

Given spaces  $X$  and  $Y$  Let  $[X, Y]$  be the set of homotopy classes of maps of  $X$  into  $Y$

a) Let  $I = [0, 1]$  show that for any  $X$ ,  $[X, I]$  has a single element

Let  $f, g \in [X, I]$ , then  $f$  and  $g$  are both continuous. Define  $F(x, t) = (1-t)f(x) + tg(x)$  then  $f$  and  $g$  are homotopic

b) Show that if  $Y$  is path connected the set  $[I, Y]$  has a single element.

Let  $f, g \in [I, Y]$  then these are both continuous, as  $Y$  is path connected then there is a path

$h$  is a path from  $f(0)$  to  $g(0)$

$$F(x, t) = \begin{cases} f(x(1-4t)) & , t \leq 1/4 \\ h(2(t-1/4)) & , 1/4 \leq t \leq 3/4 \\ g(x(4t-3)) & , 3/4 \leq t \end{cases}$$

$f(x(1-4t))$  is continuous as  $(x, t) \rightarrow x(1-4t)$  is continuous.  $h(2(t-1/4))$  and  $g(x(4t-3))$  likewise

By local formulation of continuity/pasting lemma

$F$  is continuous.