a) Show that in the finite complement topology on $\mathbb{R}$ every subspace is compact.

Let $\mathcal{A}$ be a covering of $\mathbb{R}$.

let $\mathcal{U} \in \mathcal{A}$ then $\mathcal{U}^c$ contains only finitely many points. for $x \in \mathcal{U}^c$ there is some $\mathcal{U}_x \in \mathcal{A}$ s.t $x \in \mathcal{U}_x$

let $\{\mathcal{U}, \mathcal{U}_x : x \in \mathcal{U}^c\}$ is a finite subcollection covering $\mathbb{R}$, repeat argument for any subspace

b) if $\mathbb{R}$ has the topology consisting of all sets A s.t $\mathbb{R} \setminus A$ is countable or all of $\mathbb{R}$ is $[0,1]$ a compact space?

not compact, consider the sets $C_n : \{q \in \mathbb{Q} : 0 < q \leq \frac{1}{n}\}$ closed in the given topology (countable). for any finite sub-collection $C_{n_1}, C_{n_2}, \ldots, C_{n_k}$

$C_{n_1} \cap C_{n_2} \cap \ldots \cap C_{n_k} \neq \emptyset$. but $\bigcap_{n \in \mathbb{N}} C_n = \emptyset$

$\rightarrow$ not compact by theorem 26.9