Quantum coherence, time-translation symmetry and thermodynamics

Kamil Korzekwa, Matteo Lostaglio, David Jennings, Terry Rudolph

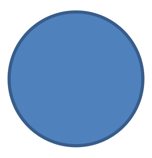
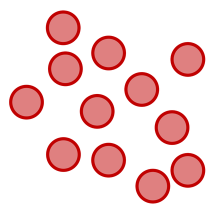
Department of Physics, Imperial College London, London SW7 2AZ, United Kingdom

Imperial College London

Controlled Quantum Dynamics

EPSRC

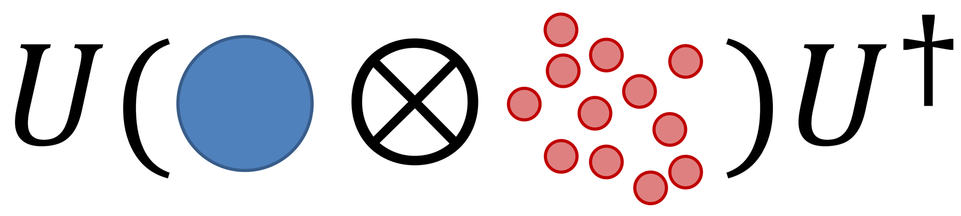
1. Thermodynamic setting

System  Environment 

Arbitrary state: ρ_S
Hamiltonian: H_S

Thermal State: $\gamma_E \propto e^{-\beta H_E}$
Hamiltonian: H_E

Joint energy-conserving unitary evolution



$$[U, H_S + H_E] = 0$$

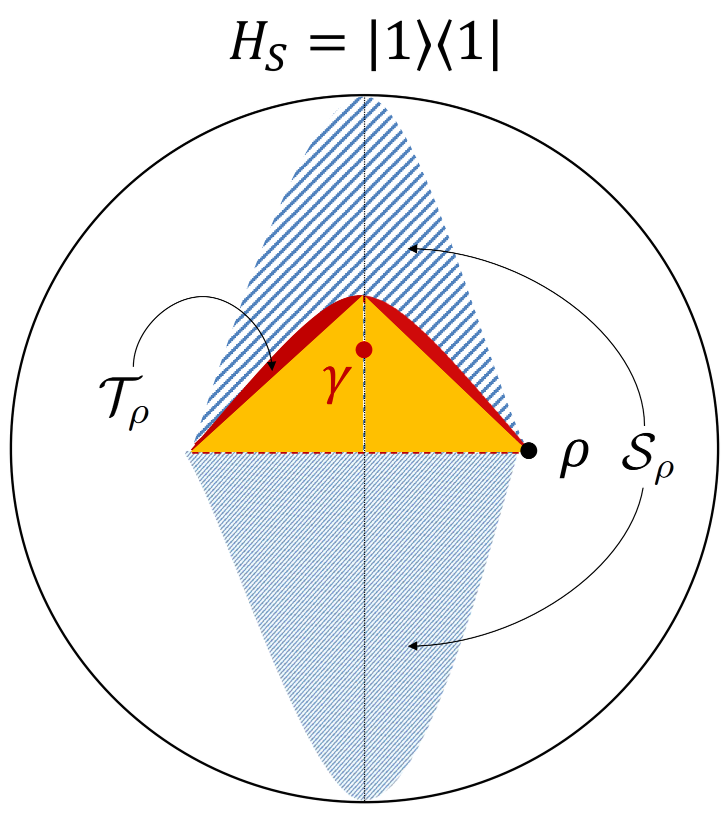
Hence the evolution of the system is described by *thermal operations*:

$$\mathcal{E}_T(\rho_S) = Tr_E\left(U(\rho_S \otimes \gamma_E)U^\dagger\right),$$

that form a subset of time-translation symmetric operations[1]:

$$\mathcal{E}_T\left(e^{-iH_S t}\rho_S \rho e^{iH_S t}\right) = e^{-iH_S t}\mathcal{E}_T(\rho_S)e^{iH_S t}$$

2. Thermal transformations of states with coherence – elementary scenario



Coherence is **actively** contributing to enlarge the set of thermodynamically accessible states.

Work is **not** the universal resource of thermodynamics.

Coherence contribution to free energy is **locked** - no trivial extension to quantum Szilard engine.

$\mathcal{T}_\rho$: Set of states accessible from ρ via thermal operations (orange region if coherence is passive)

$\mathcal{S}_\rho$: Set of states accessible from ρ via thermal operations and the access to infinite amount of work

3. How to deal with coherences? Modes of coherence

Definition:

Assuming non-degenerate Hamiltonian

$$H_S = \sum_n \hbar\omega_n |n\rangle\langle n| \qquad \rho_S = \sum_{n,m} \rho_{nm} |n\rangle\langle m|$$

The free evolution of the system is given by

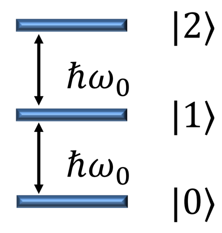
$$\rho_S(t) = e^{-iH_S t}\rho_S e^{iH_S t} = \sum_{n,m} \rho_{nm}|n\rangle\langle m| e^{-i\hbar(\omega_n - \omega_m)t}$$

We can decompose any state into *modes of coherence*[2] - 1-dimensional irreps of the U(1) time-translation group action.

$$\rho = \sum_\omega \rho^{(\omega)} \qquad \rho^{(\omega)} := \sum_{\substack{n,m \\ \omega_n - \omega_m = \omega}} \rho_{nm}|n\rangle\langle m|;$$

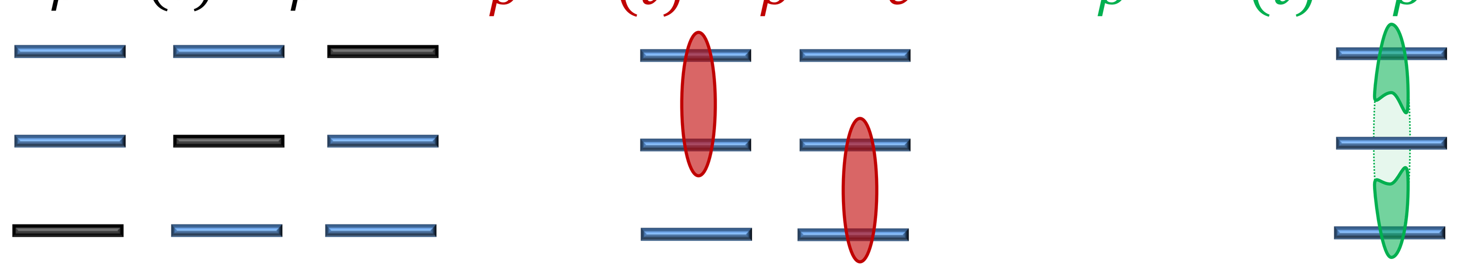
Example:

$$H_S = \sum_{n=0}^{2} n\hbar\omega_0 |n\rangle\langle n|$$



$$\rho = \begin{pmatrix} p_0 & c_{01} & c_{02} \\ c_{10} & p_1 & c_{12} \\ c_{20} & c_{21} & p_2 \end{pmatrix}$$

$$\rho^{(0)}(t) = \rho^{(0)} \qquad \rho^{(\omega_0)}(t) = \rho^{(\omega_0)}e^{-i\omega_0 t} \qquad \rho^{(2\omega_0)}(t) = \rho^{(2\omega_0)}e^{-i2\omega_0 t}$$



Decomposing thermal operations using modes:

Because of time-translation symmetry each mode in the initial state is **independently mapped** by a thermal operation to the corresponding mode of the final state:

$$\sigma = \mathcal{E}_T(\rho)$$

$$\sigma^{(\omega)} = \mathcal{E}_T\left(\rho^{(\omega)}\right)$$

Intensity of each mode has to **decrease**:

$$\left\|\sigma^{(\omega)}\right\| \leq \left\|\rho^{(\omega)}\right\|$$

4. Bounds on coherence transformation under thermal operations

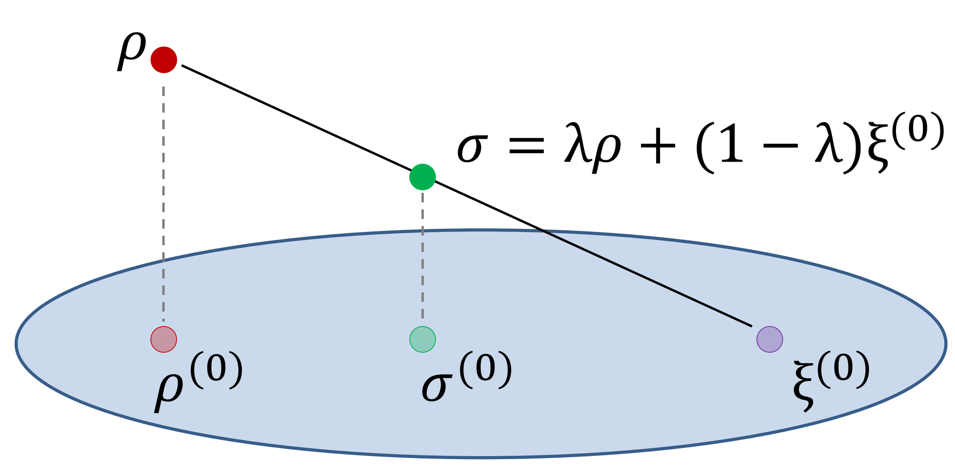
Upper bound for final coherence (based on transition probabilities for diagonal elements):

$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m}} |\rho_{cd}| \sqrt{p_{n|c}p_{m|d}}$$

$$p_{n|c} = \langle n|\mathcal{E}_T(|c\rangle\langle c|)|n\rangle$$

Guaranteed lower bound for final coherence (based on thermomajorization condition for incoherent states):

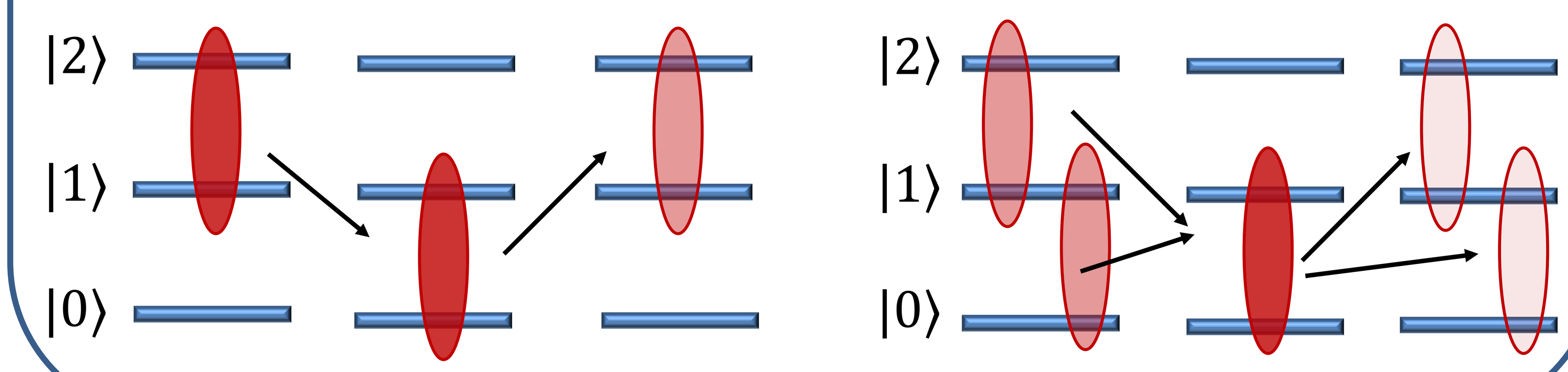
$$|\rho'_{nm}| \geq \lambda^* |\rho'_{nm}|$$



5. Irreversibility of coherence transfer

Using the fact that transition probabilities must preserve thermal state one arrives at:

$$|\rho'_{nm}| \leq \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c > \omega_n}} |\rho_{cd}| + \sum_{\substack{c,d \\ \omega_c - \omega_d = \omega_n - \omega_m \\ \omega_c \leq \omega_n}} |\rho_{cd}|\, e^{-\beta\hbar(\omega_n - \omega_c)}$$



6. Outlook

Recent results on *catalytic coherence*[3] show that coherence, unlike other quantum resources, does not have to degrade while being used to lift time-translation symmetry. This, however, requires investing work. The question that remains open is: can external coherence (reference) be used catalytically to extract work locked in the coherence of the system?

References:

1. M. Lostaglio, D. Jennings, and T. Rudolph, arXiv:1405.2188 (2014).
2. I. Marvian and R. W. Spekkens, Phys. Rev. A **90**, 062110 (2014).
3. J. Åberg, Phys. Rev. Lett. **113**, 150402 (2014).

For more details check:
M. Lostaglio, K. Korzekwa, D. Jennings, and T. Rudolph, arXiv:1410.4572 (2014).