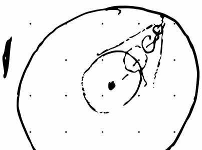
Exercise

If X is a ~~noureal~~ normed real v.s., $A \subset X$ is convex, $\operatorname{int} A \neq \emptyset$ then $\operatorname{int} A$ is convex and dense in A

Case 1. $0 \in \operatorname{int} A$



let $y \in A$. then $ty \in A$ for $0 \le t \le 1$

since $0 \in \operatorname{int} A$ $\exists\, r > 0$ s.t. $B_{2r}(0) \subset \operatorname{int} A$
$= B_{2r}$

for $x \in B_{2r}$ $(1-t)x + ty \in A$

fix t and consider $x_t = ty$

consider $B_t = B((1-t)r, x_t)$ let $a \in B_t$

let x^* s.t. $a = (1-t)x^* + ty$

$(1-t)r > \|a - x_t\| = \|(1-t)x^*\| = (1-t)\|x^*\| \Rightarrow$

$\|x^*\| < r \to x^* \in B_r \to a \in A$ ~~open a ∈~~ in

$x_t \in \operatorname{int} A$

$x_{1-1/n} \to y$ and thus $\operatorname{int} A$ is dense

Case 2 $0 \notin \operatorname{int} A$

pick $x_0 \in \operatorname{int} A$ $\operatorname{int}(A - x_0)$ is dense in $A - x_0$ so we have $x_n \to a - x_0 \Rightarrow x_n - x_0 \to a$

convex $a_1, a_2 \in \operatorname{int} A$ r_1, r_2 s.t

$B_{r_1}(a_1) \subset \operatorname{int} A$, $B_{r_2}(a_2) \subset \operatorname{int} A$