If $A \subset X$ let ∂A by

$$\partial A = \overline{A} \cap \overline{(X \setminus A)}$$

a) show that $\operatorname{int} A$ and ∂A are disjoint

if $\operatorname{int} A = \emptyset$ it is trivial.

assume $\operatorname{int} A \neq \emptyset$ let $a \in \operatorname{int} A$ then $\exists$ a neighborhood U of a contained in $\operatorname{int} A$. Then U does not intersect $X \setminus A$ so $a \notin \overline{(X \setminus A)}$ $\blacksquare$

b) $\partial A = \emptyset \iff A$ is both open and closed:

"$\Rightarrow$" Let $a \in \overline{A}$ then $\exists$ a neighborhood of a that does not intersect $X \setminus A$ thus A is open. Let a be a limit point of A. Then $a \in \overline{A}$ so by the arguments from before $a \in \operatorname{int} A$ thus $A = \overline{A}$ so A is closed

"$\Leftarrow$"

let $a \in A = \overline{A}$ then $\exists U \in \mathcal{T}$ s.t

$a \in U \subset A$ thus $a \notin \overline{(X \setminus A)}$

c) U is open $\iff \partial U = \bar{U} \setminus U$

"$\Rightarrow$" since U is open U^c is closed thus

$U^c = (X \setminus U) = \overline{(X \setminus U)} \to \bar{U} \cap \overline{(X \setminus U)} = \bar{U} \cap U^c = \bar{U} \setminus U$

"$\Leftarrow$" let $a \in U$ then $a \notin \cancel{(X} \setminus U\ \partial U$ so $a \notin$ $\overline{(X \setminus U)}$ thus $\exists$ a neighborhoor $\mathcal{U}$ of a contained in U so U is open

d) If U is open is it true that $\text{int}\,\bar{U} = U$?

$\bar{U} \supset U$ since U is open $U \subset \text{int}\,\bar{U}$.

"$\subset$" $\bar{U} = U \cup \partial U \to \text{int}(U \cup \partial U) = \text{int}\,\bar{U}$

$W = \text{int}\,\bar{U}$ if $a \in U^c$ then a is not in any open set contained in U