콜 옵션의 가격은 약 $9.6105이다.

12.9)

$S_0 = 50, S_u = 53, S_d = 48, T = 2/12, r = 0.1, K = 49$

$$f_u = max(S_u - K, 0), f_u = 4$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{4 - 0}{53 - 48}$$
$$\Delta = 0.8$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 50 * 0.8 - (53 * 0.8 - 4)e^{-0.1*2/12}$$
$$f \approx 2.2347$$

따라서 유로피언 콜 옵션의 가격은 약 $2.2347이다.
12.10)

$S_0 = 80, S_u = 85, S_d = 75, T = 4/12, r = 0.05, K = 80$

$$f_u = max(K - S_u, 0), f_u = 0$$
$$f_d = max(K - S_d, 0), f_d = 5$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{0 - 5}{85 - 75}$$
$$\Delta = -0.5$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 80 * (-0.5) - (85 * (-0.5) - 0)e^{-0.05*4/12}$$
$$f \approx 1.7975$$

따라서 유로피언 풋 옵션의 가격은 약 $1.7975이다.
12.11)

$S_0 = 40, S_u = 45, S_d = 35, T = 3/12, r = 0.08, K = 40$

$$f_u = max(K - S_u, 0), f_u = 0$$
$$f_d = max(K - S_d, 0), f_d = 5$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{0 - 5}{45 - 35}$$
$$\Delta = -0.5$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 40 * (-0.5) - (45 * (-0.5) - 0)e^{-0.08*3/12}$$
$$f \approx 2.0545$$

따라서 유로피언 풋 옵션의 가격은 약 $2.0545이다.
12.11)

$S_0 = 40, S_u = 45, S_d = 35, T = 3/12, r = 0.08, K = 40$

$$f_u = max(K - S_u, 0), f_u = 0$$
$$f_d = max(K - S_d, 0), f_d = 5$$

$$p = \frac{e^{rT} - d}{u - d}$$
$$p = \frac{S_0 e^{rT} - S_d}{S_u - S_d}$$
$$p \approx 0.5808$$

$$f = e^{-rT}(pf_u + (1 - p)f_d)$$
$$f \approx 2.0545$$

따라서 유로피언 풋 옵션의 가격은 약 $2.0545이다.
12.12)

$S_0 = 50, u = 1.06, d = 0.95, N = 2, \Delta t = 3/12, T = 6/12, r = 0.05, K = 51$

$$S_{uu} = S_0 * u * u, S_{uu} = 56.18$$
$$S_{ud} = S_0 * u * d, S_{ud} = 50.35$$
$$S_{dd} = S_0 * d * d, S_{dd} = 45.125$$

$$f_{uu} = max(S_{uu} - K, 0), f_{uu} = 5.18$$
$$f_{ud} = max(S_{ud} - K, 0), f_{ud} = 0$$
$$f_{dd} = max(S_{dd} - K, 0), f_{dd} = 0$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.05*3/12} - 0.95}{1.06 - 0.95}$$
$$p \approx 0.5689$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.05*6/12}(0.5689^2 5.18 + 2 * 0.5689(1 - 0.5689)0 + (1 - 0.5689)^2 0)$$
$$f \approx 1.6351$$

따라서 유로피언 콜 옵션의 가격은 약 $1.6351이다.
12.13)

$S_0 = 50, u = 1.06, d = 0.95, N = 2, \Delta t = 3/12, T = 6/12, r = 0.05, K = 51$

$$S_{uu} = S_0 * u * u, S_{uu} = 56.18$$
$$S_{ud} = S_0 * u * d, S_{ud} = 50.35$$
$$S_{dd} = S_0 * d * d, S_{dd} = 45.125$$

$$f_{uu} = max(K - S_{uu}, 0), f_{uu} = 0$$
$$f_{ud} = max(K - S_{ud}, 0), f_{ud} = 0.65$$
$$f_{dd} = max(K_{Sdd}, 0), f_{dd} = 5.875$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.05*3/12} - 0.95}{1.06 - 0.95}$$
$$p \approx 0.5689$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.05*6/12}(0.5689^2 0 + 2 * 0.5689(1 - 0.5689)0.65 + (1 - 0.5689)^2 5.875)$$
$$f \approx 1.3759$$

따라서 유로피언 풋 옵션의 가격은 약 $1.3759이다.
12.14)

$S_0 = 25, S_u = 27, S_d = 23, T = 2/12, r = 0.1$

$$f_u = S_u^2, f_u = 729$$
$$f_d = S_d^2, f_d = 529$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{729 - 529}{27 - 23}$$
$$\Delta = 50.0$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 25 * 50.0 - (27 * 50.0 - 729)e^{-0.1*2/12}$$
$$f \approx 639.2642$$

따라서 파생증권의 가격은 약 $639.2642이다.
12.15)

$\Delta_t = 1/12, r = 0.05, rf = 0.08, \sigma = 0.12$

$$u = e^{\sigma\sqrt{\Delta_t}}, u = 1.0352$$
$$d = e^{-\sigma\sqrt{\Delta_t}}, u = 0.966$$
$$p = \frac{e^{(r-rf)\Delta_t} - d}{u - d}$$
$$p = 0.4553$$

12.16)

$S_0 = 50, S_u = 60, S_d = 42, T = 6/12, r = 0.12, K = 48$

$$f_u = max(S_u - K, 0), f_u = 12$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$
$$\Delta = \frac{12 - 0}{60 - 42}$$
$$\Delta = 0.6667$$

$$f = S_0\Delta - (S_u * \Delta - f_u)e^{-rT}$$
$$f = 50 * 0.6667 - (60 * 0.6667 - 12)e^{-0.12*6/12}$$
$$f \approx 6.9637$$

따라서 유로피언 콜 옵션의 가격은 약 $6.9637이다.
12.16)

$S_0 = 50, S_u = 60, S_d = 42, T = 6/12, r = 0.12, K = 48$

$$f_u = max(S_u - K, 0), f_u = 12$$
$$f_d = max(S_d - K, 0), f_d = 0$$

$$p = \frac{e^{rT} - d}{u - d}$$
$$p = \frac{S_0e^{rT} - S_d}{S_u - S_d}$$
$$p \approx 0.6162$$

$$f = e^{-rT}(pf_u + (1 - p)f_d)$$
$$f \approx 6.9638$$

따라서 유로피언 콜 옵션의 가격은 약 $6.9638이다.
12.17 - a)

$S_0 = 40, u = 1.1, d = 0.9, N = 2, \Delta t = 3/12, T = 6/12, r = 0.12, K = 42$

$$S_{uu} = S_0 * u * u, S_{uu} = 48.4$$
$$S_{ud} = S_0 * u * d, S_{ud} = 39.6$$
$$S_{dd} = S_0 * d * d, S_{dd} = 32.4$$

$$f_{uu} = max(K - S_{uu}, 0), f_{uu} = 0$$
$$f_{ud} = max(K - S_{ud}, 0), f_{ud} = 2.4$$
$$f_{dd} = max(K_{Sdd}, 0), f_{dd} = 9.6$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.12*3/12} - 0.9}{1.1 - 0.9}$$
$$p \approx 0.6523$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.12*6/12}(0.6523^2 0 + 2 * 0.6523(1 - 0.6523)2.4 + (1 - 0.6523)^2 9.6)$$
$$f \approx 2.1183$$

따라서 유로피언 풋 옵션의 가격은 약 $2.1183이다.
12.17 - b)

$S_0 = 40, K = 42, r = 0.12, u = 1.1, d = 0.9, \Delta t = 3/12, T = 6/12, q = 0$

$$S_u = S_0 * u, S_u = 44.0$$
$$S_d = S_0 * d, S_d = 36.0$$

$$S_{uu} = S_0 * u * u, S_{uu} = 48.4$$
$$S_{ud} = S_0 * u * d, S_{ud} = 39.6$$
$$S_{dd} = S_0 * d * d, S_{dd} = 32.4$$

$$f_{uu} = max(K - S_{uu}, 0), f_{uu} = 0$$
$$f_{ud} = max(K - S_{ud}, 0), f_{ud} = 2.4$$
$$f_{dd} = max(K - S_{dd}, 0), f_{dd} = 9.6$$

$$p = \frac{e^{(r-q)*\Delta t} - d}{u - d}$$

$$p = \frac{e^{(0.12-0)*3/12} - 0.9}{1.1 - 0.9}$$
$$p \approx 0.6523$$

$$f_u = max(e^{-(r-q)\Delta t}(pf_{uu} + (1-p)f_{ud}), K - S_u)$$
$$f_u = max(0.8099, -2.0)$$
$$f_u = 0.8099$$

$$f_d = max(e^{-(r-q)\Delta t}(pf_{ud} + (1-p)f_{dd}), K - S_d)$$
$$f_d = max(4.7587, 6.0)$$
$$f_d = 6.0$$

$$f = max(e^{-(r-q)\Delta t}(pf_u + (1-p)f_d), K - S_0)$$
$$f = max(2.5374, 2)$$
$$f = 2.5374$$

따라서 아메리칸 풋 옵션의 가격은 약 $2.5374이다.
12.19 - a)

$S_0 = 30, u = 1.08, d = 0.9, N = 2, \Delta t = 2/12, T = 4/12, r = 0.05$

$$S_{uu} = S_0 * u * u, S_{uu} = 34.992$$
$$S_{ud} = S_0 * u * d, S_{ud} = 29.16$$
$$S_{dd} = S_0 * d * d, S_{dd} = 24.3$$

$$f_{uu} = (max(30 - S_{uu}, 0))^2, f_{uu} = 0$$
$$f_{ud} = (max(30 - S_{ud}, 0))^2, f_{ud} = 0.7056$$
$$f_{dd} = (max(30 - S_{dd}, 0))^2, f_{dd} = 32.49$$

$$p = \frac{e^{r*\Delta t} - d}{u - d}$$
$$p = \frac{e^{0.05*2/12} - 0.9}{1.08 - 0.9}$$

$$p \approx 0.602$$

$$f = e^{-rT}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-0.05*4/12}(0.602^2 0 + 2 * 0.602(1-0.602)0.7056 + (1-0.602)^2 32.49)$$
$$f \approx 5.394$$

따라서 파생상품의 가격은 약 $5.394이다.
12.20)

$S_0 = 40, K = 40, \sigma = 0.3, r = 0.04, \Delta t = 3/12, T = 6/12, q = 0$

$$u = e^{\sigma\sqrt{\Delta t}}, u = 1.1618$$
$$d = e^{-\sigma\sqrt{\Delta t}}, d = 0.8607$$

$$S_{uu} = S_0 * u * u, S_{uu} = 53.9944$$
$$S_{ud} = S_0 * u * d, S_{ud} = 40.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 29.6327$$

$$f_{uu} = max(S_{uu} - K, 0), f_{uu} = 13.9944$$
$$f_{ud} = max(S_{ud} - K, 0), f_{ud} = 0$$
$$f_{dd} = max(S_{dd} - K, 0), f_{dd} = 0$$

$$p = \frac{e^{(r-q)*\Delta t} - d}{u - d}$$
$$p = \frac{e^{(0.04-0)*3/12} - 0.8607}{1.1618 - 0.8607}$$
$$p \approx 0.4959$$

$$f = e^{-(r-q)T}(p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd})$$
$$f = e^{-(0.04-0)*6/12}(0.4959^2 13.9944 + 2 * 0.4959(1-0.4959)0 + (1-0.4959)^2 0)$$
$$f \approx 3.3739$$

따라서 유로피언 콜 옵션의 가격은 약 $3.3739이다.

12.21)

$S_0 = 50, K = 50, \sigma = 0.4, r = 0.1, \Delta t = 3/12, T = 6/12, q = 0$

$$u = e^{\sigma\sqrt{\Delta t}}, u = 1.2214$$
$$d = e^{-\sigma\sqrt{\Delta t}}, d = 0.8187$$

$$S_u = S_0 * u, S_u = 61.0701$$
$$S_d = S_0 * d, S_d = 40.9365$$

$$S_{uu} = S_0 * u * u, S_{uu} = 74.5912$$
$$S_{ud} = S_0 * u * d, S_{ud} = 50.0$$
$$S_{dd} = S_0 * d * d, S_{dd} = 33.516$$

$$f_{uu} = max(K - S_{uu}, 0), f_{uu} = 0$$
$$f_{ud} = max(K - S_{ud}, 0), f_{ud} = 0.0$$
$$f_{dd} = max(K - S_{dd}, 0), f_{dd} = 16.484$$

$$p = \frac{e^{(r-q)*\Delta t} - d}{u - d}$$
$$p = \frac{e^{(0.1-0)*3/12} - 0.8187}{1.2214 - 0.8187}$$
$$p \approx 0.513$$

$$f_u = max(e^{-(r-q)\Delta t}(pf_{uu} + (1-p)f_{ud}), K - S_u)$$
$$f_u = max(0.0, -11.0701)$$
$$f_u = 0.0$$

$$f_d = max(e^{-(r-q)\Delta t}(pf_{ud} + (1-p)f_{dd}), K - S_d)$$
$$f_d = max(7.829, 9.0635)$$
$$f_d = 9.0635$$

$$f = max(e^{-(r-q)\Delta t}(pf_u + (1-p)f_d), K - S_0)$$
$$f = max(4.3046, 0)$$
$$f = 4.3046$$

따라서 아메리칸 풋 옵션의 가격은 약 $4.3046이다.