

A Comprehensive Comparison between a Hill Climbing and a Genetic Algorithm for Solving the Graph Coloring Problem

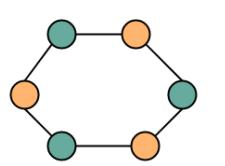
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Abstract

In this work, we compare the performance of two algorithms, a genetic algorithm and a hill climbing algorithm, for solving the graph coloring problem (vertex coloring). The algorithms are tested against the standard DIMACS benchmark graphs [1],[2],[3]. The instances used include a mix of easy, medium and difficult problems (myciel3.col,myciel4.col,mug88_1.col,mug88_25.col, mug100_1.col,mug100_25.col,myciel5.col,queen5_5.col,jean.col,queen6_6.col,huck.col,myciel6.col ,david.col,DSJC125.1.col,miles250.col,queen7_7.col,anna.col,games120.col,queen8_8.col,queen9_9.col,miles500.col,myciel7.col,queen8_12.col,queen10_10.col,zeroin.i.3.col,zeroin.i.2.col,DSJC250_1.col,mulsol.i.2.col,DSJC125.5.col,queen11_11.col,mulsol.i.3.col,mulsol.i.4.col,mulsol.i.1.col,mulsol.i.5.col,zeroin.i.1.col,miles750.col,queen12_12.col,miles1000.col,DSJC125.9.col,queen13_13.col ,will199GPIA.col,queen14_14.col,fpsol2.i.3.col,fpsol2.i.2.col,miles1500.col,queen15_15.col,fpsol2.i_1.col,queen16_16.col,DSJC500.1.col,inithx.i.3.col,inithx.i.2.col,school1_nsh.col,DSJC250.5.col,le_450_15c.col,le450_25c.col,inithx.i.1.col,school1.col,flat300_28_0.col,DSJC1000.1.col,DSJC500.5.col ,DSJC1000.5.col and DSJC1000.9.col). Our findings reveal that, for small size graphs, the hill climbing algorithm is able to achieve results of similar quality to the genetic algorithm, but in a shorter computational time. In contrast, the genetic algorithm demonstrates superior performance on large size graphs, consistently yielding better solutions than the hill climbing algorithm.

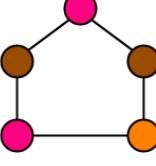
1 Introduction

The graph-coloring problem is an NP complete problem - one of the hardest problems in the class NP (non-deterministic polynomial problems). The problem can be formalized as follows. An assignment of colors to a graph $G = (V, E)$ is a mapping $C:V \rightarrow S$, where S is a finite set of colors, which are normally represented as integers, such that if $(v, w) \in E$ then $C(v) \neq C(w)$; in other words, the same color is not assigned to adjacent vertices. Given a graph, the problem is to find the minimum number of colors needed for coloring the graph according to the above requirement. This minimum number is also referred to as the chromatic number of the given graph $\chi(G)$ [4].



Chromatic Number = 2

(a) A graph with $\chi=2$



Chromatic Number = 3

(b) A graph with $\chi=3$

This study aims to evaluate and compare the performance of two algorithms, a Genetic Algorithm (GA) and a Hill Climbing with First Improvement (HCFI) algorithm, for solving the graph coloring problem. The GA, a heuristic optimization method inspired by natural evolution, and the HCFI, a local search approach that iteratively improves a solution by making small, incremental changes and selecting the first improvement encountered, will be analyzed based on a range of metrics, including the time required to discover the first valid coloring, the time required to find the optimal coloring, the first valid coloring discovered, the optimal coloring discovered, the total execution time, and the number of conflicts that must be resolved to locate the next valid coloring. In addition to presenting these quantitative results, we will also provide informative plots to illustrate the progression of the algorithms and facilitate the analysis of the data.

2 Proposed approach

2.1 GA

In this work, we utilize an adjacency matrix to represent the graphs under consideration. It is well-known that the chromatic number of a graph is always less than or equal to the maximum degree of all vertices in the graph plus one. With this in mind, we initialize the population of the GA with randomly generated chromosomes featuring colors in the range of $[0, \text{max_degree}]$. The variable **nr_of_colors** represents the coloring that the algorithm is attempting to solve at any given time, and is initially set to **max_degree**. However, upon the discovery of a valid coloring, **nr_of_colors** is updated to the found coloring minus one. The process is repeated for a predetermined number of generations, with the population size remaining constant across these iterations. In each generation, the half of the population with the poorest fitnesses are eliminated and replaced with random chromosomes featuring colors in the range of $[0, \text{nr_of_colors} - 1]$. There are two distinct selection methods (**selection1** and **selection2**), two different mutation methods (**mutation1** and **mutation2**), and one crossover method available for use in the GA. The choice of selection and mutation methods is dependent on the state of the population and the proximity of the current solution to a valid coloring. When the best fitness of the population exceeds 4, **selection1** and **mutation1** are employed. Otherwise, when the best fitness of the population is less than or equal to 4, **selection2** and **mutation2** are utilized. Empirical observations have shown that, when the number of conflicts is low (approaching a valid coloring), the combination of **selection2** (which copies the best chromosome as the new child) and **mutation2** (which randomly selects a color for the violating vertices of the child) is more likely to yield a valid coloring in a shorter amount of time than the other selection and mutation methods. It is worth noting that crossover is not applied when the best fitness of the population is less than or equal to 4. The resulting child chromosome

is then incorporated into the population. If the GA discovers a chromosome with a valid coloring, the chromosome is saved, the value of **nr_of.colors** is updated to the found coloring minus one, and the population is regenerated using chromosomes in the new $[0, nr_of_colors - 1]$ range.

2.1.1 Fitness

The fitness score of a chromosome is determined by the number of bad edges (conflicts) it has. A bad edge is an edge that connects two vertices that are colored the same. A chromosome with a lower fitness score is considered better than one with a higher score.

Algorithm 1 Fitness

```

1: procedure FITNESS(chromosome)
2:   conflicts  $\leftarrow$  0
3:   num_nodes  $\leftarrow$  len(chromosome)
4:   for i in range(num_nodes) do
5:     for j in range(num_nodes) do
6:       if adjacency_matrix[i][j] == 1 and chromosome[i] == chromosome[j] then
7:         conflicts  $\leftarrow$  conflicts + 1
8:       end if
9:     end for
10:   end for
11:   return conflicts
12: end procedure
```

2.1.2 Selection1 - Tournament 2

Creates two tournaments of size 2, where the candidate chromosomes are chosen at random. The winners of the tournaments are picked as the parents.

Algorithm 2 Selection1

```

1: procedure SELECTION1(population)
2:   temp_parents  $\leftarrow$  two randomly selected chromosomes from population
3:   parent1  $\leftarrow$  the fitter of temp_parents
4:   temp_parents  $\leftarrow$  two randomly selected chromosomes from population
5:   parent2  $\leftarrow$  the fitter of temp_parents
6:   return parent1, parent2
7: end procedure
```

2.1.3 Selection2

Selects the chromosome with the best fitness score in the population as the parent.

Algorithm 3 Selection2

```
1: procedure SELECTION2(population)
2:   parent  $\leftarrow$  the top performing chromosome in population
3:   return parent
4: end procedure
```

2.1.4 Crossover - Single Point

Combines two parent chromosomes to produce a child chromosome. It does this by randomly selecting a point in the two chromosomes. The child will receive the genetic information from **parent1** before the **crosspoint**, and the genetic information from **parent2** after the **crosspoint**.

Algorithm 4 Crossover

```
1: procedure CROSSOVER(parent1, parent2)
2:   crosspoint  $\leftarrow$  a random integer between 1 and the length of parent1 minus 1
3:   child  $\leftarrow$  the concatenation of the first crosspoint+1 elements of parent1 and the elements of
   parent2 after crosspoint+1
4:   return child
5: end procedure
```

2.1.5 Mutation1

Mutates a chromosome by changing the color of any vertex that has the same color as one of its adjacent vertices to a random color that is not already used by an adjacent vertex, in order to avoid creating additional conflicts.

Algorithm 5 mutation1

```
1: procedure MUTATION1(chromosome, adjacency_matrix, mutation_chance, all_colors)
2:   if random.random() < mutation_chance then
3:     for i, color in enumerate(chromosome) do
4:       adjacent_colors  $\leftarrow$  [chromosome[j] for j, is_adjacent in enumerate(adjacency_matrix[i])
      if is_adjacent and i  $\neq$  j]
       if color in adjacent_colors then
9:         valid_colors  $\leftarrow$  [c for c in all_colors if c not in adjacent_colors]
10:        if len(valid_colors) > 0 then
11:          new_color  $\leftarrow$  random.choice(valid_colors)
12:          chromosome[i]  $\leftarrow$  new_color
13:        end if
14:      end if
15:    end for
16:  end if
17:  return chromosome
18: end procedure
```

2.1.6 Mutation2

Mutates a chromosome by changing the color of any vertex that has the same color as one of its adjacent vertices to a random color.

Algorithm 6 Mutation2

```
1: function MUTATION2(chromosome, adjacency_matrix, mutation_chance, all_colors)
2:   if random.random() < mutation_chance then
3:     for i, color in enumerate(chromosome) do
4:       adjacent_colors ← [chromosome[j] for j, is_adjacent in enumerate(adjacency_matrix[i])
      if is_adjacent and i ≠ j]
5:       if color in adjacent_colors then
6:         new_color ← random.choice(all_colors)
7:         chromosome[i] ← copy.deepcopy(new_color)
8:       end if
9:     end for
10:   end if
11:   return chromosome
12: end function
```

2.2 HCFI

The process is repeated for a predetermined number of iterations, during which the algorithm endeavors to locate an improvement solution (one with fewer conflicts) within its neighborhood space. If a locally optimal solution is reached and further improvement is not possible, the solution is regenerated with colors in the range of $[0, nr_of_colors - 1]$. If a solution with a valid coloring is found, it is saved, the value of **nr_of_colors** is updated to the found coloring minus one, and the solution is regenerated using colors in the new $[0, nr_of_colors - 1]$ range.

2.2.1 Fitness

The fitness score of a solution is determined by the number of bad edges (conflicts) it has. A bad edge is an edge that connects two vertices that are colored the same. A solution with a lower fitness score is considered better than one with a higher score.

Algorithm 7 Fitness

```
1: procedure FITNESS(chromosome)
2:   conflicts  $\leftarrow$  0
3:   num_nodes  $\leftarrow$  len(chromosome)
4:   for i in range(num_nodes) do
5:     for j in range(num_nodes) do
6:       if adjacency_matrix[i][j] == 1 and chromosome[i] == chromosome[j] then
7:         conflicts  $\leftarrow$  conflicts + 1
8:       end if
9:     end for
10:   end for
11:   return conflicts
12: end procedure
```

2.2.2 Improved_neighbor

Finds the first neighbor coloring that improves the fitness of the original coloring. If no such neighbor coloring is found, the function returns an empty list.

```
procedure IMPROVED_NEIGHBOR(coloring, nr_of_colors)
  neighbors  $\leftarrow$  []
  for i  $\leftarrow$  0 to len(coloring) do
    node_conflicts  $\leftarrow$  fitness_node(coloring, i)
    if node_conflicts > 0 then
      for color  $\leftarrow$  0 to nr_of_colors do
        if coloring[i]  $\neq$  color then
          new_coloring  $\leftarrow$  coloring.copy()
          new_coloring[i]  $\leftarrow$  color
          new_conf_node  $\leftarrow$  fitness_node(new_coloring, i)
          if new_conf_node < node_conflicts then
            return new_coloring
          end if
        end if
      end for
    end if
  end for
  return neighbors
end procedure
```

3 Setup

3.1 Legend

Name	Notation
Algorithm	A
Number of vertexes	N
Number of edges	E
Maximum degree of the Graph	Δ
Generations	G
Iterations	I
Cromatic number	$\chi(G)$
Best coloring known	BCK
Best coloring found	BC
Time taken to find the best coloring	T:BC
First coloring found	FC
Time taken to find the first coloring	T:FC
Number of conflicts left for the next coloring	CL
System used for the problem	SYS
Total execution time of the algorithm	T:T
Genetic Algorithm	GA
Hill Climbing with First Improvement	HCFI
Not known NULL Undefined	*

Table 1: Legend

3.2 Parameters and Restrictions

Algorithm	Parameters	Restrictions
GA	CC=100%,MC=70%, Pop size=200, Generations=2000	MAX Generations=2000, MAX Pop=200
HCFI	[100000, 1000000] Iterations	*

Table 2: Parameters and Restrictions

3.3 Computational Systems

System 1:

CPU: AMD Ryzen 7 3750H
 RAM: 16 GB DDR4
 HDD|SSD: SSD
 OS: Windows 10 Pro

System 2:

CPU: Intel i7 6700HQ
 RAM: 16 GB DDR4
 HDD|SSD: SSD
 OS: Windows 10 Pro

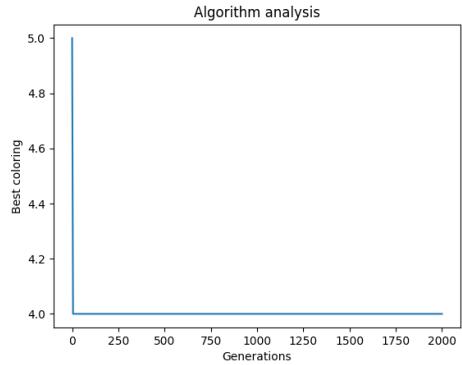
The algorithms described are implemented in Python 3.10. The plots are created using the **networkx** and **matplotlib** libraries.

Results

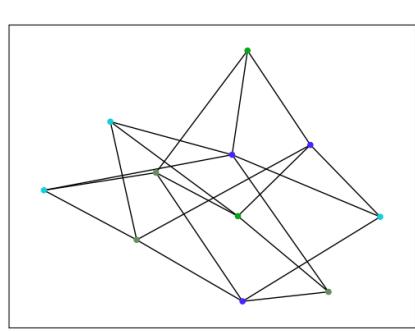
myciel3	myciel4	mug88_1
mug88_25	mug100_1	mug100_25
myciel5	queen5_5	jean
queen6_6	huck	myciel6
david	DSJC125.1_001	miles250
queen7_7	anna	games120
queen8_8	queen9_9	miles500
myciel7	queen8_12	queen10_10
zeroin.i.3	zeroin.i.2	DSJC250.1
mulsol.i.2	DSJC125.5	queen11_11
mulsol.i.3	mulsol.i.4	mulsol.i.1
mulsol.i.5	zeroin.i.1	miles750
queen12_12	miles1000	queen13_13
DSJC125.9	will199GPIA	queen14_14
fpsol2.i.3	fpsol2.i.2	miles1500
queen15_15	fpsol2.i.1	queen16_16
DSJC500.1	inithx.i.3	inithx.i.2
school1_nsh	DSJC250.5	le450_15c
le450_25c	inithx.i.1	school1
flat300_28_0	DSJC1000.1	DSJC500.5
DSJC1000.5	DSJC1000.9	

1. myciel3

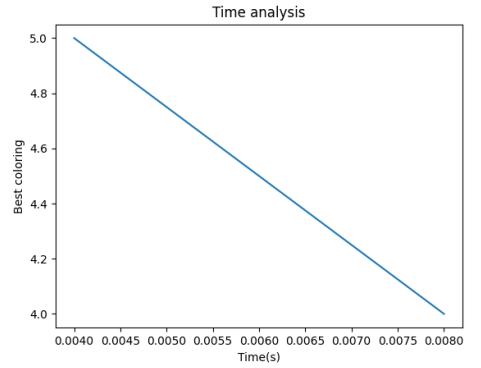
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel3	GA	11	20	5	2000	4	4	4	0.0370	6	0.0070	2	1	153
	HCFI				100000		4	4	0.0040	5	0	24	1	42



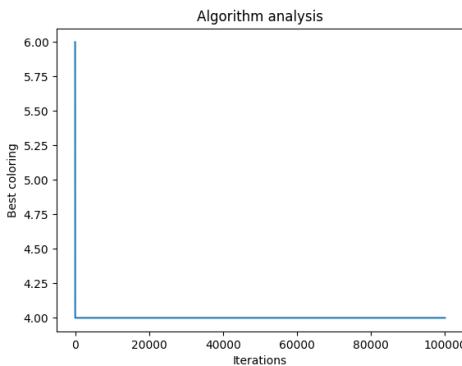
(a) GA-Algorithm analysis



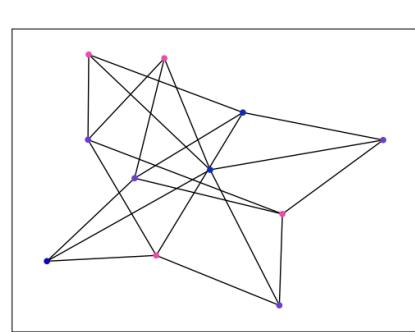
(b) GA-Graph coloring



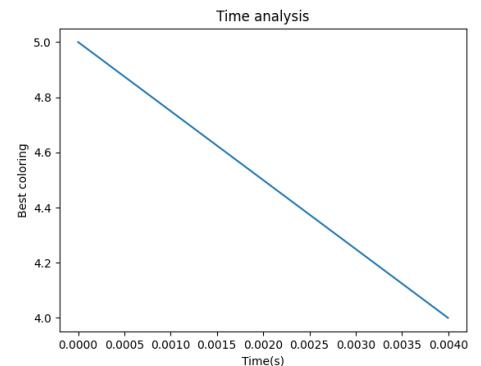
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



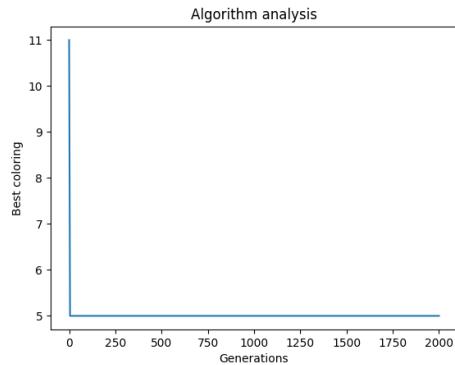
(c) HCFI-Time analysis

Results

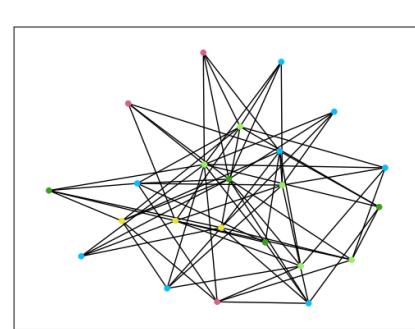
Compact Results Table

2. myciel4

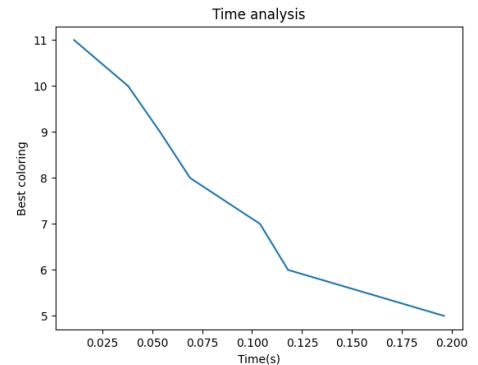
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel4	GA	23	71	11	2000	5	5	5	0.1960	11	0.0110	2	1	463
	HCFI				100000			5	0.02	12	0	24	1	42



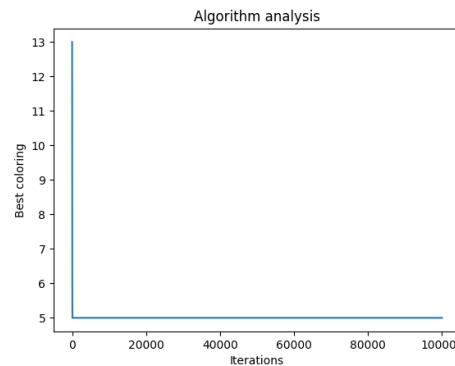
(a) GA-Algorithm analysis



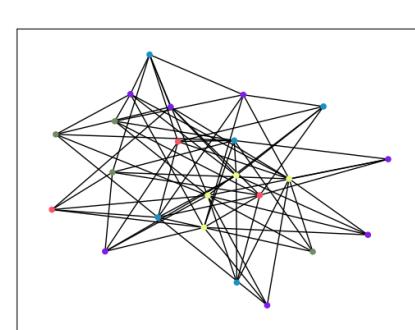
(b) GA-Graph coloring



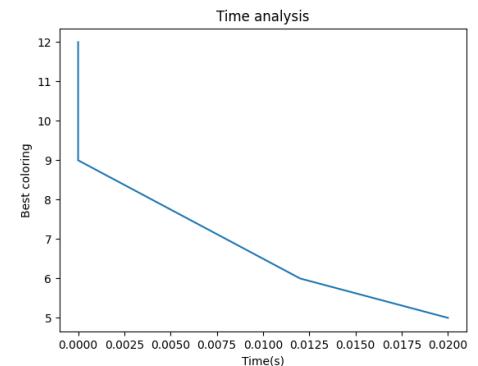
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



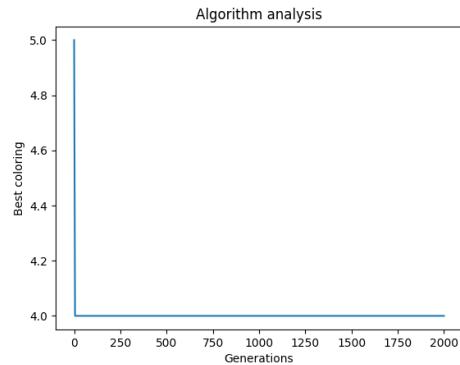
(c) HCFI-Time analysis

Results

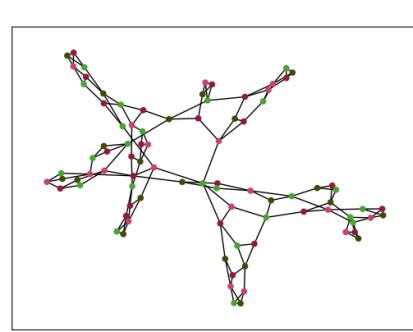
Compact Results Table

3. mug88_1

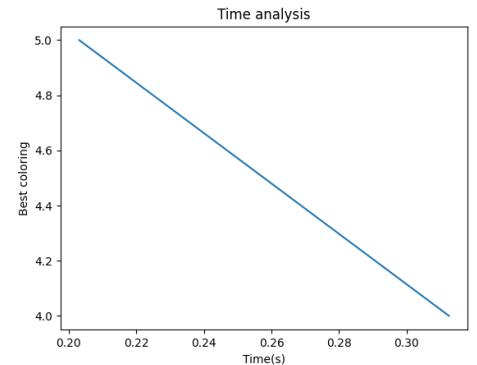
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mug88_1	GA	88	146	4	2000	4	4	4	0.3125	5	0.2031	6	1	3859
	HCFI				100000				0.0705	5	0.0403	18	1	132



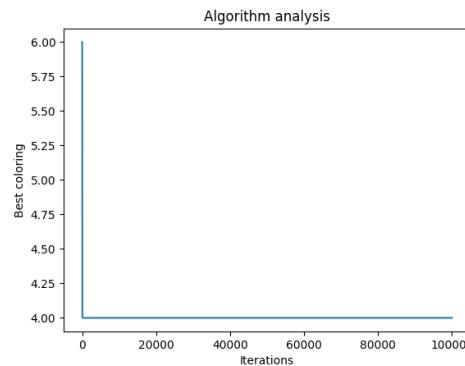
(a) GA-Algorithm analysis



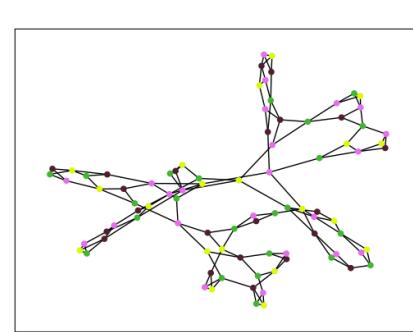
(b) GA-Graph coloring



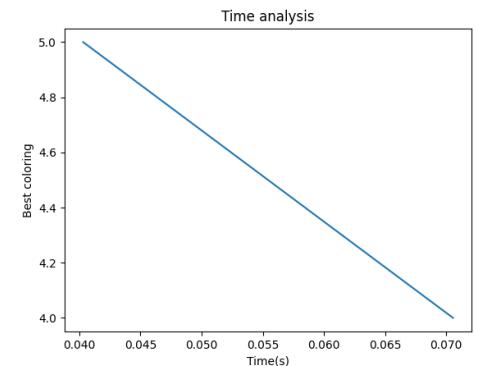
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



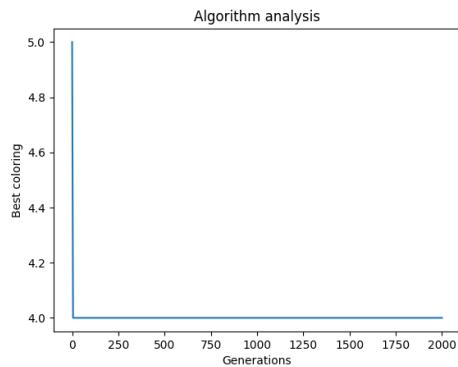
(c) HCFI-Time analysis

Results

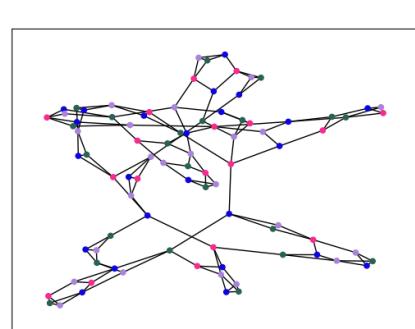
Compact Results Table

4. mug88_25

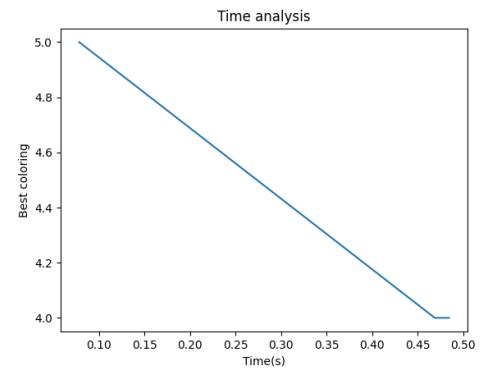
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mug88_25	GA	88	146	4	2000	4	4	4	0.4844	5	0.0781	6	1	3810
	HCFI				100000			4	0.0685	5	0.0202	68	1	68



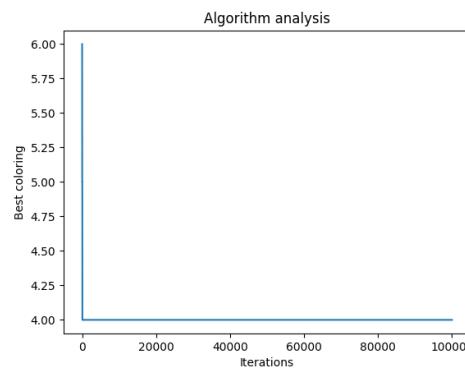
(a) GA-Algorithm analysis



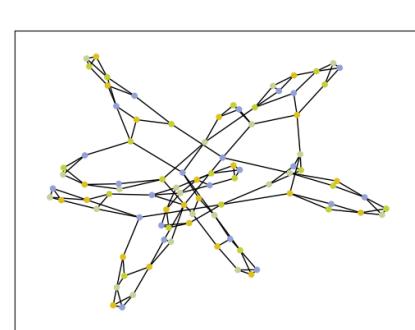
(b) GA-Graph coloring



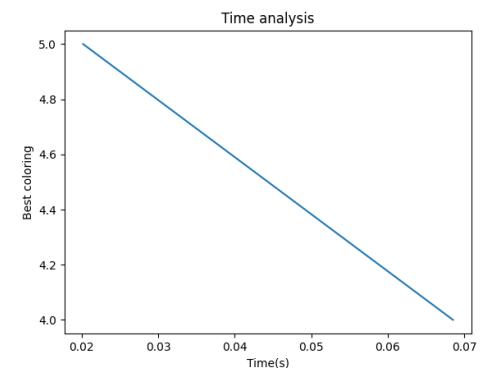
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



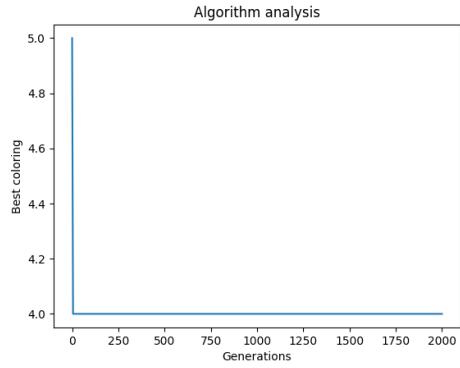
(c) HCFI-Time analysis

Results

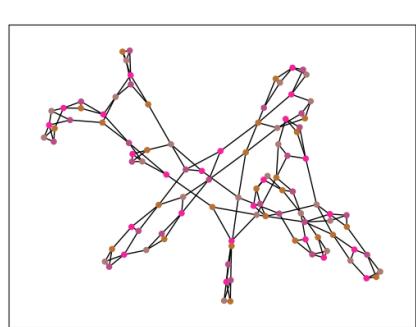
Compact Results Table

5. mug100_1

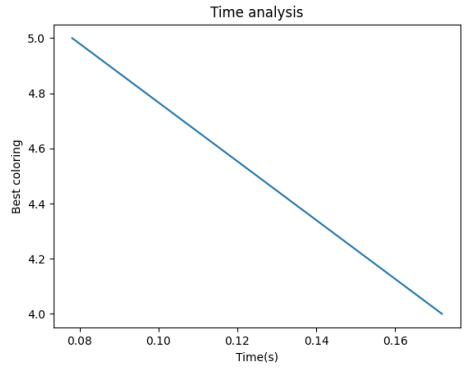
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mug100_1	GA	100	166	4	2000	4	4	4	0.1719	5	0.0781	6	1	5211
	HCFI				100000			4	0.1232	5	0.0608	46	1	168



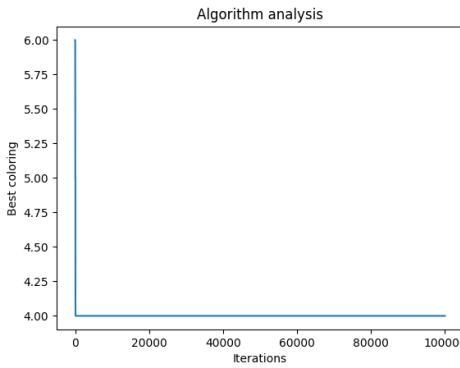
(a) GA-Algorithm analysis



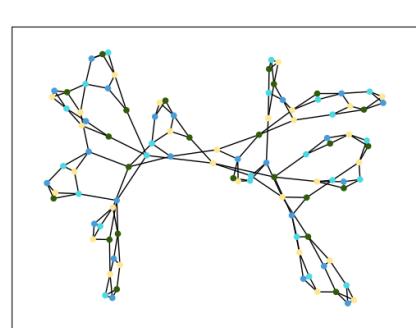
(b) GA-Graph coloring



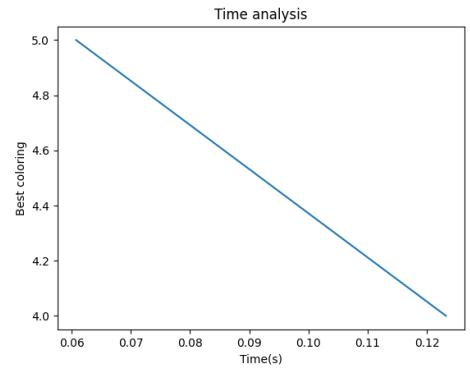
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



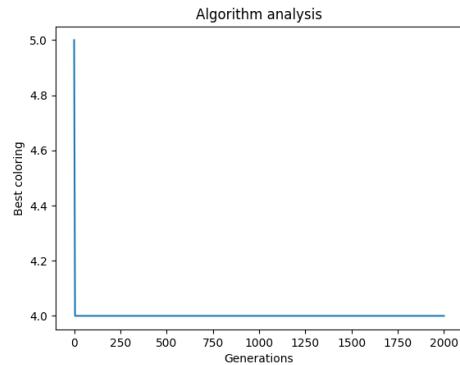
(c) HCFI-Time analysis

Results

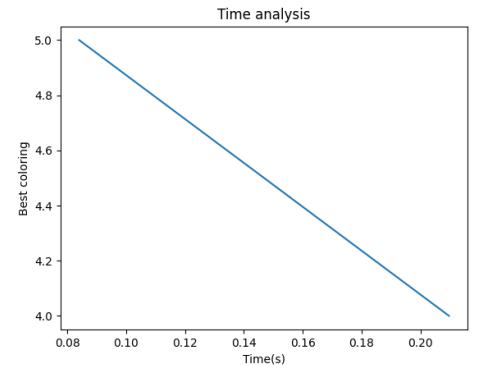
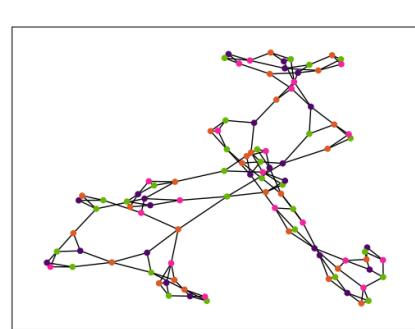
Compact Results Table

6. mug100_25

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mug100_25	GA	100	166	4	2000	4	4	4	0.2097	5	0.084	6	1	5438
	HCFI				100000				0.1091	5	0.0506	54	1	169

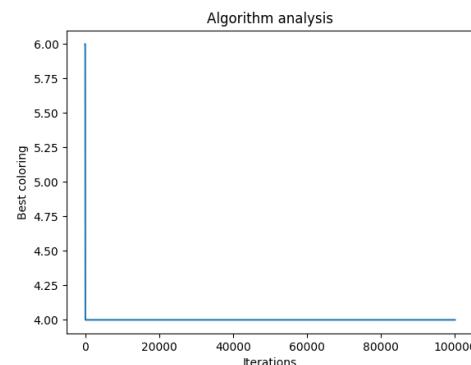


(a) GA-Algorithm analysis

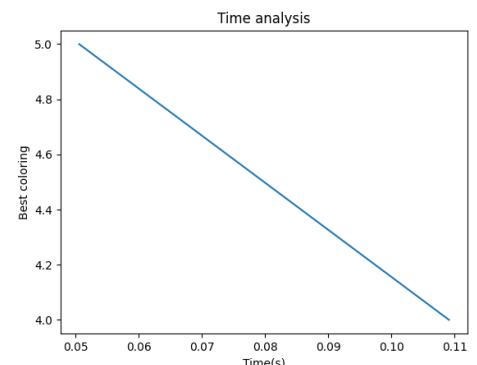
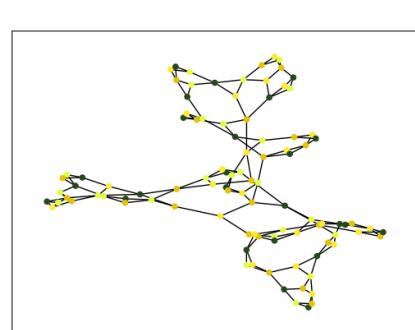


(b) GA-Graph coloring

(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring

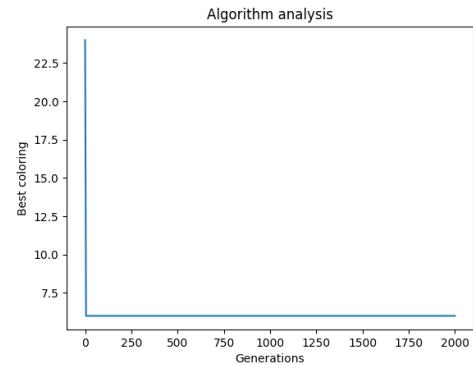
(c) HCFI-Time analysis

Results

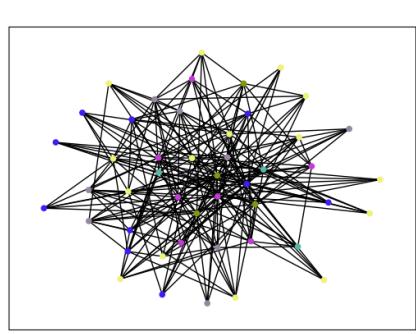
Compact Results Table

7. myciel5

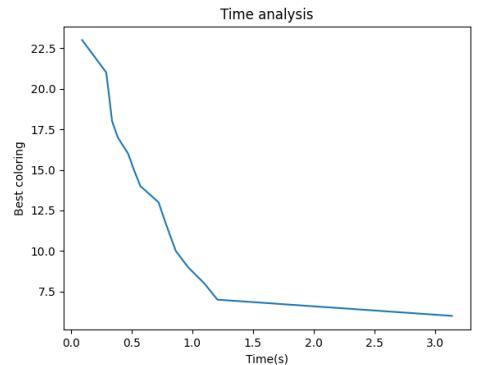
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel5	GA	47	236	23	2000	6	6	6	3.141	23	0.093	2	1	1793
	HCFI				100000				0.312	21	0.008	60	1	80



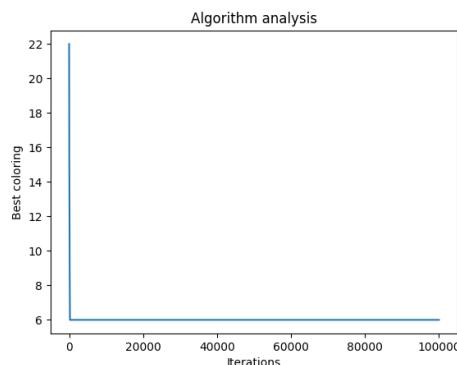
(a) GA-Algorithm analysis



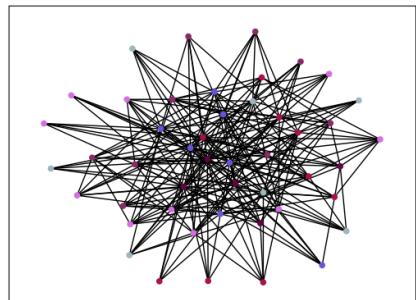
(b) GA-Graph coloring



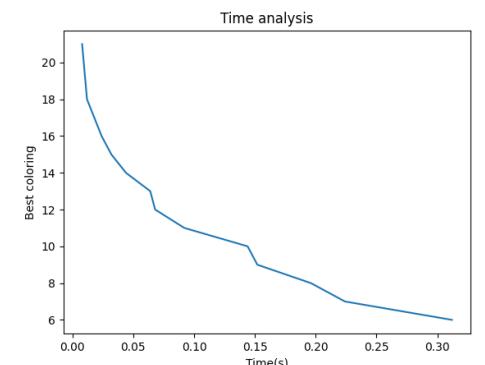
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



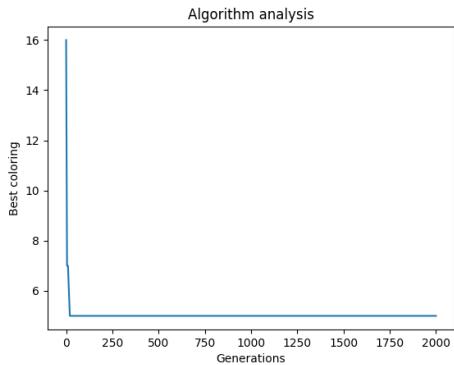
(c) HCFI-Time analysis

Results

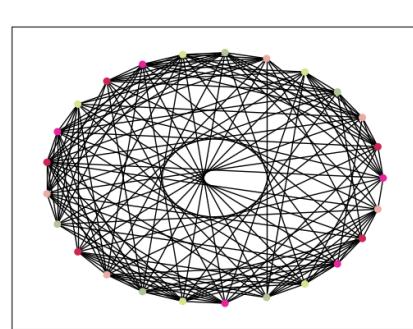
Compact Results Table

8. queen5_5

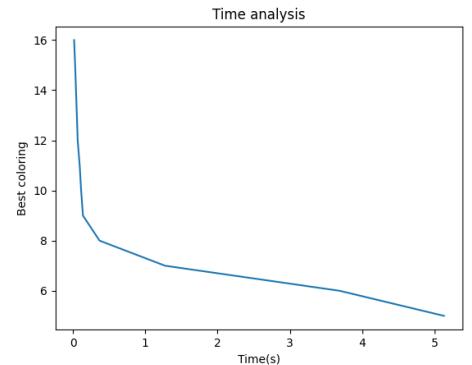
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen5_5	GA	25	320	16	2000	5	5	5	5.13	16	0.017	30	1	422
	HCFI				100000				0.1875	15	0	56	1	52



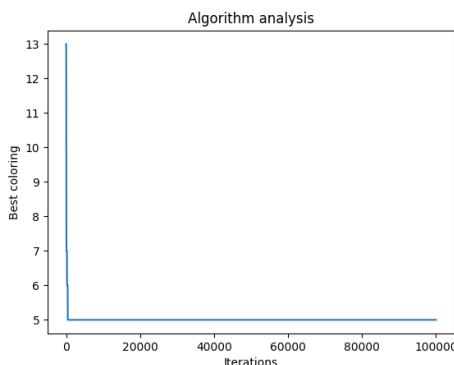
(a) GA-Algorithm analysis



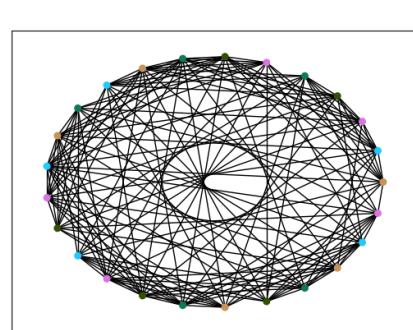
(b) GA-Graph coloring



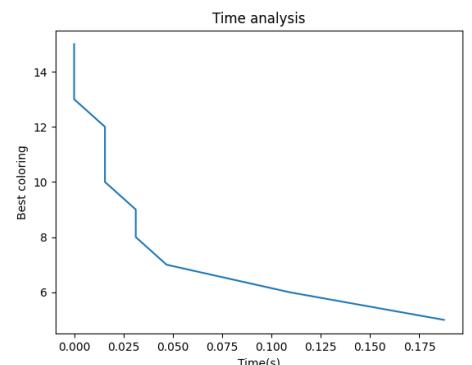
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



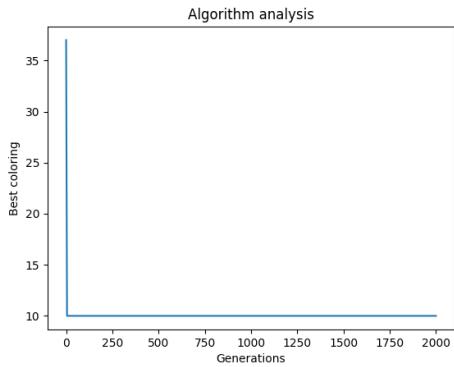
(c) HCFI-Time analysis

Results

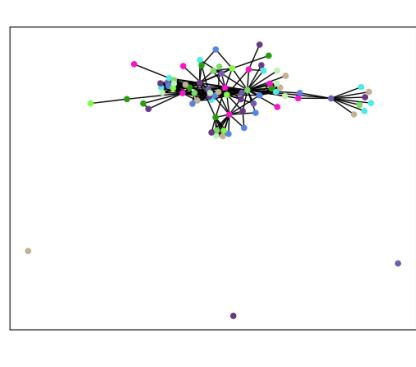
Compact Results Table

9. jean

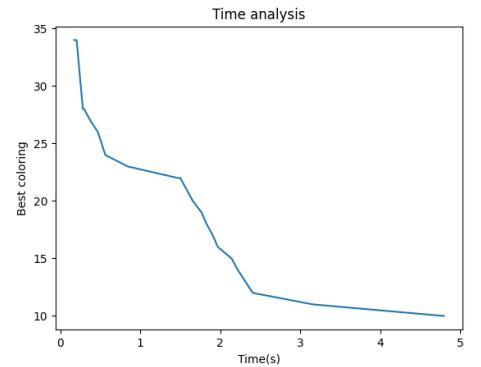
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
jean	GA	80	508	36	2000	10	10	10	4.798	34	0.1719	2	1	3683
	HCFI				100000		10	10	0.3594	34	0.0156	24	1	125



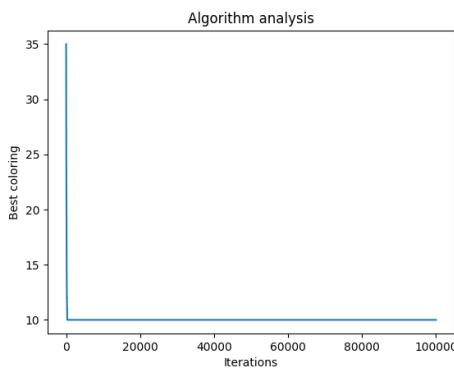
(a) GA-Algorithm analysis



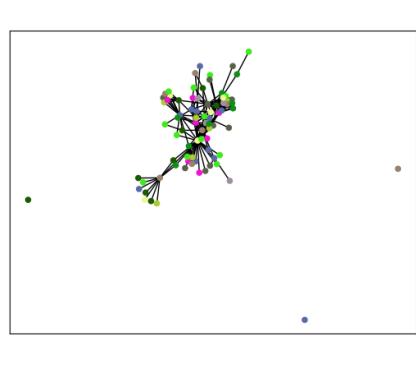
(b) GA-Graph coloring



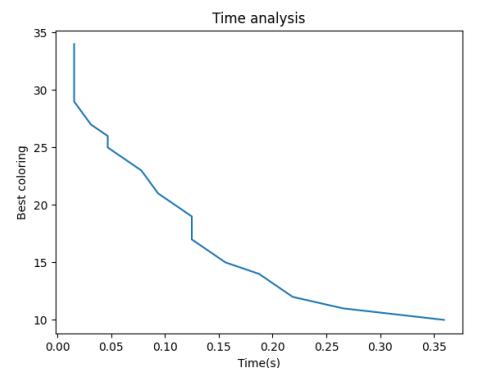
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



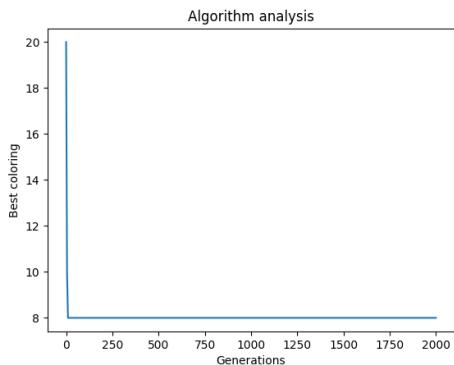
(c) HCFI-Time analysis

Results

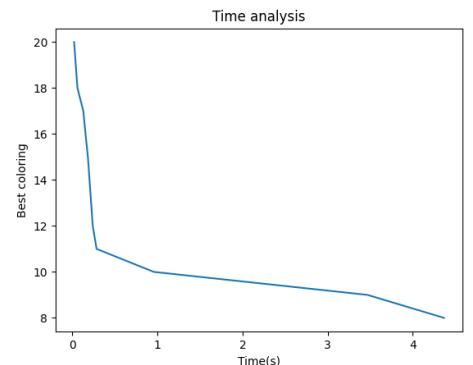
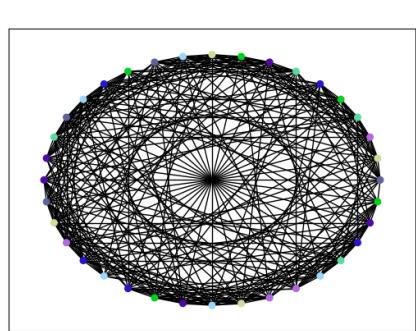
Compact Results Table

10. queen6_6

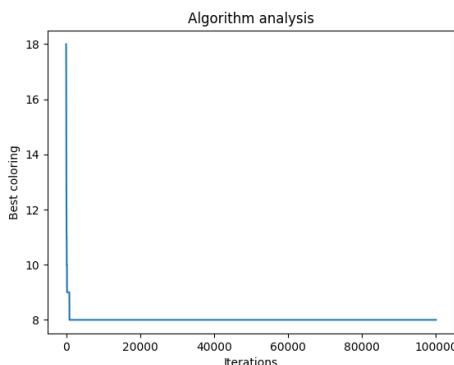
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen6_6	GA	36	580	19	2000	7	7	8	4.366	20	0.02	4	1	919
	HCFI				100000			8	0.6875	17	0	36	1	85



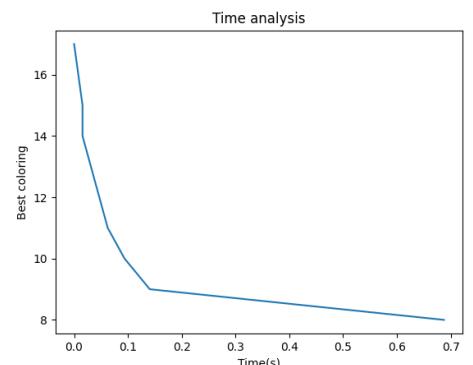
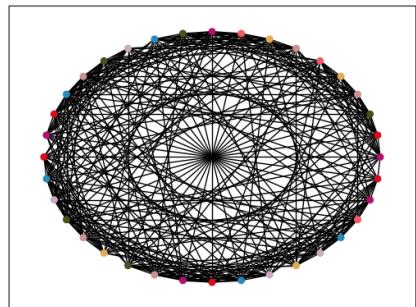
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



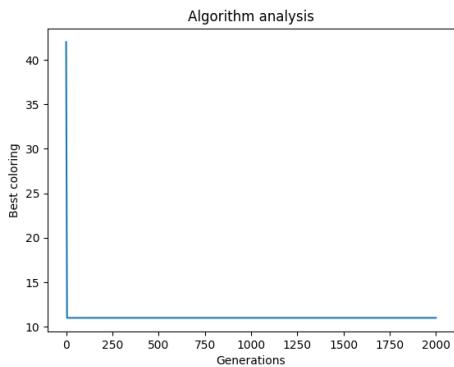
(c) HCFI-Time analysis

Results

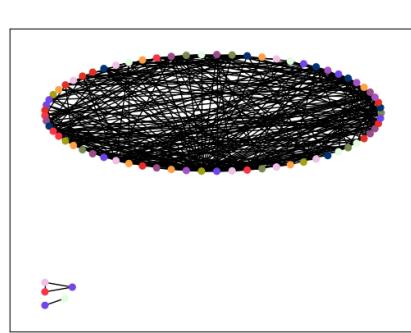
Compact Results Table

11. huck

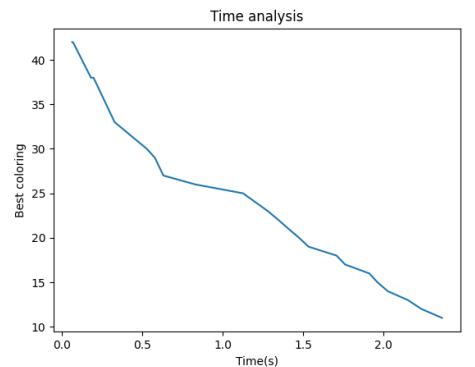
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
huck	GA	74	602	53	2000	11	11	11	2.362	42	0.064	6	1	2397
	HCFI				100000			11	0.353	37	0.013	10	1	113



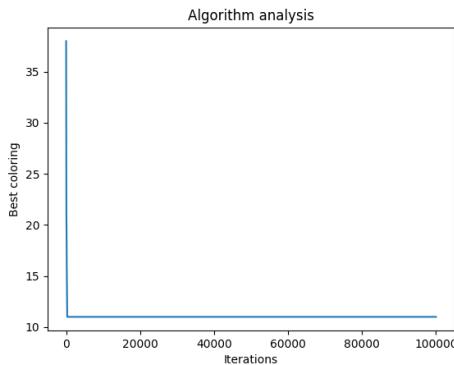
(a) GA-Algorithm analysis



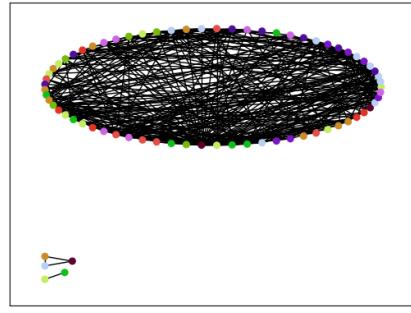
(b) GA-Graph coloring



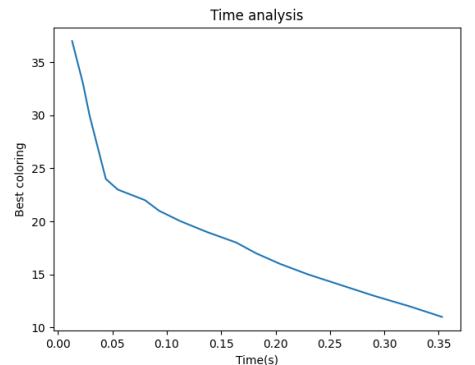
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



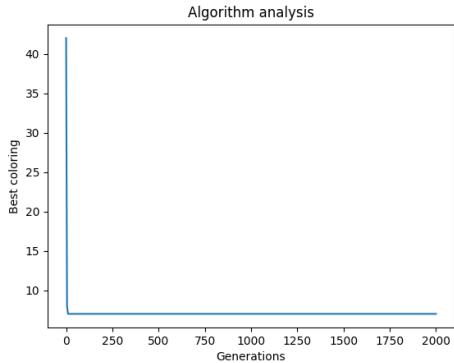
(c) HCFI-Time analysis

Results

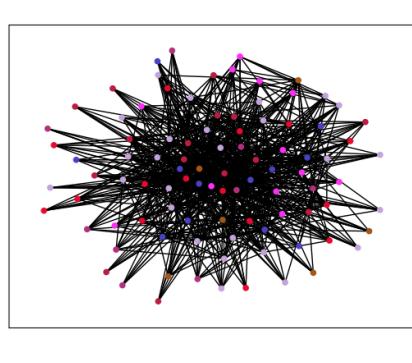
Compact Results Table

12. myciel6

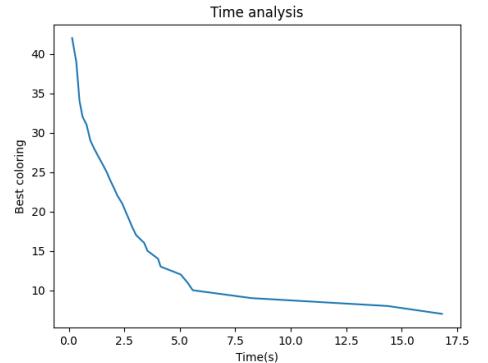
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel6	GA	95	755	47	2000	7	7	7	16	42	0.149	2	1	5786
	HCFI				100000			7	2.417	42	0.023	42	1	238



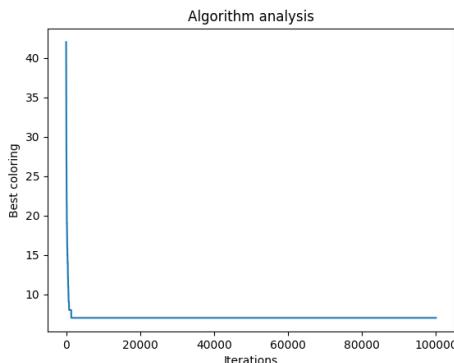
(a) GA-Algorithm analysis



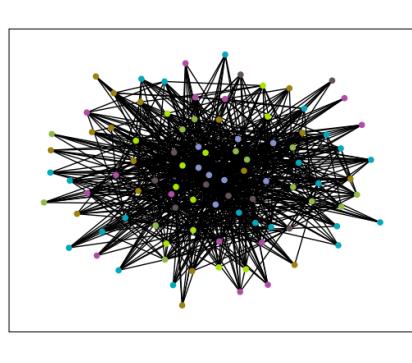
(b) GA-Graph coloring



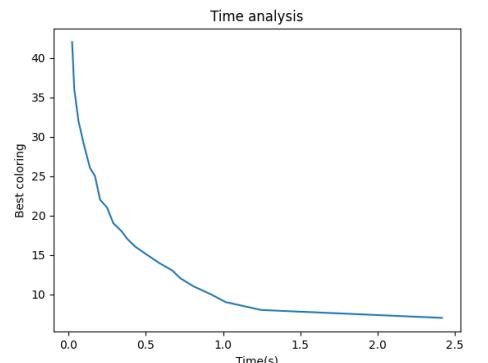
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



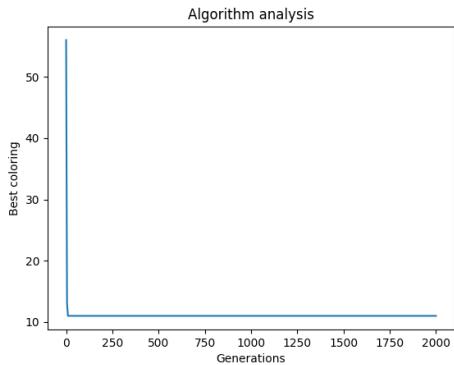
(c) HCFI-Time analysis

Results

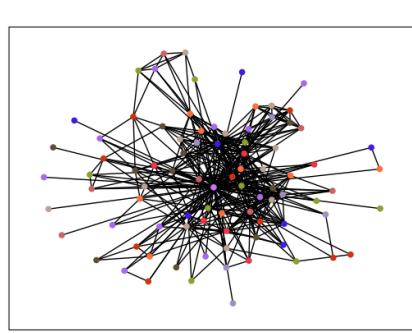
Compact Results Table

13. david

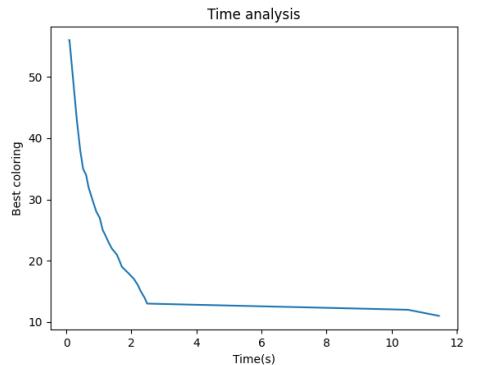
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
david	GA	87	812	82	2000	11	11	11	11	56	0.089	2	1	3963
	HCFI				100000			11	0.939	51	0.015	20	1	175



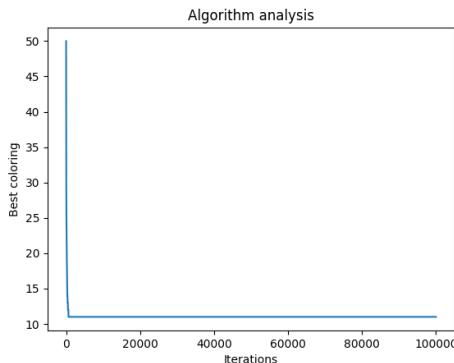
(a) GA-Algorithm analysis



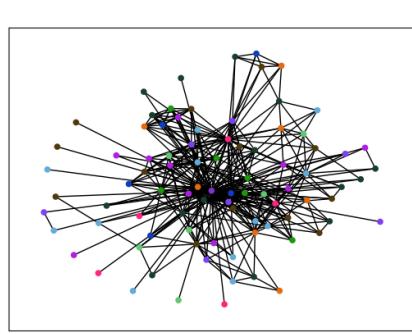
(b) GA-Graph coloring



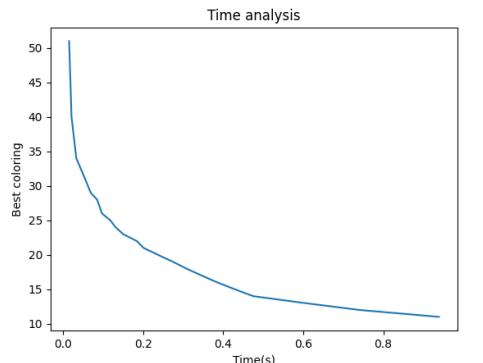
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



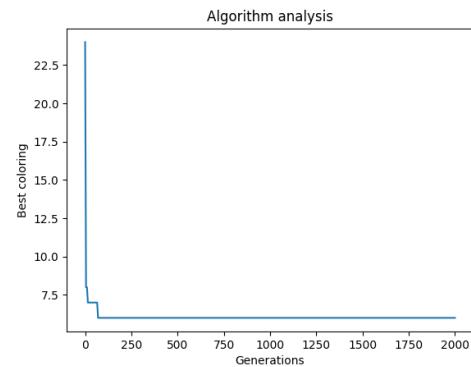
(c) HCFI-Time analysis

Results

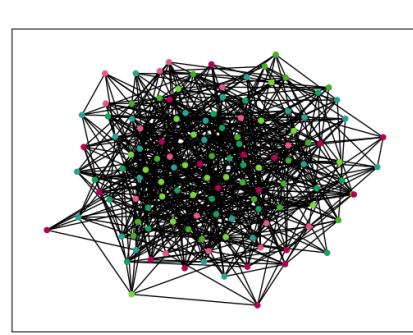
Compact Results Table

14. DSJC125.1_001

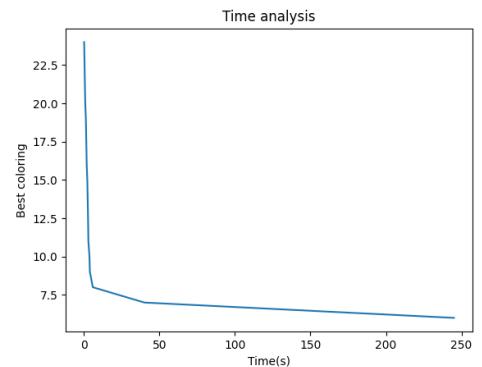
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC125.1_001	GA	125	736	23	2000	5	5	6	245	24	0.155	30	1	6030
	HCFI													



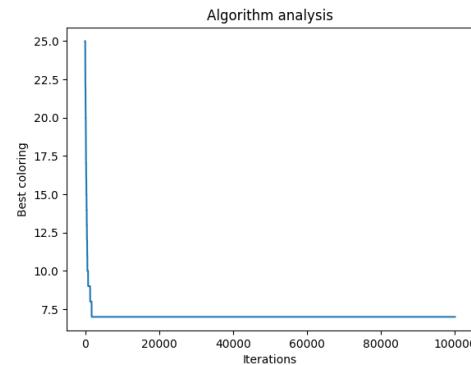
(a) GA-Algorithm analysis



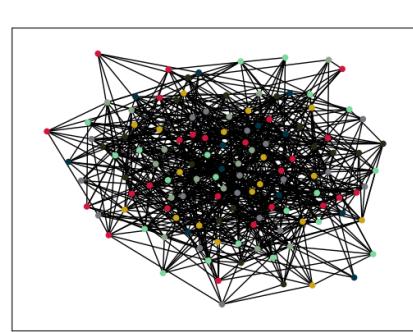
(b) GA-Graph coloring



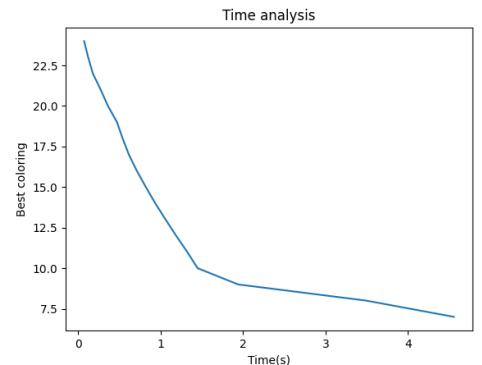
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



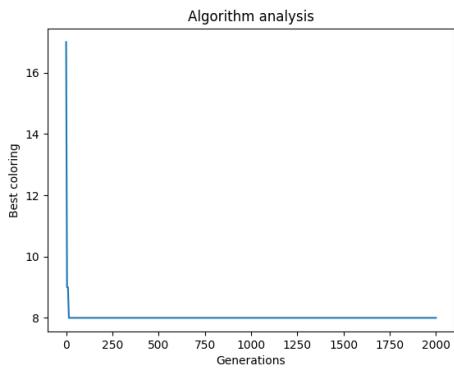
(c) HCFI-Time analysis

Results

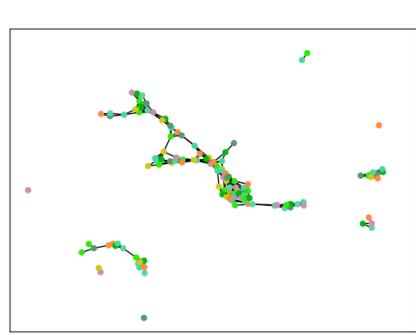
Compact Results Table

15. miles250

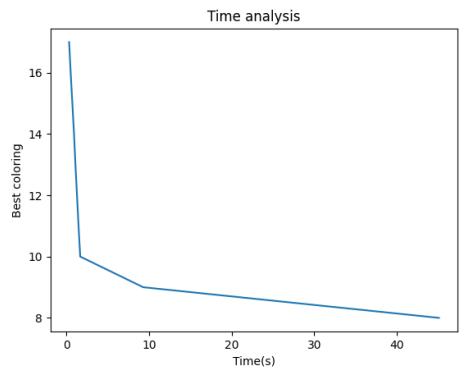
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles250	GA HCFI	128	774	16	2000 100000	8	8	8	45 7.249	17	0.3281 0.052	6 30	1 1	7745 250



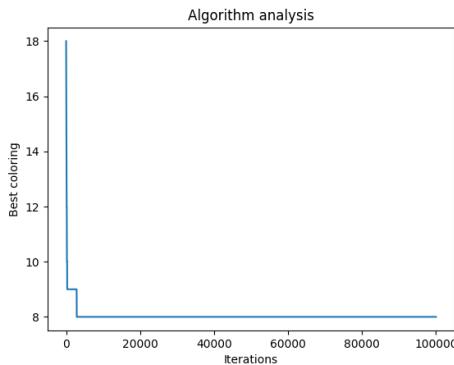
(a) GA-Algorithm analysis



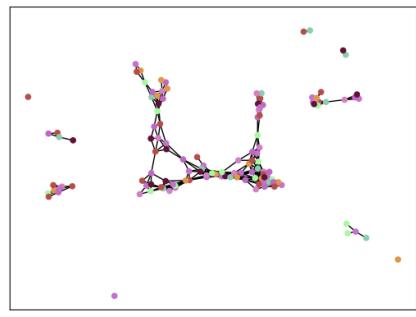
(b) GA-Graph coloring



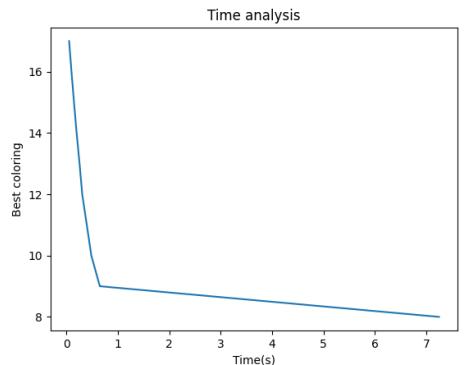
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



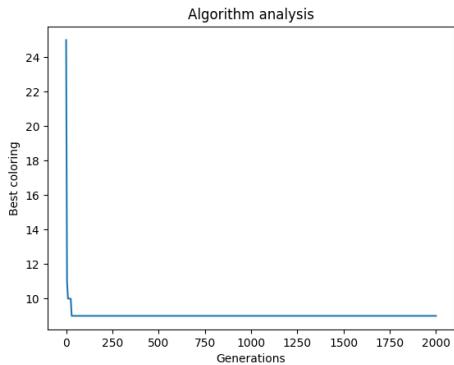
(c) HCFI-Time analysis

Results

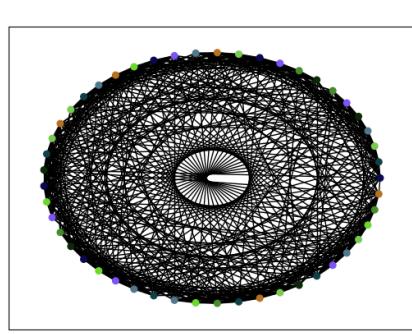
Compact Results Table

16. queen7_7

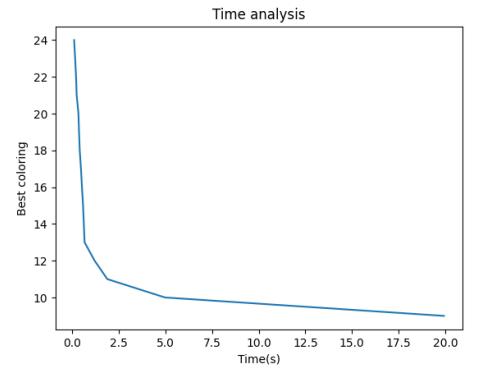
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen7_7	GA	49	952	24	2000	7	7	9	19	24	0.099	4	1	1557
	HCFI				100000			9	1.53	23	0.013	44	1	109



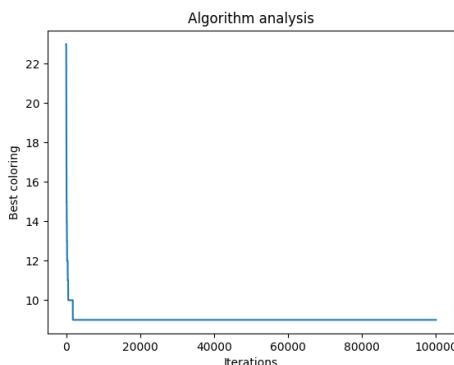
(a) GA-Algorithm analysis



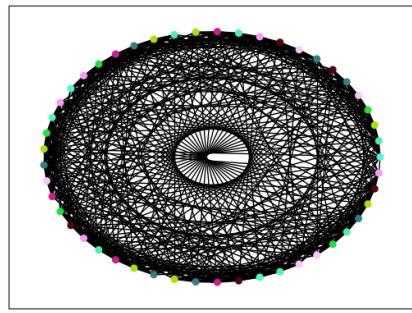
(b) GA-Graph coloring



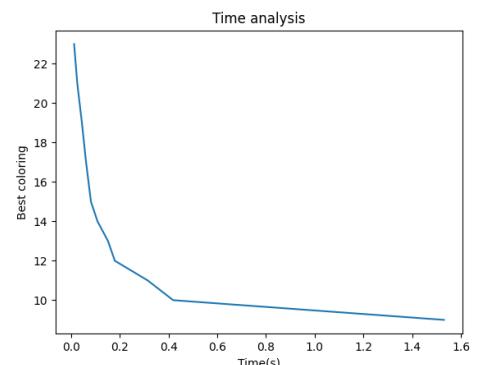
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



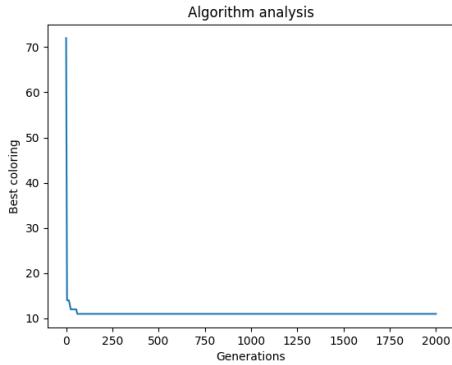
(c) HCFI-Time analysis

Results

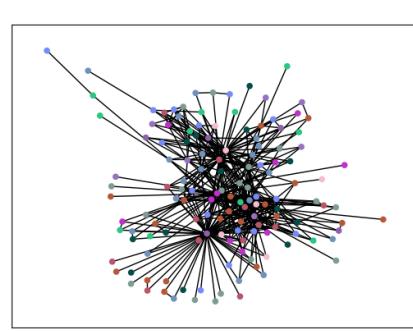
Compact Results Table

17. anna

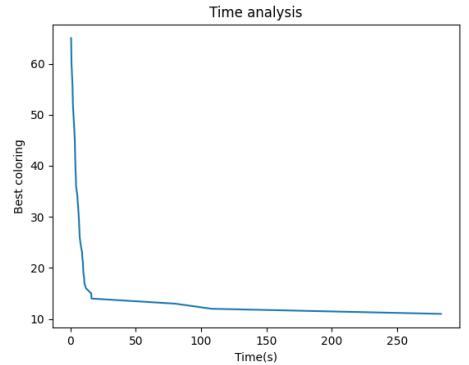
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
anna	GA	138	986	71	2000	11	11	11	283	65	0.5330	2	1	9095
	HCFI				100000				4.02	64	0.028	46	1	298



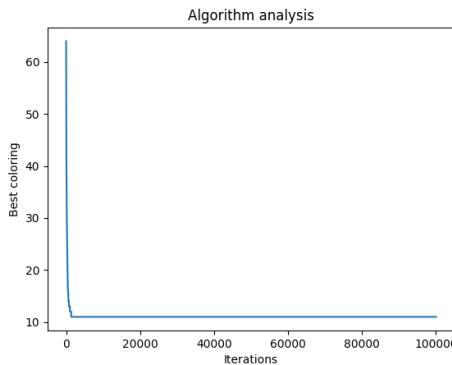
(a) GA-Algorithm analysis



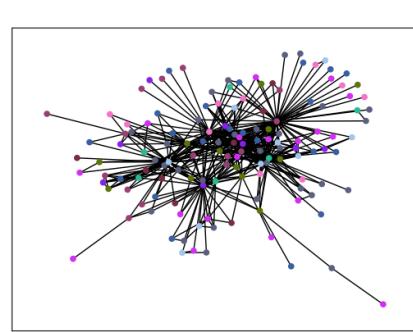
(b) GA-Graph coloring



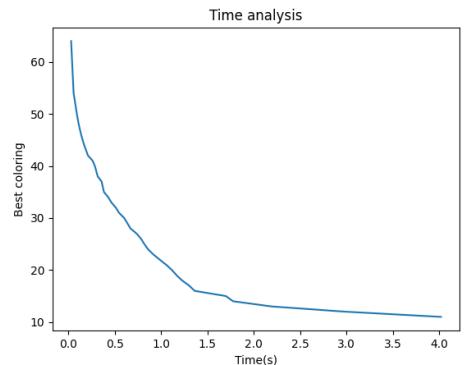
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



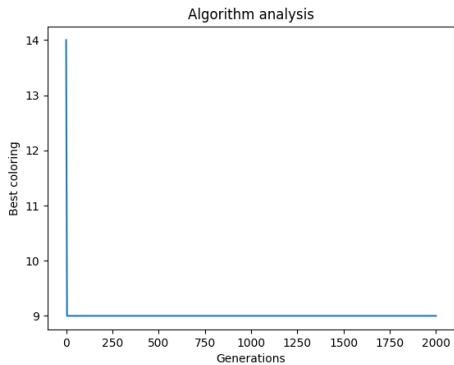
(c) HCFI-Time analysis

Results

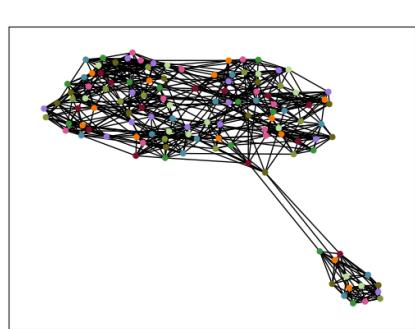
Compact Results Table

18. games120

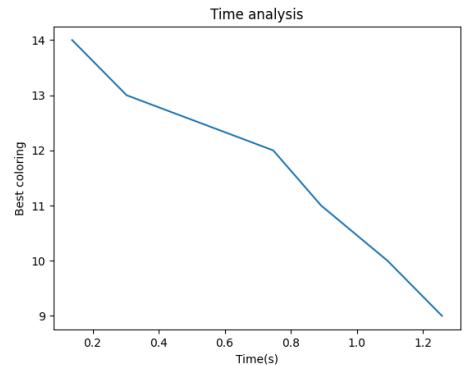
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
games120	GA	120	1276	13	2000	9	9	9	1.2580	14	0.1380	4	1	5379
	HCFI				100000		9	9	0.87	14	0.109	28	1	239



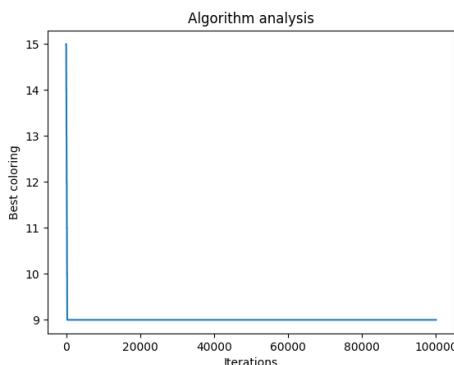
(a) GA-Algorithm analysis



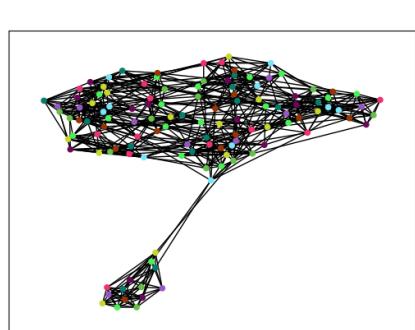
(b) GA-Graph coloring



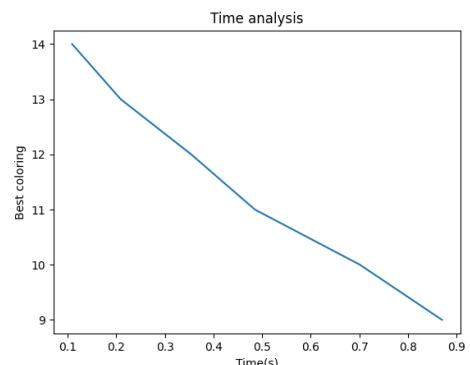
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



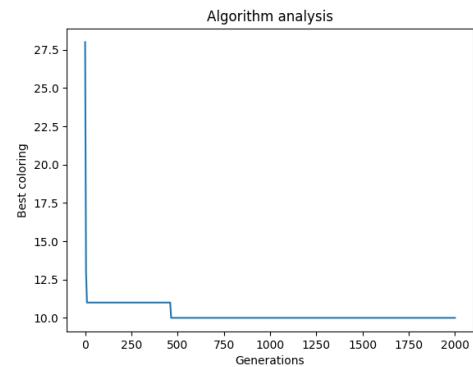
(c) HCFI-Time analysis

Results

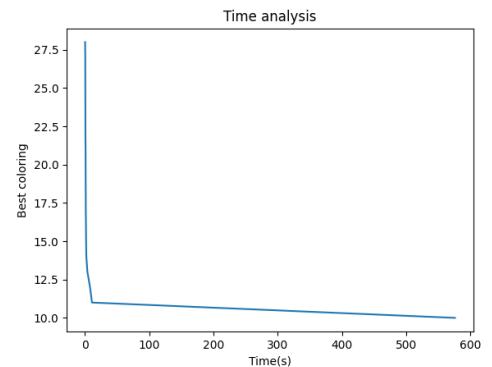
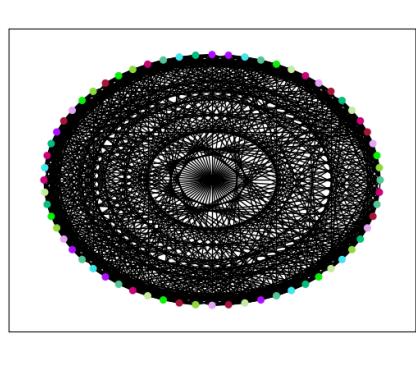
Compact Results Table

19. queen8_8

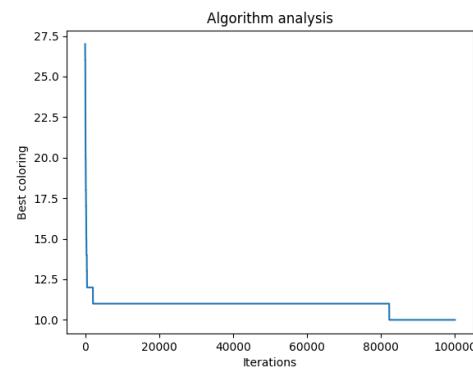
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen8_8	GA	64	1456	27	2000	9	9	10	576	28	0.2	4	1	2486
	HCFI				100000		10		119	26	0.025	64	1	150



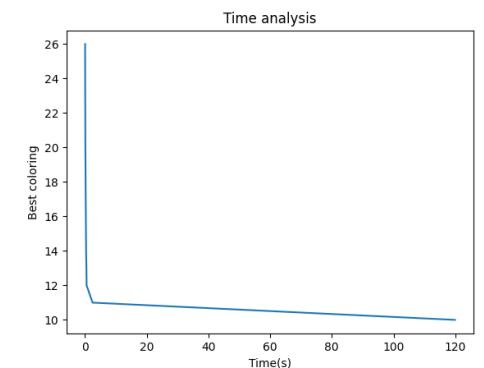
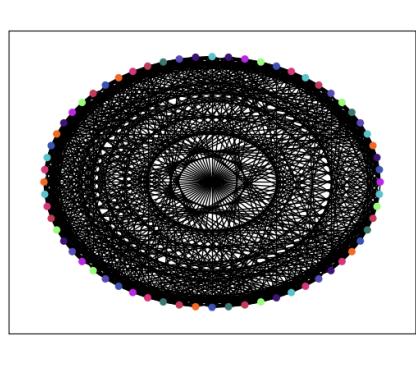
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



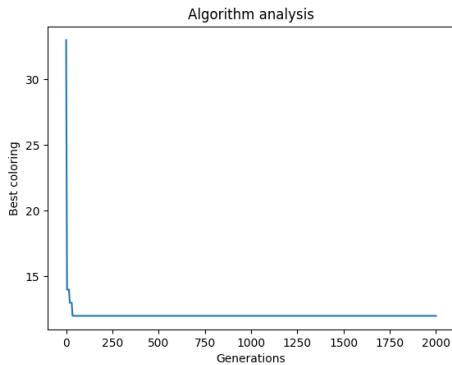
(c) HCFI-Time analysis

Results

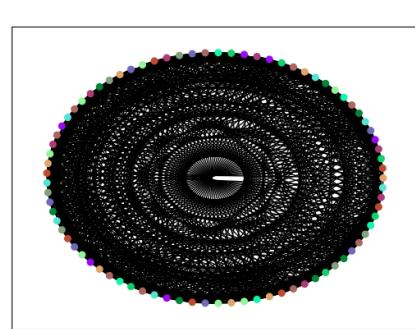
Compact Results Table

20. queen9_9

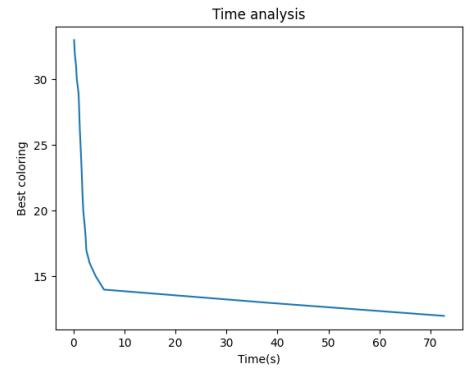
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen9_9	GA	81	2112	32	2000	10	10	12	72	33	0.1082	6	1	4337
	HCFI				100000			12	23	33	0.059	46	1	212



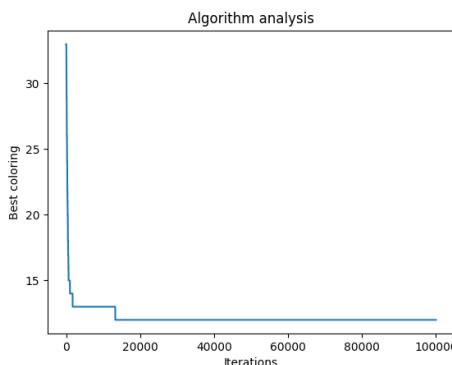
(a) GA-Algorithm analysis



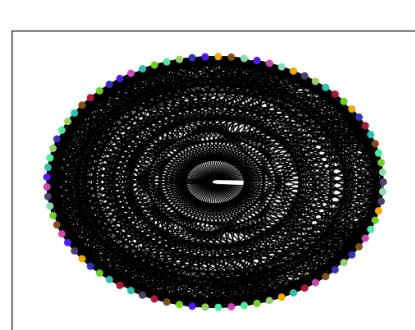
(b) GA-Graph coloring



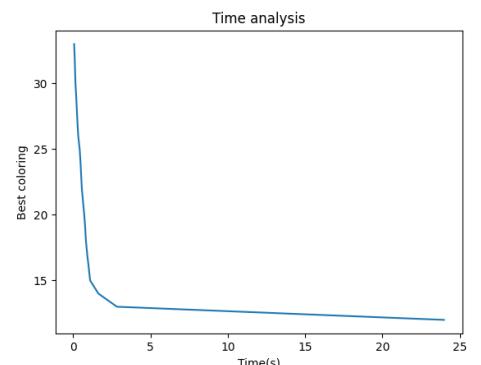
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



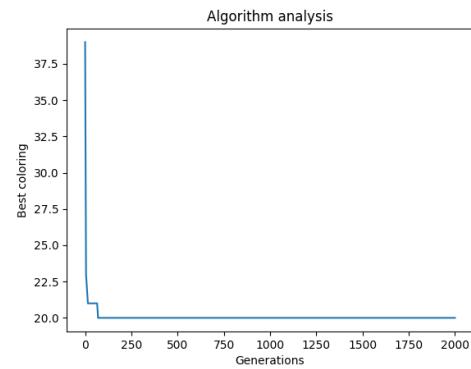
(c) HCFI-Time analysis

Results

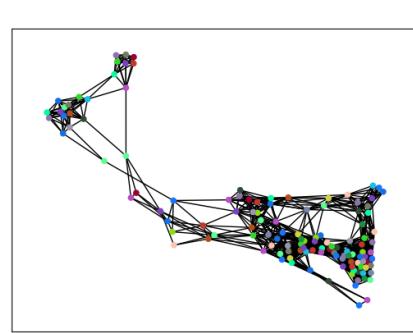
Compact Results Table

21. miles500

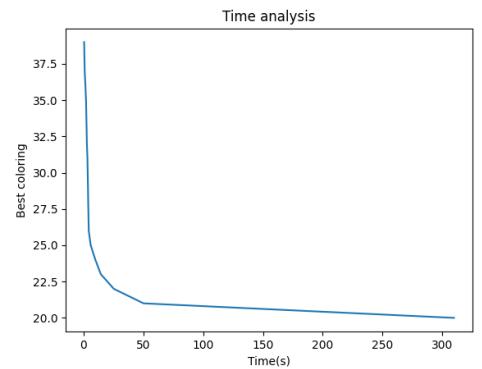
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles500	GA	128	2340	38	2000	20	20	20	309	39	0.5	6	1	9225
	HCFI				100000				141	39	0.121	62	1	309



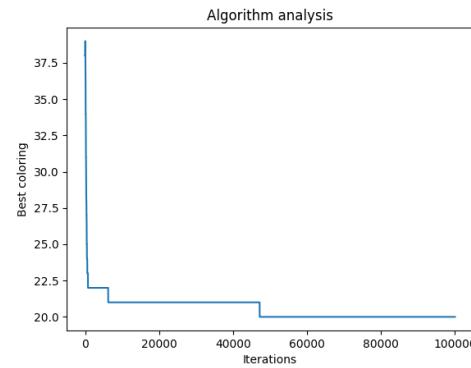
(a) GA-Algorithm analysis



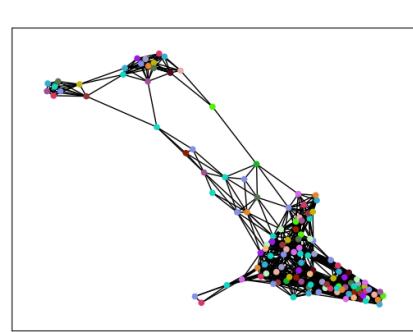
(b) GA-Graph coloring



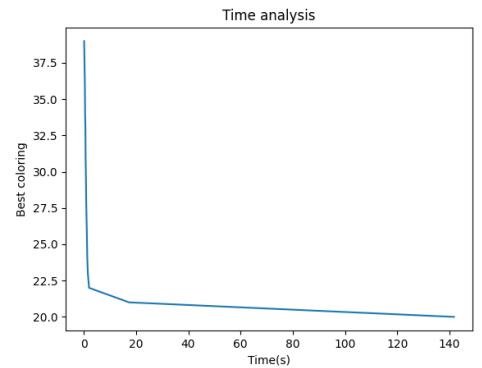
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



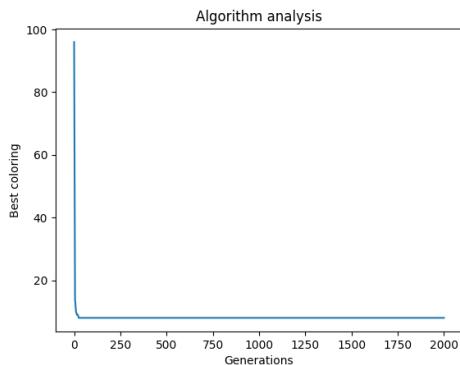
(c) HCFI-Time analysis

Results

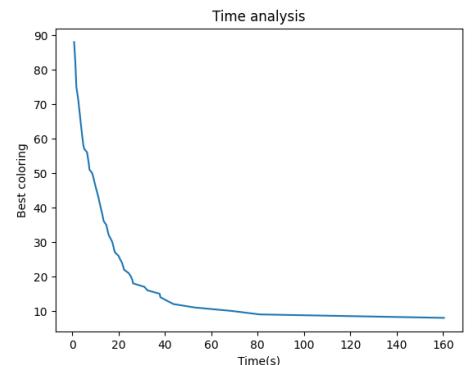
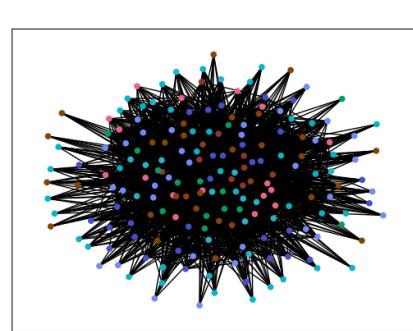
Compact Results Table

22. myciel7

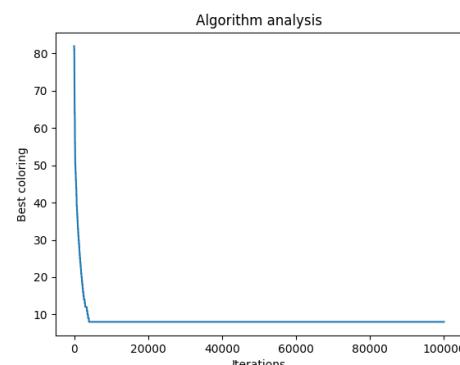
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel7	GA	191	2360	95	2000	8	8	8	160	88	0.875	6	1	16047
	HCFI				100000			8	22.92	81	0.123	446	1	801



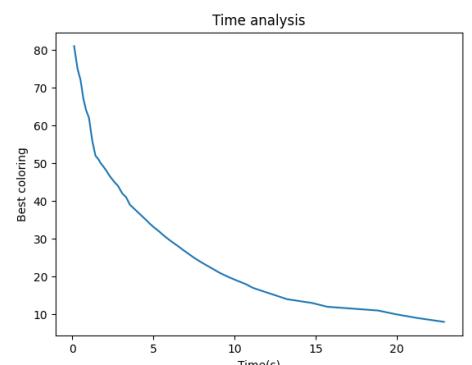
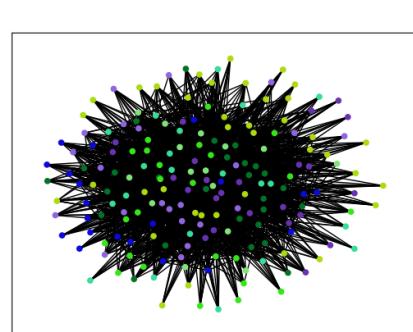
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



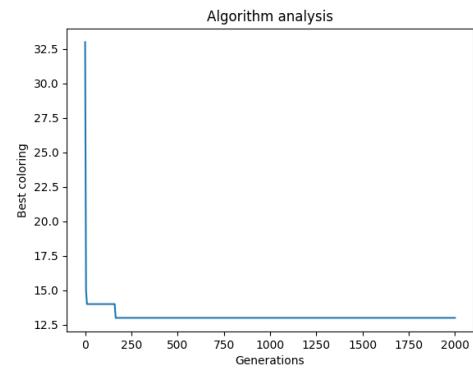
(c) HCFI-Time analysis

Results

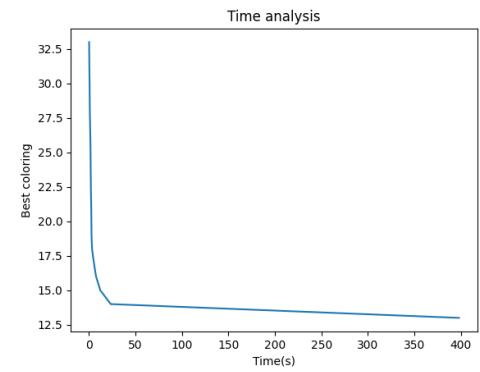
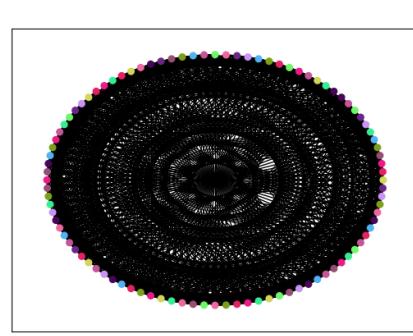
Compact Results Table

23. queen8_12

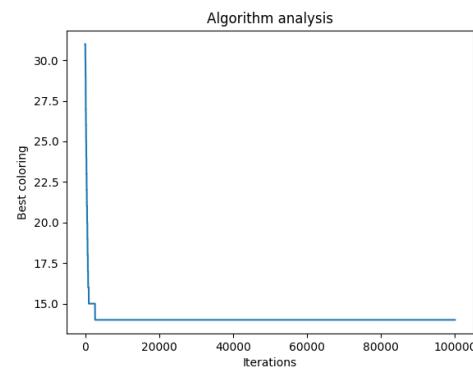
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen8_12	GA	96	2736	32	2000	12	12	13	398	33	0.103	20	1	4106
	HCFI				100000			14	7.482	31	0.082	80	1	278



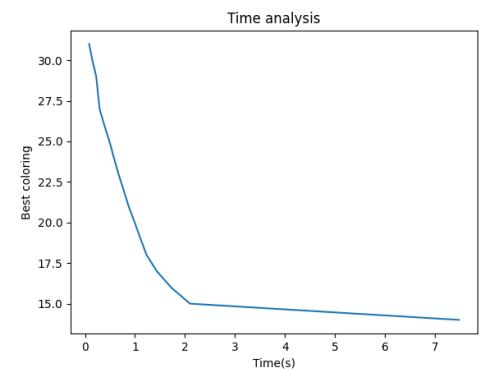
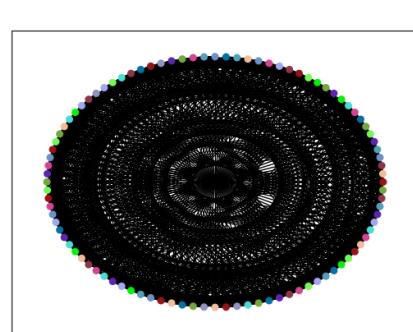
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



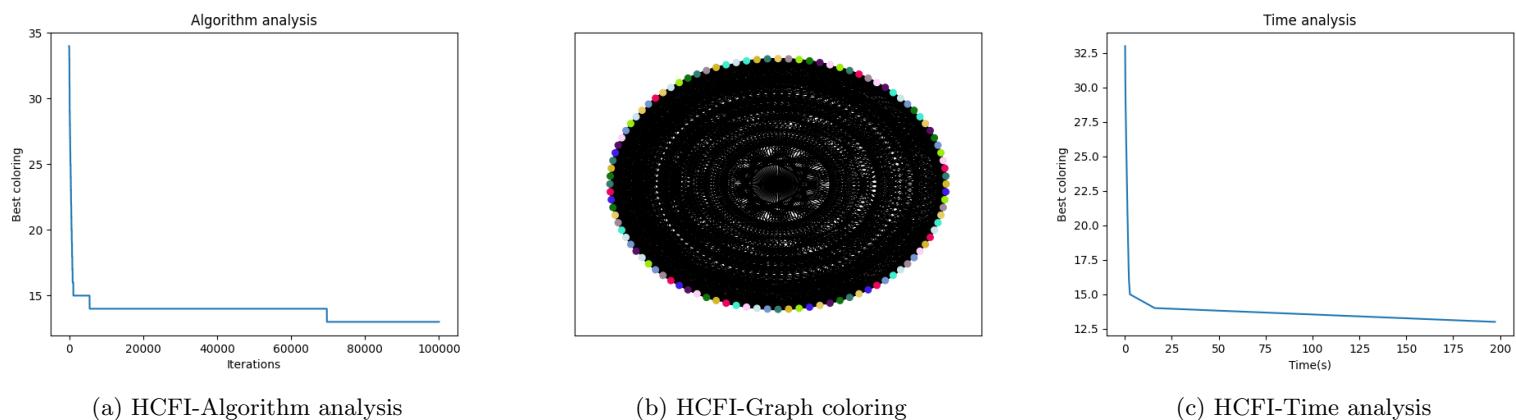
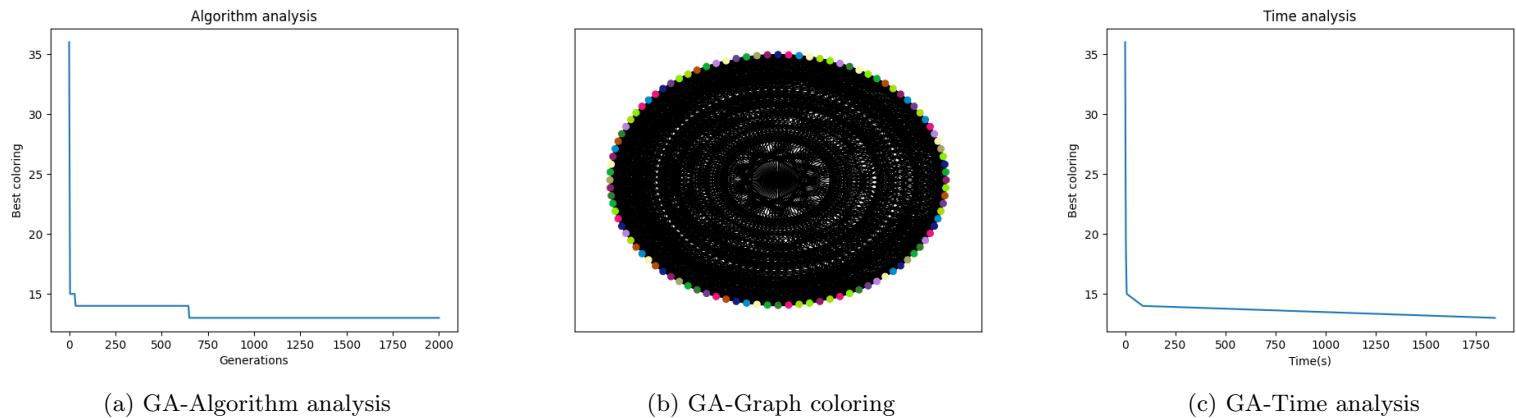
(c) HCFI-Time analysis

Results

Compact Results Table

24. queen10_10

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen10_10	GA	100	2940	35	2000	11	11	13	1845	36	0.103	6	1	5694
	HCFI				100000				197	33	0.095	206	1	299

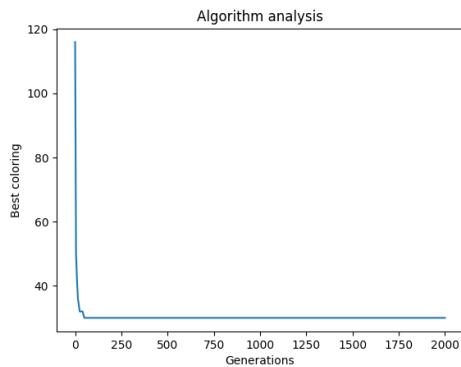


Results

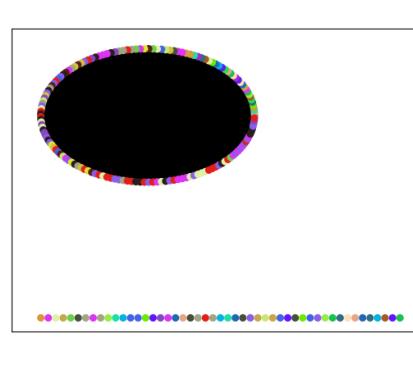
Compact Results Table

25. zeroin.i.3

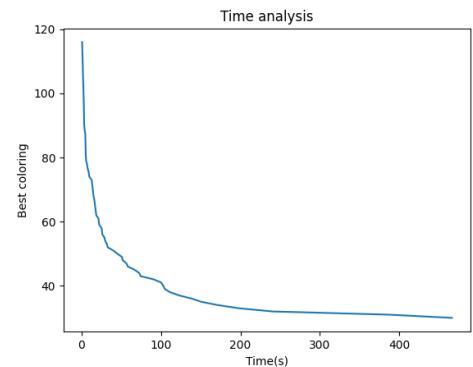
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
zeroin.i.3	GA	206	3540	140	2000	30	30	30	466	116	0.6278	6	1	18883
	HCFI				1000000			45	3499	116	0.165	38	1	16834



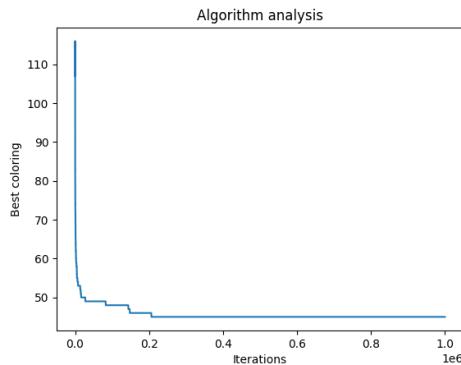
(a) GA-Algorithm analysis



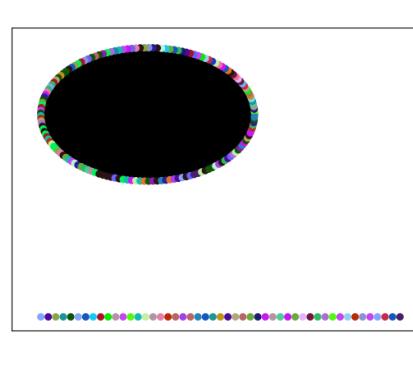
(b) GA-Graph coloring



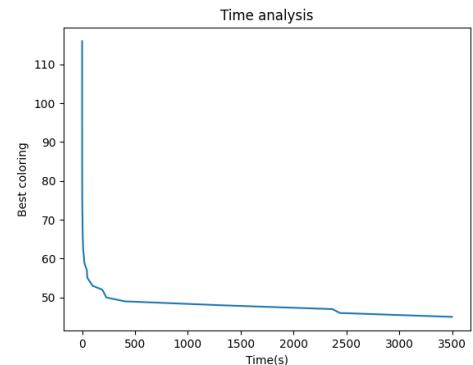
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



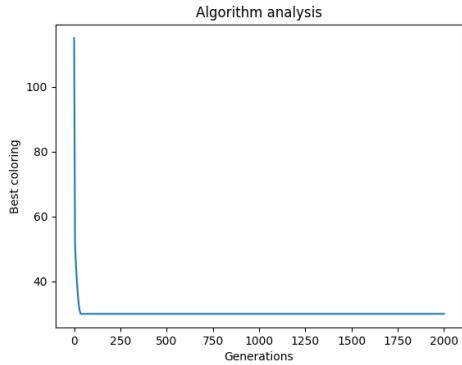
(c) HCFI-Time analysis

Results

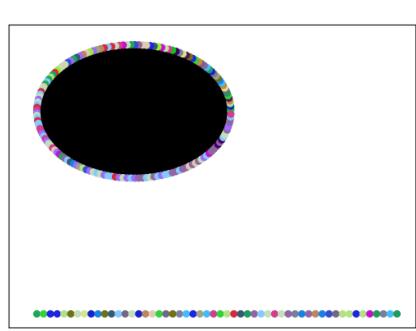
Compact Results Table

26. zeroin.i.2

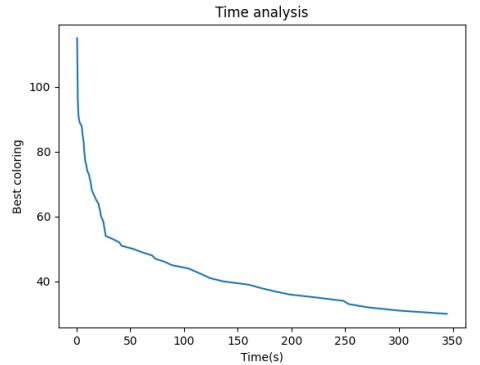
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
zeroin.i.2	GA	211	3541	140	2000	30	30	30	344	115	0.6641	6	1	19612
	HCFI				100000		45		3732	119	0.125	148	1	17224



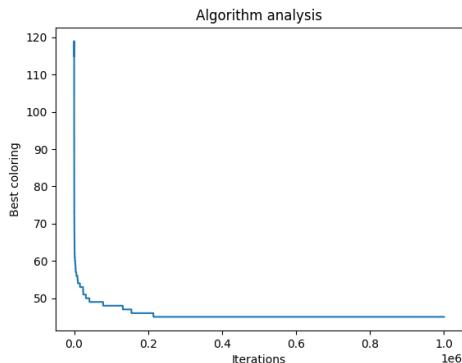
(a) GA-Algorithm analysis



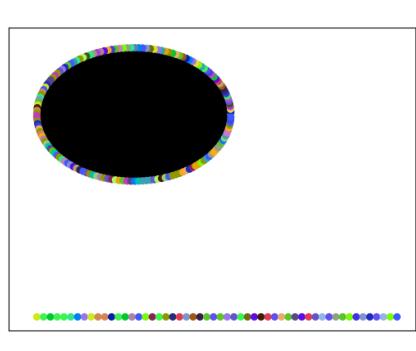
(b) GA-Graph coloring



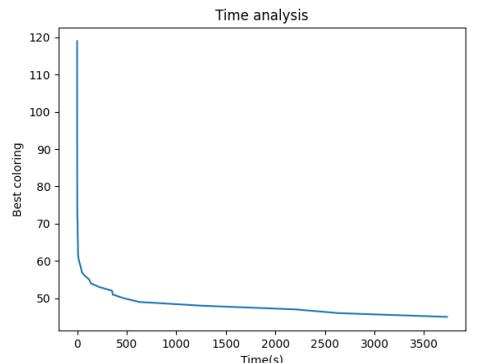
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



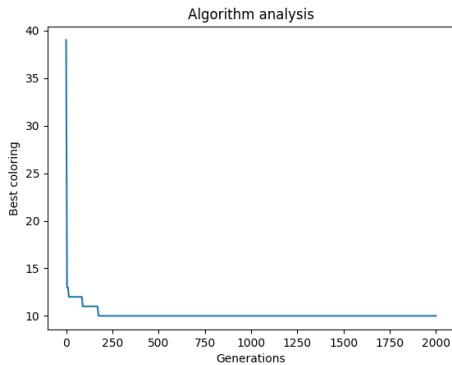
(c) HCFI-Time analysis

Results

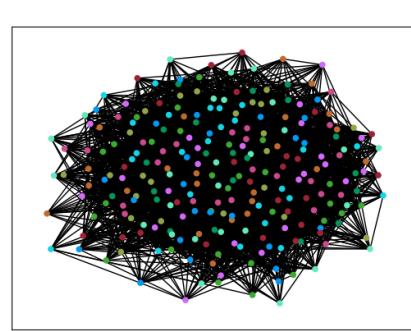
Compact Results Table

27. DSJC250.1

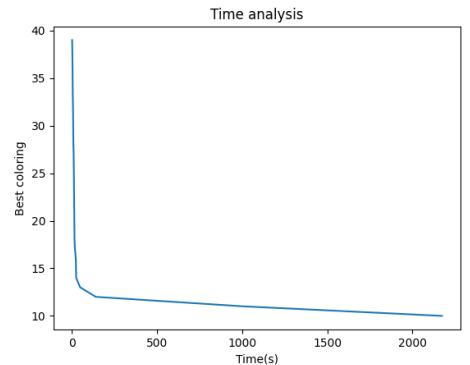
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC250.1	GA HCFI	250	3218	38	2000 100000	8	9	10 12	2173 168	39	1.188 0.737	50 234	1 1	23129 1126



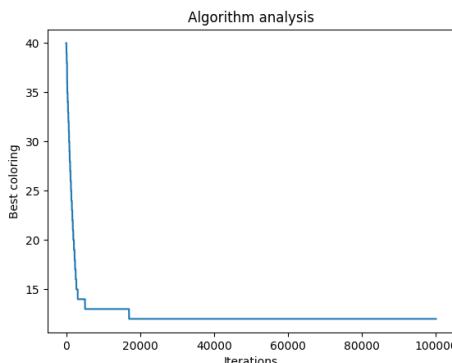
(a) GA-Algorithm analysis



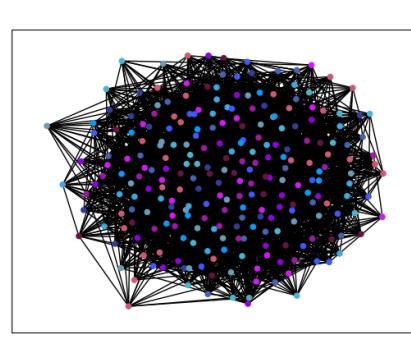
(b) GA-Graph coloring



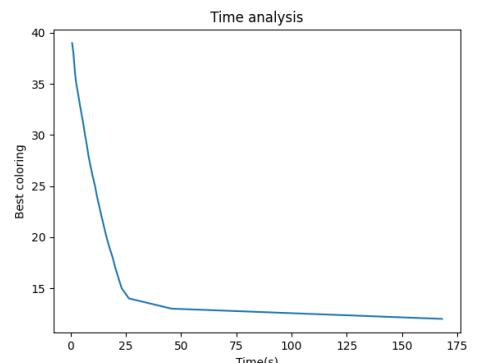
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



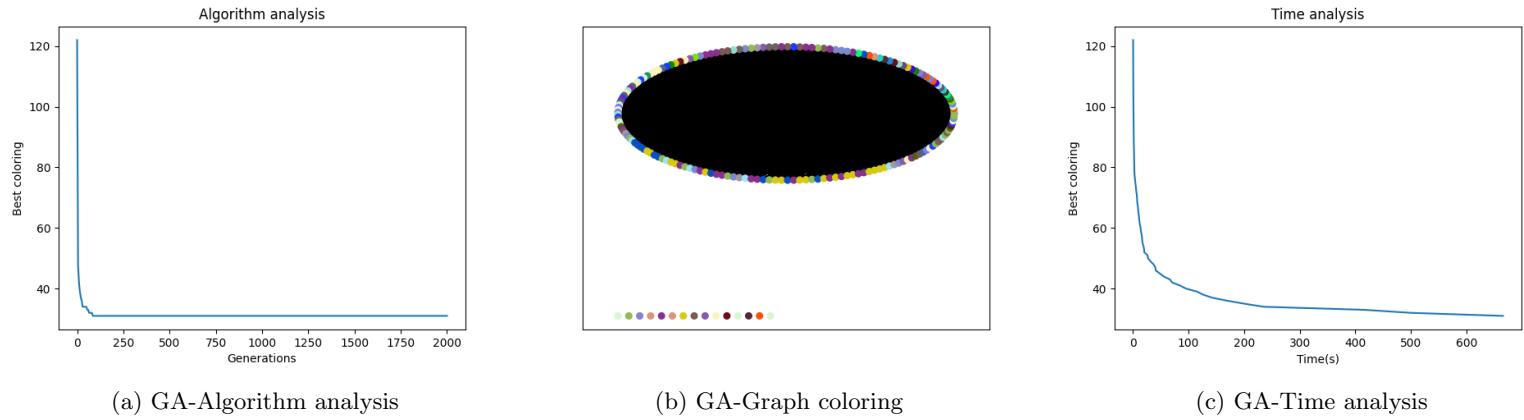
(c) HCFI-Time analysis

Results

Compact Results Table

28. mulso.i.2

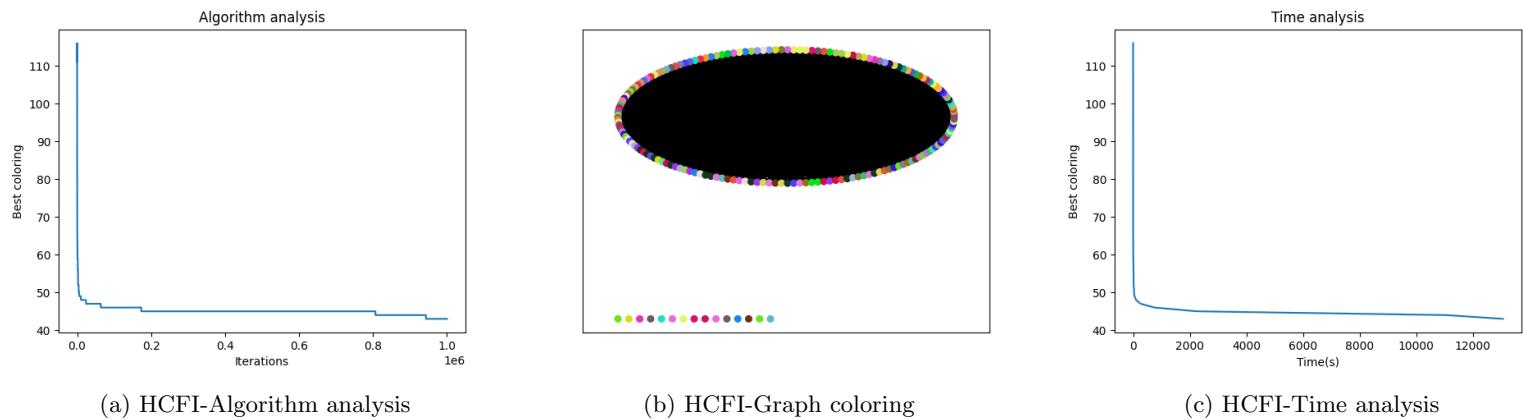
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mulso.i.2	GA	188	3885	156	2000	31	31	31	665	122	0.4561	6	1	16086
	HCFI				1000000		43		13064	116	0.157	50	1	13941



(a) GA-Algorithm analysis

(b) GA-Graph coloring

(c) GA-Time analysis



(a) HCFI-Algorithm analysis

(b) HCFI-Graph coloring

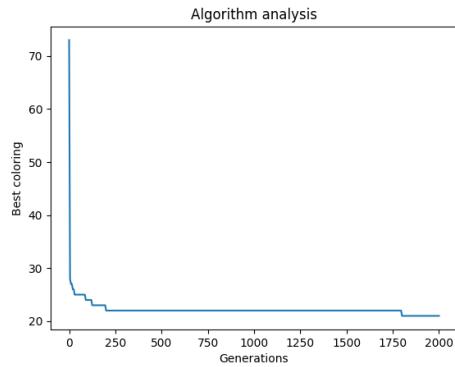
(c) HCFI-Time analysis

Results

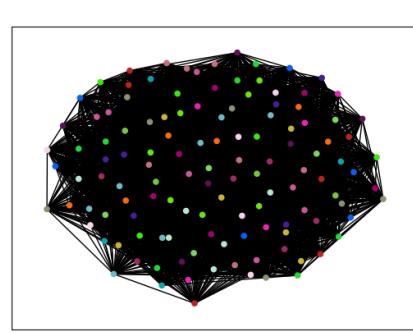
Compact Results Table

29. DSJC125.5

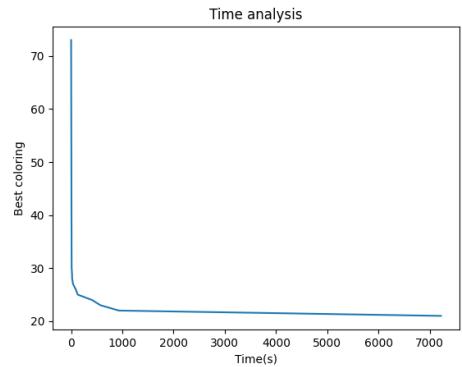
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)	
DSJC125.5	GA	125	3891	75	2000	100000	12	17	21	7219	73	0.226	28	1	7993
	HCFI						23		381	63	0.1406	70	1	494	



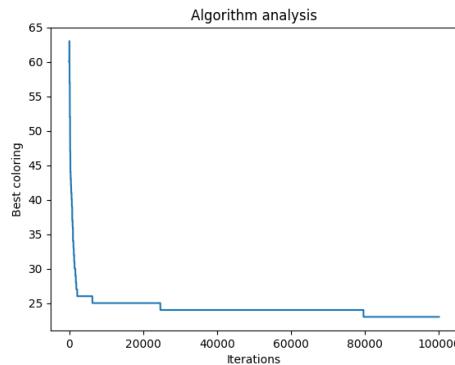
(a) GA-Algorithm analysis



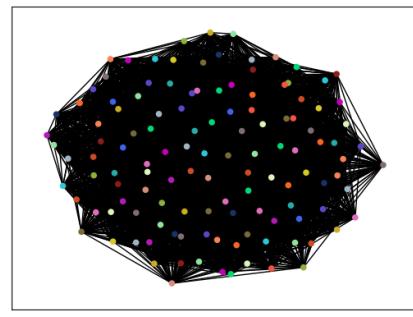
(b) GA-Graph coloring



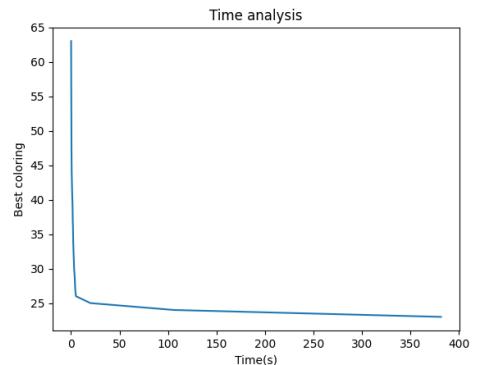
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



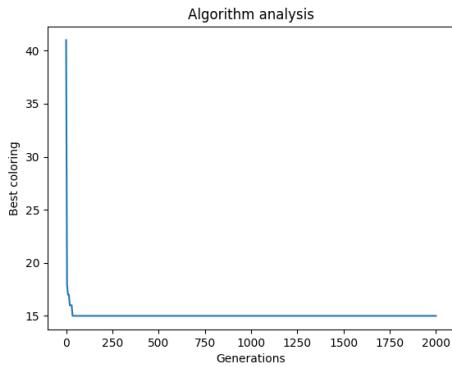
(c) HCFI-Time analysis

Results

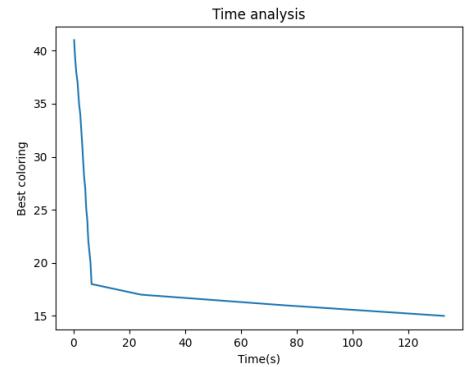
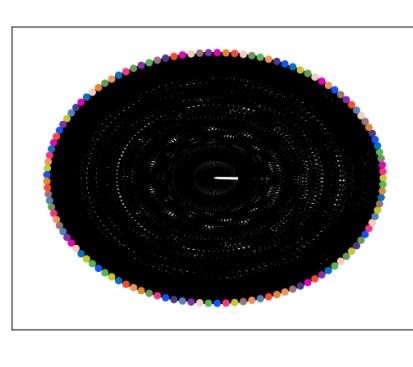
Compact Results Table

30. queen11_11

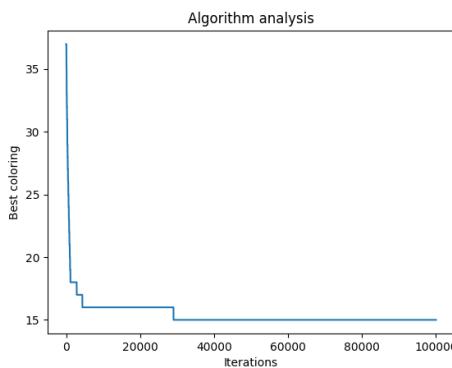
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen11_11	GA	121	3960	40	2000	11	12	15	132	41	0.196	4	1	7932
	HCFI				100000			15	87	37	0.1108	72	1	323



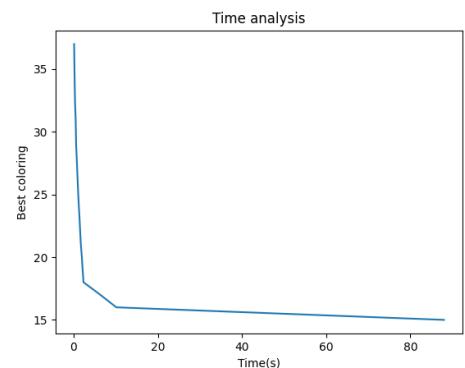
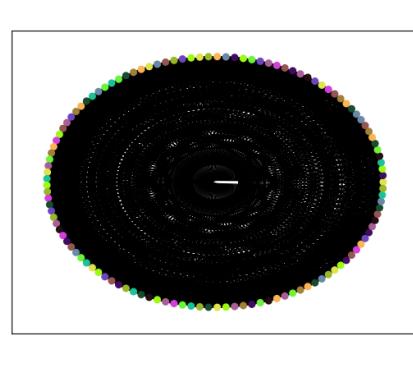
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



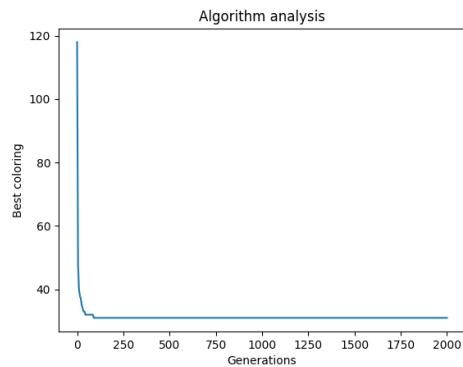
(c) HCFI-Time analysis

Results

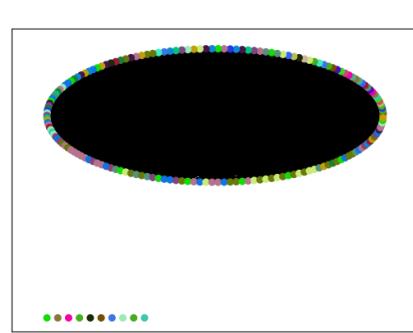
Compact Results Table

31. mulsol.i.3

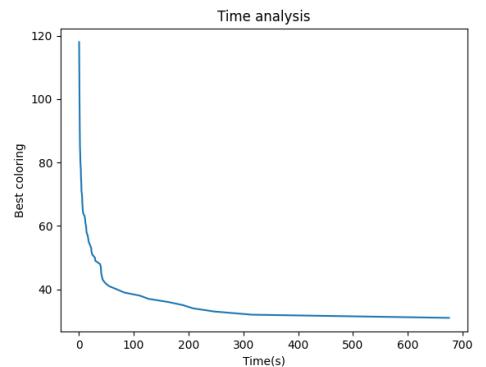
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mulsol.i.3	GA	184	3916	157	2000	31	31	31	675	118	0.424	6	1	15627
	HCFI				1000000				10295	118	0.194	76	1	15228



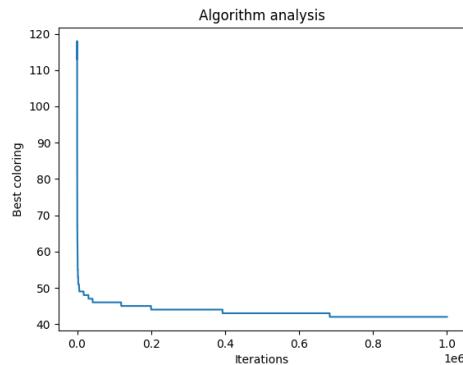
(a) GA-Algorithm analysis



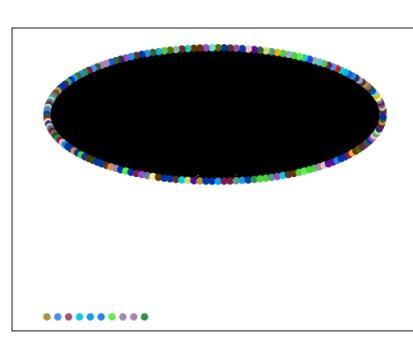
(b) GA-Graph coloring



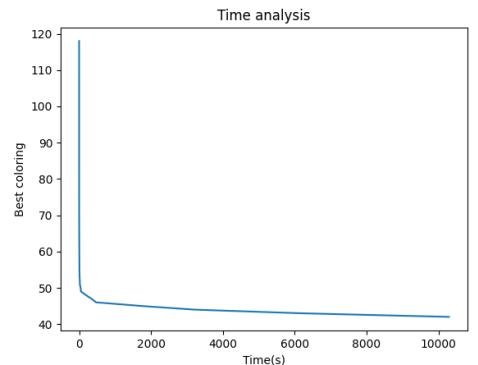
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



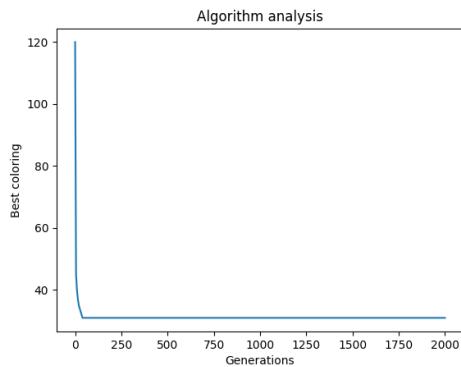
(c) HCFI-Time analysis

Results

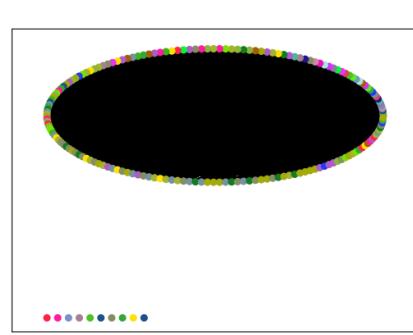
Compact Results Table

32. mulsol.i.4

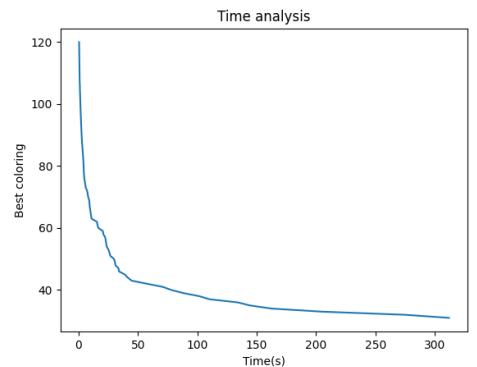
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mulsol.i.4	GA	185	3946	158	2000	31	31	31	311	120	0.452	6	1	16394
	HCFI				1000000				5955	120	0.122	24	1	14507



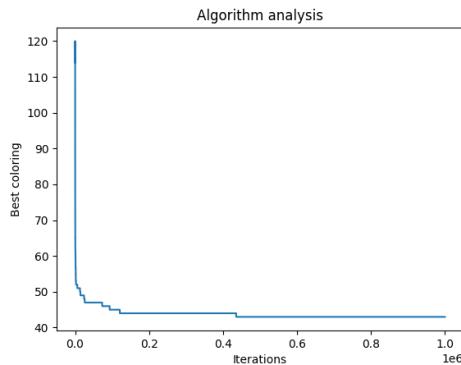
(a) GA-Algorithm analysis



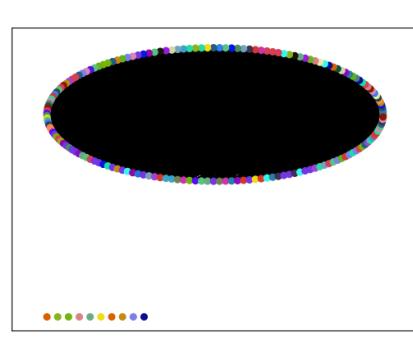
(b) GA-Graph coloring



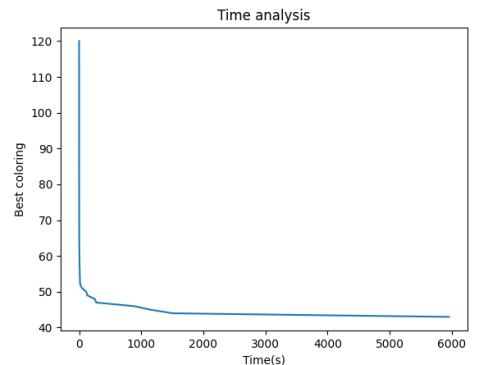
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



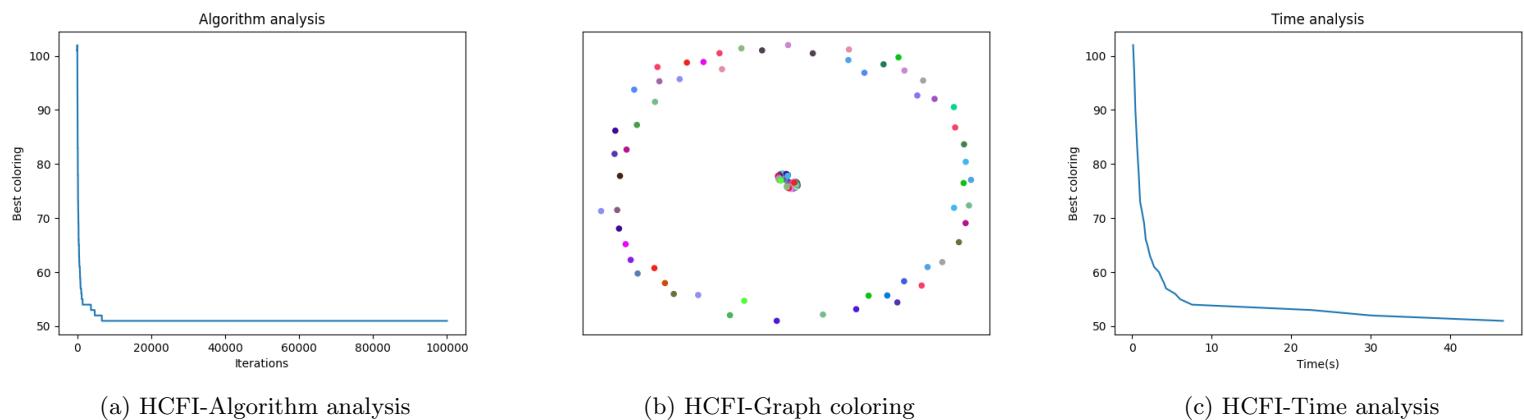
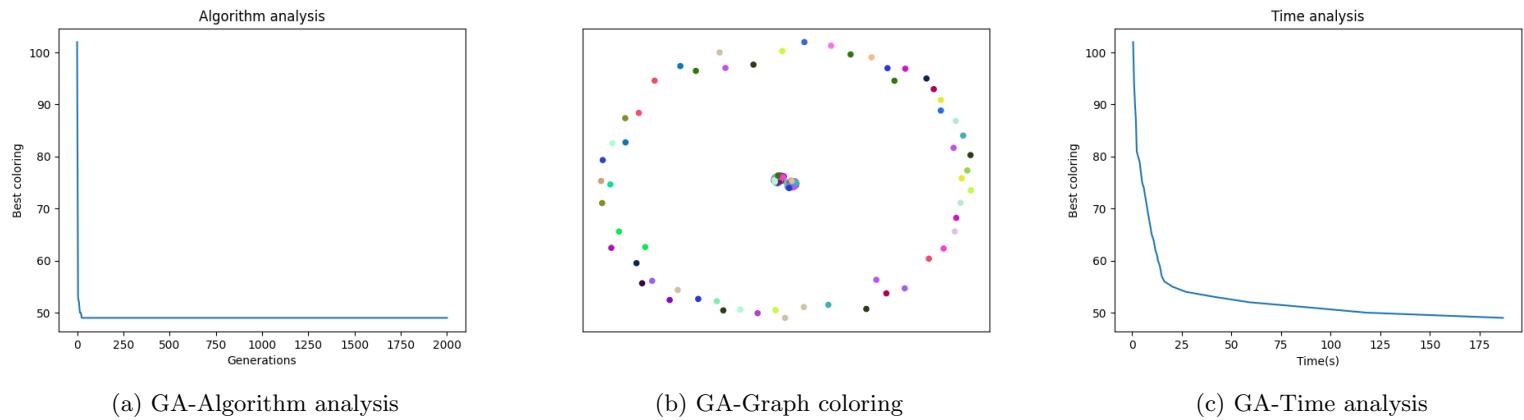
(c) HCFI-Time analysis

Results

Compact Results Table

33. mulsol.i.1

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mulsol.i.1	GA	197	3925	121	2000	49	49	49	186	102	0.5150	6	1	17597
	HCFI				100000		51		46	102	0.156	142	1	891

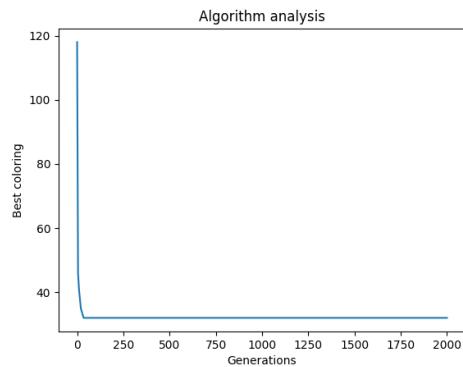


Results

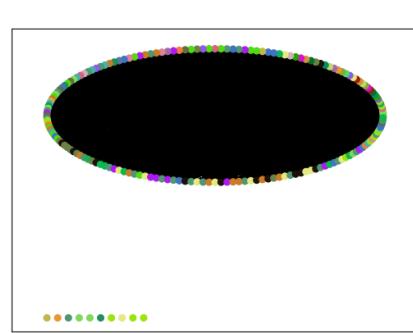
Compact Results Table

34. mulsol.i.5

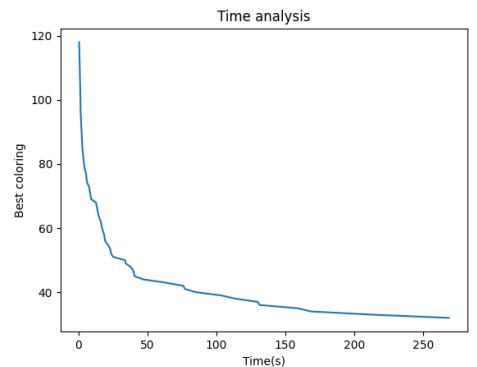
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
mulsol.i.5	GA	186	3973	159	2000	31	31	32 44	268	118	0.4584	6	1	0.4584
	HCFI				1000000				9701	116	0.07	70	1	13201



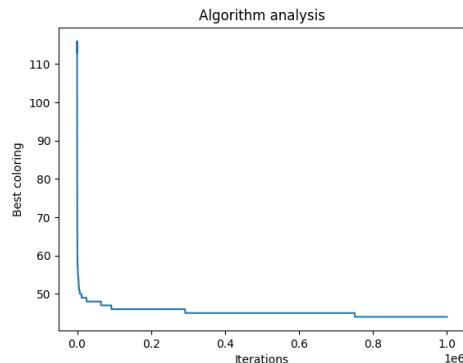
(a) GA-Algorithm analysis



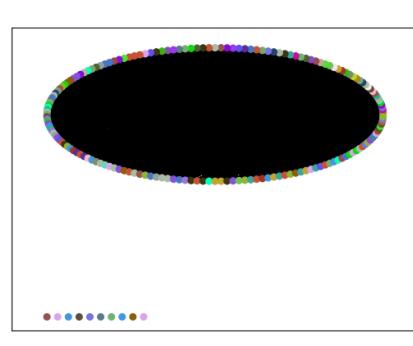
(b) GA-Graph coloring



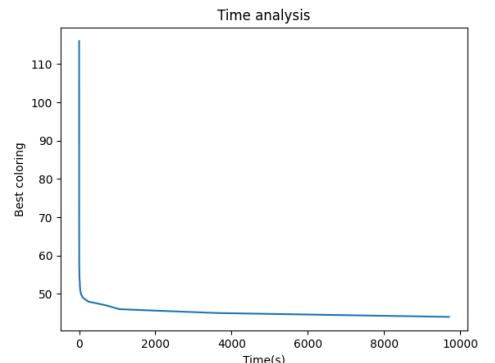
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



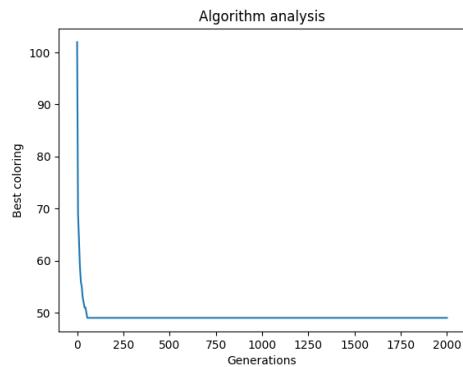
(c) HCFI-Time analysis

Results

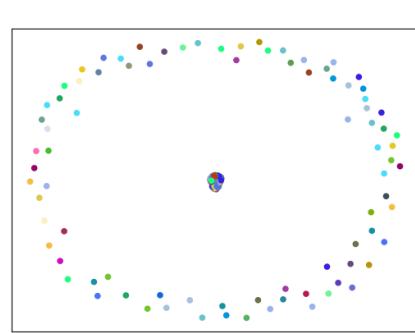
Compact Results Table

35. zeroin.i.1

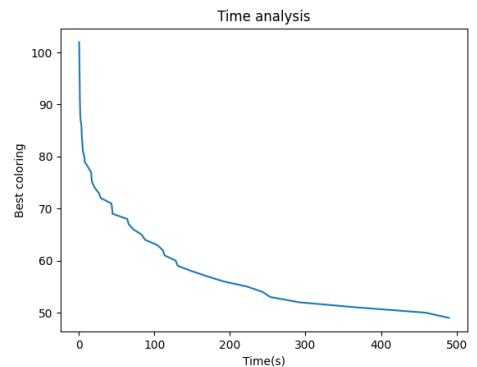
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
zeroin.i.1	GA	211	4100	111	2000	49	49	49	489	102	0.5678	6	1	19164
	HCFI				1000000			59	8055	59	0.1941	62	1	12071



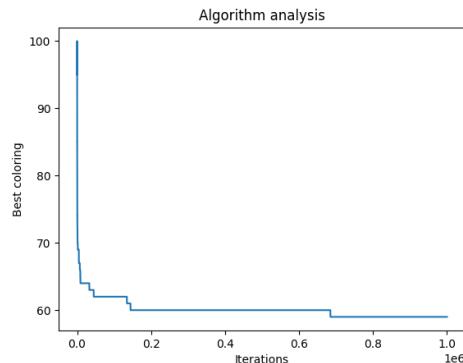
(a) GA-Algorithm analysis



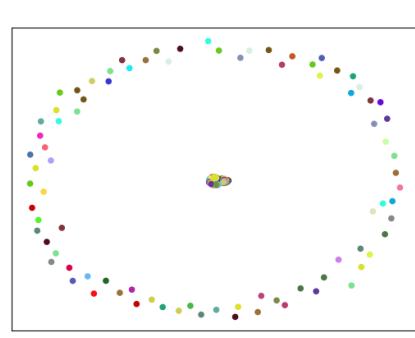
(b) GA-Graph coloring



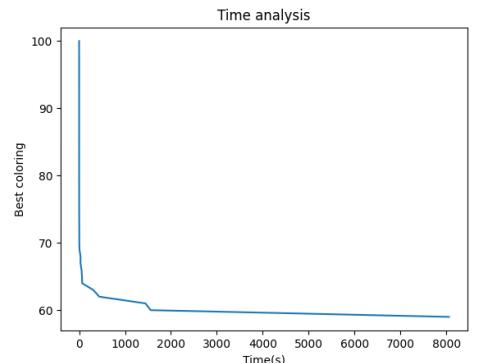
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



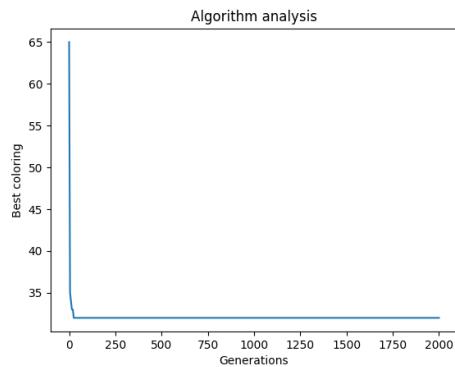
(c) HCFI-Time analysis

Results

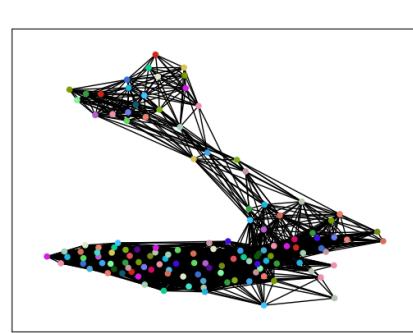
Compact Results Table

36. miles750

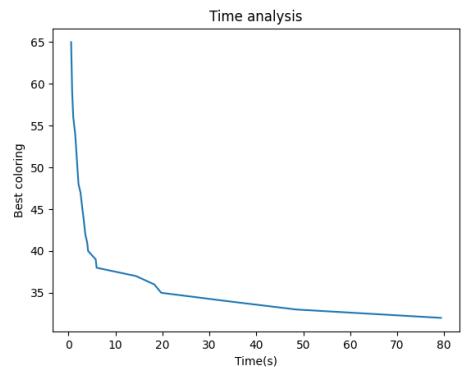
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles750	GA	128	4226	64	2000	31	31	32	79	65	0.5475	6	1	7831
	HCFI				100000			32	48	61	0.08	118	1	303



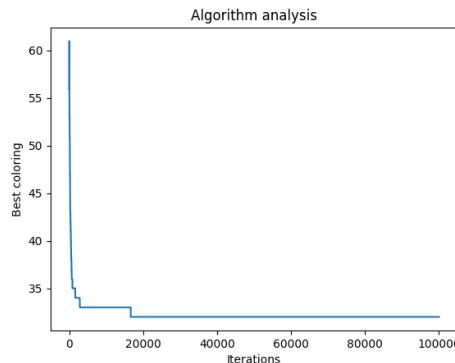
(a) GA-Algorithm analysis



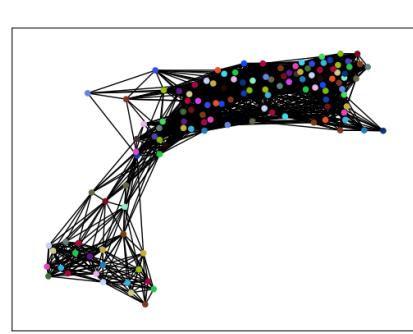
(b) GA-Graph coloring



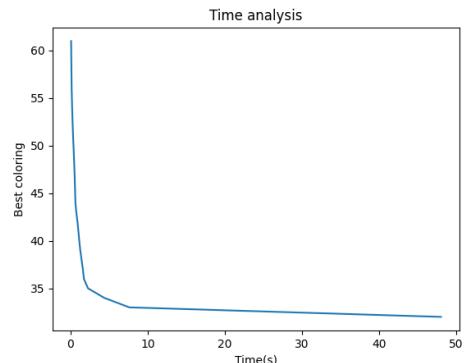
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



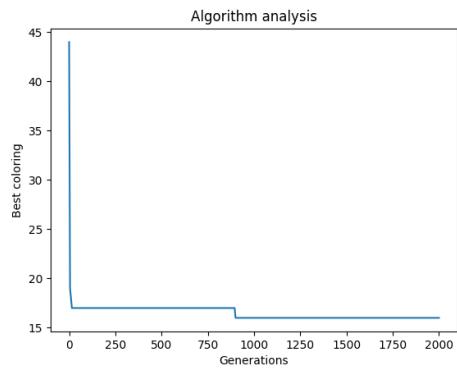
(c) HCFI-Time analysis

Results

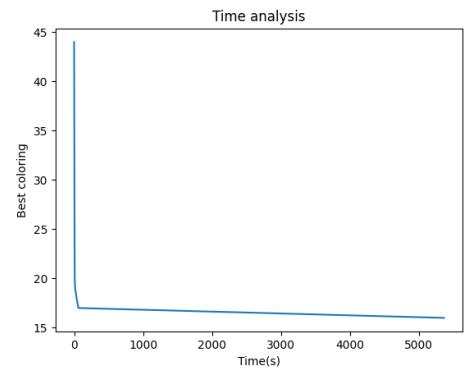
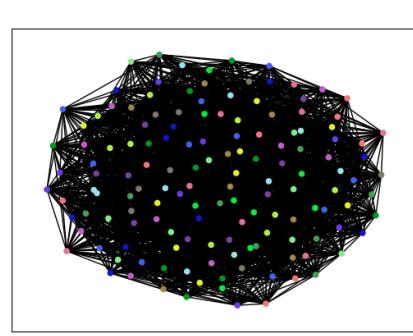
Compact Results Table

37. queen12_12

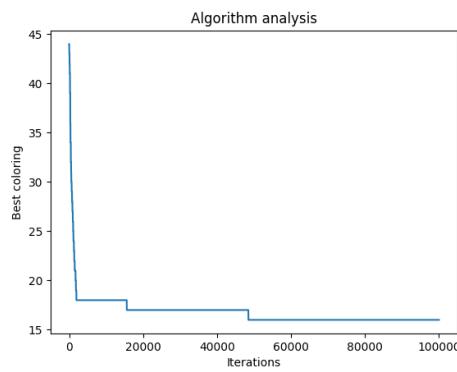
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen12_12	GA	144	5192	43	2000	12	12	16	5364	44	0.2	8	1	12650
	HCFI				100000			16	192	43	0.1301	198	1	436



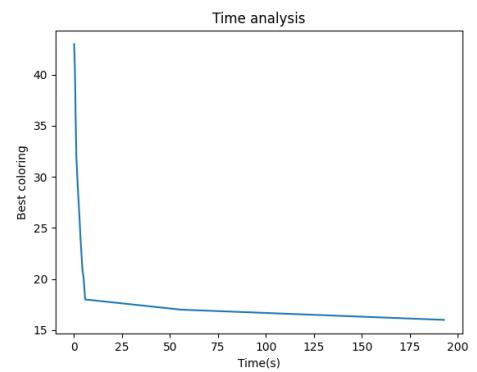
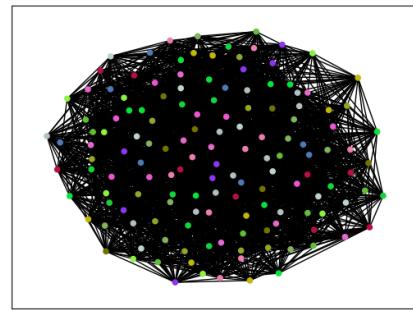
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



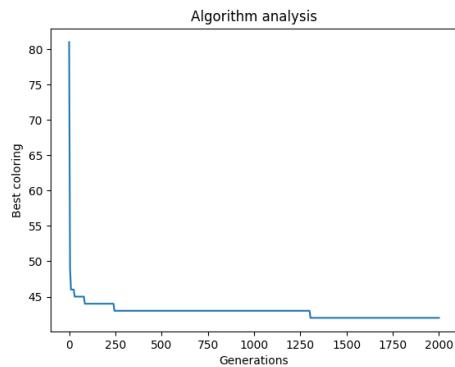
(c) HCFI-Time analysis

Results

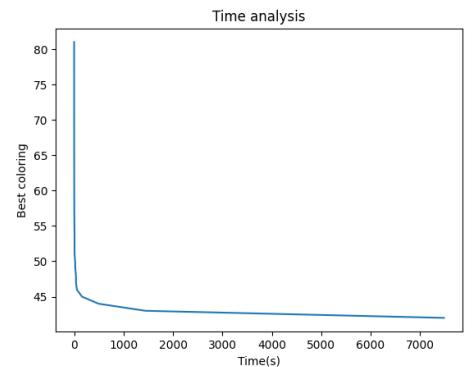
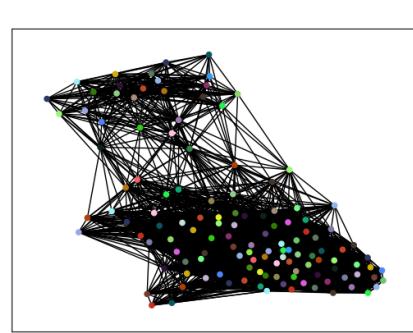
Compact Results Table

38. miles1000

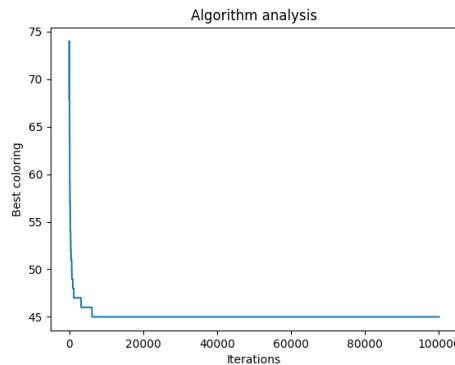
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles1000	GA	128	6432	86	2000	42	42	42	7485	81	0.25	6	1	11416
	HCFI				100000		45	45	18.663	74	0.1109	108	1	356



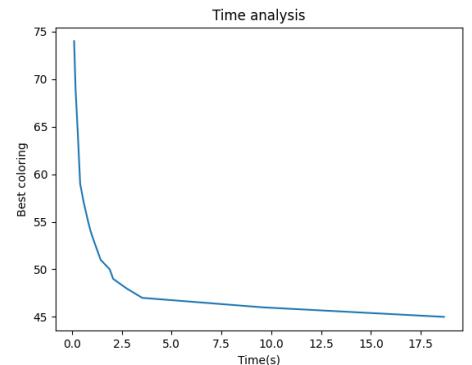
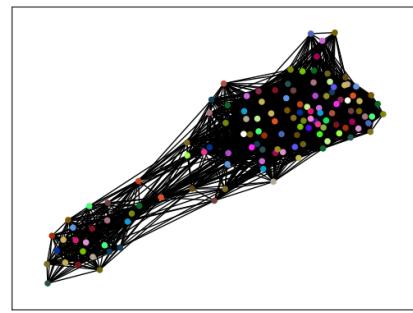
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



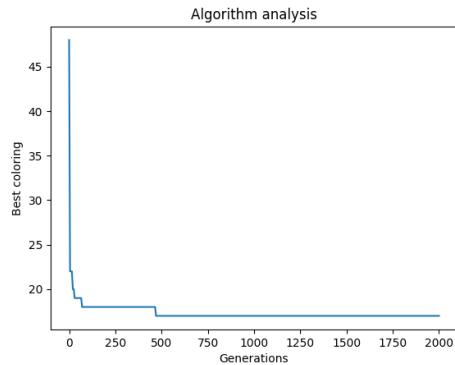
(c) HCFI-Time analysis

Results

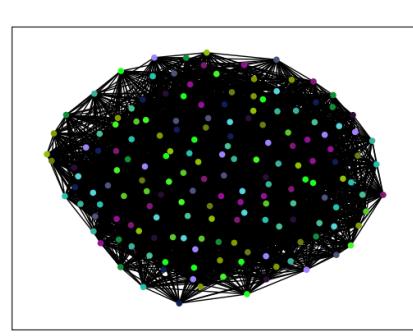
Compact Results Table

39. queen13_13

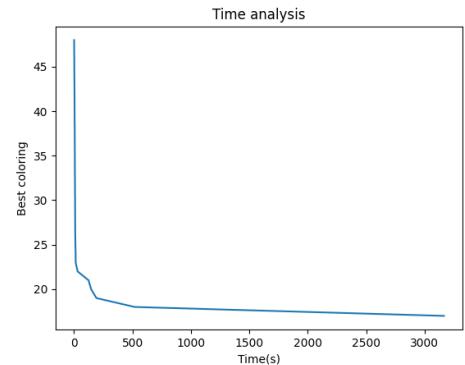
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen13_13	GA	169	6656	48	2000	13	14	17	3166	48	0.325	6	1	
	HCFI				100000			18	86	48	0.346	324	1	580



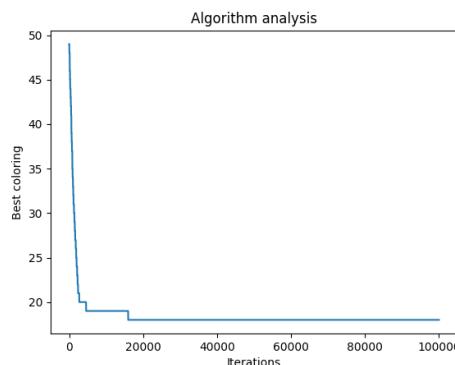
(a) GA-Algorithm analysis



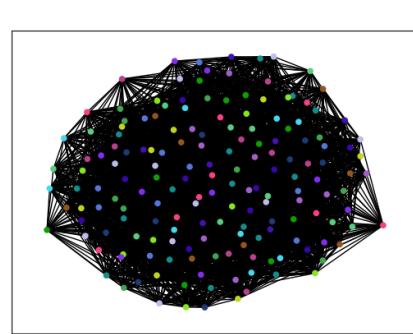
(b) GA-Graph coloring



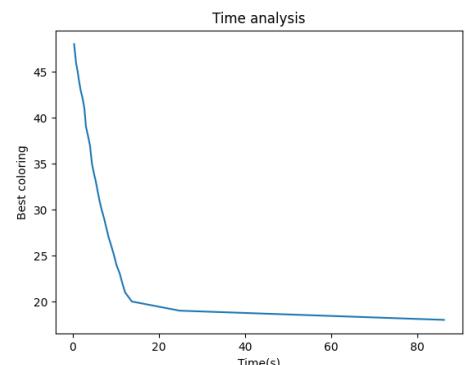
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



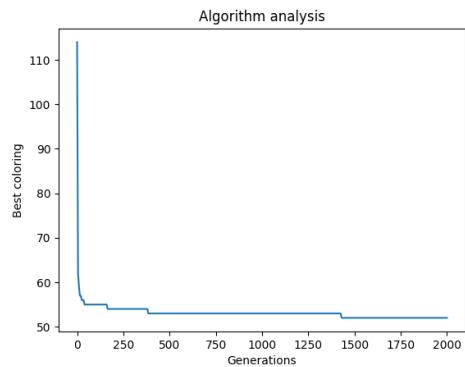
(c) HCFI-Time analysis

Results

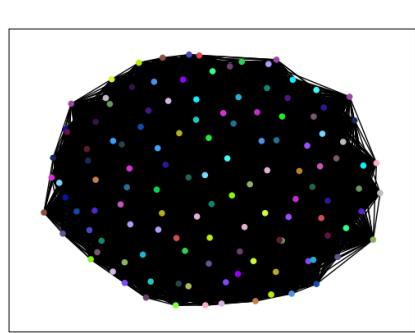
Compact Results Table

40. DSJC125.9

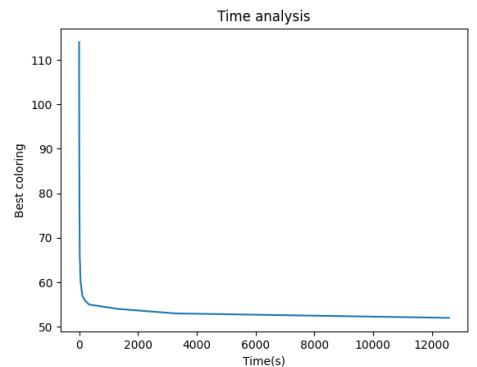
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC125.9	GA HCFI	125	6961	120	2000 100000	30	42	52 53	12584	114	0.2346	6	1	17924 742



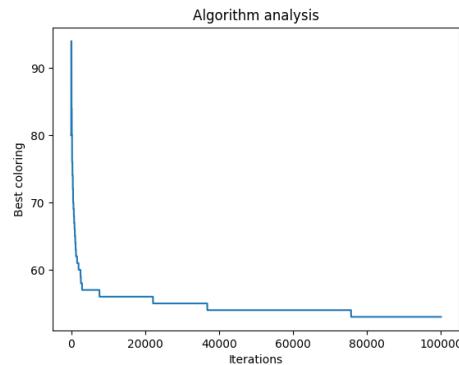
(a) GA-Algorithm analysis



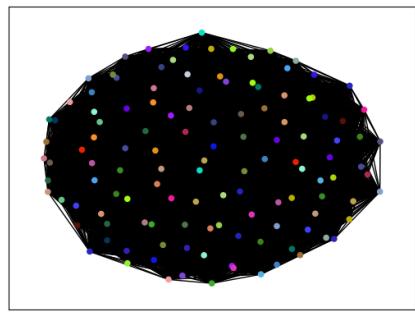
(b) GA-Graph coloring



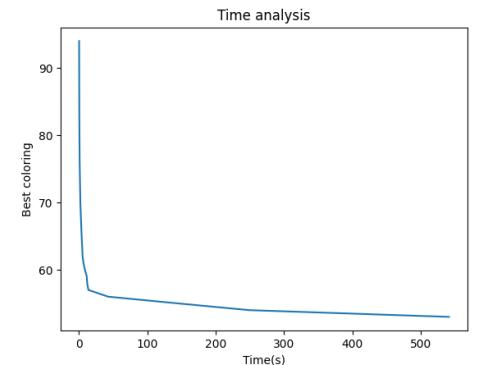
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



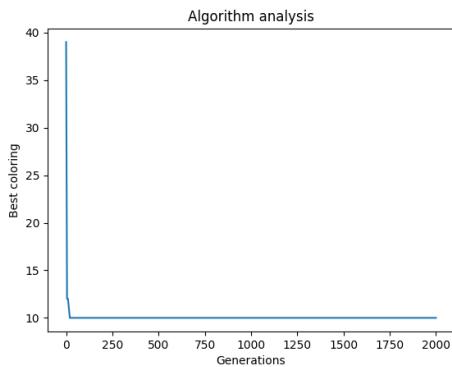
(c) HCFI-Time analysis

Results

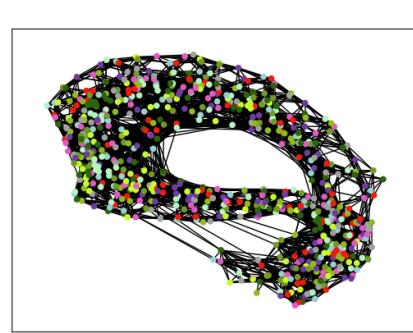
Compact Results Table

41. will199GPIA

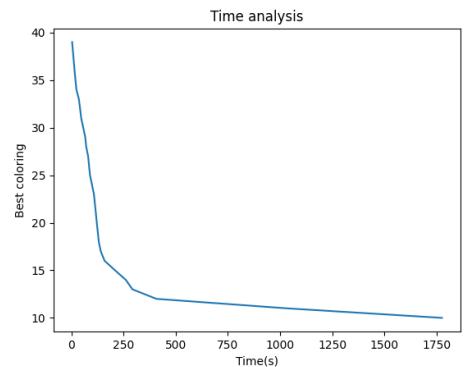
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
will199GPIA	GA	701	7065	38	2000	7	7	10	1774	39	4.0332	12	1	194357
	HCFI				100000		10		2072	39	11.359	290	1	7137



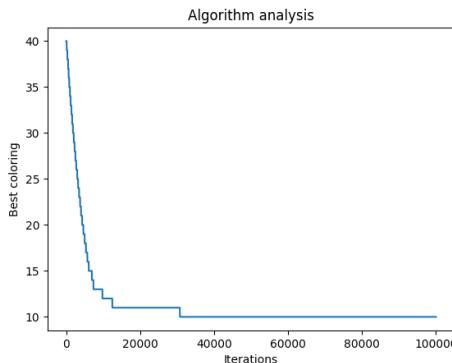
(a) GA-Algorithm analysis



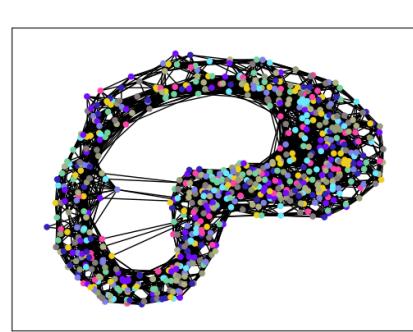
(b) GA-Graph coloring



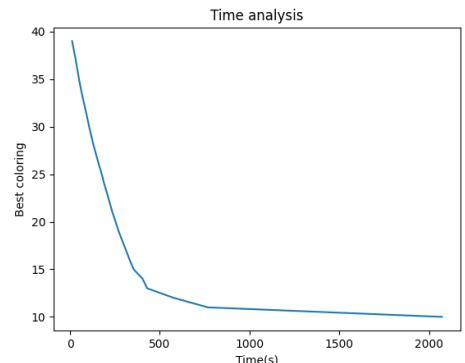
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



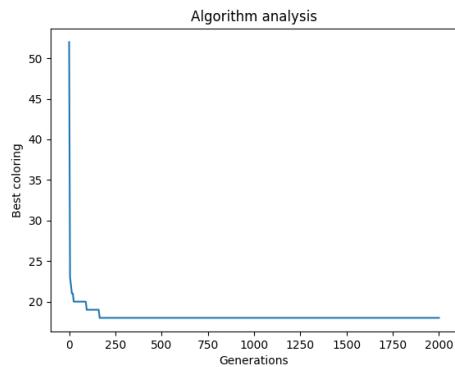
(c) HCFI-Time analysis

Results

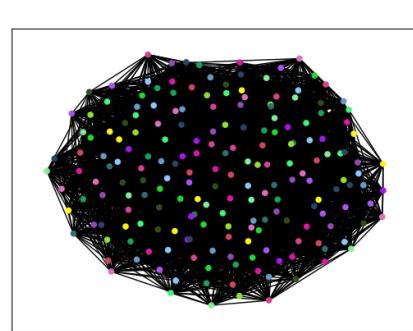
Compact Results Table

42. queen14_14

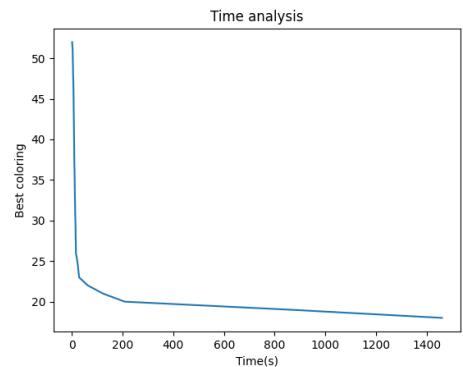
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen14_14	GA	196	8372	51	2000	14	14	18	1460	52	0.806	16	1	15187
	HCFI				100000			19		63	51	0.46	262	1



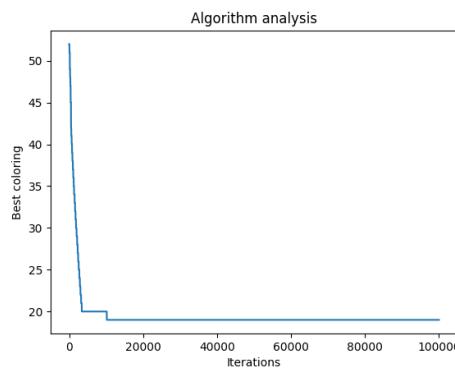
(a) GA-Algorithm analysis



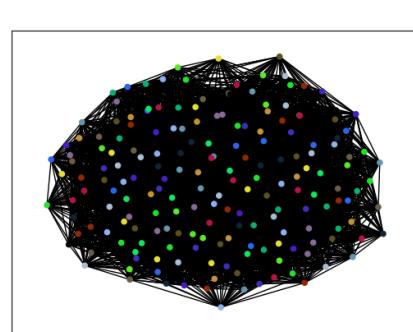
(b) GA-Graph coloring



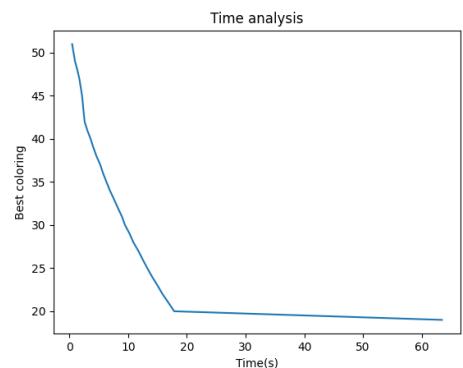
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



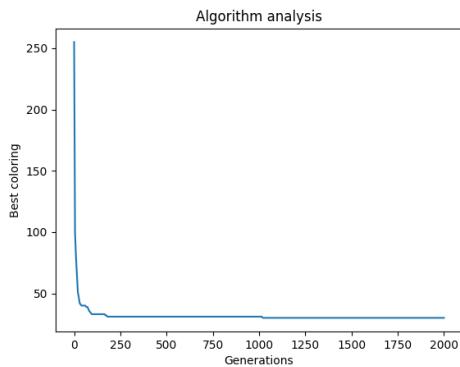
(c) HCFI-Time analysis

Results

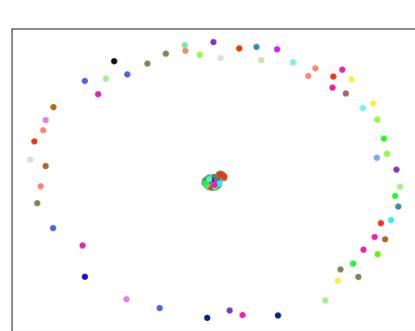
Compact Results Table

43. fpsol2.i.3

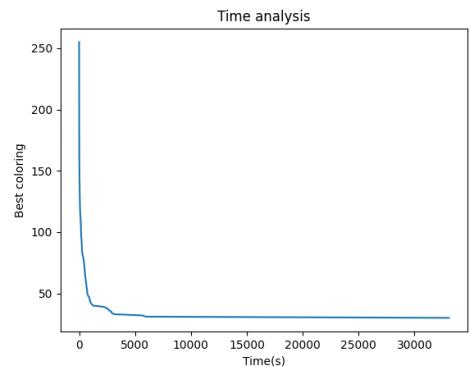
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
fpsol2.i.3	GA	425	8688	346	2000	30	30	30	33083	255	1.502	6	1	64880
	HCFI				1000000		81		54104	257	0.3717	126	1	54630



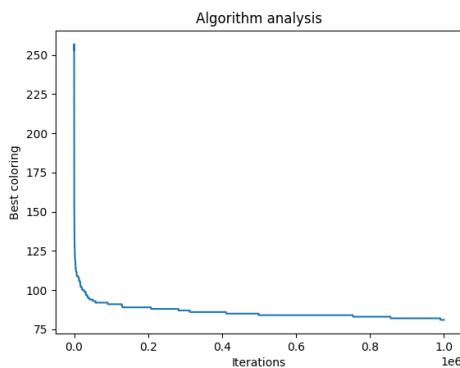
(a) GA-Algorithm analysis



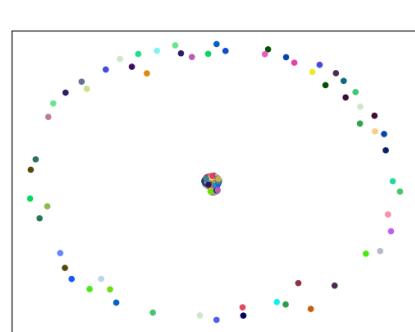
(b) GA-Graph coloring



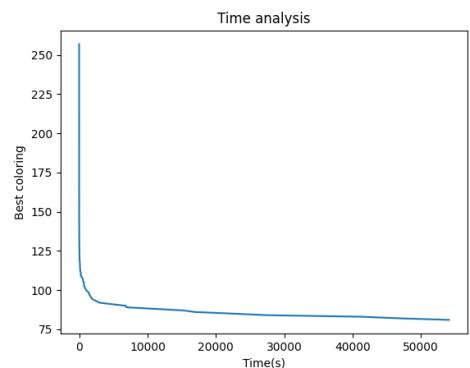
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



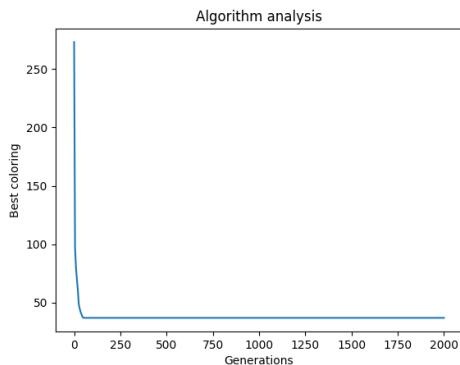
(c) HCFI-Time analysis

Results

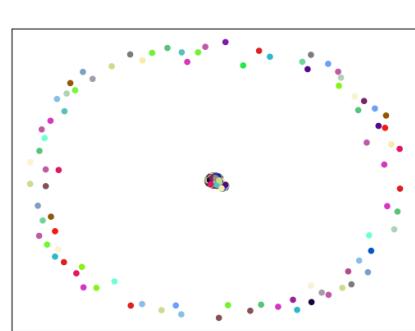
Compact Results Table

44. fpsol2.i.2

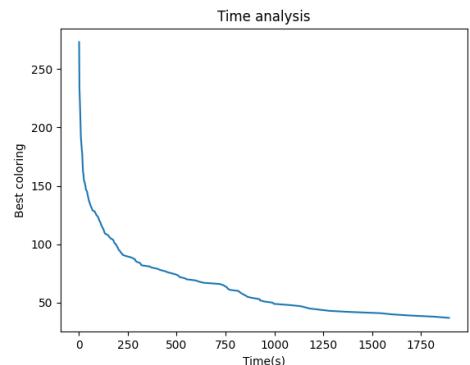
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
fpsol2.i.2	GA	451	8691	346	2000	30	30	37	1892	273	1.68	6	1	83175
	HCFI				1000000			83	26899	253	0.7610	36	1	58391



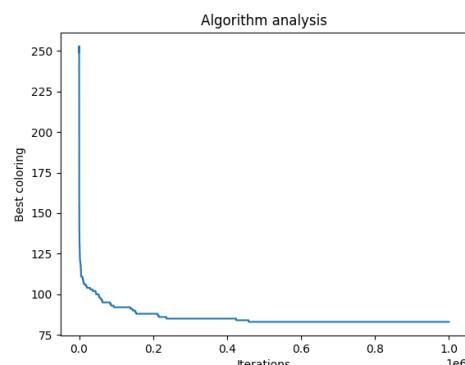
(a) GA-Algorithm analysis



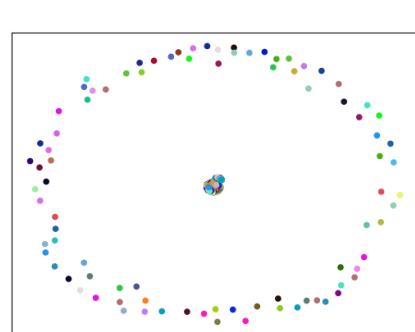
(b) GA-Graph coloring



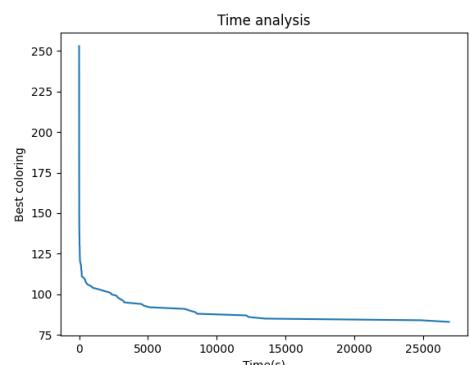
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



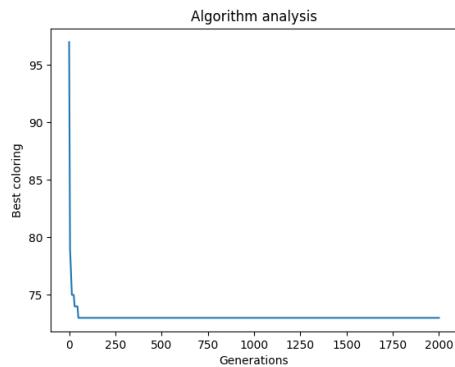
(c) HCFI-Time analysis

Results

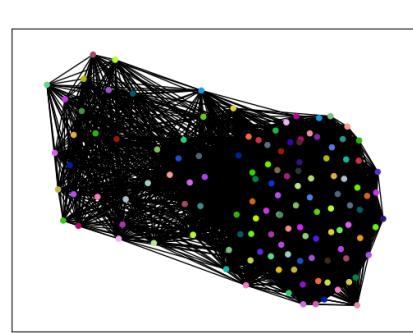
Compact Results Table

45. miles1500

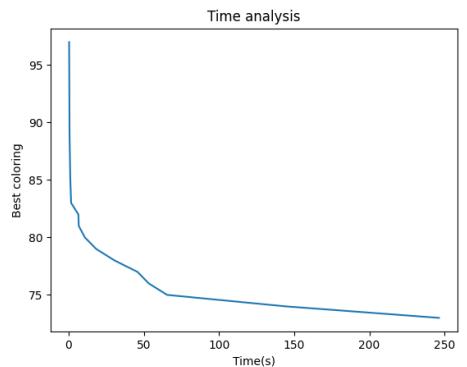
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles1500	GA	128	10396	106	2000	73	73	73	246	97	0.3689	4	1	10182
	HCFI				100000		73	74	77	91	0.2324	26	1	474



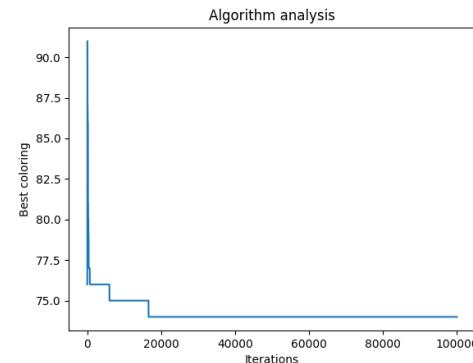
(a) GA-Algorithm analysis



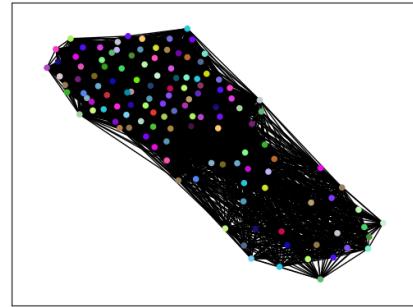
(b) GA-Graph coloring



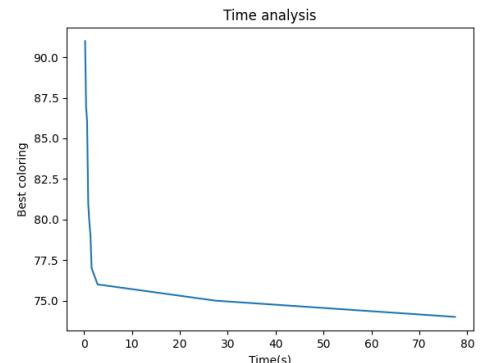
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



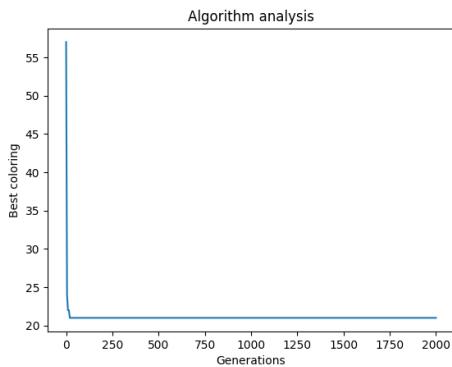
(c) HCFI-Time analysis

Results

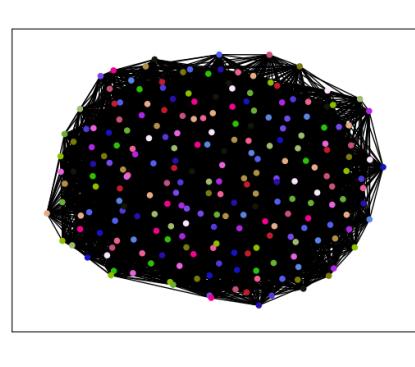
Compact Results Table

46. queen15_15

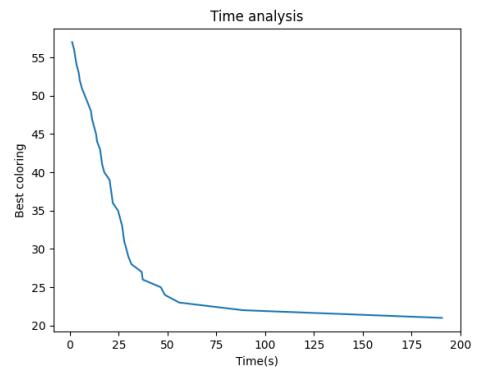
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen15_15	GA	225	10360	56	2000	15	17	21	190.581	57	1.1701	6	1	21944
	HCFI				100000		21		103	56	0.8830	512	1	1042



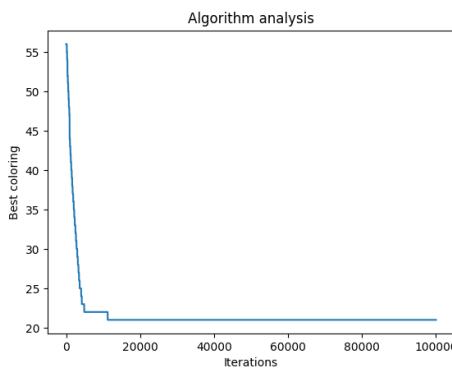
(a) GA-Algorithm analysis



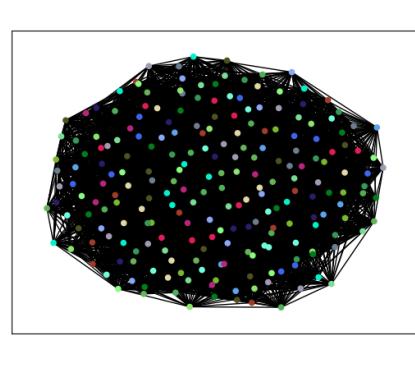
(b) GA-Graph coloring



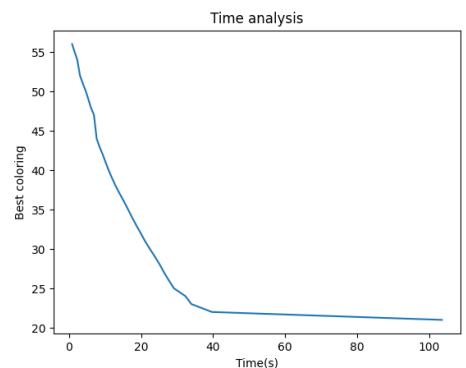
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



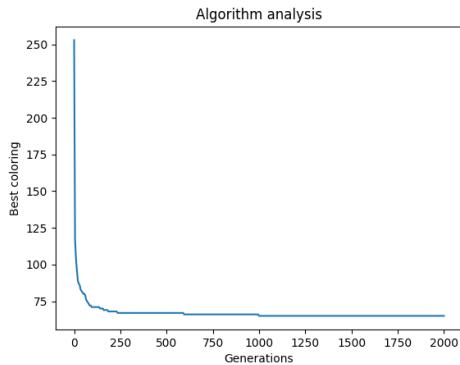
(c) HCFI-Time analysis

Results

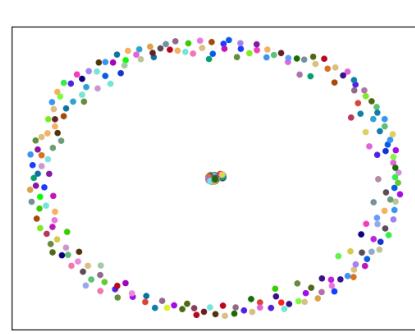
Compact Results Table

47. fpsol2.i.1

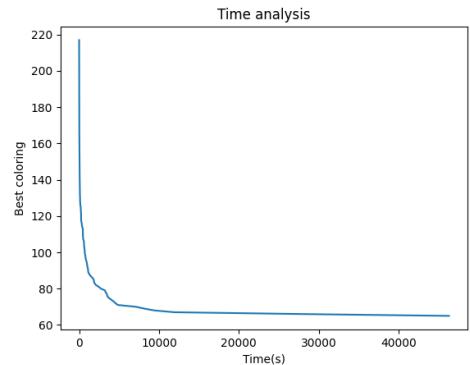
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
fpsol2.i.1	GA	496	11654	252	2000	65	65	65	46301	217	4.6228	6	1	91728
	HCFI				1000000			106	45404	106	1.4127	106	1	54307



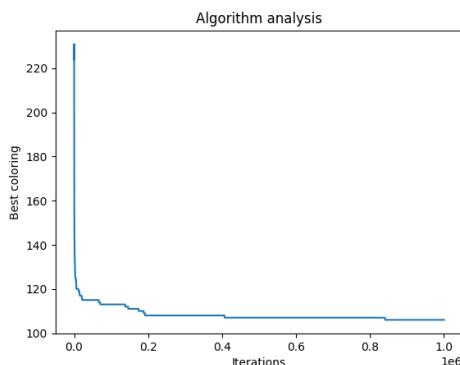
(a) GA-Algorithm analysis



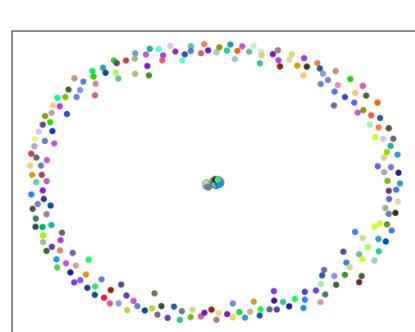
(b) GA-Graph coloring



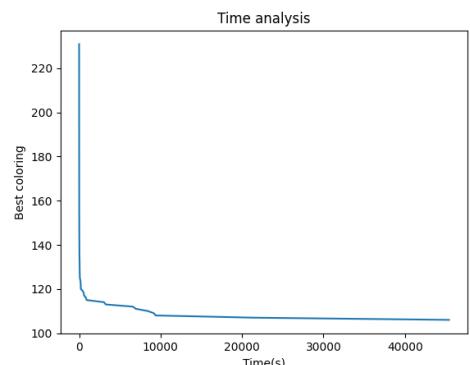
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



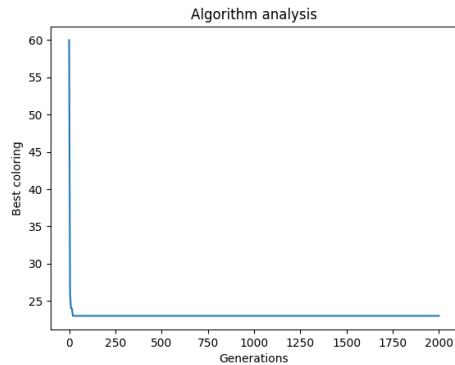
(c) HCFI-Time analysis

Results

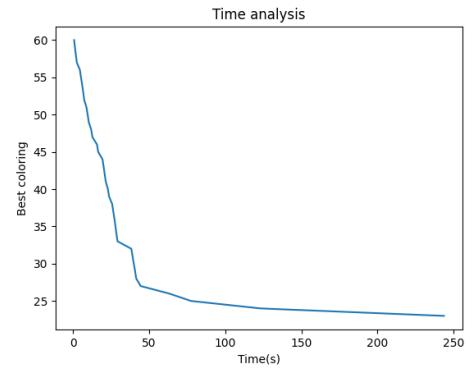
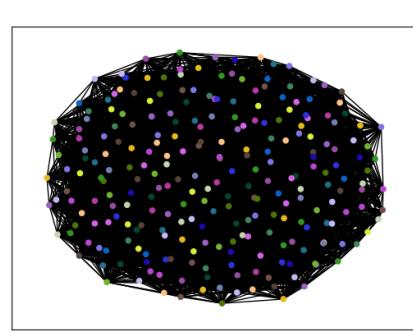
Compact Results Table

48. queen16_16

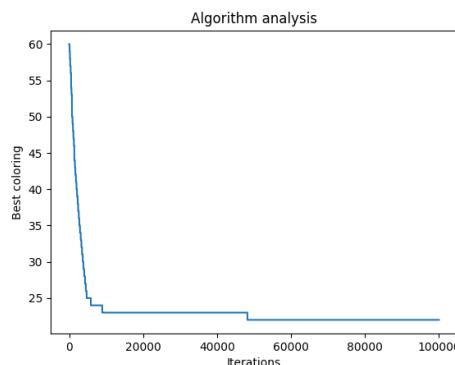
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
queen16_16	GA	256	12640	59	2000	*	18	23	243	60	0.632	6	1	29197
	HCFI				100000		22	22	574	59	1.1198	176	1	1296



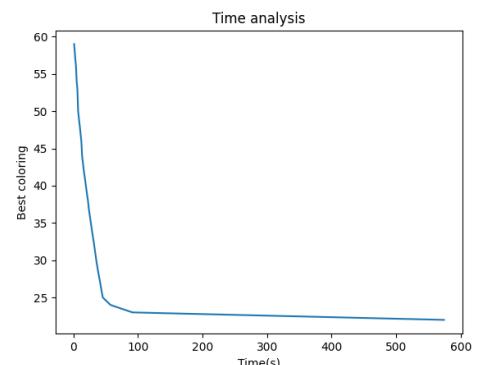
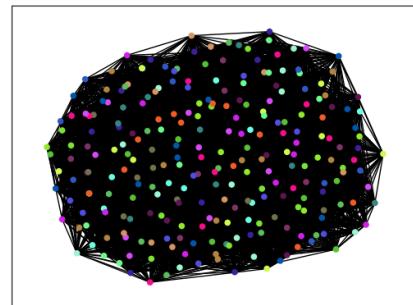
(a) GA-Algorithm analysis



(c) GA-Time analysis



(a) HCFI-Algorithm analysis



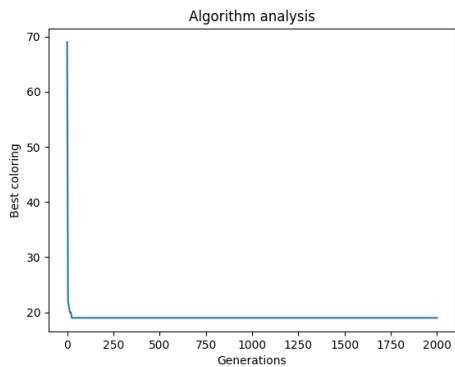
(c) HCFI-Time analysis

Results

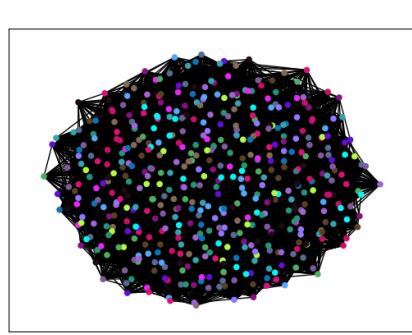
Compact Results Table

49. DSJC500.1

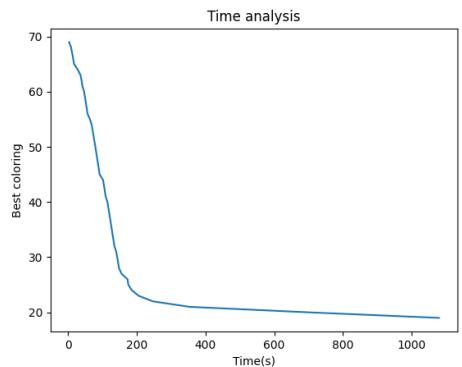
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC500.1	GA	500	12458	68	2000	*	12	19	1080	69	2.6428	8	1	105518
	HCFI				100000		20	20	866	69	5.199	1230	1	4267



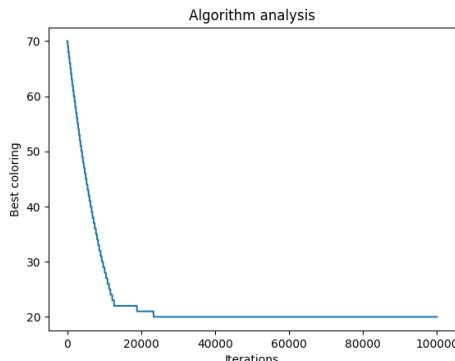
(a) GA-Algorithm analysis



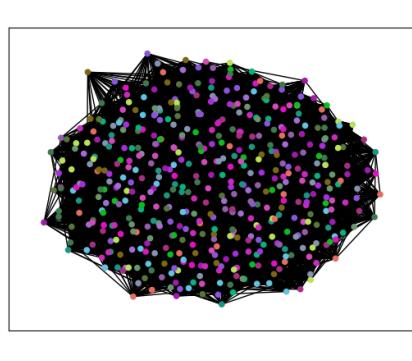
(b) GA-Graph coloring



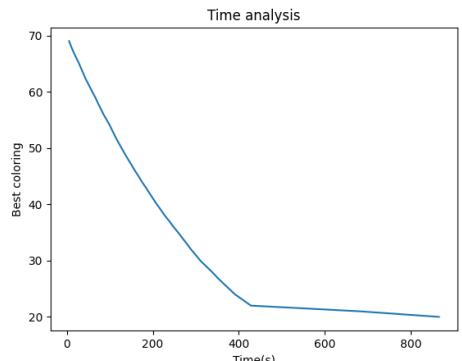
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



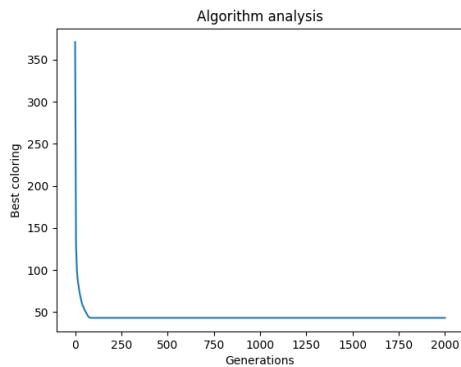
(c) HCFI-Time analysis

Results

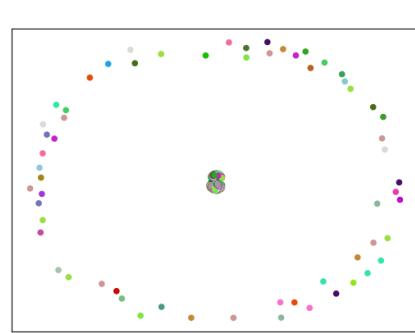
Compact Results Table

50. inithx.i.3

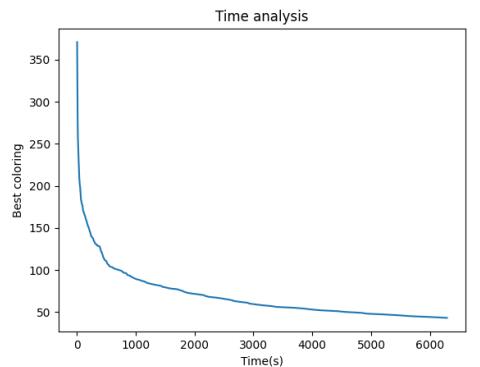
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
inithx.i.3	GA	621	13969	542	2000	31	31	43	6288	371	5.2135	6	1	153878
	HCFI				1000000				77721	382	1.406	282	1	124143



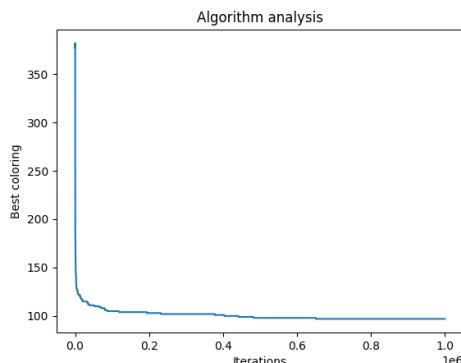
(a) GA-Algorithm analysis



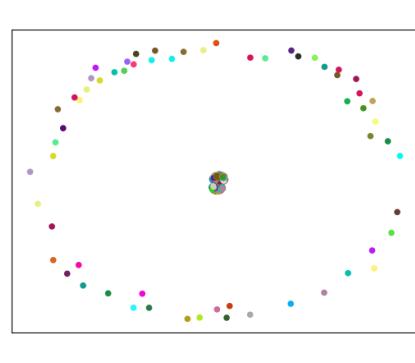
(b) GA-Graph coloring



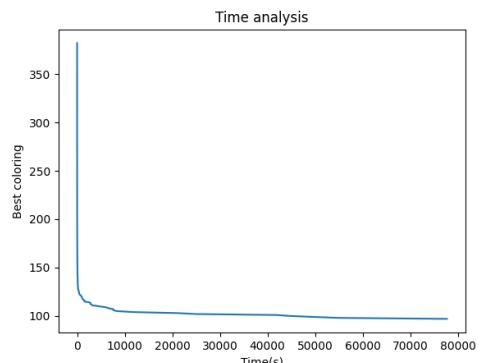
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



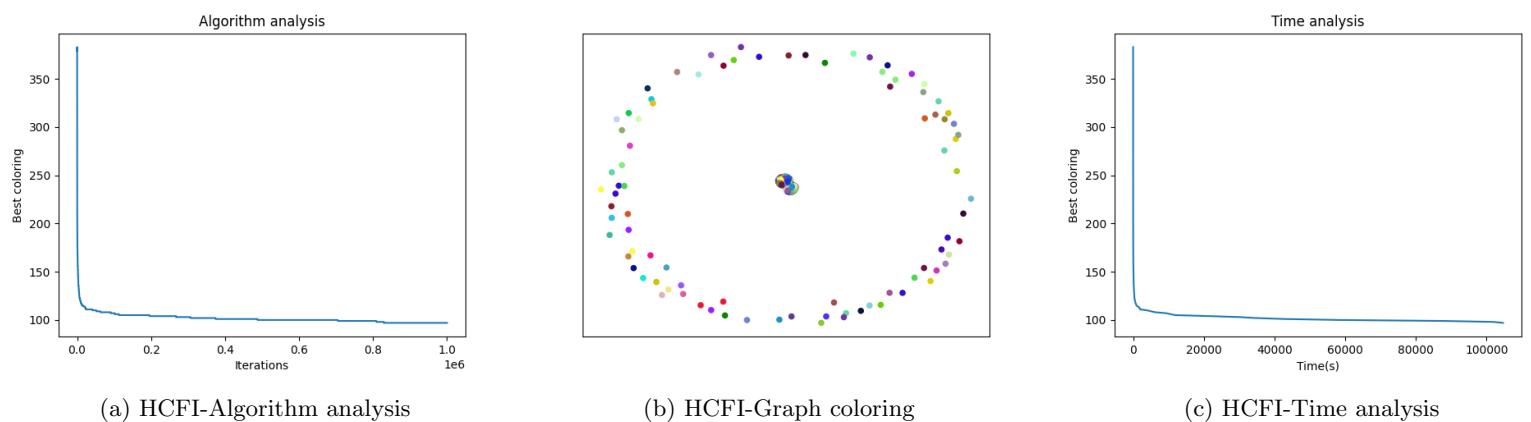
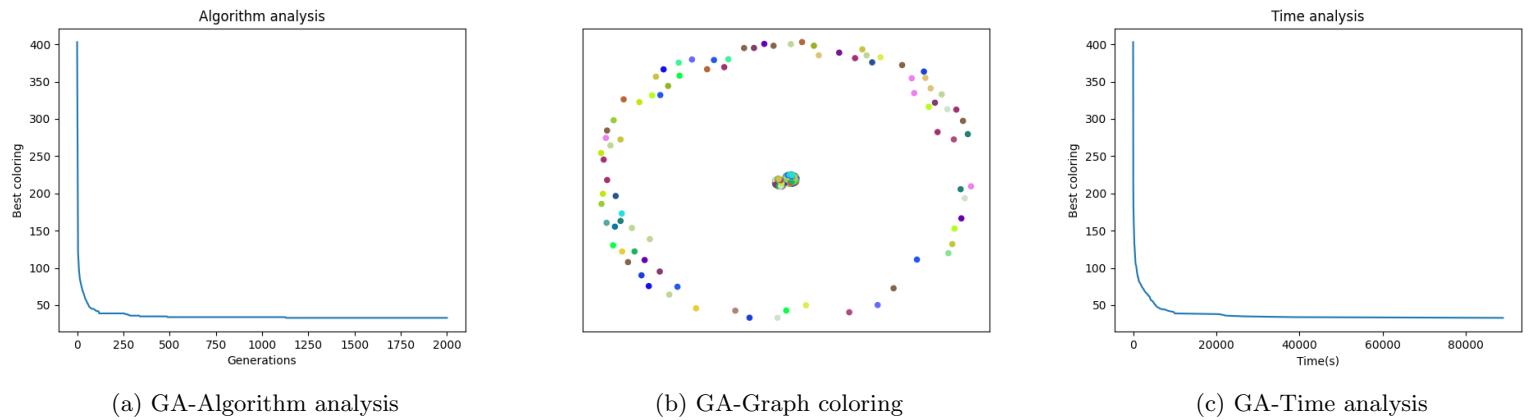
(c) HCFI-Time analysis

Results

Compact Results Table

51. inithx.i.2

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
inithx.i.2	GA	645	13979	541	2000	31	31	33 97	88885	403	2.6341	6	1	153683
	HCFI				1000000									

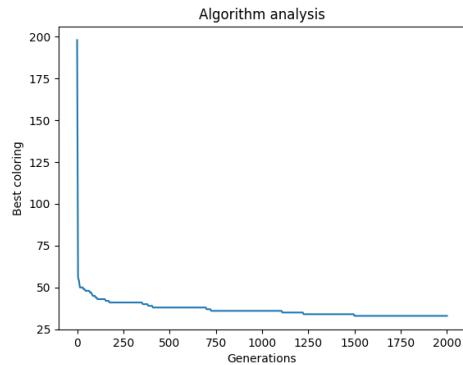


Results

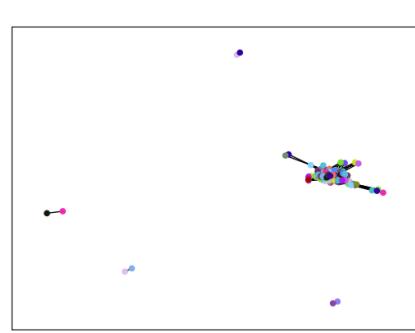
Compact Results Table

52. school1_nsh

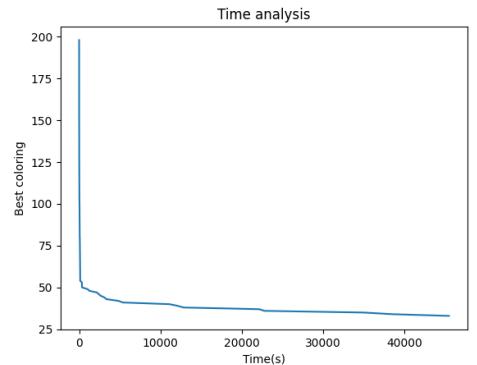
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
school1_nsh	GA HCFI	352 1000000	14612 14	232 14	2000 33		45452 18276	198 186	1.202 0.8792	6 572	1 1	58262 28564		



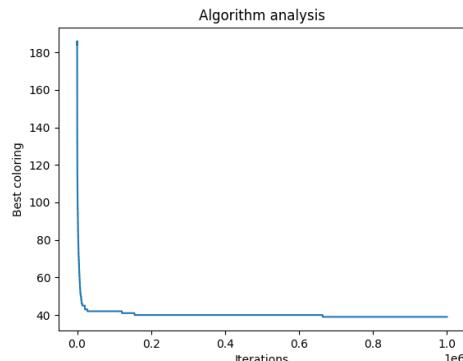
(a) GA-Algorithm analysis



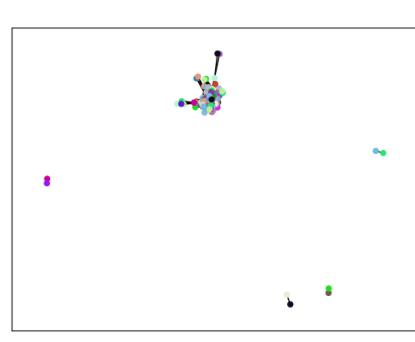
(b) GA-Graph coloring



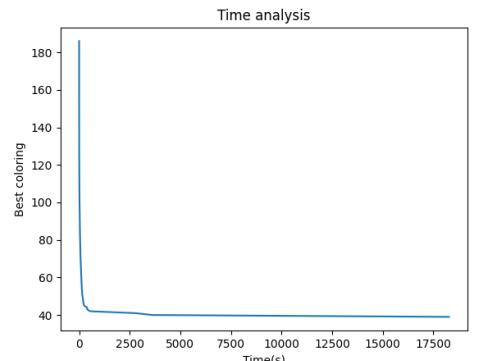
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



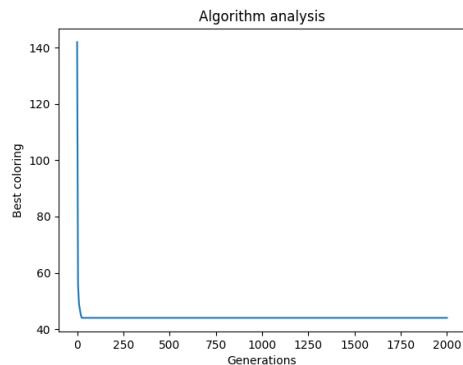
(c) HCFI-Time analysis

Results

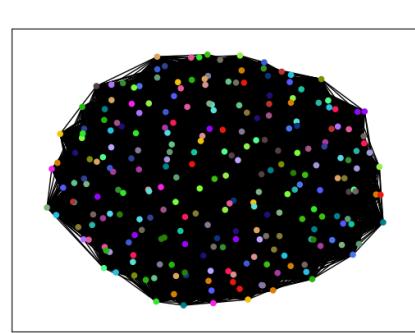
Compact Results Table

53. DSJC250.5

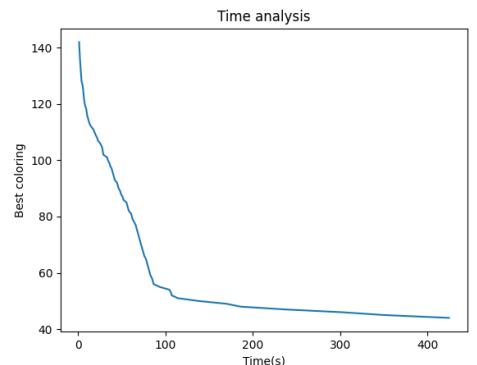
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC250.5	GA	250	15668	147	2000	*	28	44	424	142	1.053	6	1	35769
	HCFI				1000000		40	40	14399	127	1	612	1	23909



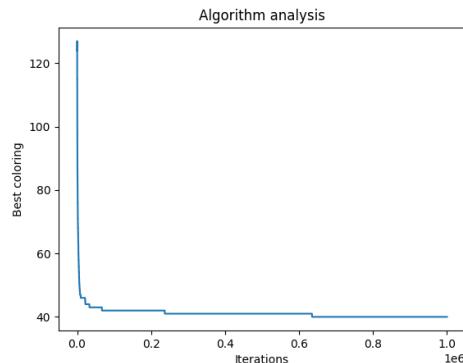
(a) GA-Algorithm analysis



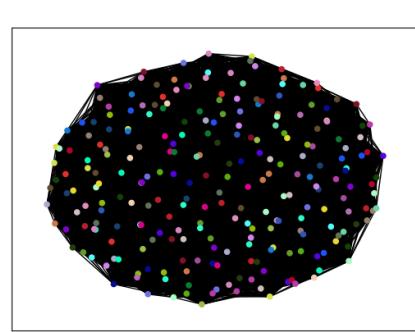
(b) GA-Graph coloring



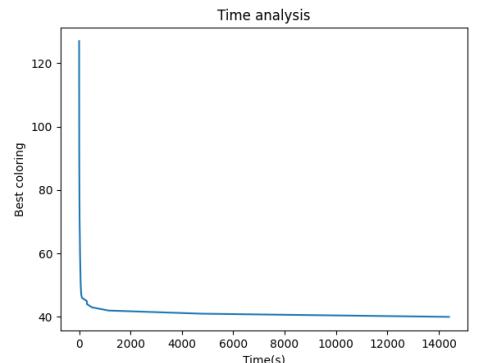
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



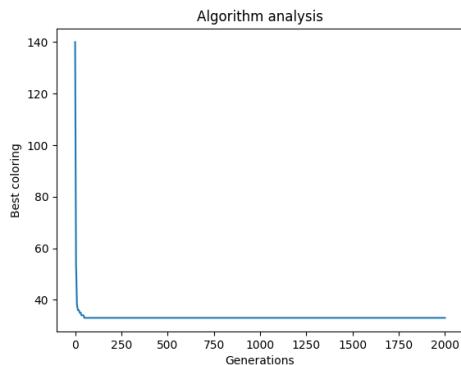
(c) HCFI-Time analysis

Results

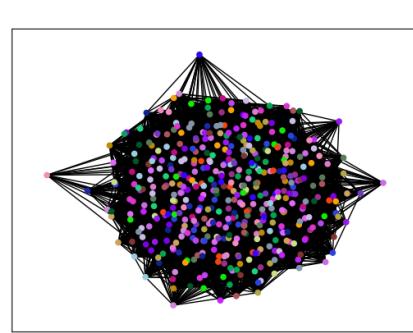
Compact Results Table

54. le450_15c

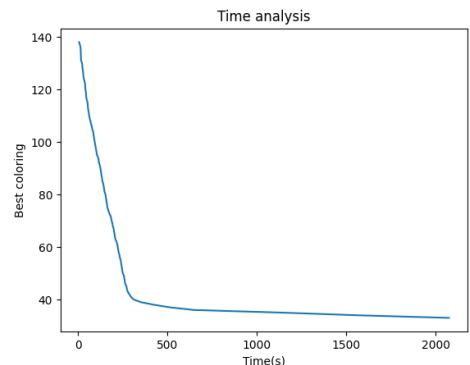
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
le450_15c	GA HCFI	450	16680	139	2000 1000000	15	15	33 31	2075 6595	138 137	5.5545 3	6 110	1 1	90493 41576



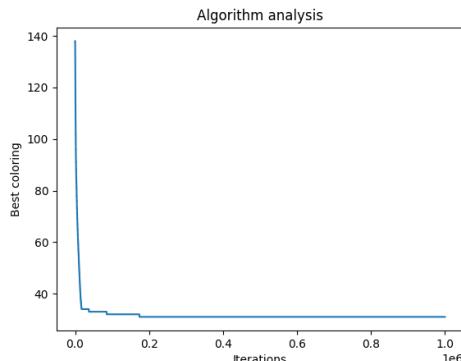
(a) GA-Algorithm analysis



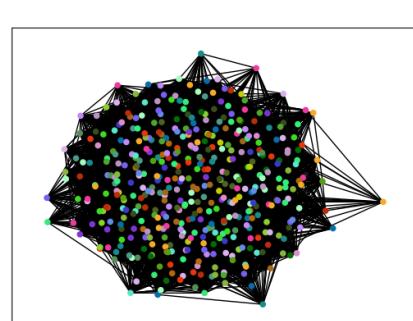
(b) GA-Graph coloring



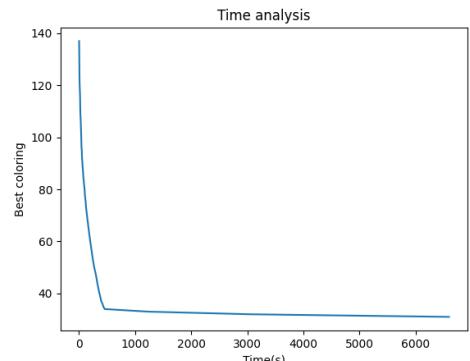
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



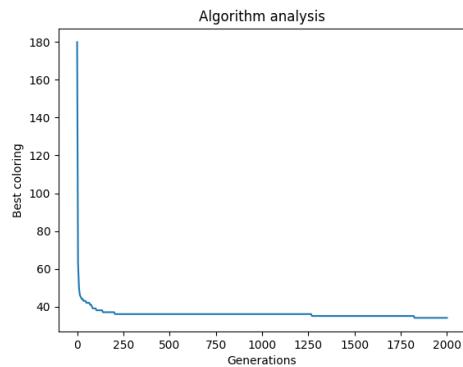
(c) HCFI-Time analysis

Results

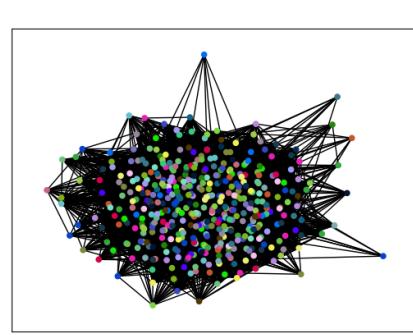
Compact Results Table

55. le450_25c

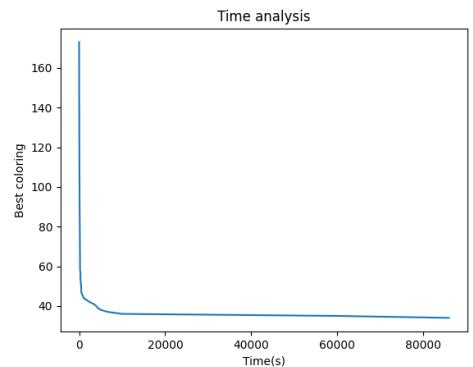
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
le450_15c	GA	450	16680	139	2000	15	15	33	2075	138	5.5545	6	1	90493
	HCFI				1000000		31		6595	137	3	110	1	41576



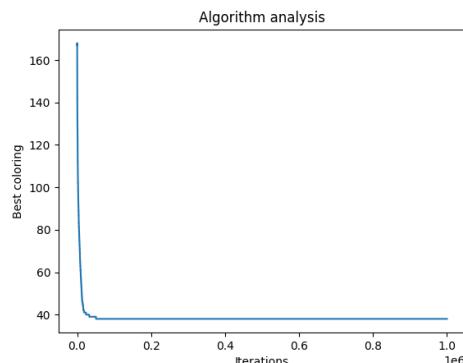
(a) GA-Algorithm analysis



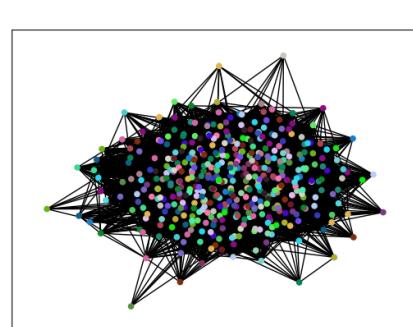
(b) GA-Graph coloring



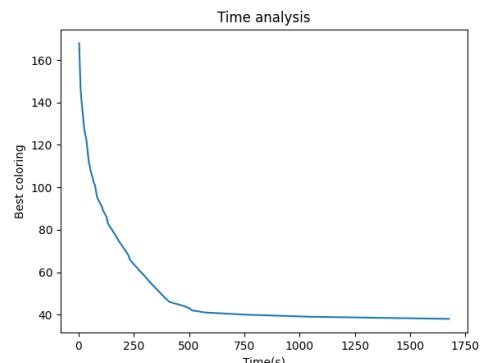
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



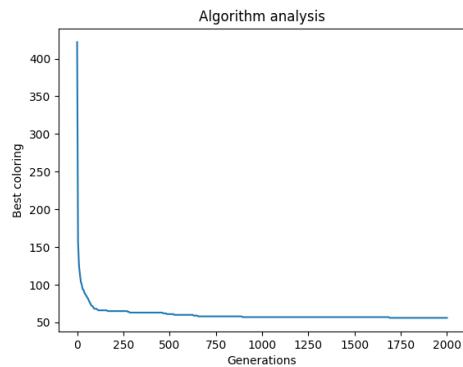
(c) HCFI-Time analysis

Results

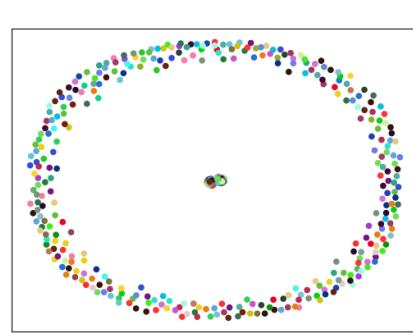
Compact Results Table

56. inithx.i.1

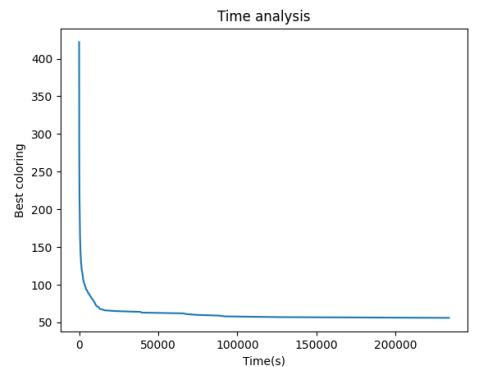
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
inithx.i.1	GA	864	18707	502	2000	54	54	56	233691	422	5.381	6	1	277190
	HCFI				1000000		117		146689	412	3	44	1	171859



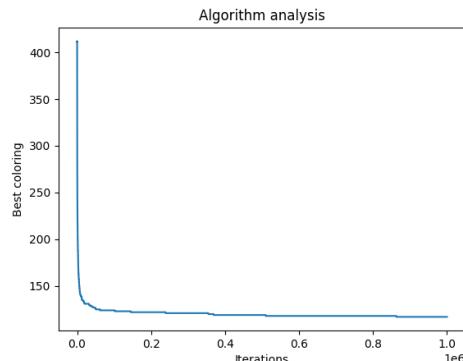
(a) GA-Algorithm analysis



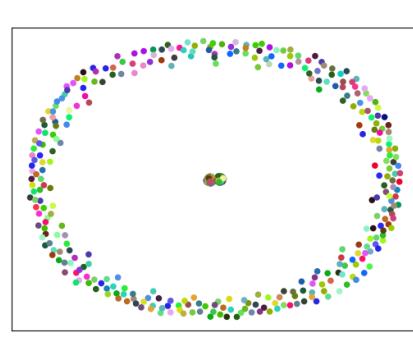
(b) GA-Graph coloring



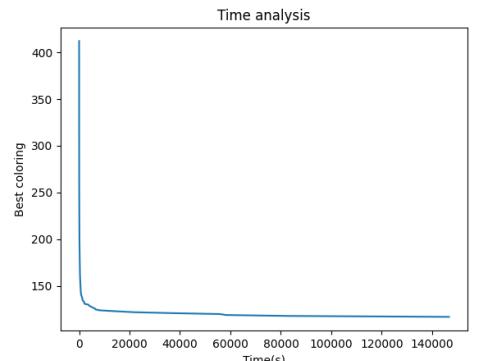
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



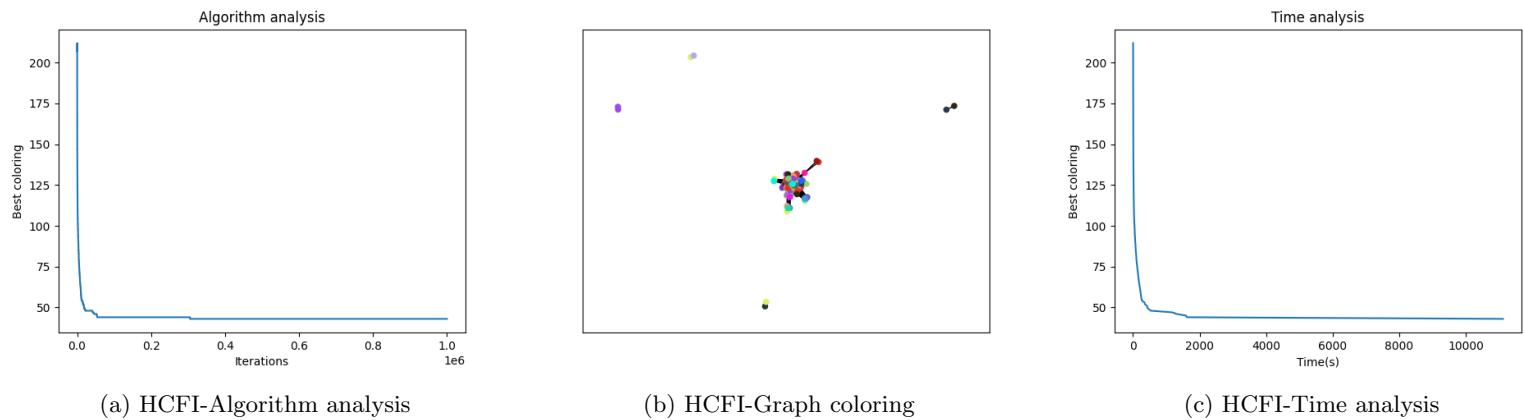
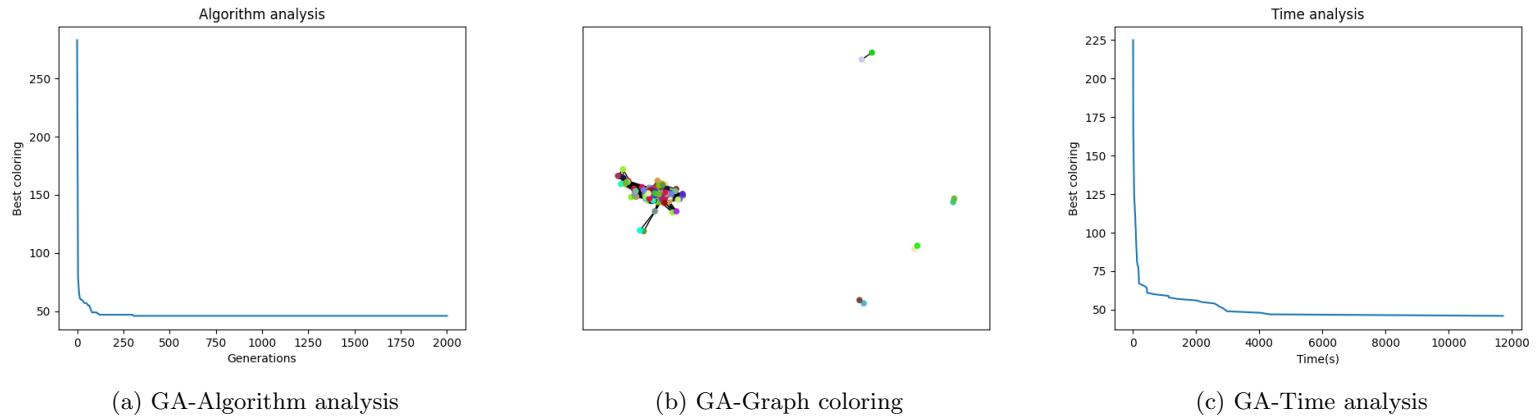
(c) HCFI-Time analysis

Results

Compact Results Table

57. school1

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
school1	GA	385	19095	282	2000	14	14	46	11725	225	5.162	4	1	78306
	HCFI				1000000			43	11121	212	1	538	1	38039

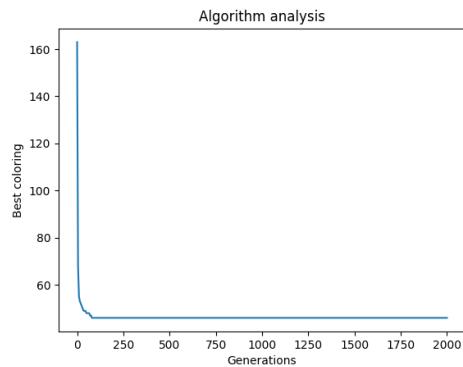


Results

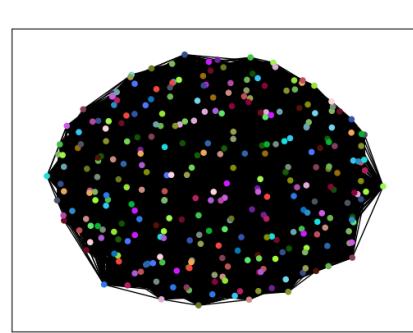
Compact Results Table

58. flat300_28_0

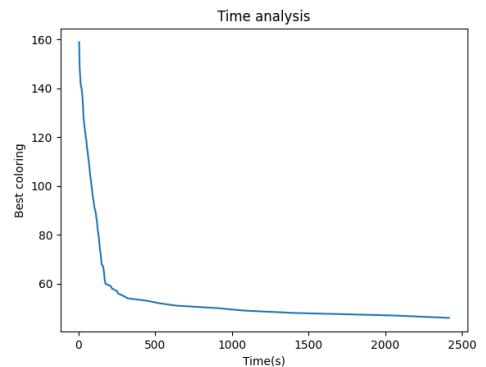
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
flat300_28_0	GA HCFI	300	21695	162	2000 1000000	28	28	46 45	2414 17558	159 142	5.1974 2	6 412	1 1	54255 33131



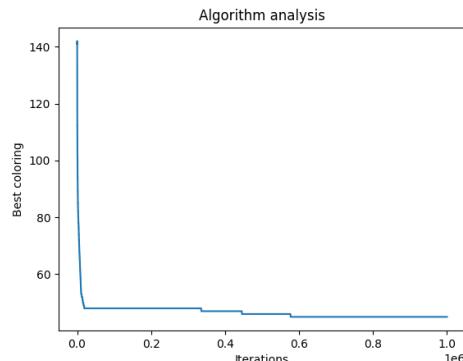
(a) GA-Algorithm analysis



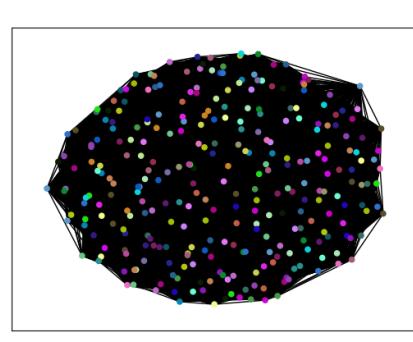
(b) GA-Graph coloring



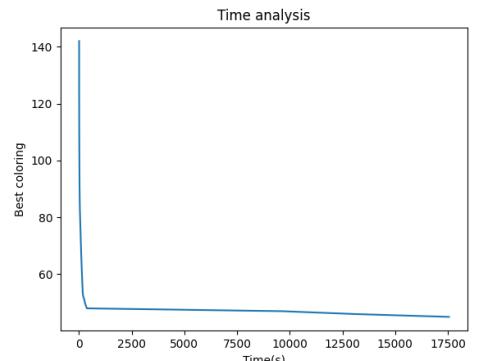
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



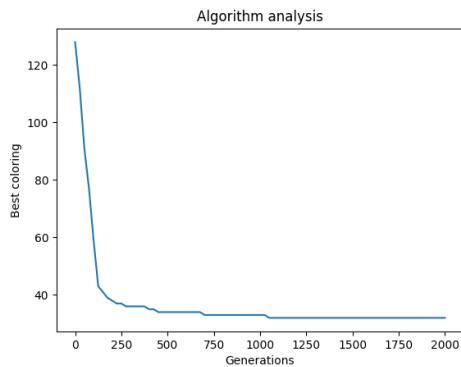
(c) HCFI-Time analysis

Results

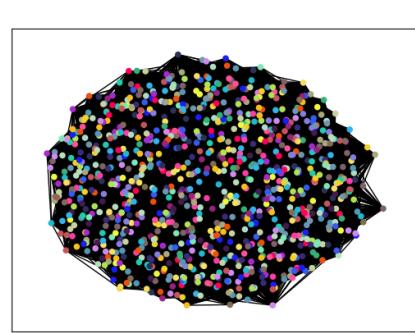
Compact Results Table

59. DSJC1000.1

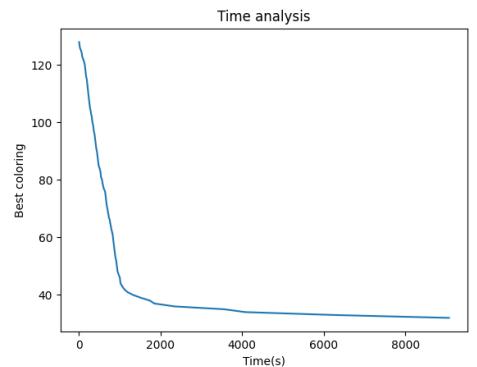
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC1000.1	GA	1000	49629	127	2000	*	20	32	9070	128	8.755	6	1	163562
	HCFI				1000000		33	21560	128	44	2810	1	191020	



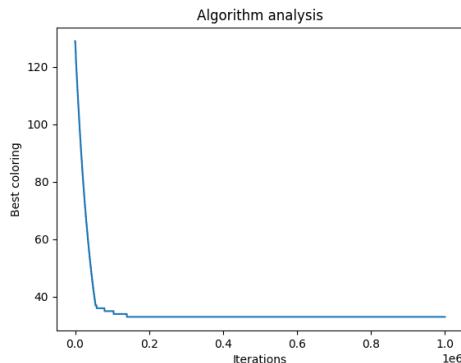
(a) GA-Algorithm analysis



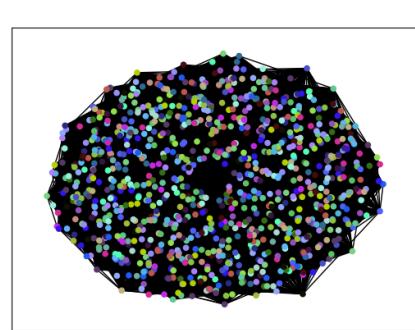
(b) GA-Graph coloring



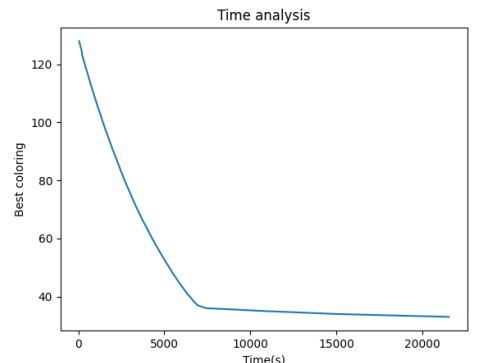
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



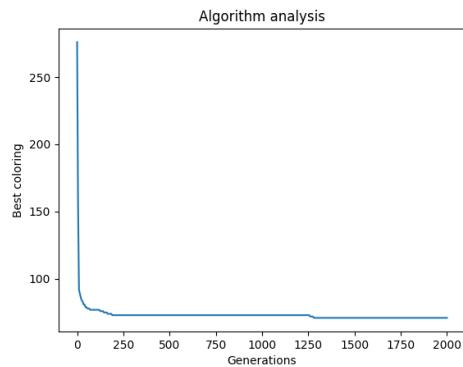
(c) HCFI-Time analysis

Results

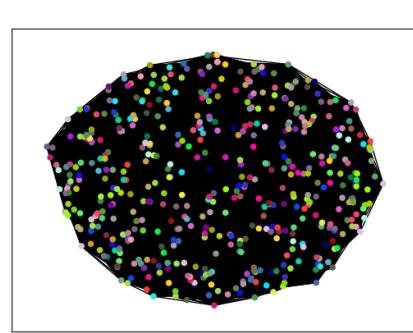
Compact Results Table

60. DSJC500.5

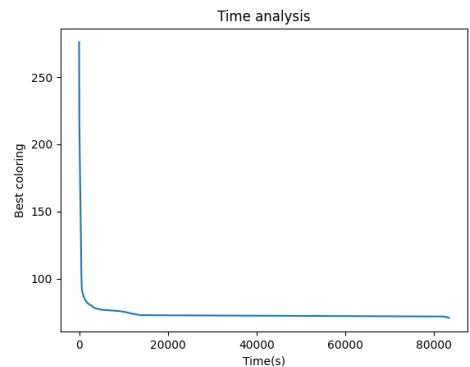
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC500.5	GA	500	62624	286	2000	*	48	71	83360	276	3.327	6	1	130812
	HCFI				1000000		75	75	66060	245	10	1146	1	98004



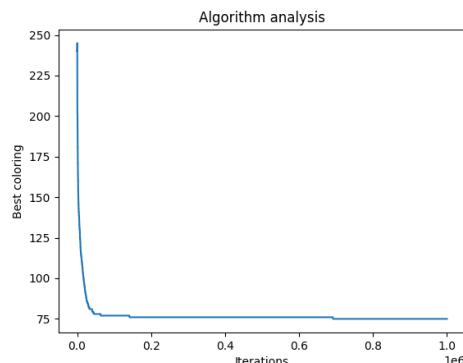
(a) GA-Algorithm analysis



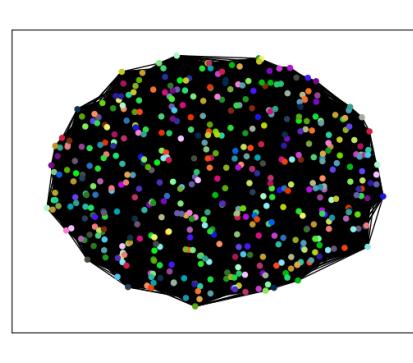
(b) GA-Graph coloring



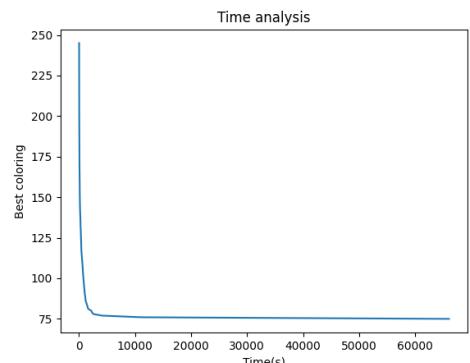
(c) GA-Time analysis



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



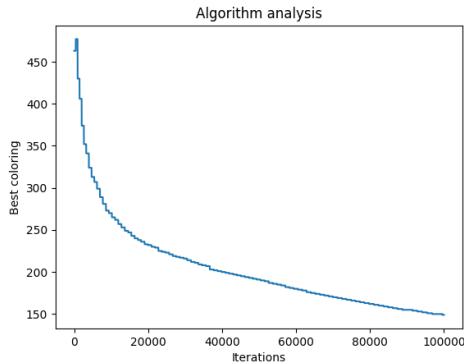
(c) HCFI-Time analysis

Results

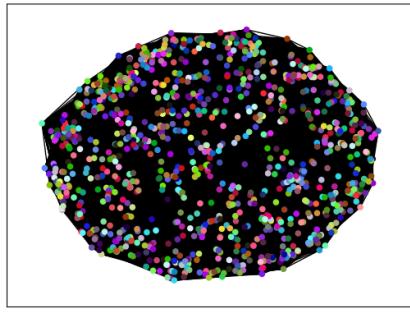
Compact Results Table

61. DSJC1000.5

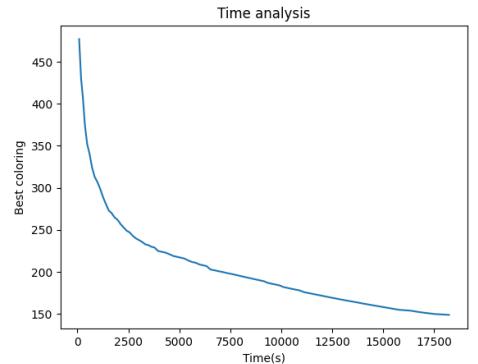
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC1000.5	GA	1000	249826	551	2000	*	83	*	*	*	*	*	*	*
	HCFI				100000		149	18231	477	83	2130	1	18319	



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



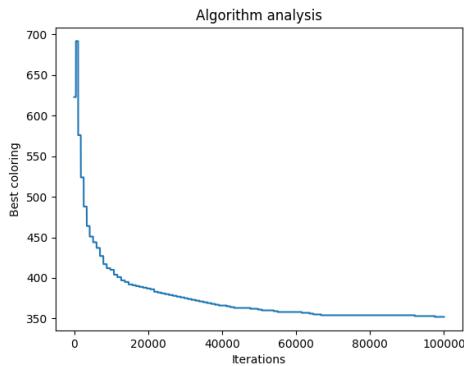
(c) HCFI-Time analysis

Results

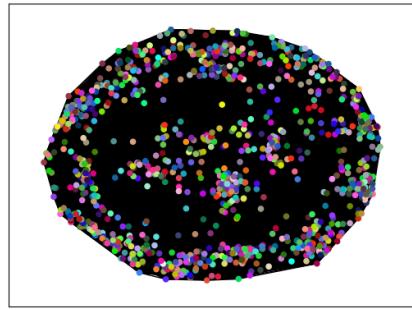
Compact Results Table

62. DSJC1000.9

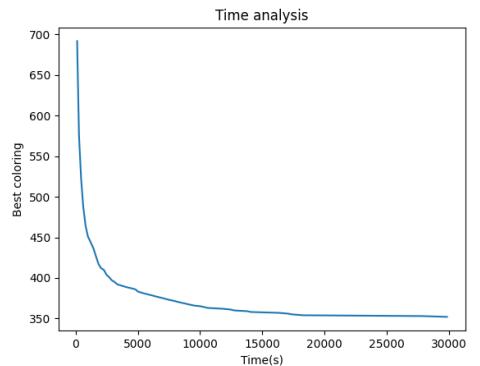
Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC1000.9	GA	1000	449449	924	2000	*	222	*	*	*	*	*	*	*
	HCFI				100000		352	29850	692	119	1656	1	30770	



(a) HCFI-Algorithm analysis



(b) HCFI-Graph coloring



(c) HCFI-Time analysis

Results

Compact Results Table

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
myciel3	GA	11	20	5	2000	4	4	4	0.0370	6	0.0070	2	1	153
	HCFI				100000		4	4	0.0040	5	0	24	1	42
myciel4	GA	23	71	11	2000	5	5	5	0.1960	11	0.0110	2	1	463
	HCFI				100000		5	5	0.02	12	0	24	1	42
mug88_1	GA	88	146	4	2000	4	4	4	0.3125	5	0.2031	6	1	3859
	HCFI				100000		4	4	0.0705	5	0.0403	18	1	132
mug88_25	GA	88	146	4	2000	4	4	4	0.4844	5	0.0781	6	1	3810
	HCFI				100000		4	4	0.0685	5	0.0202	68	1	68
mug100_1	GA	100	166	4	2000	4	4	4	0.1719	5	0.0781	6	1	5211
	HCFI				100000		4	4	0.1232	5	0.0608	46	1	168
mug100_25	GA	100	166	4	2000	4	4	4	0.2097	5	0.084	6	1	5438
	HCFI				100000		4	4	0.1091	5	0.0506	54	1	169
myciel5	GA	47	236	23	2000	6	6	6	3.141	23	0.093	2	1	1793
	HCFI				100000		6	6	0.312	21	0.008	60	1	80
queen5_5	GA	25	320	16	2000	5	5	5	5.13	16	0.017	30	1	422
	HCFI				100000		5	5	0.1875	15	0	56	1	52
jean	GA	80	508	36	2000	10	10	10	4.798	34	0.1719	2	1	3683
	HCFI				100000		10	10	0.3594	34	0.0156	24	1	125
queen6_6	GA	36	580	19	2000	7	7	8	4.366	20	0.02	4	1	919
	HCFI				100000		7	7	0.6875	17	0	36	1	85
huck	GA	74	602	53	2000	11	11	11	2.362	42	0.064	6	1	2397
	HCFI				100000		11	11	0.353	37	0.013	10	1	113
myciel6	GA	95	755	47	2000	7	7	7	16	42	0.149	2	1	5786
	HCFI				100000		7	7	2.417	42	0.023	42	1	238
david	GA	87	812	82	2000	11	11	11	11	56	0.089	2	1	3963
	HCFI				100000		11	11	0.939	51	0.015	20	1	175
DSJC125.1_001	GA	125	736	23	2000	5	5	6	245	24	0.155	30	1	6030
	HCFI				100000		5	5	4.557	24	0.071	242	1	295
miles250	GA	128	774	16	2000	8	8	8	45	17	0.3281	6	1	7745
	HCFI				100000		8	8	7.249	17	0.052	30	1	250
queen7_7	GA	49	952	24	2000	7	7	9	19	24	0.099	4	1	1557
	HCFI				100000		7	9	1.53	23	0.013	44	1	109
anna	GA	138	986	71	2000	11	11	11	283	65	0.5330	2	1	9095
	HCFI				100000		11	11	4.02	64	0.028	46	1	298
games120	GA	120	1276	13	2000	9	9	9	1.2580	14	0.1380	4	1	5379
	HCFI				100000		9	9	0.87	14	0.109	28	1	239
queen8_8	GA	64	1456	27	2000	9	9	10	576	28	0.2	4	1	2486
	HCFI				100000		9	10	119	26	0.025	64	1	150
queen9_9	GA	81	2112	32	2000	10	10	12	72	33	0.1082	6	1	4337
	HCFI				100000		10	12	23	33	0.059	46	1	212

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
miles500	GA	128	2340	38	2000	20	20	20	309	39	0.5	6	1	9225
	HCFI				100000				141	39	0.121	62	1	309
myciel7	GA	191	2360	95	2000	8	8	8	160	88	0.875	6	1	16047
	HCFI				100000				22.92	81	0.123	446	1	801
queen8_12	GA	96	2736	32	2000	12	12	13	398	33	0.103	20	1	4106
	HCFI				100000				7.482	31	0.082	80	1	278
queen10_10	GA	100	2940	35	2000	11	11	13	1845	36	0.103	6	1	5694
	HCFI				100000				197	33	0.095	206	1	299
zeroin.i.3	GA	206	3540	140	2000	30	30	30	466	116	0.6278	6	1	18883
	HCFI				1000000				3499	116	0.165	38	1	16834
zeroin.i.2	GA	211	3541	140	2000	30	30	30	344	115	0.6641	6	1	19612
	HCFI								3732	119	0.125	148	1	17224
DSJC250.1	GA	250	3218	38	2000	8	9	10	2173	39	1.188	50	1	23129
	HCFI				100000				168	39	0.737	234	1	1126
mulsol.i.2	GA	188	3885	156	2000	31	31	31	665	122	0.4561	6	1	16086
	HCFI				1000000				13064	116	0.157	50	1	13941
DSJC125.5	GA	125	3891	75	2000	12	17	21	7219	73	0.226	28	1	7993
	HCFI				100000				381	63	0.1406	70	1	494
queen11_11	GA	121	3960	40	2000	11	12	15	132	41	0.196	4	1	7932
	HCFI				100000				87	37	0.1108	72	1	323
mulsol.i.3	GA	184	3916	157	2000	31	31	31	675	118	0.424	6	1	15627
	HCFI				1000000				10295	118	0.194	76	1	15228
mulsol.i.4	GA	185	3946	158	2000	31	31	31	311	120	0.452	6	1	16394
	HCFI				1000000				5955	120	0.122	24	1	14507
mulsol.i.11	GA	197	3925	121	2000	49	49	49	186	102	0.5150	6	1	17597
	HCFI				100000				46	102	0.156	142	1	891
mulsol.i.5	GA	186	3973	159	2000	31	31	32	268	118	0.4584	6	1	0.4584
	HCFI				1000000				9701	116	0.07	70	1	13201
zeroin.i.11	GA	211	4100	111	2000	49	49	49	489	102	0.5678	6	1	19164
	HCFI				1000000				8055	59	0.1941	62	1	12071
miles750	GA	128	4226	64	2000	31	31	32	79	65	0.5475	6	1	7831
	HCFI				100000				48	61	0.08	118	1	303
queen12_12	GA	144	5192	43	2000	12	12	16	5364	44	0.2	8	1	12650
	HCFI				100000				192	43	0.1301	198	1	436
miles1000	GA	128	6432	86	2000	42	42	42	7485	81	0.25	6	1	11416
	HCFI				100000				18.663	74	0.1109	108	1	356
queen13_13	GA	169	6656	48	2000	13	14	17	3166	48	0.325	6	1	11879
	HCFI				100000				86	48	0.346	324	1	580
DSJC125.9	GA	125	6961	120	2000	30	42	52	12584	114	0.2346	6	1	17924
	HCFI				100000				541	94	0.2760	32	1	742

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
will199GPIA	GA	701	7065	38	2000	7	7	10	1774	39	4.0332	12	1	194357
	HCFI				100000									
queen14_14	GA	196	8372	51	2000	14	14	18	1460	52	0.806	16	1	15187
	HCFI				100000									
fpsol2.i.3	GA	425	8688	346	2000	30	30	30	33083	255	1.502	6	1	64880
	HCFI				1000000									
fpsol2.i.2	GA	451	8691	346	2000	30	30	37	1892	273	1.68	6	1	83175
	HCFI				1000000									
miles1500	GA	128	10396	106	2000	73	73	73	246	97	0.3689	4	1	10182
	HCFI				100000									
queen15_15	GA	225	10360	56	2000	15	17	21	190.581	57	1.1701	6	1	21944
	HCFI				100000									
fpsol2.i.1	GA	496	11654	252	2000	65	65	65	46301	217	4.6228	6	1	91728
	HCFI				1000000									
queen16_16	GA	256	12640	59	2000	*	18	23	243	60	0.632	6	1	29197
	HCFI				100000									
DSJC500.1	GA	500	12458	68	2000	*	12	19	1080	69	2.6428	8	1	105518
	HCFI				100000									
inithx.i.3	GA	621	13969	542	2000	31	31	43	6288	371	5.2135	6	1	153878
	HCFI				1000000									
inithx.i.2	GA	645	13979	541	2000	31	31	33	88885	403	2.6341	6	1	153683
	HCFI				1000000									
school1_nsh	GA	352	14612	232	2000	14	14	33	45452	198	1.202	6	1	58262
	HCFI				1000000									
DSJC250.5	GA	250	15668	147	2000	*	28	44	424	142	1.053	6	1	35769
	HCFI				1000000									
le450_15c	GA	450	16680	139	2000	15	15	33	2075	138	5.5545	6	1	90493
	HCFI				1000000									
le450_25c	GA	450	17343	179	2000	25	25	34	85929	173	4.498	30	2	93260
	HCFI				1000000									
inithx.i.1	GA	864	18707	502	2000	54	54	56	233691	422	5.381	6	1	277190
	HCFI				1000000									
school11	GA	385	19095	282	2000	14	14	46	11725	225	5.162	4	1	78306
	HCFI				1000000									
flat300_28_0	GA	300	21695	162	2000	28	28	46	2414	159	5.1974	6	1	54255
	HCFI				1000000									
DSJC1000.1	GA	1000	49629	127	2000	*	20	32	9070	128	8.755	6	1	163562
	HCFI				1000000									
DSJC500.5	GA	500	62624	286	2000	*	48	71	83360	276	3.327	6	1	130812
	HCFI				1000000									

Instance	A	N	E	Δ	G I	$\chi(G)$	BCK	BC	T:BC(s)	FC	T:FC(s)	CL	SYS	T:T(s)
DSJC1000.5	GA	1000	249826	551	2000	*	83	*	*	*	*	*	*	*
	HCFI				100000		149	18231	477	83	2130	1	18319	
DSJC1000.9	GA	1000	449449	924	2000	*	222	*	*	*	*	*	*	*
	HCFI				100000		352	29850	692	119	1656	1	30770	

4 Conclusion

In this study, we conducted a comprehensive comparison of two algorithms for solving the graph coloring problem: a Genetic Algorithm (GA) and a Hill Climbing (HC) algorithm. By applying these algorithms to a diverse set of DIMACS benchmark graphs, including instances of varying difficulty, we were able to thoroughly evaluate their performance across a range of scenarios. Our results demonstrated that, on small size graphs, the GA and HC achieved comparable results in terms of solution quality, though the HC demonstrated a shorter computational time. However, on large size graphs, the GA outperformed the HC, consistently yielding superior solutions and demonstrating improved performance in terms of solution quality. These findings suggest that the GA is a more effective approach for tackling the graph coloring problem on larger size graphs, while the HC may be more suitable for smaller size instances. Overall, our study provides valuable insights into the relative effectiveness of these two algorithms and contributes to the ongoing efforts to optimize graph coloring approaches.

5 Instances

- [1] <https://mat.gsia.cmu.edu/COLOR/instances.html>
- [2] <http://cedric.cnam.fr/~porumbed/graphs/>
- [3] <https://sites.google.com/site/graphcoloring/vertex-coloring>

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- [4] Isabel Méndez-Díaz, Paula Zabala. "A Branch-and-Cut algorithm for graph coloring." *Discrete Applied Mathematics*, vol. 154, 2006, pp. 826-847.
- [5] Hindi, Musa ,Roman Yampolskiy. "Genetic Algorithm Applied to the Graph Coloring Problem." *Midwest Artificial Intelligence and Cognitive Science Conference*, 2012. <https://www.researchgate.net/publication/256169514>.
- [6] Gary Lewandowski, Anne Condon. "Experiments with Parallel Graph Coloring Heuristics and Applications of Graph Coloring" , 1997. <https://www.researchgate.net/publication/2311223>.
- [7] <https://www.gatevidyalay.com/graph-coloring-chromatic-number/> (the colored graph images)