

Let X be locally path connected. Show that every connected open set in X is path connected.

Let C be connected open set in X .

$x \in C$, P - path components, Q not path components \rightarrow both are open

$C = P \cup Q \rightarrow$ both closed. This contradicts C being connected.