Let Y be compact. Show that $\pi_1 : X \times Y \to X$ is a closed map.

Let C be closed, pick $x \in \pi_1(C)^c$

$\pi^{-1}(x) = x \times Y \in C^c$ so there is an open set N containing $x \times Y$ then we have an open neighborhood W of x s.t $W \times Y$ is open $\pi(W)$ is open thus $\pi_1(C)^c$ is open so $\pi(C)$ is closed