a real r.v X partition function

is $F(t) = p(X \leq t)$

Show that 2 r.v.'s have same partition function $\Leftrightarrow$ they have the same distribution

X, Y are r.v's

Let μ_X, μ_Y be their respective laws

for any t $A_{X,t} = \{a \in \Omega : X(a) \leq t\}$

$A_{Y,t} = \{a \in \Omega : Y(a) \leq t\}$

" $\Leftarrow$ "

$\{x \in \Omega : X(x) \leq t\} = \{y \in \Omega : Y(y) \leq t\}$

$\Rightarrow$

$P(X \leq t) = P(Y \leq t) = F(t)$

or $\mu_X(-\infty, t) = \mu_Y(-\infty, t) = p(X \leq t)$

$= P(Y \leq t)$

" $\Rightarrow$ " We have that $P(a \leq X \leq b) = F(b) - F(a)$

Since (a, b) generates the borel-σ-alg on $\mathbb{R}$

and $\cancel{\mu_X(\mathbb{R}) =}$ $\mu_X(-\infty, \infty) = \mu_Y(-\infty, \infty) = 1$

Caratheodory's criterion gives us that they are equal