Let $f: X \to Y$. Let Y be compact hausdorff. Then f is continuous iff

$$G_f = \{x \times f(x) \mid x \in X\} \text{ is closed in } X \times Y.$$

Hint: If G_f is closed and V is a neighborhood of $f(x_0)$, then the intersection of G_f and $X \times (Y - V)$ is closed. Apply exercise 7.

"$\Rightarrow$" let $x \times y \in G_F^c$ that is $f(x) \neq y$

let y_0 s.t. $f(x) = y_0$ consider disjoint neighborhoods $\mathcal{V}_0$ of y_0, $\mathcal{V}$ of y, $f^{-1}(\mathcal{V}_0) \cap f^{-1}(\mathcal{V}) = \phi$

$x \in f^{-1}(\mathcal{V}_0)$, $f^{-1}(\mathcal{V}_0) \times \mathcal{V} \subset G_f^c$ is open so

G_f is closed

"$\Leftarrow$" $X \times (Y \setminus V)$ is closed (V is open)

$\to X \times (Y \setminus V) \cap G_F$ is closed

$= \{x \times f(x) : f(x) \notin V\}$

$\pi_1(X \times (Y \setminus V) \cap G_F) = \{x : f(x) \notin V\} = f^{-1}(V^c)$

is closed so $f^{-1}(V)$ is open. as x_0, V are arbitrary this is true for any basis element so f is continuous.