Joint Base Station Selection and Adaptive Slicing in Virtualized Wireless Networks with Log-Normal Distributed Traffic Demand

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Abstract

The abstract goes here once written.

I. Introduction

Provide background and expound.

The rest of this paper is organized as follows. In Section II, we detail and define the system model assumed for our resource allocation methods. In Section **??**, we consider our resource selection and demand allocation approaches. In Section **??**, we simulate the described approaches and evaluate their performance. Finally, in Section **??**, we discuss our conclusions and possible future extensions.

II. System Model

We consider a geographical area of width X (m) and length Y (m) that contains a set $\mathcal{S} = \{1, 2, \ldots, S\}$ of base stations (BSs) available to the Virtual Network Builder (VNB) as provided by the Resource Providers (RPs). The rate capacity of BS $s \in \mathcal{S}$ is denoted by r_s, its cost is denoted by c_s, and its broadcasting range is denoted by b_s.

A Service Provider (SP) seeking a Virtualized Wireless Network (VWN) from the VNB is assumed to know the distribution of traffic demand within the region the VWN would cover. It has been shown that a log-normal distribution or a mixture of log-normal distributions can approximate traffic demand in real-world cellular networks [**?**], [**?**]. It has also been shown that traffic distribution is spatially correlated [**?**], [**?**]. We model the spatial traffic demand of a single SP using a similar, continuous form of the SSLT (Scalable, Spatially-correlated, and Log-normally distributed Traffic) model as proposed by Lee, Zhou, and Niu [**?**].

To generate this spatial distribution over the area of consideration, an initial Gaussian field, $\rho^G = \rho^G(x, y)\,, x \in [0, X]\,, y \in [0, Y]$, is generated by

$$\rho^G\,(x,\,y) = \frac{1}{L}\sum_{l=1}^{L} \cos\left(i_l x + \phi_l\right)\,\cos\left(j_l y + \psi_l\right) \tag{1}$$

where $\mathcal{L} \stackrel{\text{def}}{=} \{1,\,2,\,\dots,\,L\}$ is a set of the products of two cosines with angular frequencies $i_l,\,j_l \sim \mathcal{U}\left(0,\,\omega_{\max}\right),\,l \in \mathcal{L}$ and phases $\phi_l,\,\psi_l \sim \mathcal{U}\left(0,\,2\pi\right),\,l \in \mathcal{L}$. As L increases, ρ^G approaches a Gaussian random field with a spatial autocorrelation dependent on $\omega_{\max}$ according to the central limit theorem.

The approximate Gaussian distribution ρ^G is then normalized to $\rho^S = \rho^S(x,\,y), x \in [0,\,X], y \in [0,\,Y]$, which has a standard normal distribution. The final log-normal distribution, $\rho = \rho(x,\,y), x \in [0,\,X], y \in [0,\,Y]$, is determined by assigning location and scale parameters μ and σ

$$\rho\,(x,\,y) = \exp\left(\sigma\,\rho^S\,(x,\,y) + \mu\right) \tag{2}$$

$\rho\,(x,\,y)$ can be sampled over the space into individual pixels as per Lee with each pixel's value indicating the number of homogeneous demand points within the pixel [**?**]. In contrast, we allow $\rho(x,\,y)$ to provide a continuous, spatially-correlated log-normal distribution depicting the demand density over the region for the SP.

Let $\mathcal{M} \stackrel{\text{def}}{=} \{1,\,2,\,\dots,\,M\}$ be the set of the SP's demand points seeking to connect to the VWN; the value of total traffic demand at each point is denoted by d_m. Further let $u_{ms} \in [0,\,1]$, $m \in \mathcal{M}$, $s \in \mathcal{S}$, represent the normalized capacity (with respect to r_s) of BS s at point m, i.e., the normalized maximum rate that a user can receive at point m from BS s. $u_{ms} = 0$ when m is outside the coverage area of s and $u_{ms} = 1$ when m is within a small distance of s. The specific position of point m, and therefore the value of u_{ms}, is determined stochastically by the demand field ρ.

We assume that a BS $s \in \mathcal{S}$ can be allocated between multiple demand points, and $\delta_{ms} \in [0,\,r_s]$, $m \in \mathcal{M}$, $s \in \mathcal{S}$, represents the rate of BS s that is allocated to point m.

Throughout this paper, stochastic variables will be differentiated from deterministic variables with a tilde (~) placed above the symbol.

III. Solution Approach

In this section, we detail our approaches for selecting the subset of resources within $\mathcal{S}$ to create a network with the minimum cost while allocating the selected resources to the demand points within the region such that it maximizes demand satisfaction. First, we formulate the problem of BS selection and their allocation to individual demand points as a two-stage stochastic optimization problem. Second, the optimization problem is approximated to a computationally solvable form as a deterministic equivalent program. Finally, a genetic algorithm is used as a separate approximation to the BS selection portion of the problem. In Section **??**, we will consider the efficacy of these latter two approaches.

A. Problem Formulation

We formulate the presented problem as a two-stage stochastic optimization problem. We introduce $z_s, s \in \mathcal{S}$ as a binary decision variable defined as

$$z_s = \begin{cases} 1, & \text{if BS } s \text{ is selected for the created network,} \\ 0, & \text{otherwise} \end{cases}$$

To balance the interest of maximizing demand satisfaction against minimizing cost, we introduce the positive real number α as a weighting coefficient between the two stages.

Problem 1 (Two-Stage Stochastic Optimization Problem)

$$\underset{\{z_s,\, s\in\mathcal{S}\}}{\text{minimize}} \left\{ \sum_{s\in\mathcal{S}} c_s\ z_s + \alpha\mathbb{E}\left[h\left(\boldsymbol{z},\ \tilde{\boldsymbol{u}}\right)\right] \right\} \tag{3}$$

subject to:

$$z_s \in \{0,1\}, \forall s \in \mathcal{S} \tag{4}$$

where $h(\boldsymbol{z}, \tilde{\boldsymbol{u}})$ is the optimal value of the second-stage problem, which is given by:

$$\underset{\{\delta_{ms}, m\in\mathcal{M}, s\in\mathcal{S}\}}{\text{minimize}} \left\{ -\sum_{m\in\mathcal{M}} \sum_{s\in\mathcal{S}} \delta_{ms}\ \tilde{u}_{ms} \right\} \tag{5}$$

subject to:

$$x_s = \mathbb{1}_{\left\{\sum_{m\in\mathcal{M}} \delta_{ms} > 0\right\}}, \forall s \in \mathcal{S} \tag{6}$$

$$\sum_{s\in\mathcal{S}} \delta_{ms}\ \tilde{u}_{ms} \leq d_m, \forall m \in \mathcal{M} \tag{7}$$

$$\sum_{m\in\mathcal{M}} \delta_{ms} \leq r_s, \forall s \in \mathcal{S} \tag{8}$$

The first stage objective function (**??**) minimizes the total cost of the selected network in context to that network's ability to satisfy the demand contained within the region, as determined by ρ. The second stage objective function (**??**) maximizes demand satisfaction by maximizing the total demand allocated to the resources comprising the network, as specified by δ_{ms} as the decision variable of the second stage. In this context, we define demand satisfaction as the ratio of the total demand allocated to the selected network to the total demand contained within the region.

Constraints (**??**), (**??**), and (**??**) implement the defined ranges and values of the decision variables z_s and δ_{ms}, with (**??**) ensuring that demand is allocated only to selected resources. For constraint (**??**), $\mathbb{1}_{\{*\}}$ is defined by

$$\mathbb{1}_{\{*\}} = \begin{cases} 1, & \text{if condition } * \text{ is true,} \\ 0, & \text{otherwise} \end{cases}$$

Constraint (**??**) ensures a demand point $m \in \mathcal{M}$ is not allocated more resource capacity than it demands.

B. Deterministic Equivalent Reformulation

One major obstacle with solving Problem 1 is that stochastic equations are impossible to solve with available optimization computational tools. In order to be practical, Problem 1 must be reformulated such that it does not contain any stochastic variables.

Our approach for solving the proposed stochastic optimization formulation is to derive their deterministic equivalent programs (DEPs). The DEP is an equivalent reformulation of the original stochastic program, but only contains deterministic variables [?].

Let $\Omega \stackrel{\text{def}}{=} \{1, 2, \ldots, O\}$ be defined as the set of discrete scenarios, each of which contains a sampled version of the stochastic variables within Problem 1. As O approaches infinity, Ω contains the entire scope of the stochastic variables. With sufficiently large O, Ω approximates the stochastic variables of Problem 1 with deterministic variables. The probability a given scenario $\omega \in \Omega$ occurs is denoted by $p^{\{\omega\}}$, $\omega \in \Omega$, where $\sum_{\omega \in \Omega} p^{\{\omega\}} = 1$. Variables that are dependent on Ω are shown with a superscript $\{\omega\}$ with the specific scenario it is dependent on indicated by ω.

Problem 2 (Deterministic Equivalent Program of Problem 1)

$$\underset{\left\{\substack{z_s,\, \delta_{ms}^{\{\omega\}},\\ s \in \mathcal{S},\, m \in \mathcal{M},\\ \omega \in \Omega}\right\}}{\text{minimize}} \sum_{s \in \mathcal{S}} c_s\, z_s - \alpha \sum_{\omega \in \Omega} p^{\{\omega\}} \left(\sum_{m \in \mathcal{M}} \sum_{s \in \mathcal{S}} \delta_{ms}^{\{\omega\}}\, u_{ms}^{\{\omega\}} \right) \tag{9}$$

subject to:

$$\sum_{s \in \mathcal{S}} \delta_{ms}^{\{\omega\}}\, u_{ms}^{\{\omega\}} \leq d_m,\ \forall m \in \mathcal{M},\ \forall \omega \in \Omega \tag{10}$$

$$\sum_{m \in \mathcal{M}} \delta_{ms}^{\{\omega\}} \leq r_s\, z_s,\ \forall s \in \mathcal{S},\ \forall \omega \in \Omega \tag{11}$$

$$z_s \in \{0, 1\},\ \forall s \in \mathcal{S} \tag{12}$$

The objective function (**??**) combines both objective functions (**??**) and (**??**) of the initial formulation into a deterministic form. Constraints (**??**) and (**??**) ensure demand is not overallocated and is only allocated to selected resources and within capacity for all scenarios.

Within each scenario ω, the SSLT demand field ρ is sampled to provide a set of M discrete demand points. Each sampling of ρ is generated by creating a non-stationary 2D Poisson point process (PPP) with M points over the region using ρ as the spatial intensity function. To generate this non-stationary PPP, we use an acceptance-rejection method. That is, each point of a stationary PPP with an intensity of $\rho_{\max} = \max_i \rho\left(x_i,\, y_i\right)$ is retained with probability $\frac{\rho\left(x_i,\, y_i\right)}{\rho_{\max}}$, where x_i and y_i are the x- and y-coordinates of the i^{th} point of the stationary PPP.

C. Adaptive Slicing within a Formed VWN

After the solution to the DEP of Section ?? has been found, the VNB has determined the joint BS selection that forms the VWN and a proposed resource slicing of considered possible scenarios, Ω, that allocates the resources to the SP's demand points. Since O is not infinite, any given scenario present in the formed VWN is exceedingly unlikely to be an element of Ω. Further, as demand points move between BSs or enter or exit the VWN, a new scenario $\omega \notin \Omega$ is formed with a possibly new set $\mathcal{M}' \stackrel{\text{def}}{=} \{1, 2, \dots, M'\}$. The VWN must adapt its resource slicing to these new demand points to maintain maximum demand satisfaction. With the VWN built, the joint BS selection, z_s, becomes a constant of the network, simplifying Problem 2 to a single-stage optimization problem

Problem 3 (Deterministic Adaptive Slicing)

$$\underset{\left\{\substack{\delta_{ms}^{\{\omega\}},\, s\in\mathcal{S},\\ m\in\mathcal{M}',\, \omega\in\Omega'}\right\}}{\text{minimize}} \; -\sum_{\omega\in\Omega'} p^{\{\omega\}} \left(\sum_{m\in\mathcal{M}'} \sum_{s\in\mathcal{S}} \delta_{ms}^{\{\omega\}}\, u_{ms}^{\{\omega\}} \right) \tag{13}$$

subject to:

$$\sum_{s\in\mathcal{S}} \delta_{ms}^{\{\omega\}}\, u_{ms}^{\{\omega\}} \le d_m,\ \forall m \in \mathcal{M}',\ \forall \omega \in \Omega' \tag{14}$$

$$\sum_{m\in\mathcal{M}'} \delta_{ms}^{\{\omega\}} \le r_s\, z_s,\ \forall s \in \mathcal{S},\ \forall \omega \in \Omega' \tag{15}$$

where $\Omega' \stackrel{\text{def}}{=} \{1, 2, \dots, O'\}$ is the set of O' scenarios independent of the original set Ω. In practice, $O' = 1$ with $p^{\{1\}} = 1$, as only the currently existing scenario of the network is of interest for slicing the resources at that moment. Higher values of O' are useful for simulating multiple scenarios with a homogeneous $\mathcal{M}'$. It is worth noting that Problem 3 is more tractable than Problem 2 as it only contains one decision variable for resource slicing, simplifying the objective function (??) and constraint (??).

D. Genetic Algorithm Approach

The full DEP formulation is notably intractable as its components increase in size. Most importantly, the accuracy of the DEP is directly dependent on the size of Ω, directly causing a trade off between the accuracy of the DEP and its computability in a reasonable amount of time. In this subsection, we reformulate the problem of joint BS selection for the VWN as a genetic algorithm, circumventing the need to discretize demand or to establish Ω, thereby simplifying the original problem into a more scalable form.

A genetic algorithm is an iterative metaheuristic in which an approximate solution to a given optimization problem is arrived at via a series of progressive generations. Each generation contains a number of candidate solutions, called individuals, each of which is defined by a chromosome. During a given generation, each individual is assessed a fitness heuristic based on its chromosome. Then individuals are selected at random, with more fit individuals being selected with greater probability. Pairs of selected individuals will crossover with probability p_{xov}, a process similar to genetic recombination in biology. The resulting chromosomes then have probability p_{mut} to mutate, altering the chromosome slightly. Once enough new individual chromosomes have been selected and possibly undergone crossover and mutation, this set of new individuals, called children, forms the next generation to repeat the process.

For the genetic algorithm, ρ is not sampled for discrete demand points. Instead, we assume that all demand over the region is allocated to the closest resource. The subset of $\mathcal{S}$, $\mathcal{S}'$, that is selected for a given possible VWN forms a Voronoi tessellation from the point locations of the selected resources. The total demand allocated to a selected resource $s \in \mathcal{S}' \subseteq \mathcal{S}$ is $\iint_{V_s} \rho\,(x,\, y)\; dx\, dy$, where V_s is the region bounded by the cell of resource s in the Voronoi tessellation. If the total demand allocated to s exceeds r_s, s is considered to be *overcapacity*. If V_s is not wholly contained within the coverage area of resource s, s is considered to be *overcoverage*.

Let $\mathcal{G} \stackrel{\text{def}}{=} \{1,\, 2,\, \dots,\, G\}$ be the set of generations used in the genetic algorithm and $\mathcal{I}_g \stackrel{\text{def}}{=} \{1,\, 2,\, \dots,\, I\}\,, g \in \mathcal{G}$ be the set of individuals within generation g. Each individual $i \in \mathcal{I}_{g \in \mathcal{G}}$ has a binary chromosome $z^{\{ig\}}$ of length S. $z_s^{\{ig\}}, s \in \mathcal{S}$, denoting each individual bit of the chromosome, is defined as follows:

$$z_s^{\{ig\}} = \begin{cases} 1, & \text{if BS } s \text{ is selected for the VWN for individual } i \text{ in generation } g, \\ 0, & \text{otherwise} \end{cases}$$

The fitness heuristic of each individual chromosome, $z^{\{ig\}}$, is assessed as the reciprocal of the chromosome's cost, which is defined as

$$\text{fitness}\left(z^{\{ig\}}\right) = \frac{1}{\text{cost}\left(z^{\{ig\}}\right)} \tag{16}$$

$$\text{cost}\left(z^{\{ig\}}\right) = \sum_{s \in \mathcal{S}} \left(c_s\; z_s^{\{ig\}} + c_{\text{cov}}\; \mathbb{1}_{\{V_s \not\subseteq R_s\}} + \left(c_{\text{cap}}^{g} - 1\right)\; \max \left(0,\, \iint_{R_s} \rho\,(x,\, y)\; dx\, dy - r_s \right) \right) \tag{17}$$

where R_s is the coverage area region of resource $s \in \mathcal{S}$.

The cost function (**??**) indicates cost increases not only based on the cost of the resources selected, but also with imperfection costs c_{cov} and c_{cap}, the costs of a selected resource being overcoverage or overcapacity, respectively. The overcapacity cost grows with each successive generation. For early generations, this allows for imperfect solutions to temporarily exist to seed later generations and improve diversity to increase the probability of finding a better final approximate solution.

Elitism is used, where the n most fit individuals of a given generation are automatically selected without crossover or mutation to be the first children of the next generation. Selection occurs via the roulette wheel selection method. Every individual i of a given generation g has a probability of being selected given by

$$\frac{\text{fitness}\left(z^{\{ig\}}\right)}{\sum_{i \in \mathcal{I}} \text{fitness}\left(z^{\{ig\}}\right)}$$

When crossover is performed on selected individuals, it is via the uniform crossover method with a mixing ratio of 0.5. That is, if two selected parent individuals crossover, each equivalent bit in the parents will swap with a probability of 50%. It has been suggested that uniform crossover is more exploratory than n-point crossover *(cite)*. Mutation occurs on a bit-by-bit level, with each bit mutating (i.e., flipping) with probability $\frac{1}{S}$. The uniqueness property is then enforced on the resulting children to ensure diversity; if a child chromosome is identical to another child chromosome in the next generation, the child is discarded and a new child generated, ensuring that each individual of any given generation is unique within that generation.

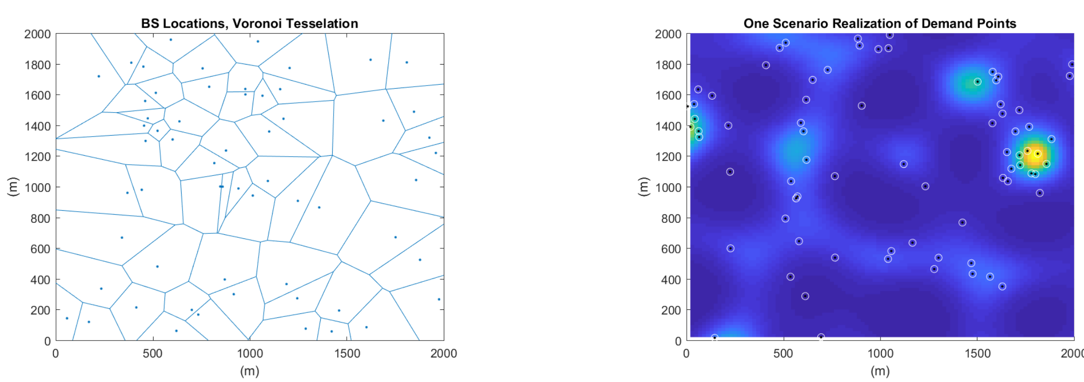
The genetic algorithm iterates for a number of generations G. If the genetic algorithm settles on a single individual for a number of continuous generations, G_{halt}, it will halt and present that individual's chromosome as the final approximate solution for z_s. Otherwise, the chromosome of the fittest individual of generation G determines z_s.

The genetic algorithm only determines an approximate solution to the BS selection forming the VWN, informing the VNB of which BSs to obtain from the RPs. With this selection, z_s, the SP's demand points can be dynamically allocated resource slices as described by Problem 3 in Section **??**.

IV. Simulation and Evaluation

In this section, we evaluate the DEP and genetic algorithm approaches as approximations of Problem 1. We will compare the cost, demand satisfaction, and time to generate of the resultant networks.

A. Setup



(a) RP BS Locations in $\mathcal{S}$ with associated Voronoi tesselation

(b) SP SSLT demand density field with one scenario of SP demand points

Fig. 1: Visualization of network area. (**??**) visualizes the resources available to VNB for creating the VWN, and (**??**) visualizes the demand to be satisfied by the VWN.

Unless stated otherwise, we use the default parameter values shown in Table **??**. BS locations are determined as a stationary PPP. Demand point locations are generated independently for each scenario as a non-stationary PPP using $\rho\left(x,\, y\right),\, x \in [0,\, X],\, y \in [0,\, Y]$, as the spatial intensity function, as described in Section II. Fig. **??** provides a visualization of the simulation network area. (**??**) shows the BS locations

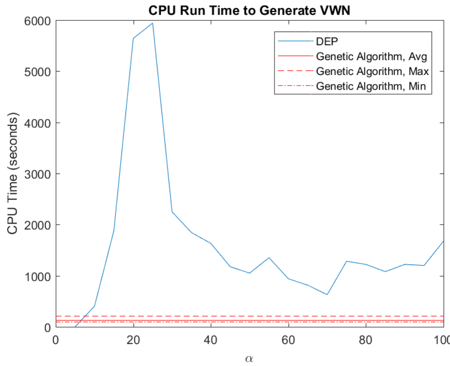
TABLE I: Numerical Values of Relevant Parameters

Parameter	**Value**
Width of Geographic Area (X)	2 km
Height of Geographic Area (Y)	2 km
Number of BSs (S)	60
Number of Demand Points (M)	75
Number of Scenarios (O)	25
BS cost ($c_s, \forall s \in \mathcal{S}$)	1
BS capacity ($r_s, \forall s \in \mathcal{S}$)	1.50 Mbps
BS range ($b_s, \forall s \in \mathcal{S}$)	500 m
Demand Point Demand ($d_m, \forall m \in \mathcal{M}$)	0.178 Mbps
Set of Two-Stage Model Weights (α)	{5, 10, ... , 100}
SSLT Approximation Depth (L)	50
SSLT Maximum Angular Frequency ($\omega_{\max}$)	$\frac{2\,\pi}{30}$
SSLT Location Parameter (σ)	0
SSLT Scale Parameter (μ)	1
Pixel Grid Size	100 x 100, 20 m side
Maximum Number of Generations (G)	3000
Minimum Number of Generations	300
Number of Unchanged Generations Before Halt (G_{halt})	150
Number of Individuals per Generation (I)	80
Number of Elite Individuals per Generation	4
Probability of Crossover (p_{xov})	0.7
Probability of Mutation per bit (p_{mut})	$\frac{1}{S} = 0.0167$
Overcoverage Cost (c_{cov})	3
Overcapacity Cost (c_{cap})	1.015

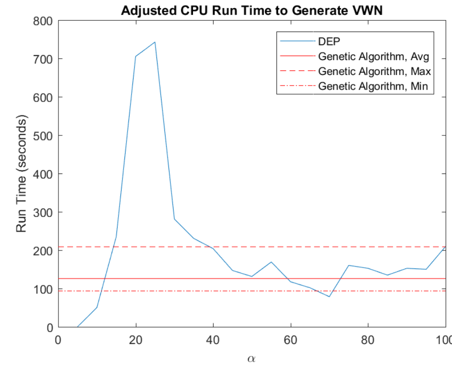
of $\mathcal{S}$ with the associated Voronoi Tesselation showing the coverage areas of the BSs when all are active with respect to the genetic algorithm. (**??**) shows the SSLT demand density field with one example scenario of demand points, which acts as a single sample of the demand density field. To compute $u_{ms}^{\{\omega\}}$, it is assumed that there is perfect propagation between the demand points and BSs. Unlike as described in Section II, $u_{ms}^{\{\omega\}} = 1$ if the distance between demand point $m \in \mathcal{M}$ of scenario $\omega \in \Omega$ and BS $s \in \mathcal{S}$ is less than b_s, and 0 otherwise. To compute the integral of the fitness function (**??**), $\rho\,(x,\,y)$ is discretized into a grid of congruent pixels, and the demands of all pixels within a Voronoi cell of interest are summed together.

We ran our simulations on an Intel Core i7-4790K 4.00 GHz 4 real core/8 virtual core CPU with 16 GB of DDR3 RAM running Windows 10 Pro. We used CPLEX [**?**] to solve the DEP optimization problems and we used MATLAB to simulate the genetic algorithm and to generate the demand field and stochastic data (i.e., $\rho\,(x,\,y)$ and $u_{ms}^{\{\omega\}}$). During the simulations, extraneous processes were culled to allow maximal use of computer resources. Average values for the performance of the genetic algorithm are provided from 50 independent runs using the identical data set except for the set of initial individuals. The DEP solutions were solved across multiple values of α as cost, time, and demand satisfaction are directly related to α.

B. Results



(a) True CPU Time



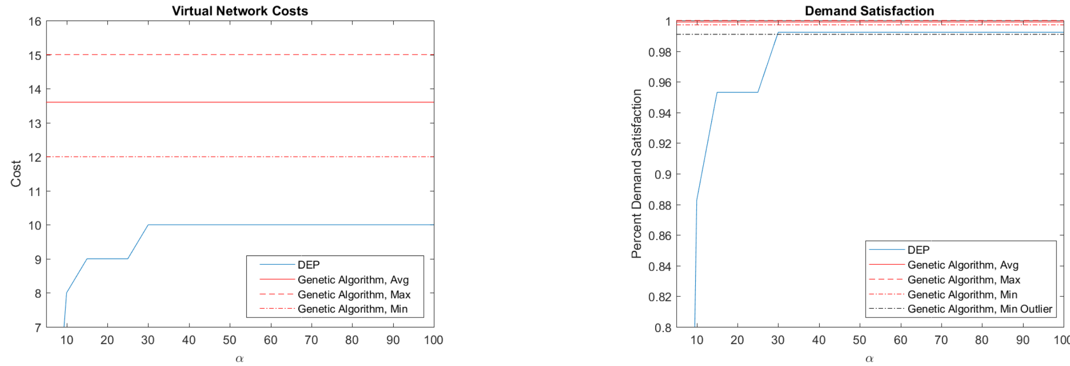
(b) Adjusted (Pseudo-Real) Run Time

Fig. 2: Run time comparison. The DEP run time is the solid blue line, while the minimum, average, and maximum genetic algorithm run times are provided as dot-dashed, solid, and dahsed red reference lines, respectively.

In Fig. **??** is a comparison of the time to run. (**??**) shows the absolute overall CPU time taken to converge to a solution in both the DEP and genetic algorithm. (**??**) shows an adjusted form of the run time the DEP took to terminate to a single solution. CPLEX is capable of parallelizing across the 8 CPU cores, allowing for the real run time to be, at minimum, one-eighth the CPU run time. With this adjustment providing an improvement to the DEP run time, the genetic algorithm converges in around the same time as the DEP for higher levels of α, when a single solution dominates so heavily it simplifies the overall set. Without the adjustment, the genetic algorithm outperforms the DEP, converging to a solution in approximately 13% of the time for $\alpha \geq 30$ and in approximately 2% of the time for $\alpha \in \{20, 25\}$.

The trade off for the genetic algorithm's improved run time is that the solution provided is less optimal than the DEP, as indicated by an increased cost for the VNB to build the VWN. Fig. **??** compares the increasing cost of the DEP as α increases with the cost of the various genetic algorithm solutions. On average, the genetic algorithm incurs a 36% increased cost in selecting the BSs for the VWN. At minimum, the incurred cost is only 20% than the DEP, which implies the genetic algorithm might be terminating early, and a tighter solution might be found by increasing G_{halt}. It should also be noted that one unit of cost is one additional BS being selected for the VWN, and the DEP selections for $\alpha \geq 30$ have a cost of only 10 BSs. Any variance that incurs one additional BS for the genetic algorithm incurs 10% increased cost. Increasing the number of BSs required to comprise the VWN would introduce additional granularity in $\mathcal{S}$ that might decrease the inefficiency of the genetic algorithm. This was not done as this data set was chosen specifically so the DEP would terminate within 15 minutes (i.e., in a reasonable amount of time); increasing the number of BSs available to the VNB drastically increases the time it would take the DEP to converge to a solution.

There is a direct correlation between the number of BS in the VWN and its capability for satisfying



(a) VWN costs

(b) SP demand point satisfaction

Fig. 3: Comparison of VWN costs and SP demand point satisfaction. Note that cost and demand satisfaction show a direct correlation.

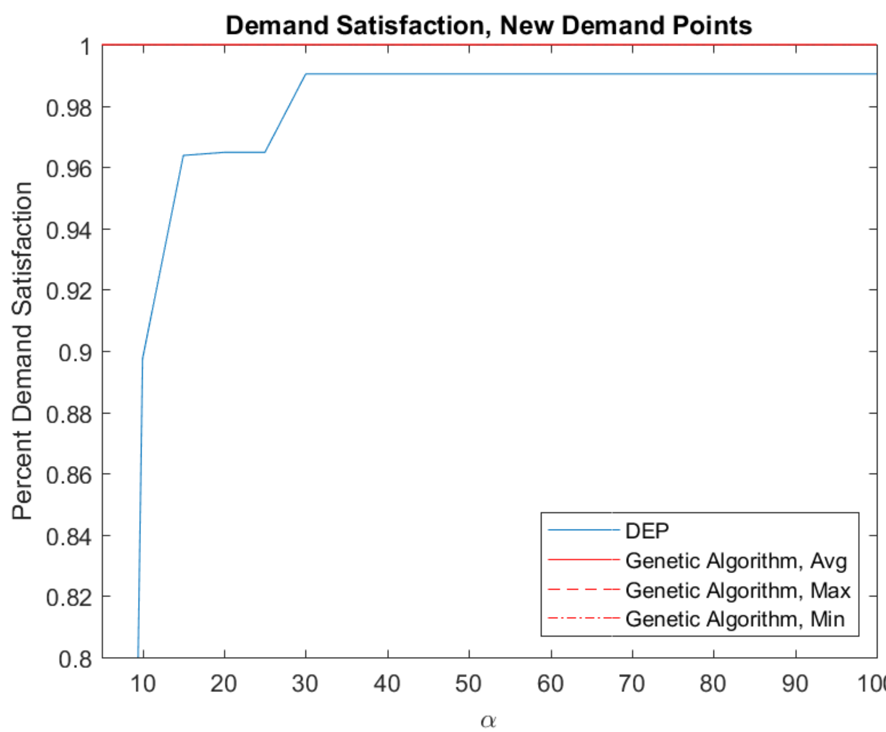


Fig. 4: Comparison of demand satisfaction with 50 scenarios of 200 SP demand points

demand. As the cost increases of the DEP solution, more BS are selected, and the demand satisfaction the DEP solution similarly trends towards 100%, as shown in Fig. **??**. Because of this, the genetic algorithm solutions have a very high demand satisfaction, averaging 99.9% demand satisfaction, even reaching 100% for some solutions. The most expensive, 10-BS DEP solutions reach 99.2% demand satisfaction when slicing the same set of resources that determined the DEP BS selection.

When the set of demand points change to a scenario no longer in Ω, the DEP performs very similarly. Fig. **??** shows the demand satisfaction for both the DEP and genetic algorithm BS selections when sliced to a new set of scenarios. Here, the number of demand points increases to 200 points per scenario, each with 66.8 kbps rate demand, across 50 independent scenarios. The demand satisfaction trend of the DEP BS selection follows very closely to the original set of scenarios, but hits a maximum of 99.0% demand satisfaction with 10 BSs. In comparison, the SP demand point scenarios are far more beneficial to the genetic algorithm, which reaches greater than 99.99% demand satisfaction for all generated VWNs. This is expected as a side effect of the increased number of points and scenarios more accurately describing as a sampling of the original SSLT demand density field of the SP, $\rho(x, y)$.

V. Conclusion

The following are draft notes for this section:

The conclusion goes here. Look back on results and reiterate main takeaways. Include possible avenues for further research and expansion to the model. Perhaps power control, more nuanced demand-resource allocation (other than basic, simple voronoi), path-loss addition, applying slicing to the genetic algorithm more directly, integrating over the regions in the voronoi GA instead of summing (could be faster; math could be interesting in this or further papers), etc.

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