Let $f: A \to B$. If there are functions $g: B \to A$ and $h: B \to A$ s.t $g(f(a)) = a$ for every $a \in A$ and $f(h(b)) = b$ for every $b$ in $B$ then $f$ is bijective and $g = h = f^{-1}$

first show that $f$ is surjective.

for every $b \in B$ $f(h(b)) = b$ thus $f$ is surjective

injective

need to show that there is only one value

let $a_1, a_2$ be s.t $f(a_1) = f(a_2) = b$

then $g(b) = g(f(a_1)) = a_1$

$= g(f(a_2)) = a_2$

so $a_1 = a_2$

thus $f$ is bijective.

we have $g(f(a)) = a$, $f(g(\underbrace{f(a)}_{\in B})) = f(a)$

so $g = f^{-1}$

$f(h(b)) = b$ and $h(f(\underbrace{h(b)}_{\in A})) = h(b)$.

so $h = f^{-1} = g$ ∎