



Algorithm Analysis / Design

Practical Report

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2021UCI8036

CSE (Internet of Things)
Semester 3

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1 Counting Sort

1.1 Description

This sorting technique doesn't perform sorting by comparing elements. It performs sorting by counting objects having distinct key values like hashing. After that, it performs some arithmetic operations to calculate each object's index position in the output sequence. Counting sort is not used as a general-purpose sorting algorithm.

1.2 Algorithm

Algorithm 1 Counting Sort

```

1: procedure COUNTINGSORT(array, size)
2:   max = findMax(array, size)
3:   initialize countArray(max + 1) with 0
4:   for i = 0 to size do
5:     countArray[array[i]] ++
6:   end for
7:   for i = 1 to max do
8:     countArray[i] = countArray[i] + countArray[i - 1]
9:   end for
10:  for i = size to 1 do
11:    output[countArray[array[i]] - 1] = array[i]
12:    countArray[array[i]] = countArray[array[i]] - 1
13:  end for
14: end procedure

```

1.3 Code

```

1  // Using counting sort to sort the elements in the basis of significant places
2  void countSort(vector<int> &arr, int n, int exp) {
3      vector<int> output(n); // output array
4      int i, count[10] = {0};
5
6      // Store count of occurrences in count[]
7      for (i = 0; i < n; i++)
8          count[(arr[i] / exp) % 10]++;
9
10     // Change count[i] so that count[i] now contains actual
11     // position of this digit in output[]
12     for (i = 1; i < 10; i++)
13         count[i] += count[i - 1];
14
15     // Build the output array
16     for (i = n - 1; i >= 0; i--) {
17         output[count[(arr[i] / exp) % 10] - 1] = arr[i];
18         count[(arr[i] / exp) % 10]--;

```

```

19     }
20
21     // Copy the output array to arr[], so that arr[] now
22     // contains sorted numbers according to current digit
23     for (i = 0; i < n; i++)
24         arr[i] = output[i];
25 }

```

1.4 Output

Sorted array: 1 2 3 4 5 6 7 8 9 10

2 Radix Sort

2.1 Description

Radix sort is the linear sorting algorithm that is used for integers. In Radix sort, there is digit by digit sorting is performed that is started from the least significant digit to the most significant digit.

2.2 Algorithm

Algorithm 2 Radix Sort

```

1: procedure RADIXSORT(array, size)
2:   max = findMax(array, size)
3:   for exp = 1 to max do
4:     countSort(array, size, exp)
5:   end for
6: end procedure

```

2.3 Code

```

1  /** An implementation of Radix sort. */
2  #include <iostream>
3  #include <vector>
4  using namespace std;
5
6  extern void countSort(vector<int> &arr, int n, int exp);
7
8  // * Function to get the largest element from an array
9  int getMax(vector<int> &arr, int n) {
10     int mx = arr[0];
11     for (int i = 1; i < n; i++)
12         if (arr[i] > mx)

```

```

13     mx = arr[i];
14     return mx;
15 }
16
17 void radixsort(vector<int> &arr, int n) {
18     // Find the maximum number to know number of digits
19     int m = getMax(arr, n);
20
21     // Do counting sort for every digit. Note that instead
22     // of passing digit number, exp is passed. exp is 10^i
23     // where i is current digit number
24     for (int exp = 1; m / exp > 0; exp *= 10)
25         countSort(arr, n, exp);
26 }
27
28 // * Driver program to test above functions
29 int main() {
30     int n;
31     cout << "Enter the number of elements: ";
32     cin >> n;
33     vector<int> arr(n);
34     cout << "Enter the elements: ";
35     for (int i = 0; i < n; i++)
36         cin >> arr[i];
37     radixsort(arr, n);
38     cout << "Sorted array: ";
39     for (int i = 0; i < n; i++)
40         cout << arr[i] << " ";
41     return 0;
42 }

```

2.4 Output

```

Enter the number of elements: 5
Enter the elements: 170 45 75 90 802
Sorted array: 45 75 90 170 802

```

3 Bucket Sort

3.1 Description

Bucket sort is a sorting algorithm that separate the elements into multiple groups said to be buckets. Elements in bucket sort are first uniformly divided into groups called buckets, and then they are sorted by any other sorting algorithm. After that, elements are gathered in a sorted manner.

3.2 Algorithm

Algorithm 3 Bucket Sort

```

1: procedure BUCKETSORT( $A$ )
2:   Let  $B[0 \dots n-1]$  be a new array
3:    $n = \text{length of } A$ 
4:   for  $i \leftarrow 0$  to  $n-1$  do
5:     make  $B[i]$  an empty list
6:   end for
7:   for  $i \leftarrow 1$  to  $n$  do
8:     insert  $A[i]$  into list  $B[na[i]]$ 
9:   end for
10:  for  $i \leftarrow 0$  to  $n-1$  do
11:    sort list  $B[i]$  with insertion-sort
12:  end for
13:  Concatenate lists  $B[0], B[1], \dots, B[n-1]$  together in order
14: end procedure

```

3.3 Code

```

1  #include <iostream>
2  #include <vector>
3  using namespace std;
4
5  void bucketSort(float arr[], int n) {
6      // Create n empty buckets
7      vector<float> b[n];
8
9      // Put array elements in different buckets
10     for (int i = 0; i < n; i++) {
11         int bi = n * arr[i]; // Index in bucket
12         b[bi].push_back(arr[i]);
13     }
14
15     // Sort individual buckets
16     for (int i = 0; i < n; i++)
17         sort(b[i].begin(), b[i].end());
18
19     // Concatenate all buckets into arr[]
20     int index = 0;
21     for (int i = 0; i < n; i++)
22         for (int j = 0; j < b[i].size(); j++)
23             arr[index++] = b[i][j];
24 }
25
26 // * Driver program to test above functions
27 int main() {
28     int n;

```

```

29     cout << "Enter the number of elements: ";
30     cin >> n;
31     float arr[n];
32     cout << "Enter the elements: ";
33     for (int i = 0; i < n; i++)
34         cin >> arr[i];
35     bucketSort(arr, n);
36
37     cout << "Sorted array is ";
38     for (int i = 0; i < n; i++)
39         cout << arr[i] << " ";
40     return 0;
41 }

```

3.4 Output

```

Enter the number of elements: 5
Enter the elements: 0.897 0.565 0.656 0.1234 0.665
Sorted array is 0.1234 0.565 0.656 0.665 0.897

```

4 Quick Sort

4.1 Description

Quicksort picks an element as pivot, and then it partitions the given array around the picked pivot element. In quick sort, a large array is divided into two arrays in which one holds values that are smaller than the specified value (Pivot), and another array holds the values that are greater than the pivot. After that, left and right sub-arrays are also partitioned using the same approach. It will continue until the single element remains in the sub-array.

4.2 Algorithm

Algorithm 4 Quick Sort

```

1: procedure QUICKSORT(array, left, right)
2:   if left < right then
3:     pivot = partition(array, left, right)
4:     QuickSort(array, left, pivot - 1)
5:     QuickSort(array, pivot + 1, right)
6:   end if
7: end procedure

```

4.3 Code

```

1  #include <iostream>
2  using namespace std;
3
4  /* This function takes last element as pivot, places
5  the pivot element at its correct position in sorted
6  array, and places all smaller (smaller than pivot)
7  to left of pivot and all greater elements to right
8  of pivot */
9
10 int partition (int arr[], int low, int high)
11 {
12     int pivot = arr[high]; // pivot
13     int i = (low - 1); // Index of smaller element
14
15     for (int j = low; j <= high - 1; j++) {
16         // If current element is smaller than or
17         // equal to pivot
18         if (arr[j] <= pivot) {
19             i++; // increment index of smaller element
20             swap(&arr[i], &arr[j]);
21         }
22     }
23     swap(&arr[i + 1], &arr[high]);
24     return (i + 1);
25 }
26
27 // The main function that implements QuickSort
28 // arr[] --> Array to be sorted,
29 // low --> Starting index,
30 // high --> Ending index
31
32 void quickSort(int arr[], int low, int high)
33 {
34     if (low < high) {
35         /* pi is partitioning index, arr[p] is now
36         at right place */
37         int pi = partition(arr, low, high);
38
39         // Separately sort elements before
40         // partition and after partition
41         quickSort(arr, low, pi - 1);
42         quickSort(arr, pi + 1, high);
43     }
44 }
45
46 // Function to print an array
47 void printArray(int arr[], int size)
48 {
49     for (int i = 0; i < size; i++)
50         cout << arr[i] << " ";
51     cout << endl;
52 }

```

```

53
54 int main()
55 {
56     int arr[] = { 10, 7, 8, 9, 1, 5 };
57     int n = sizeof(arr) / sizeof(arr[0]);
58     quickSort(arr, 0, n - 1);
59     cout << "Sorted array: n";
60     printArray(arr, n);
61     return 0;
62 }

```

4.4 Output

```

Enter the number of elements: 5
Enter the elements: 170 45 75 90 802
Sorted array is 45 75 90 170 802

```

5 Merge Sort

5.1 Description

Merge sort is similar to the quick sort algorithm as it uses the divide and conquer approach to sort the elements. It is one of the most popular and efficient sorting algorithm. It divides the given list into two equal halves, calls itself for the two halves and then merges the two sorted halves. We have to define the merge() function to perform the merging.

The sub-lists are divided again and again into halves until the list cannot be divided further. Then we combine the pair of one element lists into two-element lists, sorting them in the process. The sorted two-element pairs is merged into the four-element lists, and so on until we get the sorted list.

5.2 Algorithm

Algorithm 5 Merge Sort

```

1: procedure MERGESORT(array, left, right)
2:   if left < right then
3:     middle = (left + right)/2
4:     MergeSort(array, left, middle)
5:     MergeSort(array, middle + 1, right)
6:     merge(array, left, middle, right)
7:   end if
8: end procedure

```

5.3 Code

```
1  #include <iostream>
2  using namespace std;
3
4  // merge two sorted subarrays arr[low..mid] and arr[mid+1..high]
5  void merge(int arr[], int low, int mid, int high)
6  {
7      // create a temp array
8      int temp[high - low + 1];
9
10     // crawlers for both intervals and for temp
11     int i = low, j = mid + 1, k = 0;
12
13     // traverse both arrays and in each iteration add smaller of both elements in temp
14     while (i <= mid && j <= high)
15     {
16         if (arr[i] <= arr[j])
17             temp[k++] = arr[i++];
18         else
19             temp[k++] = arr[j++];
20     }
21
22     // add elements left in the first interval
23     while (i <= mid)
24         temp[k++] = arr[i++];
25
26     // add elements left in the second interval
27     while (j <= high)
28         temp[k++] = arr[j++];
29
30     // copy temp to original interval
31     for (int i = low; i <= high; i++)
32         arr[i] = temp[i - low];
33 }
34
35 // Recursive implementation of merge sort
36 void mergeSort(int arr[], int low, int high)
37 {
38     // base condition
39     if (low == high)
40         return;
41
42     // find the mid value
43     int mid = (low + (high - low) / 2);
44
45     // recursively split the array into two halves until it can no more be split
46     mergeSort(arr, low, mid);
47     mergeSort(arr, mid + 1, high);
48
49     merge(arr, low, mid, high);
50 }
51
52 int main()
```

```

53 {
54     int arr[] = { 3, 5, 8, 4, 1, 9, -2 };
55     int n = sizeof(arr) / sizeof(arr[0]);
56
57     mergeSort(arr, 0, n - 1);
58     for (int i = 0; i < n; i++)
59         cout << arr[i] << " ";
60     return 0;
61 }

```

5.4 Output

```

Enter the number of elements: 5
Enter the elements: 170 45 75 90 802
Sorted array is 45 75 90 170 802

```

6 Heap Sort

6.1 Description

Heap sort processes the elements by creating the min-heap or max-heap using the elements of the given array. Min-heap or max-heap represents the ordering of array in which the root element represents the minimum or maximum element of the array.

6.2 Algorithm

Algorithm 6 Heap Sort

```

1: procedure HEAPSORT(array, n)
2:   buildMaxHeap(array, n)
3:   for i ← n to 1 do
4:     swap array[0] and array[i]
5:     heapify(array, i, 0)
6:   end for
7: end procedure

```

6.3 Code

```

1  #include <iostream>
2  using namespace std;
3
4  // Function to heapify the tree
5  void heapify(int arr[], int n, int i)
6  {

```

```

7      int largest = i; // Initialize largest as root
8      int l = 2 * i + 1; // left = 2*i + 1
9      int r = 2 * i + 2; // right = 2*i + 2
10
11     // If left child is larger than root
12     if (l < n && arr[l] > arr[largest])
13         largest = l;
14
15     // If right child is larger than largest so far
16     if (r < n && arr[r] > arr[largest])
17         largest = r;
18
19     // If largest is not root
20     if (largest != i) {
21         swap(&arr[i], &arr[largest]);
22         // Recursively heapify the affected sub-tree
23         heapify(arr, n, largest);
24     }
25 }
26
27 // Function to build a Max-Heap from the given array
28 void buildHeap(int arr[], int n)
29 {
30     // Index of last non-leaf node
31     int startIdx = (n / 2) - 1;
32
33     // Perform reverse level order traversal from last non-leaf node and heapify each
34     ↪ node
35     for (int i = startIdx; i >= 0; i--) {
36         heapify(arr, n, i);
37     }
38
39 // Function to sort an array using Heap Sort
40 void heapSort(int arr[], int n)
41 {
42     // Build a max heap
43     buildHeap(arr, n);
44
45     // Heap sort
46     for (int i = n - 1; i > 0; i--) {
47         // Swap
48         swap(&arr[0], &arr[i]);
49         // Heapify root element to get highest element at root again
50         heapify(arr, i, 0);
51     }
52 }
53
54 // Function to print an array
55 void printArray(int arr[], int n)
56 {
57     for (int i = 0; i < n; ++i)
58         cout << arr[i] << " ";
59     cout << endl;

```

```
60 }
61
62 // Driver code
63 int main()
64 {
65     int arr[] = { 1, 12, 9, 5, 6, 10 };
66     int n = sizeof(arr) / sizeof(arr[0]);
67
68     heapSort(arr, n);
69
70     cout << "Sorted array is " << endl;
71     printArray(arr, n);
72 }
```

6.4 Output

```
Enter the number of elements: 5
Enter the elements: 170 45 75 90 802
Sorted array is 45 75 90 170 802
```

7 Depth First Search

7.1 Description

The depth-first search (DFS) algorithm starts with the initial node of graph G and goes deeper until we find the goal node or the node with no children.

7.2 Algorithm

Algorithm 7 Depth First Search

```
1: procedure DFS( $G, v$ )
2:   mark  $v$  as visited
3:   for each unvisited neighbor  $w$  of  $v$  do
4:     DFS( $G, w$ )
5:   end for
6: end procedure
```

7.3 Code

```

1  #include <iostream>
2  #include <vector>
3  #include <stack>
4  using namespace std;
5
6  struct Edge {
7      int src, dest;
8  };
9
10 class Graph
11 {
12 public:
13     vector<vector<int>> adjList;
14
15     Graph(vector<Edge> const &edges, int N)
16     {
17         // resize the vector to N elements of type vector<int>
18         adjList.resize(N);
19
20         // add edges to the directed graph
21         for (auto &edge: edges)
22         {
23             // insert at the end
24             adjList[edge.src].push_back(edge.dest);
25             // Uncomment below line for undirected graph
26             // adjList[edge.dest].push_back(edge.src);
27         }
28     }
29 };
30
31 // Perform DFS on graph starting from vertex v
32 bool DFS(Graph const &graph, int v, vector<bool> &discovered)
33 {
34     // create a stack used to do DFS
35     stack<int> stack;
36
37     // push the source node into stack
38     stack.push(v);
39
40     // loop till stack is empty
41     while (!stack.empty())
42     {
43         // we pop a vertex from stack and print it
44         v = stack.top();
45         stack.pop();
46
47         // if the vertex is already discovered yet, ignore it
48         if (discovered[v])
49             continue;
50
51         // we print the vertex and mark it as discovered
52         discovered[v] = true;

```

```

53     cout << v << " ";
54
55     // do for every edge (v -> u)
56     for (int u : graph.adjList[v])
57     {
58         // if vertex u is not discovered, push it into stack
59         if (!discovered[u])
60             stack.push(u);
61     }
62 }
63
64 return true;
65 }
66
67 // Depth First Search Algorithm
68 int main()
69 {
70     // vector of graph edges as per above diagram
71     vector<Edge> edges = {
72         {1, 2}, {1, 3}, {1, 4}, {2, 5}, {2, 6}, {5, 9},
73         {5, 10}, {4, 7}, {4, 8}, {7, 11}, {7, 12}
74         // vertex 0, 13 and 14 are single nodes
75     };
76
77     // Number of nodes in the graph
78     int N = 15;
79
80     // create a graph from edges
81     Graph graph(edges, N);
82
83     // stores vertex is discovered or not
84     vector<bool> discovered(N);
85
86     // Do DFS traversal from all undiscovered nodes to
87     // cover all unconnected components of graph
88     for (int i = 0; i < N; i++)
89         if (discovered[i] == false)
90             // start DFS traversal from vertex i
91             DFS(graph, i, discovered);
92
93     return 0;
94 }

```

7.4 Output

```
1 4 8 12 7 11 3 2 6 10 9 5
```

8 Breadth First Search

8.1 Description

Breadth-first search is a graph traversal algorithm that starts traversing the graph from the root node and explores all the neighbouring nodes. Then, it selects the nearest node and explores all the unexplored nodes.

8.2 Algorithm

Algorithm 8 Breadth First Search

```

1: procedure BFS( $G, s$ )
2:   for each vertex  $u \in V[G]$  do
3:      $d[u] \leftarrow \infty$ 
4:      $\pi[u] \leftarrow NIL$ 
5:   end for
6:    $d[s] \leftarrow 0$ 
7:    $Q \leftarrow \emptyset$ 
8:   ENQUEUE( $Q, s$ )
9:   while  $Q \neq \emptyset$  do
10:     $u \leftarrow DEQUEUE(Q)$ 
11:    for each vertex  $v \in Adj[u]$  do
12:      if  $d[v] = \infty$  then
13:         $d[v] \leftarrow d[u] + 1$ 
14:         $\pi[v] \leftarrow u$ 
15:        ENQUEUE( $Q, v$ )
16:      end if
17:    end for
18:  end while
19: end procedure

```

8.3 Code

```

1  #include <iostream>
2  #include <vector>
3  #include <queue>
4  using namespace std;
5
6  struct Edge {
7      int src, dest;
8  };
9
10 class Graph
11 {
12 public:
13     vector<vector<int>>> adjList;
14
15     Graph(vector<Edge> const &edges, int N)

```

```

16     {
17         // resize the vector to N elements of type vector<int>
18         adjList.resize(N);
19
20         // add edges to the directed graph
21         for (auto &edge: edges)
22         {
23             // insert at the end
24             adjList[edge.src].push_back(edge.dest);
25             // Uncomment below line for undirected graph
26             // adjList[edge.dest].push_back(edge.src);
27         }
28     }
29 };
30
31 // Perform BFS on graph starting from vertex v
32 bool BFS(Graph const &graph, int v, vector<bool> &discovered)
33 {
34     queue<int> q;
35     // mark source vertex as discovered
36     discovered[v] = true;
37     // push source vertex into the queue
38     q.push(v);
39
40     while (!q.empty())
41     {
42         // dequeue front node and print it
43         v = q.front();
44         q.pop();
45         cout << v << " ";
46
47         // do for every edge (v -> u)
48         for (int u : graph.adjList[v])
49         {
50             // if vertex u is not discovered yet
51             if (!discovered[u])
52             {
53                 discovered[u] = true;
54                 q.push(u);
55             }
56         }
57     }
58 }
59
60 int main()
61 {
62     // vector of graph edges as per above diagram
63     vector<Edge> edges = {
64         {1, 2}, {1, 3}, {1, 4}, {2, 5}, {2, 6}, {5, 9},
65         {5, 10}, {4, 7}, {4, 8}, {7, 11}, {7, 12}
66         // vertex 0, 13 and 14 are single nodes
67     };
68
69     // total number of nodes in the graph

```

```
70  int N = 15;
71
72  // build a graph from the given edges
73  Graph graph(edges, N);
74
75  // to keep track of whether a vertex is discovered or not
76  vector<bool> discovered(N);
77
78  // Do BFS traversal from all undiscovered nodes to
79  // cover all unconnected components of graph
80  for (int i = 0; i < N; i++)
81      if (discovered[i] == false)
82          // start BFS traversal from vertex i
83          BFS(graph, i, discovered);
84
85  return 0;
86 }
```

8.4 Output

```
1 4 8 12 7 11 3 2 6 10 9 5
```

9 Prim's Algorithm

9.1 Description

Prim's algorithm is a greedy algorithm that finds a minimum spanning tree for a weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. The algorithm operates by building this tree one vertex at a time, from an arbitrary starting vertex, at each step adding the cheapest possible connection from the tree to another vertex.

9.2 Algorithm

Algorithm 9 Prim's Algorithm

```

1: procedure PRIM( $G$ )
2:   for each vertex  $u \in V[G]$  do
3:      $key[u] \leftarrow \infty$ 
4:      $\pi[u] \leftarrow NIL$ 
5:   end for
6:    $key[s] \leftarrow 0$ 
7:    $Q \leftarrow V[G]$ 
8:   while  $Q \neq \emptyset$  do
9:      $u \leftarrow EXTRACT - MIN(Q)$ 
10:    for each vertex  $v \in Adj[u]$  do
11:      if  $v \in Q$  and  $w(u, v) < key[v]$  then
12:         $\pi[v] \leftarrow u$ 
13:         $key[v] \leftarrow w(u, v)$ 
14:      end if
15:    end for
16:  end while
17: end procedure

```

9.3 Code

```

1  struct Edge {
2      int src, dest, weight;
3  };
4
5  class Graph
6  {
7  public:
8      // construct a vector of vectors of Edge to represent an adjacency list
9      vector<vector<Edge>> adjList;
10
11     // Graph Constructor
12     Graph(vector<Edge> const &edges, int N)
13     {
14         // resize the vector to N elements of type vector<Edge>
15         adjList.resize(N);
16
17         // add edges to the directed graph
18         for (auto &edge: edges)
19         {
20             // insert at the end
21             adjList[edge.src].push_back(edge);
22         }
23     }
24 };
25

```

```

26 // Function to print adjacency list representation of graph
27 void printGraph(Graph const &graph, int N)
28 {
29     for (int i = 0; i < N; i++)
30     {
31         // print current vertex number
32         cout << i << " --> ";
33
34         // print all neighboring vertices of vertex i
35         for (Edge e : graph.adjList[i])
36             cout << "(" << e.dest << ", " << e.weight << ") ";
37
38         cout << endl;
39     }
40 }
41
42 // Function to perform Prim's algorithm on a graph
43 int prim(Graph const &graph, int N)
44 {
45     // create a min-heap using std::set in STL
46     // we use `std::pair` as the data type of the element stored in the heap
47     // the first element of the pair stores the key and the second element stores the
48     ↪ vertex number
49     set<pair<int, int>> pq;
50
51     // create a vector to store key of the vertices which have been found by the
52     ↪ algorithm
53     // initialize all keys to infinite (INT_MAX)
54     vector<int> key(N, INT_MAX);
55
56     // create a vector to store parent node of a vertex
57     // it keeps track of the minimum spanning tree
58     vector<int> parent(N, -1);
59
60     // to keep track of vertices included in MST
61     vector<bool> inMST(N, false);
62
63     // insert source vertex in the set and make its key 0
64     pq.insert(make_pair(0, 0));
65     key[0] = 0;
66
67     // run till `pq` is not empty
68     while (!pq.empty())
69     {
70         // find the vertex with the minimum key
71         // extract it from the set
72         int u = pq.begin()->second;
73         pq.erase(pq.begin());
74
75         // include vertex in MST
76         inMST[u] = true;
77
78         // do for each adjacent vertex `v` of `u`
79         for (Edge e : graph.adjList[u])

```

```

78     {
79         int v = e.dest;
80         int weight = e.weight;
81
82         // if `v` is not in MST and weight of (u, v) is smaller than current key of
↪ `v`
83         if (!inMST[v] && key[v] > weight)
84         {
85             // update the key of `v` in the set
86             pq.erase(make_pair(key[v], v));
87             key[v] = weight;
88             pq.insert(make_pair(key[v], v));
89
90             // update the parent of `v`
91             parent[v] = u;
92         }
93     }
94 }
95
96 // print edges of the MST
97 for (int i = 1; i < N; i++)
98     cout << parent[i] << " - " << i << endl;
99
100 // return weight of the MST
101 return accumulate(key.begin(), key.end(), 0);
102 }
103
104 int main()
105 {
106     // vector of graph edges as per above diagram
107     vector<Edge> edges =
108     {
109         // (x, y, w) -> edge from x to y having weight w
110         { 0, 1, 6 }, { 1, 2, 7 }, { 2, 0, 5 }, { 2, 1, 4 },
111         { 3, 2, 10 }, { 5, 4, 1 }, { 4, 5, 3 }, { 4, 0, 1 }
112     };
113
114     // Number of nodes in the graph
115     int N = 6;
116
117     // construct graph
118     Graph graph(edges, N);
119
120     // print adjacency list representation of graph
121     printGraph(graph, N);
122
123     // run Prim's algorithm on graph
124     cout << "Weight of the MST is " << prim(graph, N);
125
126     return 0;
127 }

```

9.4 Output

```

0 --> (1, 6) (2, 5) (4, 1)
1 --> (2, 7) (0, 6)
2 --> (1, 4) (0, 5) (3, 10)
3 --> (2, 10)
4 --> (5, 3) (0, 1)
5 --> (4, 1)
0 - 4
4 - 5
2 - 1
1 - 2
3 - 2
Weight of the MST is 16

```

10 Kruskal's Algorithm

10.1 Description

Kruskal's algorithm is a minimum spanning tree algorithm that finds an edge of the least possible weight that connects any two trees in the forest. It is a greedy algorithm in graph theory as it finds a minimum spanning tree for a connected weighted graph adding increasing cost arcs at each step.

10.2 Algorithm

Algorithm 10 Kruskal's Algorithm

```

1: procedure KRUSKAL( $G$ )
2:    $A \leftarrow \emptyset$ 
3:   for each vertex  $v \in V[G]$  do
4:      $MAKE - SET(v)$ 
5:   end for
6:   Sort the edges of  $E[G]$  into nondecreasing order by weight  $w$ 
7:   for each edge  $(u, v) \in E[G]$ , taken in nondecreasing order by weight do
8:     if  $FIND - SET(u) \neq FIND - SET(v)$  then
9:        $A \leftarrow A \cup \{(u, v)\}$ 
10:       $UNION(u, v)$ 
11:    end if
12:  end for
13:  return  $A$ 
14: end procedure

```

10.3 Code

```

1  #include <iostream>
2  #include <vector>
3
4  // data structure to store graph edges
5  struct Edge {
6      int src, dest, weight;
7  };
8
9  // class to represent a graph object
10 extern class Graph;
11
12 // Function to print adjacency list representation of graph
13 extern void printGraph(Graph const &graph, int N);
14
15 int kruskal(Graph const &graph, int N)
16 {
17     // stores edge list of the MST
18     vector<Edge> MST;
19
20     // initialize a Union-Find data structure
21     DisjointSet ds(N);
22
23     // get all edges of the graph in a vector
24     vector<Edge> edges;
25     for (int i = 0; i < N; i++)
26     {
27         for (Edge e : graph.adjList[i])
28             edges.push_back(e);
29     }
30
31     // sort edges by increasing weight
32     sort(edges.begin(), edges.end(), [](Edge const &e1, Edge const &e2) {
33         return e1.weight < e2.weight;
34     });
35
36     // process edges in increasing weight order
37     for (Edge const &edge: edges)
38     {
39         // check if the selected edge creates a cycle or not
40         if (ds.find(edge.src) != ds.find(edge.dest))
41         {
42             // include the current edge in MST
43             MST.push_back(edge);
44
45             // merge two components
46             ds.merge(edge.src, edge.dest);
47         }
48     }
49
50     // print MST
51     for (Edge const &e: MST)
52         cout << "(" << e.src << ", " << e.dest << ", " << e.weight << ") ";

```



```

53     return 0;
54 }
55
56 // main function
57 int main()
58 {
59     // vector of graph edges as per above diagram
60     vector<Edge> edges = {
61         {0, 1, 7}, {1, 2, 8}, {0, 3, 5}, {1, 3, 9}, {1, 4, 7}, {2, 4, 5},
62         {3, 4, 15}, {3, 5, 6}, {4, 5, 8}, {4, 6, 9}, {5, 6, 11}
63     };
64
65     // total number of nodes in the graph
66     int N = 7;
67
68     // build a graph from the given edges
69     Graph graph(edges, N);
70
71     // print adjacency list representation of graph
72     printGraph(graph, N);
73
74     // run Kruskal's algorithm on the graph
75     kruskal(graph, N);
76
77     return 0;
78 }
79

```

10.4 Output

```

0 --> (1, 7) (3, 5)
1 --> (0, 7) (2, 8) (3, 9) (4, 7)
2 --> (1, 8) (4, 5)
3 --> (0, 5) (1, 9) (4, 15) (5, 6)
4 --> (1, 7) (2, 5) (3, 15) (5, 8) (6, 9)
5 --> (3, 6) (4, 8) (6, 11)
6 --> (4, 9) (5, 11)
(0, 3, 5) (2, 4, 5) (3, 5, 6) (0, 1, 7) (1, 4, 7) (4, 6, 9)

```