Show that if X is an infinite set, it is connected in the finite complement topology.

$\tau_f = \{U \subset X : X \setminus U$ is finite or all of $X\}$

let $\emptyset \neq F \subset X$ be finite. then F is closed.

$X \setminus F$ is infinite so F is not open.

let $X \neq U \in \tau_f$, U is not closed as its complement is not open. Then there is no separation of X so it is connected.