Let $f: A \to B$ and $g: C \to D$ be continuous functions. Let us define a map $f \times g: A \times C \to B \times D$ by

$$(f \times g)(a \times c) = f(a) \times g(c)$$

Show that $f \times g$ is continuous.

Let V_b be open in B, V_d open in D then $V_b \times V_d$ is a basis element of $B \times D$ it's inverse image is $U_a \times U_c$, a basis element in $A \times C$ thus $f \times g$ is continuous

or define $\pi_b(b \times d) = a \times C$

$\pi_d(b \times d) = A \times c$ (continuous)

these are both open and

$$(f \times g)^{-1}(b \times d) = \pi_b(b \times d) \cap \pi_d(b \times d)$$

so continuous