Show that if $\prod X_\alpha$ is Hausdorff or regular, or normal, then so is X_α. (Assume each X_α is nonempty).

<u>Hausdorff:</u> pick any $(x_\alpha)_{\alpha \in J}$. for any $\beta \in J$ Pick $(y_\alpha^\beta)_\alpha$ s.t $y_\alpha^\beta = x_\alpha$ $\forall$ α except β. Then as $\prod X_\alpha$ is hausdorff there must be disjoint open sets in X_β containing y_β^β and x_β ∎

<u>regular:</u> let $(x_\alpha)_\alpha \in \prod_{\alpha \in J} X_\alpha$, for any $\beta \in J$ consider $\mathcal{U} = \prod \mathcal{U}_\alpha$, $\mathcal{U}_\alpha = X_\alpha$ except for $\alpha = \beta$ Where $\mathcal{U}_\beta$ is a closed set disjoint from x_β. then $\mathcal{U}$ is closed as it's complement is open. We can make disjoint neighborhoods V, O of $x, \mathcal{U}$. Then $V_\alpha = O_\alpha = X_\alpha$, $\alpha \neq \beta$ and V_β, O_β are disjoint open sets containing x_β, $\mathcal{U}_\beta$, thus X_α is regular for all $\alpha \in J$

<u>Normal:</u> We argue as in the regular case. let $A = \prod A_\alpha$, $A_\alpha = X_\alpha$ except for some $\beta \in J$ s.t A_β is closed in X_β. similar for $B = \prod B_\alpha$. Then By normality of X we have disjoint open sets $\mathcal{U}, V$ containing A, B, we see that $\mathcal{U}_\beta$ and V_β are then

disjoint open sets containing A_{β}, B_{β} respectively so X_{β} is normal $\blacksquare$