Assume $\mathbb{R}$ is uncountable. Show that if A is a countable subset of $\mathbb{R}^2$, then $\mathbb{R}^2 - A$ is path connected. [Hint: How many lines are there passing through a given point of $\mathbb{R}^2$]

consider $a, b \in \mathbb{R}^2 - A$. Let $C = \{c \in \mathbb{R} : |c-a| = |c-b|\}$

$C \simeq \mathbb{R}$ and is thus uncountable

for $c \in C$ consider the circle $\{x \in \mathbb{R}^2 : |c-x| = |c-a|\}$

this circle intersects both a and b, and are all disjoint (except a,b). If $\mathbb{R}^2 - A$ is not path connected then these circles would all intersect A at at least one point, but then the circles would be countable as they are disjoint (except a,b), thus $\mathbb{R}^2 - A$ is path connected. Note that circles is irrellevant, we could have chosen many other figures that would be disjoint. f.x parabelas.

sketct