Show that if X is a Hausdorff space that is locally compact at x then for each neighborhood U of x there is a neighborhood V of x s.t V is compact and $\bar{V} \subset U$

let C be the compact space containing a neighborhood of x, if $C \subset U$ we are done. Assume $U \subset C$, as U is open $U^c \cap C$ is closed and thus compact as a subspace of C. then pick an open neighborhood V of x, and an open set containing $U^c \cap C$ F s.t $V \cap F = \Phi$ then $\bar{V} \subset U$ and is compact