

Show that the product of two Hausdorff spaces is Hausdorff.

let  $X, Y$  be Hausdorff.

let  $(a_1, b_1), (a_2, b_2) \in X \times Y$  be distinct points. Then either  $a_1 \neq a_2$  or  $b_1 \neq b_2$ .

if  $a_1 \neq a_2$  then we have  $U_1, U_2 \in \mathcal{T}_X$

s.t.  $U_1 \cap U_2 = \emptyset$  then for any  $V \in \mathcal{T}_Y$

$$V_1, V_2 \in \mathcal{T}_Y, (U_1 \times V_1) \cap (U_2 \times V_2)$$

$$= (U_1 \cap U_2) \times (V_1 \cap V_2) = \emptyset$$