Prove that for $f: \mathbb{R} \to \mathbb{R}$ the ε-δ definition of continuity implies the open set definition

let $f: \mathbb{R} \to \mathbb{R}$ be continuous that is for $x \in \mathbb{R}$, $\varepsilon > 0$ $\exists$ δ s.t

$|f(x) - f(y)| < \varepsilon$ when $|x-y| < \delta$

let $B \subset \mathbb{R}$ be an open set $A = f^{-1}(B)$

then for $x \in B$ $\exists$ ε s.t $(x-\varepsilon, x+\varepsilon) \in B$

then if $\tilde{x} \in \mathbb{R}$ s.t $f(\tilde{x}) = x$ $\exists$ δ s.t

$f(\tilde{x}-\delta, \tilde{x}+\delta) \subset (x-\varepsilon, x+\varepsilon)$ thus $f^{-1}(B)$ is open.

another way Assume $f^{-1}(B)$ is not open

then $\exists$ some $x_0 \in f^{-1}(B)$ s.t $\forall$ $\delta > 0$

$(x_0-\delta, x_0+\delta) \not\subset f^{-1}(B)$ but $\exists$ some $\exists$

s.t $(f(x_0)-\varepsilon, f(x_0)+\varepsilon) \subset B$ $\to$ f is not continuous.