Examine the proof of the Urysohn lemma, and show that for given r,

$$f^{-1}(r) = \bigcap_{p>r} U_p \setminus \left(\bigcup_{q<r} U_q \right), \quad p,q \text{ rational}$$

$f(x) = \inf\{p : x \in U_p\}$

$f^{-1}(r) = \{x : \inf\{p : x \in U_p\} = r\}$ so x is in $f^{-1}(r)$ an p is a rational number $> r$ then there is some rational numbers s.t $r < \tilde{p} < p$ s.t $x \in U_{\tilde{p}}$. Bu consruction $U_{\tilde{p}} \subset U_p$. thus $x \in U_p \ \forall p > r$.
from definition of f it is clear that x is not in U_q for $q < r$. thus $x \in \bigcap_{p>r} U_p \setminus \left(\bigcup_{q<r} U_q \right)$.
"$\supset$" if $x \in \bigcap_{p>r} U_p \setminus \left(\bigcup_{q<r} U_q \right)$ then

$f(x) = r$ ∎