Show that every compact subspace of a metric space is bounded in that metric and is closed. Find a metric space ~~that~~ in which not every closed and bounded set is compact.

Bounded: Assume a subspace V is not bounded.
Conside $x \in V$, $A_1 = B(x,1)$

$A_n = \{ y \in V, n-2 < d(x,y) < n \}$, $n \geq 2$

then $\bigcup_{n=1}^{\infty} A_n = V$ but there is no finite subset covering V as for any N we can find y s.t $d(x,y) \geq N$, thus V is not compact.

Closed: assume V is not open. let $x \in \overline{V} \setminus V$
consider $V_n = \{ y \in V : d(x,y) > \frac{1}{n} \}$

$\bigcup_{n \in \mathbb{N}} V_n = V$ but again no finite subcollection covering V

$\mathbb{R}$ with the "uniform" metric $\bar{d} = \min\{|x-y|, 1\}$ is both closed and bounded.

take $A_1 = B(0,1)$, for $n \geq 2$, $A_n = B(n-1,1) \cup B(-(n-1),1)$

then $\bigcup_{n \in \mathbb{N}} A_n = \mathbb{R}$ but there is no finite subcollection covering $\mathbb{R}$.