Let X be a compact Hausdorff space. Suppose that for $x \in X$ there is a neighbor-hood U of x and a positive integer k s.t U can be imbedded in $\mathbb{R}^k$. Show that X can be imbedded in $\mathbb{R}^N$ for some positive N.

We take some inspiration from the proof of 36.2:

Let $\{U_1, \dots, U_n\}$ be a finite cover of X s.t U_i can be imbedded in $\mathbb{R}^{k_i}$. Let g_i be the imbeddings.

As X is normal let $\phi_1, \dots, \phi_n$ be a partition of unity dominated By $\{U_i\}$

Again let $A_i = \operatorname{supp} \phi_i$ and

$$h_i(x) = \begin{cases} \phi_i(x) \cdot g_i(x) ; & x \in U_i \\ 0 & ; x \in X \setminus A_i \end{cases}$$

and $F : X \to \underbrace{\mathbb{R} \times \dots \times \mathbb{R}}_{n} \times \underbrace{\mathbb{R}^{k_1} \times \dots \times \mathbb{R}^{k_n}}_{n}$

$$F(x) = (\phi_1(x), \dots, \phi_n(x), h_1(x), \dots, h_n(x))$$

by similar argument as in the proof this is continuous and injective

(the only difference is the k_i's)