Prove that every manifold is regular and hence metrizable. Where do you use the Hausdorff condition?

Let X be an m-manifold, $x \in X$, B a closed set disjoint from x. Let U be a neighborhood of x that is homeomorphic to an open subset of $\mathbb{R}^m$. Let f be the homeomorphism. Let $y = f(x)$. If $B \cap U = \Phi$ then let V be a neighborhood of y s.t $\bar{V} \subset f(U)$. then $f^{-1}(V)$ and $f^{-1}(\bar{V})^c$ satisfies our requirements.

otherwise $B \cap U$ is closed in U as a subspace thus $f(B \cap U)$ is closed in $f(U)$. Let W, V be disjoint open sets of $f(U)$ containing y and $f(B \cap U)$ then $f^{-1}(W)$ and $f^{-1}(V)$ are the sets we are looking for.

Then by urysohn it is metrizable