Show that a connected metric space having more than one point is uncountable

Let X be a connected metric space with more than 1 element. Let $a, b \in X$. Define $d_a(x)$ by $d(a, x)$ this is a continuous function (by the intermediate value theorem) thus for any $r \in [0, d(a,b)]$ there is x s.t $d(a,x) = r$ hence X is uncountable.