A space is <u>totally disconnected</u> if its only connected subspaces are one-point sets. Show that if X has the discrete topology it is totally disconnected.

Clearly any one-point subspace of X is connected. (This is true for all topologies)

let $\mathcal{U}$ be a subset of X containing more than one element and give $\mathcal{U}$ the subspace topo. let $x \in \mathcal{U}$. $\{x\} = \{x\} \cap \mathcal{U}$ so $\{x\}$ is both open and closed in $\mathcal{U}$ thus $\mathcal{U} \setminus \{x\}$ is open and closed

so $\{x\}, \mathcal{U} \setminus \{x\}$ is a separation of $\mathcal{U}$.

Conversely take lower limit topology to see that it is not true.

let $X^i_{B_r}$ be the collection of tuples $\tilde{x}$ s.t $\tilde{x}_k = x_k$, $k \neq i$, $\tilde{x}_i \in B(x_i, r)$