MAT 171 - CLASS NOTES - Section 3.5: Rational Functions and Their Graphs

1. How to find the **Vertical Asymptote(s) (VA)** of a graph:
   Set the denominator equal to zero and solve.

2. How to find the **Horizontal Asymptote(s) (HA)** of a graph:

   In general, $f(x) = \dfrac{a_m x^m + ... + a_1 x + a_0}{b_n x^n + ... + b_1 x + b_0}$

   (a) If $m < n$, then the Horizontal Asymptote is the equation $y = 0$.

   (b) If $m > n$, then no Horizontal Asymptote exists.

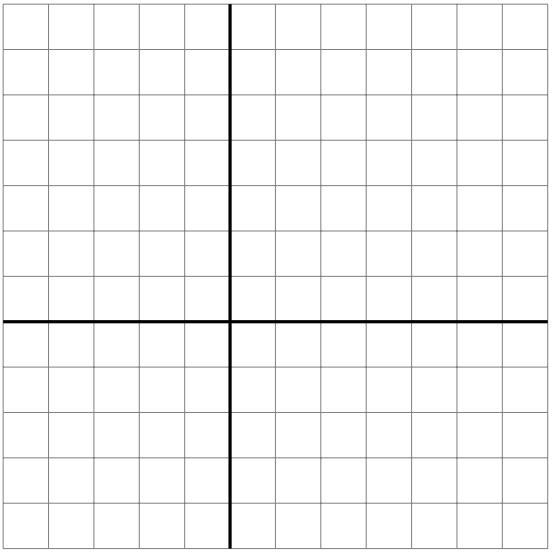
   (c) If $m = n$, then the Horizontal Asymptote is the equation $y = \dfrac{a_m}{b_n}$.
   Are there any other possibilities?

3. How to find the **x-intercepts**.
   Set the numerator equal to zero and solve.

4. How to find the **y-intercept**.
   Find $f(0)$. In other words, plug zero into the function and evaluate.

5. How to **graph the rational function**.
   Set up a chart with intervals from left to right using the x-intercepts and/or Vertical Asymptotes as boundaries to start and stop your intervals. Then find whether y is positive or negative in those intervals. More techniques will be discovered with practice and help from Calculus in the future, but this is a mighty fine rough sketch.

6. Find all the pieces listed above (HA, VA, x-int, and y-int) and graph $f(x) = \dfrac{2x+1}{x-2}$.



7. Find the domain of the function and identify any horizontal and vertical asymptotes.

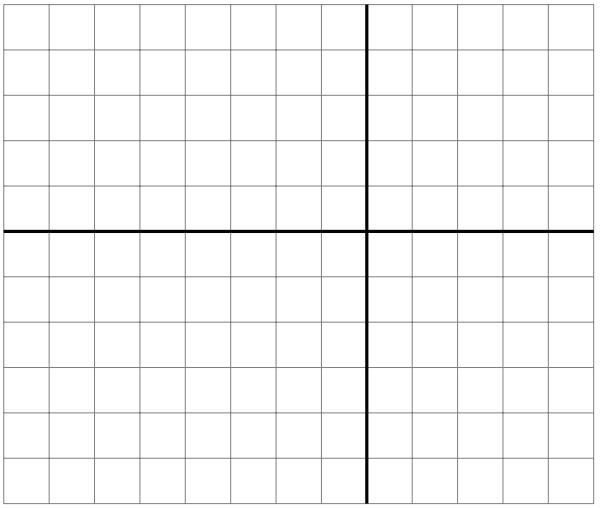
(a) $f(x) = \dfrac{1}{x^2 - 4}$

(b) $f(x) = \dfrac{2x^3}{x^2 + 3x + 2}$

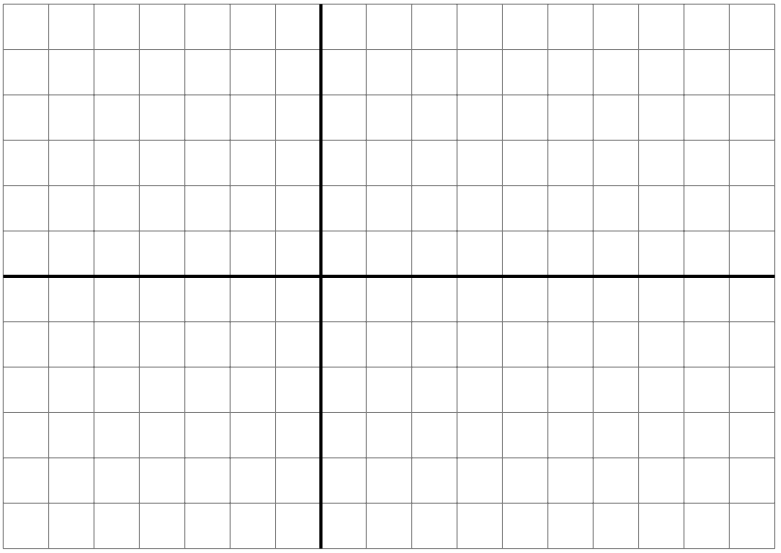
(c) $f(x) = \dfrac{x + 4}{x^2 - 8x - 48}$

(d) $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2}$

8. Graph $f(x) = \dfrac{1}{x+3}$



9. Graph $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9}$



10. Graph $f(x) = \dfrac{x^2}{x^2 - 16}$