$X \sim N(0,1)$ $\sigma, b \in \mathbb{R}$, $x, K > 0$

show that

$$E[(xe^{b+\sigma X} - K)^+] = xe^{b+\frac{1}{2}\sigma^2}\Phi(-\xi+\sigma) - K\phi(\xi_0)$$

Where $\xi_0 = \frac{1}{\sigma}(\log\frac{K}{x} - b)$ and Φ is the partition function of $N(0,1)$ distribution

$$\text{"} E(xe^{b+\sigma X} > K) \text{"}$$

$$\cancel{x}e^{b+\sigma X} > K \to b + \sigma X > \log\frac{K}{x}$$

$$X > \frac{1}{\sigma}\left(\log\frac{K}{x} - b\right)$$

$$E[(xe^{b+\sigma X} - K)^+] = \int_{\xi_0}^{\infty}(xe^{b+\sigma z} - K)e^{-\frac{z^2}{2}}\,dz$$

$$= \int_{\xi_0}^{\infty} xe^{b+\sigma z}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\,dz - \overbrace{\int_{\xi_0}^{\infty}\frac{K}{\sqrt{2\pi}}e^{-z^2/2}\,dz}^{1-K\Phi(\xi_0) = K\Phi(-\xi_0)}$$

$$= \int_{\xi_0}^{\infty}\frac{1}{\sqrt{2\pi}}x\,e^{b}\,e^{-\left(\frac{z^2}{2}-\sigma z\right)}dz - K\Phi(-\xi_0) = \int_{\xi_0}^{\infty} xe^{b}e^{-\frac{(z-\sigma)^2}{2}}e^{+\frac{\sigma^2}{2}}dz - K\Phi(-\xi_0)$$

$$\left(\frac{z^2}{2} - \sigma z\right) = \left(\frac{z}{\sqrt{2}} - \frac{\sigma}{\sqrt{2}}\right)^2 - \frac{\sigma^2}{2}$$

$$= xe^{b+\frac{\sigma^2}{2}}\int_{\xi_0}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(z-\sigma)^2}{2}}dz - K\Phi(-\xi_0) = xe^{b+\frac{\sigma^2}{2}}\int_{\xi-\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz - K\Phi(-\xi_0)$$

$$= x e^{b+\frac{\sigma^2}{2}}\left(1-\phi(\xi_0-\sigma)\right)-K\phi(-\xi_0)$$

$$= x e^{b+\frac{\sigma^2}{2}}\phi(\sigma-\xi_0)$$