$\mathcal{A}$ is a basis for a topology on X then the topology generated by $\mathcal{A}$ equals the intersection of all topologies on X that contain $\mathcal{A}$

---

let $\mathcal{T}_{\mathcal{A}}$ be the topologies generated by $\mathcal{A}$, $\mathcal{T}$ the intersection $\bigcap_{i \in I} \mathcal{T}_{\mathcal{A}_i}$ of all topologies $\mathcal{T}_{\mathcal{A}_i}$ containing $\mathcal{A}$

Cleraly $\mathcal{T} \subset \mathcal{T}_{\mathcal{A}_i}$ $\forall$ i and thus $\mathcal{T} \subset \mathcal{T}_{\mathcal{A}}$

"$\supset$"

let $\mathcal{U} \in \mathcal{T}_{\alpha}$ then for $x \in \mathcal{U}$ $\exists$ $B_x \in \mathcal{A}$ s.t $x \in B_x \subset \mathcal{U}$, but B_x in $\mathcal{T}$ so $\mathcal{U} \in \mathcal{T}$ ∎

---

let now $\mathcal{A}$ be a subbasis let $\mathcal{T}_{\mathcal{A}}, \mathcal{T}$ as be fore

again $\mathcal{T} \subset \mathcal{T}_{\mathcal{A}}$

"$\supset$"

let $\mathcal{U} \subset \mathcal{T}_{\mathcal{A}}$ then $\mathcal{U}$ is a union of finite intersections from $\mathcal{A}$

$\mathcal{U} = \bigcup_{i \in I} \left(\bigcap_{n=1}^{N_i} A_n^{(i)} \right)$ let T be a topology containing $\mathcal{A}$ then $\underbrace{\bigcap_{n=1}^{N_i} A_n^{(i)}}_{A_i} \in T$ and thus $\bigcup_{i \in I} A_i \in T$ so

$\mathcal{U} \in T$ thus $\mathcal{U} \in \mathcal{T}$