Let X be locally compact. If $f: X \to Y$ is continuous, does it follow that $f(X)$ is locally compact? What if f is both continuous and open? Justify your answer.

Does not necessarily follow from continuity.

$X = \mathbb{Q}$ with discrete topology. then every finite subset is open and compact. $Y = \mathbb{Q}$ with subspace topology from the standard on $\mathbb{R}$.

f = identity.

if f is continuous then it holds. Let u be a neighborhood of x, V a compact set containing u then $f(u)$ is an open neighborhood of $f(x)$ and $f(V)$ is compact containing $f(u)$.