Quantum information and thermodynamics: a resource-theoretic approach

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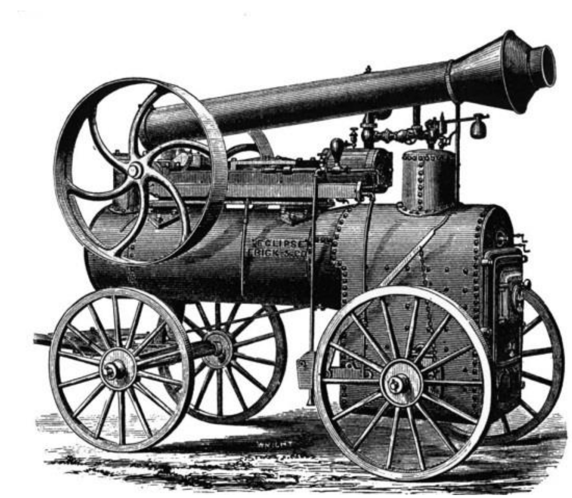
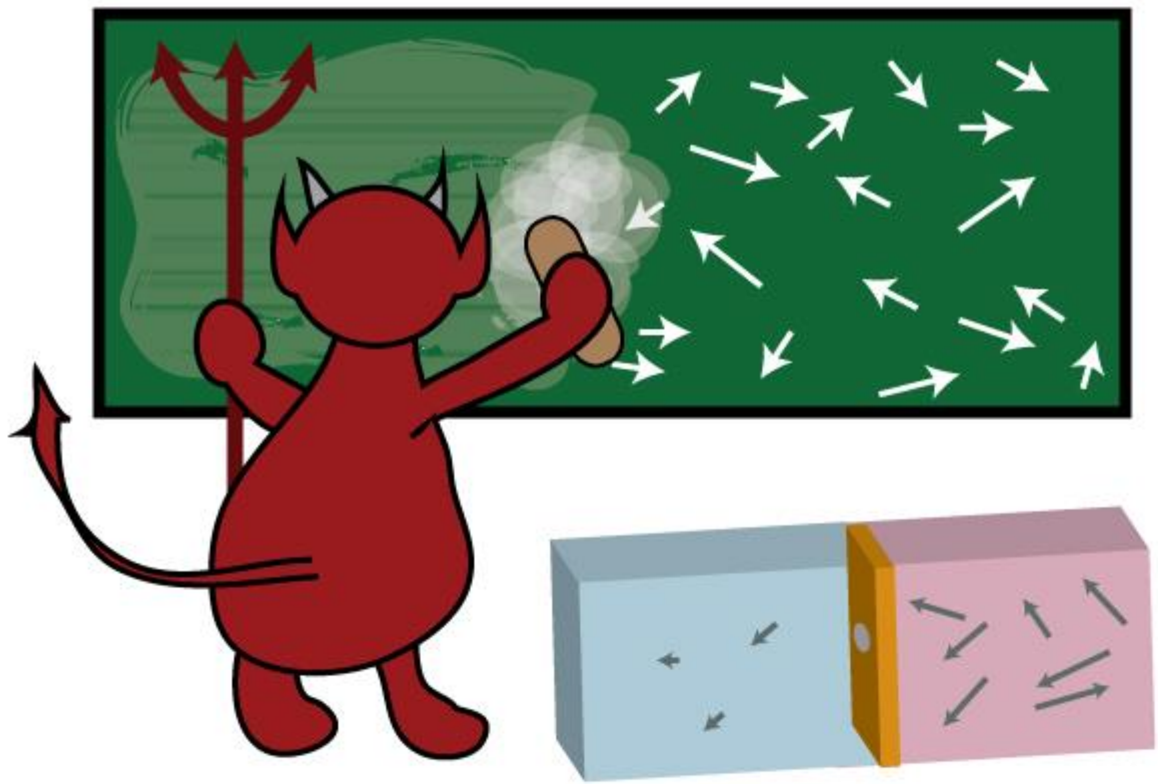


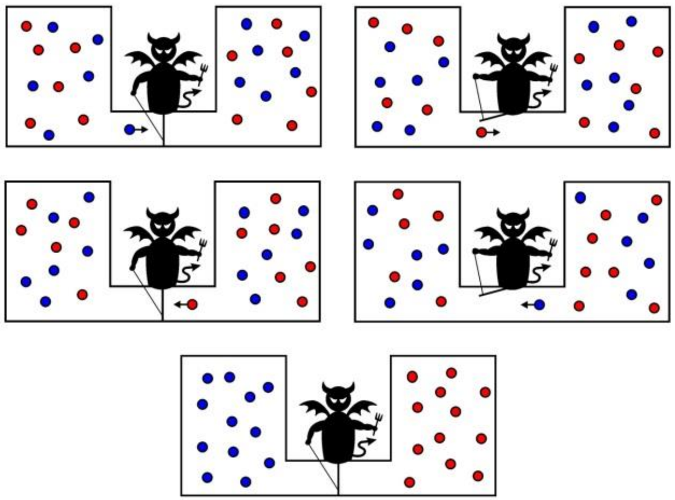
1. Motivation



*Picture from Lidia del Rio PhD Thesis

Thermodynamics and information theory

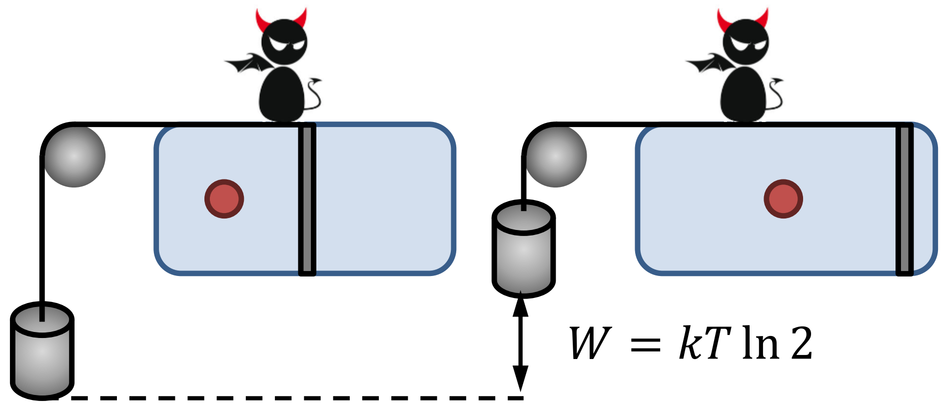
1874 - Maxwell's demon



Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality

Nature Physics **6**, 988–992 (2010)

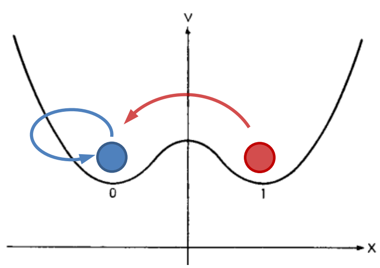
1929 - Szilard engine



Experimental realization of a Szilard engine with a single electron

Proceedings of the National Academy of Sciences **111**, 13786 (2014)

1961 – Landauer erasure



Experimental verification of Landauer's principle linking information and thermodynamics

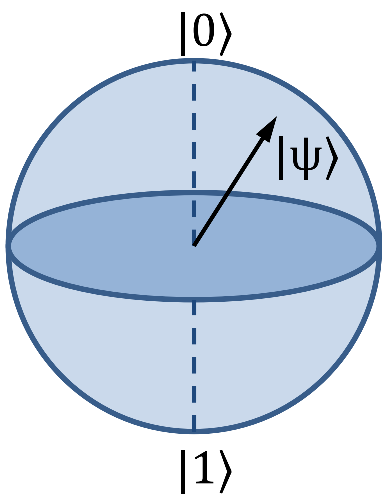
Nature **483**, 187–189 (2012)

Classical information: key messages

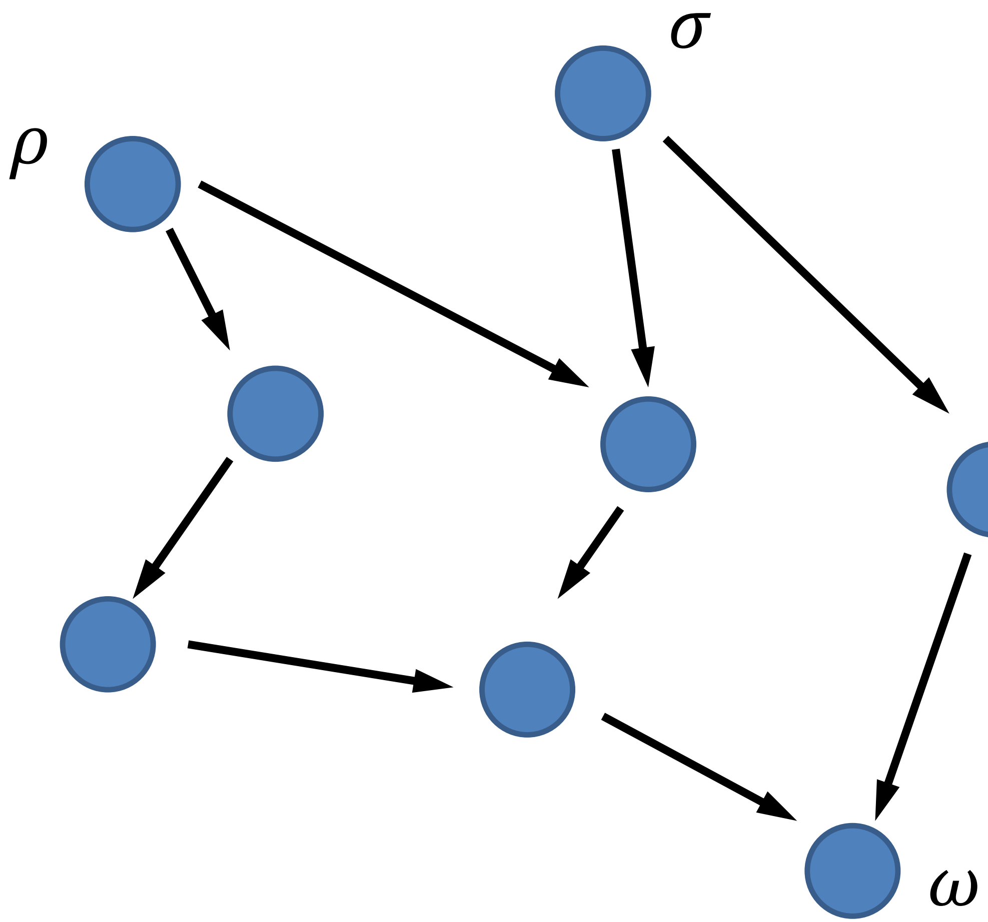
- Irreversible information processing requires heat dissipation and *vice versa* heat can be seen as a loss of information.
- Correspondence between thermodynamic and information entropy.
- One bit of information has an ***energetic value*** of $\mathrm{kT}\ln 2$.

Quantum information: central questions

- What about quantum information and qubits?
- Do entanglement and **coherence** play any special role?



2. Thermodynamics as a resource theory



General framework

Thermodynamics ultimately concerns the accessibility/inaccessibility of one physical state from another.

The mathematical foundations of thermodynamics, R. Giles (1964)

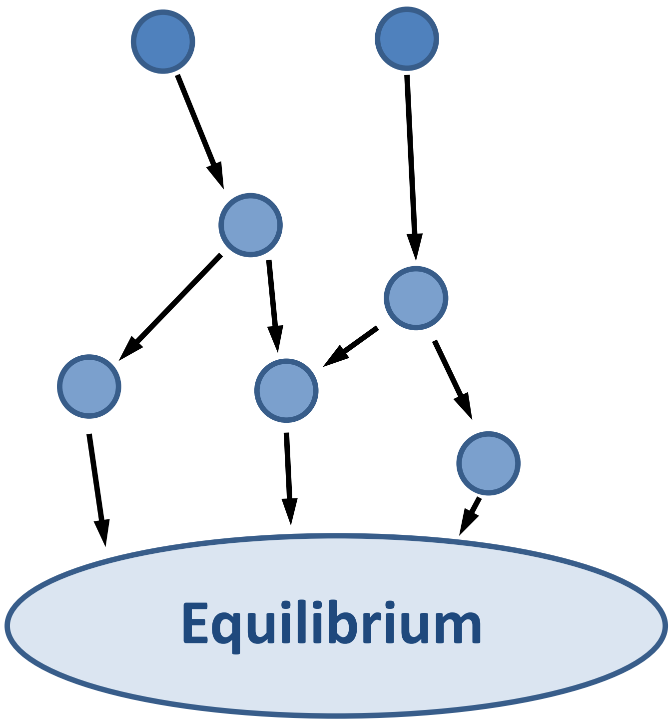
$$\boldsymbol{T = 300K}$$

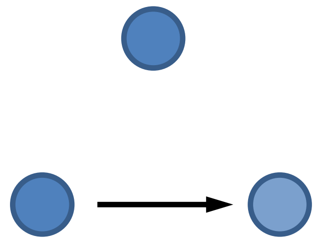
Out-of-equilibrium state $\longrightarrow$ Equilibrium state

Equilibrium state $\longleftarrow$ Out-of-equilibrium state

$$\boldsymbol{T = 200K}$$

Ordering of states





Quantum states

Allowed transformations

General resource theory:

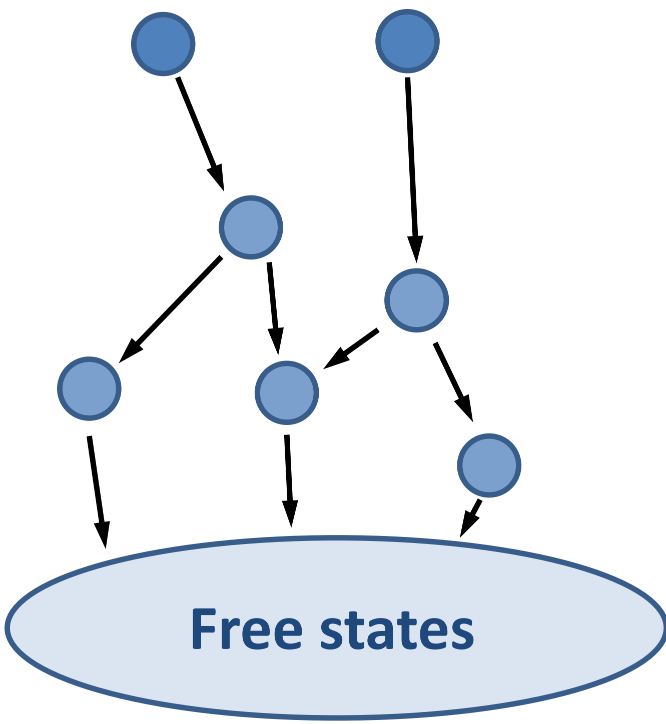
1. Define the set of **free states.**
2. Define the set of **free operations.**

Free states and free operations together imply **resource measures.** These allow a quantitative approach.

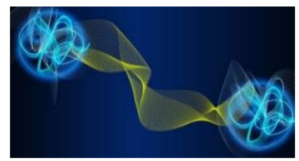
Define measure M:

If $\rho \to \sigma$
Then $M(\rho) \geq M(\sigma)$

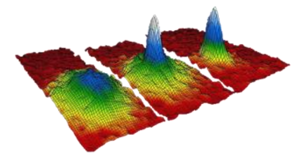
General framework



Recently developed resource theories



Entanglement



Coherence

Asymmetry

Entanglement	Coherence	Asymmetry
Non-local	**Coherent**	**Asymmetric**
↓	↓	↓
Separable	**Incoherent**	**Symmetric**

Resource theory of thermodynamics:

Free states – equilibrium states

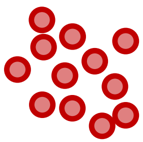
Free operations - ….

Free operations in thermodynamics

Given

 System in a state ρ_S described by Hamiltonian H_S

We can choose

 Environment described by an arbitrary Hamiltonian H_E prepared in a thermal state γ_E

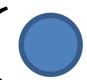
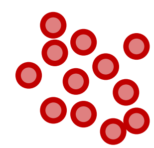
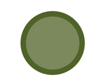
"Encoding" 2nd Law

$$\gamma_E = \frac{e^{-\beta H_E}}{Tr(e^{-\beta H_E})}$$

And couple it with the system through an energy-preserving unitary U:

$$[U, H_S + H_E] = 0$$

"Encoding" 1st Law

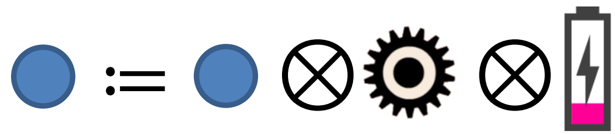
$Tr_E\Big(U($  $\otimes$  $)U^\dagger\Big) =$ 

Formal definition of free operations (known as **thermal operations**):

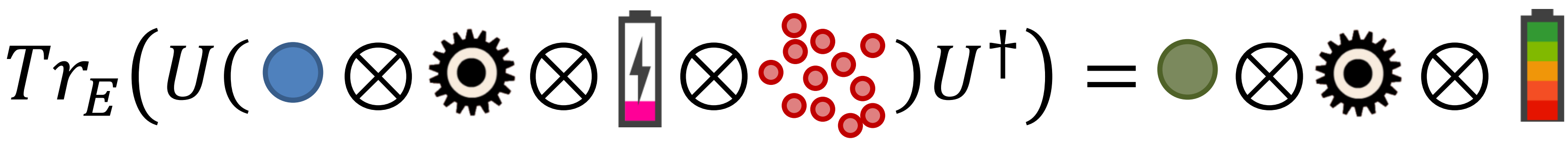
$$\mathcal{E}_T(\rho_S) = Tr_E\big(U(\rho_S \otimes \gamma_E)U^\dagger\big)$$

Explicit extension of free operations

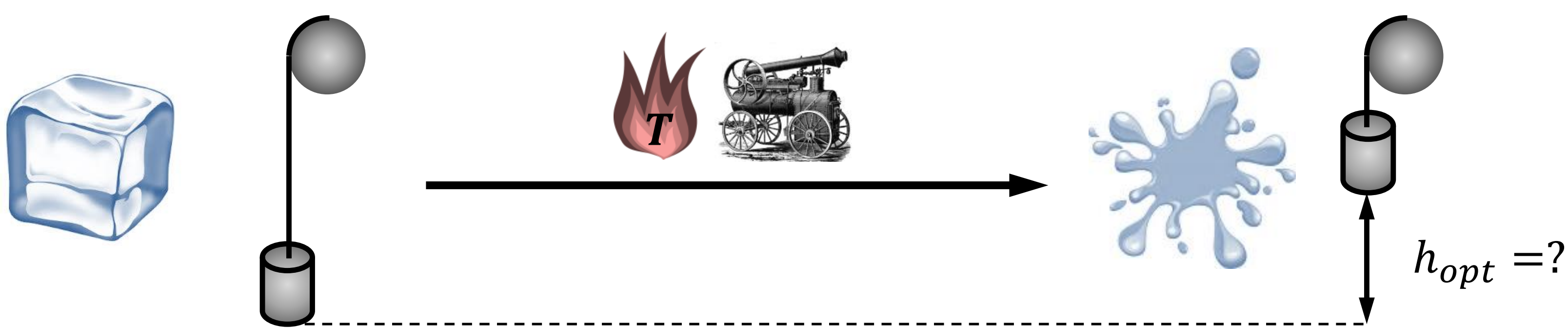
We can include the use of cyclically working thermal machines and describe processes requiring work (not only heat from thermal environment) by explicitly modelling the machines and work storage systems:



And consider a thermal operation on the joint system:

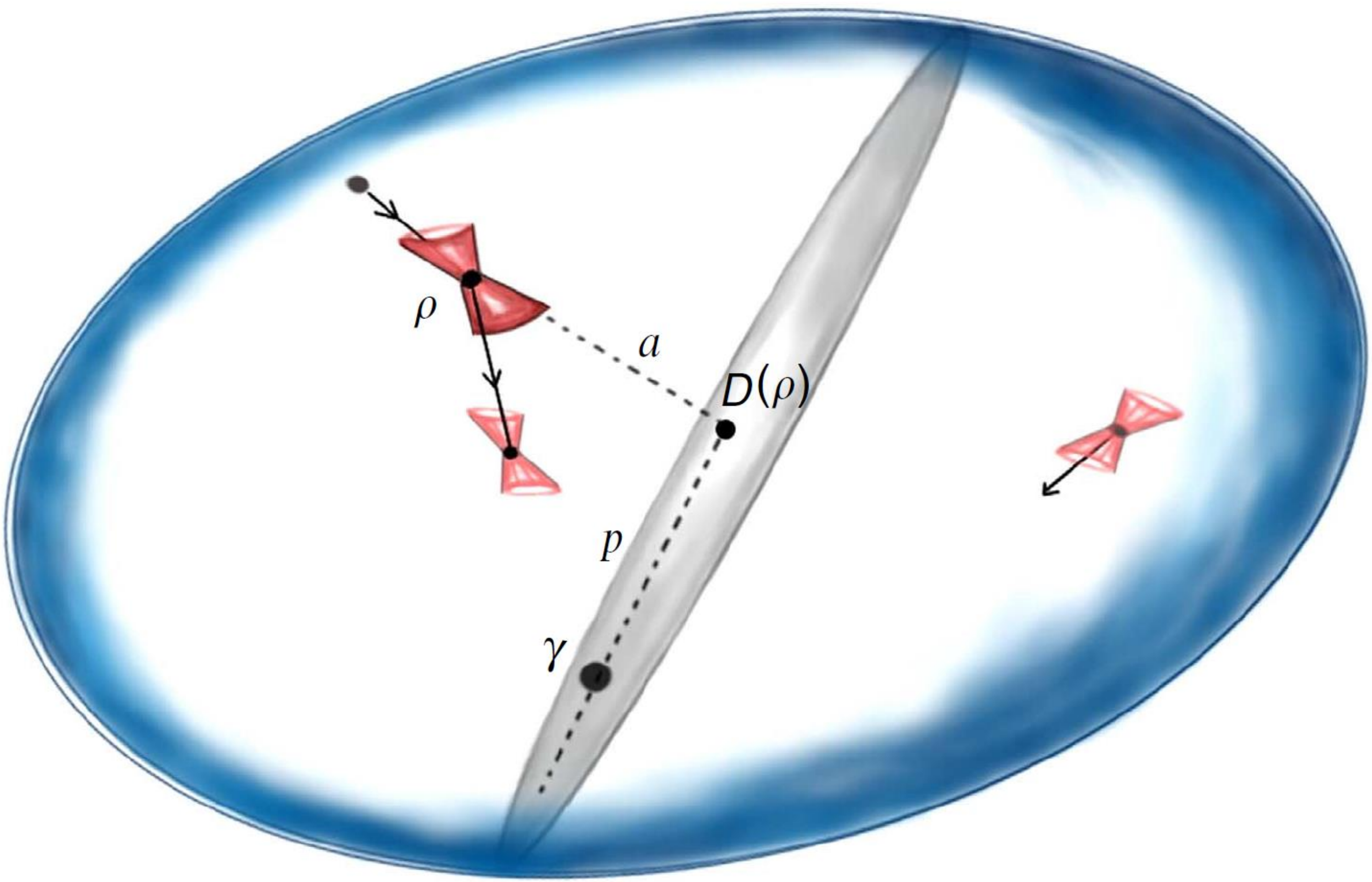


This way we can study all sorts of questions like:



But instead of ice cubes we can study single quantum systems.

3. Results



Thermodynamic ordering of quantum states

To find the ordering of states is to answer the interconversion problem:

$$\begin{pmatrix} p_0 & c_{01} & c_{02} & \cdots & c_{0n} \\ c_{10} & p_1 & c_{12} & \cdots & c_{1n} \\ c_{20} & c_{21} & p_2 & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n0} & c_{n1} & c_{n2} & \cdots & p_n \end{pmatrix} \xrightarrow{\mathcal{E}_T} \begin{pmatrix} q_0 & 0 & 0 & \cdots & 0 \\ 0 & q_1 & 0 & \cdots & 0 \\ 0 & 0 & q_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & q_n \end{pmatrix}$$

Solved: Necessary and sufficient condition for $\rho_S \to \sigma_S$: $\forall_\alpha : F_\alpha(\rho_S) \geq F_\alpha(\sigma_S)$

$$F_\alpha(\rho_S) := -kT \ln Z + kT S_\alpha(\rho_S || \gamma_S)$$

Standard expression for free energy at equilibrium (Z is the partition function)

Nonequilibrium contribution

$$S_\alpha(\rho_S || \gamma_S) := \frac{1}{\alpha - 1} \ln \sum_i p_i^\alpha \gamma_i^{1-\alpha}$$

The second laws of quantum thermodynamics
F. Brandão, *et. al*, PNAS **112**, 3275 (2015)

Thermodynamic coherence processing

To find the ordering of states is to answer the interconversion problem:

$$\begin{pmatrix} p_0 & c_{01} & c_{02} & \cdots & c_{0n} \\ c_{10} & p_1 & c_{12} & \cdots & c_{1n} \\ c_{20} & c_{21} & p_2 & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n0} & c_{n1} & c_{n2} & \cdots & p_n \end{pmatrix} \xrightarrow{\mathcal{E}_T} \begin{pmatrix} q_0 & ? & ? & \cdots & ? \\ ? & q_1 & ? & \cdots & ? \\ ? & ? & q_2 & \cdots & ? \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & ? & ? & \cdots & q_n \end{pmatrix}$$

Observation: thermal operations are a subset of **time-translation symmetric** operations:

$$\mathcal{E}_T\left(e^{-iH_S t}\rho_S e^{iH_S t}\right) = e^{-iH_S t}\mathcal{E}_T(\rho_S)e^{iH_S t}$$

As a result all measures of coherence must be non-increasing. E.g.,

$S_\alpha(\rho||D(\rho))$ Information theoretic “distance” between a state and its dephased version

Dephasing operation $$D(\rho) = \sum_n \langle n|\rho|n\rangle |n\rangle\langle n|$$

Description of quantum coherence in thermodynamic processes requires constraints beyond free energy,
M. Lostaglio, D. Jennings, T. Rudolph,
Nat. Commun. **6**, 6383 (2015)

Mode structure of time-translation symmetric maps

System described by nondegenerate Hamiltonian:

$$H_S = \sum_n \hbar\omega_n \, |n\rangle\langle n| \qquad\qquad \rho_S(0) = \sum_{n,m} \rho_{nm} \, |n\rangle\langle m|$$

Modes of coherence ${\rho_S}^{(\omega)}$:

$${\rho_S}^{(\omega)} := \sum_{\substack{n,m \\ \omega_n - \omega_m = \omega}} \rho_{nm} |n\rangle\langle m|; \qquad \rho_S(0) = \sum_\omega {\rho_S}^{(\omega)} \qquad \rho_S(t) = \sum_\omega {\rho_S}^{(\omega)} e^{-i\hbar\omega t}$$

Each mode transforms independently and its *intensity* cannot increase:

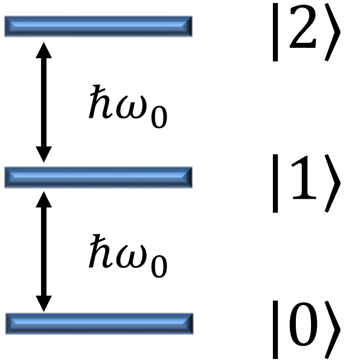
Given: $\sigma_S = \mathcal{E}_T(\rho_S)$ We have:

$${\sigma_S}^{(\omega)} = \mathcal{E}_T\left({\rho_S}^{(\omega)}\right)$$

$$\left\|{\sigma_S}^{(\omega)}\right\| \le \left\|{\rho_S}^{(\omega)}\right\|$$

Modes of asymmetry…
I. Marvian, R. Spekkens,
Phys. Rev. A 90, 062110 (2014)

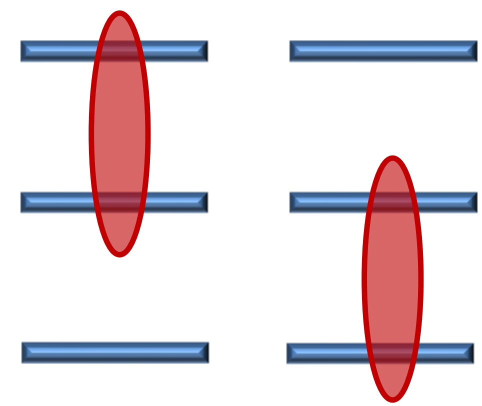
Modes: Qutrit example



$$H_S = \sum_{n=0}^{2} n\hbar\omega_0 \, |n\rangle\langle n|$$

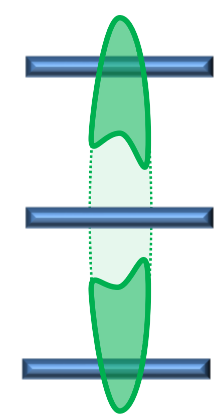
$$\rho_S = \begin{pmatrix} p_0 & c_{01} & c_{02} \\ c_{10} & p_1 & c_{12} \\ c_{20} & c_{21} & p_2 \end{pmatrix}$$

Mode 0: $\rho_S{}^{(0)}(t) = \rho_S{}^{(0)}$



Mode ω_0 and $-\omega_0$: $\rho_S{}^{(\omega_0)}(t) = \rho_S{}^{(\omega_0)} e^{-i\omega_0 t}$;

$\rho_S{}^{(-\omega_0)}(t) = \rho_S{}^{(-\omega_0)} e^{i\omega_0 t}$



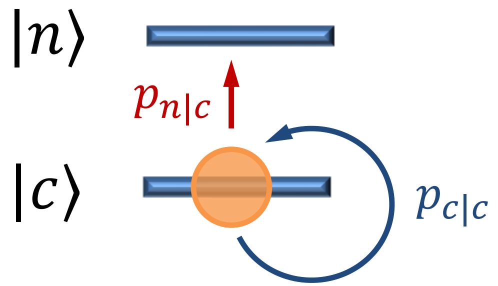
Mode $2\omega_0$ and $-2\omega_0$: $\rho_S{}^{(2\omega_0)}(t) = \rho_S{}^{(2\omega_0)} e^{-i2\omega_0 t}$;

$\rho_S{}^{(-2\omega_0)}(t) = \rho_S{}^{(-2\omega_0)} e^{i2\omega_0 t}$

Mode structure of time-translation symmetric maps

Action of $\mathcal{E}_T$ on mode zero (occupation of energy eigenstates) described by stochastic matrix Λ_T (energy transfers independent of coherence):

$$[\Lambda_T]_{nc} = p_{n|c} := \langle n|\mathcal{E}_T(|c\rangle\langle c|)|n\rangle$$



What are the allowed transformations of non-zero (coherence) modes?

1. We need to enforce CPTP condition.

2. We note that Gibbs state as a fixed point of all thermal maps:
$\mathcal{E}_T(\gamma_S) = \gamma_S$

"Encoding" 2nd Law

Otherwise one could build a *perpetuum mobile*

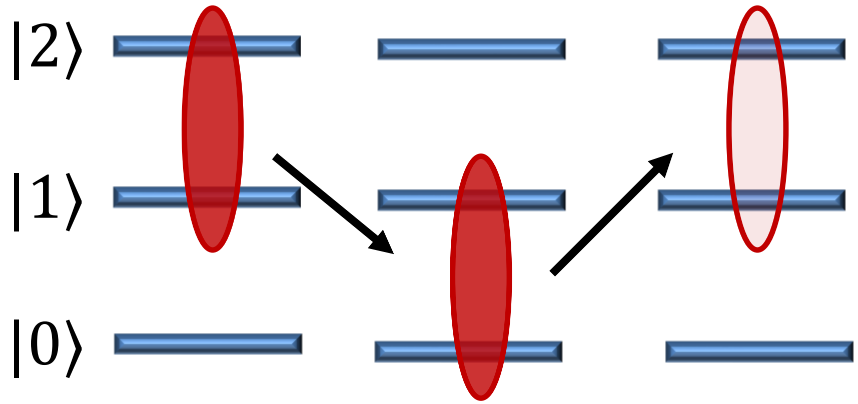
Bounds on coherence transformations

From the CPTP condition we get:
(Inequality is saturated by a time-translation symmetric map.)

$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m}} |\rho_{cd}| \sqrt{p_{n|c} p_{m|d}}$$

Gibbs preserving property leads to irreversibility of coherence transfer:

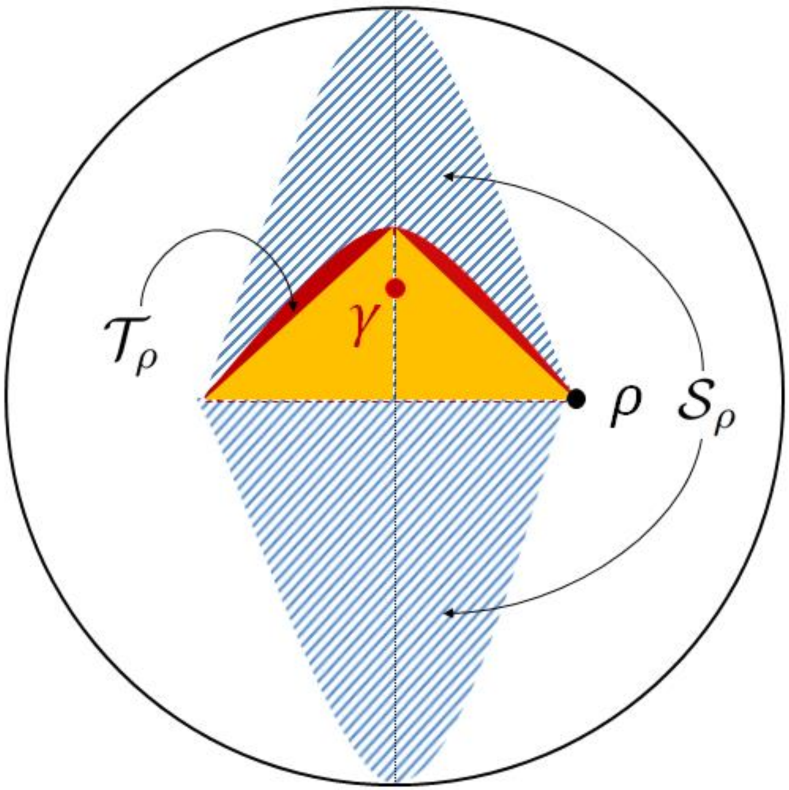
$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c > \omega_n}} |\rho_{cd}| \quad + \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c \leq \omega_n}} |\rho_{cd}| \, e^{-\beta\hbar(\omega_n - \omega_c)}$$



Quantum coherence, time-translation symmetry and thermodynamics
M. Lostaglio, K. Korzekwa, D. Jennings, T. Rudolph, Phys. Rev. X **5**, 021001 (2015)

Elementary scenario - ordering of qubit states (or: coherence processing for qubit systems)

$H_S = |1\rangle\langle 1|$



Coherence is **actively** contributing to enlarge the set of thermodynamically accessible states.

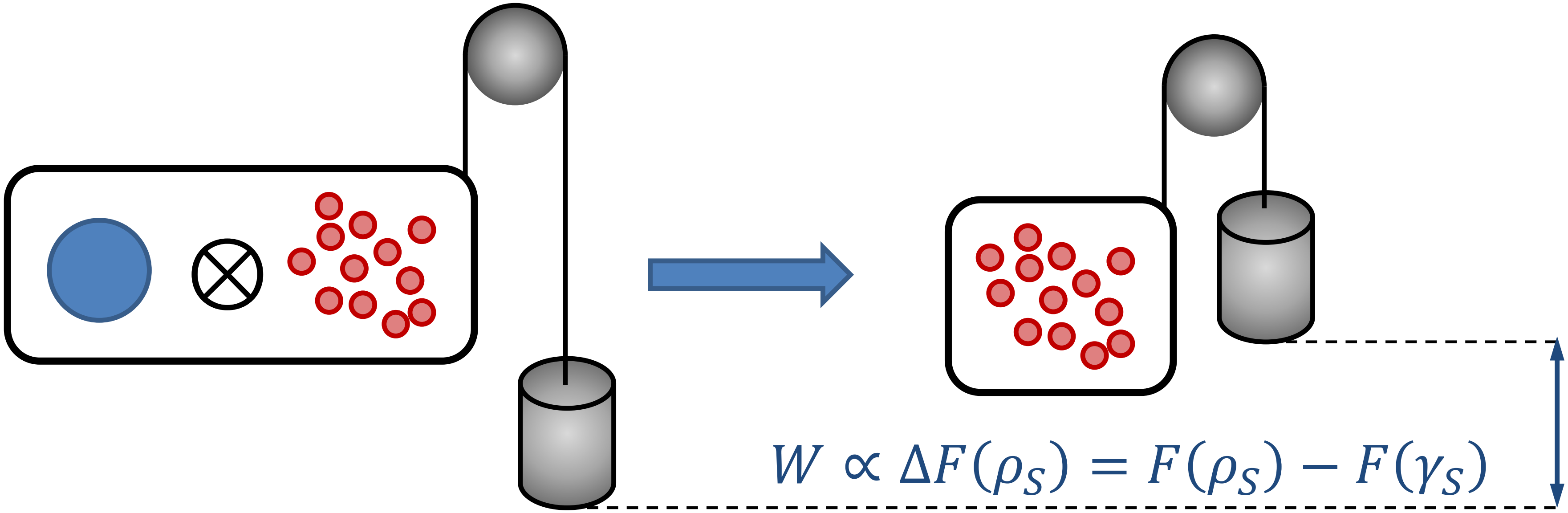
Work is **not** the universal resource of thermodynamics.

Coherence contribution to free energy is **locked** - no trivial extension to quantum Szilard engine.

$\mathcal{T}_\rho$: Set of states accessible from ρ via thermal operations (orange region if coherence is passive)

$\mathcal{S}_\rho$: Set of states accessible from ρ via thermal operations and the access to infinite amount of work

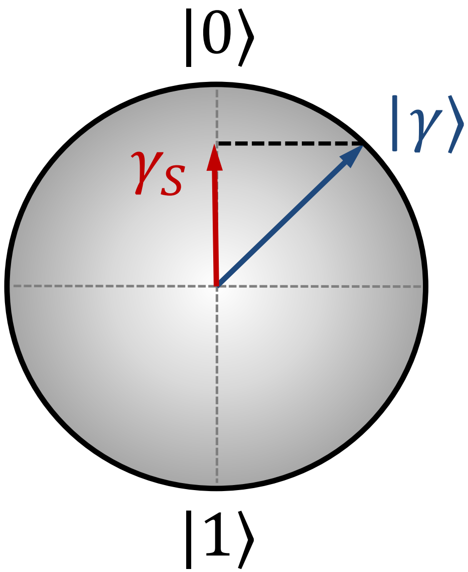
Work extraction and work-locking



$$W \propto \Delta F(\rho_S) = F(\rho_S) - F(\gamma_S)$$

Work ∝ Free energy difference

With: $F(\rho) = Tr(\rho H_S) - kTS(\rho)$



Coherence part of free energy is locked!

$$\rho_S \to W \Leftrightarrow D(\rho_S) \to W$$

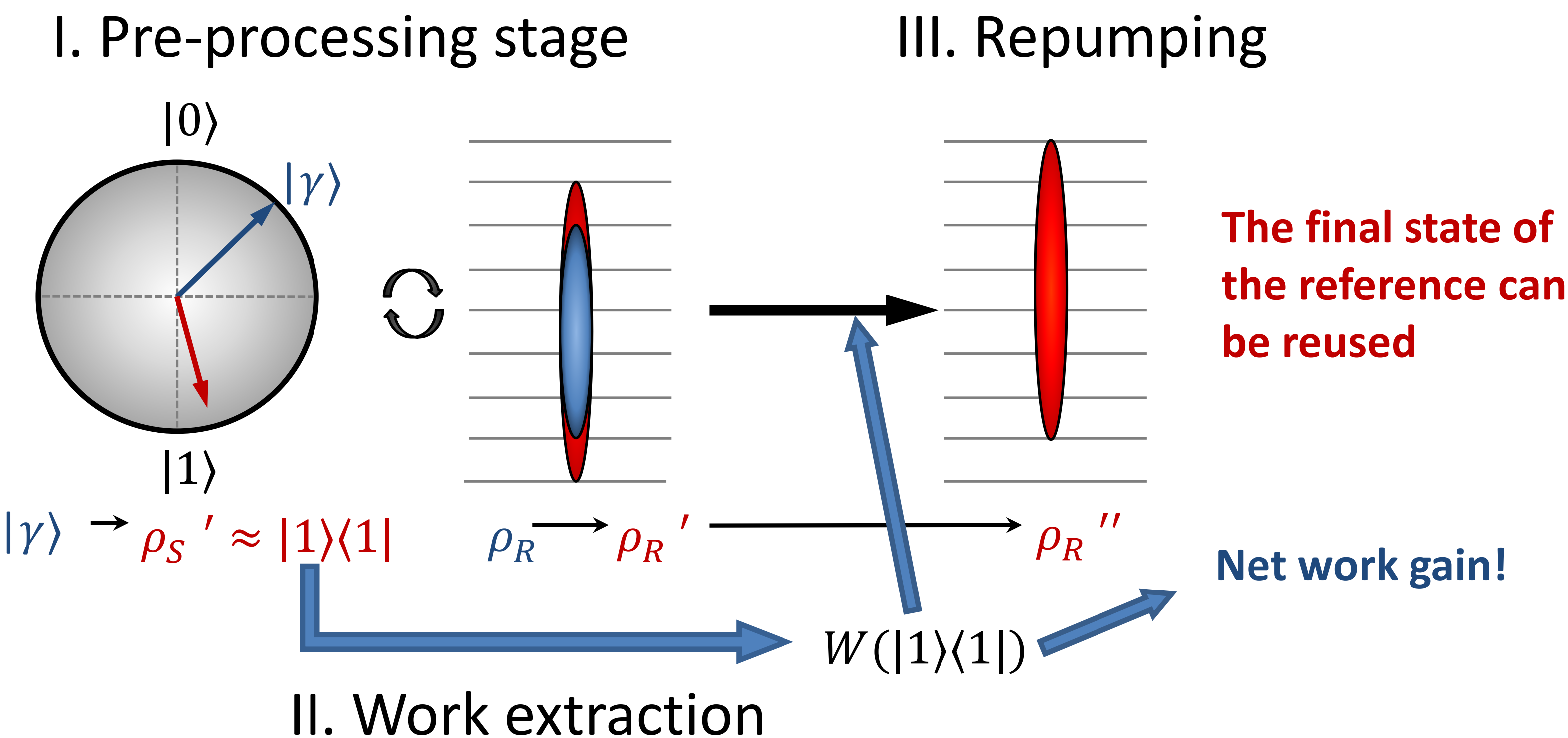
E.g. The amount of work that can be extracted from pure qubit state $|\gamma\rangle$ is zero.

Unlocking work with a repeatable resource

Idea: Introduce an ancillary ladder system (**reference**) with coherence that can be reused infinitely many times:

$$H_R = \sum_n \hbar\omega_0 n\,|n\rangle\langle n|$$

E.g. Single-mode bosonic field in a coherent state $|\alpha\rangle$ or a uniform superposition of energy eigenstates $|\psi_L\rangle \propto \sum_{n=0}^{L} |n\rangle$.

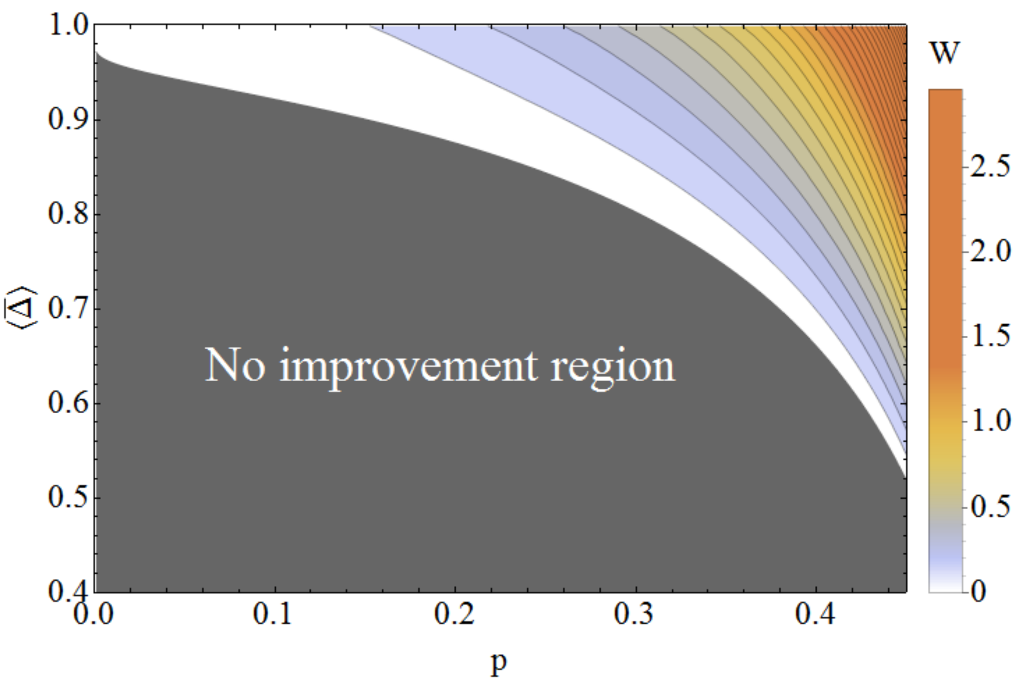


Extraction of work from quantum coherence

In the limit of a classical (unbounded) reference (properly defined size $N \to \infty$) all work can be extracted from coherence: $W(\rho_S) \to \Delta F(\rho_S)$

Using a bounded reference the amount of work that can be extracted is strictly smaller than the free energy difference: $W(\rho_S) < \Delta F(\rho_S)$

Explicit protocols for bounded references:



$\langle\Delta\rangle$ - quality of the reference

$\langle\Delta\rangle = 1 \Leftrightarrow$ unbounded coherence

$\langle\Delta\rangle = 0 \Leftrightarrow$ no coherence

p – thermal occupation of excited state

$p = 0 \Leftrightarrow T = 0$ $\qquad$ $p = \frac{1}{2} \Leftrightarrow T = \infty$

The extraction of work from quantum coherence
K. Korzekwa, M. Lostaglio, J. Oppenheim, D. Jennings
New J. Phys. **18**, 023045 (2016)

Conclusions

- The First Law of Thermodynamics imposes symmetry constraints that affect the thermodynamic processing of quantum coherence. As a result coherences do not transform individually, but in *chunks*, called **modes**.

- Each mode is subject to individual constraints under thermal transformations. The known α-free energy relations correspond to constraints only on the zero mode of a state and we have found lower and upper bounds for all other modes.

- Thermodynamic transformations within each mode display irreversibility.

- Work extraction from coherence: limitations arise in the quantum regime, one needs a coherence resource to act as a reference.

Thank you!