

$f: A \rightarrow B$ is a surjective function
define a relation on A by

$$a_0 \sim a_1 \text{ if } f(a_0) = f(a_1)$$

• Show that it is an equivalence relation

for $a \in A$ $f(a) = f(a)$ so $a \sim a$

if $f(a_1) = f(a_2)$ then $f(a_2) = f(a_1)$

$$\text{so } a_1 \sim a_2 \rightarrow a_2 \sim a_1$$

if $a_1 \sim a_2$ and $a_2 \sim a_3$ then

$$f(a_1) = f(a_2) = f(a_3) \text{ so } a_1 \sim a_3$$

• let A^* be the set of equivalence classes
Show there is a bijective correspondence
between A^* and B .

$$[a] \in A^* \text{ s.t. } [a] = \{\tilde{a} \in A, \tilde{a} \sim a\}$$

$$\tilde{f}([a]) = f(a), \text{ since } f \text{ is surjective}$$

We know that for each $b \in B$ \exists at least
one $a \in A$ s.t. $f(a) = b$

$$\text{let } g: B \rightarrow A^* \text{ s.t. } g(f(a)) = [a]$$

then $g(\tilde{f}([a])) = [a]$ for all $[a] \in A^*$ and

$$\tilde{f}(g(b)) = \tilde{f}(g(f(a))) = \tilde{f}([a]) = f(a) = b \quad \forall b \in B \quad \text{hence } b \text{ is fixed}$$

