Имеем такие наблюдения:

$$I_k = A_k h_{d\,k} \sum_{l=0}^{L-1} \cos(\omega_0 t_{k,l} + \omega_{d\,k} l T_d + \varphi_k) \cdot \cos(\omega_0 t_{k,l} + \breve{\omega}_{d\,k} l T_d + \breve{\varphi}_k) + n_{I\,k}$$

$$Q_k = A_k h_{d\,k} \sum_{l=0}^{L-1} \cos(\omega_0 t_{k,l} + \omega_{d\,k} l T_d + \varphi_k) \cdot \sin(\omega_0 t_{k,l} + \breve{\omega}_{d\,k} l T_d + \breve{\varphi}_k) + n_{Q\,k}$$

Введем обозначения:

$$\bar{I}_k = A_k h_{d\,k} \sum_{l=0}^{L-1} \cos(\Phi_{k,l}) \cdot \cos(\breve{\Phi}_{k,l})$$

$$\bar{Q}_k = A_k h_{d\,k} \sum_{l=0}^{L-1} \cos(\Phi_{k,l}) \cdot \sin(\breve{\Phi}_{k,l})$$

$$\Phi_{k,l} = \omega_0 t_{k,l} + \omega_{d\,k} l T_d + \varphi_k$$

$$\breve{\Phi}_{k,l} = \omega_0 t_{k,l} + \breve{\omega}_{d\,k} l T_d + \breve{\varphi}_k$$

Тогда наблюдения перепишем так

$$I_k = \bar{I}_k + n_{I\,k}$$

$$Q_k = \bar{Q}_k + n_{Q\,k}$$

Запишем функцию правдоподобия $p(I_k, Q_k \mid \boldsymbol{\lambda}_k, h_{d\,k})$, где $\boldsymbol{\lambda}_k = \begin{vmatrix} \varphi_k \\ \omega_{\partial\,k} \end{vmatrix}$

$$p(I_k, Q_k \mid \boldsymbol{\lambda}_k, h_{d\,k}) = \frac{1}{\sqrt{2\pi\sigma_{IQ}^2}} \exp\left(-\frac{(I_k - \bar{I}_k(\boldsymbol{\lambda}_k, h_{d\,k}))^2}{2\sigma_{IQ}^2} \right) \cdot \frac{1}{\sqrt{2\pi\sigma_{IQ}^2}} \exp\left(-\frac{(Q_k - \bar{Q}_k(\boldsymbol{\lambda}_k, h_{d\,k}))^2}{2\sigma_{IQ}^2} \right) =$$

$$= \frac{1}{2\pi\sigma_{IQ}^2} \exp\left(-\frac{1}{2\sigma_{IQ}^2} \left[(I_k - \bar{I}_k(\boldsymbol{\lambda}_k, h_{d\,k}))^2 + (Q_k - \bar{Q}_k(\boldsymbol{\lambda}_k, h_{d\,k}))^2 \right] \right)$$

$$\left\{ \frac{1}{2\pi\sigma_{IQ}^2} = B \right\}$$

Усредним функцию правдоподобия по символу навигационного сообщения, считая, что он принимает значения $h_{d\,k} = \pm 1$ с равной вероятностью.

$$p(I_k, Q_k \mid \boldsymbol{\lambda}_k, h_{d\,k}) = \frac{1}{2} p(I_k, Q_k \mid \boldsymbol{\lambda}_k, h_{d\,k} = 1) + \frac{1}{2} p(I_k, Q_k \mid \boldsymbol{\lambda}_k, h_{d\,k} = -1) =$$

$$= \frac{B}{2} \left[\exp\left(-\frac{1}{2\sigma_{IQ}^2} \left[(I_k - \bar{I}_k(\boldsymbol{\lambda}_k))^2 + (Q_k - \bar{Q}_k(\boldsymbol{\lambda}_k))^2 \right] \right) + \exp\left(-\frac{1}{2\sigma_{IQ}^2} \left[(I_k + \bar{I}_k(\boldsymbol{\lambda}_k))^2 + (Q_k + \bar{Q}_k(\boldsymbol{\lambda}_k))^2 \right] \right) \right]$$

Запишем по - иному исходную функцию правдоподобия.

Для чего представим

$$\frac{1}{\sqrt{2\pi\sigma_{IQ}^2}}\exp\left(-\frac{(I_k-\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right))^2}{2\sigma_{IQ}^2}\right)=$$

$$=\frac{1}{\sqrt{2\pi\sigma_{IQ}^2}}\exp\left(-\frac{1}{2\sigma_{IQ}^2}I_k^2\right)\exp\left(\frac{1}{\sigma_{IQ}^2}\left(I_k\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)^2\right)\right)=$$

$$=\frac{1}{\sqrt{2\pi\sigma_{IQ}^2}}\exp\left(-\frac{1}{2\sigma_{IQ}^2}I_k^2\right)\exp\left(\frac{1}{\sigma_{IQ}^2}\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(I_k-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)\right)=$$

$$=\frac{1}{\sqrt{2\pi\sigma_{IQ}^2}}c_I\exp\left(\frac{1}{\sigma_{IQ}^2}\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(I_k-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)\right),\quad c_I=\exp\left(-\frac{1}{2\sigma_{IQ}^2}I_k^2\right)$$

Перепишем функцию правдоподобия

$$p(I_k,Q_k\,|\,\boldsymbol{\lambda}_k,h_{d\,k})=Bc_Ic_Q\exp\left(\frac{1}{\sigma_{IQ}^2}\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(I_k-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)\right)\exp\left(\frac{1}{\sigma_{IQ}^2}\bar{Q}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(Q_k-0.5\bar{Q}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)\right)=$$

$$=Bc_Ic_Q\exp\left(\frac{1}{\sigma_{IQ}^2}\left[\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(I_k-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)+\bar{Q}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\left(Q_k-0.5\bar{Q}_k\left(\boldsymbol{\lambda}_k,h_{d\,k}\right)\right)\right]\right)$$

Усредним по навигационному сообщению

$$p(I_k,Q_k\,|\,\boldsymbol{\lambda}_k)=\frac{Bc_Ic_Q}{2}\left[\exp\left(\frac{1}{\sigma_{IQ}^2}\left[\bar{I}_k\left(\boldsymbol{\lambda}_k\right)\left(I_k-0.5\bar{I}_k\left(\boldsymbol{\lambda}_k\right)\right)+\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)\left(Q_k-0.5\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)\right)\right]\right)+\right.$$

$$\left.+\exp\left(\frac{1}{\sigma_{IQ}^2}\left[-\bar{I}_k\left(\boldsymbol{\lambda}_k\right)\left(I_k+0.5\bar{I}_k\left(\boldsymbol{\lambda}_k\right)\right)-\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)\left(Q_k+0.5\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)\right)\right]\right)\right]$$

$$p(I_k,Q_k\,|\,\boldsymbol{\lambda}_k)=\frac{Bc_Ic_Q}{2}\left[\exp\left(\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)I_k}{\sigma_{IQ}^2}\right)\exp\left(-\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)^2}{2\sigma_{IQ}^2}\right)\exp\left(\frac{\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)Q_k}{\sigma_{IQ}^2}\right)\exp\left(-\frac{\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)^2}{2\sigma_{IQ}^2}\right)+\right.$$

$$\left.+\exp\left(-\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)I_k}{\sigma_{IQ}^2}\right)\exp\left(-\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)^2}{2\sigma_{IQ}^2}\right)\exp\left(\frac{-\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)Q_k}{\sigma_{IQ}^2}\right)\exp\left(-\frac{\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)^2}{2\sigma_{IQ}^2}\right)\right]$$

Итого

$$p(I_k,Q_k\,|\,\boldsymbol{\lambda}_k)=\tilde{C}\exp\left(-\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)^2+\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)^2}{2\sigma_{IQ}^2}\right)ch\left(\frac{\bar{I}_k\left(\boldsymbol{\lambda}_k\right)I_k+\bar{Q}_k\left(\boldsymbol{\lambda}_k\right)Q_k}{\sigma_{IQ}^2}\right),\ \tilde{C}=Bc_Ic_Q.$$

Возьмем ln:

$$\ln p(I_k, Q_k \mid \boldsymbol{\lambda}_k) = \ln(\tilde{C}) - \frac{\bar{I}_k(\boldsymbol{\lambda}_k)^2 + \bar{Q}_k(\boldsymbol{\lambda}_k)^2}{2\sigma_{IQ}^2} + \ln\left(ch\left(\frac{\bar{I}_k(\boldsymbol{\lambda}_k) I_k + \bar{Q}_k(\boldsymbol{\lambda}_k) Q_k}{\sigma_{IQ}^2} \right) \right)$$

Теперь нужно взять производную по φ_k. Учитываем, что $\bar{I}_k^2(\boldsymbol{\lambda}_k) + \bar{Q}_k^2(\boldsymbol{\lambda}_k)$ - мощность «отсчета» в корреляторе, от начальной фазы не зависит.

Получилось так:

$$u_{\partial\, \varphi_k} = \frac{\partial}{\partial \varphi_k} \ln p(I_k, Q_k \mid \boldsymbol{\lambda}_k)\big|_{\varphi_k = \tilde{\varphi}_k} = \frac{\partial}{\partial \varphi_k} \ln\left(ch\left(\frac{\bar{I}_k(\boldsymbol{\lambda}_k) I_k + \bar{Q}_k(\boldsymbol{\lambda}_k) Q_k}{\sigma_{IQ}^2} \right) \right)\Bigg|_{\varphi_k = \tilde{\varphi}_k} =$$

$$= th\left(\frac{\bar{I}_k(\boldsymbol{\lambda}_k) I_k + \bar{Q}_k(\boldsymbol{\lambda}_k) Q_k}{\sigma_{IQ}^2} \right) \left(I_k \frac{\partial \bar{I}_k(\boldsymbol{\lambda}_k)}{\partial \varphi_k} + Q_k \frac{\partial \bar{Q}_k(\boldsymbol{\lambda}_k)}{\partial \varphi_k} \right)\Bigg|_{\varphi_k = \tilde{\varphi}_k} =$$

$$= -\frac{A_k L \operatorname{sinc}\left(\frac{\tilde{\delta}\omega_{\partial\, k} T}{2} \right)}{2\sigma_{IQ}^2} th\left(\frac{\bar{I}_k(\boldsymbol{\lambda}_k) I_k + \bar{Q}_k(\boldsymbol{\lambda}_k) Q_k}{\sigma_{IQ}^2} \right)\Bigg|_{\varphi_k = \tilde{\varphi}_k} \left[I_k \cdot \sin\left(\frac{\tilde{\delta}\omega_{\partial\, k} T}{2} + \tilde{\delta}\varphi_k \right) + Q_k \cdot \cos\left(\frac{\tilde{\delta}\omega_{\partial\, k} T}{2} + \tilde{\delta}\varphi_k \right) \right]$$

где обозначено $\tilde{\delta}\omega_{\partial\, k} = \tilde{\omega}_{\partial\, k} - \breve{\omega}_{\partial\, k}$, $\tilde{\delta}\varphi_k = \tilde{\varphi}_k - \breve{\varphi}_k$