INTEGRALES DE LÍNEA

$u \cdot v = \|u\| \|v\| \cos\theta$

$dr = \begin{bmatrix} dx \\ dy \end{bmatrix}$

$\int_C F \cdot dr$, donde $F(x,y) = \begin{bmatrix} x-y \\ x^2 \end{bmatrix}$ y C es la recta que va de $(0,0)$ a $(2,2)$

$y = x \qquad dy = dx$

$\int_C \left((x-y)\,\mathrm{i} + x^2\mathrm{j}\right) \cdot (dx\mathrm{i} + dy\mathrm{j})$

$\int_C \begin{bmatrix} x-y \\ x^2 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \int (x-y)\,dx + \int x^2 dy$

$= \int (x-x)\,dx + \int x^2 dx = \int_0^2 x^2 dx = \frac{8}{3}$

$\int_C F \cdot dr$ donde $F = \begin{bmatrix} x^2+y \\ x-y \end{bmatrix}$ y C es la curva $y = x^2$ desde $(0,0)$ hasta $(1,1)$

$y = x^2 \qquad dy = 2xdx$

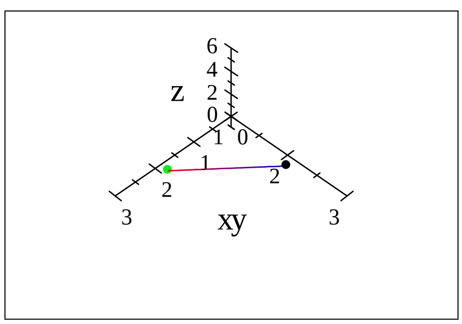
$\int_C \left(x^2+y\right) dx + (x-y)\,dy$

$\int_C \left(x^2+x^2\right) dx + \left(x-x^2\right) 2xdx$

$\int_0^1 \left(\left(x^2+x^2\right) + \left(x-x^2\right) 2x\right) dx = \frac{5}{6}$

$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\int_C F \cdot dr$ donde $F = \begin{bmatrix} x-z \\ x-y \\ 2y+1 \end{bmatrix}$ y C es la recta que va de $(0,2,-1)$ a $(3,2,6)$



$x = 3t + 0 = 3t \qquad dx = 3dt$

$y = 0t + 2 = 2 \qquad dy = 0$

$z = 7t - 1 \qquad dz = 7dt \qquad 0 \leq t \leq 1 \begin{bmatrix} 3t \\ 2 \\ 7t-1 \end{bmatrix}$

$\int (x-z)\,dx + (x-y)\,dy + (2y+1)\,dz$
$\int_0^1 (3t-(7t-1))\,(3dt) + (3t-2)\,(0) + (2\,(2)+1)\,(7dt)$
$\int_0^1 ((3t-(7t-1))\,(3) + (2\,(2)+1)\,(7))\,dt = 32$
$(a,b,c) - > (d,e,f)$
$x = (d-a)\,t + a \qquad y = (e-b)\,t + b \qquad\qquad z = (f-c)\,t + c$

Integral de línea sobre funciones escalares
$ds = \|dr\|$
$ds = \sqrt{(dx)^2 + (dy)^2}$
$ds = \sqrt{(dx)^2 + \left(dx\frac{dy}{dx}\right)^2} = \sqrt{(dx)^2 + (dx)^2\left(\frac{dy}{dx}\right)^2} =$
$= \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)(dx)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx$
$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}dy$
$ds = \sqrt{\left(\frac{dx}{dt}dt\right)^2 + \left(\frac{dy}{dt}dt\right)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2(dt)^2 + \left(\frac{dy}{dt}\right)^2(dt)^2}$
$= \sqrt{\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)(dt)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$
$\int_C yds$, donde C es $y = \sin x$ desde $(0,0)$ a $(\pi,0)$
$\frac{dy}{dx} = \cos x$
$\int_C y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx = \int_0^\pi \sin x\sqrt{1 + (\cos x)^2}dx = 1.338\,487\,7$

$\int_C (x+y)\,ds$, donde C es la curva $r = \left(t, t^2, \sqrt{t}\right)$ con $1 \leq t \leq 4$
$x = t \qquad\qquad \frac{dx}{dt} = 1$
$y = t^2 \qquad\qquad \frac{dy}{dt} = 2t$
$z = \sqrt{t} \qquad\qquad \frac{dz}{dt} = \frac{1}{2\sqrt{t}}$
$\int_C (x+y)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}dt$
$\int_1^4 \left(t+t^2\right)\sqrt{(1)^2 + (2t)^2 + \left(\frac{1}{2\sqrt{t}}\right)^2}dt = 172.357\,79$

¿Cómo hacer una integral de línea cuando el campo es conservativo?
$\int_C F \cdot dr$, donde $F = \begin{bmatrix} 2xy^2 \\ 2x^2y \end{bmatrix}$, donde C es $y = 2x^2 - 1$ con $0 \leq x \leq 1$
$(0,-1)$ a $(1,1)$
Método convencional:
$y = 2x^2 - 1, dy = 4xdx$
$\int 2xy^2dx + 2x^2ydy = \int_0^1 2x\left(2x^2-1\right)^2 dx + 2x^2\left(2x^2-1\right)4xdx =$
$= \int_0^1 \left(2x\left(2x^2-1\right)^2 + 2x^2\left(2x^2-1\right)4x\right)dx = 1$
Verifiquemos si F es conservativo:

$\frac{\partial}{\partial y}2xy^2 = 4xy$ $\qquad$ $\frac{\partial}{\partial x}2x^2y = 4xy \qquad SI\ ES$

$\int 2xy^2dx = x^2y^2$ $\qquad$ $\int 2x^2ydy = x^2y^2$

$f(x,y) = x^2y^2$

$\int_C F \cdot dr = \left[x^2y^2\right]_{(1,1)} - \left[x^2y^2\right]_{(0,-1)} = \left(1^21^2\right) - \left(0^2\left(-1\right)^2\right) = 1$