MAT 171 (Illenye) - Section 1.5 Quadratic Equations

1: Quadric Equations

A **quadratic equation** in x an equation that can be written in the **general form**

$$ax^2 + bx + c = 0$$

Where a,b and c exist in the real numbers and $a \neq 0$. These equations are also known as **second degree polynomials** in x.

2: Zero-Product Principle

If the product of two expression algebraic expressions is zero, then at least one of these expressions is equal to zero. Let A and B both be algebraic expressions.

If $AB = 0$, then we know $A = 0$ or $B = 0$.

3: Solving Quadratic Equations by Factoring

Step 1: rewrite the equation in general form moving all non-zero terms to one side.
Step 2: Factor completely.
Step 3: Apply the zero product principle, setting each factor containing a variable equal to zero.
Step 4: Solve the equations in the previous step.
Step 5: check the solutions in the original equations.

4: Square Root Property

If A is an algebraic and C is a non-zero number, then $A^2 = C$ has exactly two solutions:

If $A^2 = C$, then $A = \sqrt{C}$ or $A = \sqrt{C}$.

Often denoted as:

If $A^2 = C$, then $A = \pm\sqrt{C}$.

5: Completing the Square

If $x^2 + bx$ is a binomial, then by adding $(\frac{b}{2})^2$, which is the square of half of the coefficient of x, a perfect square trinomial will result.

$$x^2 + bx + (\frac{b}{2})^2 = (x + \frac{b}{2})^2$$

6: Solving a Quadratic by Completing the Square

Step 1: If a, the leading coefficient, is not 1. divide bot sides by a.
Step 2: Isolate the variable terms on one side of the equation and the constant term on the other side of the equation.
Step 3: Complete the square.
Step 4: Use the square root property to solve for x.

7: Quadratic Formula

The solutions of a quadratic equation in general form $ax^2 + bx + c = 0$, with $a \neq 0$, are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

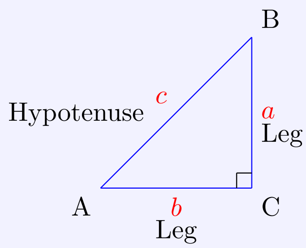
8: The Discriminant

The discriminant of a quadratic equation is $b^2 - 4ac$. It can be used to determine the number of solutions for a quadratic equation.

$b^2 - 4ac > 0$	Two unequal real number solutions
$b^2 - 4ac = 0$	One real number solution (or repeated real number)
$b^2 - 4ac < 0$	No real solution, Two imaginary solutions

9: Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the legs have lengths a and b, and the hypotenuse has length c, then $a^2 + b^2 = c^2$.



Solve the following by Factoring: (Note: Move everything to one side.)

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1) $8r^2 = 72r$

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2) $25x^2 + 4 = 20x$

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3) $12x^2 - 19x - 18 = 0$

Solve each of the following by using the square root property (extracting square roots).

1) $3x^2 - 1 = 47$

2) $(x - 5)^2 = 49$

3) $3(x + 4)^2 = 21$

4) $(7p - 2)^2 = -9$

Solve by completing the square.

1) $x^2 + 8x + 36 = 0$

2) $3x^2 - 4x + 10 = 0$

Solve the following with the Quadratic formula.

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1) $x^2 - 2x + 6 = 0$

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2) $3y^2 - 3 = 10y$

Solve the following with any method.

1) $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{4}$

2) $\frac{3x}{x-9} - \frac{x}{x-8} = \frac{9}{x^2-17x+72}$

1) A rectangular park is 6 miles long and 3 miles wide. how long is a pedestrian route that runs diagonally across the park?

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2) The base of a 28 foot ladder is 8 feet from the building. If the ladder reaches the flat roof, how tall is the building?

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3) Each side of a square is lengthened by 2 inches. The area of this new larger square is 36 square inches. Find the length of a side of the original square.
