a) Let $p: X \to Y$ be a continuous map. Show that if $\exists$ a continuous map $f: Y \to X$ s.t. $p \circ f$ equals the identity map of Y then p is a quotient map.

Since $p \circ f = I_Y$ we get that p is surjective and f is injective

1. A open in $Y \to p^{-1}(A)$ open in X
   p is continuous so this is true
2. $p^{-1}(A)$ open in $X \to A$ open in Y

let $A \in Y$, $A = \underbrace{f^{-1}(\underbrace{p^{-1}(A)}_{\text{open}})}_{\text{open}} \in \mathcal{T}_Y$

b) If $A \subset X$, a retraction of X onto A is a continuous map $r: X \to A$ s.t. $r(a) = a$ $\forall a \in A$. Show that a retraction is a quotient map.

Clearly r is surjective. define $\tilde{r}: A \to X$ by $\tilde{r}(a) = a$ then $r \circ \tilde{r} = I_A$

need to show that $\tilde{r}$ is continuous.

Let $\mathcal{U}$ be open in X, $\mathcal{U} = (\mathcal{U} \cap A) \cup (\mathcal{U} \cap A^c)$

$\tilde{r}_a^{-1}(\mathcal{U}) = \tilde{r}_a^{-1}(\mathcal{U} \cap A) \cup \tilde{r}_a^{-1}(\mathcal{U} \cap A^c)$

$= \mathcal{U} \cap A \cup \emptyset = \mathcal{U} \cap A$, open in A as a subspace of X ∎