eQus

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

THE UNIVERSITY OF SYDNEY

Coherifying quantum states and channels

Kamil Korzekwa

Centre for Engineered Quantum Systems, School of Physics,
University of Sydney, Sydney NSW 2006, Australia

Team

Karol Życzkowski

Jagiellonian University, Kraków

Polish Academy of Sciences, Warsaw

Zbigniew Puchała

Jagiellonian University, Kraków

Polish Academy of Sciences, Gliwice

Stanisław Czachórski

Jagiellonian University, Kraków

Outline

1. Motivating examples

2. What does it mean to coherify a state or a channel?

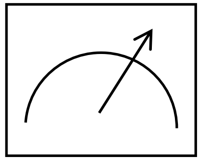
3. How coherent can a quantum channel be?

4. How well can we distinguish classically indistinguishable states and channels?

5. Outlook

Motivating examples

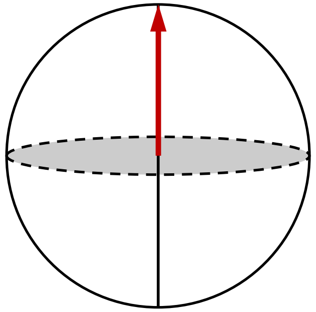
Quantum state ρ

$\rho \longrightarrow$  $\longrightarrow p_j = \langle j|\rho|j\rangle$

What $\boldsymbol{p}$ tells us about ρ?

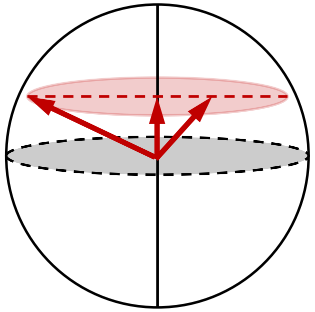
$\boldsymbol{p} = [1, 0]$ $\quad\quad$ $\boldsymbol{p} = [3/4, 1/4]$

$|0\rangle\langle 0|$



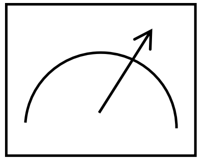
$|1\rangle\langle 1|$

$|0\rangle\langle 0|$



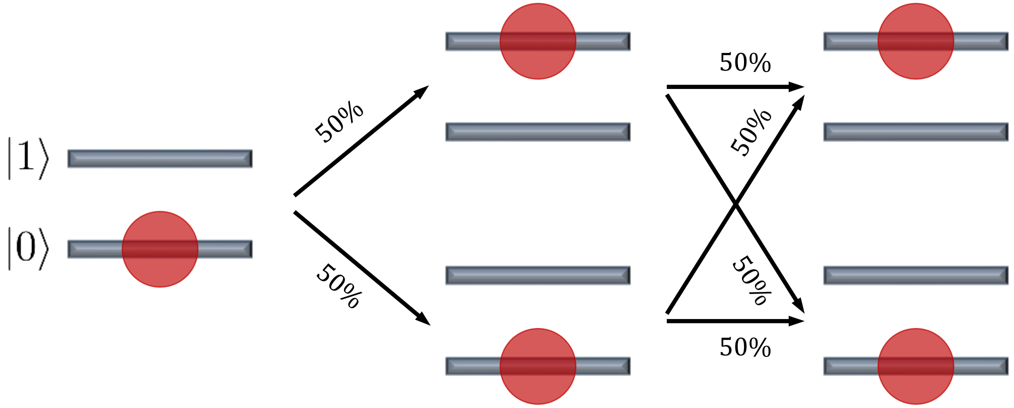
$|1\rangle\langle 1|$

Quantum channel Φ

$|k\rangle\langle k| \longrightarrow \boxed{\Phi} \longrightarrow$  $\longrightarrow T_{jk} = \langle j|\Phi(|k\rangle\langle k|)|j\rangle$

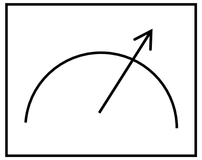
What T tells us about Φ?

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad \Phi(\cdot) = \frac{\mathbb{1}}{2}$$



Motivating examples

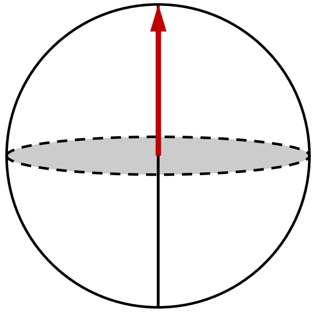
Quantum state ρ

$\rho \longrightarrow$  $\longrightarrow p_j = \langle j|\rho|j\rangle$

What $\boldsymbol{p}$ tells us about ρ?

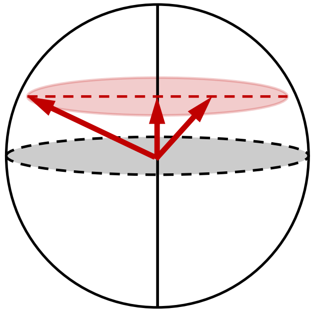
$\boldsymbol{p} = [1, 0]$ $\qquad$ $\boldsymbol{p} = [3/4, 1/4]$

$|0\rangle\langle 0|$



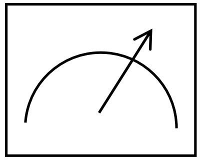
$|1\rangle\langle 1|$

$|0\rangle\langle 0|$



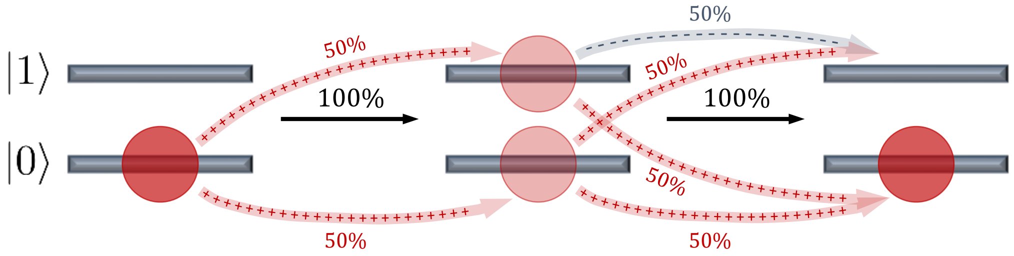
$|1\rangle\langle 1|$

Quantum channel Φ

$|k\rangle\langle k| \longrightarrow \boxed{\Phi} \longrightarrow$  $\longrightarrow T_{jk} = \langle j|\Phi(|k\rangle\langle k|)|j\rangle$

What T tells us about Φ?

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad \Phi(\cdot) = H(\cdot)H^\dagger \qquad H = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



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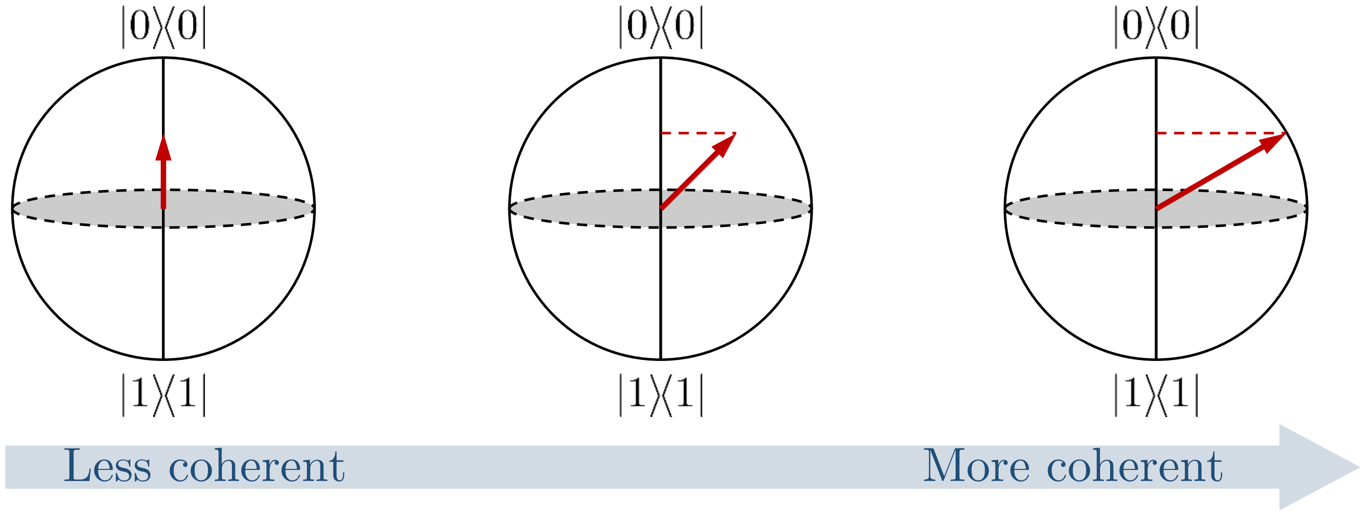
Coherence of quantum states

Given a fixed basis $\{|j\rangle\}$ with $j \in \{1, \ldots, d\}$:

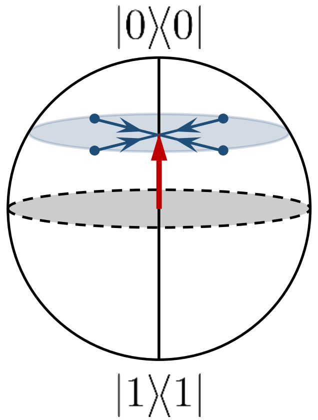
$\langle j|\rho|j\rangle$: occupations p_j $\quad$ $\langle j|\rho|k\rangle$: coherences c_{jk}



$$\rho_1 = \begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \qquad \rho_2 = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \qquad \rho_3 = \begin{bmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix}$$

Decohering channel $\mathcal{D}$:

$$\mathcal{D}(\rho) = \sum_j \langle j|\rho|j\rangle\, |j\rangle\langle j|$$



$$c_{jk} \to 0, \quad p_j \to p_j$$

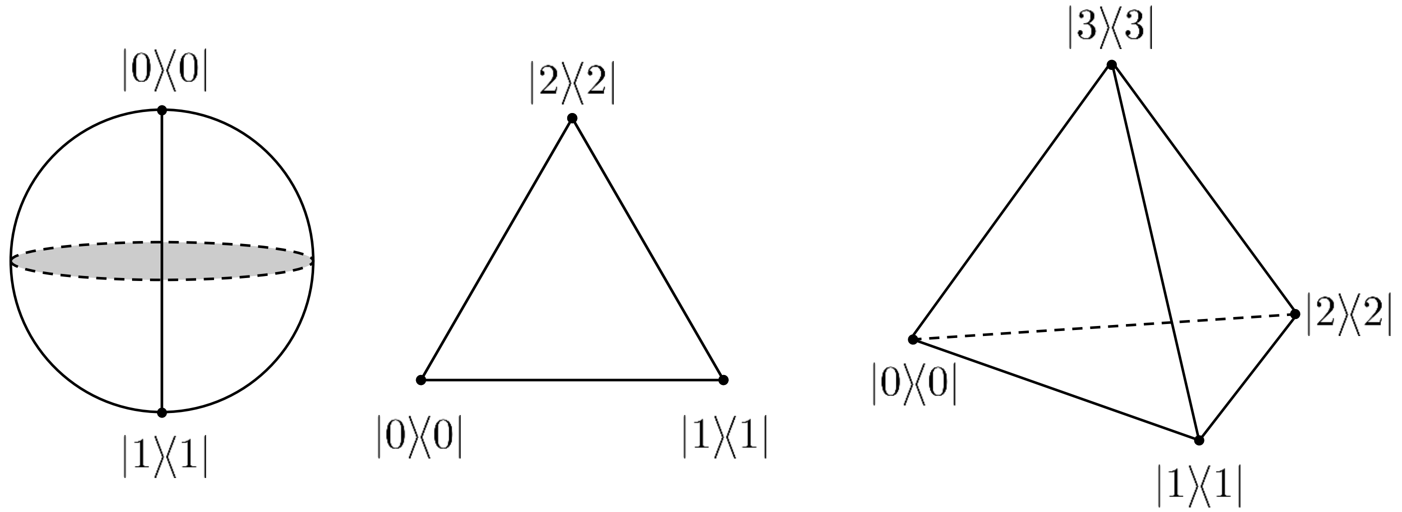
$$\mathcal{D}(\rho_3) = \mathcal{D}(\rho_2) = \mathcal{D}(\rho_1) = \rho_1$$

Coherence of quantum states

Incoherent (classical) state ρ identified with probability distribution $\boldsymbol{p}$:

$$\rho = \mathcal{D}(\rho) = \sum_j p_j \, |j\rangle\langle j|$$

Classical state space
=
probability simplex



Coherence measures (distance from incoherent states):

$$C_{\mathrm{e}}(\rho) := S(\rho||\mathcal{D}(\rho)) = S(\boldsymbol{p}) - S(\boldsymbol{\lambda}(\rho))$$

with $\boldsymbol{\lambda}(\rho)$ denoting the eigenvalues of ρ

$$C_2(\rho) := ||\rho - \mathcal{D}(\rho)||_2 = \boldsymbol{\lambda}(\rho) \cdot \boldsymbol{\lambda}(\rho) - \boldsymbol{p} \cdot \boldsymbol{p}$$

Coherifying quantum states

Decohering channel $\mathcal{D}$:

$$\rho \text{ with } \langle j|\rho|j\rangle = p_j \xrightarrow{\mathcal{D}} \rho^{\mathcal{D}} = \mathrm{diag}(\boldsymbol{p})$$

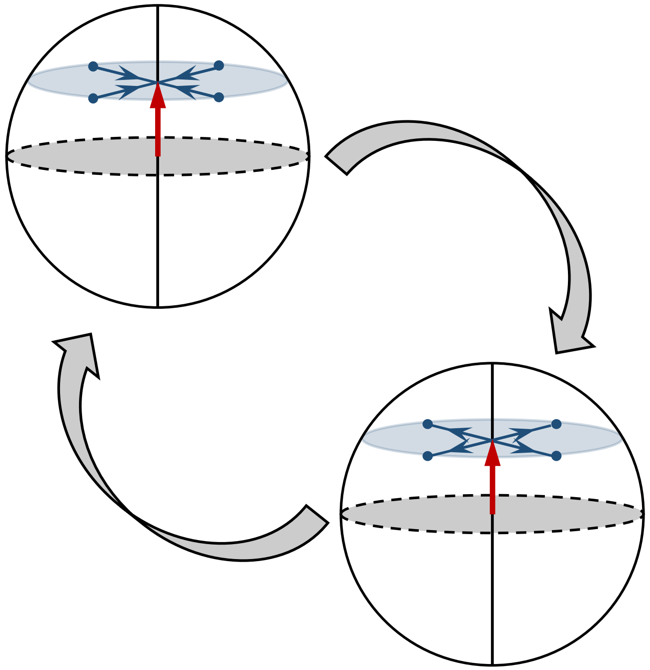
Coherification $\mathcal{C}$ is a formal (not unique!) inverse of $\mathcal{D}$:

$$\rho = \mathrm{diag}(\boldsymbol{p}) \xrightarrow{\mathcal{C}} \rho^{\mathcal{C}} \text{ with } \langle j|\rho|j\rangle = p_j$$

One can always optimally coherify a classical state $\boldsymbol{p}$:

$$\mathrm{diag}(\boldsymbol{p}) \xrightarrow{\mathcal{C}} |\psi\rangle\langle\psi| \text{ with } |\psi\rangle = \sum_j \sqrt{p_j} e^{i\phi_j} |j\rangle$$

$$C_{\mathrm{e}}(|\psi\rangle\langle\psi|) = S(\boldsymbol{p}) \qquad C_2(|\psi\rangle\langle\psi|) = 1 - \boldsymbol{p}\cdot\boldsymbol{p}$$



How many distinct ways to coherify?

Coherence of quantum channels

Given a fixed basis $\{|j\rangle\}$ with $j \in \{1, \ldots, d\}$:

$\langle j|\Phi(|k\rangle\langle k|)|j\rangle$: classical action T_{jk}

$\langle j|\Phi(|m\rangle\langle n|)|k\rangle$: action involving coherences

Choi-Jamiołkowski isomorphism (channel $\Phi \leftrightarrow$ bipartite state J_Φ):

$$J_\Phi = \tfrac{1}{d}(\Phi \otimes \mathcal{I})\,|\Omega\rangle\langle\Omega|, \quad |\Omega\rangle = \textstyle\sum_j |jj\rangle$$

CPTP conditions are translated into:

$$J_\Phi \geq 0, \qquad \mathrm{Tr}_1\left(J_\Phi\right) = \frac{\mathbb{1}}{d}$$

Relation between J_Φ and T:

$$\langle jk|J_\Phi|jk\rangle = \tfrac{1}{d}T_{jk}$$

Vectorising classical action:

$$\mathrm{diag}(J_\Phi) = \tfrac{1}{d}|T\rangle\rangle, \qquad |T\rangle\rangle = T \otimes \mathbb{1}|\Omega\rangle$$

E.g. $\left|\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right\rangle\!\!\Big\rangle = [1,2,3,4,5,6,7,8,9]^\top$

Coherence of quantum channels

Classical channels defined as channels with incoherent (classical) Jamiołkowski state.

Effect of a classical channel with classical action T:

$$\rho \implies \mathcal{D}(\rho) = \sum_j p_j \, |j\rangle\langle j| \implies \sigma = \sum_j q_j \, |j\rangle\langle j| \text{ with } \boldsymbol{q} = T\boldsymbol{p}$$

Define coherence measures of Φ through coherence measures of J_Φ:

$$C_{\mathrm{e}}(\Phi) = S(\tfrac{1}{d}|T\rangle\rangle) - S(\boldsymbol{\lambda}(J_\Phi)), \qquad C_2(\Phi) = \boldsymbol{\lambda}(J_\Phi) \cdot \boldsymbol{\lambda}(J_\Phi) - \tfrac{1}{d^2}\langle\langle T|T\rangle\rangle$$

Cf. $\quad C_{\mathrm{e}}(\rho) = S(\boldsymbol{p}) - S(\boldsymbol{\lambda}(\rho)), \qquad C_2(\rho) = \boldsymbol{\lambda}(\rho) \cdot \boldsymbol{\lambda}(\rho) - \boldsymbol{p} \cdot \boldsymbol{p}$

Optimising coherence of Φ with fixed $T \iff$ optimising $\boldsymbol{\lambda}(J_\Phi)$

Coherifying quantum channels

Decohering operation $\mathcal{D}$:

$$\Phi \text{ with } \mathrm{diag}(J_\Phi) = \tfrac{1}{d}|T\rangle\rangle \xrightarrow{\mathcal{D}} \Phi^{\mathcal{D}} \text{ with } J_{\Phi^{\mathcal{D}}} = \mathcal{D}(J_\Phi) = \tfrac{1}{d}\mathrm{diag}(|T\rangle\rangle)$$

Coherification $\mathcal{C}$ is a formal (not unique!) inverse of $\mathcal{D}$:

$$\Phi \text{ with } J_\Phi = \mathcal{D}(J_\Phi) = \tfrac{1}{d}\mathrm{diag}(|T\rangle\rangle) \xrightarrow{\mathcal{C}} \Phi^{\mathcal{C}} \text{ with } \mathrm{diag}(J_{\Phi^{\mathcal{C}}}) = \tfrac{1}{d}|T\rangle\rangle$$

Can one always optimally coherify a classical map T?

$$\tfrac{1}{d}|T\rangle\rangle \xrightarrow{\mathcal{C}} |\psi\rangle\langle\psi| \text{ with } |\psi\rangle = \tfrac{1}{\sqrt{d}}\textstyle\sum_{j,k}\sqrt{T_{jk}}e^{i\phi_{jk}}|jk\rangle$$

No! TP condition requires $\mathrm{Tr}_1\left(|\psi\rangle\langle\psi|\right) = \frac{\mathbb{1}}{d}$

E.g.

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

$$\mathrm{Tr}_1\left(|\psi\rangle\langle\psi|\right) = |+\rangle\langle+|$$

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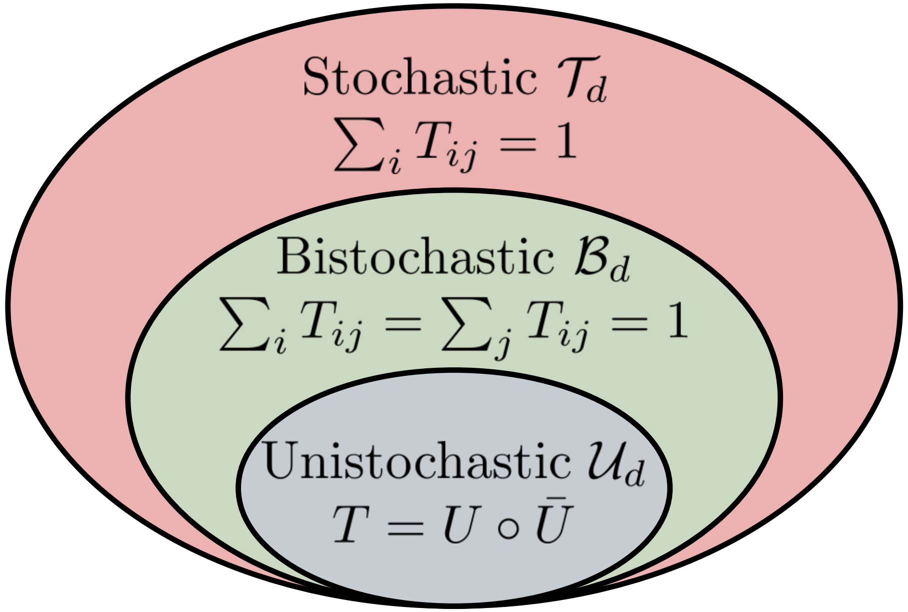
Categories of classical actions



Hadamard product:

$(A \circ B)_{jk} = A_{jk} B_{jk}$

Example of $T \in \mathcal{B}_d$ and $T \notin \mathcal{U}_d$:

$$T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}, \qquad U = \begin{bmatrix} 0 & \frac{e^{i\theta_{12}}}{\sqrt{2}} & \frac{e^{i\theta_{13}}}{\sqrt{2}} \\ \frac{e^{i\theta_{21}}}{\sqrt{2}} & 0 & \frac{e^{i\theta_{23}}}{\sqrt{2}} \\ \frac{e^{i\theta_{31}}}{\sqrt{2}} & \frac{e^{i\theta_{32}}}{\sqrt{2}} & 0 \end{bmatrix}$$

Not a unitary!

Kraus decomposition and classical action:

$$\Phi(\cdot) = \sum_j K_j (\cdot) K_j^\dagger, \qquad \sum_j K_j \circ \bar{K}_j = T$$

$$\Phi \text{ can be completely coherified} \iff T \text{ is unistochastic}$$

Coherification upper-bound

Majorisation partial order: $\boldsymbol{p} \succ \boldsymbol{q} \iff \forall k: \ \sum_{j=1}^{k} p_j^{\downarrow} \geq \sum_{j=1}^{k} q_j^{\downarrow}$

Important because: $\boldsymbol{p} \succ \boldsymbol{q} \implies S(\boldsymbol{p}) \leq S(\boldsymbol{q}) \quad \text{and} \quad \boldsymbol{p} \cdot \boldsymbol{p} \geq \boldsymbol{q} \cdot \boldsymbol{q}$

Look for $\boldsymbol{\mu}^{\succ}(T)$ such that: $\forall \Phi \text{ with } \mathrm{diag}(J_\Phi) = \frac{1}{d}|T\rangle\rangle : \ \boldsymbol{\mu}^{\succ}(T) \succ \boldsymbol{\lambda}(J_\Phi)$

Why? Cf. $C_{\mathrm{e}}(\Phi) = S(\frac{1}{d}|T\rangle\rangle) - S(\boldsymbol{\lambda}(J_\Phi)), \quad C_2(\Phi) = \boldsymbol{\lambda}(J_\Phi) \cdot \boldsymbol{\lambda}(J_\Phi) - \frac{1}{d^2}\langle\langle T|T\rangle\rangle$

Procedure to obtain upper-bounding $\boldsymbol{\mu}^{\succ}(T)$:

$$T = \begin{bmatrix} 0.7 & 0.2 & 0.6 \\ 0.1 & 0.6 & 0.4 \\ 0.2 & 0.2 & 0 \end{bmatrix} \xrightarrow[\text{columns}]{\text{Sum over}} \begin{bmatrix} 1.5 \\ 1.1 \\ 0.4 \end{bmatrix} \xrightarrow[\text{parts } \leq 1]{\text{Distribute into}} \begin{bmatrix} 1 & 0.5 & 0 \\ 1 & 0.1 & 0 \\ 0.4 & 0 & 0 \end{bmatrix} \xrightarrow[\text{normalise}]{\text{Sum over rows and}} \begin{bmatrix} 0.8 \\ 0.2 \\ 0 \end{bmatrix}^{\top} = \boldsymbol{\mu}^{\succ}(T)$$

Coherification lower-bound

Explicit construction of non-optimally coherified channel:

$$\Phi^{\mathcal{C}}(\cdot) = \sum_j K_j(\cdot)K_j^\dagger,$$

$$T = \begin{bmatrix} 0.7 & 0.2 & 0.6 \\ 0.1 & 0.6 & 0.4 \\ 0.2 & 0.2 & 0 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} \sqrt{0.7} & 0 & 0 \\ 0 & \sqrt{0.6} & 0 \\ 0 & \sqrt{0.2} & 0 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0 & 0 & \sqrt{0.6} \\ 0 & 0 & \sqrt{0.4} \\ \sqrt{0.2} & 0 & 0 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 0 & \sqrt{0.2} & 0 \\ \sqrt{0.1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Leave only the square root of the largest entry in each row

Leave only the square root of the 2nd largest entry in each row

Leave only the square root of the 3rd largest entry in each row

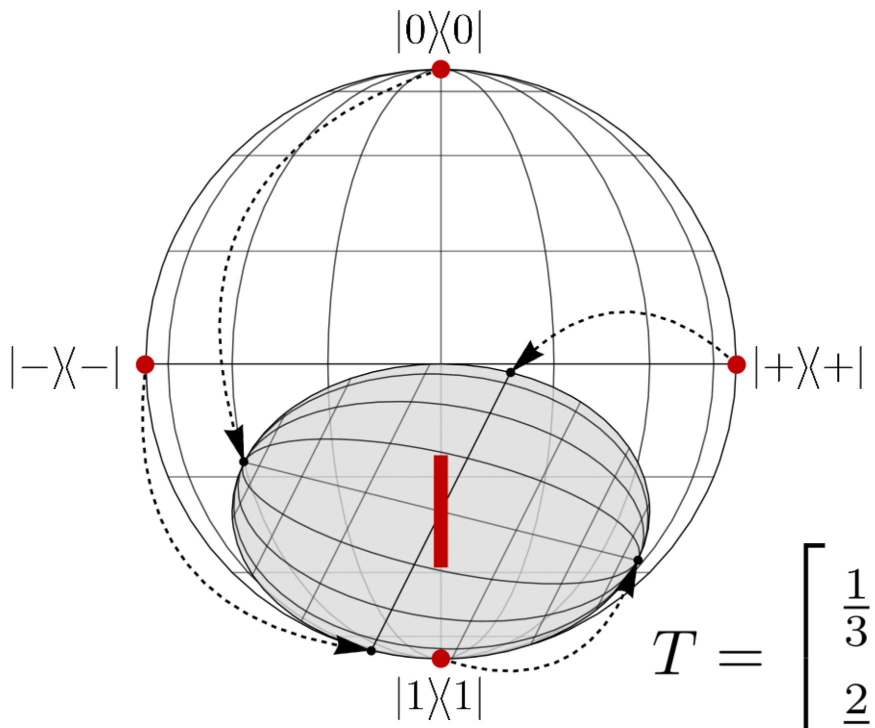
Procedure to obtain lower-bounding $\boldsymbol{\mu}^{\prec}(T)$:

$$T = \begin{bmatrix} 0.7 & 0.2 & 0.6 \\ 0.1 & 0.6 & 0.4 \\ 0.2 & 0.2 & 0 \end{bmatrix} \xrightarrow[\text{rows}]{\text{Order within}} \begin{bmatrix} 0.7 & 0.6 & 0.2 \\ 0.6 & 0.4 & 0.1 \\ 0.2 & 0.2 & 0 \end{bmatrix} \xrightarrow[\text{normalise}]{\text{Sum over rows and}} \begin{bmatrix} 0.5 \\ 0.4 \\ 0.1 \end{bmatrix}^\top = \boldsymbol{\mu}^{\prec}(T)$$

Optimal coherification of qubit channels

Classical action of a qubit channel:

$$T = \begin{bmatrix} a & 1-b \\ 1-a & b \end{bmatrix} =: \begin{bmatrix} a & \tilde{b} \\ \tilde{a} & b \end{bmatrix}$$



$$T = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{2}{3} & \frac{5}{6} \end{bmatrix}$$

Optimally coherified channel:

$$\Phi^{\mathcal{C}}(\cdot) = \Psi(U(\cdot)U^{\dagger})$$

- Extremal
- Min output entropy=0

with unitary:

$$U = \frac{1}{\sqrt{a+\tilde{b}}} \begin{bmatrix} \sqrt{a} & -\sqrt{\tilde{b}} \\ \sqrt{\tilde{b}} & \sqrt{a} \end{bmatrix},$$

and $\Psi(\cdot) = L_1(\cdot)L_1^{\dagger} + L_2(\cdot)L_2^{\dagger}$ with:

$$L_1 = \begin{bmatrix} \sqrt{a+\tilde{b}} & 0 \\ 0 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 & 0 \\ \sqrt{b-a} & 0 \end{bmatrix}.$$

Bistochastic classical action

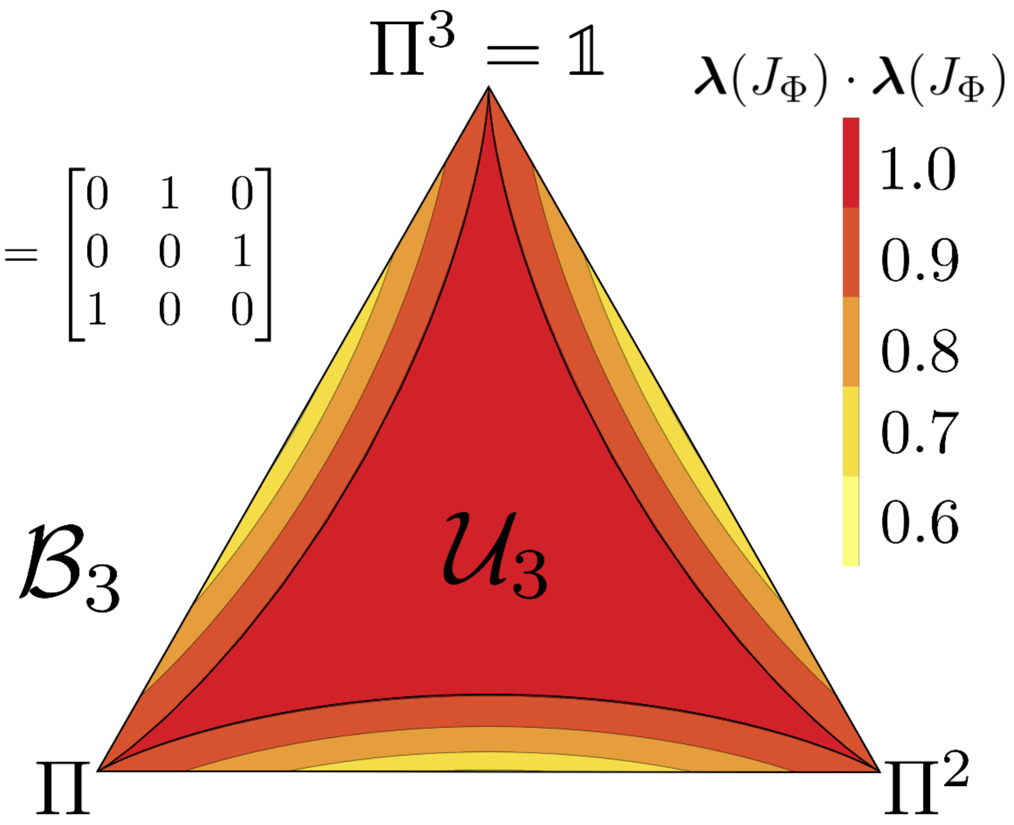
For bistochastic T majorisation upper-bound becomes trivial:

$$[1,0,\ldots,0]^\top = \boldsymbol{\mu}^\succ(T) \succ \boldsymbol{\lambda}(J_\Phi)$$

Any non-trivial bound describes the unistochastic-bistochastic boundary (known to be complex: characterised so far only for $d < 4$)

Developed a family of bounds based on *polygon* constraints.

$$\Pi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Coherence and classical randomness

Evolution of a pure quantum state $|\psi\rangle$ under the action of a channel Φ,

$\Phi(|\psi\rangle\langle\psi|) = \sum_j K_j \, |\psi\rangle\langle\psi| \, K_j^\dagger$

can be interpreted as incoherent mixture of pure state transformations:

$|\psi\rangle \xrightarrow{\Phi} \frac{1}{\sqrt{q_j}} K_j |\psi\rangle$ with probability q_j, $\qquad q_j = \mathrm{Tr}\left(K_j \, |\psi\rangle\langle\psi| \, K_j^\dagger\right)$

Path probability averaged over all pure states, $\langle\cdot\rangle_\psi = \int d\psi(\cdot)$,

$\langle q_j\rangle_\psi = \mathrm{Tr}\left(K_j \langle|\psi\rangle\langle\psi|\rangle_\psi K_j^\dagger\right) = \mathrm{Tr}\left(K_j K_j^\dagger\right) = \lambda_j(J_\Phi)$

Also note a bound on unitarity: $u(\Phi) \leq \frac{d^2}{d^2-1}\left[\boldsymbol{\lambda}(J_\Phi)\cdot\boldsymbol{\lambda}(J_\Phi) - \frac{1}{d^2}\right]$

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Perfectly distinguishable state coherifications

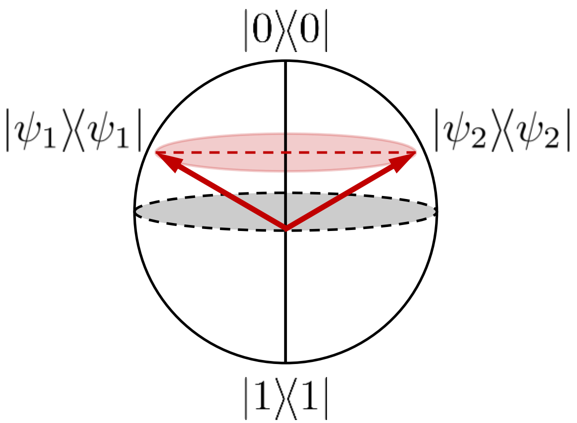
One can always optimally coherify a classical state $\boldsymbol{p}$:

$$\mathrm{diag}(\boldsymbol{p}) \xrightarrow{\mathcal{C}} |\psi_j\rangle\langle\psi_j| \text{ with } |\psi_j\rangle = \sum_k \sqrt{p_k} e^{i\phi_{jk}} |k\rangle$$

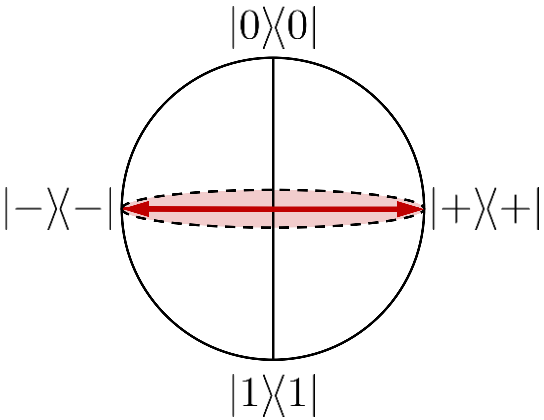
Classical versions of such states $|\psi_j\rangle$ are the same and thus indistinguishable.
However, $|\psi_j\rangle$ may be distinguished by measurements in different bases.

Question:

How many perfectly distinguishable states with classical version $\boldsymbol{p}$ are there?



$\boldsymbol{p} = [3/4, 1/4]$



$\boldsymbol{p} = [1/2, 1/2]$

Necessary condition for N-distinguishability

$$\begin{array}{c} N \text{ perfectly distinguishable states } \{\rho^{(j)}\} \\ \text{with } \langle k|\rho^{(j)}|k\rangle = p_k \end{array} \iff \begin{array}{c} N \text{ orthogonal states } \{|\psi_j\rangle\} \\ \text{with } |\langle k|\psi_j\rangle|^2 = p_k \end{array}$$

$$\Downarrow$$

$$\forall k: \; p_k \leq \frac{1}{N}$$

Orthogonal $\{|\psi_j\rangle\}$ could form columns of a unitary matrix:

$$U = \begin{bmatrix} \sqrt{p_1}e^{i\phi_{11}} & \dots & \sqrt{p_1}e^{i\phi_{1N}} & \dots \\ \sqrt{p_2}e^{i\phi_{21}} & \dots & \sqrt{p_2}e^{i\phi_{2N}} & \dots \\ \vdots & \ddots & \vdots & \\ \underbrace{\sqrt{p_d}e^{i\phi_{N1}} \quad \dots \quad \sqrt{p_d}e^{i\phi_{dN}}}_{N} & & & \dots \end{bmatrix}$$

Corresponding unistochastic matrix:

$$U \circ \bar{U} = \begin{bmatrix} p_1 & \dots & p_1 & \dots \\ p_2 & \dots & p_2 & \dots \\ \vdots & \ddots & \vdots & \dots \\ \underbrace{p_d \quad \dots \quad p_d}_{N} & & & \dots \end{bmatrix}$$

But rows must sum to 1!

(In)sufficiency of the condition

$N = 2:\quad \langle\psi_1|\psi_2\rangle = \sum_j p_j e^{i\phi_j} = 0 \quad\Longleftrightarrow\quad \exists$ polygon with sides of length $\{p_j\}$

$$\forall k:\ p_k \leq \frac{1}{2}$$

Triangle inequality

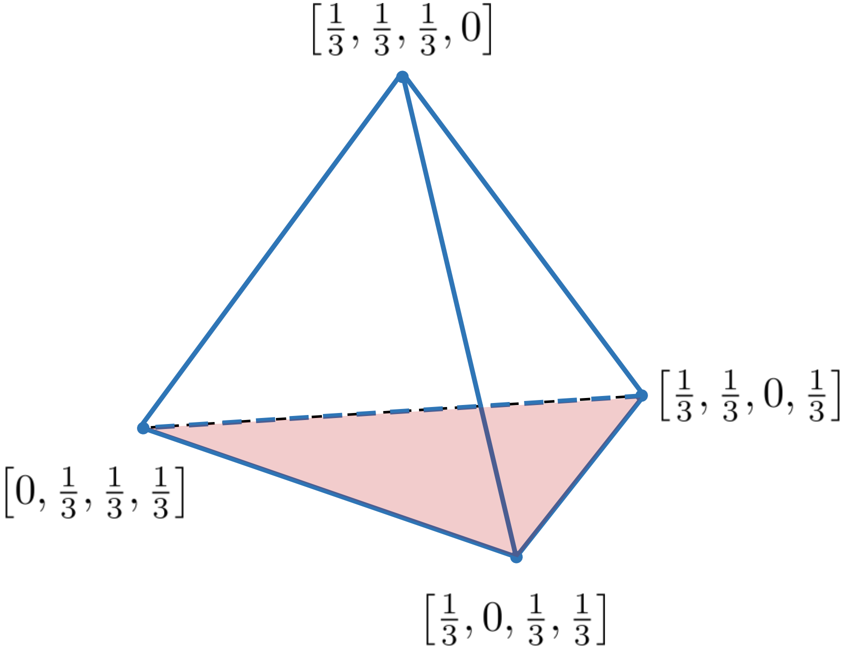
$N = d:\quad$ Choose d vectors from the unbiased basis (connected via Fourier matrix)

$N = 3:$

For $d = 4$ necessary condition means that:

$$\boldsymbol{p} = \lambda_1[0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}] + \lambda_1[\tfrac{1}{3}, 0, \tfrac{1}{3}, \tfrac{1}{3}] + \lambda_1[\tfrac{1}{3}, \tfrac{1}{3}, 0, \tfrac{1}{3}] + \lambda_1[\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0]$$

with $\lambda_j \geq 0$ and $\sum_j \lambda_j = 1$

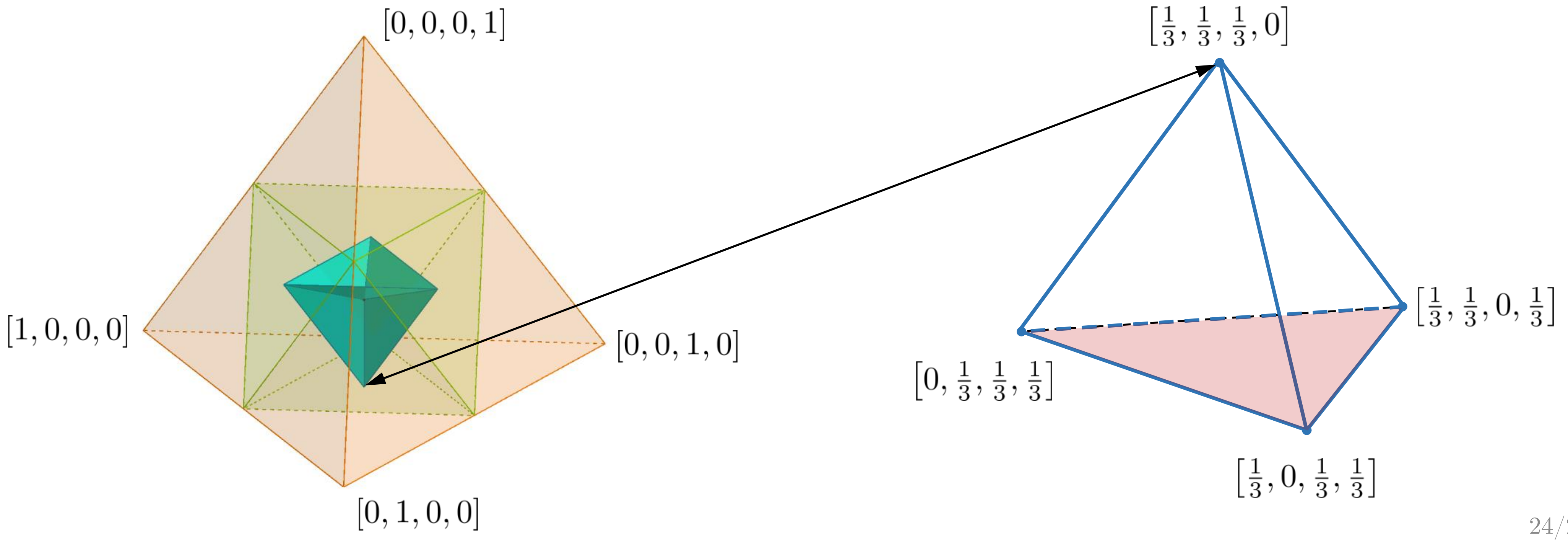


(In)sufficiency of the condition

$N = 2:$ $\quad \langle\psi_1|\psi_2\rangle = \sum_j p_j e^{i\phi_j} = 0 \quad \Longleftrightarrow \quad \exists$ polygon with sides of length $\{p_j\}$

$$\forall k: \; p_k \leq \frac{1}{2} \quad \Updownarrow \text{ Triangle inequality}$$

$N = d:$ $\quad$ Choose d vectors from the unbiased basis (connected via Fourier matrix)



Distinguishing channel coherifications

Channels $\{\Phi^{(j)}\}$ with fixed classical action T are perfectly distinguishable iff:

$\exists \rho_{AB}$: $\{\Phi^{(j)} \otimes \mathcal{I}(\rho_{AB})\}$ are perfectly distinguishable

If $\exists \rho$: $\{\Phi^{(j)}(\rho)\}$ are perfectly distinguishable then no entanglement needed

Type of classical action	**Number of perfectly distinguishable channels**	**Requires entanglement**
Unistochastic	$\geq d$	Yes for $> d$
Circulant	d	Yes
Bistochastic	2	Yes
Such that $\exists j,k: \; T_{jk} \leq \frac{1}{2}$	2	No

Outlook

- Are optimally coherified channels extremal? Have vanishing minimum output entropy? Other special properties?
- Stronger links between coherifications and channel irreversibility? Random walks?
- Formal relation between the number of perfectly distinguishable states and information loss due to decoherence? Energy-time uncertainty?
- Applications in cryptographic protocols? Usefulness in the presence of SSR?

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Thank you!