Let $\{A_\alpha\}$ be a collection of subsets of X: let $X = \bigcup A_\alpha$. Let $f: X \to Y$, suppose that $f|_{A_\alpha}$ is continuous for each α

a) Show that if the collection $\{A_\alpha\}$ is finite and each set A_α is closed then f is continuous.

Suppose that for some $N \in \mathbb{N}$ $\{A_\alpha\} = \{A_n\}_{n \le N}$ f is continuous. Want to show that this holds for $N+1$

$X = \bigcup_{n=1}^{N+1} A_n$ A_n is closed let $f_n = f|_{A_n}$

then for a closed set $C \subset Y$, $g = f|_{\bigcup^N A_n}$

$f^{-1}(C) = \bigcup f_n^{-1}(C) = g^{-1}(C) \cup f_{N+1}^{-1}(C)$ which is closed. Since this holds for $N=2$ it holds for all $N < \infty$. so f is continuous

b) Find an example where $\{A_\alpha\}$ is countable and each A_α is closed but f is not continuous

$A_n = \bigcup_{k \in \mathbb{Z}} [k, k+\frac{1}{n+1}]$, $\bigcup_{n \in \mathbb{N}} A_n = \mathbb{R}$

$f(x) = \{k : k \le x < k+1\}$ f is continuous

on each A_n but not their union

An indexed family of sets $\{A_\alpha\}$ is said to be locally finite if each $x \in X$ has a neighborhood that intersects A_α for finitely many values α. Show that if $\{A_\alpha\}$ is locally finite and all A_α are closed then f is continuous.

Let $\mathcal{U}_x$ be ~~the~~ a neighborhood of x s.t. $\mathcal{U}_x$ intersects only finitely many A_α. by (a) f is continuous in this finite union of A_α so f is continuous in $\mathcal{U}_x$ but $X = \bigcup_{x \in X} \mathcal{U}_x$ so f is continuous by "local formulation of continuity"