a) $\Omega = \{1,2,3,4,5\}$

$\mathcal{U} = \{\{1,2,3\}, \{3,4,5\}\}$

find $\sigma(\mathcal{U})$

$\sigma(\mathcal{U}) = \{\phi, \Omega, \{1,2,3\}, \{3,4,5\}, \{4,5\}\{1,2\}$

$, \{1,2,4,5\}, \{3\}\} = \mathcal{H}_{\mathcal{U}}$

b) let $X: \Omega \to \mathbb{R}$ by $X(1) = X(2) = 0, X(3) = 10$

$X(4) = X(5) = 1$

is X measurable wrt $\mathcal{H}_{\mathcal{U}}$?

$X^{-1}(0) = \{1,2\} \in \mathcal{H}_{\mathcal{U}}, \quad X^{-1}(10) = \{3\} \in \mathcal{H}_{\mathcal{U}}$

$X^{-1}(1) = \{4,5\} \in \mathcal{H}_{\mathcal{U}} \to$ measurable

c) $Y: \Omega \to \mathbb{R}$ by $Y(1) = 0, Y(2) = Y(3) = Y(4) = Y(5) = 1$

$Y^{-1}(0) = \{1\}, \quad Y^{-1}(1) = \{2,3,4,5\}$

$\mathcal{H}_Y(\mathcal{U}) = \{\phi, \Omega, \{1\}, \{2,3,4,5\}\}$