

X is f.d normed X , $A \subset X$ is a convex subset s.t. $0 \in A$ and the linear span of A is X show that $\text{int } A$ is not empty.

want to show that $\text{int } A \neq \emptyset$

let D be the dimension of X since X is spanned by A we can write $x_i = (0, \dots, \underset{i}{1}, \dots, 0)$

as $c_i a_i$ where $a_i \in A$. since we can always multiply by a constant we can assume

$\|x_i\| = 1$ thus $\|a_i\| = \frac{1}{|c_i|} \geq 0$ by convexity

all points $ta_i \in A$, $t \in [0, 1]$ for $t < 1$

$ta_i \in B(0, \frac{1}{|c_i|})$ thus pick $C = \max_{1 \leq i \leq D} \{ |c_i| \}$

then $B(0, \frac{1}{C}) \subset A$ and thus $\text{int } A \neq \emptyset$