Let A, B, A_α denote subsets of a space X. Prove the following:

a) if $A \subset B$ then $\overline{A} \subset \overline{B}$

Since $A \subset B$, $A \subset \overline{B}$.

Let $a \in \overline{A}$ then a is in all closed sets containing A, in particular $a \in \overline{B}$ $\blacksquare$

b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

"$\subset$"

$A \subset A \cup B \to \overline{A} \subset \overline{A \cup B}$ Similarly $\overline{B} \subset \overline{A \cup B}$

thus $\overline{A} \cup \overline{B} \subset \overline{A \cup B}$ as unions preserve inclusions

"$\supset$" $A \cup B \subset \overline{A} \cup \overline{B} \to \overline{A \cup B} \subset \overline{A} \cup \overline{B}$

c) $\overline{\bigcup A_\alpha} \supset \bigcup \overline{A_\alpha}$

again $A_{\alpha_i} \subset \overline{\bigcup A_\alpha} \to \overline{A}_{\alpha_i} \subset \overline{\bigcup A_\alpha}$ $\forall\ i \in I$

$\to \bigcup \overline{A}_\alpha \subset \overline{\bigcup A_\alpha}$

let $A_n = (\frac{1}{n}, 1)$ then $\bigcup \overline{A}_n = \bigcup [\frac{1}{n}, 1] = (0, 1]$

however $\overline{\bigcup A_n} = [0, 1]$