

Using the closed set definition of continuity show that the following sets are closed subsets of \mathbb{R}^2 :

$$A = \{x \times y : xy = 1\}$$

we know from lemma 21.4 that

$f(x, y) = xy$ is a continuous function

$\{1\}$ is closed thus $f^{-1}(\{1\})$ is closed

$$S = \{x \times y : x^2 + y^2 = 1\}$$

by lemma 21.5 $f(x, y) = x^2 + y^2$ is continuous
so $f^{-1}(\{1\})$ is closed

$$B = \{x \times y : x^2 + y^2 \leq 1\}$$

again $f^{-1}([0, 1])$ is closed as $f(x, y) = x^2 + y^2$ is continuous.