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Tunneling Transfer Protocol in a Quantum Dots Chain Immune to Inhomogeneity

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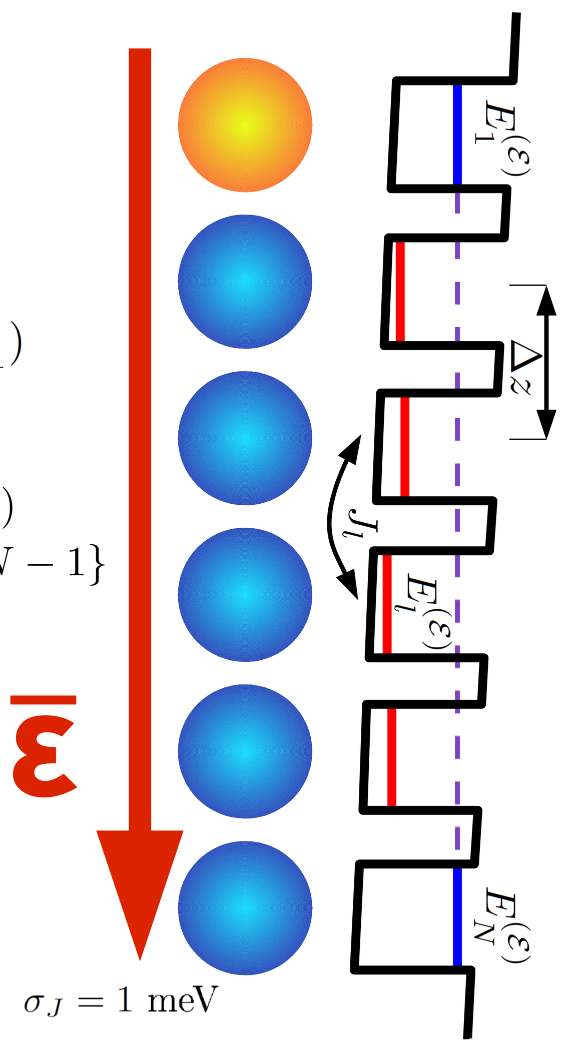
1. Motivation:

- To develop a physical implementation of a quantum state transfer channel
- To elaborate a transfer protocol independent of inhomogeneity and satisfying the requirement of simplicity
- To achieve quantum transfer on demand

2. System:

- Chain of N vertically stacked QDs with the nearest neighbour tunnel coupling
- Axial electric field
- The QD chain is doped with one electron
- Higher electron energies in terminal QDs



3. Model:

- Hamiltonian:

$$H = \sum_{l=1}^{N} \overbrace{(E_l^{(0)} + \mathcal{E} l \Delta z)}^{E_l^{(\mathcal{E})}} \sigma_l^z + \sum_{l=1}^{N-1} J_l (\sigma_l^+ \sigma_{l+1}^- + \sigma_l^- \sigma_{l+1}^+)$$

- Parameters given by Gaussian distribution:
  - Electron energy of terminal QDs: $E_1^{(0)}, E_N^{(0)} \sim \mathcal{N}(E_T, \sigma_E^2)$
  - Energies inside the chain: $E_l^{(0)} \sim \mathcal{N}(0, \sigma_E^2),\ l = \{2 \dots N-1\}$
  - Tunnel coupling: $J_l \sim \mathcal{N}(J, \sigma_J^2)$
- The basis (number of electrons is conserved):

$|1\rangle = |100\dots000\rangle\ ;\ \dots\ ;\ |N\rangle = |000\dots001\rangle$

4. Parameters for the investigated chains:

A $N = 10,\quad E_T = 50\text{ meV},\quad \sigma_E = 10\text{ meV},\quad J = 10\text{ meV},\quad \sigma_J = 1\text{ meV}$

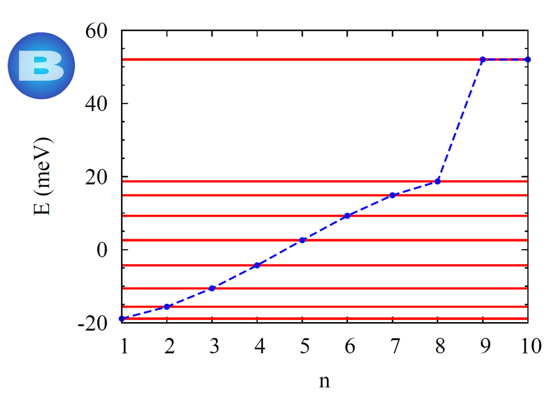
B $N = 10,\quad E_T = 50\text{ meV},\quad \sigma_E = 0\text{ meV},\quad J = 10\text{ meV},\quad \sigma_J = 0\text{ meV}$

C $N = 5,\quad E_T = 50\text{ meV},\quad \sigma_E = 10\text{ meV},\quad J = 10\text{ meV},\quad \sigma_J = 1\text{ meV}$

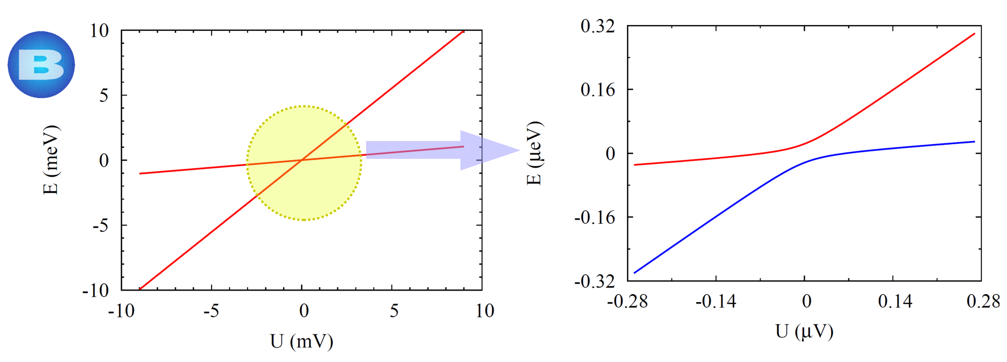
5. Method

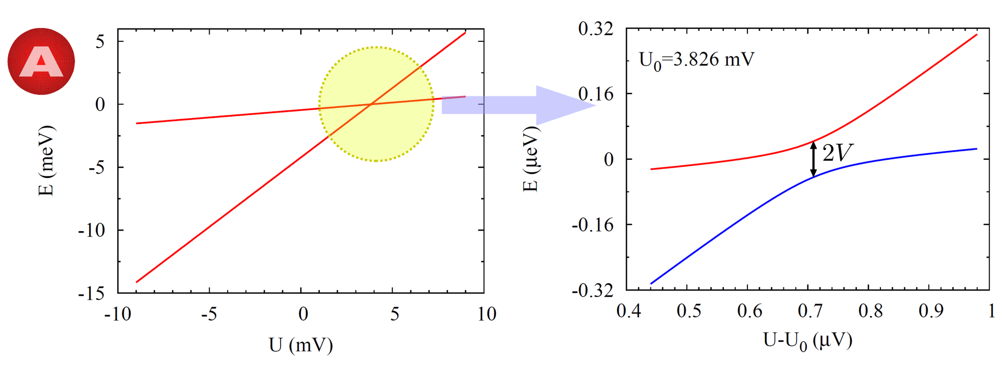
- Numerical diagonalization of the Hamiltonian: finding eigenvalues and eigenstates
- Decomposition of the initial state into eigenstates of the Hamiltonian in order to evolve them and obtain the final state (for constant or no electric field)
- Numerical solution of the Schrödinger differential equation using Runge-Kutta method (for a time dependent electric field)

6. Spectral properties:

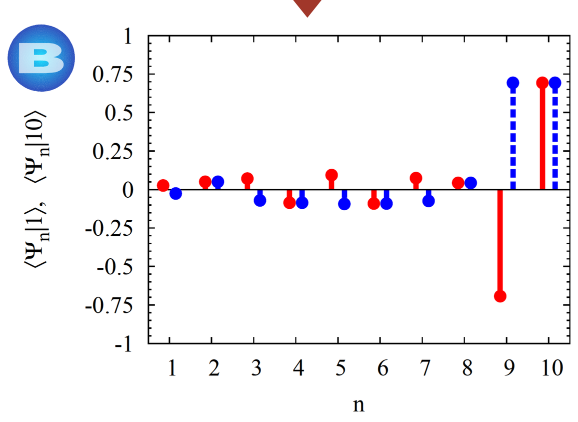


- Energetic separation of the eigenstates $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ due to difference between electron energies of terminal QDs and the ones inside the chain
- Inhomogeneity affects mainly the energy levels of the states delocalized in the central part of the chain. It shifts the resonance between $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$, but only weakly changes its width.
- Dependence of these energy levels on the voltage between initial and final QDs is shown on the graphs (for both homogenous and inhomogenous chain):



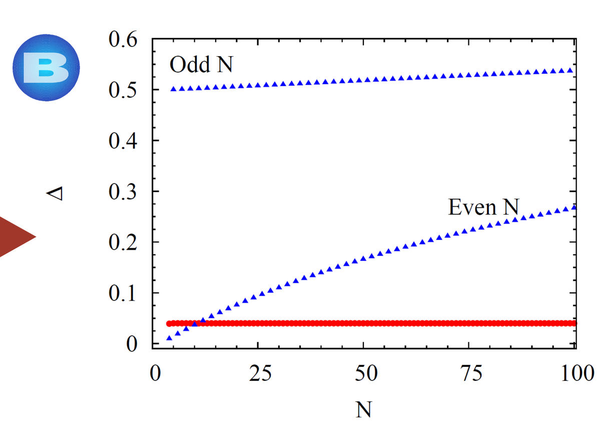


- At resonance, states with the electron on the initial (red) and final (blue) QD mainly composed of $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$:



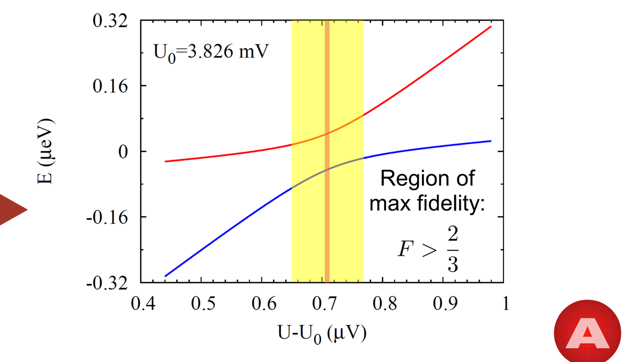
- Transfer fidelity parameter Δ independent of the chain length (red circles) in contrast to the "weak coupling" protocol* (blue triangles) with thermal couplings 10 times smaller, i.e. equal to 1 meV:

$$\Delta = 1 - |\langle\Psi_N|1\rangle|^2 - |\langle\Psi_{N-1}|1\rangle|^2$$

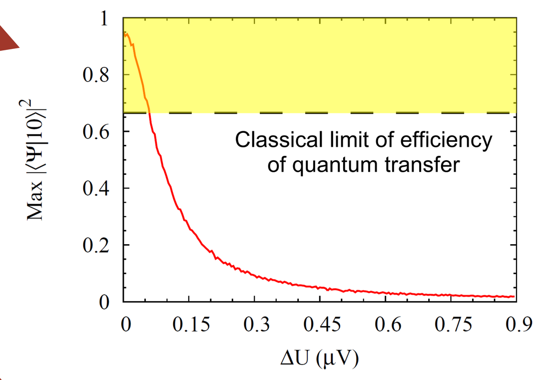


7. Free transfer:

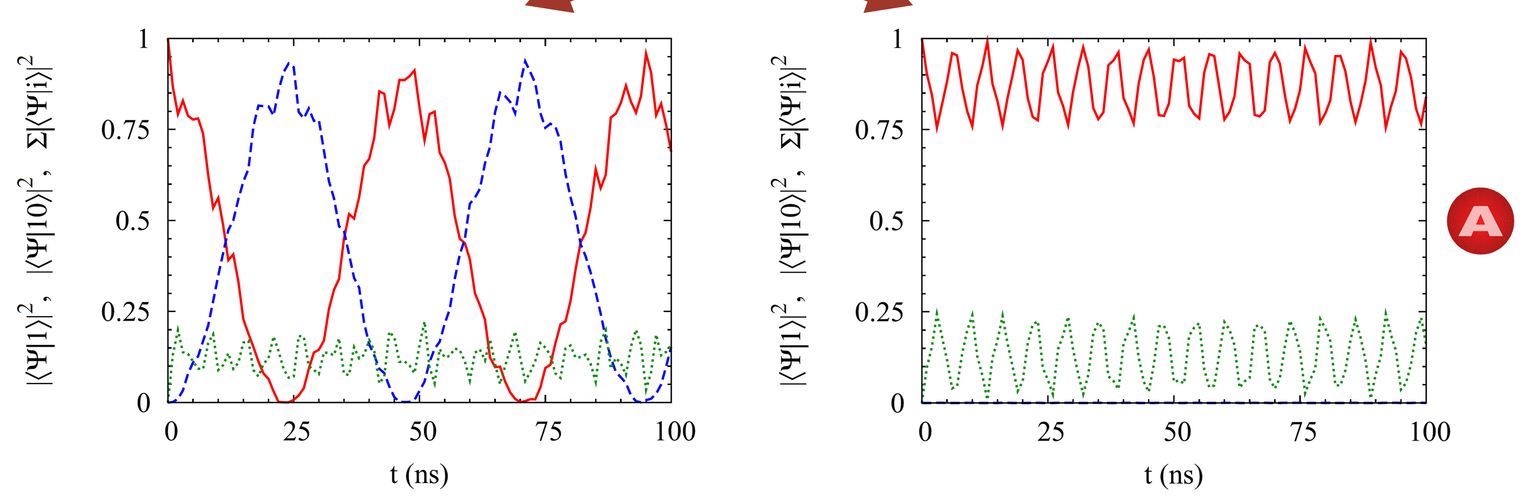
- Compensation of inhomogeneity by bringing $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ to exact resonance



- Transfer fidelity $F = |\langle\Psi|N\rangle|^2$ very sensitive to small deviations from exact resonance

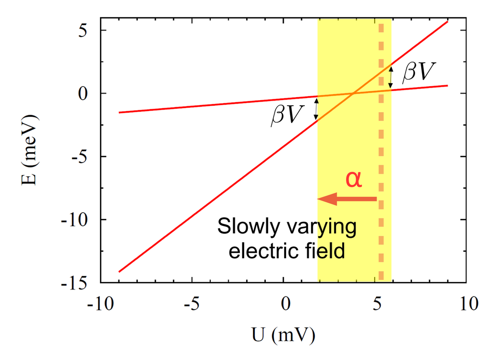


- Evolution of the occupation of the initial QD (solid line), the final QD (dashed line) and the QDs inside the chain (dotted line) on the left in comparison to the "weak coupling" model (tunnel coupling of terminal dots 10 times weaker) on the right:



- Transfer time: $\tau_f = \frac{\pi\hbar}{2} \cdot \frac{1}{V}$

8. Adiabatic transfer:



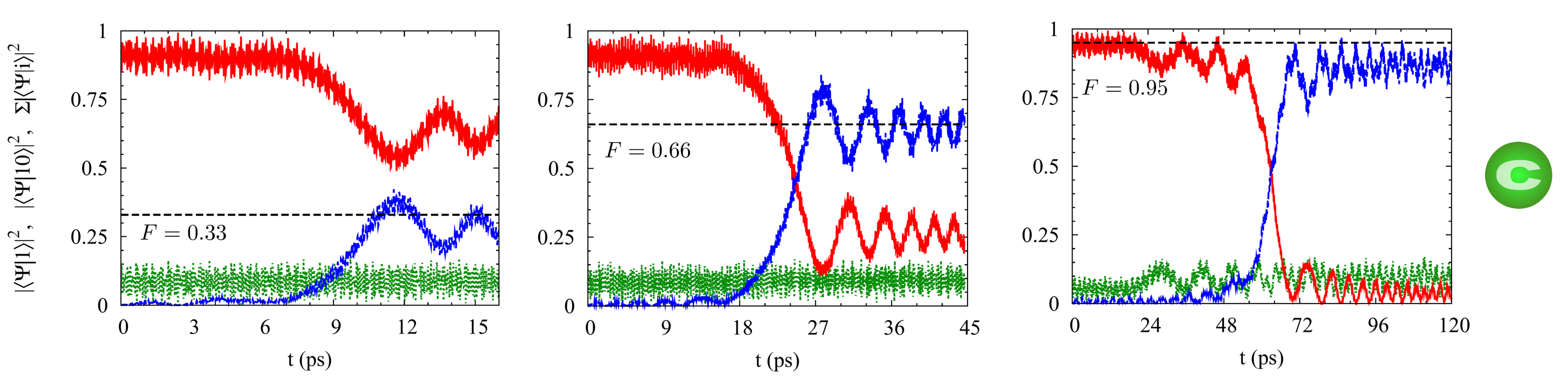
- Adiabatic transfer of a quantum state obtained by slowly changing the electric field, which sweeps energy levels of $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ through the resonance
- Effective 2-level model: $|\Psi_N\rangle$ and $|\Psi_{N-1}\rangle$ with the coupling given by half of the energy splitting at resonance: $H_{12} = V$
- Finite speed of the electric field sweep α obtained from the Landau-Zener formula for nonadiabatic tansition probabilities P_{na}:

$$P_{na} = \exp\left(-\frac{2\pi}{\hbar} \cdot \frac{|H_{12}|^2}{\alpha}\right) \quad ; \quad F = 1 - P_{na}$$

- Finite transfer time obtained by narrowing the limits of the electric field sweep to the area where energy separation of the states is smaller than βV (assumption: for $\Delta E > \beta V$ interaction is negligible):

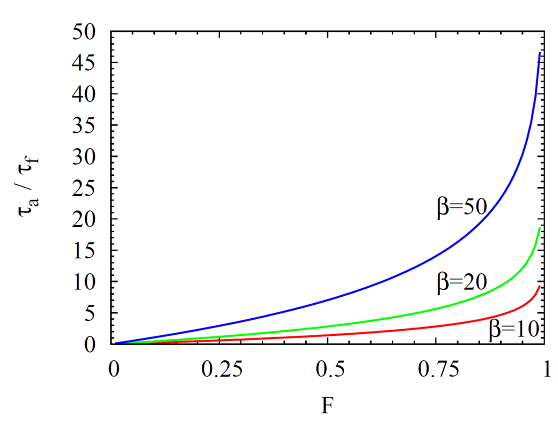
$$\tau_a = \frac{\hbar}{\pi} \cdot \frac{\beta}{V} \ln\left(\frac{1}{1-F}\right)$$

- Landau-Zener result for the effective two-level model shows good agreement with the simulation of the evolution of the full system for β large enough (here $\beta = 20$). The graphs show the simulated evolution of the occupation of QDs (colors meaning as in the free transfer section) and the calculated fidelity from the L-Z formula (black dashed line):



- Adiabatic transfer time to free transfer time ratio for given β is only a function of fidelity we want to obtain adiabatically:

$$\frac{\tau_a}{\tau_f} = \frac{2\beta}{\pi^2} \ln\left(\frac{1}{1-F}\right)$$



9. Conclusions:

- The negative impact of the inhomogeneity of the QD chain on the quantum transfer efficiency can be overcome by shifting energy levels to the resonance with exactly chosen electric field
- Adiabatic transfer in a QD chain can be well described within an approximate model of a two-level system and the transfer time can be obtained by using Landau-Zener formula for a given desired fidelity
- The adiabatic transfer protocol makes it possible to achieve quantum transfer on demand

*A. Wójcik, T. Łuczak, P. Kurzynski, A. Grudka, T. Gdala and M. Bednarska, Phys. Rev. A **72**, 034303