

Let $f: X \rightarrow X$ be continuous. Show that if $X = [0, 1]$, there is a point x s.t. $f(x) = x$. The point x is called a fixed point of f . What happens if $X = (0, 1)$ or $[0, 1)$?

$g(x) = f(x) - x$. if $g = 0$ we are done
 $g(0) \geq 0$, $g(1) \leq 0$ so $g(x) = f(x) - x = 0$
for some x .

Does not work for the other two examples
(for $(0, 1)$ let $f(x) = \frac{x}{2}$)

(for $[0, 1)$ let $f(x) = 1 - e^{-(x+1)}$)

$$e^{-(x+1)} < \frac{x}{1-x} \quad \text{and} \quad x = 1 - (1-x)$$