Show that U is an open connected subspace of $\mathbb{R}^2$, then U is path connected.
[Hint: Show that given $x_0 \in U$, the set of points that can be joined to x_0 by a path in U is both open and closed in U.]

Let $x_0 \in U$. Let J be the set of points that can be joined to x_0 by a path in U.
let $y \in J$. Then as U is $\exists$ some ball $B(y, \varepsilon) \subset U$. We know that balls are path connected so J is open.
let $y \in J^c$ again we have a ball $B(y, \varepsilon') \subset U$ that is path connected. Thus J^c is open $\to J$ is closed.
As U is connected the only subsets that are both open and closed are Φ and U.
Assuming $U \neq \Phi$, $x_0 \in J$ so $J = U$
if $U = \Phi$, $J = \Phi$.
Thus $J = U$ so U is path connected.