Show that if X is a countable product of spaces having countable dense subsets then X has a countable dense subset

let $X = \prod_{n=1}^{\infty} X_n$, Y_n is a countable set dense in X_n , let $(a_n)_{n=1}^{\infty}$ be a sequence of numbers s.t $a_n \in X_n$.

Let $A_n = \prod_{k=1}^{n} Y_n \times \prod_{k=n+1}^{\infty} \{a_k\}$, A_n is homeomorphic to $\prod_{k=1}^{n} Y_n$ an is countable.

Let $A = \bigcup_{n=1}^{\infty} A_n$, this is countable being a countable union of countable sets.

take any basis element $B = \prod_{n=1}^{\infty} U_n \subset X$

then for some N $U_n = X_n$, $n \geq N$.

for $n < N$ $U_n \cap Y_n \neq \emptyset$ by density of Y_n.

then $B \cap \cancel{A}_N \neq \emptyset$ so B intersects ~~A~~

and thus $\overline{A} = X$ so A is dense in X.