Show that a finite union of compact spaces is compact

Start with 2 compact spaces $(X_1, \mathcal{T}_1), (X_2, \mathcal{T}_2)$

Consider an open covering of $X_1 \cup X_2$

This is an open cover of both X_1 and X_2

Thus we have a finite subcollection covering X_1 and one covering X_2. Their union is finite and a covering of $X_1 \cup X_2$.

Proceed by induction.