A is all subsets of topological X

a) show starting with a set A that one can form no more than 14 sets applying complementation and closure successively

start with $A \to \overline{A} \to (\overline{A})^c \to \overline{\overline{A}^c} \to \overline{\overline{A}^c}^c \to \overline{\overline{\overline{A}^c}^c}$

closed, open, closed, open

$A^c \to \overline{A^c} \to \overline{A^c}^c \to \overline{\overline{A^c}^c} \to \overline{\overline{A^c}^c}^c \to \overline{\overline{\overline{A^c}^c}^c} = \overline{A^c}$

c ~ closure, i ~ interior, x ~ complement

$A \to cA \to xcA = ixA \to cixA \to xcixA$
$= ixixA$
$= icxxA$
$= icA$
$\to cicA$
$\to xcicA = ixicA$
$= icxc$
$= icixA$
$\to cicixA$

$xA \to cxA \to xcxA = iA$
$\to ciA \to xciA = ixiA$
$= icxA$
$\to cixiA = cicxA$
$\to icicx$

$ciciA = cicixxA = cic\cancel{x}cxA$

$= cixi\cancel{cx} = ccxicx = xicx = \cancel{c}xixi =$

$= cxcx$

$= cixx$

$\cancel{icic = icicxx = icixix = icixxc}$

$\cancel{= icxcxc}$

$\cancel{= icxcix}$

$ic \subset cic$

$i(ic) \subset icic$

$ic \subset c$

$cic \subset cc = c$

$icic \subset ic \quad \to icic = ic$

$(0,1) \cup (1,2) \cup \{3\} \cup (4,5) \cup \{6\}$

$\to [0,2] \cup \{3\} \cup [4,5] \cup \{6\}$

$\to (-\infty,0) \cup (2,3) \cup (3,4) \cup (5,6) \cup (6,\infty)$

$\to (-\infty,0] \cup [2,4] \cup [5,\infty]$

$\to (0,2) \cup (4,5)$

$\to [0,2] \cup [4,5]$

$\to (-\infty,0] \cup \{1\} \cup [2,3) \cup (3,4] \cup [5,6)$

$\to (-\infty,0] \cup \{1\} \cup [2,4] \cup [5,6]$

$\to (0,1) \cup (1,2) \cup (4,5)$

$\to [0,2] \cup [4,5]$

$\to (-\infty,0)$