

Let $\{A_n\}$ be a sequence of connected subspaces of X , s.t. $A_n \cap A_{n+1} \neq \emptyset$ for all n . Show that $\bigcup A_n$ is connected.

Assume U, V is a separation of $\bigcup_{n \in \mathbb{N}} A_n$.

let $a \in A_1$ and assume that $a \in U$. Then by lemma 23.2 $A_1 \subset U$. Assuming that U contains A_n then as $\exists a \in A_n \cap A_{n+1}$, U also contains A_{n+1} . Thus $U = \bigcup_{n \in \mathbb{N}} A_n \rightarrow V = \emptyset$ which contradicts our assumption \hookrightarrow