Let $P: X \to Y$ be a closed continuous surjective map. Show that if X is normal, then so is Y. Hint: If U is an open set containing $p^{-1}(y)$, show there is a neighborhood W of y s.t. $p^{-1}(W) \subset U$.

if Y contains less than 2 elements this is clear. Assume Y contains at least 2 elements. Let $U \neq X$ be as described. then $P(U^c)$ is closed in Y. Let $P(U^c)^c = W$. then W is open and contains y. We also have that $p^{-1}(W) \subset U$ as all points mapping to W lie in U.

Show $\{y\}$ is closed: there must be $x \in X$ s.t. $P(x) = y$. Then $\{y\}$ must be closed as $\{x\}$ is closed.

Show normality: Let A, B be disjoint closed sets in Y. Then $p^{-1}(A), p^{-1}(B)$ are closed in X. Then we have $\mathcal{U}, V$ disjoint and open in X. For y in $P(\mathcal{U})$ pick a neigborhood W_y s.t. $p^{-1}(W_y) \subset \mathcal{U}$. then $\bigcup_{y \in P(\mathcal{U})} W_y$ is open and it's inverse image is contained in $\mathcal{U}$. similarly for V. then $\bigcup_{y \in P(\mathcal{U})} W_y$ and $\bigcup_{y \in P(V)} W_y$ are disjoint and open containing A, B ∎