Show that if $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic then $k \circ h$ and $k' \circ h'$ are homotopic

Let $F_1$ be the homotopy between $h$ and $h'$, $F_2$ the homotopy between $k$ and $k'$.

Define $G : X \times I \to Z$ by

$$G(x,t) = F_2(F_1(x,t),t)$$

$$G(x,0) = F_2(F_1(x,0),0) = F_2(h(x),0) = k \circ h(x)$$

$$G(x,1) = k' \circ h'(1)$$

note that

$$X \times I \to Y \times I \quad \text{by } (x,t) \to (F_1(x,t),t)$$

is continuous in the product topology as it is continuous coordinatewise.

(It's inverse image is $F^{-1}(\pi_1(B)) \cap (X \times \pi_2(B))$)
both open

thus $G$ is the composite of 2 continuous functions so $G$ is continuous.