

Let $f: S^1 \rightarrow \mathbb{R}$ be a continuous map. Show that there exists a point x of S^1 s.t. $f(x) = f(-x)$

$g(x) = f(x) - f(-x)$ is continuous

$g(-x) = -g(x)$ assuming first f is constant the statement is clear. if f is not constant then

$\exists x^0$ s.t. $g(x^0) \neq 0$ then $g(x^0) < 0$. Then

we have x_0 s.t. $g(x_0) = 0 = f(x_0) - f(-x_0) \Leftrightarrow$
 $f(x_0) = f(-x_0)$

