a) Show that no two of the spaces $(0,1)$, $(0,1]$, $[0,1]$ are homeomorphic.

suppose we have a homeomorphism $f:(0,1] \to (0,1)$ then $\tilde{f}:(0,1) \to (0,1) \setminus f(1)$ by $\tilde{f}(x) = f(x)$ is a homeomorphism. but $(0,1) \setminus f(1)$ is not connected $\lightning$

argue similarly for the remaining two cases.

b) suppose that there exists imbeddings $f: X \to Y$ and $g: Y \to X$. Show by means of example that X and Y need not be homeomorphic

$f:(0,1) \to [0,1]$ by $f(x) = x$. $g:[0,1] \to \mathbb{R} \simeq (0,1)$ $g(x) = x$ but then we have an imbedding into $(0,1)$. As $(0,1) \not\simeq [0,1]$ we are done

c) Show that $\mathbb{R}$ and $\mathbb{R}^n$ are not homeomorphic.

Removing a point from $\mathbb{R}^2$ it is still path connected and thus connected, $\mathbb{R}$ is not. By induction it follows that $\mathbb{R}^n$, $n > 1$ is not homeomorphic to $\mathbb{R}$