If a set A has 2 elements show that $P(A)$ has 4 elements. How many elements does $P(A)$ have when A has 1, 3, 0 elements

---

A has to elements X_1, X_2

$\emptyset$ is a subset of A, so is $\{X_1\}, \{X_2\}$, $\{X_1, X_2\} \rightarrow 4$ subsets

---

if A has 1 element -> 2 subsets

$A = \emptyset \rightarrow 1$ subset

$A = \{X_1, X_2, X_3\}$

$\rightarrow \emptyset, \{X_1\}, \{X_2\}, \{X_3\}, \{X_1, X_2\}, \{X_2, X_3\}$
$, \{X_1, X_3\}, \{X_1, X_2, X_3\} \rightarrow 8$

So now we guess that if A has n elements $|P(A)| = 2^n$

We argue by induction

we suppose that it is true for some n

then check if this is true for $n+1$

We have $A \cup X_{n+1} = \{X_1, \ldots, X_{n+1}\}$

if $E \subset A$ then $E \subset (A \cup X_{n+1})$

let $E \subset (A \cup X_{n+1})$ then if

$X_{n+1} \notin E$, $E \subset A$, there are 2^n such sets

if $X_{n+1} \in E$ then E is the union of

some subset of A and X_{n+1}. there are again

2^n of these. Thus $|\mathcal{P}(A \cup X_{n+1})| = 2 \cdot 2^n = 2^{n+1}$ $\boxtimes$