

Which sets of  $\mathbb{R}^\omega$  can be expressed as the cartesian product of subsets of  $\mathbb{R}$

$$a) \{x \mid x_i \text{ is an integer for all } i\}$$

$$= \mathbb{Z} \times \mathbb{Z} \times \dots \times \mathbb{Z} = \mathbb{Z}^\omega$$

$$b) \{x \mid x_i \geq i \text{ for all } i\} = [1, \infty) \times [2, \infty) \times \dots \times [i, \infty) \times \dots$$

$$c) \{x \mid x_i \text{ is an integer for } i \geq 100\}$$

$$= \mathbb{R}^{99} \times \mathbb{Z}^\omega$$

$$d) \{x \mid x_2 = x_3\}$$

assume that there is such a subset  $A$  then  $\mathbb{R} \times A \times A \times \mathbb{R}^\omega$

since  $x_2$  can take any value in  $\mathbb{R}$   
 $A = \mathbb{R}$  but  $(1, 2, 3, \dots) \in \mathbb{R}^\omega \not\subseteq$