

Let $f, g: X \rightarrow Y$ be continuous. Assume that Y is Hausdorff. Show that $\{x: f(x) = g(x)\}$ is closed in X .

Let $C = \{x: f(x) = g(x)\}$. Let $x \in C^c$

Let $y_1 = f(x)$, $y_2 = g(x)$ choose disjoint open neighborhoods V_1, V_2 of y_1, y_2 . Then $g^{-1}(V_2) \cap f^{-1}(V_1)$ is an open neighborhood of x contained in C^c , thus C is closed.