

Self tuning, Exponentially Weighted Moving Average, applied to the input of PID Controller

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ABSTRACT

The use of a Proportional, Integral and Derivative (PID) controller, to drive the speed and direction, in a mobile robot, is limited by the precision and noise of the on board input sensors, particularly severe problems arise in computing the derivative term. In this work a PID whose terms are computed from filtered input with an Exponentially Weighted Moving Average (EWMA) is used. In order to solve the tradeoff between speed and noise rejection, parameters of the filter are dynamically adjusted using statistical criteria.

Keywords: *PID, Moving Average, Self Tuning, IIR, EWMA.*

1. INTRODUCTION

The PID control is the most popular control system; it is versatile and can be tuned adjusting three constants. PID is a well proved and successfully applied in many control systems and there are a lot of information and application notes (Wescott, 2000).

In a digital controller, its discrete form is used, (Kuchen, et al. 1988).

$$y[n] = K_p \cdot e[n] + K_i \cdot i[n] + K_d \cdot d[n], \quad (1)$$

Where:

$$d[n] = (e[n] - e[n-1])/T_s, \quad (2)$$

$$i[n] = T_s \cdot \sum e[n]; \quad (3)$$

The derivative term is computed as difference between current sample of error $e[n]$, and the last sample $e[n-1]$, T_s is the sampling time, and the integral term is computed as summation.

To set T_s , it is necessary to bear in mind the time response of the system. As a rule of thumb next relationship is used:

$$T_s \leq T_p/10, \quad (4)$$

Where, T_p is the main time constant of the system (Wescott, 2000). It is needed that the control loop acts ten times, at least, in response to step input, to smooth the output response.

To tune the response of the controller, it is needed to select the parameters K_p , K_i and K_d in an appropriated way. There are several rules to do this job; the most popular is the Ziegler-Nichols method.

In practice, several problems arise, mainly due to the electrical noise at the input. The error variable $e[n]$ is computed as a difference between input signal (and noise) and the set point. When the output of the system reaches the steady state, $e[n]$ is near zero, but the relative error is high. In the case of the derivative variable computed as $e[n] - e[n-1]$, the situation is worst and could lead unstable results; usually low pass filters are employed to mitigate the noise effects.

In this work, it is proposed the use of exponentially weighted moving average (EWMA), using statistical criteria and the noise reduction properties of the EWMA, to adjust the filter. In Fig. 1 it is shown the proposed control loop.

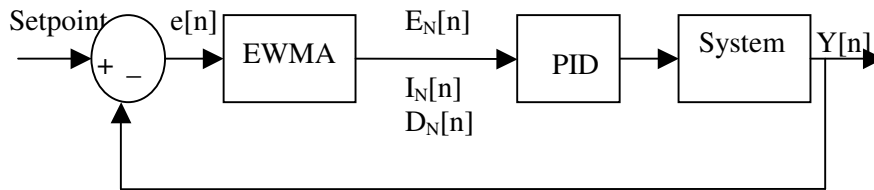


Figure 1: EWMA-PID controller, $P_N[n]$, $I_N[n]$, $D_N[n]$ are filtered variables computed from $e[n]$, and will be used to calculate the control ec. (1)

2. DEVELOPMENT

2.1. The Exponentially Weighted Moving Average

The *moving average* technique is often used in order to obtain an average of samples at the time they are “arriving” to the control system. The idea is to add the last N samples acquired, and then divide the sum by N . The average is “moving” because it is computed each time a new sample is obtained. To save memory and to make faster calculations, the recursive formula is used.

$$P_N[n] = P_N[n-1] + (e[n] - P_N[n-1])/N, \quad (5)$$

Called Exponentially Weighted Moving Average (EWMA), because the recent samples are more weighted than the oldest, and as it could be seen in Proakis y Manolakis, (1998), it behaves like a IIR (Infinite-duration Impulse Response) low pass filter. Analytical expression of the response to an unitary step is:

$$P_N[n] = e[0] \cdot (1 - ((N-1)/N)^{n+1})$$

In an approximated way, it is possible to describe the temporal step response (Crenshaw, 1996) as

$$P_N[n] = e[0] \cdot (1 - e^{-n/N})$$

The EWMA will reach the $1-1/e$ of his final value after N samples. N is called "averaging time constant" of the filter and it produces a delay τ :

$$\tau = N \cdot T_s, \quad (6)$$

The EWMA is used because of its noise filtering capabilities. In those cases where the noise is added to the signal and has zero mean value the filter cancels the effects of noise at the output (Smith., 1997). In this analysis, the noise superimposed to the input signal $e[n]$ is supposed Gaussian, zero mean value and variance σ^2 .

The noise reduction of the EWMA is more effective than the average of N samples, the variance of the EWMA could be expressed as (Hunter, 1986):

$$\sigma_{PN}^2 = \sigma^2 / 2N-1, \quad (7)$$

An optimal value of N , that reduces the noise of the EWMA, and at the same time, introduced a delay compatible with the normal operation of the system, should be found.

2.2. Proposed Solution

To determine the optimum averaging constant N of the filter during the system evolution, $e[n]$ measurements are carried out at constant time T_s , called *instantaneous* measurement, also an EWMA $P_N[n]$ is calculated using Ec. (5); the standard deviation of $P_N[n]$ is:

$$\sigma_{PN}[n] = \sigma / \sqrt{2N-1}, \quad (8)$$

At the instant n , $P_N[n]$ will be considered the "current average" and the best available estimation of the input variable. To be valid, $P_N[n]$ should fulfill the following requirement:

$$|P_N[n] - e[n]| \leq \sigma, \quad (9)$$

If this requirement is not fulfilled, $N[n+1] = N[n]/2$ will be adopted.

On the other hand, a simultaneous EWMA $P_{2N}[n]$ with averaging time constant $2.N$ is calculated, and if the previous requirement is satisfied, it could be possible to adopt a better filtered current average provided that the following requirement is fulfilled.

$$|P_N[n] - P_{2N}[n]| \leq \sigma / \sqrt{2N-1}, \quad (10)$$

To adopt $N[n+1] = 2.N[n]$ the average $P_{2N}[n]$ should fall inside the band of noise of the current average. With this procedure fast (and noisy) measurements are achieved when the system is evolving faster. On the other hand, measurements very well filtered (and slow) are achieved when the system is inside the band of noise of the instantaneous measurements. Should be noted that, only the value of averaging constant N , is changed, not the actual value of $P_N[n]$.

On the other hand, the standard deviation of the EWMA σ_{PN} , can be seen as the uncertainty of the estimation of the mean value of $e[n]$.

2.3. Calculation of EWMA-PID Controller variables

2.3.1. Calculation of the proportional variable $E[n]$

The current EWMA, $P_N[n]$, is adopted as the better estimation of $e[n]$.

$$E[n] = P_N[n] \quad (11)$$

2.3.2. Calculation of the derivative variable $D[n]$

The instantaneous derivative term $d[n]$, is calculated in the same way that standard PID by eq. (2). As $e[n]$ and $e[n-1]$ are independent measures of the error, the variance of $d[n]$ can be expressed as:

$$\sigma_d^2 = 2 \cdot \sigma^2 \quad (12)$$

For the calculation of the EWMA of the differential variable $D[n]$, two EWMA are computed, defined in the same way that in the Ec. (5); the first, defined over the odd samples of $e[n]$; the second, over even samples.

$$\text{Odd}[n] = \text{Odd}[n-1] + [e[n] - \text{Odd}[n-1]]/N_d, \quad \text{for } n \text{ odd} \quad (13)$$

$$\text{Even}[n] = \text{Even}[n-1] + [e[n] - \text{Even}[n-1]]/N_d, \quad \text{for } n \text{ even} \quad (14)$$

Where N_d , is averaging time constant for derivative term $D[n]$.

In this way, $D[n]$ can be defined as:

$$D[n] = \text{Even}[n] - \text{Odd}[n], \quad (15)$$

It is computed only when n is even. It is assumed equal to last calculated value for odd n . The standard deviation σ_D , of $D[n]$ can be calculated, bearing in mind eq.(8) and that $\text{Even}[n]$ and $\text{Odd}[n]$ are EWMA with averaging time constant N_d .

$$\sigma_D = \sigma / \sqrt{N_d - 1/2} \quad (16)$$

In a similar way that for the case of proportional variable $P_N[n]$, the double constant filter $D_{2N}[n-1]$ is calculated. The averaging time constant N_d , for derivative variable $D[n]$, is adjusted for even n , using σ_d and σ_D in eq.(9) and (10).

2.3.3. Calculation of the integral variable $I[n]$

The integral variable $i[d]$, for standard PID is calculated as the accumulation of samples using eq. (3), when the system is far from the set

point the summation saturate and produce an effect called wind up. For these reason, the EWMA integral variable $I[n]$ is computed as:

$$I[n] = N_i \cdot P_{N_i}[n] \quad (17)$$

Where N_i is averaging time constant for the integral variable; bearing in mind that the variance of $I[n]$ is $\sigma_I^2 = N \sigma^2$ and using the same criteria of eq. (9) and (10); can be seen that N_i will be adjusted at the same time that N , then it is possible to use $N_i = N$.

2.3.4. EWMA-PID Controller

Using eqs. (11), (15) and (17), in eq. (1), it is possible to write the final form of the EWMA-PID Controller.

$$y[n] = (K_p + N \cdot K_i) \cdot P_N[n] + K_d \cdot D_N[n], \quad (18)$$

The integral term of the EWMA-PID Controller, act as a reinforcement of the proportional term, to reach the set point, and is shortened when the system is far away from the steady state, this behaviour can be seen as an automatic gain control to increase the stability of the system; N could be used as an indicator of the state of the system.

3. REALIZATION

The EWMA-PID controller was implemented as position controller (direction) and velocity controller (traction) in a robotic inspection vehicle (PMIR). Each motor, one for traction and other for direction, has its own AVR microcontroller, connected by wireless link to a Master device, sending position (or velocity) to each motor.

It is desired to implement a position and velocity control without over dumping. The conditions of the mobile platform such as the weight, friction and noise can change, when it happens, it is necessary to adjust the values of K_p , K_i and K_d to obtain an optimal response of standard PID. In Fig. 2 and 3 it is shown the response of standard PID and the EWMA-PID. The EWMA-PID controller show a fast response, and at the same time, more stable behavior with noise.

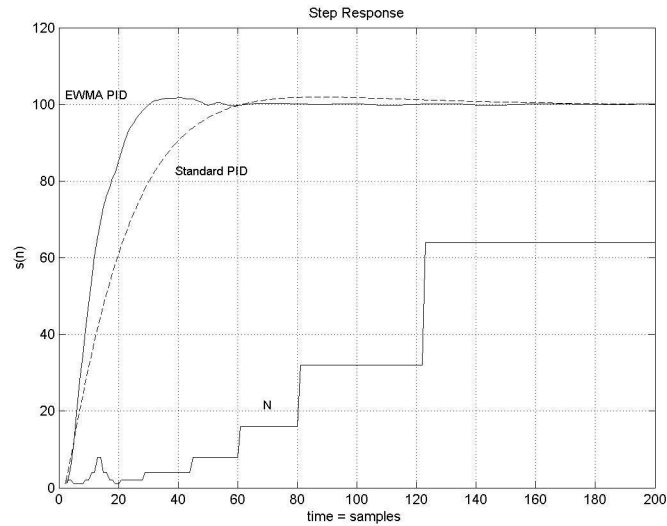


Figure 2: Step response without noise.

- a) Filled line, EWMA-PID controller with: $K_p=7; K_d=45; K_i=0.5$.
- b) Dashed line, standard PID with: $K_p=1.3; K_d=20; K_i=.001$.
- c) Stair case line, state of the averaging constant N .

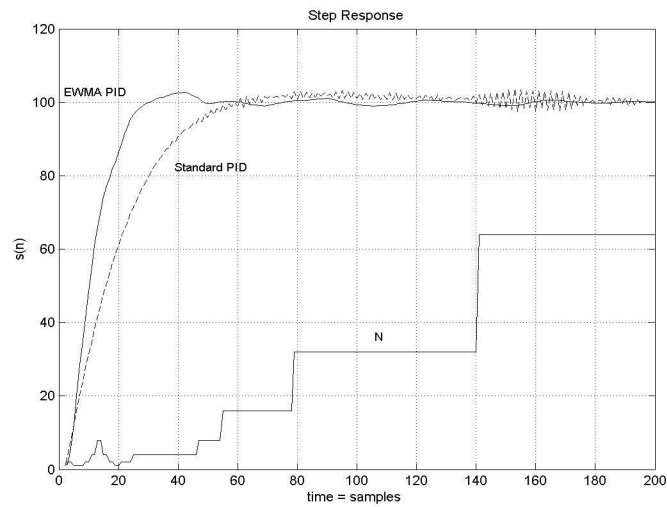


Figure 3: Step response with 1% of noise.

- a) Filled line, EWMA-PID controller with: $K_p=7; K_d=45; K_i=0.5$.
- b) Dashed line, standard PID with: $K_p=1.3; K_d=20; K_i=.001$.
- c) Staircase line, state of the averaging constant N .

4. DISCUSSION

The algorithm developed could be tailored in order to meet different control needs. The obvious parameters; N , K_p , K_d , K_i , and the band limits can be tuned up.

The criteria used to change the filtering conditions could be modified too. As an example: "Wait for three consecutive times the value is out the band to change to a lesser filtered condition".

Another interesting alternative is to change the filtering conditions more than one step at the time. Furthermore, the criteria used do not need to be the same depending on if it is going to better or to worse filtering conditions. Looking for a lesser measurement error or a faster controller reaction respectively.

5. CONCLUSION

A control method has been developed, using a EWMA input filter of PID, whose time response depends, upon the deviation of the system from the set point. The averaging time constant N , is revised each T_s . The instantaneous measure $e[n]$, is used as "guard" measurement with known noise and precision. This criterion allows reducing the level of noise in the input signal, decreasing the permanent error and time response of the system. The same method, has been extended to the calculation of the differential variable, with improvements in stability and to the integral variable, avoiding the use of an anti windup strategy, with excellent results as far as simplicity in the implementation and robustness.

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