Show that if Y is a subspace of X and A is a subset of Y the the topology A inherits as a subspace of Y is the same as A inherits as a subspace of X

$$\tau_{A_Y} = \{A \cap U_Y \mid U_Y \in \tau_Y\}$$

$$\tau_{A_X} = \{A \cap U_X \mid U_X \in \tau\}$$

let $U \in \tau_{A_Y}$ $\exists$ some $U_Y \in \tau_Y$ s.t $U = A \cap U_Y$

$U_Y = Y \cap U_X$, $U_X \in \tau$ so

$$U = A \cap (Y \cap U_X) = A \cap U_X \in \tau_{A_X}$$

let $U \in \tau_{A_Y}$. $\exists$ some $U_X \in \tau$ s.t

$$U = A \cap U_X = A \cap (Y \cap U_X) = A \cap U_Y \to U_X \in \tau_Y$$