Avoiding irreversibility: resonant conversion of quantum resources

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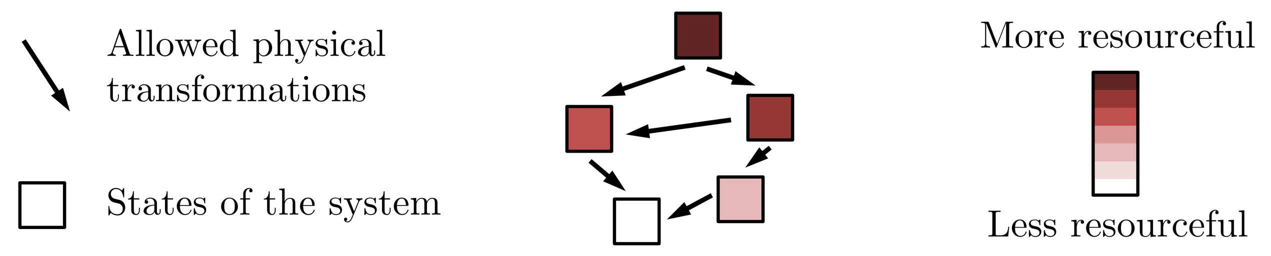
Background

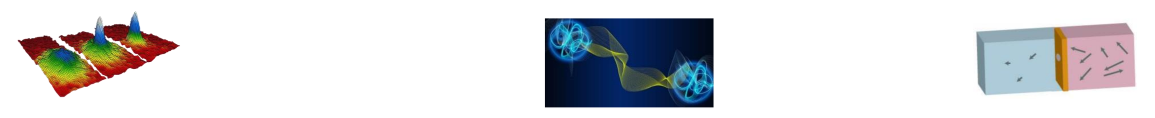
Quantum resources

Motivation: Quantum resources, such as entanglement and coherence, are crucial components of emerging quantum technologies.

Goal: To understand the potential and limitations of manipulating and interconverting quantum resources.

Framework: Resource-theoretic ordering of quantum states.

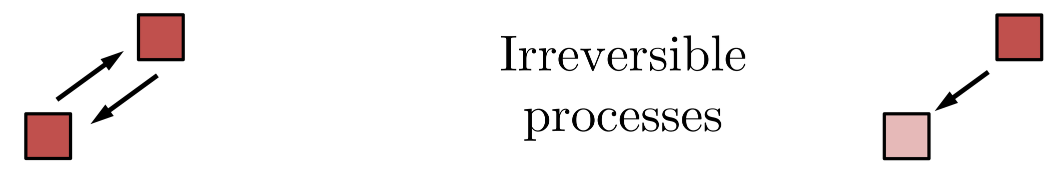




Coherence	Entanglement	Thermodynamics
Coherent	Non-local	Out-of-equilibrium
↓ Incoherent operations	↓ LOCC	↓ Thermal operations
Incoherent	Separable	Equilibrium

Interconversion regimes

Reversible processes | Irreversible processes



Regime		
Single-shot regime ($n=1$)	■ $\xrightarrow{\text{Yes/No?}}$ ■	All non-trivial transformations are irreversible
Asymptotic regime ($n\to\infty$)	n ■ $\xrightarrow{R_\infty=?}$ $R_\infty n$ ■	Reversibility in the asymptotic limit $R_\infty(\rho\to\sigma\to\rho)=1$
Beyond asymptotics (Large finite n)	n ■ $\xrightarrow{R_n=?}$ $R_n n$ ■	Quantifying irreversibility (dissipation of resources) $R_n(\rho\to\sigma\to\rho)\leq 1$

Asymptotic analysis not suitable for:

- Near-future quantum technologies (limited resources)
- Studying quantum effects beyond the thermodynamic limit

Optimal resource interconversion

Setting the scene

Technical goal: Find the optimal trade-off between the rate R_n and transformation error ϵ_n (infidelity between the final and target states).

Initial and final states represented by probability vectors $\boldsymbol{p}$ and $\boldsymbol{q}$

Resource theory	Initial state	Final state
Entanglement (pure bipartite states)	$\lvert\Psi\rangle=\sum_j\sqrt{p_j}\lvert\psi_j\rangle\otimes\lvert\psi'_j\rangle$	$\lvert\Phi\rangle=\sum_j\sqrt{q_j}\lvert\phi_j\rangle\otimes\lvert\phi'_j\rangle$
Coherence (pure states)	$\lvert\psi\rangle=\sum_j\sqrt{p_j}e^{i\alpha_j}\lvert j\rangle$	$\lvert\phi\rangle=\sum_j\sqrt{q_j}e^{i\beta_j}\lvert j\rangle$
Thermodynamics (energy incoherent states)	$\rho=\sum_j p_j\lvert E_j\rangle\langle E_j\rvert$	$\sigma=\sum_j q_j\lvert E_j\rangle\langle E_j\rvert$

Relevant information-theoretic quantities	
$H(\boldsymbol{p})=-\sum_j p_j\ln p_j$	$D(\boldsymbol{p}\lvert\lvert\boldsymbol{q})=\sum_j p_j\ln\frac{p_j}{q_j}$
$V(\boldsymbol{p})=\sum_j p_j[\ln p_j+H(\boldsymbol{p})]^2$	$V(\boldsymbol{p}\lvert\lvert\boldsymbol{q})=\sum_j p_j\left[\ln\frac{p_j}{q_j}-D(\boldsymbol{p}\lvert\lvert\boldsymbol{q})\right]^2$
$\gamma_j=e^{-\frac{E_j}{k_BT}}/Z$	$Z=\sum_j e^{-\frac{E_j}{k_BT}}$

Results

Vanishing error: $\epsilon_n=\exp(-nt_n^2)$, $\quad t_n\sim n^{-\alpha}$ with $\alpha\in(0,1/2)$

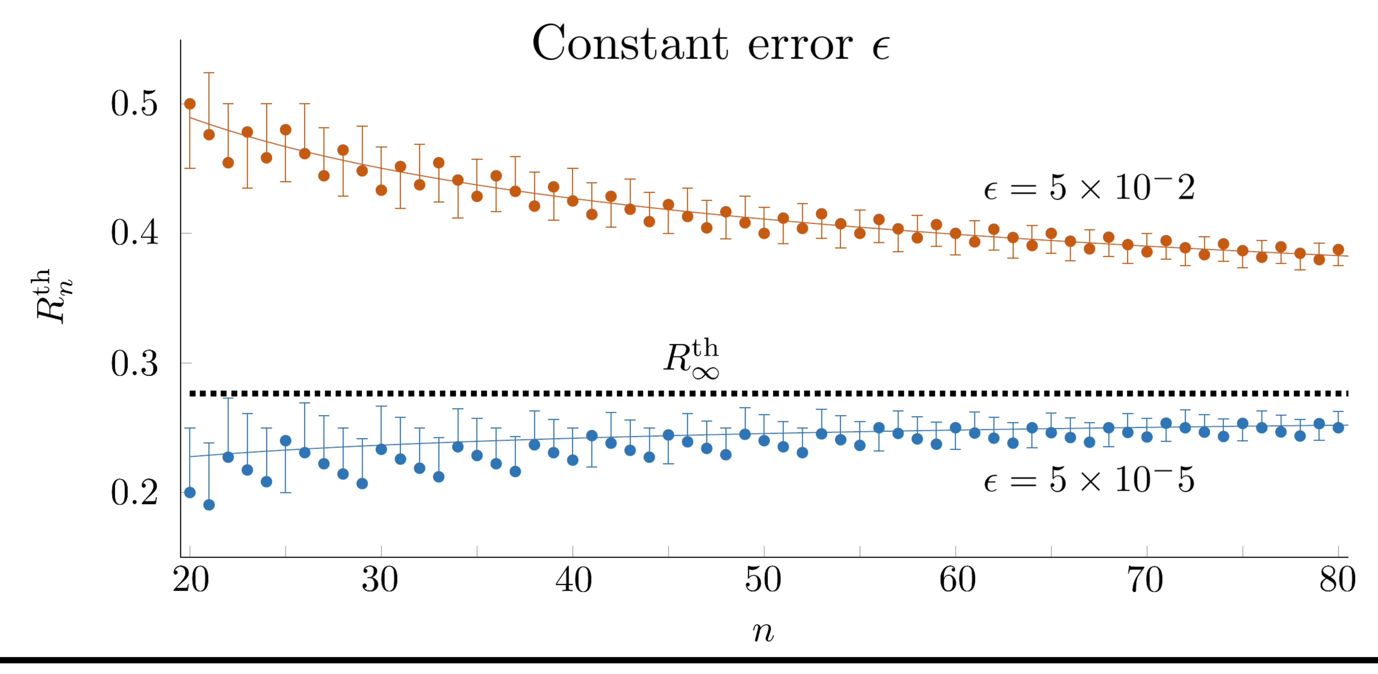
$$R_n^{\mathrm{ent}}(\epsilon_n)\simeq R_\infty^{\mathrm{ent}}-\sqrt{\frac{2V(\boldsymbol{p})}{H(\boldsymbol{q})^2}}\left|1-1/\sqrt{\nu^{\mathrm{ent}}}\right|t_n$$

$$R_n^{\mathrm{th}}(\epsilon_n)\simeq R_\infty^{\mathrm{th}}-\sqrt{\frac{2V(\boldsymbol{p}\lvert\lvert\boldsymbol{\gamma})}{D(\boldsymbol{q}\lvert\lvert\boldsymbol{\gamma})^2}}\left|1-1/\sqrt{\nu^{\mathrm{th}}}\right|t_n$$

$$R_\infty^{\mathrm{ent}}=\frac{H(\boldsymbol{p})}{H(\boldsymbol{q})}\qquad \nu^{\mathrm{ent}}=\frac{V(\boldsymbol{p})/H(\boldsymbol{p})}{V(\boldsymbol{q})/H(\boldsymbol{q})}$$

$$R_\infty^{\mathrm{th}}=\frac{D(\boldsymbol{p}\lvert\lvert\boldsymbol{\gamma})}{D(\boldsymbol{q}\lvert\lvert\boldsymbol{\gamma})}\qquad \nu^{\mathrm{th}}=\frac{V(\boldsymbol{p}\lvert\lvert\boldsymbol{\gamma})/D(\boldsymbol{p}\lvert\lvert\boldsymbol{\gamma})}{V(\boldsymbol{q}\lvert\lvert\boldsymbol{\gamma})/D(\boldsymbol{q}\lvert\lvert\boldsymbol{\gamma})}$$

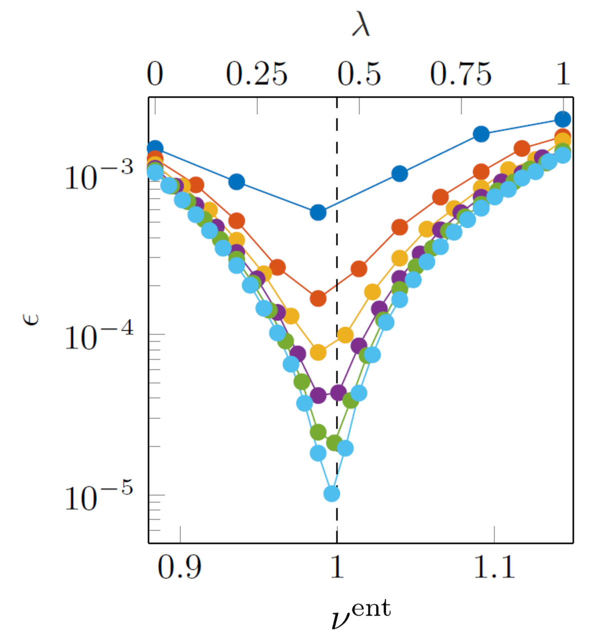
R_∞ - Asymptotic rate, $\quad\nu$ - Irreversibility parameter



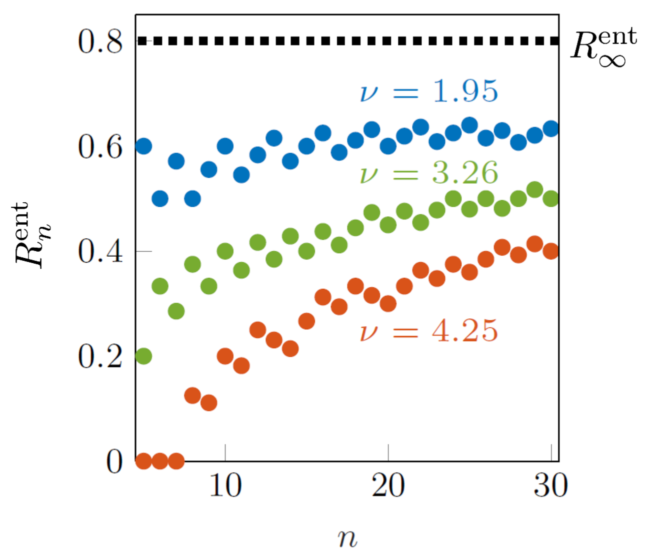
Resource resonance

Entanglement transformations

Tuning resources to resonance



Approaching reversibility



$\lvert\Psi_1\rangle^{\otimes\lambda n}\otimes\lvert\Psi_2\rangle^{\otimes(1-\lambda)n}\xrightarrow{\text{LOCC}}\lvert\Phi\rangle^{\otimes n}$

$\lvert\Psi_1\rangle$, $\lvert\Psi_2\rangle$: same $H(\boldsymbol{p})$, different $V(\boldsymbol{p})$

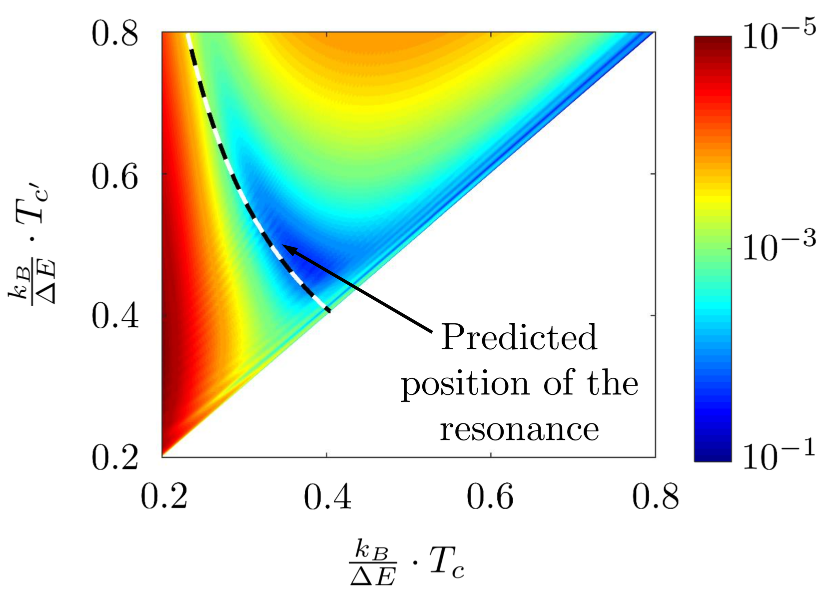
Different colours: $n\in\{5,10,\dots,30\}$

$\lvert\Psi_i\rangle^{\otimes n}\xrightarrow{\text{LOCC}}\lvert\Phi\rangle^{\otimes R_n^{\mathrm{ent}}n}$

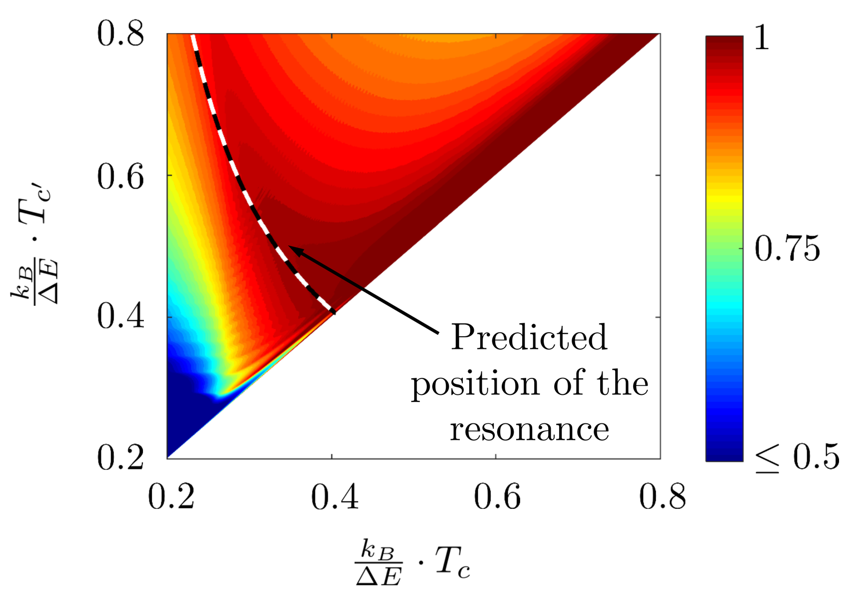
Constraint: $\epsilon<0.01$

Thermodynamic work extraction

Infidelity in work extraction with 95% Carnot efficiency



Optimal fraction of Carnot efficiency with quality $\epsilon<0.001$



Working body: $n=200$ non-interacting qubits with energy gap ΔE

Initial temperature T_c, final temperature $T_{c'}$

Heat bath: $T_h=10\Delta E/k_B$