8 (apply lemma 13.2) to show that the countable ~~union~~ collection

$\mathcal{B} = \{(a,b) \mid a<b,\ a,b \in \mathbb{Q}\}$ is a basis that generates the standard topology on $\mathbb{R}$

using lemma 13.2
let (a,b) be an open set on $\mathbb{R}$
by well-ordering $\exists$ $c \in \mathbb{R}$ s.t
$a<c<b$, by density of $\mathbb{Q}$ in $\mathbb{R}$ $\exists$
$a^*, b^* \in \mathbb{Q}$ s.t $a < a^* < c < b^* < b$
then $(a^*, b^*) \subset (a,b)$ and thus by lemma 13.2 $\mathcal{B}$ ~~gene~~ is a basis for ~~the~~ standard topology

W.O.
let (a,b) be an non-empty open set on $\mathbb{R}$
let $x \in (a,b)$. By density of $\mathbb{Q}$ in $\mathbb{R}$ and well ordering $\exists$ $a^*, b^* \in \mathbb{Q}$ s.t $a<a^*<x<b^*<b$
then $(a^*, b^*) \subset (a,b)$ and $(a^*, b^*)\ \mathcal{B}$ ☒

$\longrightarrow$ bad solution

Standard topology is the open sets on $\mathbb{R}$

let $(a,b) \in \mathcal{T}$ $x \in (a,b)$

let $r = \min\{d(x,a), d(x,b)\}$

if $x \in \mathbb{Q}$ let $B = \bigcup_{\substack{\varepsilon \in \mathbb{Q} \\ \varepsilon < r/2}} B(x,\varepsilon)$ $\leftarrow \in \mathcal{B}$

else find $x^* \in \mathbb{Q}$ s.t. $d(x,x^*) < r/2$

then $x \in \bigcup_{\substack{\varepsilon \in \mathbb{Q} \\ \varepsilon < r/2}} B(x,\varepsilon)$, $B(x,\varepsilon) \in \mathcal{B}$ $\forall$ ε $<$