A Genetic Algorithm Approach for Approximating Resource Allocation in Virtualized Wireless Networks with Log-Normal Distributed Traffic Demand

Kory Teague School of Electrical and
Computer Engineering
Virginia Tech
Blacksburg, VA
Email: koryt@vt.edu
Mohammad Abdel-Rahman School of Electrical and
Computer Engineering
Virginia Tech
Blacksburg, VA
Email: mo7ammad@vt.edu
Allen MacKenzie School of Electrical and
Computer Engineering
Virginia Tech
Blacksburg, VA
Email: mackenab@vt.edu

Abstract

The abstract goes here once written.

I. Introduction

Provide background and expound.

The rest of this paper is organized as follows. In section II, we detail and define the system model assumed for our resource allocation methods. In section III, we consider our resource selection and demand allocation approaches. In section IV, we simulate the described approaches and evaluate their performance. Finally, in section V, we discuss our conclusions and possible future extensions.

II. System Model

We consider a geographical area of width X and length Y that contains a set $\mathcal{S} = \{1, 2, \ldots, S\}$ of base stations (BS). The rate capacity of BS $s \in \mathcal{S}$ is denoted by r_s and its cost is denoted by c_s.

Within this area is a continuous spatial distribution indicating the overall traffic demand. It has been shown that a log-normal distribution or a mixture of log-normal distributions can approximate traffic demand in real-world cellular networks [1], [2]. It has also been shown that traffic distribution is spatially correlated [2], [3]. We model this spatial traffic demand using a similar, continuous form of the SSLT (Scalable, Spatially-correlated, and Log-normally distributed Traffic) model as proposed by Lee, Zhou, and Niu [4].

To generate this spatial distribution over the area of consideration, an initial Gaussian field, $\rho^G = \rho^G(x, y), x \in [0, X], y \in [0, Y]$, is generated by:

$$\rho^G(x, y) = \frac{1}{L} \sum_{l=1}^{L} \cos(i_l x + \phi_l) \cos(j_l y + \psi_l) \tag{1}$$

where $\mathcal{L} \stackrel{\text{def}}{=} \{1, 2, \ldots, L\}$ is a set of the products of two cosines with angular frequencies $i_l, j_l \sim \mathcal{U}(0, \omega_{max}), l \in \mathcal{L}$ and phases $\phi_l, \psi_l \sim \mathcal{U}(0, 2\pi), l \in \mathcal{L}$. As L increases in size, ρ^G approaches a Gaussian random field with a spatial autocorrelation dependent on ω_{max}.

The approximate Gaussian distribution ρ^G is then normalized to $\rho^S = \rho^S(x, y), x \in [0, X], y \in [0, Y]$, which has a standard normal distribution. The final log-normal distribution, $\rho = \rho(x, y), x \in [0, X], y \in [0, Y]$, is determined by assigning location and scale parameters μ and σ:

$$\rho(x, y) = \exp(\sigma \rho^S(x, y) + \mu) \tag{2}$$

$\rho(x, y)$ can be sampled over the space into individual pixels as per Lee with each pixel's value indicating the number of homogeneous demand points within the pixel [4]. In contrast, we allow it to provide a continuous, spatially-correlated log-normal distribution depicting the demand density over the region.

Let $\mathcal{M} \stackrel{\text{def}}{=} \{1, 2, \ldots, M\}$ be the set of demand points within the region; the value of total traffic demand at each point is denoted by d_m. Further let $u_{ms} \in [0, 1]$, $m \in \mathcal{M}$, $s \in \mathcal{S}$ represent the normalized capacity (with respect to r_s) of BS s at point m, i.e., the normalized maximum rate that a user can receive at point m from BS s. $u_{ms} = 0$ when m is outside the coverage area of s and $u_{ms} = 1$ when m is within a small distance of s.

We assume that a BS $s \in \mathcal{S}$ can be allocated between multiple demand points, and $\delta_{ms} \in [0, r_s]$, $m \in \mathcal{M}$, $s \in \mathcal{S}$ represents the rate of BS s that is allocated to point m.

Throughout this paper, stochastic variables will be differentiated from deterministic variables with a tilde ($\sim$) placed above the symbol.

III. Solution Approach

In this section, we detail our approaches for optimally selecting the subset of resources within $\mathcal{S}$ to create a network with the minimum cost while allocating the demand within the region to the selected resources

such that it maximizes demand satisfaction. First, we formulate the problem as a capacitated set cover problem solved optimally via a two-stage stochastic optimization problem. Second, the optimization problem is approximated to a more computationally solvable form as a deterministic equivalent program. Finally, a genetic algorithm is used to further approximate the original problem to a more tractable form. In section IV, we will consider the efficacy of these latter two approaches.

A. Problem Formulation

We formulate the presented problem as a two-stage stochastic optimization problem. We introduce z_s, $s \in \mathcal{S}$ as a binary decision variable defined as follows:

$$z_s = \begin{cases} 1, & \text{if BS } s \text{ is selected for the created network,} \\ 0, & \text{otherwise} \end{cases}$$

To balance the interest of maximizing demand satisfaction against minimizing cost, we introduce the positive real number α as a weighting coefficient between the two stages.

Problem 1 (Two-Stage Stochastic Optimization Problem)

$$\underset{\{s \in \mathcal{S}\}}{\text{minimize}} \left\{ \sum_{s \in \mathcal{S}} c_s z_s + \alpha \mathbb{E}\left[h\left(\boldsymbol{z},\, \tilde{\boldsymbol{u}}\right)\right] \right\} \tag{3}$$

subject to:

$$z_s \in \{0,\, 1\},\ \forall s \in \mathcal{S} \tag{4}$$

where $h\left(\boldsymbol{z},\, \tilde{\boldsymbol{u}}\right)$ is the optimal value of the second-stage problem, which is given by:

$$\underset{\{\delta_{ms},\, m \in \mathcal{M},\, s \in \mathcal{S}\}}{\text{minimize}} \left\{ - \sum_{m \in \mathcal{M}} \sum_{s \in \mathcal{S}} \delta_{ms} \tilde{u}_{ms} \right\} \tag{5}$$

subject to:

$$z_s = \mathbb{1}_{\left\{\sum_{m \in \mathcal{M}} \delta_{ms} > 0\right\}},\ \forall s \in \mathcal{S} \tag{6}$$

$$\sum_{s \in \mathcal{S}} \tilde{u}_{ms} \delta_{ms} \leq d_m,\ \forall m \in \mathcal{M} \tag{7}$$

$$\sum_{m \in \mathcal{M}} \delta_{ms} \leq r_s,\ \forall s \in \mathcal{S} \tag{8}$$

The first stage objective function (3) minimizes the total cost of the selected network in context to that network's ability to satisfy the demand contained within the region, as determined by ρ. The second stage objective function (5) maximizes the total demand allocated to the resources comprising the network, as specified by σ_{ms} as the decision variable of the second stage, which maximizes demand satisfaction. In this context, we define demand satisfaction as the ratio of the total demand allocated to the selected network to the total demand contained within the region.

Constraints (4), (6), and (8) implement the defined ranges and values of the decision variables z_s and δ_{ms}, with (6) ensuring that demand is allocated only to selected resources. For constraint (6), $\mathbb{1}_{\{*\}}$ is defined by:

$$\mathbb{1}_{\{*\}} = \begin{cases} 1, & \text{if condition } * \text{ is true,} \\ 0, & \text{otherwise} \end{cases}$$

Constraint (7) ensures a demand point $m \in \mathcal{M}$ is not allocated more resource capacity than it demands.

B. Deterministic Equivalent Reformulation

One major obstacle with solving Problem 1 is that stochastic equations are difficult to solve with typical computational libraries. In order to be practical, Problem 1 may be reformulated such that it does not contain any stochastic variables.

Our approach for solving the proposed stochastic optimization formulation is to derive their deterministic equivalent programs (DEPs). The DEP is an equivalent reformulation of the original stochastic program, but only contains deterministic variables [5].

Let $\Omega \stackrel{\text{def}}{=} \{1, 2, \ldots, O\}$ be defined as the set of discrete scenarios, each of which contains a sampled version of the stochastic variables within Problem 1. As O approaches infinity, Ω contains the entire scope of the stochastic variables. With sufficiently large O, Ω approximates the stochastic variables of Problem 1 with deterministic variables. The probability a given scenario $\omega \in \Omega$ occurs is denoted by $p^{\{\omega\}}$, $\omega \in \Omega$, where $\sum_{\omega \in \Omega} p^{\{\omega\}} = 1$. Variables that are dependent on Ω are shown with a superscript $\{\omega\}$ with the specific scenario it is dependent on indicated by ω.

Problem 2 (DEP of Problem 1)

$$\underset{\substack{\{z_s, \delta_{ms}^{\{\omega\}}, \\ s \in \mathcal{S}, m \in \mathcal{M}, \\ \omega \in \Omega\}}}{\text{minimize}} \sum_{s \in \mathcal{S}} c_s z_s - \alpha \sum_{\omega \in \Omega} p^{\{\omega\}} \left(\sum_{m \in \mathcal{M}} \sum_{s \in \mathcal{S}} \delta_{ms}^{\{\omega\}} u_{ms}^{\{\omega\}} \right) \tag{9}$$

subject to:

$$\sum_{s \in \mathcal{S}} u_{ms}^{\{\omega\}} \delta_{ms}^{\{\omega\}} \leq d_m, \; \forall m \in \mathcal{M}, \; \forall \omega \in \Omega \tag{10}$$

$$\sum_{m \in \mathcal{M}} \delta_{ms}^{\{\omega\}} \leq r_s z_s, \; \forall s \in \mathcal{S}, \; \forall \omega \in \Omega \tag{11}$$

$$z_s \in \{0, 1\}, \; \forall s \in \mathcal{S} \tag{12}$$

The objective function (9) combines both objective functions (3) and (5) of the initial formulation into a deterministic form. Constraints (10) and (11) ensure demand is not overallocated and is only allocated to selected resources and within capacity for all scenarios.

Within each scenario ω, the log-normal demand field ρ is sampled to provide a set of M discrete demand points. Each sampling of ρ is generated by creating a non-stationary 2D Poisson point process (PPP) with M points over the region using ρ as the spatial intensity function. To generate this non-stationary PPP, we use an acceptance-rejection method. That is, each point of a stationary PPP with an intensity of $\rho_{max} = \max_i \rho(x_i, y_i)$ is retained with probability $\frac{\rho(x_i, y_i)}{\rho_{max}}$, where x_i and y_i are the x- and y-coordinates of the i^{th} point of the stationary PPP.

C. Genetic Algorithm Approach

The DEP formulation is notably intractable as its components increase in size. Most importantly, the accuracy of the DEP is directly dependent on the size of Ω, directly causing a trade off between the accuracy of the DEP and its computability in a reasonable amount of time. In this subsection, we reformulate the original problem as a genetic algorithm, circumventing the need to discretize demand or to establish Ω, thereby simplifying the original problem into a more scalable form.

A genetic algorithm is an iterative metaheuristic in which an approximate solution to a given optimization problem is arrived at via a series of progressive generations. Each generation contains a number of candidate solutions, called individuals, each of which is defined by a chromosome. During a given generation, each individual is assessed a fitness heuristic based on its chromosome. Then individuals are selected at random, with more fit individuals being more probable to be selected. Pairs of selected individuals will crossover with a probability of p_{xov}, a process similar to genetic recombination in biology. The resulting chromosomes then have a probability p_{mut} to mutate, altering the chromosome slightly. Once enough new individual chromosomes have been selected and possibly undergone crossover and mutation, this set of new individuals, called children, forms the next generation to repeat the process.

For the genetic algorithm, ρ is not sampled for discrete demand points. Instead, we assume that all demand over the region is allocated to the closest resource. The subset of $\mathcal{S}$, $\mathcal{S}'$, that is selected for a given network forms a Voronoi tessellation from the point locations of the selected resources. The total demand allocated to a selected resource $s \in \mathcal{S}' \subseteq \mathcal{S}$ is $\iint_{V_s} \rho\left(x,\, y\right)\, dx\, dy$, where V_s is the region bounded by the cell of resource s in the Voronoi tessellation. If the total demand allocated to s exceeds r_s, s is considered to be *overcapacity*. If V_s is not wholly contained within the coverage area of resource s, s is considered to be *overcoverage*.

Let $\mathcal{G} \stackrel{\text{def}}{=} \{1,\, 2,\, \ldots,\, G\}$ be the set of generations used in the genetic algorithm and $\mathcal{I}_g \stackrel{\text{def}}{=} \{1,\, 2,\, \ldots,\, I\}, g \in \mathcal{G}$ be the set of individuals within generation g. Each individual $i \in \mathcal{I}_{g\in\mathcal{G}}$ has a binary chromosome $z^{\{ig\}}$ of length S. $z_s^{\{ig\}}, s \in \mathcal{S}$, denoting each individual bit of the chromosome, is defined as follows:

$$z_s^{\{ig\}} = \begin{cases} 1, & \text{if BS } s \text{ is selected for the created network for individual } i \text{ in generation } g, \\ 0, & \text{otherwise} \end{cases}$$

The fitness heuristic of each individual chromosome, $z^{\{ig\}}$, is assessed as the reciprocal of the chromosome's cost, which is defined as:

$$fitness\left(z^{\{ig\}}\right) = \frac{1}{cost\left(z^{\{ig\}}\right)} \tag{13}$$

$$cost\left(z^{\{ig\}}\right) = \sum_{s\in\mathcal{S}} \left(c_s z_s^{\{ig\}} + c_{cov} \mathbb{1}_{\{V_s \not\subseteq R_s\}} + (c_{cap}^g - 1) \max\left(0,\, \iint_{R_s} \rho\left(x,\, y\right)\, dx\, dy - r_s\right)\right) \tag{14}$$

where R_s is the coverage area region of resource $s \in \mathcal{S}$.

The cost function (14) indicates cost increases not only based on the cost of the resources selected, but also with imperfection costs c_{cov} and c_{cap}, the costs of a selected resource being overcoverage or overcapacity,

respectively. The overcapacity cost also grows with each successive generation. For early generations, this allows for imperfect solutions to temporarily exist to seed later generations and improve diversity to increase the probability of finding a better final approximate solution.

Elitism is used, where a number of the most fit individuals of a given generation are automatically selected without crossover or mutation to be the first children of the next generation. Selection occurs via the roulette wheel selection method. Every individual i of a given generation g has a probability of being selected of:

$$\frac{fitness\left(z^{\{ig\}}\right)}{\sum_{i\in J} fitness\left(z^{\{ig\}}\right)}$$

When crossover is performed on selected individuals, it is via the uniform crossover method with a mixing ratio of 0.5. That is, each equivalent bit in the selected parent individuals will swap with a probability of 50%. It has been suggested that uniform crossover is more exploratory than n-point crossover (*cite*). Mutation occurs on a bit-by-bit level, with each bit mutating (i.e., flipping) with probability $\frac{1}{S}$. The uniqueness property is then enforced on the resulting children to ensure diversity; if a child chromosome is identical to another child chromosome in the next generation, the child is discarded and a new child generated, ensuring that each individual of any given generation is unique within that generation.

The genetic algorithm iterates for a number of generations G. If the genetic algorithm settles on a single individual chromosome for a number of continuous generations, it will halt and present that individual chromosome as the final approximate solution.

IV. Simulation Results

The following are draft notes for this section:

In this section, we detail the simulation procedures. This includes data generation, assumptions, differing models (small and larger scale data sets?), resulting data, how the data is evaluated. Include the resulting data and potential takeaways from the data. If solutions are as expected, expand and expound upon it. If not, then hypothesize why.

Most of this section cannot be worked on until the appropriate data is generated via simulations. Need optimization and approximation results to compare/contrast against each other. Approximation data currently seems good, but need optimization results to ensure satisfactory results, especially over a larger data set.

V. Conclusion

The following are draft notes for this section:

The conclusion goes here. Look back on results and reiterate main takeaways. Include possible avenues for further research and expansion to the model. Perhaps power control, more nuanced demand-resource allocation (other than basic, simple voronoi), path-loss addition, applying slicing to the genetic algorithm

more directly, integrating over the regions in the voronoi GA instead of summing (could be faster; math could be interesting in this or further papers), etc.

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