if Y sub X

then $A \subset Y$ is closed $\iff$ $A = Z \cap Y$, Z closed in X

"$\Rightarrow$"

$Y \setminus A$ is open (in Y) so $Y \setminus A = \mathcal{U}$, $\mathcal{U} \in \mathcal{T}_X$

$\mathcal{U}^c$ is closed, $Y \cap \mathcal{U}^c = A$ $\blacksquare$

A is closed in X

"$\Leftarrow$" ~~$Z A X$~~ Z^c open $Z^c \cap Y$ is open

$\underline{Y \setminus Z^c}$ is closed $= Y \cap Z$

17.5

Y sub X

$A^c = X \setminus A = X \setminus (Y \cap Z)$ ~ open $\rightarrow$

closed

Y sub X , $A \subset Y$

Theorem 17.4 show that

$\bar{A}_Y = \bar{A}_X \cap Y$

"$\subset$" clear

"$\supset$" $a \in \bar{A}_X \cap Y$ then $a \in$ Y and a in all closed sets from X intersecting Y but these are closed in Y so

17.5

$A \subset X$, $x \in \bar{A} \iff$ every neighborhood of x intersects A

"$\Rightarrow$"

x has a neighborhood that does not intersect A then x is in c

so $x \in U \subset A^c$ so $x \in \operatorname{int} A^c = \bar{A}^c$

$\overline{A}^c = \text{int}(A^c)$ (1)

"$\subset$" $x \in \overline{A}^c$ $\exists$ $\mathcal{U}$ of x that does not intersect A so $x \in \text{int}(A^c)$

"$\supset$" $x \in \text{int}(A^c)$, same argument, $x \notin \overline{A} \to x \in \overline{A}^c$

$\overline{(\text{int}A^c)}^c = \text{int}\,\overline{A}$ (2)

use (1): $\overline{(\text{int}A^c)}^c = \text{int}((\text{int}A^c)^c)$

show $(\text{int}(A^c))^c = \overline{A}$:

"$\supset$"

$a \in \overline{A}$, all neighborhoods intersect A

So a is not in $\text{int}(A^c)$ so $a \in \text{int}(A^c)^c$

"$\subset$"

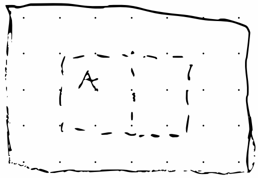
$a \in \text{int}(A^c)^c$ a not in $\text{int}(A^c)$ so there is no neighborhood that does not intersect A

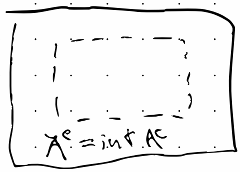
i.e $a \in \overline{A}$

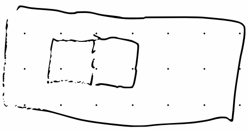
$\text{int}A^c \to \overline{(\text{int}A^c)} \to \text{int}\,\overline{A} \to \overline{A} \to \text{int}A^c$

take som A:

$A \to \overline{A} \to \overline{A}^c = \text{int}A^c \to \overline{\text{int}A^c} \to \text{int}\,\overline{A} \to \overline{A}$







$A = (\quad)(\quad)\quad\{\}\ \{\}$

$\bar{A} = [\quad]\quad\{\}\ \{\}$

$\bar{A}^c = (\quad)\quad(\quad)(\quad)(\quad)$

$(0,1)\cup(1,2)\cup\{3\}\cup\{4\}$