On-demand quantum state transfer in spin chains with limited control and high resilience against imperfections

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1. Motivation:

- To elaborate a transfer protocol independent of inhomogeneity and satisfying the requirement of simplicity
- To achieve quantum transfer on demand

2. System:

- Chain of N linearly arranged spins with the nearest neighbour XY coupling
- The terminal spins are controlled by external magnetic fields, while the fields on all the nodes (including the terminal ones) are random, reflecting the effects of an inhomogeneous environment
- The model is restricted to at most one spin-flip excitation in the chain

3. Model:

- Hamiltonian:

$$H = \sum_{l=1}^{N-1} J_l \left(|l\rangle\langle l+1| + \text{h.c.} \right) + \sum_{l=1}^{N} B_l |l\rangle\langle l|$$

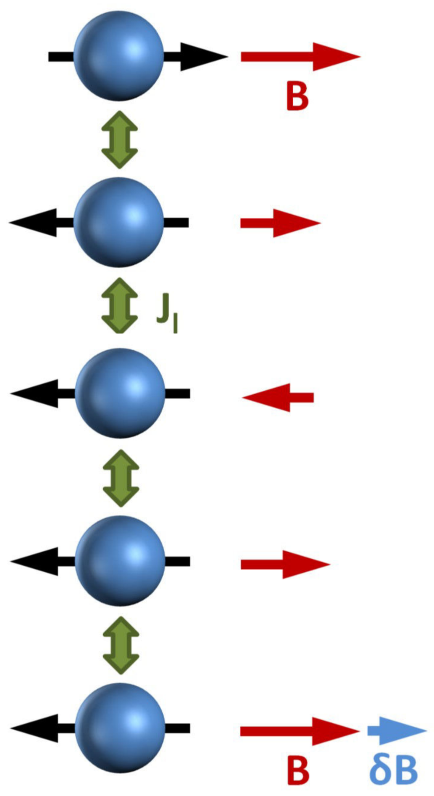
- The basis (number of excitations is conserved):

$|1\rangle = |100\ldots000\rangle\ ;\ \ \ldots\ ;\ |N\rangle = |000\ldots001\rangle$

- Parameters given by Gaussian distribution:
  - Coupling strengths: $J_l \sim \mathcal{N}(J, \sigma_J^2)$
  - Magnetic field: $B_1 \sim \mathcal{N}(B, \sigma_B^2)$

$$B_N \sim \mathcal{N}(B + \delta B, \sigma_B^2)$$
$$B_l \sim \mathcal{N}(0, \sigma_B^2),\ l = \{2 \ldots N-1\}$$

- In the simulations we set $B/J = 5$

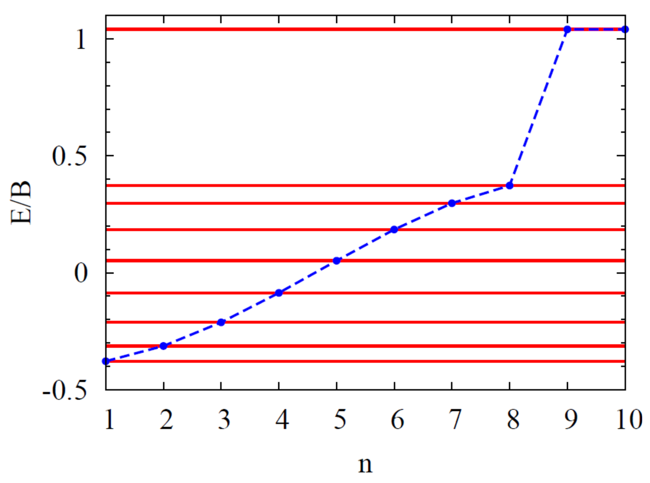


4. Method

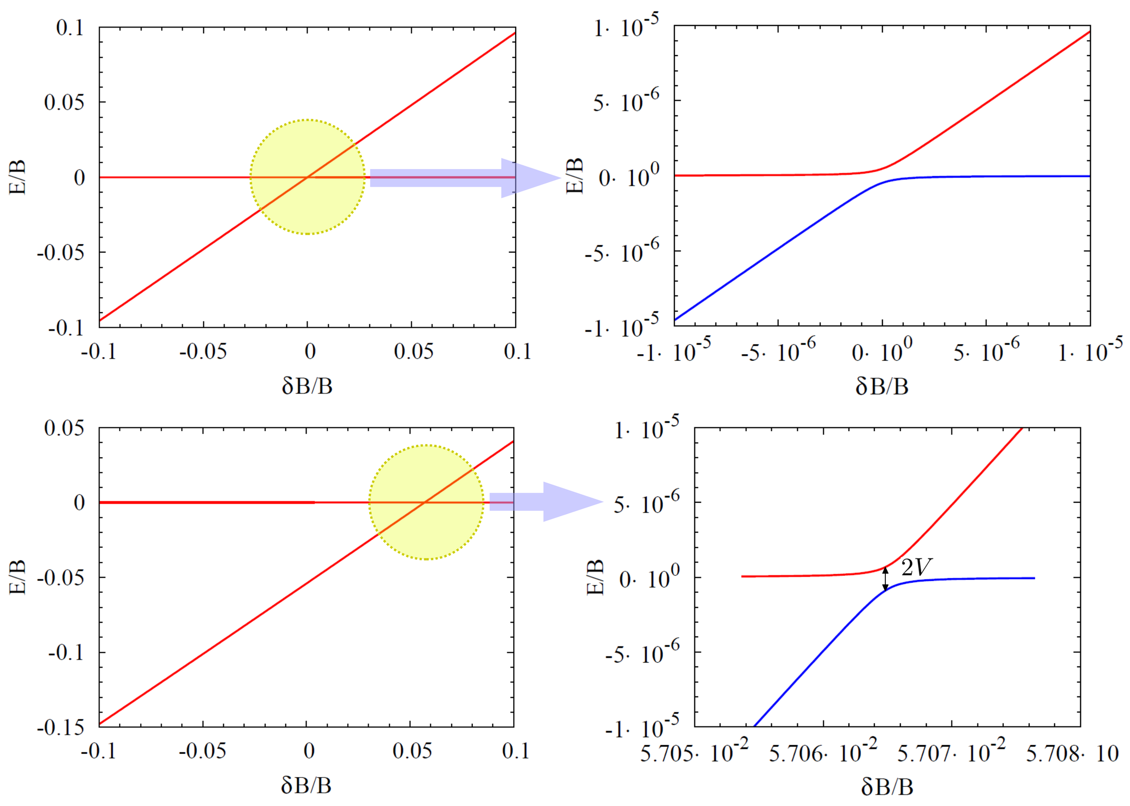
- Numerical diagonalization of the Hamiltonian: finding eigenvalues and eigenstates
- Decomposition of the initial state into eigenstates of the Hamiltonian in order to evolve them and obtain the final state (for constant or no external magnetic field)
- Numerical solution of the Schrödinger differential equation using Runge-Kutta method (for a time dependent external magnetic field)

5. Spectral properties

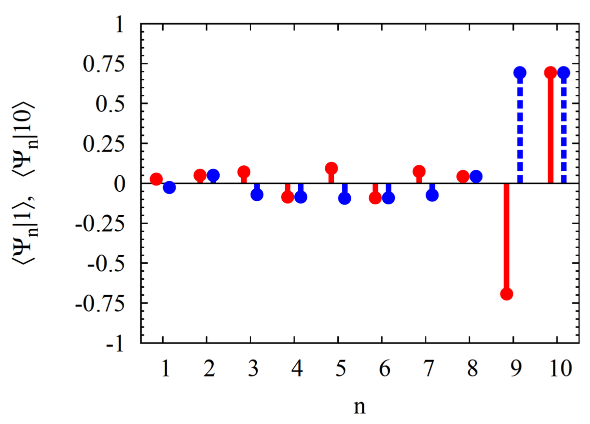
- Energetic separation of the eigenstates $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ due to external magnetic field applied at the terminal sites of the chain
- Inhomogeneity affects mainly the energy levels of the states delocalized in the central part of the chain. It shifts the resonance between $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$, but only weakly changes its width.
- Dependence of these energy levels on the difference between external magnetic field at the initial and final nodes, δB, is shown on the graphs (top: homogenous chain of length 10, bottom: inhomogeneous chain of length 10):



Energy levels of homogeneous spin chain of length 10

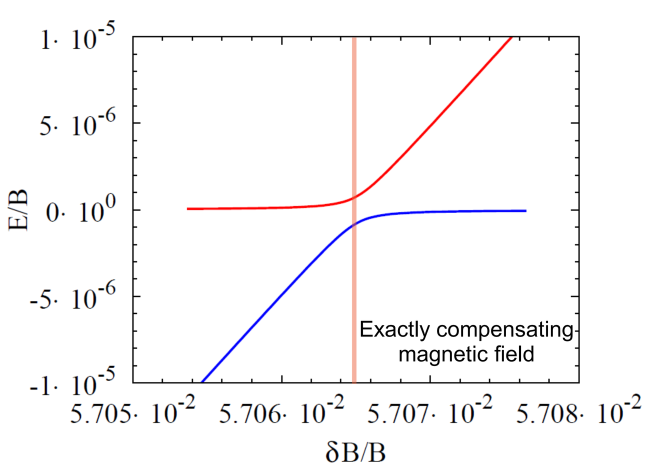


- At resonance, states with the inverted spin at the initial (red) and final (blue) nodes are mainly composed of $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$:



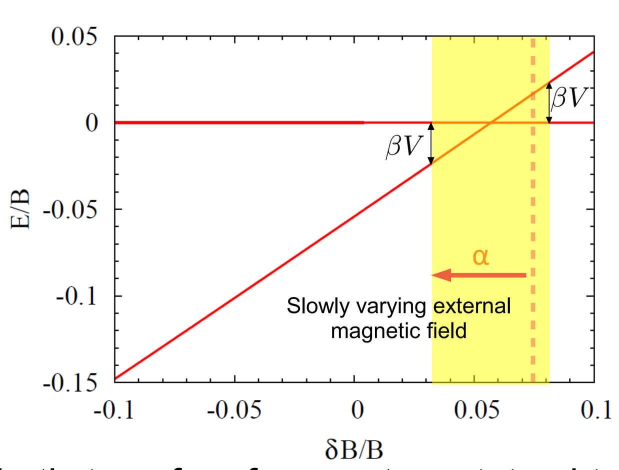
- Due to the energy separation of states $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ the dynamics can be approximately restricted to the two-dimensional subspace of terminal states coupled indirectly via the central part of the chain. This allows for two protocols of quantum state transfer:

Free transfer



Compensation of inhomogeneity by bringing $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ to exact resonance

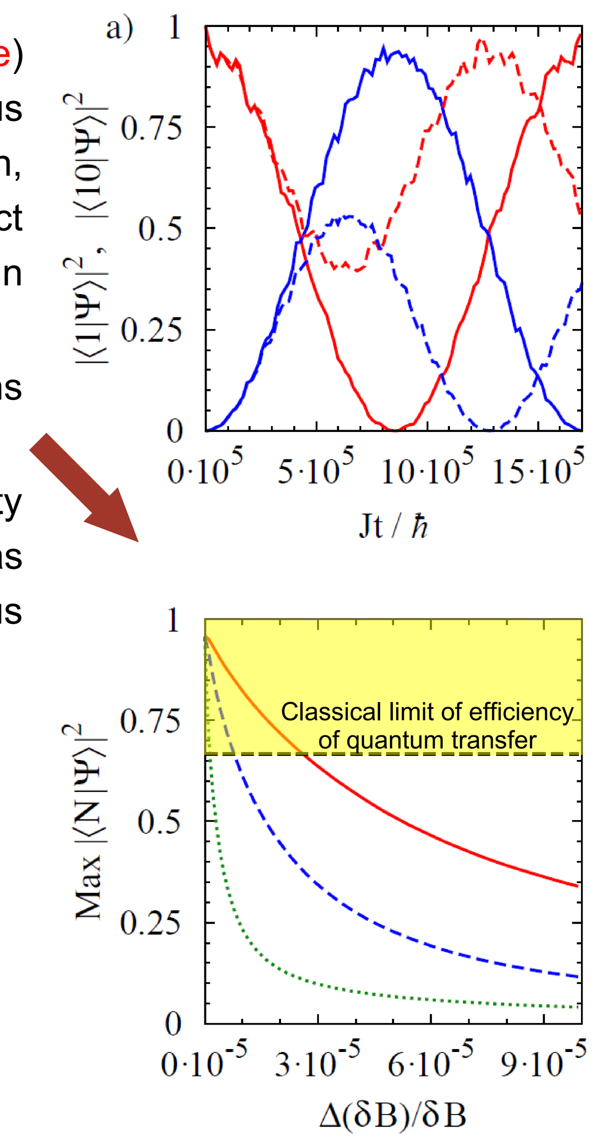
Adiabatic transfer

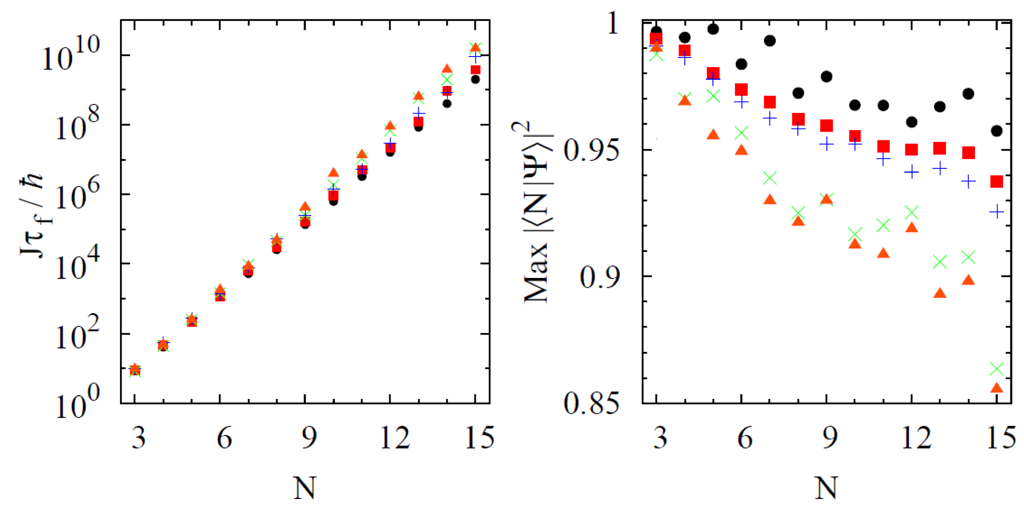


Adiabatic transfer of a quantum state obtained by slowly changing external magnetic field at one end of the chain, which sweeps energy levels of $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ through the resonance

6. Free transfer:

- Evolution of the occupation probabilities for the initial (red line) and final (blue line) nodes for a compensated inhomogenous spin chain. Solid lines correspond to perfect compensation, whereas dashed lines to 0.002% deviation from from perfect compensation. Parameters used in the simulations: spin chain length $N = 10$, $\sigma_J^2/J^2 = 0.1$, $\sigma_B^2/J^2 = 0.5$.
- Transfer fidelity $F = |\langle\Psi|N\rangle|^2$ very sensitive to small deviations from exact resonance
- Transfer time $\tau_f = (\pi\hbar)/(2V)$ and maximum achieved fidelity for homogeneous and compensated inhomogeneous chains as a function of the chain length (the data for the inhomogeneous chains are averaged over 100 random realizations):





■ $\sigma_J^2/J^2 = 0.1,\ \sigma_B^2/J^2 = 0.5$ + $\sigma_J^2/J^2 = 0.2,\ \sigma_B^2/J^2 = 0.5$ ● $\sigma_J^2/J^2 = 0,\ \sigma_B^2/J^2 = 0$
× $\sigma_J^2/J^2 = 0.1,\ \sigma_B^2/J^2 = 1$ ▲ $\sigma_J^2/J^2 = 0.2,\ \sigma_B^2/J^2 = 1$

7. Adiabatic transfer:

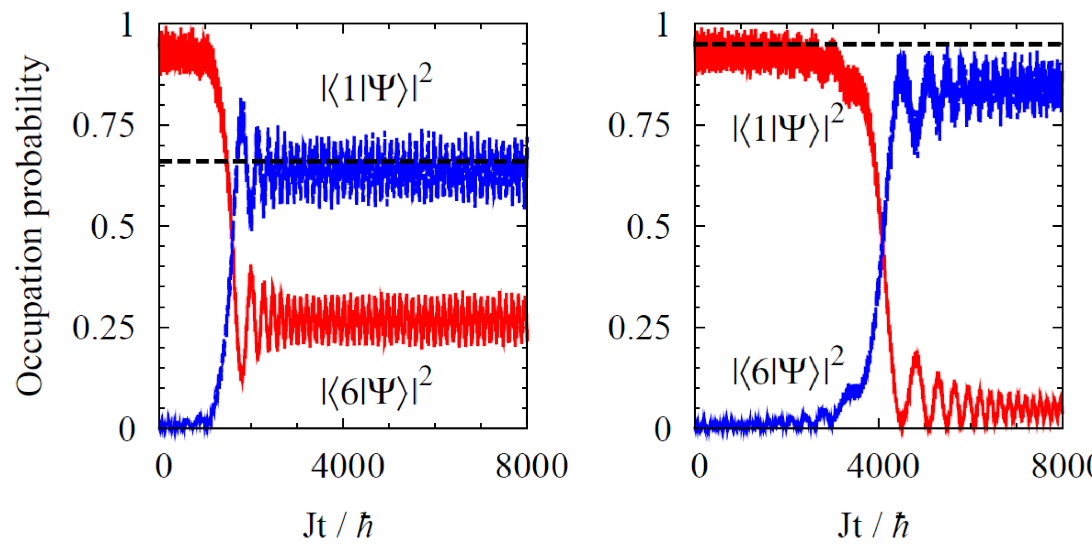
- Effective 2-level model: $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ with the coupling given by half of the energy splitting at resonance: $H_{12} = V$
- Finite speed of the magnetic field sweep α obtained from the Landau-Zener formula for nonadiabatic transition probabilities P_{na}:

$$P_{na} = \exp\left(-\frac{2\pi}{\hbar}\cdot\frac{|H_{12}|^2}{\alpha}\right)\quad ;\quad F = 1 - P_{na}$$

- Finite transfer time obtained by narrowing the limits of the magnetic field sweep to the area where energy separation of the states is smaller than βV (assumption: for $\Delta E > \beta V$ interaction is negligible):

$$\tau_a = \frac{\hbar}{\pi}\cdot\frac{\beta}{V}\ln\left(\frac{1}{1-F}\right)$$

- Landau-Zener result for the effective two-level model shows good agreement with the simulation of the evolution of the full system for β large enough (here $\beta = 20$). The graphs show the simulated evolution of the occupation probabilities for the initial (red line) and final (blue line) nodes of the spin chain of length 6 and the calculated fidelity F from the L-Z formula (black dashed line):



- Adiabatic transfer time to free transfer time ratio for given β is only a function of fidelity we want to obtain adiabatically:

$$\frac{\tau_a}{\tau_f} = \frac{2\beta}{\pi^2}\ln\left(\frac{1}{1-F}\right)$$

- For large F the actual fidelity is lower than expected from L-Z formula (further increase of β or decrease of the speed of magnetic field α does not bring a considerable improvement). The origin of the problem with achieving very high fidelity is leakage to the nodes inside the chain.

8. Leaking information:

- The measure of the information leaking to the nodes inside the chain can be defined as the trace distance between the reduced density matrices of the central part of the chain in the case of one and zero spin-up states in the whole system:

$$I = 1 - |\langle 1|\Psi\rangle|^2 - |\langle N|\Psi\rangle|^2$$

- For both protocols I remains very small (around 10%) and is mainly determined by the magnitude of the external magnetic field responsible for isolating terminal states. Information in the central part of the chain averaged over the free transfer time and 100 random realizations of a chain of length 10 is shown in the table.

σ_J^2/J^2 \ σ_B^2/J^2	0	0.5	1
0	7.65%	8.26%	10.65%
0.1	7.80%	7.89%	11.12%
0.2	8.30%	8.23%	9.80%

9. Conclusions:

- The negative impact of the inhomogeneity caused both by the local magnetic fields and disorder in exchange couplings can be overcome by separating terminal states with external magnetic field that brings them to resonance
- Adiabatic transfer in a spin chain can be well described within an approximate model of a two-level system and the transfer time can be obtained by using Landau-Zener formula for a given desired fidelity
- The adiabatic transfer protocol makes it possible to achieve quantum transfer on demand