Let $f_n : \mathbb{R} \to \mathbb{R}$ by

$$f_n(x) = \frac{1}{n^3[x-(1/n)]^2+1}$$

let $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = 0$

a) show that $f_n(x) \to 0$ for each $x \in \mathbb{R}$

we see that $f(0) = \frac{1}{n^3(-\frac{1}{n})^2+1} = \frac{1}{n+1}$

and that $[x - \frac{1}{n}]^2$ increases as x goes from zero to minus ∞ thus

$f(x) \le \frac{1}{n+1}$ for $x > 0$ so clearly $f_n(x) \to 0$

$\forall\, x \le 0$. We also see that $f_n(\frac{2}{n}) = \frac{1}{n+1}$

and $f_n(x) \le \frac{1}{n+1}$, $x \ge \frac{2}{n}$

thus for $\varepsilon > 0$, $x > 0$ let n_1 be s.t.

$\frac{2}{n_1} < x$ $\quad n_2$ s.t. $\frac{1}{n_2+1} < \varepsilon$ let $N = \max\{n_1, n_2\}$

then $f_n(x) < \varepsilon$ when $n \ge N$

b) $f_n(\frac{1}{n}) = 1$ $\forall\, n$ so $f_n \not\to 0$