$X:(\Omega, F) \to (E, \mathcal{E})$, $D \subset \mathcal{E}$ a a family of subsets E s.t $\sigma(D) = \mathcal{E}$. Assume $X^{-1}(A) \in F \ \forall A \in D$ show that X is measurable

Let $B \in \mathcal{E}$. Then B is a countable number of intersekt, union and complement -operations from D

$$B = \bigcup_{i=1}^{\infty} A^{(i)} \quad \text{where} \quad A^{(i)} = \bigcap_{n=1}^{N_i} A_n^{(i)}$$

Where $A_n^{(i)}$ is either an element in D or the complement of an element

X^{-1} preserves Unions thus

$X^{-1}(B) = \bigcup X^{-1}(A^{(i)})$ want to check that $X^{-1}(A^{(i)}) \in F$. since $X^{-1}(A) \in F \ \forall A \in D$ $X^{-1}(A)^c \in F$. $X^{-1}(A)^c = \{\omega \in \Omega \mid X(\omega) \notin A\}$

$$= X^{-1}(A^c)$$

thus for $A \in D$, $X^{-1}(A^c) \in F$ so.

let us then writ $A_n^{(i)}$ as $A_n^{(i)c}$ since this is still in D or the complement of some element in D. then $X^{-1}\left(\bigcap_{n=1}^{N} A_n^{(i)c}\right) \neq \left(\bigcup_{n=1}^{N} X^{-1}(A_n^{(i)})\right)^c$

$X^{-1}(A_n^{(i)}) \in \mathcal{F}$, thus their union is
and thus the complement of their union