a) show that a connected normal space having more than one point is uncountable

argue contrapositively

Assume X is countable. Using Urysohn's lemma take any 2 disjoint closed sets A and B and construct $f: X \to [0,1]$ s.t $f(a)=0$, $f(b)=1$, $a \in A$, $b \in B$. Now f cannot be injective as X is countable. Thus we have some element $r \in [0,1]$ s.t $r \notin f(X)$ then $f^{-1}([0,r))$ and $f^{-1}((r,1])$ is a separation of X so X is not connected ∎