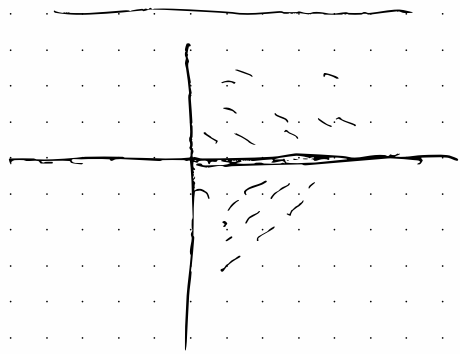
find the boundary and interior of following subsets of $\mathbb{R}^2$

a) $A = \{x \times y \mid y = 0\}$. $\partial A = A$

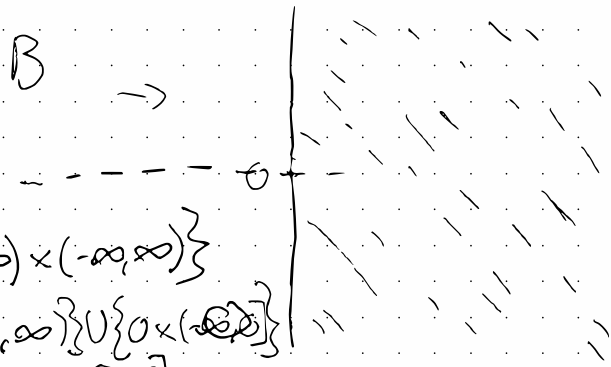


b) $B = \{x \times y \mid x > 0, y \neq 0\}$

$\operatorname{int} B = B$

$\partial B = \{0 \times (-\infty, \infty)\} \cup \{[0, \infty) \times 0\}$

c) $C = A \cup B$



$\operatorname{int} C = \{(0, \infty) \times (-\infty, \infty)\}$

$\partial C = \{0 \times (-\infty, \infty)\} \cup \{0 \times (-\infty, 0]\}$

d) $D = \{x \times y \mid x \text{ is rational}\}$

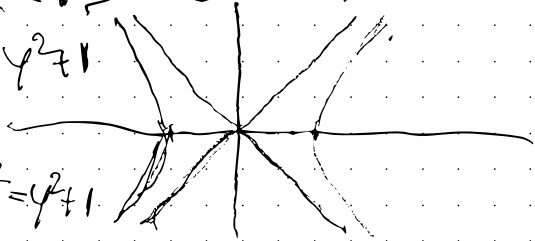
$y > \sqrt{x^2 - 1}$

$\operatorname{int} D = \emptyset$ ~~(rationals are closed?)~~

∂.

e) $E = \{x \times y \mid 0 < x^2 - y^2 < 1\}$ $0 < x^2 - y^2 \Leftrightarrow$

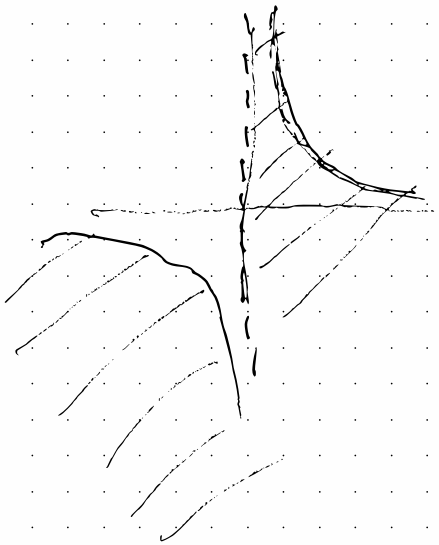
$x^2 > y^2 \Leftrightarrow |x| > |y|$, $x^2 < y^2 + 1$



$\operatorname{int} E = E$

$\partial E = \{x \times y \mid x^2 = y^2\} \cup \{x \times y \mid x^2 = y^2 + 1\}$

$F = \{x \times y \mid x \neq 0,\ y \leq 1/x\}$



$\text{int } F = \{x \times y \mid x \neq 0,\ y < 1/x\}$

$\partial F = \{x \times y \mid x = 0\}$

$\cup \{x \times y \mid x \neq 0,\ y = 1/x\}$