X, X' are single sets in $\mathcal{T}, \mathcal{T}'$

Y, Y' ——— | ——— $\mathcal{U}, \mathcal{U}'$

Assume ~~the~~ X, X', Y, Y' are nonempty

(a) show that if $\mathcal{T}' \supset \mathcal{T}$ and $\mathcal{U}' \supset \mathcal{U}$ then the product topology on $X' \times Y'$ is finer than the product topology on $X \times Y$

$\mathcal{T}_X$ ~ topology on X

$A \in \mathcal{T}_X$ then $A = X \cap T$, $T \in \mathcal{T}$ but $T \in \mathcal{T}'$

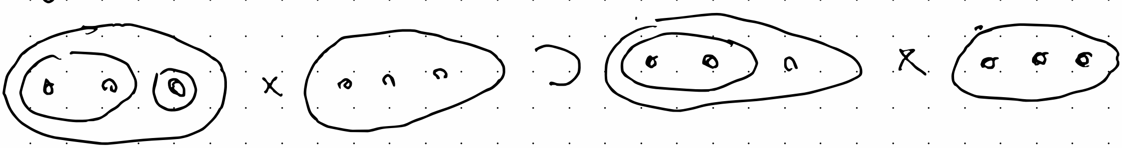
so $\mathcal{T}_X \subset \mathcal{T}_X'$, similarly $\mathcal{U}_Y \subset \mathcal{U}_Y'$

thus $\mathcal{T}_X \times \mathcal{U}_Y \subset \mathcal{T}_X' \times \mathcal{U}_Y'$

b) is the converse true? i.e. if

$X' \times Y' \supset X \times Y$ is then $X' \supset X$ and $Y' \supset Y$

again



but