Show that $\mathbb{Q}$ is not locally compact.

if $(a,b)\setminus\mathbb{R}$ is contained in a compact set then $[a,b]\setminus\mathbb{R}$ is compact. Let $c\in\mathbb{R}\setminus\mathbb{Q}$ s.t $a<c<b$ then

$$\left(\left(\bigcup_{n=2}^{\infty}\left(a-1,\, c-\frac{c-a}{n}\right)\right)\cup\left(\bigcup_{n=2}^{\infty}\left(c+\frac{b-c}{n},\, b+1\right)\right)\right)\setminus\mathbb{R}$$ is an open

cover of $[a,b]\setminus\mathbb{R}$ with no finite subcover.