Markovian evolution of quantum coherence under symmetric dynamics

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Motivation

Resource theory meets applications

Quantum coherence as a resource

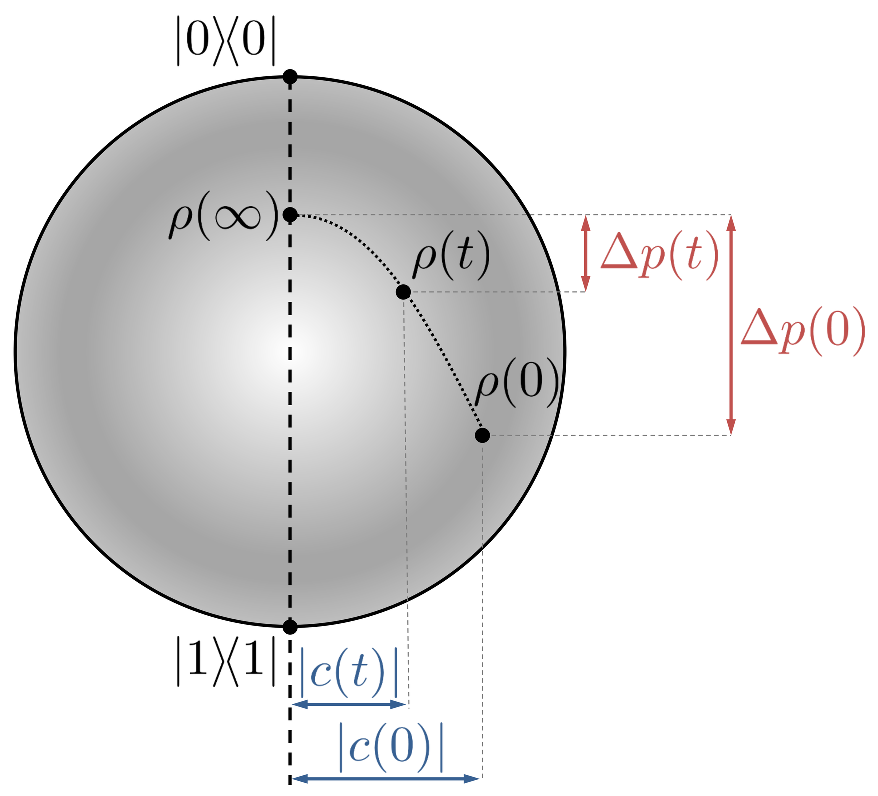
- Coherence in LOCC scenarios
- Coherence in thermodynamics
- Incoherent ops, symmetric ops, etc.

?

Quantum coherence in applications

- Decoherence and relaxation
- Master equation formalism
- Secular approximation

Elementary scenario



Population dynamics:

$$\Delta p(t) = \Delta p(0) e^{-t/T_1}$$

Coherence dynamics:

$$|c(t)| = |c(0)| e^{-t/T_2}$$

Relaxation and decoherence times are linked:

$$T_2 \leq 2T_1$$

Setting the scene

Assumptions

(A1) Master equation: $\frac{d\rho}{dt} = -i\mathcal{H}(\rho) + \mathcal{L}(\rho)$

(A2) Time-translation symmetry: $[\mathcal{L}, \mathcal{H}] = 0$

Unitary evolution:

$$\mathcal{H}(\rho) = [H, \rho], \quad H = \sum_{x=0}^{d-1} \hbar\omega_x |x\rangle\langle x|$$

Dissipative evolution:

$$\mathcal{L}(\rho) = \mathcal{A}(\rho) - \tfrac{1}{2}\{\mathcal{A}^\dagger(\mathbb{1}), \rho\}, \quad \mathcal{A} \text{ – CP map}$$

Physical relevance of (A2)

1. Rotating wave approximation
$$g(\sigma_+ + \sigma_-)(a + a^\dagger) \longrightarrow g(\sigma_+ a + \sigma_- a^\dagger)$$
2. Secular approximation
$$\mathcal{L} \longrightarrow \mathcal{L}', \quad [\mathcal{L}', \mathcal{H}] = 0$$
3. Absence of a reference frame for time
4. Superselection rule
5. Global conservation law

Mode structure

Due to symmetry constraint

$$\langle x'|\mathcal{L}(|x\rangle\langle y|)|y'\rangle = 0 \text{ unless } \omega_{xy} = \omega_{x'y'},$$

where $\omega_{xy} := \omega_x - \omega_y$.

Matrix elements evolve in independent modes

$$\rho^{(\omega)} = \sum_{x,y}^{(\omega)} \rho_{xy} |x\rangle\langle y| \text{ with } \sum_{x,y}^{(\omega)} := \sum_{\substack{x,y \\ \omega_{xy}=\omega}}$$

Mode $\omega = 0$: populations
Modes $\omega \neq 0$: coherences

Minimal decoherence theorem

Notation

Population transition rates:

$$L_{x|x'} := \langle x|\mathcal{L}(|x'\rangle\langle x'|)|x\rangle$$

Coherence damping rates:

$$\gamma_{xy} = (|L_{x|x}| + |L_{y|y}|)/2$$

Coherence transport rates:

$$t^{x|x'}_{y|y'} = \sqrt{L_{x|x'} L_{y|y'}}$$

Statement

For every symmetric Master equation

$$|\rho_{xy}(t)| \leq \sigma_{xy}(t)$$

with

$$\frac{d\sigma_{xy}}{dt} = -\gamma_{xy}\sigma_{xy} + \sum_{x',y'}^{(\omega_{xy})} t^{x|x'}_{y|y'} \sigma_{x'y'}$$

Observations

1. Symmetry-induced mode structure constrains coherence transport to a given mode.
2. Coherence damping and transport rates are directly linked with population dynamics.
3. Tightness conditions are known and yield optimal coherence-preserving maps.

Applications

Generalisation of $T_2 \leq 2T_1$

Definition: Population dynamic is ergodic if it has a unique fixed point.

Ergodicity and no matching energy gaps

$$\rho_{xx}(t) = \rho_{xx}(\infty) + \sum_y a_{xy} e^{-t/T_1^y}$$

$$|\rho_{xy}(t)| = |\rho_{xy}(0)| e^{-t/T_2^{xy}}$$

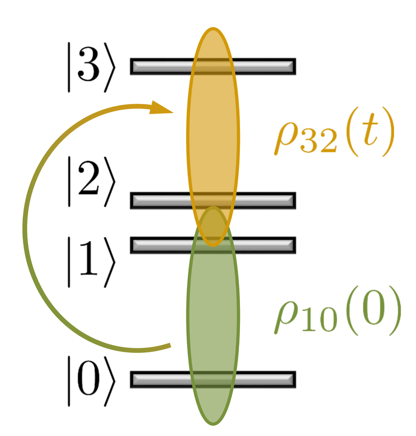
With relaxation and decoherence times related by

$$\langle T_2 \rangle_h \leq \frac{d}{d-1} \langle T_1 \rangle_h, \quad \langle \cdot \rangle_h \text{ - harmonic average}$$

Ergodicity and arbitrary Hamiltonian

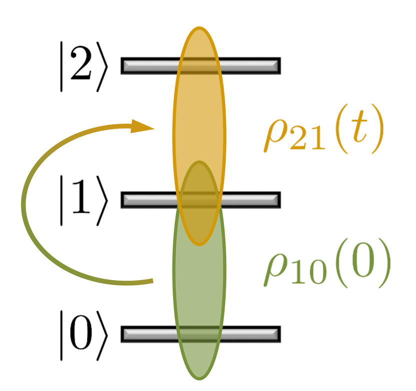
$$|\rho_{xy}(t)| \xrightarrow{t\to\infty} 0$$

Non-Markovianity as a resource for coherence processing?



Perfect transfer of coherence possible via Markovian dynamics

$$\rho_{32}(\infty) = \rho_{10}(0)$$

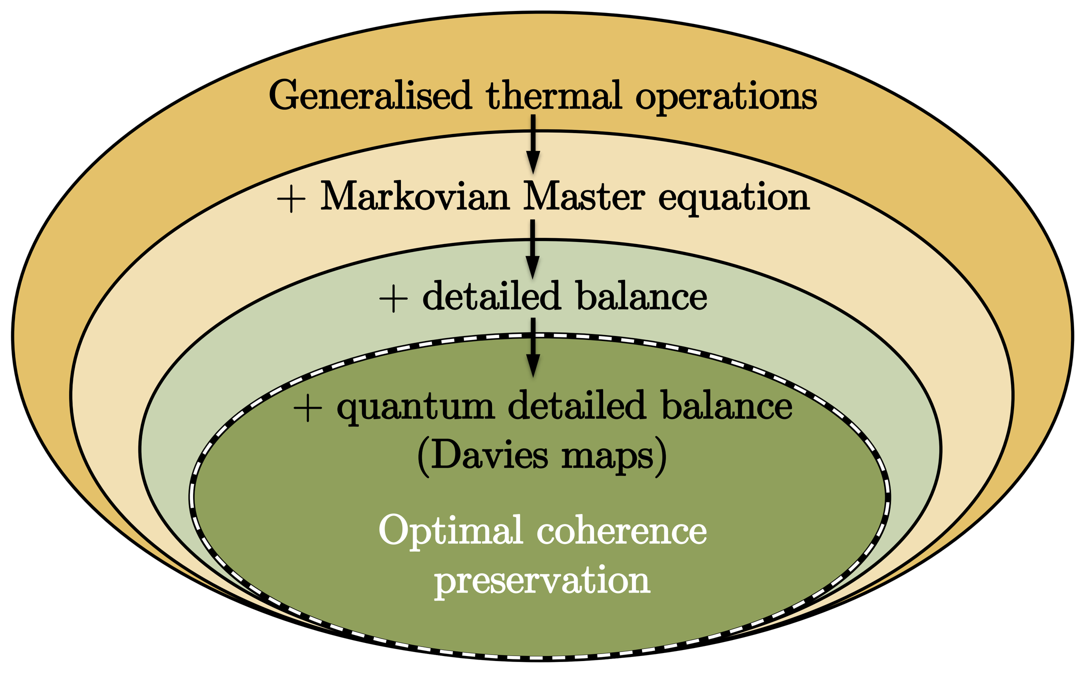


Perfect transfer of coherence possible only via non-Markovian dynamics

$$\rho_{21}(t) < \rho_{10}(0)$$

Answer depends crucially on mode structure of the system and is connected to the existence of non-ergodic dynamics

From resource theory to thermalisation models



Resource theory of thermodynamics as a non-Markovian generalisation of thermalisation models

Tighter and physically more relevant bounds on possible transformations under thermal operations

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