

Show that if X is normal every pair of disjoint closed sets have neighborhoods whose closures are disjoint.

Let A, B be disjoint closed subsets of X .
Let U_1, V_1 be disjoint open sets containing A, B . Then $\overline{U_1} \cap B = \emptyset$. Let U_2, V_2 be disjoint open subsets containing $\overline{U_1}, B$ then $\overline{V_2} \cap \overline{U_1} = \emptyset$ thus $\overline{U_1}, \overline{V_2}$ are the needed sets. \square