Consider the map $h: \mathbb{R}^\omega \to \mathbb{R}^\omega$ defined in exercise 8 of chap (19)

$h((x_1, x_2, \ldots)) = (a_1 x_1 + b_1, a_2 x_2 + b_2, \ldots)$ $\quad a_i > 0$

give $\mathbb{R}^\omega$ the uniform topology. Under what conditions on a_i, b_i is h continuous? a homeomorphism.

Consider ~~an~~ a basis element $B(x, \varepsilon$

$y = ax + b \to x = \frac{y-b}{a}$

$|a_i x_i + b_i - (a_i y_i + b_i)| = |a_i (x_i - y_i)| < \varepsilon \iff$

$|x_i - y_i| < \frac{\varepsilon}{a_i}$ if $a_i \to \infty$ then the inverse image of h is not open since $\frac{\varepsilon}{a_i} \to 0$ for all ε. In other words any ~~nbh~~ neighborhood of $h^{-1}(B(x, \varepsilon))$ contains points not in $h^{-1}(B(x, \varepsilon))$

For a homeomorphism we need h^{-1} to be continuous, similar to before

$h^{-1}(x) = \left(\frac{x_1 - b_1}{a_1}, \ldots\right) = \left(\frac{1}{a_1} x_1 - \frac{b_1}{a_1}, \ldots\right) = (\tilde{a}_1 x_1 - \tilde{b}_1, \ldots$

so $\tilde{a}_n$ must be bounded.