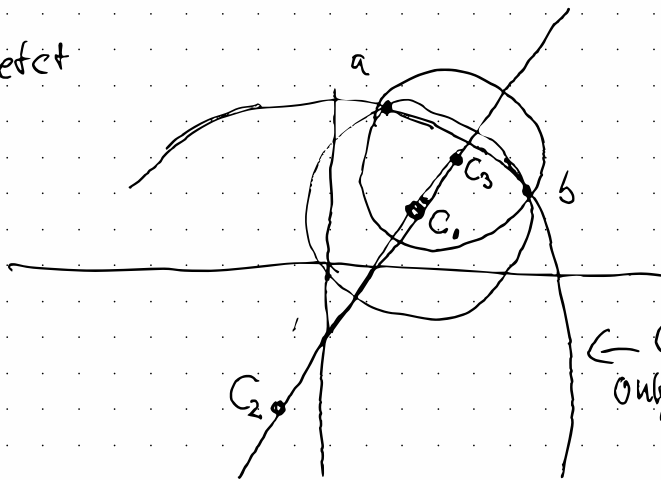


Assume \mathbb{R} is uncountable. Show that if A is a countable subset of \mathbb{R}^2 , then $\mathbb{R}^2 - A$ is path connected. [Hint: How many lines are there passing through a given point of \mathbb{R}^2 .]

consider $a, b \in \mathbb{R}^2 - A$. Let $C = \{c \in \mathbb{R} : |c-a| = |c-b|\}$
 $C \cong \mathbb{R}$ and is thus uncountable

for $c \in C$ consider the circle $\{x \in \mathbb{R}^2 : |c-x| = |c-a|\}$
 this circle intersects both a and b , and are all disjoint. If $\mathbb{R}^2 - A$ is not path connected then these circles would all intersect A at at least one point, but then the circles would be countable as they are disjoint, ^(except a, b) thus $\mathbb{R}^2 - A$ is path connected. Note that circles is irrelevant, we could have chosen many other figures that would be disjoint. f.x parabolas.

sketch



← Circles intersect only at a, b