Let $\mathcal{C}$ be a collection of subsets of the set X. Suppose $\emptyset$ and X are in $\mathcal{C}$ and that finite unions and arbitrary intersections of elements in $\mathcal{C}$ are in $\mathcal{C}$. Show that

$$\mathcal{T} = \{X \setminus C \mid C \in \mathcal{C}\} \text{ is a topology on } X$$
$$= \{C^c \mid C \in \mathcal{C}\}$$

1) $\emptyset \in \mathcal{C}$ so $X \in \mathcal{T}$, $X \in \mathcal{C}$ so $\emptyset \in \mathcal{T}$

2) let $C_1, \ldots, C_n \in \mathcal{C}$

$$\bigcap C_k^c = \left(\bigcup_{k=1}^{n} C_k\right)^c \in \mathcal{T} \text{ since } \bigcup_{k=1}^{n} C_k \in \mathcal{C}$$

3) let $\{C_i\}_I$ be a sequence from $\mathcal{C}$

$$\bigcup_{i \in I} C_i^c = \left(\bigcap C_i\right)^c \in \mathcal{C}$$

so $\mathcal{T}$ is a topology on X