Prove theorem 19.2:

Suppose the topology on each space X_α is given by a basis $\mathcal{B}_\alpha$. The collection of all sets of the form $\prod_{\alpha \in J} B_\alpha$

where $B_\alpha \in \mathcal{B}_\alpha$ $\forall \alpha$ will serve as a basis for the box topology $\prod_{\alpha\in J} X_\alpha$.

The collection of all sets on the same form where $B_\alpha \in \mathcal{B}_\alpha$ for finitely many indices α and $B_\alpha = X_\alpha$ for all remaining indices will serve as a basis for the product topology $\prod_{\alpha\in J} X_\alpha$

The general basis for the box topology $\prod_{\alpha\in J} X_\alpha$ is the collection of all sets U_α s.t U_α is open in X_α. let U_α be such a set

then for any α, $x \in U_\alpha$ $\exists$ B s.t $x \in B \subset U_\alpha$ denote this by $(B_\alpha)_{\alpha\in J}$ thus by lemma 13.2 the collection of all such sets is a basis for the box topology

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by Theorem 19.1 the product topology has basis of all sets of the form: $\prod_{\alpha\in J} \mathcal{U}_\alpha$ wher $\mathcal{U}_\alpha$ is open in X_α $\forall \alpha$ and $\mathcal{U}_\alpha = X_\alpha$ except for finitely many values of α

Conside such a set

$\prod_{\alpha \in J} U_\alpha$. consider α s.t $U_\alpha \neq X_\alpha$ then

for $x \in U_\alpha$ $\exists$ $B \in \mathcal{B}_\alpha$ s.t $x \in B \subset U_\alpha$

for all these ~~at~~ α let $B = B_\alpha$ and for the

rest $B_\alpha = X_\alpha$ then we have a set as described

this is included in $\prod_{\alpha \in J} U_\alpha$ thus by lemma

13.2 again This is a basis element. $\blacksquare$