$A \subset X$, $f: A \to Y$ be continuous Y hausdorf show that if f may be extended to a continuous $g: \bar{A} \to Y$ then g is uniquely determined by f.

Clearly $g(a) = f(a)$ $\forall a \in A$. Let $g, \tilde{g}$ be to extensions of f let $a \in \partial A$

we know that since g and $\tilde{g}$ are continuous then for every V of $g(a)$, $\tilde{V}$ of $\tilde{g}(a)$ $\exists$ $\mathcal{U}$ of a s.t $g(\mathcal{U}) \subset V$ and $\tilde{g}(\mathcal{U}) \subset \tilde{V}$

we also know that $\mathcal{U}$ intersects A so we have some $x \in \mathcal{U}$ s.t $x \in A$. then $g(x) = \tilde{g}(x)$

thus $g(\mathcal{U}) \cap \tilde{g}(\mathcal{U}) \neq \Phi$ since Y is Hausdorf

$g(a) = \tilde{g}(a)$