$\tau_f := \{ u \subseteq X \mid X - u$ or X is finite$\}$

a) $\Phi \in \tau_f$, $X \in \tau_f$

ii) $\bigcap_{n=1}^{N} \tau_n$, $\tau_n \in \tau_f$ so either $X - \tau_n$ is finite

or $X - \tau_n = X$ (τ_n is Φ)

if $\Phi \in \Rightarrow \bigcap_{n=1}^{N} \tau_n = \Phi \in \tau_f$

else: $X \setminus \left(\bigcap \tau_n\right) = \bigcup \left(X \setminus \tau_n\right)$

$\rightarrow$ finite

iii) $\bigcup_{i \in I} \tau_i$. $X \setminus \left(\bigcup \tau_i\right) = \bigcap_{i \in I} \underbrace{\left(X \setminus \tau_i\right)}_{\text{finite}}$

each contained so true.