X, Y are independent $N(0,1)$ distributed

Determine the laws of $(X, X+Y)$ and $(X, \sqrt{2}X)$

show that these have the same marginals

$$(X, X+Y) = \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}$$

$$(X, \sqrt{2}X) = \begin{pmatrix}1 & 0\\ \sqrt{2} & 0\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix} \sim (X, X+Y) \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix}\right)$$

$$\begin{pmatrix}1 & 0\\ \sqrt{2} & 0\end{pmatrix}\begin{pmatrix}1 & \sqrt{2}\\ 0 & 0\end{pmatrix} = \begin{pmatrix}1 & \sqrt{2}\\ \sqrt{2} & 2\end{pmatrix} \to (X, \sqrt{2}X) \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}0 & \sqrt{2}\\ \sqrt{2} & 1\end{pmatrix}\right)$$

marginal of first is

$\cancel{f_1(x) =}$ $\quad \sqrt{2}X = \sqrt{2}N(0,1) = N(0,2)$

$$N(\mu, \sigma^2) = \sigma N + \mu$$

$$X + Y = N(0,1) + N(0,1) = N(0,2)$$

Same marginals.