Give a direct proof of urysohn's lemma for a metric space (X, d) by defining

$$f(x) = \frac{d(x,A)}{d(x,A) + d(x,B)}$$

It is clear that $f(a) = 0$, $a \in A$, $f(b) = 1$, $b \in B$ as A and B are disjoint the function is well defined and a product of continuous functions so it is continuous.

It is also clear that $0 \leq f(x) \leq 1$ $\blacksquare$