Define $f_n : [0,1] \to \mathbb{R}$ by $f_n(x) = x^n$. Show that the sequence $(f_n(x))$ converges for each $x \in [0,1]$ but that the sequence does not converge uniformly.

- pointwise: $f_n(0) = 0 \ \forall n$, $f_n(1) = 1 \ \forall n$

if $x \in (0,1)$ then for any $\varepsilon > 0$ pick N s.t $x^n < \varepsilon$ for $n \geq N$. then $f_n(x) \to 0$

- Assume that it does converge uniformly.

then for any $\varepsilon > 0$ $\exists$ $\exists N \in \mathbb{N}$ s.t

$d(f_n(x), f(x)) < \varepsilon$, $\forall n \geq N$, $x \in [0,1]$

assuming that $\varepsilon < 1$ then $\varepsilon^{1/N} < 1$

pick $x \in (\varepsilon^{1/N}, 1)$ then $f_n(x) \to 0$

but $x^N > \varepsilon$ $\lightning$