# Table of Physical Constants

| Some important physical constants | (CODATA values from Dec. 2014) | |
|---|---|---|
| Avogadro constant | $N_A$, | $= 6.022140857 \cdot 10^{23}\ \text{mol}^{-1}$ |
| Gravitational constant | $G$ | $= 6.667_4 \cdot 10^{-11}\ \text{N m}^2/\text{kg}^2$ |
| Electric field constant | $\varepsilon_0$ | $= 8.854188 \cdot 10^{-12}\ \text{A s/V m}$ |
| Magnetic field constant | $\mu_0$ | $= 12.566370 \cdot 10^{-7}\ \text{V s/A m}$ |
| Velocity of light in vacuum | $c$ | $= (\varepsilon_0\mu_0)^{-1/2} = 2.997925 \cdot 10^8\ \text{m/s}$ |
| Wave resistance of vacuum | $Z_{el}$ | $= (\mu_0/\varepsilon_0)^{1/2} = 376.73\ \text{Ohm}$ |
| Relative atomic mass of the proton | $(A)_p$ | $= 1.007277\, u^*$ |
| Relative atomic mass of the neutron | $(A)_n$ | $= 1.008665\, u^*$ |
| Rest mass of the proton | $m_p$ | $= 1.672621 \cdot 10^{-27}\ \text{kg}$ |
| Rest energy of the proton | $(W_p)_0$ | $= 9.382720 \cdot 10^8\ \text{eV}$ |
| Rest mass of the electron | $m_0$ | $= 9.101383 \cdot 10^{-31}\ \text{kg}$ |
| Rest energy of the electron | $(W_e)_0$ | $= 5.10999 \cdot 10^5\ \text{eV}$ |
| Ratio of proton mass/electron mass | $m_p/m_0$ | $= 1836.152$ |
| Elementary electric charge | $e$ | $= 1.602177 \cdot 10^{-19}\ \text{A s}$ |
| Specific charge of the electron | $e/m_0$ | $= 1.760366 \cdot 10^{11}\ \text{A s/kg}$ |
| Faraday constant | $F$ | $= 96\,485.332_9\ \text{A s/mol}$ |
| Boltzmann's constant | $k$ | $= 1.380648 \cdot 10^{-23}\ \text{W s/K}$<br>$= 8.617325 \cdot 10^{-5}\ \text{eV/K}$ |
| Stefan-Boltzmann constant | $\sigma$ | $= 5.670367 \cdot 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4}$ |
| Planck's constant | $h$ | $= 6.626070 \cdot 10^{-34}\ \text{W s}^2$<br>$= 4.135667 \cdot 10^{-15}\ \text{eV s}$ |
| Bohr magneton | $\mu_B$ | $= he/4\pi m_0$<br>$= 9.274010 \cdot 10^{-24}\ \text{A m}^2\ (= \text{J/T})$ |
| Classical electron radius | $r_{el}$ | $= \mu_0 e^2/4\pi m_0 = 2.820\,419 \cdot 10^{-15}\ \text{m}$ |
| Rydberg frequency | $R_y$ | $= e^4 m_0/8\varepsilon_0^2 h^3 = 3.289842 \cdot 10^{15}\ \text{s}^{-1}$ |
| Rydberg constant | $R_y{}^*$ | $= e^4 m_0/8\varepsilon_0^2 h^3 c = 10\,973\,731.6\ \text{m}^{-1}$ |
| Compton wavelength | $\lambda_C$ | $= h/m_0 c = 2.426310 \cdot 10^{-12}\ \text{m}$ |
| Sommerfeld's fine structure constant | $\alpha$ | $= e^2/2\varepsilon_0 hc = 1/137.036$ |
| | $\alpha =$ | $\dfrac{\text{Electron's velocity } v_0 \text{ in the smallest H orbit}}{\text{Velocity of light } c}$ |

* $u$ is the atomic mass unit, $u = 1.660539 \cdot 10^{-27}$ kg.
See http://physics.nist.gov/cuu/Constants/index.html.



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# Solutions to the Exercises

## I. Electromagnetism

**1.1.** $I = 4.98\,\mathrm{A}$

**1.2.** 745 hours

**1.3.** The voltmeter is connected in series with a resistance of $99\,\Omega$.

**1.4.** $R_{\mathrm{shunt}} = 5.025\,\mathrm{m}\Omega$

**1.5.** $R = U/(I - U/R_{\mathrm{V}})$ (If OHM's law does not hold for the conductor, then $R$ depends on $I$.)

**1.6.** Two values of $R_2$ fulfill this condition: $R_2 = 1\,\Omega$ and $4\,\Omega$!

**1.7.** The battery has an internal resistance $R_{\mathrm{i}}$ of $1\,\Omega$, $U_{\mathrm{I}} = 1.25\,\mathrm{V}$

**1.8.** $N = 147$

**1.9.** $I = 0.167\,\mathrm{A}$

**1.10.** $U = 6.8\,\mathrm{kV}$

**1.11.** a) $I = 1.25\,\mathrm{A}$, b) $\dot{Q}_2/\dot{Q}_1 = 3$

**2.1.** a) $\varrho = -\sigma/h$; b) $E = -(1/\varepsilon_0)\varrho(h - x)$, up to the height $h$, $E$ is negative, and for $x \geq h$, $E = 0$; c) $U_{\mathrm{x}} = -(1/\varepsilon_0)\varrho(x^2/2 - hx)$; d) $U = (1/\varepsilon_0)\varrho h^2/2$

**2.2.** $C = 356\,\mu\mathrm{F}$

**2.3.** $C = 0.9995\,\mathrm{nF}$

**2.4.** $C_{\mathrm{V}} = 0.9\,\mathrm{nF}$

**2.5.** $N_{\mathrm{e}} = 5.54 \cdot 10^{10}\,\mathrm{cm}^{-2}$

**2.6.** $C/l = 93\,\mathrm{pF/m}$

**2.7.** a) 1.37 m from $Q_1$, on the side away from $Q_2$; b) 25 cm away from $Q_1$, between the two charges

**2.8.** $\int U\,dt = 10^{-5}$ V s

**2.9.** a) $t_{1/2} = RC \ln 2$; b) $C = 44.8$ pF

**2.10.** With $I = dQ/dt$ and $U = -Q/C$, we find $dQ/dt = -Q/RC + U_g/R$, $(U_g = 157\,\text{V})$, with the solution $Q = Ae^{-t/RC} + U_g C$, $(A = U_{max} - U_g,\ U_{max} = 220\,\text{V})$. It then follows that $\tau = 1.16$ s.

**2.11.** $W_{Batt} = U_0 \int I\,dt = U_0 Q = U_0^2 C$, $W_C = (1/2)U_0^2 C$ (Eq. (3.19)). Half of the energy supplied by the battery is therefore converted to heat in the resistor, independently of its resistance! This result however does not hold for $R = 0$, since then the condenser would be charged within an infinitely short time without allowing any energy to be converted to heat. See Chap. 10, "Electrical Oscillations".

**2.12.** 17 plates

**2.13.** $\varepsilon = 2.8$

**2.14.** 6.5 % of the volume between the plates is filled with metal. This reduces the spacing of the plates effectively by 6.5 %, i.e. the capacitance is increased by about 7 %. This leads to $\varepsilon = 1.07$.

**2.15.** Corresponding to Eqns. (2.11) and (2.25), the capacitance of the completely filled condenser would be $C = \varepsilon\varepsilon_0(a\,b/l)$, and thus the energy stored in it would be $W_e = (1/2)\varepsilon\varepsilon_0(U/l)^2 a\,bl$. In the present experiment, the energy thus increases when $h$ increases, by an amount $\Delta W_e = (1/2)(\varepsilon - 1)\varepsilon_0(U/l)^2 hbl = Fh$. The liquid is thus pulled up into the condenser with a constant force $F$. $h$ is then found by setting $F$ equal to the opposite force of the weight of the liquid which has been pulled up: $h = (1/2)(\varepsilon - 1)\varepsilon_0(U/l)^2/(\varrho g)$.

**2.16.** The electrical energy is $W_e = (1/2)\varepsilon\varepsilon_0(U/l)^2 A\,l$. This leads (compare Eqns. (3.17) and (3.11) in Sect. 3.7) to the force $F = (1/2)\varepsilon\varepsilon_0(U/l)^2 A$.

**3.1.** The weight of the brass plate is $F = 1.6$ N. Using Eq. (3.12), this leads to $l = 50\,\mu$m (this corresponds roughly to the thickness of a human hair; see Vol. 1, Sect. 1.2). The roughness of the polished stone causes an average spacing which is however certainly much smaller, so that a considerable fraction of the potential drop must occur within the stone!

**3.2.** $R = 80$ nm

**3.3.** The capacitance $C$ and thus also the energy $W_e$ vary by a factor $n$. If $n < 1$, then energy will be pumped back into the current source.

**5.1.** $\int U\,dt = 3.2 \cdot 10^{-4}$ V s, in good agreement with the observed 10 scale divisions

**5.2.** a) $u = 2.04$ m/s; b) $\int U\,dt = 3.9 \cdot 10^{-5}$ V s, in good agreement with the observed 1.2 scale divisions $(3.8 \cdot 10^{-5}$ V s)

**5.3.** a) $B_h = 2.5 \cdot 10^{-5}$ V s/m$^2$; b) $\varphi = 66°$ (In Göttingen, the horizontal component of the earth's magnetic field points to the north and its vertical component points downwards).

**5.4.** $\nu = 1.06$ Hz

**5.5.** The voltmeter indicates zero! (See also the first paragraph in small print in Sect. 5.5). The pilot could indeed detect an electric field, e.g. by carrying out the experiment of W. WIEN described at the end of Sect. 7.3, but he cannot detect the voltage of $U = 0.8$ V between the wing tips as given by Eq. (5.9), since it will be compensated by the charges which collect on the wing tips. (The "insulated wire" corresponds to the conductor *CA* in Fig. 7.2).

**5.6.** The angle between the field vector $\boldsymbol{B}$ and the surface vector $\boldsymbol{A}$ is $\alpha = 2\pi\nu t$. From this, it follows from Eq. (5.10) that $U = -\mathrm{d}/\mathrm{d}t(BAN\cos(2\pi\nu t)) = 2\pi\nu BAN\sin(2\pi\nu t)$, i.e. an AC voltage, as described in Chap. 8 (Fig. 9.3). Note that this result does not depend on the frame of reference.

**6.1.** a) $I = 93$ A ($\int H\,\mathrm{d}s = 1.85$ scale divisions); b) measured: $H = 1360$ A/m, calculated with Eq. (4.1): $H = 1610$ A/m; c) $\int H\,\mathrm{d}s = 500$ A

**6.2.** a) Since the magnetic potentiometer measured outside all the remaining coils along a closed path (Fig. 6.7), we find $U_{\text{mag}} = 0$; b) From a), it follows that $U_{\text{mag,i}} = -U_{\text{mag,a}}$.

**6.3.** If we combine the two methods of measuring the magnetic potentials $U_{\text{mag,a}}$ and $U_{\text{mag,i}}$, a closed path results. Around this path, the magnetic potential is always equal to zero unless the path encloses a current. This is the case here, since, as mentioned in Sect. 6.3, each tunnel bored through the material does not pass through individual molecules but rather between them; i. e. no molecular currents can be included. We thus have $U_{\text{mag,i}} = -U_{\text{mag,a}}$. The same result follows also from the MAXWELL equation (14.12). (See also Exercise 6.2.)

**8.1.** $B = 3.37 \cdot 10^{-4}$ T

**8.2.** $M_{\text{mech}} = 0.1$ N m

**8.3.** From the area which is marked with dashed lines, we obtain the flux $\Phi \approx 1.5 \cdot 10^{-6}$ V s. The magnetic flux measured in Video 5.1 is $8 \cdot 10^{-6}$ V s.

**8.4.** $W_{\text{magn}} = (1/2)\mu_0(N^2/l)I^2\pi r^2$. $W_{\text{magn}}$ increases when $l$ becomes smaller. In order to prevent this, we require a force of $F = 10^{-2}$ N.

**10.1.** $L = 0.55$ H

**10.2.** $L_N = N^2 L_1$

**10.3.** $R = 3.466\,\Omega$

**10.4.** $I_{\text{eff}} = 0.796$ A

**10.5.** $\nu = 50$ Hz

**10.6.** $L = 123$ mH

**10.7.** $\nu = 97.5\,\text{Hz}$

**10.8.** $L = 158\,\text{mH}$

**10.9.** $LC = 2.53 \cdot 10^{-8}\,\text{s}^2$

**10.10.** a) $Z_{\text{RL}} = 849\,\Omega$, $\varphi_1 = 87.4°$ (the current in the $RL$ branch "lags behind" the applied voltage); b) $I_{\text{RL},0} = 86\,\text{mA}$, $I_{\text{C},0} = 84.85\,\text{mA}$, $I_0 = 86\,\text{mA} \cdot \sin 2.6° = 3.9\,\text{mA}$, $\varphi_2 = 15.2°$; c) It follows from these values that $Z = 73\,\text{V}/3.9\,\text{mA} = 1872\,\Omega$, and from Eq. (10.33), we find $Z = 1840\,\Omega$, in agreement with the value found in b) (compare this value also with the one that can be read off at the maximum of the curve in Fig. 10.20).

**10.11.** $\Lambda = 0.139$, in agreement with the value in Fig. 10.20.

**10.12.** a) The active current is 3.77 mA and the reactive current is 1.0 mA; b) $\dot{W} = (1/2)\,U_0 I_0 \cos 15° = 0.138\,\text{W}$, $\dot{W}_{\text{RLC}} = (1/2)(I_{\text{RL},0})^2 R = 0.141\,\text{W}$, and thus within the error limits, the values are the same.

**10.13.** The current in the primary coil is $I_{\text{p}} = I_{\text{p},0} \sin \omega t$, with $I_{\text{p},0} = U_{\text{p},0} l_{\text{p}}/(\mu_0 \omega N_{\text{p}}^2 A_{\text{p}})$ (Eq. (10.4)). From this we find, with Eqns. (4.1) and (5.4), the flux density $B$ of the magnetic field from the primary coil, and with it, applying the law of induction (Sect. 5.6), the voltage across the secondary coil, $U_{\text{s}} = N_{\text{s}}(-\text{d}B/\text{d}t)A_{\text{p}}$. This leads finally to Eq. (10.39).

**10.14.** a) Owing to the phase difference of 90° between the current and the voltage, in the primary circuit we expect $\overline{\dot{W}} = 0$. b) With a finite value of $R$, a current will flow in the secondary circuit. The thermal energy from JOULE heating in the secondary circuit then must come from the primary circuit. The mechanism for this is that the current in the secondary circuit induces an additional current in the primary circuit, so that the phase difference there is no longer 90°, and a finite active current results.

**11.1.** The voltage across the condenser is $U_{\text{C}} = Q/C$. The voltage across the coil is $U_{\text{L}} = L\,\text{d}I/\text{d}t = L\,\text{d}^2Q/\text{d}t^2$. These voltages are equal and opposite at all times: $Q_{\text{C}}/C + L\,\text{d}^2Q/\text{d}t^2 = 0$. This differential equation can be solved by the trial function $Q_{\text{C}} = Q_{\text{C},0} \cos \omega t$. This then yields $\omega = \sqrt{(1/LC)}$. Thus, we find that $U_{\text{C}} = (Q_0/C) \cos \omega t$ and $I = \text{d}Q/\text{d}t = -U_{\text{C},0}\omega C \sin \omega t$.

**11.2.** $U_{\text{L}} = L\,\text{d}I/\text{d}t$ (Eq. (10.16)). Therefore, $U_{\text{L},1} = L_1\,\text{d}I/\text{d}t$ and $U_{\text{L}} = (U_{\text{L},1} + U_{\text{L},2}) = (L_1 + L_2)\,\text{d}I/\text{d}t$. From this, it follows that $|U_{\text{L},1}/U_{\text{L}}| = L_1/(L_1 + L_2)$. The coils thus act as a voltage divider. An application is shown by the experiment described in Fig. 11.7. With the correct choice of the inductance of the wire loop relative to the overall inductance of the oscillator circuit, a voltage will be induced which causes the lamp to light up without burning out.

**11.3.** With the values of the capacitance $C = 3.2\,\text{nF}$ and the inductance $L = 0.048\,\text{mH}$, we find the frequency to be $\nu_0 = 400\,\text{kHz}$.

**12.1.** The rectifier allows only half of the sinusoidal current to pass. The throttle coils in Fig. 12.21 pass only the low-frequency FOURIER components of this current (for FOURIER analysis, see Vol. 1, Sect. 11.3). These components include a direct-current contribution which is proportional to the amplitude $I_0$, and it will be indicated by the galvanometer.

**12.2.** a) Using the trial function $E_x = E_{x,0} \sin\omega(-t + z/c)$, superposition with the incident wave gives $E_{tot} = 2E_{x,0} \sin\omega(z/c)\cos\omega t$, that is a standing wave with a node at $z = 0$, as seen in Fig. 12.28. b) The magnetic component of the incident wave is described by the term $B_y = -B_{y,0} \sin\omega(t + z/c)$, and thus its amplitude points in the negative $y$ direction (right-handed coordinate system, wave propagation in the negative $z$ direction). The reflected magnetic component is given by $B_y = B_{y,0} \sin\omega(-t + z/c)$ (in phase with the electrical component; the sign follows again from the right-handed coordinate system). Superposition of the two magnetic components gives $B_{tot} = 2B_{y,0} \cos\omega(z/c)\sin\omega t$, i.e. again a standing wave, but now it has a maximum (wave crest) at the reflecting surface ($z = 0$), with the amplitude $2B_{y,0}$.

**12.3.** a) $E = 220\,\text{V/m}$, $B = 1.8 \cdot 10^{-4}\,\text{V s/m}^2$, $S = 31.5\,\text{kW/m}^2$; the POYNTING vector points towards the interior of the spiral, as if the thermal power were deposited in the spiral from outside. b) $b = 1.35\,\text{kW/m}^2$, $E = 713\,\text{V/m}$, $B = 2.38 \cdot 10^{-6}\,\text{V s/m}^2$ (also effective values).

**12.4.** The electromagnetic wave which is radiated from the outside and is used to excite the small dipoles of the water molecules (Fig. 12.6) has a wavelength of $\lambda = 3\,\text{m}$. With its velocity of propagation, $c = 3 \cdot 10^8\,\text{m/s}$, we find its frequency to be $\nu = 10^8\,\text{Hz}$. The dielectric constant of water at this frequency is $\varepsilon = 81$ (Table 13.2 and Fig. 13.11); thus its index of refraction is $n = 9.0$. It follows from this that $R = 0.64$ (at normal incidence). 36 % of the incident radiation thus penetrates into the water; this is sufficient to excite the small dipoles.

**13.1.** The energy changes from $(1/2)QU_0$ to $(1/2)QU_m$, where $U_m = U_0/\varepsilon$. The excess energy is taken up by the person who places the plate into the condenser.

**13.2.** The voltage is $U' = E(l - d) + (E/\varepsilon)d$ with the electric field $E = U/l$. Solving for $\varepsilon$, we find $\varepsilon = d/(d + l(U'/U - 1))$. In this example, the result is $\varepsilon = 5$.

**13.3.** The voltage between the points 1 and 2 is zero when the voltage drop over the capacitance $C_1$ is equal to that over the resistance $R_1$. Since the charges $Q$ on the two condensers are then equal, we have $Q/C_1 = I/R_1$ and $Q/C_x = I/R_2$. From this, it follows that $C_x = C_1R_1/R_2$.

**13.4.** The electric field $E$ experiences no depolarization (Eq. (13.11)). It follows that $D = \varepsilon D_0$. The displacement density in the rod is thus larger than in the space outside it. The concept of "depolarization" refers to the field $E$ (see also "demagnetization", Sect. 14.6)

**13.5.** $P = 3((\varepsilon - 1)/(\varepsilon + 2))\varepsilon_0 E_0$ (compare this result with the magnetization of a sphere, Eq. (14.18))

**13.6.** $p_p = 3.04 \cdot 10^{-30}\,\text{A s m}$, and thus only half a large as measured in the gas phase. The reason for this discrepancy is that in deriving the CLAUSIUS-MOSSOTTI equation within the "cavity", the interactions between neighboring molecules were not taken into account; that is, for water, the hydrogen bonds between the water molecules (see H. Fröhlich, "Theory of dielectrics", Oxford University Press (1949), p. 137).

**13.7.** In paraelectric materials, $\varepsilon$ depends on the temperature, both due to the equation of state of ideal gases and to the LANGEVIN-DEBYE theory. For the electric susceptibility, one finds $\chi_e = \varepsilon - 1 = (1/3\varepsilon_0)p_p^2/(kT)^2 \cdot p$ (to distinguish it from the pressure $p$, the dipole moment is denoted here by $p_p$). This explains the difference between the two values of $\chi_e$ to better than 1 %.

**13.8.** a) From Fig. 13.11, we read off the dielectric constant at this frequency, thus obtaining the index of refraction $n = 9.0$. It then follows that $\lambda_W = 1.36\,\text{cm}$. b) $\overline{\dot{W}} = 697\,\text{W}$. c) $S = 11.6\,\text{kW/m}^2 = 8.6\,E_e$ (this is about the same as the value of the solar constant on the planet Mercury!) (For reflection losses, see Exercise 12.4).

**14.1.** $\chi_m = -1.42 \cdot 10^{-5}$. The permeability $\mu$ is thus only slightly greater than 1.0, so that the demagnetization effect can be neglected.

**14.2.** a) Analogously to Eq. (13.15), we have here $H = H_0 + H_m$. Comparing with Eq. (14.18), we then find $H_m = -M/3 = -((\mu - 1)/(\mu + 2))H_0$. b) From Eq. (14.7), it follows that $B_m = (2/3)\mu_0 M$.

**14.3.** $H_i = 3\mu_a H_a/(\mu_i + 2\mu_a)$; from this, we find $\mu_i = 1$ and $H_a = H_0/\mu_a$: $H_i = 3H_0/(1 + 2\mu_a)$, and for $\mu_a \gg 1$: $H_i \approx (1.5/\mu_a)H_0$.

**14.4.** $F_0 = 600\,\text{N}$, $F_d = 17.6\,\text{N}$. Due to the hysteresis of the iron (Fig. 14.7), we must consider these values to be only rough estimates.

**14.5.** a) and b): Following the rapid change of the current during the first second, both measurements show an exponential variation with a relaxation time of $\tau_r = 10\,\text{s}$. c) Here, also, following an initial rapid change, an exponential variation begins, but with a relaxation time of 110 s. Only near the saturation value does the rate of change become more rapid and again approaches a relaxation time of 10 s. For a qualitative explanation: During the initial rapid variation, of which we can see an inkling even in experiment c), the magnetic domains cannot follow. The coil thus has a small inductance during this variation. Thereafter, the reversals of the domains delay the rate of change of the current in the coil in the same manner as the changes of the current in the coil delay its rate of change due to induction. The coil now has a larger inductance because of the iron core. From experiments a) and b), we obtain $L = \tau_r R = 1300\,\text{H}$. In experiment c), the delay of the current changes in the coil is even greater, since in the first 60 seconds, the magnetic domains (remanent magnetization) rotate through 180°, causing the induced voltage which delays the current change to be increased. This results in a still larger inductance: $L = 14\,000\,\text{H}$. Only near the end of the increase in the current, after the remanent magnetization has been completely overcome, does the rate of change again correspond to the same time constant as in experiments a) and b); $L$ is thus again smaller.

**14.6.** In Exercise 14.2, it was shown for a magnetized ball that the field $H_m$ in its interior is given by $H_m = -NM$ ($N$ is the demagnetization factor). Starting from Eq. (14.14), it can be shown that the same result holds for every ellipsoid of rotation which is magnetized parallel to its axis of rotational symmetry. For the disk with $l/d = 0.1$, we find $N = 0.863$ (Table 13.4). For a long rod ($l/d = \infty$), $N = 0$, and the field $H_m$ is thus zero. From this, with Eq. (14.7) we find for the field $B_m$ in the disk the value $B_m = \mu_0(M - 0.863M) = 0.137\mu_0 M$, and for the rod, $B_m = \mu_0 M$. These flux densities continue into the space outside the material, since div $B = 0$ (Maxwell's equation (14.13)).

# II. Optics

**16.1.** From the expressions for the law of refraction on both sides of the prism, $\sin\alpha_1 = n \cdot \sin\beta_1$ and $\sin\alpha_2 = n \cdot \sin\beta_2$, we obtain from the sum and the difference: $\sin\alpha_1 \pm \sin\alpha_2 = n \cdot (\sin\beta_1 \pm \sin\beta_2)$. Rearranging, this gives $\sin((\alpha_1 + \alpha_2)/2) \cdot \cos((\alpha_1 - \alpha_2)/2) = n \cdot \sin((\beta_1 +$

$\beta_2)/2) \cdot \cos((\beta_1 - \beta_2)/2)$ and $\cos((\alpha_1 + \alpha_2)/2) \cdot \sin((\alpha_1 - \alpha_2)/2) = n \cdot \cos((\beta_1 + \beta_2)/2) \cdot \sin((\beta_1 - \beta_2)/2)$; dividing these two equations yields: $\tan((\alpha_1 + \alpha_2)/2)/\tan((\alpha_1 - \alpha_2)/2) = n \cdot \tan((\beta_1 + \beta_2)/2)/\tan((\beta_1 - \beta_2)/2)$. With $\beta_1 + \beta_2 = \gamma$ and $\alpha_1 + \alpha_2 = \delta + \gamma$, Eq. (16.8) follows. b) Inserting the values of $\beta_1 = 19.47°$ and $\gamma/2 = 30°$ into Eq. (16.8), we obtain a quadratic equation for $\tan(\delta/2)$, and thus initially two solutions for $\delta$. To find the correct solution, we consider a sketch corresponding to Fig. 16.13. The result is finally $\delta = 47°$.

**16.2.** The point at which the incident beam in Fig. 16.24 is reflected by the mirror is called $A$, and the angle of reflection is $\alpha$. Then we have $AF = FZ = R/(2\cos\alpha)$. For rays near the axis of the mirror (paraxial rays), $\alpha$ is small and so $\cos\alpha \approx 1$. It follows that $f = R - FZ = R/2$.

**16.3.** A scale drawing corresponding to Fig. 18.1 for a concave mirror with an object $y$ at a distance $a$ and the associated image $y'$ at a distance $b$ allows us to compare similar triangles and obtain $y/y' = f/(b-f) = a/b$, from which it follows that $1/f = 1/a + 1/b$.

**16.4.** A parallel shift of light beams, even when they have differing wavelengths, leaves the image in the focal plane of the eye unchanged (see e. g. Fig. 18.24, with parallel light beams between the lens and the eye).

**17.1.** a) $d \approx 100\,\text{m}$; b) $B \approx 11\,\text{cm}$

**17.2.** $2\varphi_{\text{min}} = 1.04 \cdot 10^{-7}$, $\alpha = 2.28 \cdot 10^{-7}$

**18.1.** Considering two pairs of similar triangles in Fig. 18.1, we find $y/y' = a/b = f'/(b-f')$. From this, it follows that $1/a + 1/b = 1/f'$.

**20.1.** a) $m_{\text{max}} = 133$, $x = 21\,\text{cm}$, b) $x_{10} = 1.19\,\text{m}$

**20.2.** The observation is carried out at an angle of inclination $\beta_{\text{m}}$, which can be treated as constant for small changes in the order number. Then from Eq. (20.4), we find $d = (1/3)\lambda/2 = 0.1\,\mu\text{m}$.

**21.1.** a) No change, since over the whole slit, the same path differences are found; b) likewise no change, since the path difference due to the glass, $\Delta = d(n-1)$, is an integral multiple of the wavelength ($\Delta = 145.8\,\lambda$, thus nearly the "order-one position"); c) $\alpha = 8.2°$. since in general the quantities $d$, $n$ and $\lambda$ are not known with sufficient accuracy, this calculation shows how one can obtain the two positions in practice.

**21.2.** Due to the phase difference of $\lambda/2$ between the two halves of the slit, in making the construction, we must rotate the arrows 7 to 12 by 180°; we then obtain the diffraction pattern of the order-two position, that is with extrema at $\sin\alpha = N\lambda/B$, where $N = 1, 3, \ldots$ for the maxima and $N = 0, 2, 4, \ldots$ for the minima. (A more precise calculation shows that the maxima at $N = 1$ (Fig. 21.12) are slightly shifted.)

**21.3.** The angular range of the principal maximum determined by the width $B$ of the slit ranges over $\sin\alpha = \pm\,\lambda/B$. The maxima which occur through interference with immediately neighboring slits are found at $\sin\alpha_{\text{m}} = \pm\,m\lambda/D$ ($m = 0, 1, 2, \ldots$). Then for a), we observe only three maxima, but for b) there are 39 maxima.

**21.4.** a) In this case, the whole surface of the grating radiates with the same phase; diffraction occurs only at its edges. b) The diffracted waves which emerge from two neighboring pairs of beams and slits produce a diffraction pattern as shown in Fig. 21.12, lower left. The superposition of the waves emerging from all the beams and slits leads to a sharpening of the maxima (Fig. 22.6), but leaves their positions and relative heights unchanged.

**21.5.** Since in the order-two position ($\beta$), the radiant power is zero for the non-deflected beam ($\alpha = 0$), the power of the light which passes through the gaps and the "grating beams" must be the same. Thus, if we can neglect absorption in the grating beams, they must have the same width, i.e. $L/D = 1/2$.

**21.6.** $c = m\lambda\nu/(m \sin\alpha) = 875\,\mathrm{m/s}$

**22.1.** a) $\lambda = 600\,\mathrm{nm}$, b) $n = 1.33$

**26.1.** a) The equation of motion for free damped oscillations is given by $F_\mathrm{R} + F_\mathrm{D} = m\,\mathrm{d}^2 l/\mathrm{d}t^2$, with $F_\mathrm{R} = -\alpha\,\mathrm{d}l/\mathrm{d}t$ (frictional force) and $F_\mathrm{D} = -Dl$ (elastic force). That the expression $l = l_0 e^{-\delta t}$, with $\delta = (1/2)(\alpha/m)$, is a solution of this equation of motion can be demonstrated by substituting it into that equation. For the relation between $\alpha$ and $\Lambda$, we obtain with this solution $\alpha = 2\delta m = 2\lambda\nu_0 m$. b) The equation of motion is now given by $F_\mathrm{p} + F_\mathrm{R} + F_\mathrm{D} = m\,\mathrm{d}^2 l/\mathrm{d}t^2$, or, after inserting the forces and dividing by $m$: $\mathrm{d}^2 l/\mathrm{d}t^2 + 2\Lambda\nu_0\,\mathrm{d}l/\mathrm{d}t + 4\pi^2\nu_0^2 l = (F_0/m)\cos(2\pi\nu t)$. We now insert the complex trial solution $l = l_0 e^{i(2\pi\nu t)}$ together with its first and second time derivatives into the equation of motion, obtaining $4\pi^2[(\nu_0^2 - \nu^2) + i(\Lambda/\pi)\nu_0\nu] = (F_0/l_0 m)e^{i\varphi}$. From this, applying the mathematical relations $a + ib = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ (Eq. (25.27)) and $\sin^2\varphi + \cos^2\varphi = 1$, we obtain equations (26.1) and (26.3). (For literature on "forced oscillations", see e.g. The Feynman Lectures, Vol. 1, Chap. 21-5 (http://www.feynmanletures.caltech.edu/), or http://hyperphysics.phy-astr.gsu.edu/hbase/oscdr.html).

**27.1.** $(1/2)l_{0,\max}^2/l_{0,\max}^2 = (\Lambda/\pi)^2\nu_0^4/[(2\nu_0)^2(\nu_0 - \nu)^2 + (\Lambda/\pi)^2\nu_0^4]$,
$(1/2)(2\nu_0)^2(\nu_0 - \nu)^2 = (1/2)(\Lambda/\pi)^2\nu_0^4$, $2(\nu_0 - \nu) = H = (\Lambda/\pi)\nu_0$. This result holds for every harmonic oscillator, in particular for an electrical resonator.

**27.2.** Let the deflection be $x = l_0\ \sin\omega t$; taking the time derivative gives: $\dot{x} = \omega l_0 \cos\omega t$. The kinetic energy is thus $W_\mathrm{kin} = (1/2)m\dot{x}^2 = (1/2)m(\omega l_0 \cos\omega t)^2$. With the average value of $\cos^2\omega t = 1/2$ over a whole oscillation, it follows that $\overline{W}_\mathrm{kin} = (1/4)m(\omega l_0)^2$.

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