Quantum coherence, time-translation symmetry and thermodynamics

Kamil Korzekwa

Controlled Quantum Dynamics Centre Doctoral Training, Imperial College, London, UK

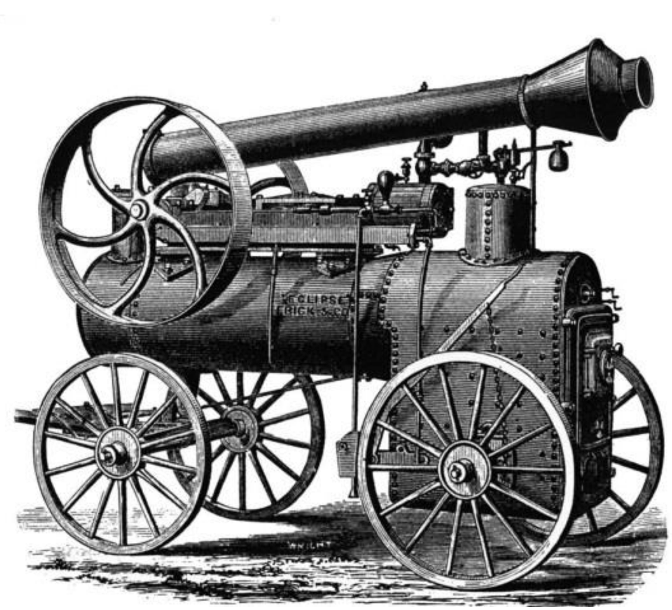
Team

Terry Rudolph

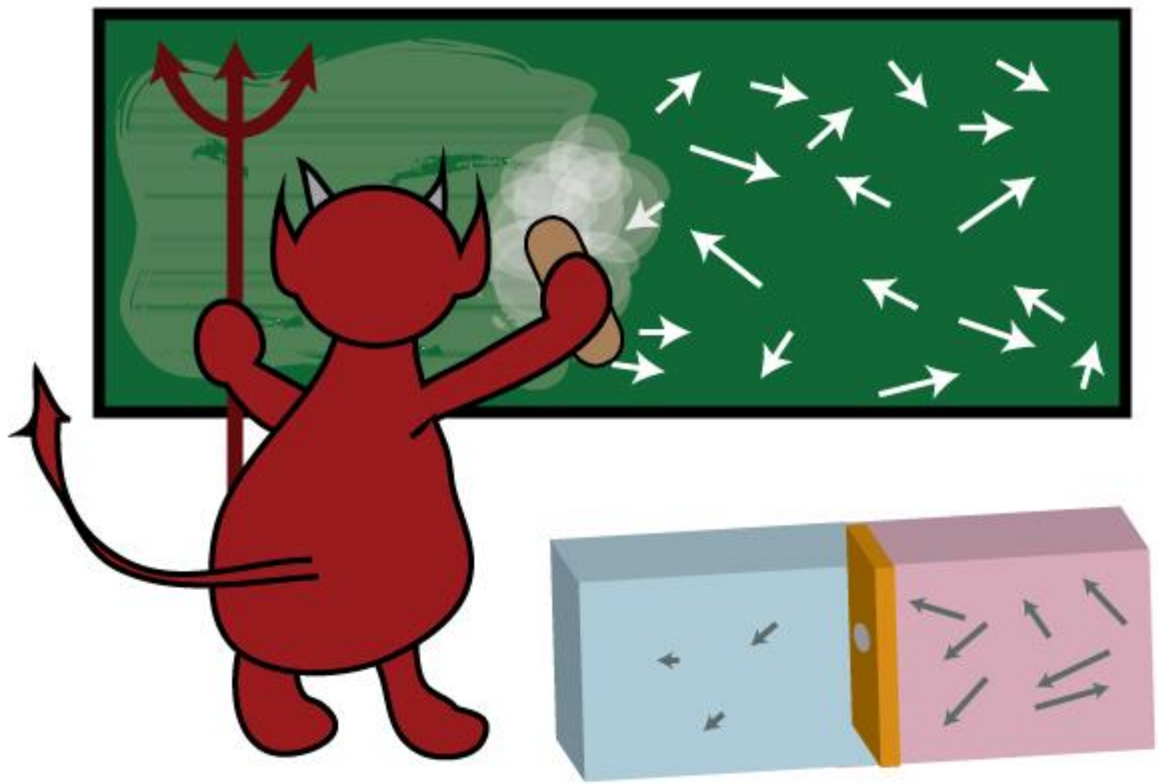
Matteo Lostaglio

David Jennings

Contents



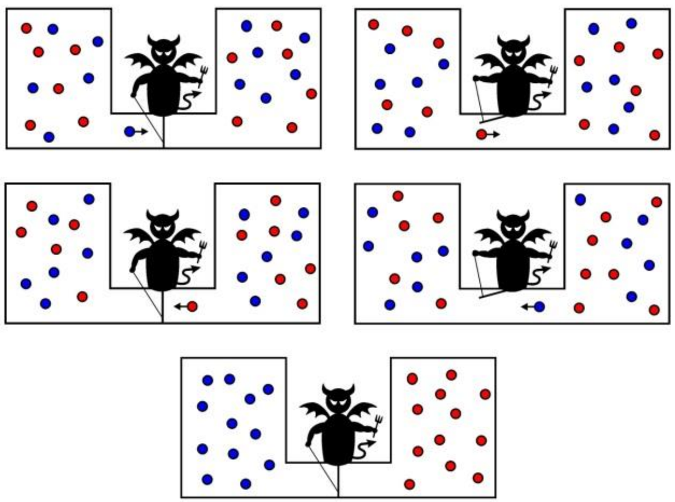
1. Motivation



*Picture from Lidia del Rio PhD Thesis

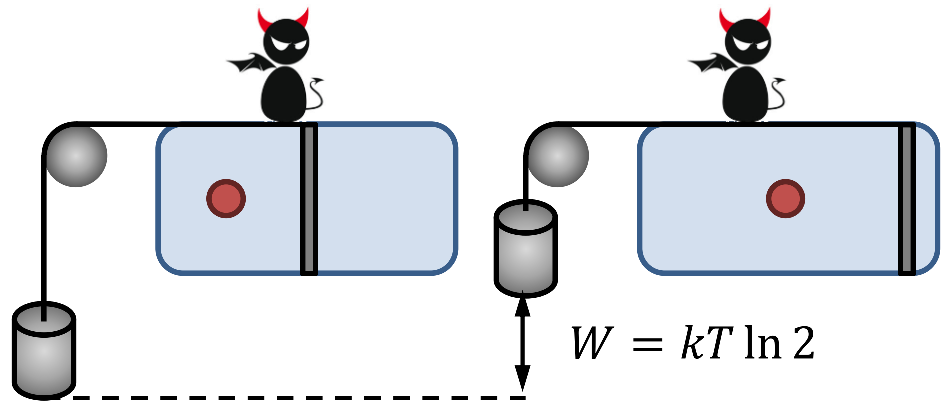
Thermodynamics and information theory

1874 - Maxwell's demon



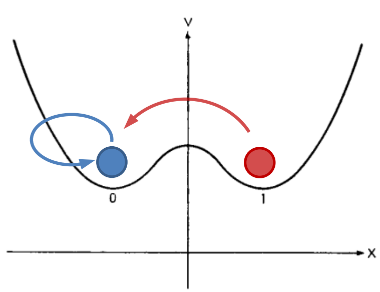
Nature Physics **6**, 988–992 (2010)

1929 - Szilard engine



Proceedings of the National Academy of Sciences **111**, 13786 (2014)

1961 – Landauer erasure



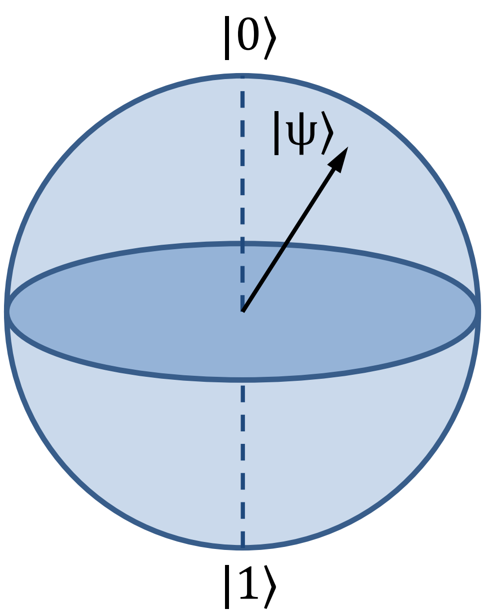
Nature **483**, 187–189 (2012)

Thermodynamics and information theory

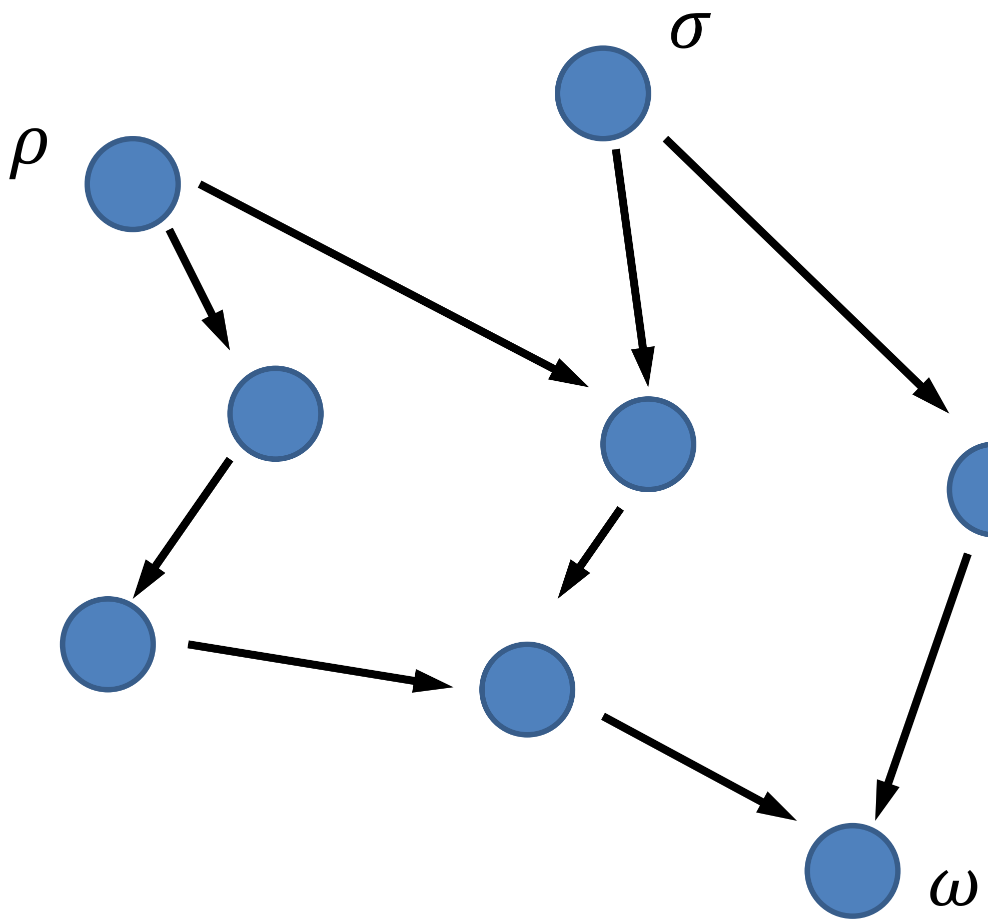
- Irreversible information processing requires heat dissipation and *vice versa* heat can be seen as a loss of information.
- Correspondence between thermodynamic and information entropy.
- One bit of information has an *energetic value* of $\mathrm{kT} \ln 2$.

Central questions:

What about quantum information and qubits?
Do entanglement and coherence play any special role?

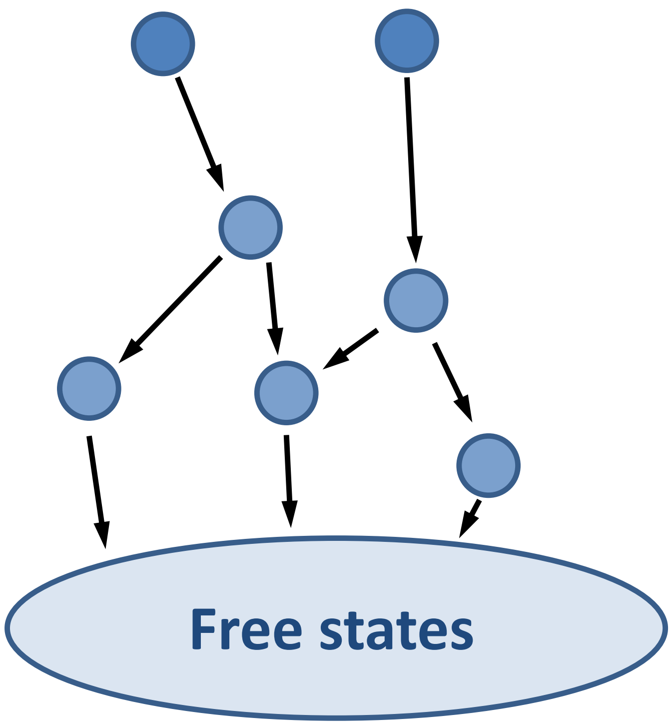


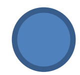
2. Thermodynamics as a resource theory



Ordering of states

1. Define the set of **free quantum operations.**
2. Define the set of **free quantum states.**
3. These imply **resource measures.**





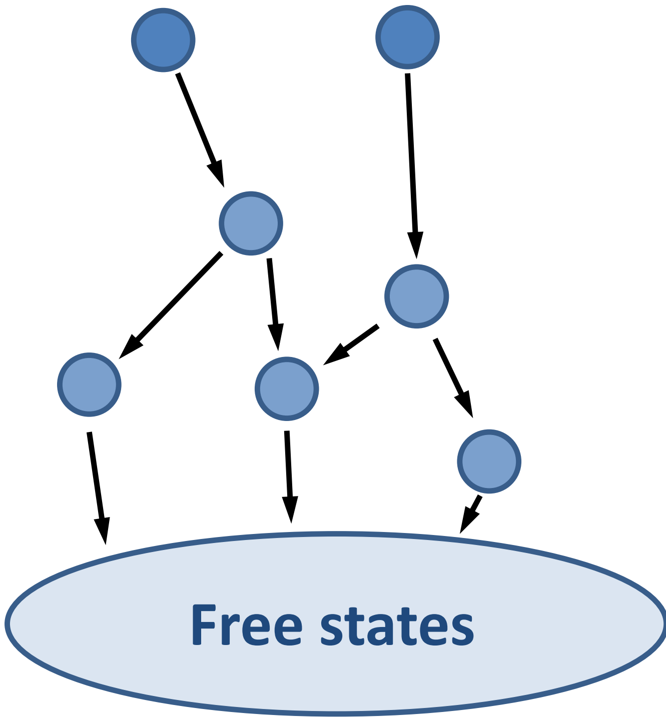
Quantum states

Allowed transformations

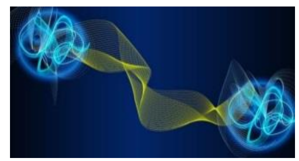
Define measure M:

If $\rho \to \sigma$
Then $M(\rho) \geq M(\sigma)$

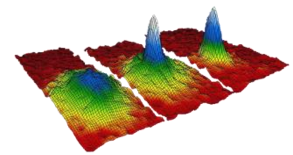
General framework



Recently developed resource theories



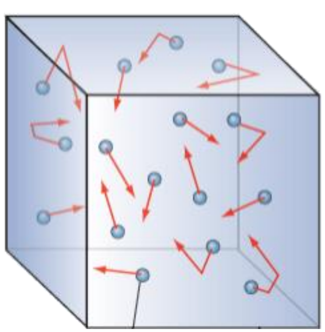
Entanglement



Coherence

Asymmetry

Entanglement	**Coherence**	**Asymmetry**
Non-local	**Coherent**	**Asymmetric**
↓	↓	↓
Separable	**Incoherent**	**Symmetric**



Thermodynamics?

Out of equilibrium ⟶ thermal equilibrium?

Thermodynamics ultimately concerns the accessibility/inaccessibility of one physical state from another.

The mathematical foundations of thermodynamics, R. Giles (1964)

Resource theory of thermodynamics

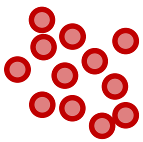
Given

 System in a state ρ_S described by Hamiltonian H_S

We can choose

“Encoding” 2nd Law

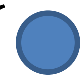
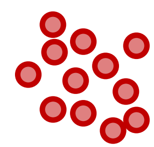
$$\gamma_E = \frac{e^{-\beta H_E}}{Tr(e^{-\beta H_E})}$$

 Environment described by an arbitrary Hamiltonian H_E prepared in a thermal state γ_E

And couple it with the system through an energy-preserving unitary U:

$$[U, H_S + H_E] = 0$$

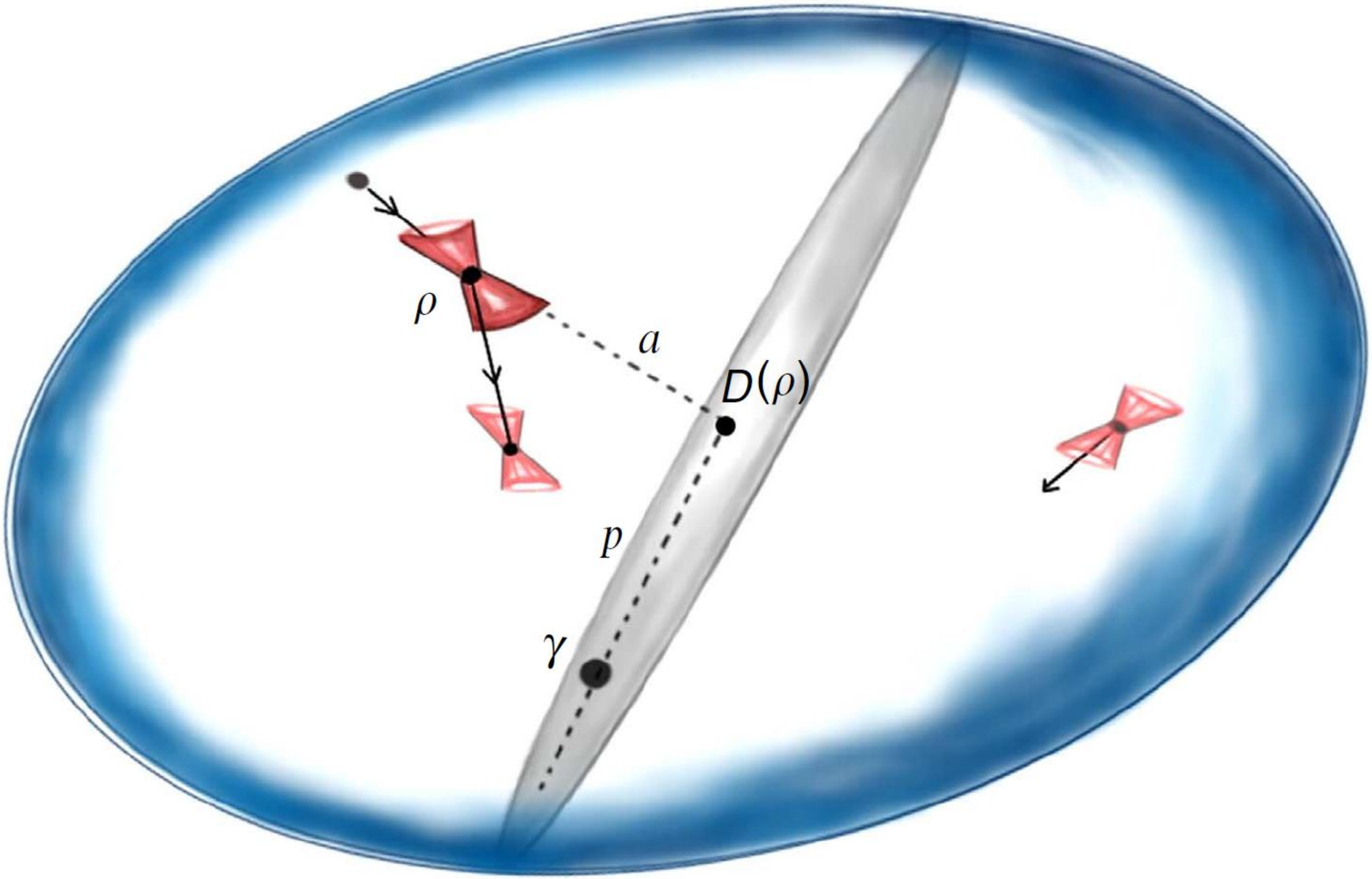
“Encoding” 1st Law

$Tr_E\Big(U($  $\otimes$  $)U^{\dagger}\Big) =$ 

Formal definition of allowed operations (known as **thermal operations**):

$$\mathcal{E}_T(\rho_S) = Tr_E\big(U(\rho_S \otimes \gamma_E)U^{\dagger}\big)$$

3. Thermodynamic constraints on coherent transformations



Simple yet powerful observation

To find the ordering of states is to answer the interconversion problem:

$$\begin{pmatrix} p_0 & c_{01} & c_{02} & \cdots & c_{0n} \\ c_{10} & p_1 & c_{12} & \cdots & c_{1n} \\ c_{20} & c_{21} & p_2 & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n0} & c_{n1} & c_{n2} & \cdots & p_n \end{pmatrix} \xrightarrow{\mathcal{E}_T} \begin{pmatrix} q_0 & 0 & 0 & \cdots & 0 \\ 0 & q_1 & 0 & \cdots & 0 \\ 0 & 0 & q_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & q_n \end{pmatrix}$$

Solved: Necessary and sufficient condition for $\rho_S \to \sigma_S$: $\forall_\alpha : F_\alpha(\rho_S) \geq F_\alpha(\sigma_S)$

The second laws of quantum thermodynamics
F. Brandão*, et. al*, PNAS **112**, 3275 (2015)

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Observation: thermal operations are a subset of **time-translation symmetric** operations:

$$\mathcal{E}_T\left(e^{-iH_St}\rho_S e^{iH_St}\right) = e^{-iH_St}\mathcal{E}_T(\rho_S)e^{iH_St}$$

As a result all measures of coherence must be non-increasing. E.g.,

$S_\alpha(\rho||D(\rho))$ Information theoretic "distance" between a state and its dephased version

Dephasing operation $D(\rho) = \sum_n \langle n|\rho|n\rangle|n\rangle\langle n|$

Description of quantum coherence in thermodynamic processes requires constraints beyond free energy,
M. Lostaglio, D. Jennings, T. Rudolph,
Nat. Commun. **6**, 6383 (2015)

Mode structure of time-translation symmetric maps

System described by nondegenerate Hamiltonian:

$$H_S = \sum_n \hbar\omega_n \,|n\rangle\langle n| \qquad\qquad \rho_S(0) = \sum_{n,m} \rho_{nm}\,|n\rangle\langle m|$$

Modes of coherence ${\rho_S}^{(\omega)}$:

$${\rho_S}^{(\omega)} := \sum_{\substack{n,m \\ \omega_n - \omega_m = \omega}} \rho_{nm}|n\rangle\langle m|; \qquad \rho_S(0) = \sum_\omega {\rho_S}^{(\omega)} \qquad \rho_S(t) = \sum_\omega {\rho_S}^{(\omega)} e^{-i\hbar\omega t}$$

Each mode transforms independently and its *intensity* cannot increase:

Given: $\sigma_S = \mathcal{E}_T(\rho_S)$ We have:

$${\sigma_S}^{(\omega)} = \mathcal{E}_T\left({\rho_S}^{(\omega)}\right)$$

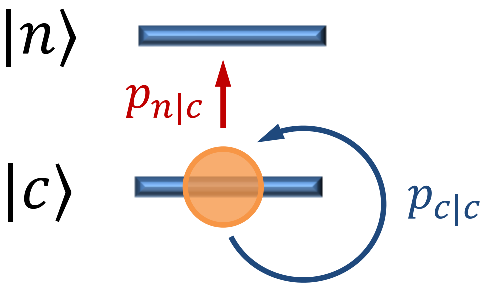
$$\left\|{\sigma_S}^{(\omega)}\right\| \le \left\|{\rho_S}^{(\omega)}\right\|$$

Modes of asymmetry…
I. Marvian, R. Spekkens,
Phys. Rev. A 90, 062110 (2014)

Mode structure of time-translation symmetric maps

Action of $\mathcal{E}_T$ on mode zero (occupation of energy eigenstates) described by stochastic matrix Λ_T (energy transfers independent of coherence):

$$[\Lambda_T]_{nc} = p_{n|c} := \langle n|\mathcal{E}_T(|c\rangle\langle c|)|n\rangle$$



What are the allowed transformations of non-zero (coherence) modes?

1. We need to enforce CPTP condition.

2. We note that Gibbs state as a fixed point of all thermal maps:
$\mathcal{E}_T(\gamma_S) = \gamma_S$

"Encoding" 2nd Law

Otherwise one could build a *perpetuum mobile*

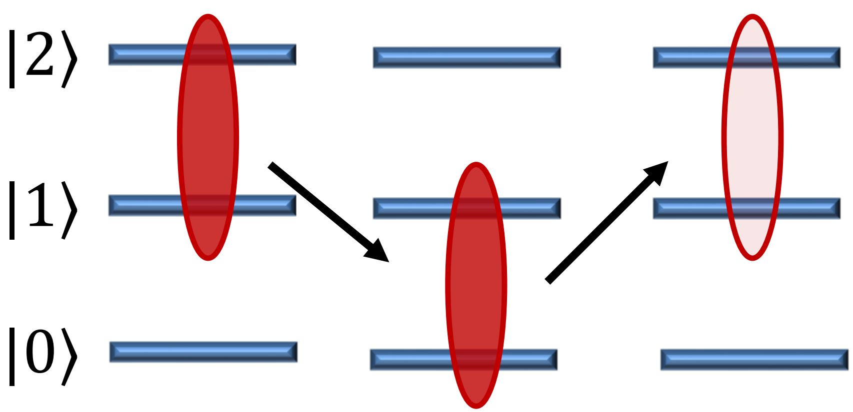
Bounds on coherence transformations

From the CPTP condition we get:
(Inequality can be saturated by a time-translation symmetric map.)

$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m}} |\rho_{cd}| \sqrt{p_{n|c} p_{m|d}}$$

Gibbs preserving property leads to irreversibility of coherence transfer:

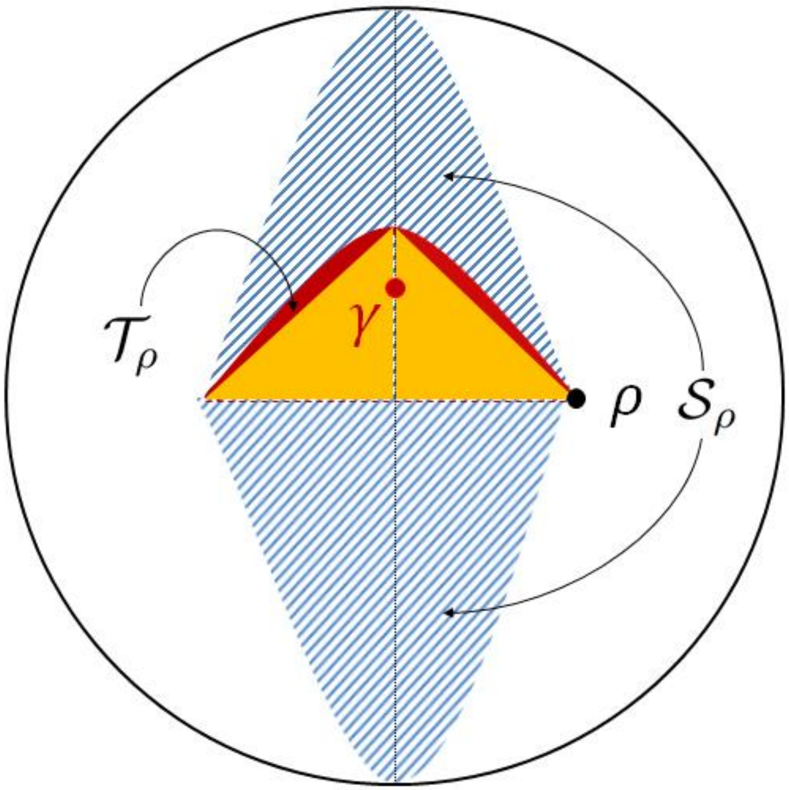
$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c > \omega_n}} |\rho_{cd}| \;+\; \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c \leq \omega_n}} |\rho_{cd}| \, e^{-\beta\hbar(\omega_n - \omega_c)}$$



Quantum coherence, time-translation symmetry and thermodynamics
M. Lostaglio, K. Korzekwa, D. Jennings, T. Rudolph, Phys. Rev. X **5**, 021001 (2015)

Elementary scenario: thermal ordering of qubit states

$$H_S = |1\rangle\langle 1|$$



Coherence is **actively** contributing to enlarge the set of thermodynamically accessible states.

Work is **not** the universal resource of thermodynamics.

Coherence contribution to free energy is **locked** - no trivial extension to quantum Szilard engine.

$\mathcal{T}_\rho$: Set of states accessible from ρ via thermal operations (orange region if coherence is passive)

$\mathcal{S}_\rho$: Set of states accessible from ρ via thermal operations and the access to infinite amount of work

The extraction of work from quantum coherence
K. Korzekwa, M. Lostaglio, J. Oppenheim, D. Jennings
New J. Phys. **18**, 023045 (2016)

Conclusions

- The First Law of Thermodynamics imposes symmetry constraints that affect the thermodynamic processing of quantum coherence. As a result coherences do not transform individually, but in *chunks*, called **modes**.

- Each mode is subject to individual constraints under thermal transformations. The known α-free energy relations correspond to constraints only on the zero mode of a state and we have found lower and upper bounds for all other modes.

- Thermodynamic transformations within each mode display irreversibility.

- Work extraction from coherence: limitations arise in the quantum regime, one needs a coherence resource to act as a reference.

Phys. Rev. X 5, 021001 (2015)   New J. Phys. 18, 023045 (2016)

Thank you!