a) $\mathcal{T}$ and $\mathcal{T}'$ are topologies on X. Suppose $\mathcal{T}' \supset \mathcal{T}$. What does compactness of X under one of these topologies imply about compactness under the other?

if $\mathcal{T}'$ is compact then $\mathcal{T}$ is compact

not the other way: take $(0,1)$, $\mathcal{T} = \{\phi, \{0,1\}\}$

$\mathcal{T}'$ the subspace topology from $\mathbb{R}$

let $B_n = (\frac{1}{n+1}, 1)$, $\bigcup_{n \in \mathbb{N}} B_n = (0,1)$ but no finite subcollection covering $(0,1)$

b) Show that if X is compact Hausdorf under both $\mathcal{T}$ and $\mathcal{T}'$ then either $\mathcal{T}$ and $\mathcal{T}'$ are equal or they are not comparable.

assume that $\mathcal{T}'$ and $\mathcal{T}$ are comparable and that $\mathcal{T}' \supset \mathcal{T}$. Want to show $\mathcal{T}' \subset \mathcal{T}$. let $\mathcal{U} \in \mathcal{T}'$ consider $\mathcal{U}^c$. This is closed in $\mathcal{T}'$ and thus compact as a subspace of X. Thus it is also compact as a subspace of X w.r.t the $\mathcal{T}$-topology. Then it is closed w.r.t $\mathcal{T}$ as X is hausdorf so $\mathcal{U} \in \mathcal{T}$ ∎