Let A and B be disjoint compact subspaces of the Hausdorff space X. show that there exists disjoint open sets U and V containing A and B respectively

We know that for $a \in A$ there exists disjoint sets $U_a, V_a$ s.t $a \in U_a$, $B \subset V_a$

$A \subseteq \bigcup_{a \in A} U_a$, but as A is compact we can pick $a_1, \ldots, a_n$ s.t $\bigcup_{k=1}^{n} U_{a_k} \supseteq A$

$U =$

$V = \bigcap_{k=1}^{n} V_a$, $V \cap U = V \cap \bigcup_{k=1}^{n} U_{a_k}$

$= \bigcup_{k=1}^{n} (V \cap U_{a_k}) = \phi$ ∎