Show that every compact ~~met~~ metrisable space X has a countable basis. Hint: Le $\mathcal{A}_n$ be a finite covering of X by $1/n$-balls.

Let d be ~~the~~ metric inducing the topology $\mathcal{T}$ on X.
As X is sequentially compact there is a finite collection of ε-Balls covering X. (Proved in Theorem 28.2).

Following the hint let then $\mathcal{A}_n$ be a finite covering of X by $1/n$-balls. Then let
$\mathcal{A} = \bigcup_{n=1}^{\infty} \mathcal{A}_n$ this is countable being a countable union of finite sets.

Let $\mathcal{U} \in \mathcal{T}$, $x \in \mathcal{U}$ then for some $\varepsilon > 0$
$B(x,\varepsilon) \subset \mathcal{U}$ pick N s.t $\frac{1}{N} < \frac{\varepsilon}{2}$. as $\mathcal{A}_N$ covers X there is an elemen $A \in \mathcal{A}_N$ s.t $x \in A$
But then $A \subset B(x,\varepsilon) \subset \mathcal{U}$ as $\operatorname{diam} A < \varepsilon$.
Thus $\mathcal{A}$ is a basis ■