If A is a connected subspace of X, does it follow that $\operatorname{int} A$ and $\operatorname{Bd} A$ are connected? does the converse hold.

∂A is not necesarily connected:

Conside: $S = \{(x,y) \in \mathbb{R}^2 : |x|^2 + |y|^2 \leq 1\} \setminus \{0\}$

then $\{0\} \cup \{(x,y) \in \mathbb{R}^2 : |x|^2 + |y|^2 = 1\}$ is a separation of ∂A

Same for $\operatorname{int} A$.

$$A = \{x \in \mathbb{R}^2 : |x - (-1,0)|^2 \leq 1\} \cup \{x \in \mathbb{R}^2 : |x - (1,0)|^2 \leq 1\}$$

A is connected but not $\operatorname{int} A$

Don't have any implications other direction