Show that the subspace $(a,b)$ of $\mathbb{R}$ is homeomorphic with $(0,1)$ and that $[a,b]$ is homeomorphic with $[0,1]$

define $f:(a,b)\to(0,1)$ by

$f(x)=\frac{x-a}{(b-a)}$, $f^{-1}(\cancel{x})=a+(b-a)x$, $f$ is bijective

$f((a_1,b_1))=\left(\frac{a_1-a}{b-a},\frac{b_1-a}{b-a}\right)=\left(\left(\frac{a_1-a}{b-a},\frac{b_1-a}{b-a}\right)\cap(0,1)\right)$

so $f^{-1}$ is continuous, similar to show that $f$ is continuous. Thus $(a,b)$ and $(0,1)$ are homeomorphic

$f:[a,b]\to[0,1]$ as before this is ~~sur~~ bijective

we have already shown that for $(a_1,b_1)\subset[a,b]$ $f$ maps to open sets and vice versa

consider then

$f([a,\tilde{b}))=[0,\frac{\tilde{b}-a}{b-a})$ open in the subspace topology for $[0,1]$ so $f^{-1}$ is continuous. Similar to show that $f$ is continuous

Thus $[a,b]$ and $[0,1]$ are homeomorphic

(we should check $[a,b]$, $(\tilde{a},b]$ as well)