Show that the onepoint compactification of $\mathbb{R}$ is homeomorphic with the circle S^1

We construct $Y = \mathbb{R} \cup \infty$ and give Y the topology $\mathcal{T} = \mathcal{T}_X \cup \{Y - C : C \text{ is compact in } X\}$

want to show this is homeomorphic to the cirkle for this case we take the circle from $-\pi \to \pi$

let $f(0) = 0$ $\quad f(x) = \frac{x}{\sqrt{1+x^2}}\pi$ $\quad$ bijective on $(-\pi, \pi)$

$$f(\pm\infty) = \pm\pi = \pi$$

f is continuous

$$y = \frac{x}{\sqrt{1+x^2}}\pi \Rightarrow \sqrt{1+x^2}\,y = x\pi$$

$$(1+x^2)y^2 = x^2\pi^2$$

$$y^2 = x^2(\pi^2 - y^2)$$

$$x^2 = \frac{y^2}{\pi^2 - y^2}$$

$$x = \frac{y}{\sqrt{\pi^2 - y^2}}$$

homeomorphic ~~continuous~~ on $\mathbb{R}$

for a set $Y \setminus C$ we have that $f(C)$ is compact in S^1 thus $S^1 \setminus f(C)$ is open on the rel topology.