Let $p: E \to B$ be a covering map, let B be connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$ then $p^{-1}(b)$ has k elements for every $b \in B$. In such a case E is called a k-fold covering of B.

first let $\mathcal{U}$ be a neighborhood of b_0 evenly covered by p. Then the partition of $p^{-1}(\mathcal{U})$ into slices must have k slices as each element in must be in all slices and cannot be in more than k as they are disjoint. As p maps each slice homeomorphically on $\mathcal{U}$ $p^{-1}(b)$ contains k elements for $b \in \mathcal{U}$.

now suppose there is some b s.t $p^{-1}(b)$ does not contain k-elements. then if $\mathcal{V}$ is an evenly covered neighborhood of b, $\mathcal{V} \cap \mathcal{U} = \Phi$.

let $K = \{b \in B : p^{-1}(b) \text{ contains } k \text{ elements}\}$. Let $\mathcal{U}_b$ denote a neighborhood evenly covered by p.

then $K = \bigcup_{b \in K} \mathcal{U}_b$ so K is open. $K^c = \bigcup_{b \in K^c} \mathcal{U}_b$ is also open $B = K \sqcup K^c$ but this means that $K = B$ as B is connected ∎