

X, X' are single sets in $\mathcal{T}, \mathcal{T}'$
 $\mathcal{Y}, \mathcal{Y}' \xrightarrow{\quad} \mathcal{U}, \mathcal{U}'$

Assume ~~the~~ $X, X', \mathcal{Y}, \mathcal{Y}'$ are nonempty

(a) show that if $\mathcal{T}' \supset \mathcal{T}$ and $\mathcal{U}' \supset \mathcal{U}$ then
 the product topology on $X' \times \mathcal{Y}'$ is finer than
 the product topology on $X \times \mathcal{Y}$

\mathcal{T}_x ~ topology on X

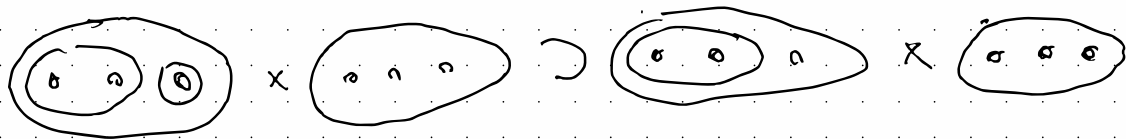
$A \in \mathcal{T}_x$ then $A = X \cap T, T \in \mathcal{T}$ but $T \in \mathcal{T}'$

so $\mathcal{T}_x \subset \mathcal{T}'_x$, similarly $\mathcal{U}_y \subset \mathcal{U}'_y$

thus $\mathcal{T}_x \times \mathcal{U}_y \subset \mathcal{T}'_x \times \mathcal{U}'_y$

b) is the converse true? i.e. if
 $X' \times \mathcal{Y}' \supset X \times \mathcal{Y}$ is then $X' \supset X$ and $\mathcal{Y}' \supset \mathcal{Y}$

again



but $\text{[diagram of two ovals with three dots each]} \not\supset \text{[diagram of two ovals with two dots each]}$