

$Y = [-1, 1]$ as a subspace of \mathbb{R}

Which of the following sets are open in Y which in \mathbb{R} ?

$$A = \{x \mid \frac{1}{2} < |x| < 1\}$$

$$B = \{x \mid \frac{1}{2} < |x| \leq 1\}$$

$$C = \{x \mid \frac{1}{2} \leq |x| < 1\}$$

$$D = \{x \mid \frac{1}{2} \leq |x| \leq 1\}$$

$$E = \{x \mid 0 < |x| < 1, \frac{1}{x} \notin \mathbb{Z}_+\}$$

We see that the basis of Y consists of sets of the form:

$$(a, b), \quad -1 \leq a < b \leq 1$$

$$[-1, a), \quad -1 < a < 1$$

$$[a, 1], \quad -1 < a < 1$$

$$\emptyset \text{ or } [-1, 1]$$

$A = (-1, -\frac{1}{2}) \cup (\frac{1}{2}, 1)$ open in \mathbb{R} and thus in Y

$B = [-1, -\frac{1}{2}) \cup (\frac{1}{2}, 1]$ not open in \mathbb{R} but open in Y

$C = \{(-1, -\frac{1}{2}] \cup [\frac{1}{2}, 1)\}$ not open in Y

$D \rightarrow$ not open in Y

$$E = (-1, 0) \cup ((0, 1) \setminus (\bigcup (\frac{1}{n}))$$

$(-1, 0)$ is open

$$(0, 1) \setminus (\bigcup_{n \in \mathbb{N}} \{\frac{1}{n}\}) = A$$

if $x \in A$ let $a = \sup \{ \frac{1}{n} < x \}$

$$b = \inf \{ \frac{1}{n} > x \}$$

then $a^* \leq x$ s.t. $a < a^* < x$

$b^* \leq x$ s.t. $x < b^* < b$

thus $x \in (a^*, b^*) \rightarrow$ open

so E is open in X