$X = \{0,1\}$ show that there is a bijective correspondance between $P(\mathbb{Z})$ and X^ω

denote $x \in X^\omega$ by $(x_1, x_2, \ldots$

$g : X^\omega \to P(\mathbb{Z}_+)$ by $g(x) = \{i \mid x_i = 1\}$

this is injective

$h : P(\mathbb{Z}) \to X^\omega$ by $h(A)_i = 1$ if $i \in A$ else 0

for any $x \in X$ $h(g(x)) = h(\{i \mid x_i = 1\})$

$$h(\{i \in \mathbb{Z}_+ \mid x_i = 1\})_j = \begin{cases} 1 & \text{if } j \in \{i \in \mathbb{Z}_+ \mid x_i = 1\} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 1 & \text{if } x_j = 1 \\ 0 & \text{if } x_j = 0 \end{cases}$$

$$= x_j$$

$\to h(g(x)) = x \to h$ is surjective

$g(h(A)) = \{i \mid h(A)_i = 1\}$

$= \{i \mid i \in A\} = A$ surjective

thus a bijection