

$$\tau_c := \{U \subseteq X \mid U^c \text{ countable or } X\}$$

$$X, \emptyset \in \tau_c$$

$$\text{ii) } \bigcap_{i=1}^{\infty} U_i = \bigcup_{i=1}^{\infty} U_i^c \text{ countable}$$

$$\text{iii) } \bigcup_{i \in I} U_i = \bigcap_{i \in I} U_i^c \subset U_i^c \text{ for any } i \in I$$

hence countable