

Let α be a path in X from x_0 to x_1 . Let β be a path in X from x_1 to x_2 . Show that, if $\gamma = \alpha \circ \beta$ then $\hat{\gamma} = \hat{\beta} \circ \hat{\alpha}$ for any $[f]$

$$\hat{\gamma}(f) = [\overline{\alpha \circ \beta}] \circ [f] \circ [\alpha \circ \beta]$$

$$= \overline{\beta \circ \alpha} \circ [f] \circ \alpha \circ \beta$$

$$= \hat{\beta}(\overline{\alpha} \circ [f] \circ \alpha) = \hat{\beta} \circ \hat{\alpha}(f)$$