Graphing Rational Functions

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Rational Expressions and Functions

Blitzer definitions:

A **Rational Expression** consists of a polynomial divided by a nonzero polynomial (denominator cannot be equal to 0).

A **Rational Function** is a function defined by a formula that is a rational expression. For Example:

$$f(x) = \frac{x+3}{x+6}$$

Domain of Rational Functions

The domain of a rational function is all real numbers except those that make the denominator equal to zero.

Example:

$$f(x) = \frac{x+3}{x^2-9}$$

When we factor the denominator we get:

$$f(x) = \frac{x+3}{(x+3)(x-3)}$$

Set the denominator equal to zero and solve.

$$(x+3)(x-3) = 0$$

Using the zero-product principle we know that $x = \pm 3$ solves this equation. So ± 3 is not in the domain.

Interval notation: $D = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Vertical Asymptotes and holes

A **vertical asymptote** is a vertical line that the graph of a function approaches, but does not touch.

Any value that would make the denominator equal to zero is either a hole or a vertical asymptote.

Lets look at our previous function:

$$f(x) = \frac{x+3}{(x+3)(x-3)}$$

at $x = 3$ we have a vertical asymptote and at $x - 3$ we have a hole. Why? can you figure it out?

Horizontal Asymptotes

a **horizontal asymptote** is a horizontal line that the graph of a function approaches as x gets very large or very small. The graph of a function may touch/cross its horizontal asymptote in multiple places.

lets define a general rational function:

$$f(x) = \frac{a_m x^m + a_{m-1} x^{m-1} + a_{m-2} x^{m-2} + ... + a_0}{b_n x^n + b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + ... + b_0}$$

m represents the degree of the polynomial in the numerator and n denotes the degree of the polynomial in the denominator.

a represents the coefficients of the polynomial in numerator and b denotes the coefficients of the polynomial in the denominator.

Horizontal Asymptote (continued)

- if $m < n$ then there is a horizontal asymptote at $y = 0$.
- if $m = n$ then there is a horizontal asymptote at $y = \frac{a_m}{b_n}$.
- if $m > n$ then no horizontal asymptote exists.

Graphing a Rational Function

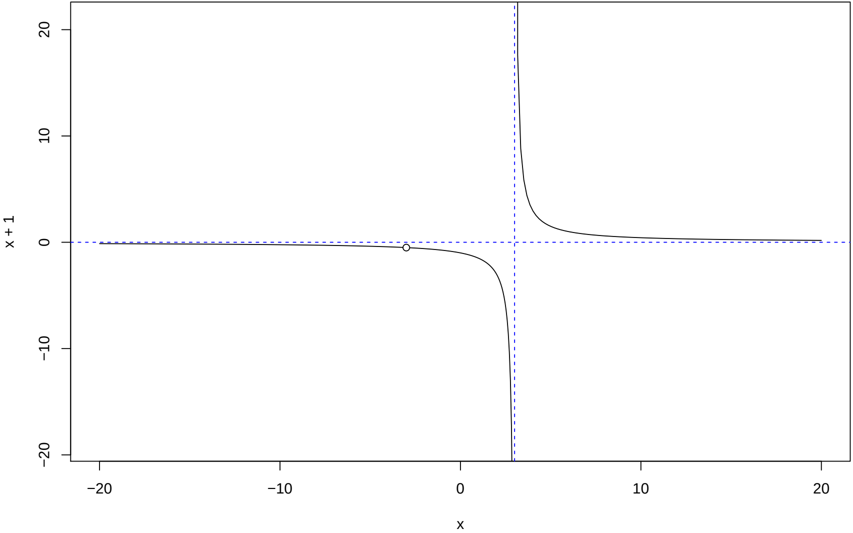
- step 1: factor both the numerator and denominator completely.
- step 2: state the domain.
- step 3: simplify if able.
- step 4: identify vertical asymptotes and holes.
- step 5: idtenify horizontal asymptors.
- step 6: find x and y intercepts.
- step 7: create a behavior table for points near all vertical asymptotes for both sides of the asymptote.
- step 8: plot graph

Graphing a Rational Function Example

$$f(x) = \frac{3x+9}{x^2-9}$$

- step 1: $f(x) = \frac{3(x+3)}{(x-3)(x+3)}$ - step 2: domain is all real numbers except 3 and -3. $D = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ - step 3: $\frac{3}{x-3}$ - step 4: vertical asymptote at $x = 3$ and a hole at $x = -3$ - step 5: horizontal asymptote at $y = 0$ - step 6: no x intercept and a y intercept at $(0, -1)$ - step 7: $f(2.9) = -20.16949$ and $f(3.1) = 19.83607$

plot



Questions

References

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