Show that $(X_1 \times X_2 \times \cdots \times X_{n-1}) \times X_n$ is homeomorphic with $X_1 \times \cdots \times X_n$.

Want to find a bijective function $f: (X_1 \times \cdots \times X_{n-1}) \times X_n \to X_1 \times \cdots \times X_n$ that is continuous and f^{-1} is continuous as well

denote by x^{n-1} an $(n-1)$ tuple in $(X_1 \times \cdots X_{n-1})$

let then $f((x^{n-1}, x_n)) = (x^{n-1}_0, x^{n-1}_2, \ldots, x^{n-1}_{n-1}, x_n)$

$f^{-1}((x_1, \ldots, x_n)) = (x^{n-1}, x_n)$ where $x^{n-1}_k = x_k \in X_k$

So f is bijective

let A_k be open in X_k for $1 \leq k \leq n$

then $f^{-1}\left(\prod_{k=1}^{n} A_k\right) = \left(\prod_{k=1}^{n-1} A_k, A_n\right)$ which is open in $(X_1 \times \cdots \times X_{n-1}) \times X_n$ ~~Simil~~ so f is continuous.

Let now A be an open set in $(X_1 \times \cdots X_{n-1})$

then it is a union of set of the form $\prod_{k=1}^{n-1} U_k$ where U_k is open in A_k i.e

$A = \bigcup_{i \in I} \left(\prod_{k=1}^{n-1} U_k^{(i)}\right)$ thus $f((A, A_n)) = f\left(\left(\bigcup_{i \in I} \left(\prod_{k=1}^{n-1} U_k^{(i)}\right), A_n\right)\right)$

$= f\left(\bigcup_{i \in I}\left(\left(\prod_{k=1}^{n-1} U_k^{(i)}, A_n\right)\right)\right) = \bigcup_{i \in I} f\left(\prod U_k^{(i)}, A_n\right)$

$= \bigcup_{i \in I} (U_1^{(i)}, \ldots, U_{n-1}^{(i)}, A_n)$ this is a union of open sets so f^{-1} is continuous ∎