Let X denote the rational points of the interval $[0,1]\times 0$ in $\mathbb{R}^2$. Let T denote the union of all line segments joining $p = 0\times 1$ to points in X

a) Show that T is path connected ~~only at p.~~

la $x, y \in T$. I Assume the question refers to straight lines.

then x, y are both on some lines

$(t, 1-at)$ wher $at = r \iff t = \frac{1}{a} \in \mathbb{Q}$ $(a \in \mathbb{Q})$

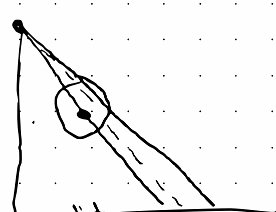
Let a_x, t_x s.t $x = (t_x, 1-a_x t_x)$,
a_y, t_y s.t $y = (t_y, 1-a_y t_y)$

then $f: [-t_x, t_y] \to T$ by

$$f(t) = \begin{cases} (-t, 1+a_x t) & , t < 0 \\ (t, 1-a_y t) & , t \geq 0 \end{cases}$$ is continuous

so T is path connected

locally connected only at p



Consider some point $x \in T$ $x \neq p$, $B(x, \varepsilon)$ then we have a line joining p and $r \times 0$, $r \notin \mathbb{Q}$ s.t this line intersects $B(x, \varepsilon)$ at more than one point. Then the line segments of T to the 'right' of this line and to the "left" forms a separation consider $B(p,\varepsilon) \to$ path connected in T

so connected.

b) find a subset of $\mathbb{R}^2$ is path connected but is locally conneted at none of its points

Define T_n as the union of all lines joining $0, \frac{1}{n}$ to elements in X, $\bigcup_{n \in \mathbb{N}} T_n$ is path connected but not locally connected at any point