Let X and X' denote a single set in the topologies $\mathcal{T}$ and $\mathcal{T}'$ respectively. Let $i: X' \to X$ be the identity function

(a) show that i is continuous $\iff \mathcal{T}' \supset \mathcal{T}$
finer than

"$\Rightarrow$" if A is open in X then $i^{-1}(A)$ is open in X'. Since we didnot specify X or X', X could be the Whole space. Thus every $A \in \mathcal{T}$ is also in $\mathcal{T}'$ so $\mathcal{T}' \supset \mathcal{T}$

"$\Leftarrow$" again we can assume X to be the entire space since this is in $\mathcal{T}$ thus for $A \in \mathcal{T}$ $i^{-1}(A) = A \in \mathcal{T}'$ so the inverse image of any open set is open thus i is continuous $\boxtimes$