Preve theorem 19.3:

Theorem 19.3: Let A_α be a subspace of X_α, for each $\alpha \in J$. Then $\prod A_\alpha$ is a subspace of $\prod X_\alpha$ if both products are given the box topology or if both products are given the product topology.

In both cases it suffices to prove that a basis in $\prod X_\alpha$ creates a basis in $\prod A_\alpha$

Box topology:

let $\prod_{\alpha \in J} U_\alpha$ be a basis element of the box topology. Consider $\left(\prod_{\alpha \in J} U_\alpha\right) \cap \left(\prod_{\alpha \in J} A_\alpha\right)$

$= \prod_{\alpha \in J} (U_\alpha \cap A_\alpha)$. since U_α is open in X_α

$U_\alpha \cap A_\alpha$ is open in A_α thus the collection of all $\prod_{\alpha \in J} (U_\alpha \cap A_\alpha)$ is a basis for the box-topology on $\prod_{\alpha \in J} A_\alpha$ ∎

Product topology:

let $\prod_{\alpha \in J} U_\alpha$ be a basis element in the product topology on $\prod_{\alpha \in J} X_\alpha$. Then $\left(\prod_{\alpha \in J} U_\alpha\right) \cap \left(\prod_{\alpha \in J} A_\alpha\right) = \prod_{\alpha \in J} (U_\alpha \cap A_\alpha)$

where $U_\alpha \cap A_\alpha$ is open in A_α $\forall \alpha$ and

$U_\alpha \cap A_\alpha = X_\alpha \cap A_\alpha = A_\alpha$ for all but finitely many values of α. Thus it is a basis element for the product topology on $\prod_{\alpha \in J} A_\alpha$