Let X be a topological space and let Y be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions. Let x_n be a sequence of points of X converging to x. Show that if the sequence f_n converges uniformly to f then $(f_n(x_n))$ converges to $f(x)$.

$d(f_n(x_n), f(x)) \leq d(f_n(x_n), f(x_n)) + d(f(x_n), f(x))$

let $\varepsilon > 0$. Let N_f s.t $\bar{d}(f_n, f) < \frac{\varepsilon}{2}$, $n \geq N_f$

let $\delta > 0$ s.t $d(f(x), f(y)) < \frac{\varepsilon}{2}$ when $d(x, y) < \delta$

let N_x s.t $d(x_n, x) < \delta$, $n \geq N_x$

let $N = \max\{N_x, N_f\}$

then $d(f_n(x_n), f(x)) < \varepsilon$ ∎