Let $f: A \to B$, $A_0 \subset A$, $B_0 \subset B$

a) show that $A_0 \subset f^{-1}(f(A_0))$ with equality if f is injective

b) Show that $f(f^{-1}(B_0)) \subset B_0$ with equality if f is surjective

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a) let $a \in A_0$ then $\exists$ some b in B s.t $b = f(a)$. $b \in f(A_0)$ then $f^{-1}(b) \in f^{-1}(f(A_0))$

So $a \in f^{-1}(b) \subset f^{-1}(f(A_0))$

if f is injective then

for any $a \in A_0$, $f^{-1}(f(a)) = \{a\}$

thus $f^{-1}(f(a)) \subset A_0 \to$ equality.

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b) Let $b \in B_0$. if $f^{-1}(b) = \emptyset$ then

$f(f^{-1}(b)) = \emptyset \subset B_0$

other wise there is at least one $a \in A$ s.t. $f(a) = b$, then $f(f^{-1}(b)) = \{b\} \in B_0$

if ~~every~~ f is surjective then for any b there is such an a and thus $\{b\} \in f(f^{-1}(b))$