MAT 171 - CLASS NOTES - Section 3.4: Zeros of Polynomial Functions

1. Find all the zeros of the function, state the degree of the function, and draw a rough sketch.

   (a) $f(x) = x^2(x+3)(x-1)^3(x+1)$

   (b) $f(x) = (x+5))(x-8)^4(x+2i)(x-2i)$

2. Find a polynomial function that has the following zeros and a leading coefficient of $a = 1$.

   (a) $4, 3i, -3i$

   (b) $-5, -5, 1+i\sqrt{3}$

   (c) zeros at -3 with a multiplicity of 2, 0 with a multiplicity of 1, 2 with a multiplicity of 5 and $2+5i$ with a multiplicity of 1. (Hint: not all roots are explicity listed.)

3. Find an n-th degree polynomial function with real coefficients satisfying the given conditions.

   $n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$

4. **Rational Zero Test** - Given a polynomial of general form $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x^1 + a_0$ we let p be a factor of a_0 and q be a factor of a_n.

   Then all **possible** rational zeros of the function comes from finding all possible $\pm \frac{p}{q}$.

   In more detail, be sure the polynomial is in proper order, then find all the factors of of the constant coefficient and list them on the top of a fraction. Do the same for the leading coefficient except place them in the denominator of the fraction you are creating. Then take every combination of a top element over a bottom element and generate a list of fractions. If the polynomial crosses the x-axis at a rational number it will be on the list you've just created!

5. Use the Rational Zero Test to list all possible rational zeros of $f(x)$.

   (a) $f(x) = x^3 - 4x^2 - 4x + 16$

   (b) $f(x) = 2x^4 - 3x^3 - 11x^2 - 9x + 15$

6. List all the possible rational zeros of $f(x)$. Use synthetic division to test the possible rational roots and find an actual root. Then find the remaining zeros of $f(x)$ and solve the equation for $f(x) = 0$.

   (a) $f(x) = -3x^3 + 20x^2 - 36x + 16$

(b) $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

(c) $f(x) = x^3 - 13x^2 + 65x - 125$

(d) $f(x) = x^3 - 4x^2 + 8x - 5$

(e) $f(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

(f) $f(x) = 2x^4 - x^3 - 2x^2 + 13x - 6$