Show that if X is regular every pair of points of X have neighborhoods whose closures are disjoint.

Let $x, y \in X$ pick disjoint neighborhoods U, V of x, y then $y \notin \overline{U}$ by regularity $\exists$ there are open disjoint sets U_1, V_1 s.t $\overline{U} \subset U_1$, $y \in V$ then $\overline{V} \cap \overline{U} = \phi$ ∎