MAT 171 - CLASS NOTES - Section 3.2: Polynomial Functions and their Graphs

1. **Degree of a function** - denoted by n, it is the highest exponent of the variable
2. **Leading Coefficient Test**
   (a) When n is odd and the leading coefficient is positive, then the ends follow the pattern of the graph $f(x) = x^3$. Thus the graph points down (falls) on the left and points up (rises) on the right.
   (b) When n is odd and the leadnig coefficient is negative, then the ends follow the pattern of the graph $f(x) = -x^3$. Thus the graph points up (rises) on the left and points down (falls) on the right.
   (c) When n is even and the leading coefficient is positive, then the ends follow the pattern of the graph $f(x) = x^2$. Thus the graph points up (rises) on the left and the right.
   (d) When n is even and the leading coefficient is positive, then the ends follow the patter of the graph $f(x) = -x^2$. Thus the graph pionts down (falls) on the left and the right.
3. **Zeros (roots, x-intercepts, solutions) of a function** - where the graph crosses the x-axis.
   (a) To find the x-intercepts from the equation, set the equation equal to 0, then solve.
   (b) The number of zeros a function has is equal to the degree of the function. The imaginary zeros cannot be graphed.
4. A factor $(x - a)^k$ where $k > 1$ yields a **repeated zero** at $x = a$ of **multiplicity** k.
   (a) If k is odd, then the graph squiggles (like x^3) through the x-axis at $x = a$.
   (b) If k is even, then the graph touches the x-axis (but does not cross the x-axis) at $x = a$.
5. If the multiplicity is one, then teh graph crosses the x-axis at $x = a$.
6. **Turning points**- the number of turning points a graph has is at most $n - 1$.

7. Describe the end behavior of the graph of the polynomial function.

(a) $f(x) = 2x^5 - 5x + 7.5$

(b) $f(x) = 3x^4 - 48x^2$

(c) $f(x) = 15x + 4x^2 - 4x^3$

(d) $f(x) = -2x^6 + 5x^4 + 3x - 1$

8. Find all the real zeros of the polynomial function, determine the multiplicity of each zero and the y-intercept, and draw a rough sketch of the function.

   (a) $f(x) = 3(x+2)^2(x+5)$

   (b) $f(x) = -3x^3(x-1)^2(x+3)$

   (c) $f(x) = -x^4 + 4x^2$

(d) $f(x) = x^4 - x^3 - 20x^2$

(e) $f(x) = x^5 - 14x^3 + 49x$

(f) $f(x) = -x^3 + 4x^2 + 25x - 100$