A space is <u>totally disconnected</u> if its only connected subspaces are one-point sets. Show that if X has the discrete topology it is totally disconnected.

Clearly any one-point subspace of X is connected. (This is true for all topologies)

Let $\mathcal{U}$ be a subset of X containing more than one element and give $\mathcal{U}$ the subspace topo let $x \in \mathcal{U}$. $\{x\} = \{x\} \cap \mathcal{U}$ so $\{x\}$ is both open and closed in $\mathcal{U}$ thus $\mathcal{U}\setminus\{x\}$ is open and closed so $\{x\}, \mathcal{U}\setminus\{x\}$ is a separation of $\mathcal{U}$.

Consider $\mathbb{R}^\omega$ in the box topology let U be a subspace of $\mathbb{R}^\omega$ with more than one element. Let $x \in U$ an $\mathcal{U}$ neighborhood of of x that is different from U.

This is possible as we can define ε_i by

$$\varepsilon_i = \inf\left(\sup_{y \in U} |y_i - x_i|, \sup\{r : x^{i}_{B_r} \in U\}\right) \text{ and let } \mathcal{U} = \prod B\left(x_i, \tfrac{\varepsilon_i}{2}\right)$$

but then ~~for~~ for $y \in U \setminus \mathcal{U}$ we can create a basis element similarly. Thus $U = \underbrace{(U\setminus\mathcal{U})}_{\text{open}} \cup \underbrace{\mathcal{U}}_{\text{open}}$

so $\mathbb{R}^\omega$ is totally disconnected in the ~~box~~ box topology.

* let $X^i_{B_r}$ beth the collection of tuples $\tilde{x}$ s.t $\tilde{X}_k = X_k$, $k \neq i$, $\tilde{X}_i \in B(X_i, r)$