$F: \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} xy/(x^2+y^2) & \text{if } x,y \neq 0,0 \\ 0 & x,y \neq 0,0 \end{cases}$$

a) show that f is continuous in each variable seperately

let $y_0 \in \mathbb{R}$ then $g(x) = \frac{xy_0}{x^2+y_0^2}$ is continuous as x and $\frac{y_0}{x^2+y_0^2}$ is continuous

let $y_0 = 0$ $g(x) = 0$ is continuous

b) Compute $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = F(x,x)$

$$g(x) = \frac{x^2}{2x^2} = \frac{1}{2}$$

c) show that F is not continuous.

let $x_n = \frac{1}{n}$, ~~$g(x_n) = \frac{1}{2}$ $\forall$~~

$F(x_n, x_n) = \frac{1}{2}$ $\forall$ n but $(x_n, x_n) \to (0,0)$

and $0 \neq \frac{1}{2}$