a) a real number is algebraic if it satisfies $x^n + a_{n-1}x^{n-1} + \dots + a_0 = 0$ with rational coefficients a_i. Assuming each polynomial has only finitely many roots show that the set of algebraic numbers is countable

For any degree "n" polynomial there is a bijection $\mathbb{Q} \times \mathbb{Q} \times \dots \times \mathbb{Q} \to P$

$$(a_0, a_1, \dots, a_{n-1}) \to x^n + a_{n-1}x^{n-1} + \dots + a_0 = 0$$

thus there are countably many polynomials of degree "n". The number of roots for a particular polynomial is finite and thus countable.

let $\{p_n^{(i)}\}_{i \in \mathbb{Z}^+}$ be the set of all polynomials for a given degree. $\{r_n^{(i,k)}\}_{k=1}^{K_i}$ the set of roots for $p_n^{(i)}$

then $\bigcup_{i \in \mathbb{Z}_+} \{r_n^{(i,k)}\}_{k=1}^{K_i} = A_n$ is countable and includes all algebraic integers that are roots up to degree "n". Since A_n is countable $\bigcup_{n \in \mathbb{Z}_+} A_n$ is countable and also the union of all algebraic integers

6) a real number is transcendental if it is not algebraic. Assuming real numbers are uncountable show that the transcendental numbers are uncountable.

$\mathbb{R} = T \cup A$, ~~as~~ if T is countable then $\mathbb{R}$ is countable $\lightning$