A, B, A_α are subsets of X Determine whether following equations hold

(a) $\overline{A\cap B} = \bar{A} \cap \bar{B}$

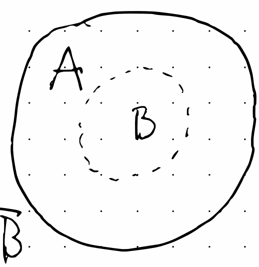
"$\supset$" let $x \in \bar{A} \cap \bar{B}$ then every neighboorhood O of x intersects both A and B so O intersects $A \cap B \to x \in \overline{A\cap B}$

"$\subset$" $x \in \overline{A \cap B}$ every neighborhood O of x intersects $A \cap B$ so it intersects both A and B so $x \in \bar{A}$ and $x \in \bar{B} \to x \in \bar{A} \cap \bar{B}$

b) $\overline{\bigcap A_\alpha} = \bigcap \bar{A}_\alpha$ true, by same arguments

c) $\overline{A \setminus B} = \bar{A} \setminus \bar{B}$?

$x \in \overline{A \setminus B} = \overline{A \cap B^c} = \bar{A} \cap \overline{B^c}$, every O of x intersects A and B^c



if A, B are open

$x \in A$, $x \in \partial B$ then

$x \in A \setminus B \to x \in \overline{A \setminus B}$ but $x \notin \bar{A} \setminus \bar{B}$

ie f.x $A = (0, 10)$, $B = (1, 2)$, $x = 1$

$\overline{A \setminus B} = [0, 1] \cup [2, 10]$, $\bar{A} \setminus \bar{B} = [0, 1) \cup (2, 10]$

"$\supset$" $x \in \bar{A} \setminus \bar{B}$ $\Rightarrow$ since $B \subset \bar{B}$

$\Rightarrow$ $x \in \bar{A} \setminus B = \bar{A} \cap B^c \subset \bar{A} \cap \overline{B^c} = \overline{A \setminus B}$