Let X and Y be metric spaces with metrics d_X and d_Y respectively. Let $f: X \to Y$ have the property that for every pair of points x_1, x_2 of X,

$$d_Y(f(x_1), f(x_2)) = d_X(x_1, x_2)$$

Show that f is an imbedding. This is called an <u>isometric imbedding</u>

f is injective since if $f(x_1) = f(x_2) = y$ then $d_Y(f(x_1), f(x_2)) = d_X(x_1, x_2) = 0 \iff x_1 = x_2$

clearly the map of any ε-ball is open since it is itself an ε-ball.

Considering the subspace $f(X) \cap Y$ of Y we see that the inverse image of any ball is a ball as well. Thus f is a homeomorphism on $f(X) \cap Y$ so it is an imbedding.