Let X be a Hausdorff space s.t each $x \in X$ has a neighborhood that is homeomorphic with an open subset of $\mathbb{R}^m$. Show that if X is compact it is an m manifold.

Want to show that X has a countable Basis.

We know that $\mathbb{R}^m$ is 2nd countable.

As X is compact we can let $\{U_1, \ldots, U_N\}$ be a collection of sets covering X s.t U_i is homeomorphic to an open subset of $\mathbb{R}^m$ by f_i. Consider the set $\underbrace{\bigcup_{i=1}^{N} \{f_i^{-1}(b) : b \text{ is a basis element of } \mathbb{R}^m\}}_{B}$

this is a countable set. Check if it's a basis. Let $\mathcal{U}$ be an open set, $x \in \mathcal{U}$, then $x \in U_i$ for some i. $U_i \cap \mathcal{U}$ is open and contains x, $f_i(U_i \cap \mathcal{U})$ is open in $f_i(U_i)$ thus there is a basis element $b \in \mathbb{R}^m$ containing $f(x)$.

then $x \in f_i^{-1}(b) \subset \mathcal{U}$ and open so B is a basis