Let $\pi_1 : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ be projection on the first coordinate. Let A be the subspace of $\mathbb{R}\times\mathbb{R}$ consisting of all points $x \times y$ for which either $x \geq 0$ or $y = 0$ (or both) let $q : A \to \mathbb{R}$ be obtained by restricting π_1. Show that q is a quotient map that is neither open nor closed.

- q is surjective.

- q is continuous: Note that for $E \subseteq A$

$q^{-1}(E) = E \times \mathbb{R} \cap A$. take any basis element (a,b) of $\mathbb{R}$ then $q^{-1}((a,b)) = \underbrace{\underbrace{(a,b)\times\mathbb{R}}_{\text{open in } \mathbb{R}^2} \cap A}_{\text{open in } A}$

so q is continuous as the inverse of basis elements are open

- $q^{-1}(E)$ is open $\Rightarrow$ E is open in A:

$q^{-1}(E) = E \times \mathbb{R} \cap A$ is open $\Leftrightarrow$ $E \times \mathbb{R}$ is open in $\mathbb{R}^2$ $\Leftrightarrow$ E is open in $\mathbb{R}$

- q is not open: let $E = [0,1) \times (1,2)$ open in A since $E = (-1,1)\times(1,2) \cap A$ but $q(E) = [0,1)$ not open in $\mathbb{R}$

- q is not closed: $(-\infty,0]\times\{0\} \cap A = \overbrace{(-\infty,0)\times\{0\}}^{\text{closed in } A}$

$q((-\infty,0) \times \{0\}) = (-\infty,0)$ is open in $\mathbb{R}$ $\blacksquare$