Avoiding irreversibility

Lossless interconversion of quantum resources

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Outline

1. Background
   a. Quantum technologies and resources
   b. Resource theories
   c. Asymptotic regime and beyond
2. Results
   a. Finite-size conversion rates
   b. Resonant conversion of resources
3. Outlook

Quantum technologies and resources

First Quantum Revolution

- laser systems
- nuclear power
- MRI imagers
- transistors
- semi-conductor electronics

Second Quantum Revolution

- quantum computing

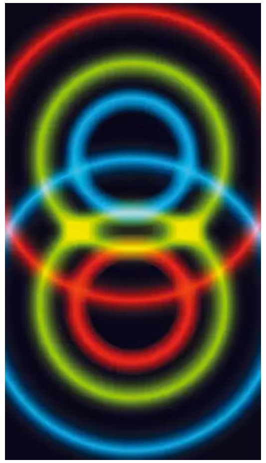
  Shor's & Grover's algorithm, simulating quantum systems

- quantum communication

  Secure BB84 & E91 protocols, quantum internet

- quantum thermodynamics

  Increased power of heat engines & efficiency of energy harvesting



New quantum technologies actively create, manipulate and read out quantum states

Quantum technologies and resources

Harnessing quantum resources

1. Identification

Technology $\Leftrightarrow$ Particular resource

$$1 \text{ teleported qubit} = 1 \text{ entangled Bell pair}$$

$$1 \text{ secure bit} = 1 \text{ coherent state}$$

Task: designing quantum protocols

2. Characterisation

Particular resource $\Leftrightarrow$ General resource

$$x \text{ entangled Bell pairs} = 5 \text{ entangled states } |\psi\rangle$$

$$1 \text{ entangled Bell pair} = y \text{ entangled states } |\psi\rangle$$

Task: finding limits on resource manipulation

3. Implementation

General resource $\Leftrightarrow$ Physical system

Task: Experimental realisation

Resource theories

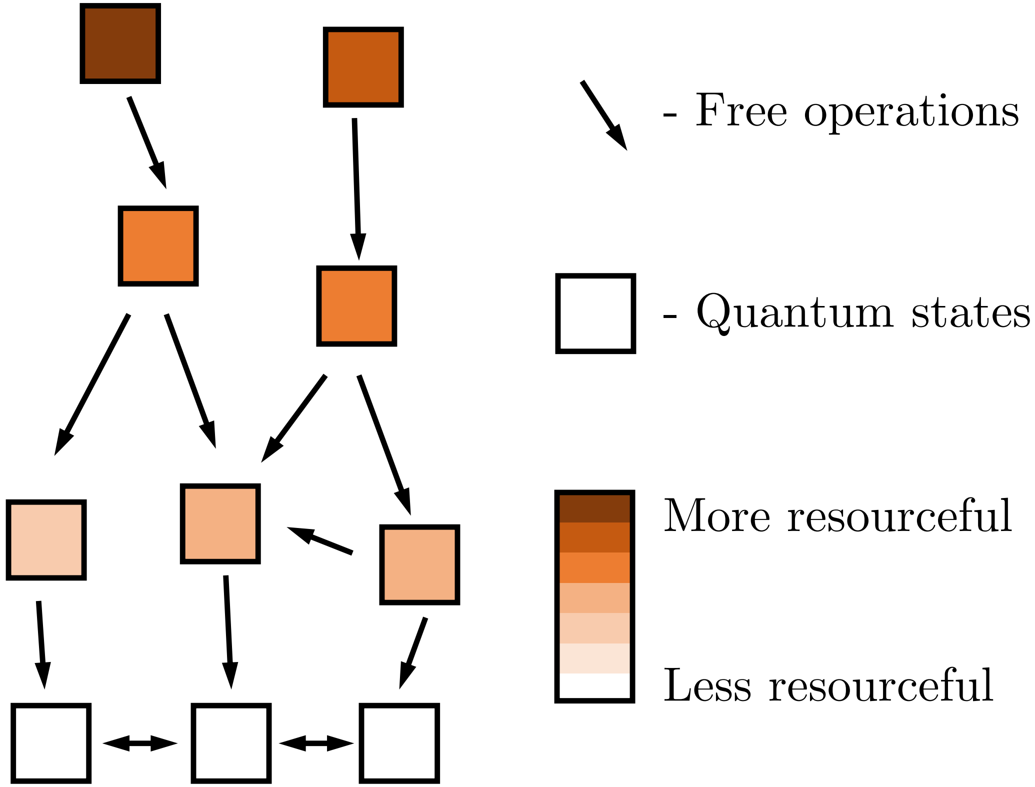
Resource theory:

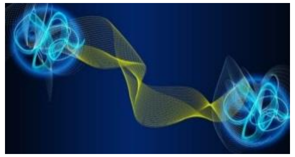
- Mathematical framework designed to study possible manipulation of resources
- Defined by physical constraints and the corresponding set of quantum operations

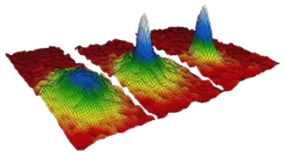
Resource theory	Physical constraint	Restricted set of operations	Free states
Entanglement	Space-like separation of distant parties	Local operations and classical communication	Separable states
Coherence	Creating superoposition of states in a particular basis is difficult	Incoherent operations	Incoherent states
Thermodynamics	1st and 2nd law of thermodynamics (energy conservation and growth of entropy)	Thermal operations	Thermal equilibrium state

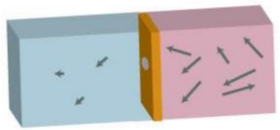
Resource theories

Restricted set of operations induces partial ordering of quantum states





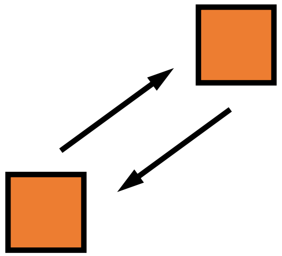




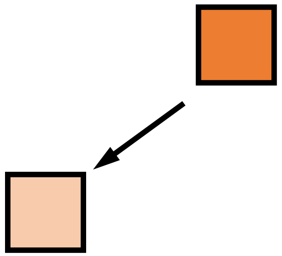
Entanglement	Coherence	Thermodynamics
Non-local	Coherent	Out-of-equilibrium
↓	↓	↓
Separable	Incoherent	Equilibrium

Resource theories

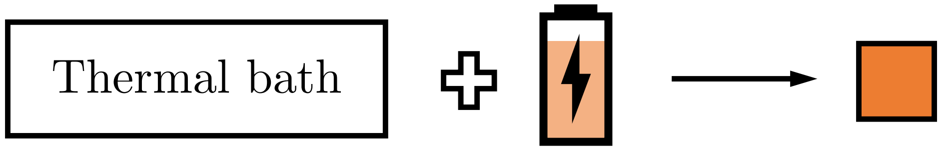
Reversible processes



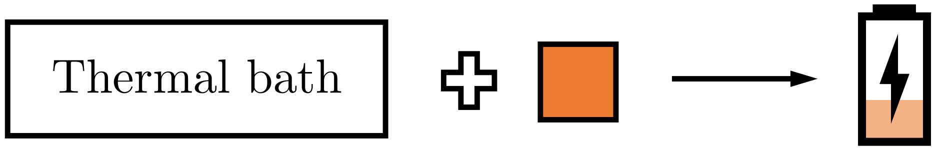
Irreversible processes



Transformations between most resource states are irreversible



Resource dilution



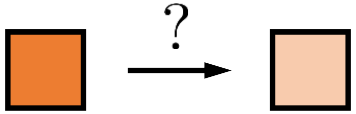
Resource distillation

Dissipation of resources – initial and final states have different resource content

Asymptotic regime and beyond

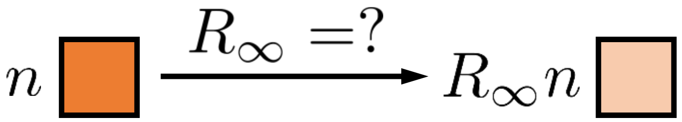
Single-shot regime

$n = 1$



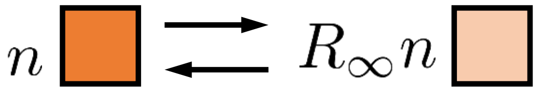
Asymptotic regime

$n \to \infty$



Reversibility in the asymptotic limit

$$R_\infty(\rho \to \sigma \to \rho) = 1$$



Optimal asymptotic rate:

$$R_\infty(\rho \to \sigma) = \frac{r(\rho)}{r(\sigma)}$$

$r(\rho)$ - asymptotic resource measure

Resource theory	Asymptotic resource measure
Entanglement	Entropy of the reduced state
Coherence	Entropy of the decohered state
Thermodynamics	Free energy of the state

Asymptotic regime and beyond

Asymptotic analysis not suitable for:

- Near-future quantum technologies (limited resources)
- Studying quantum effects beyond the thermodynamic limit

Need for second-order asymptotic expansion

$$R_n(\rho \to \sigma) = R_\infty(\rho \to \sigma) - \frac{f(\rho, \sigma, \epsilon)}{n^\alpha}$$

ϵ - accepted error level

Want: $\rho^{\otimes n} \to \sigma^{\otimes R_n n}$
Have: $\rho^{\otimes n} \to \tilde{\sigma}$, with $\delta(\tilde{\sigma}, \sigma^{\otimes R_n n}) \leq \epsilon$

Second-order asymptotics and thermodynamic limit

$$\frac{\text{energy fluctuations}}{\text{average energy}} = \frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{\langle E \rangle} \sim \frac{1}{n^{1/2}} \xrightarrow{n \to \infty} 0$$

Finite-size conversion rates

Probability distribution representation of quantum states:

We want: $\rho \to \sigma$ We consider: $\boldsymbol{p} \to \boldsymbol{q}$

Single-shot conversion laws:

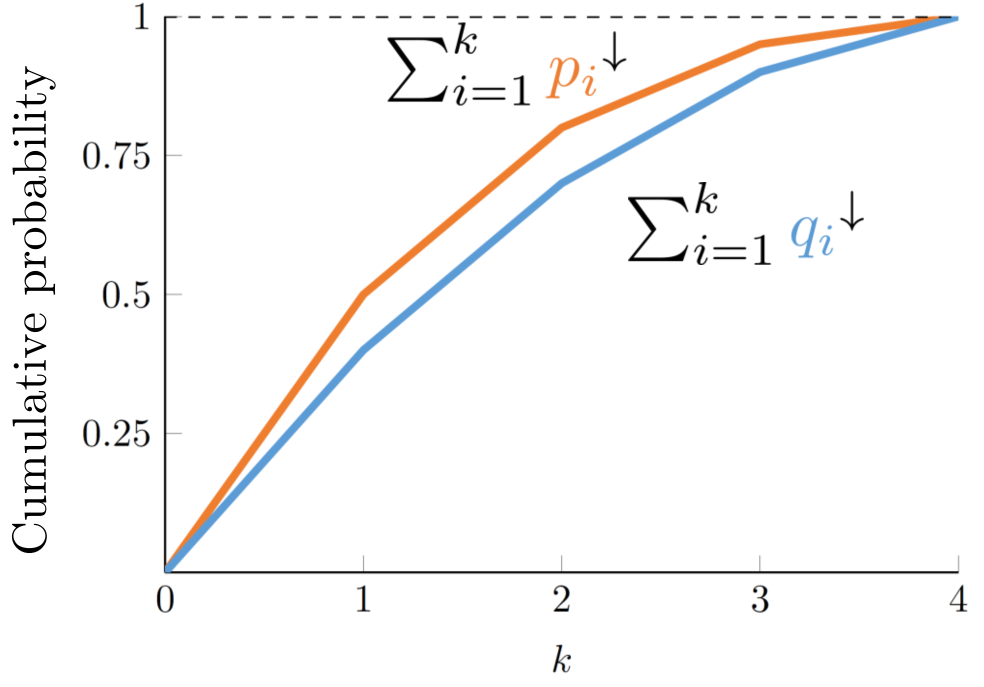
$\boldsymbol{p} \prec \boldsymbol{q}$

Majorisation condition
(entanglement & coherence)

$\boldsymbol{p} \succ^{\beta} \boldsymbol{q}$

Thermomajorisation
condition

Resource theory	Probability distribution representation
Entanglement	Reduced state spectrum
Coherence	Decohered state spectrum
Thermodynamics	Distribution over energy eigenstates



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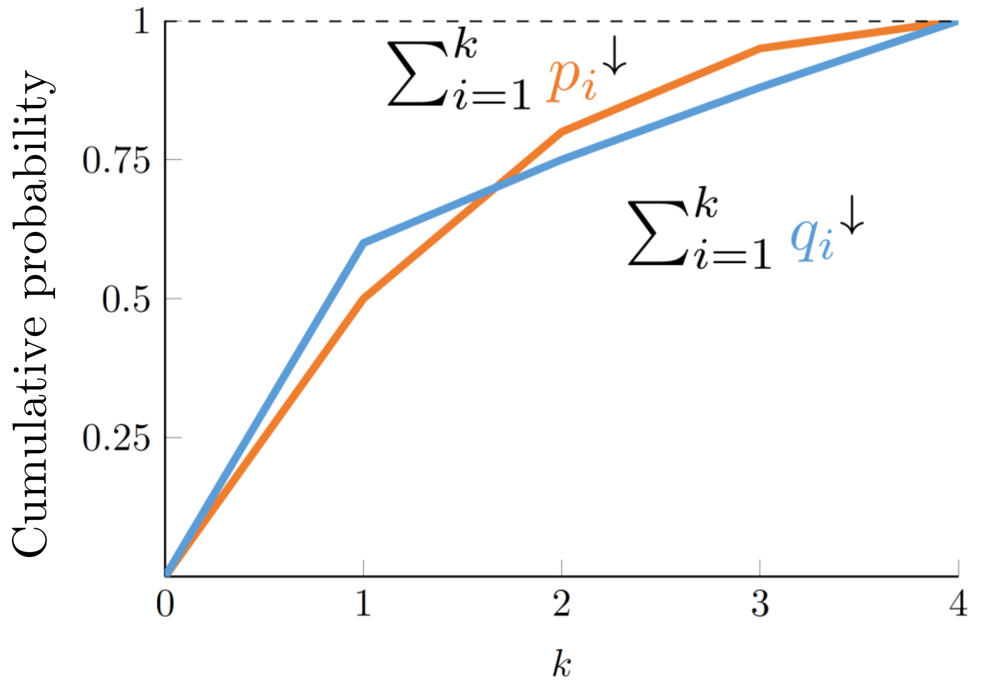


Finite-size conversion rates

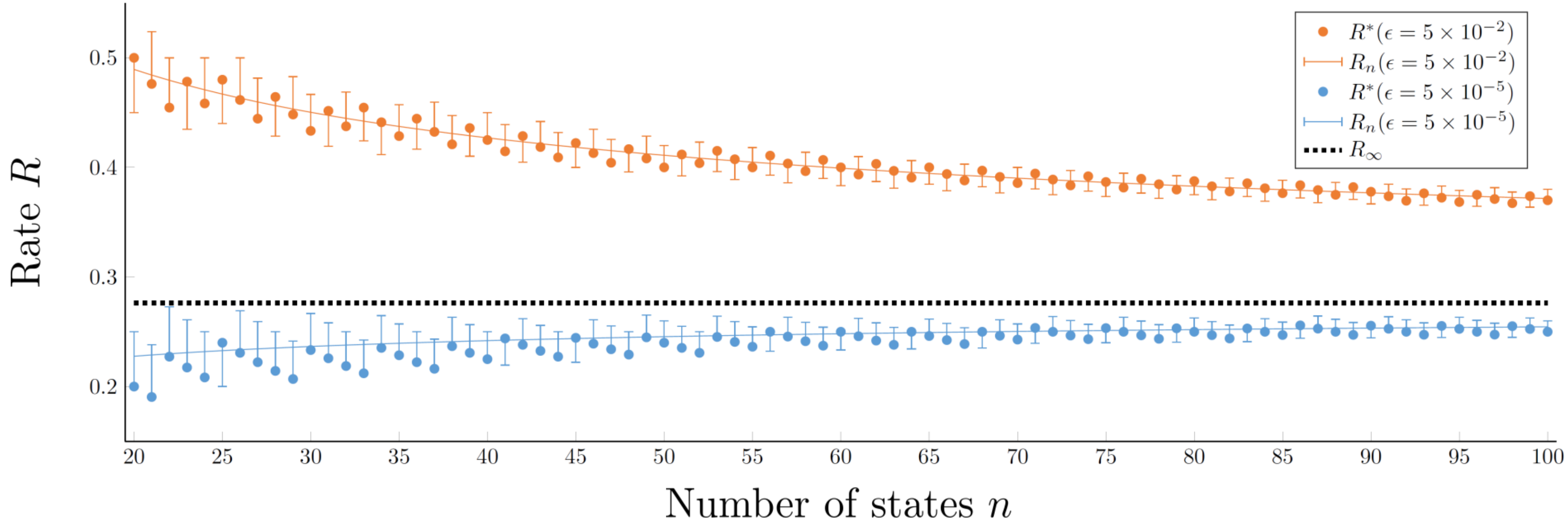
Explicit analytic expressions for correction terms:

Constant error ϵ

$$R_n \simeq R_\infty - \frac{r(\boldsymbol{p}, \boldsymbol{q}, \epsilon)}{\sqrt{n}}$$

Vanishing error $\epsilon = e^{-n^\alpha}, \ \alpha \in (0,1)$

$$R_n \simeq R_\infty - \frac{\tilde{r}(\boldsymbol{p}, \boldsymbol{q})}{\sqrt{n^{1-\alpha}}}$$



Resonant conversion of resources

Both correction terms depend on *irreversibility* parameter $\nu(\boldsymbol{p}, \boldsymbol{q})$:

$$r(\boldsymbol{p}, \boldsymbol{q}, \epsilon) \xrightarrow{\nu \to 1} 0 \qquad\qquad \tilde{r}(\boldsymbol{p}, \boldsymbol{q}) \xrightarrow{\nu \to 1} 0$$

Asymptotic rate achievable even for finite n and with vanishing error $\epsilon = 0$:

$$\nu = 1 \quad \Longrightarrow \quad R_n \simeq R_\infty$$

Conclusion: Pairs of resource states satisfying $\nu = 1$ are in resonance
(lossless conversion of resources is possible)

Can we properly tune initial and final states to avoid dissipation of resources?

Resonant conversion of resources

Example 1 - Tuning resources to resonance

2 available initial states: $|\Psi_1\rangle$ and $|\Psi_2\rangle$

1 target state: $|\Phi\rangle$

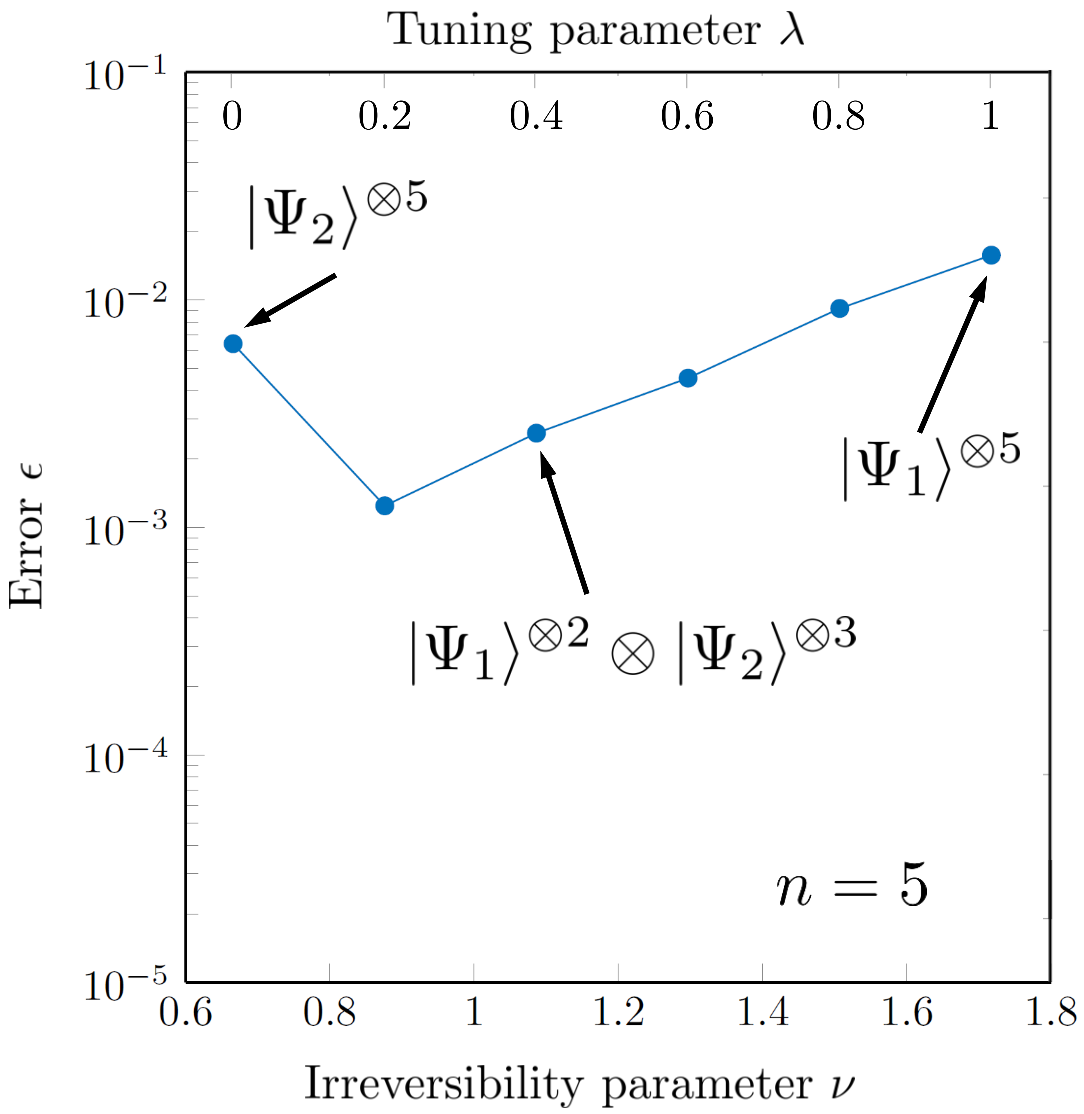
Asymptotically same resource content:

$$|\Psi_1\rangle^{\otimes n} \to |\Phi\rangle^{\otimes n}, \qquad |\Psi_2\rangle^{\otimes n} \to |\Phi\rangle^{\otimes n}$$

Hence, for all $\lambda \in [0,1]$:

$$|\Psi_1\rangle^{\otimes \lambda n} \otimes |\Psi_2\rangle^{\otimes (1-\lambda) n} \to |\Phi\rangle^{\otimes n}$$

What about finite n?



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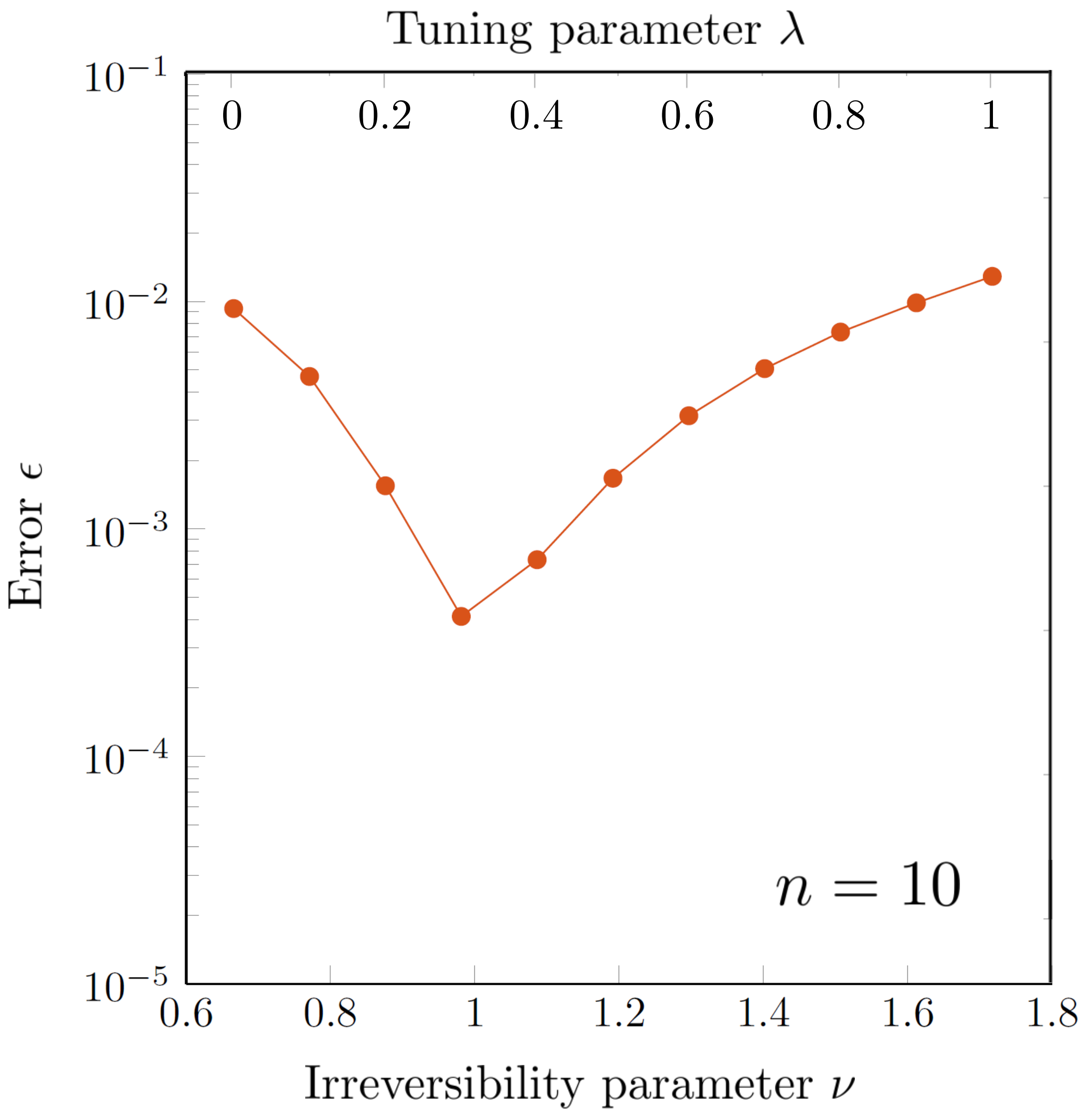

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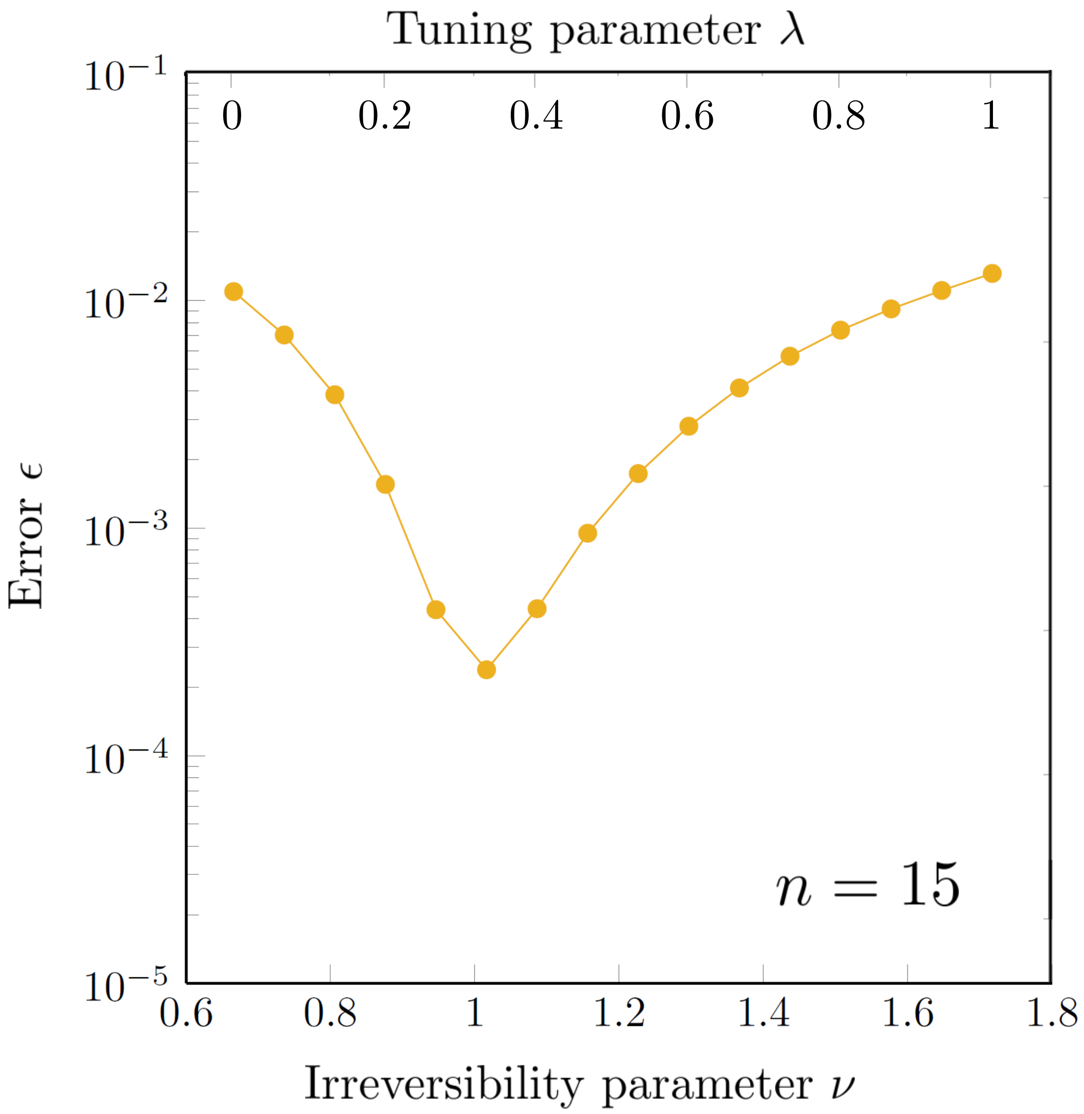

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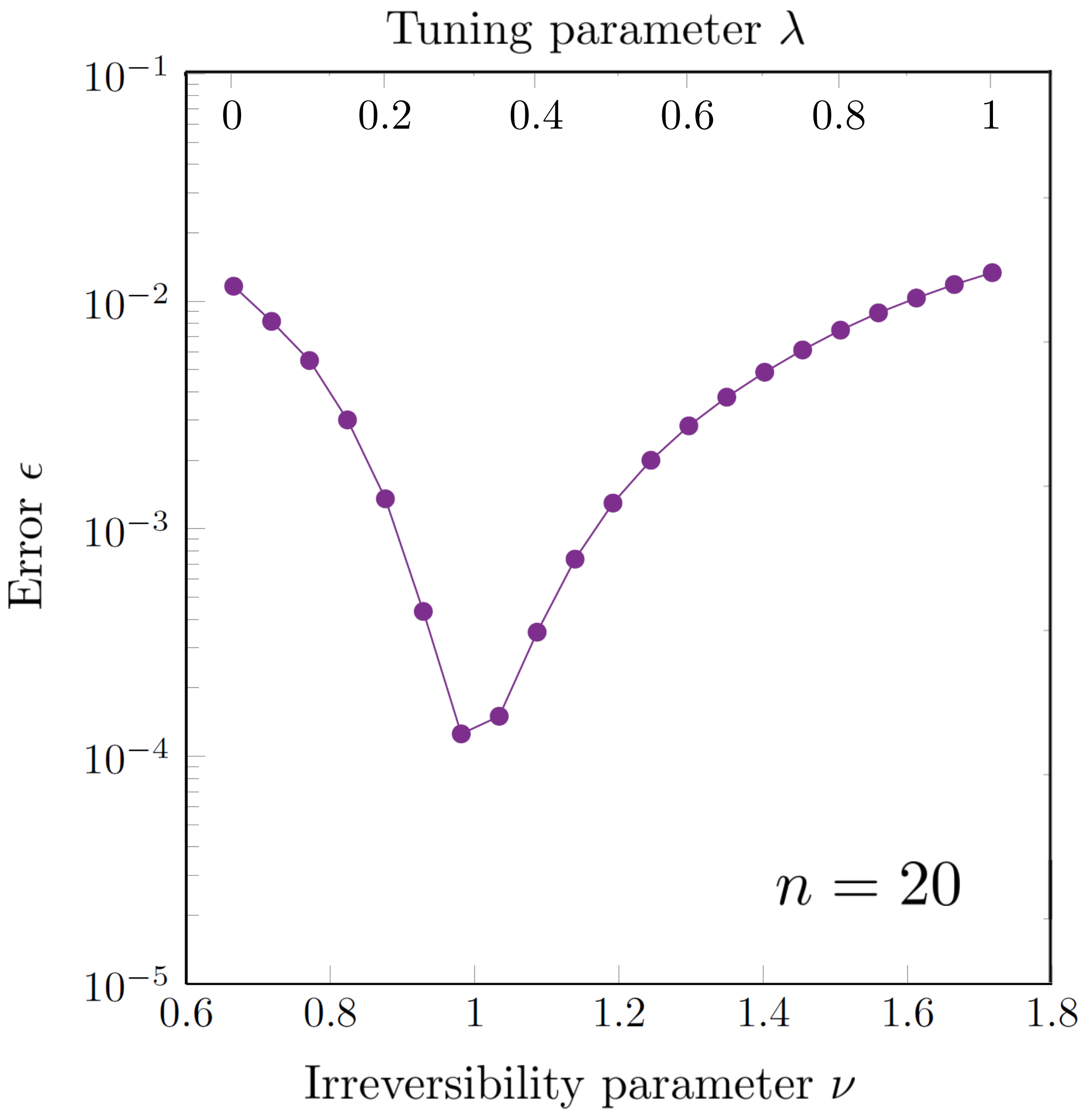

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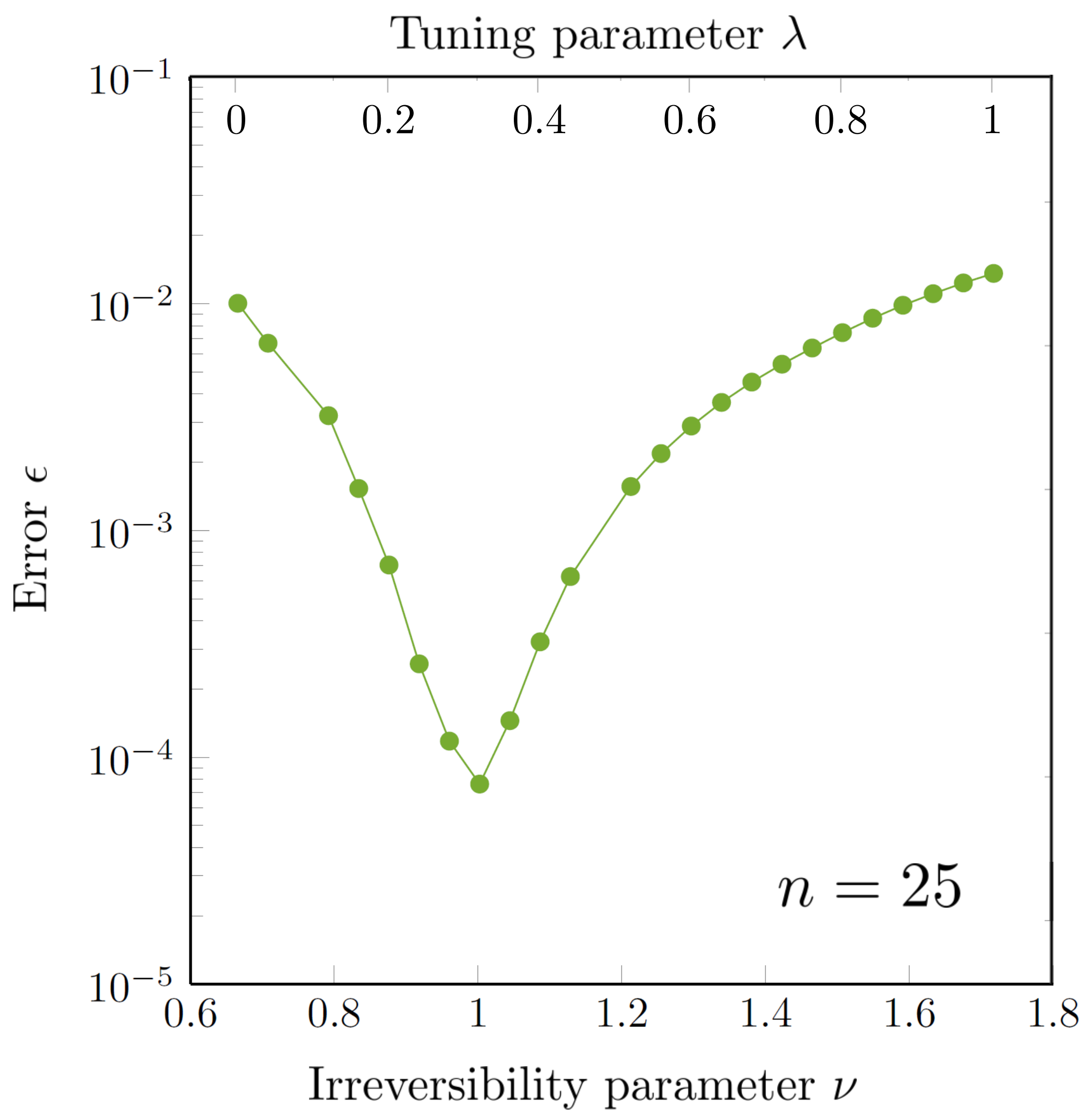

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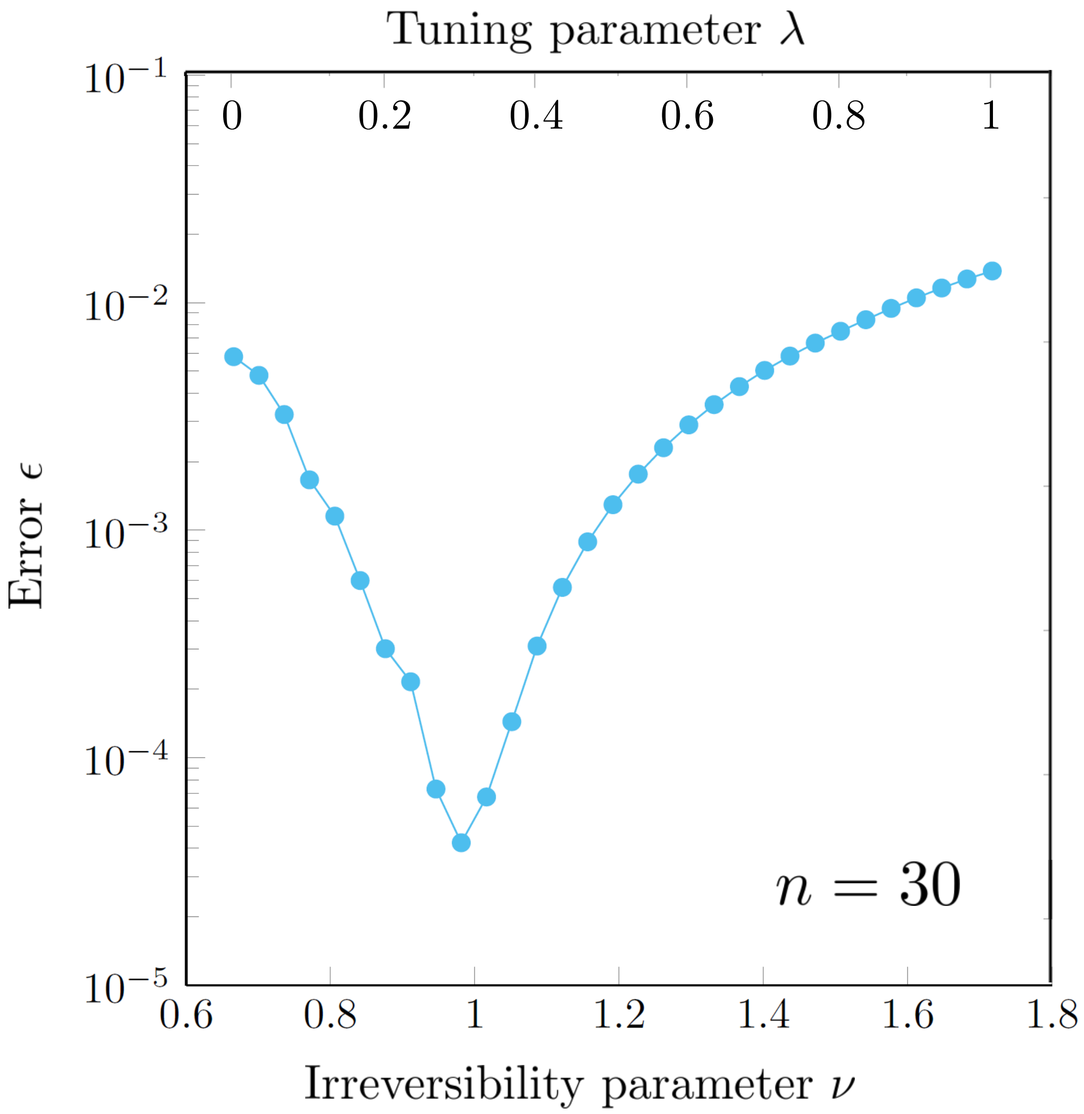

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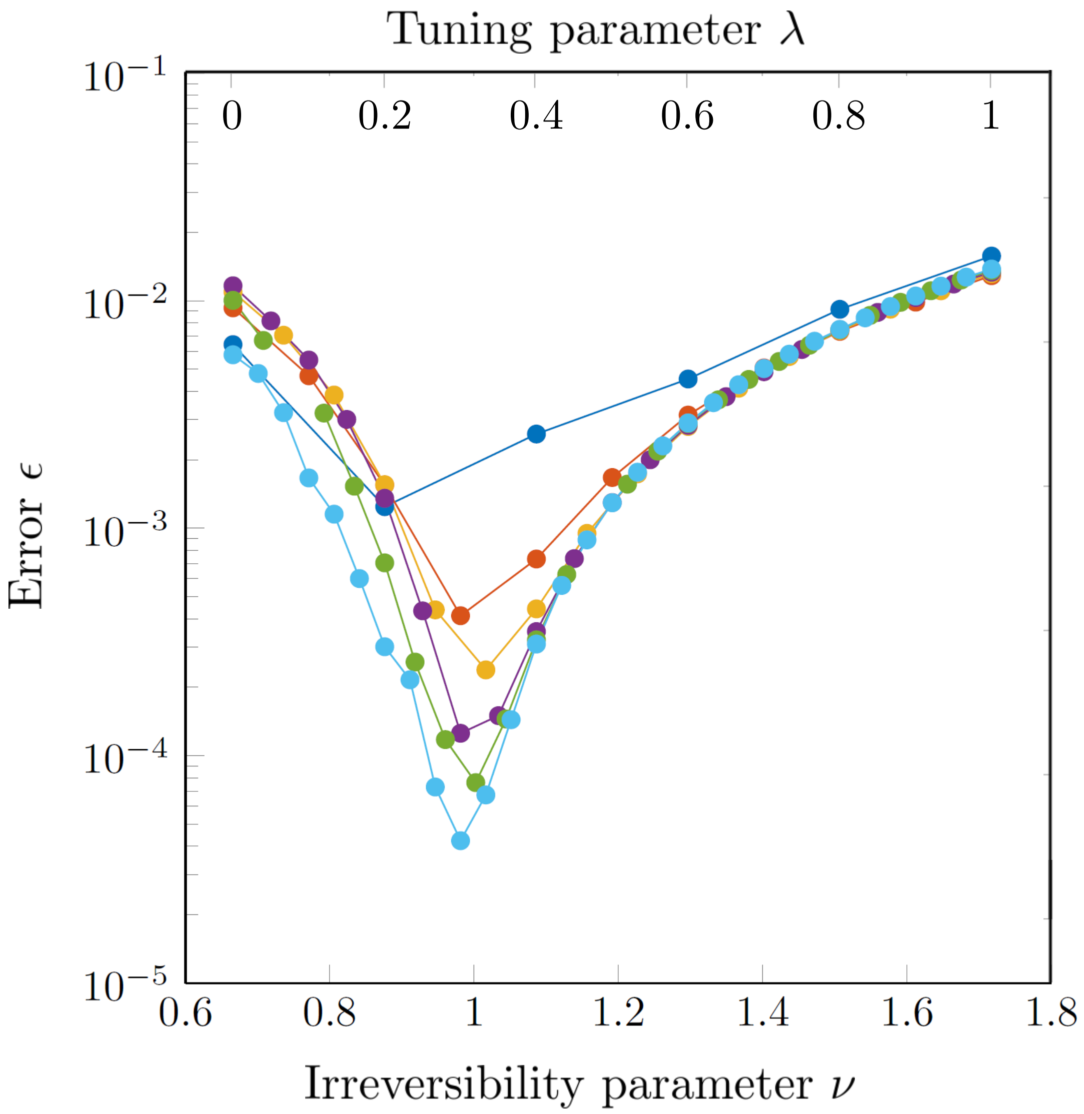


Resonant conversion of resources

Example 2 – Resonance in work extraction

Access to thermal bath at temperature T_h

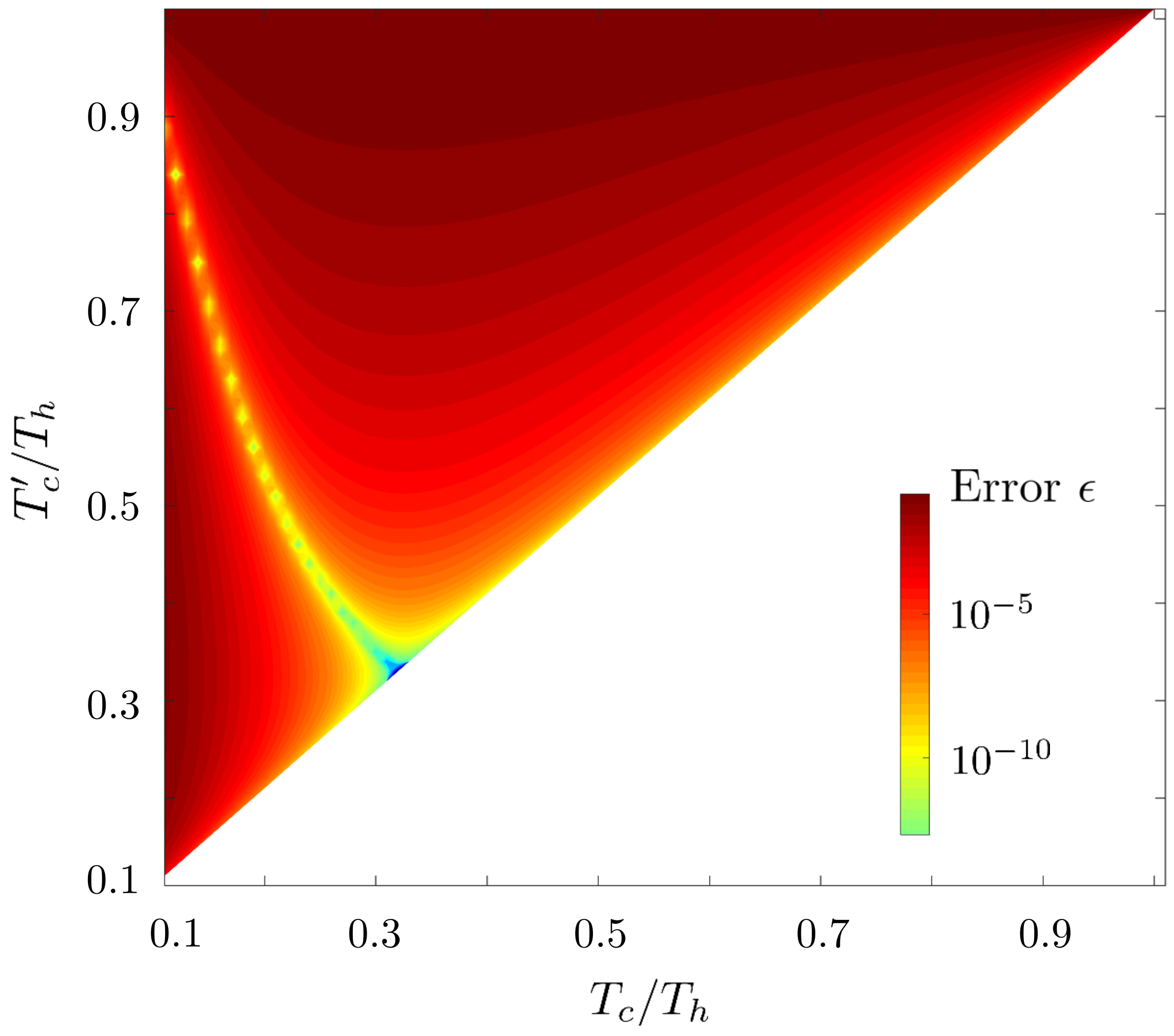
Gas of non-interacting spins at T_c

Asymptotically, perfect engine extracts:

$$W = F(\gamma_c) - F(\gamma_{c'})$$

while heating the gas to T'_c

For finite n *quality* of work is not perfect



Outlook

- Extend finite-size analysis to other resource-theories (asymmetry, contextuality)
- Design experimental protocols employing the resonance phenomenon
- Apply the results to thermodynamic problems involving finite-size heat baths (Landauer's erasure, fluctuation theorems, the third law of thermodynamics).
- Look for resonance phenomena in other quantum information processing tasks

Details:

Beyond the thermodynamic limit: finite-size corrections to state interconversion rates [arXiv:1711.01193]

Avoiding irreversibility: resonant conversion of quantum resources [soon on arXiv]

Thank you!