Beyond the thermodynamic limit

Finite-size corrections to state interconversion rates

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Outline

1. Background and motivation
2. Framework and statement of the problem
3. Result 1: Small deviation analysis
4. Result 2: Moderate deviation analysis
5. Outlook

Background and motivation

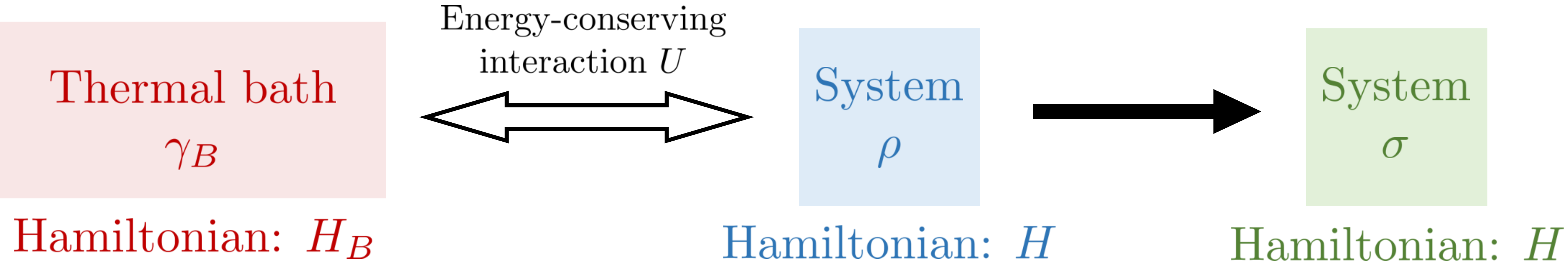
Standard thermodynamics	Our work	Quantum thermodynamics
• Wide applicability	• Intermediate regime	• Quantum regime
• Statistical nature	• Mixed nature	• Information-theoretic nature
• Thermodynamic limit	• Large but finite number of particles	• Single-shot processes
• Reversible cycles	• Irreversibility?	• Inherent irreversibility

Outline

Framework: Resource theory

Free thermodynamic transformations modelled by **thermal operations:**

$$\mathcal{E}^{T}(\cdot) = \mathrm{Tr}_B\left(U\left(\cdot \otimes \gamma_B\right)U^{\dagger}\right) \quad \text{with} \quad [U, H + H_B] = 0$$



Gibbs state γ of the system at temperature T: $\quad \gamma = e^{-\frac{H}{T}}/\mathcal{Z}, \quad \mathcal{Z} = \mathrm{Tr}\left(e^{-\frac{H}{T}}\right)$

Note: all results with units such that $k_B = 1$.

Framework: State interconversion

Setting: Initial state ρ, target state σ, background temperature T

General interconversion problem: Does there exist $\mathcal{E}^T$ such that $\mathcal{E}^T(\rho) = \sigma$?

Studied interconversion problem: Does there exist $\mathcal{E}^T$ such that $\mathcal{E}^T\left(\rho^{\otimes n}\right) \approx_\epsilon \sigma^{\otimes R_n n}$?

Optimal rate R_n for error ϵ?

Note: $\sigma \approx_\epsilon \tilde{\sigma}$ means $1 - F(\sigma, \tilde{\sigma}) \leq \epsilon$ with fidelity F

Restrictions:

Focus on many copies (large but finite n) and *energy-incoherent* states:

$$[\rho, H] = [\sigma, H] = 0 \quad \Longrightarrow \quad \text{states represented by: } \boldsymbol{p} = \mathrm{eig}(\rho), \ \boldsymbol{q} = \mathrm{eig}(\sigma).$$

$$[\gamma, H] = 0 \quad \Longrightarrow \quad \text{thermal state represented by: } \boldsymbol{\gamma} = \mathrm{eig}(\gamma)$$

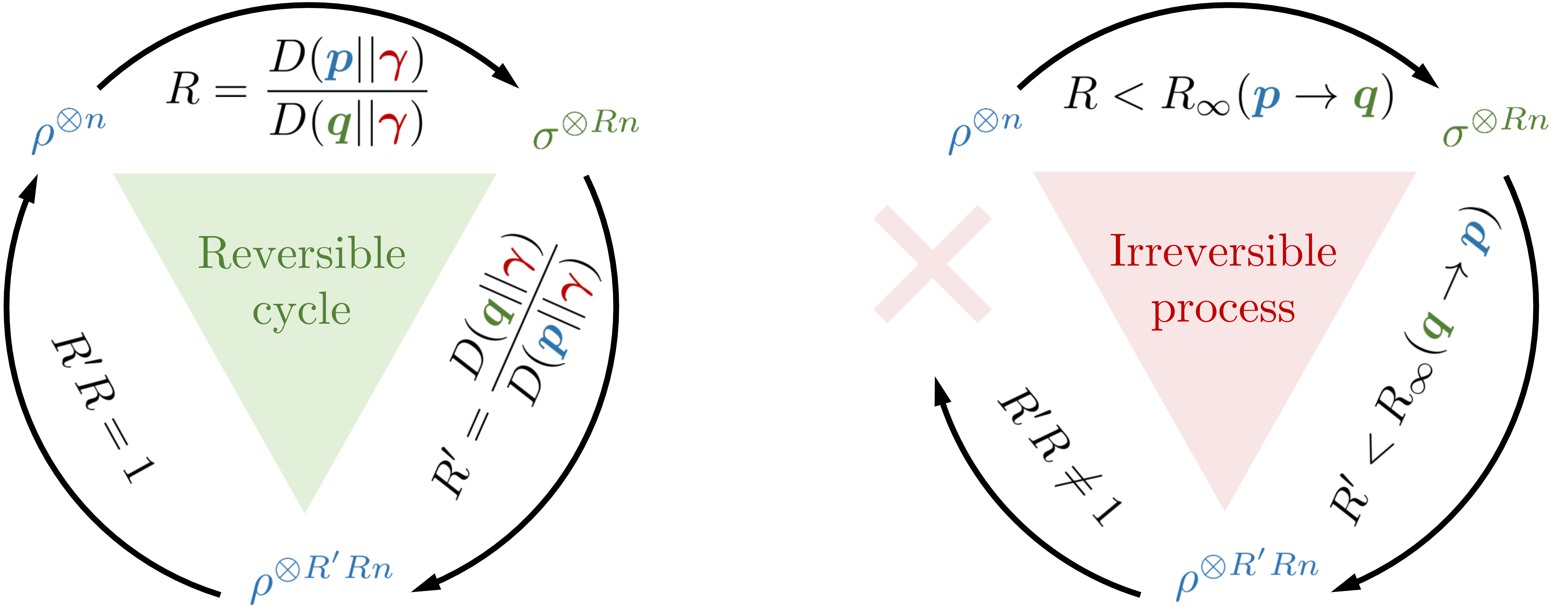
Framework: Relevant notions

	Expression	Interpretation
Relative entropy	$D(\boldsymbol{p}\|\boldsymbol{\gamma}) := \sum_{i=1}^{d} p_i \log \frac{p_i}{\gamma_i}$	$\frac{1}{T}[\underbrace{\langle E\rangle_{\boldsymbol{p}} - TH(\boldsymbol{p})}_{\substack{\text{Free energy} \\ F=U-TS}} - \underbrace{(-T\log \mathcal{Z})}_{\text{Free energy of } \gamma}]$
Relative entropy variance	$V(\boldsymbol{p}\|\boldsymbol{\gamma}) := \sum_{i=1}^{d} p_i \left(\log \frac{p_i}{\gamma_i} - D(\boldsymbol{p}\|\boldsymbol{\gamma})\right)^2$	$V(\boldsymbol{\gamma}'\|\boldsymbol{\gamma}) = \underbrace{\frac{\partial \langle E\rangle_{\boldsymbol{\gamma}'}}{\partial T'}}_{\substack{\text{Specific heat} \\ \text{capacity}}} \cdot \underbrace{\left(1-\frac{T'}{T}\right)^2}_{\text{Carnot factor}}$

Framework: Asymptotics & reversibility

Asymptotic rate: $R_\infty(\boldsymbol{p} \to \boldsymbol{q}) = \dfrac{D(\boldsymbol{p}\|\boldsymbol{\gamma})}{D(\boldsymbol{q}\|\boldsymbol{\gamma})}$

Finite n: $R_n = R_\infty - f(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\gamma}, n, \epsilon)$



*K. Ito, W. Kumagai, M. Hayashi, Phys. Rev. A **92**, 052308 (2015).

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   a. Statement
   b. Numerical verification
   c. Applications
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Result 1: Small deviation analysis

Optimal conversion rate R_n with constant error ϵ:

$$R_n(\epsilon) \simeq R_\infty + \sqrt{\frac{V(\boldsymbol{p}\|\boldsymbol{\gamma})}{D(\boldsymbol{q}\|\boldsymbol{\gamma})^2}}\,\frac{Z_\nu^{-1}(\epsilon)}{\sqrt{n}}$$

Irreversibility parameter:
$$\nu = \frac{V(\boldsymbol{q}\|\boldsymbol{\gamma})/D(\boldsymbol{q}\|\boldsymbol{\gamma})}{V(\boldsymbol{p}\|\boldsymbol{\gamma})/D(\boldsymbol{p}\|\boldsymbol{\gamma})}$$

Rayleigh-normal distribution Z_ν introduced in [*]

*W. Kumagai, M. Hayashi, IEEE Trans. Inf. Theory 63, 1829–1857 (2017).

Result 1: Numerical verification

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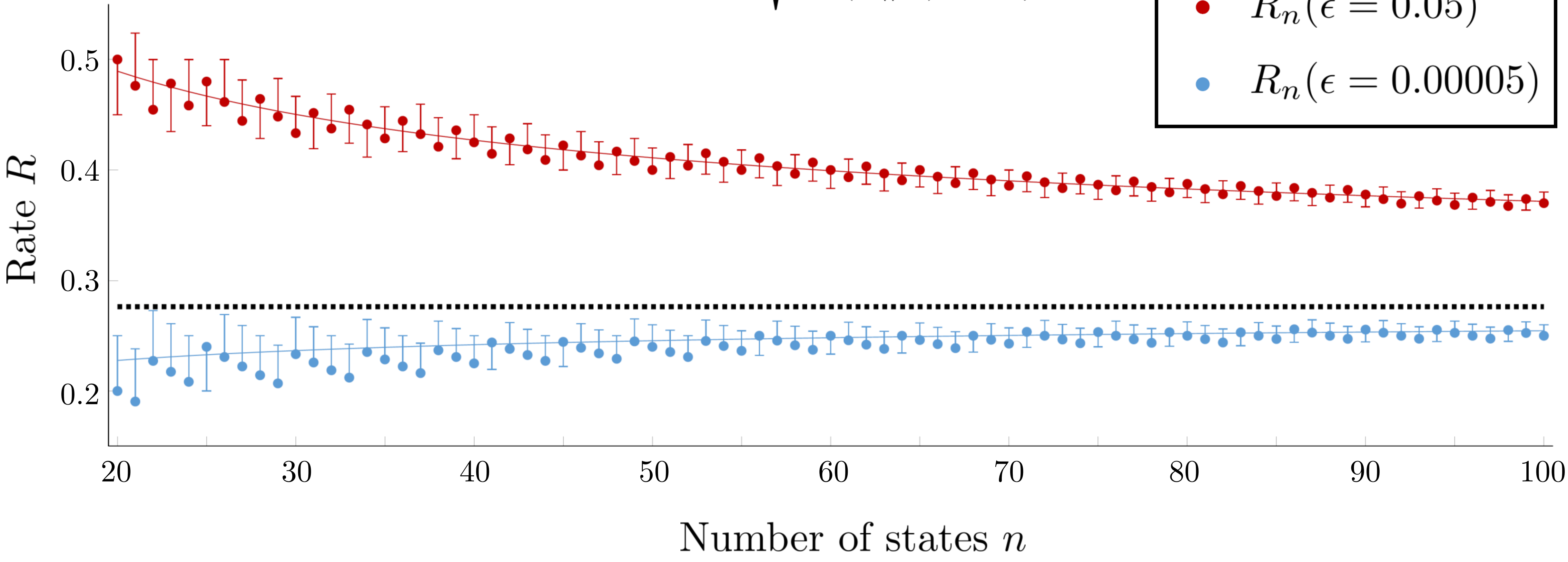
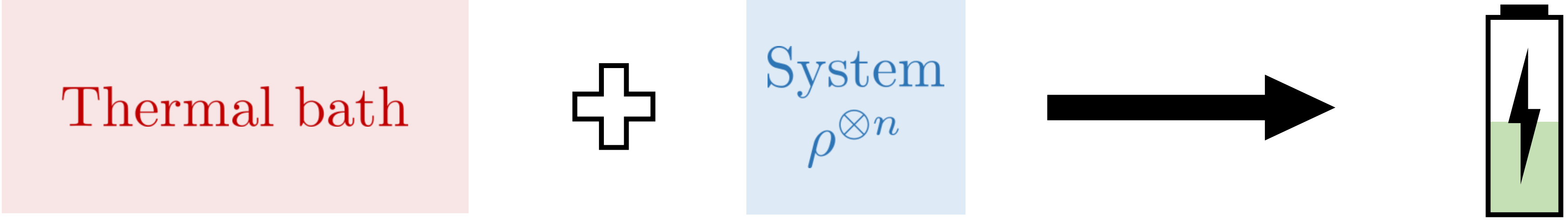


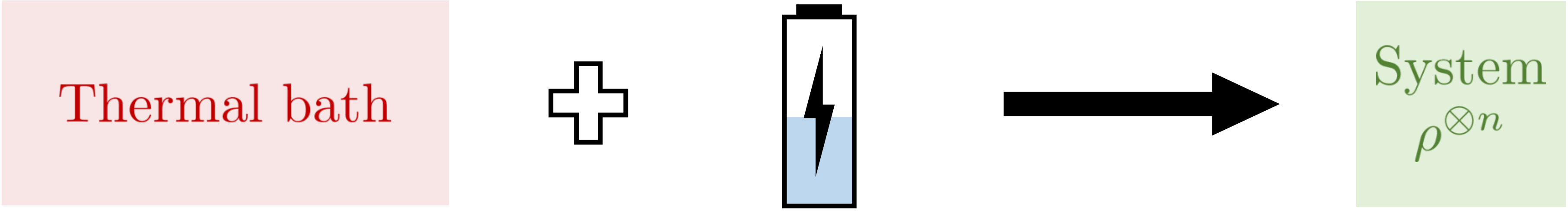
Result 1: Applications

Work distillation process:



Distillable work: $n \cdot W_D$

Work dilution process:

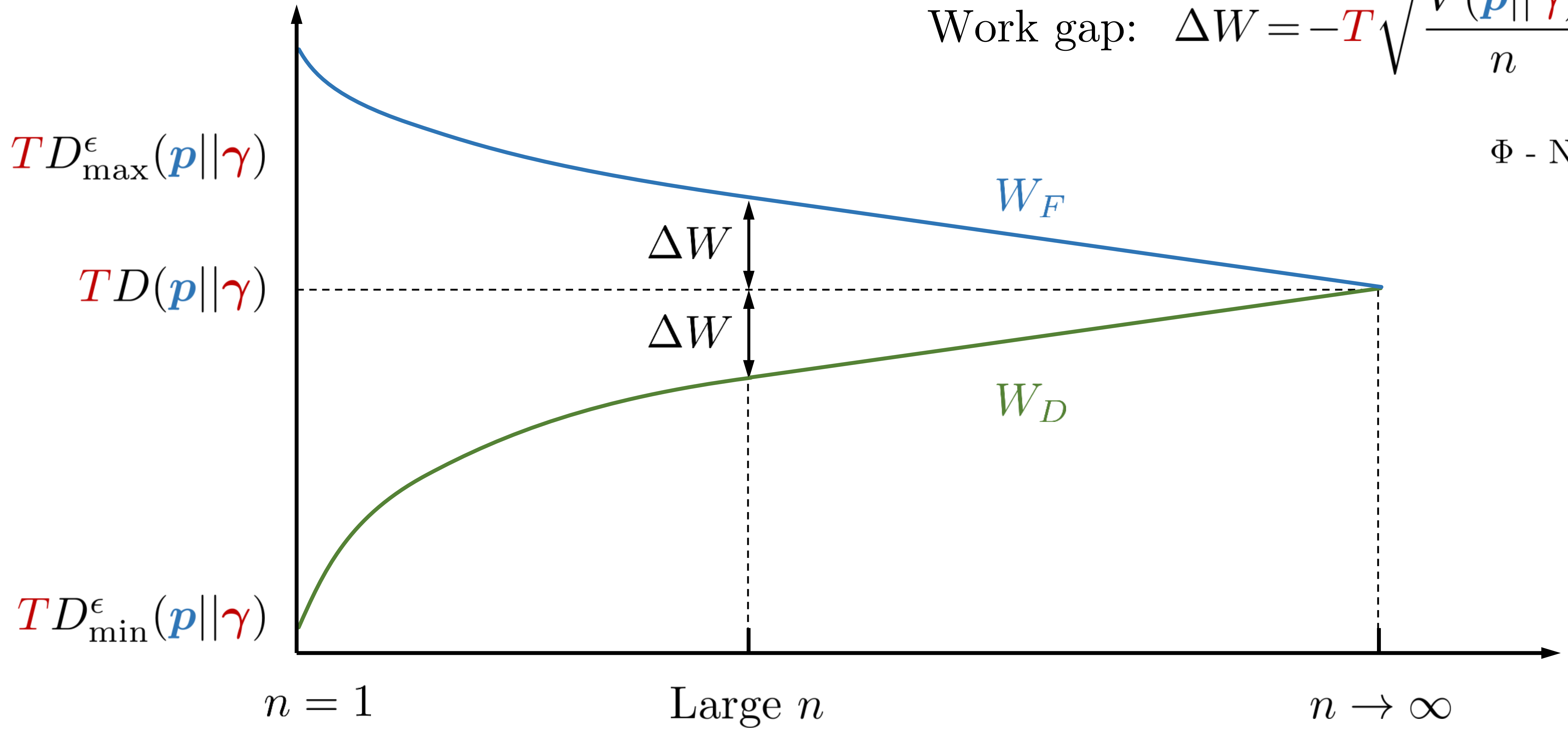


Work of formation: $n \cdot W_F$

Result 1: Applications

Work gap: $\Delta W = -T\sqrt{\frac{V(\boldsymbol{p}||\boldsymbol{\gamma})}{n}}\Phi^{-1}(\epsilon)$

Φ - Normal cdf



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Result 2: Moderate deviation analysis

Optimal conversion rate R_n with vanishing error $\epsilon = e^{-n^\alpha}$ and $\alpha \in (0,1)$:

$$R_n(\epsilon) \simeq R_\infty - \sqrt{\frac{V(\boldsymbol{p}\|\gamma)}{D(\boldsymbol{q}\|\gamma)^2}}\,\frac{\left|\sqrt{1/\nu}-1\right|}{\sqrt{n^{1-\alpha}}}$$

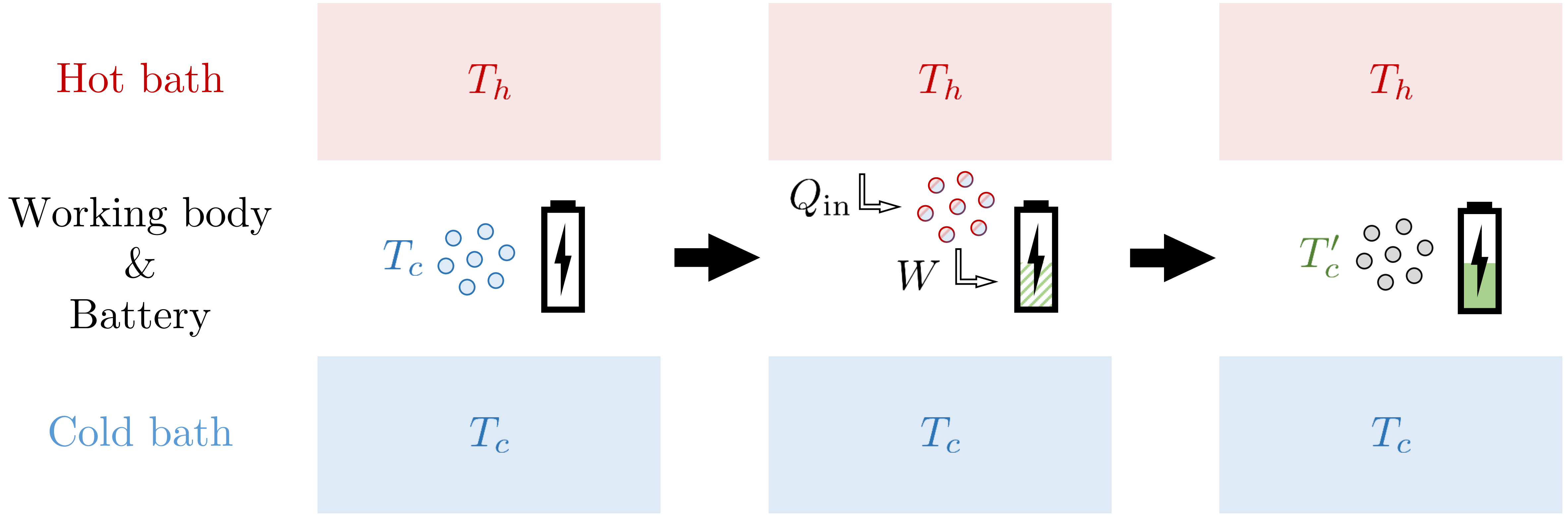
Also analogous result for entanglement and coherence transformations:

$$R_n(\epsilon) \simeq R_\infty - \sqrt{\frac{V(\boldsymbol{p})}{H(\boldsymbol{q})^2}}\,\frac{\left|\sqrt{1/\nu}-1\right|}{\sqrt{n^{1-\alpha}}}$$

$H(\boldsymbol{p})$ - Shannon entropy, $V(\boldsymbol{p})$ - entropy variance

Result 2: Applications

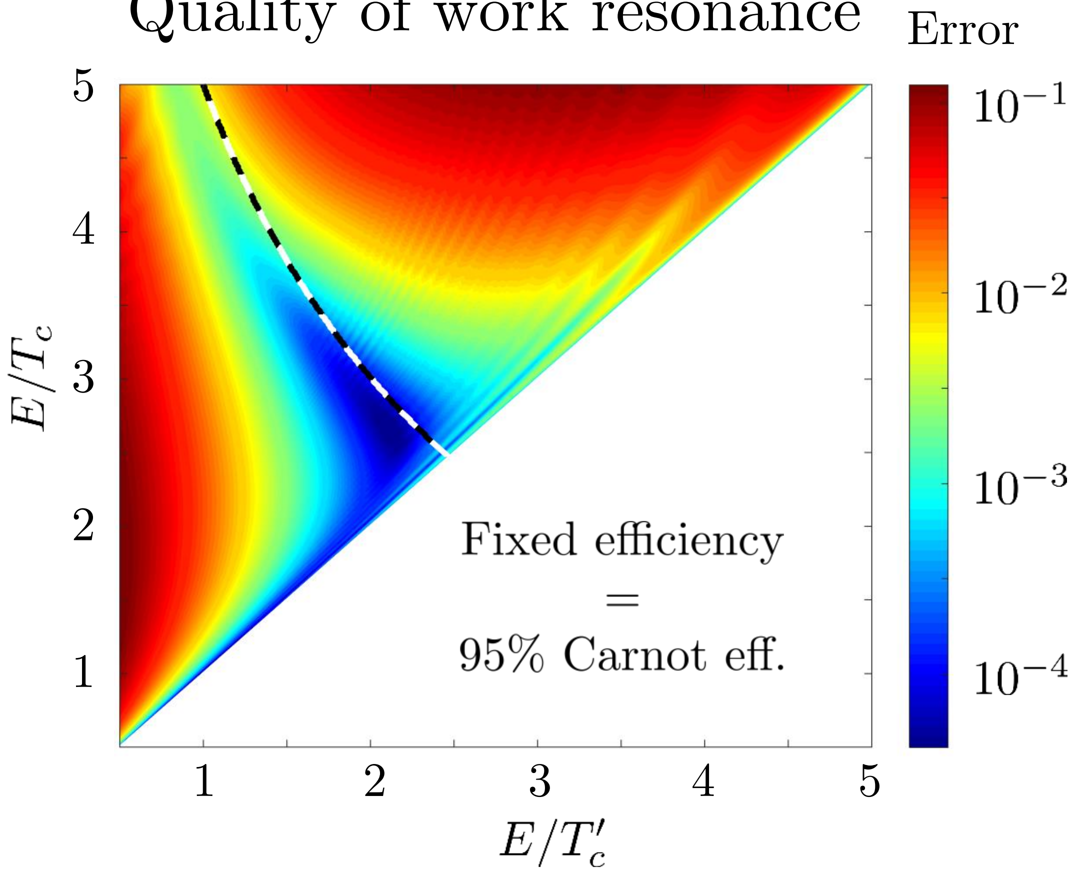
Heat engine with a finite-size working body:



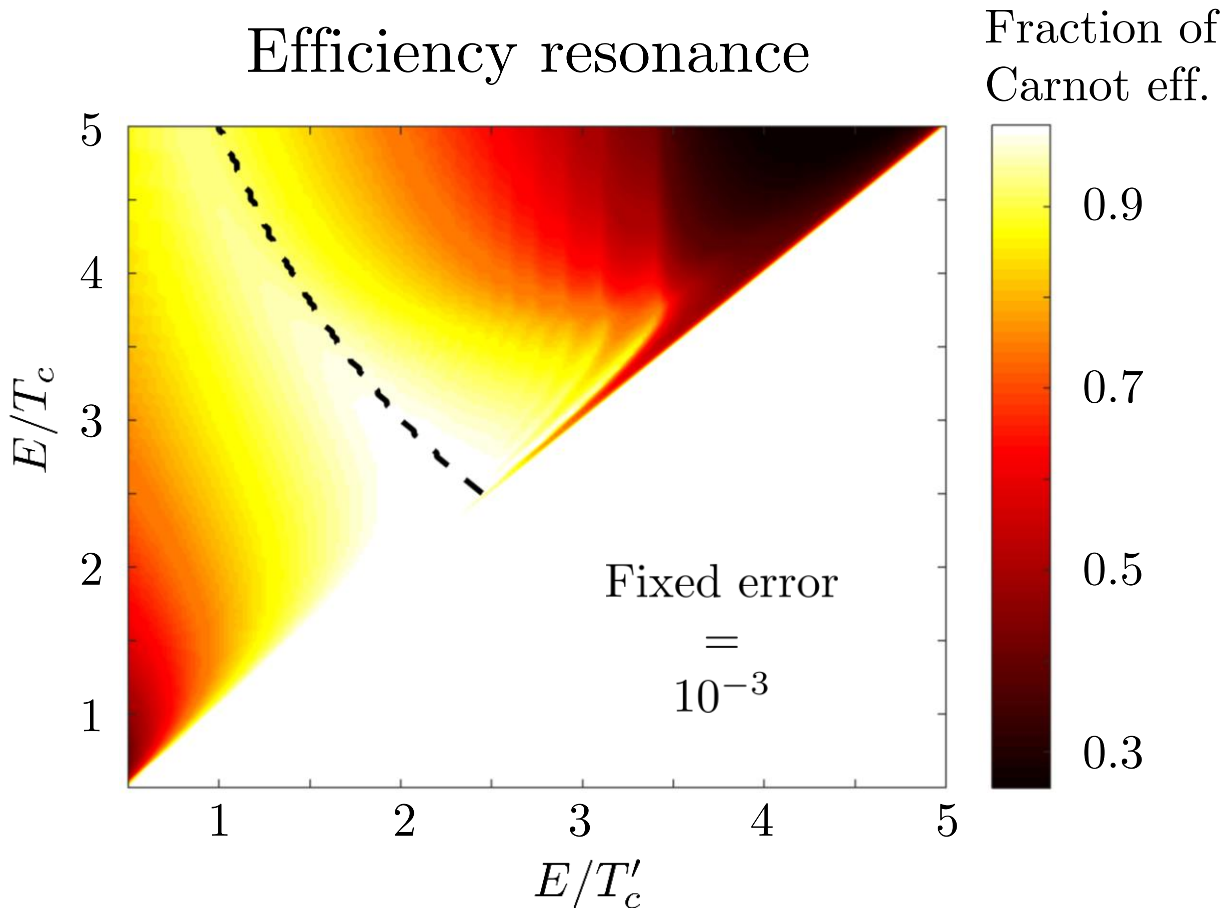
Result 2: Numerical verification

Working body: $n = 200$ qubits, energy gap E
Background (hot) bath: $T_h = 10E$

Quality of work resonance



Efficiency resonance



Result 2: Numerical verification 2

Tuning resources to resonance

2 available initial states: $|\Psi_1\rangle$ and $|\Psi_2\rangle$

1 target state: $|\Phi\rangle$

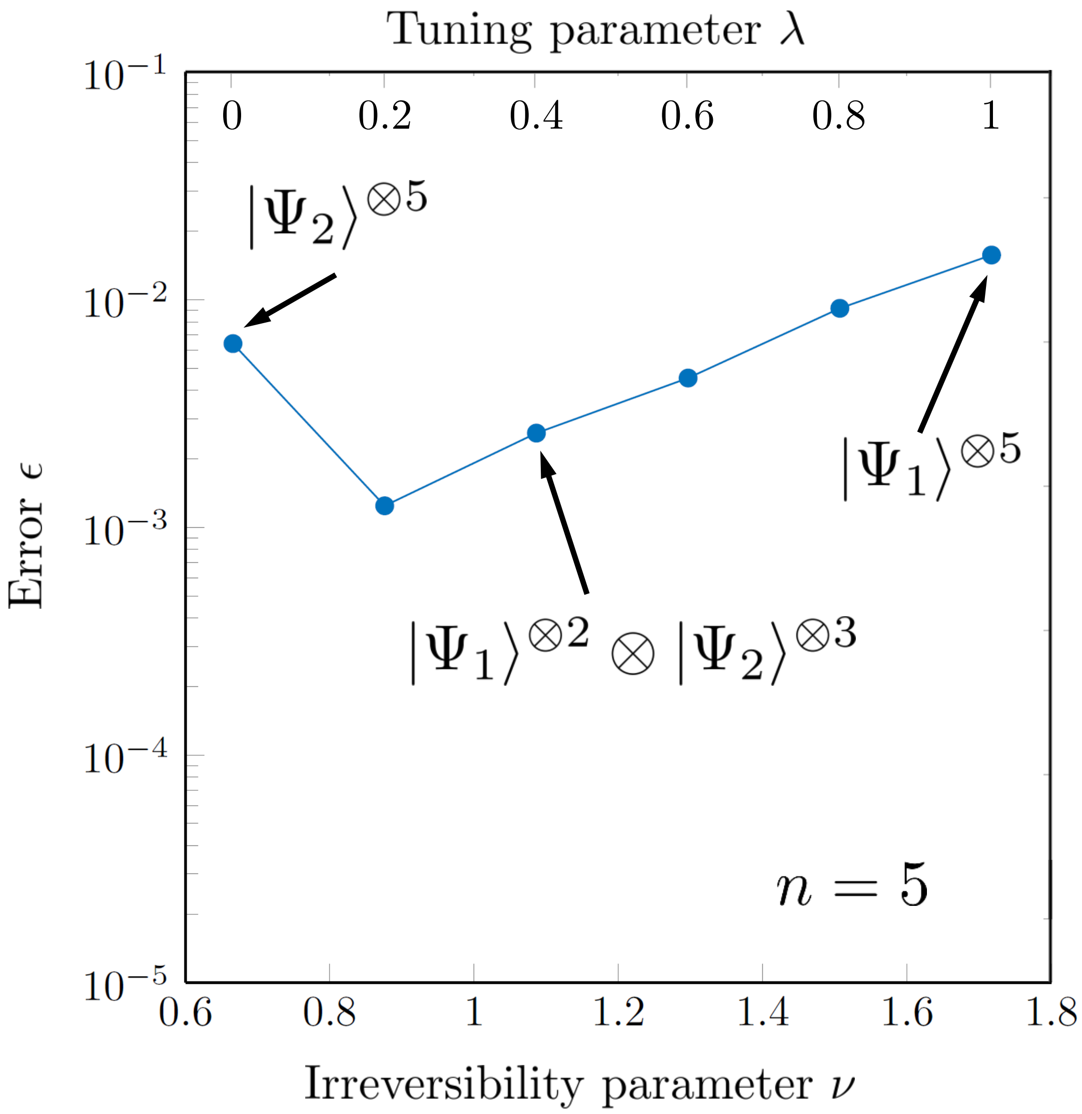
Asymptotically same resource content:

$$|\Psi_1\rangle^{\otimes n} \to |\Phi\rangle^{\otimes n}, \qquad |\Psi_2\rangle^{\otimes n} \to |\Phi\rangle^{\otimes n}$$

Hence, for all $\lambda \in [0,1]$:

$$|\Psi_1\rangle^{\otimes \lambda n} \otimes |\Psi_2\rangle^{\otimes (1-\lambda) n} \to |\Phi\rangle^{\otimes n}$$

What about finite n?



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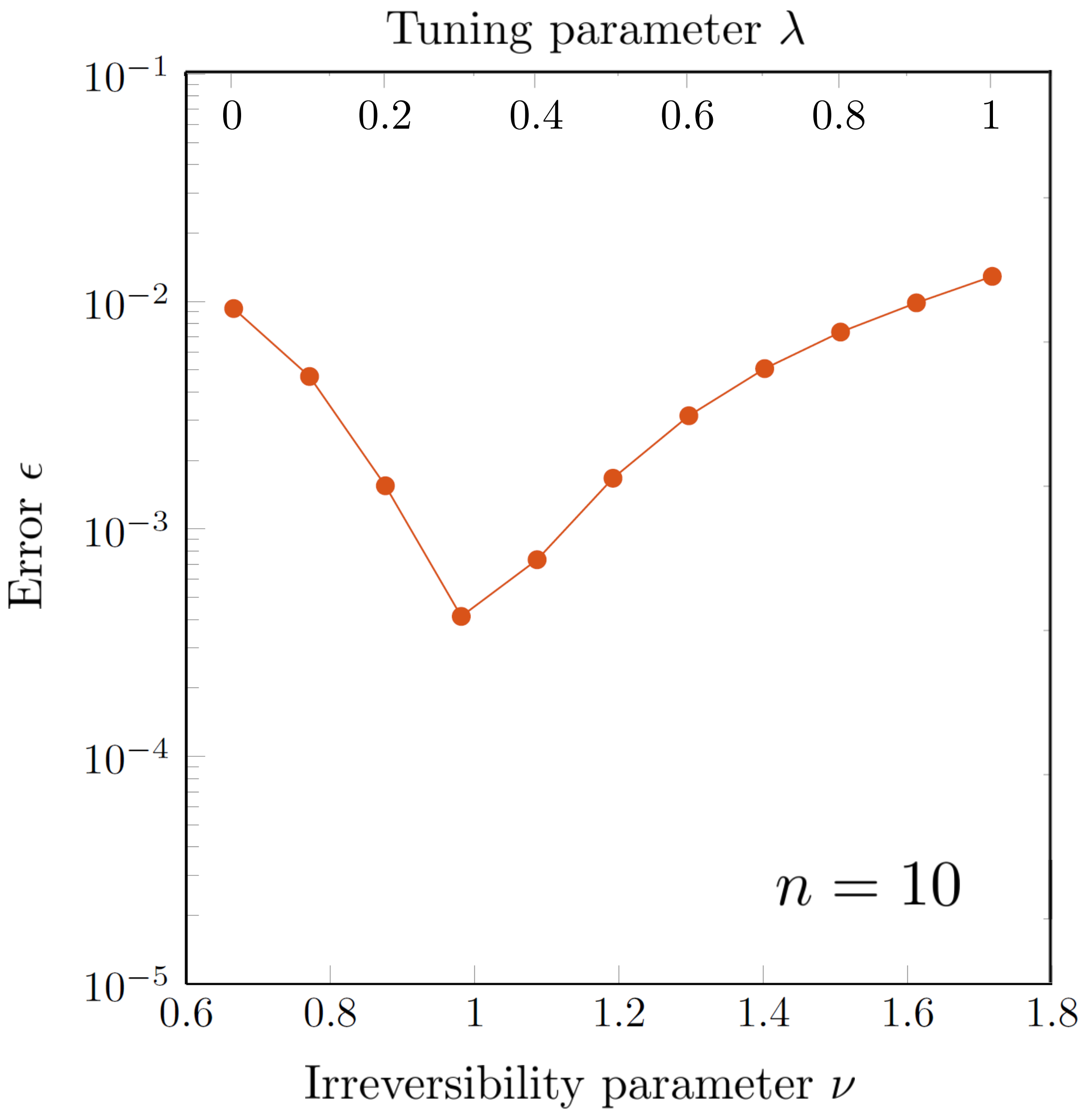

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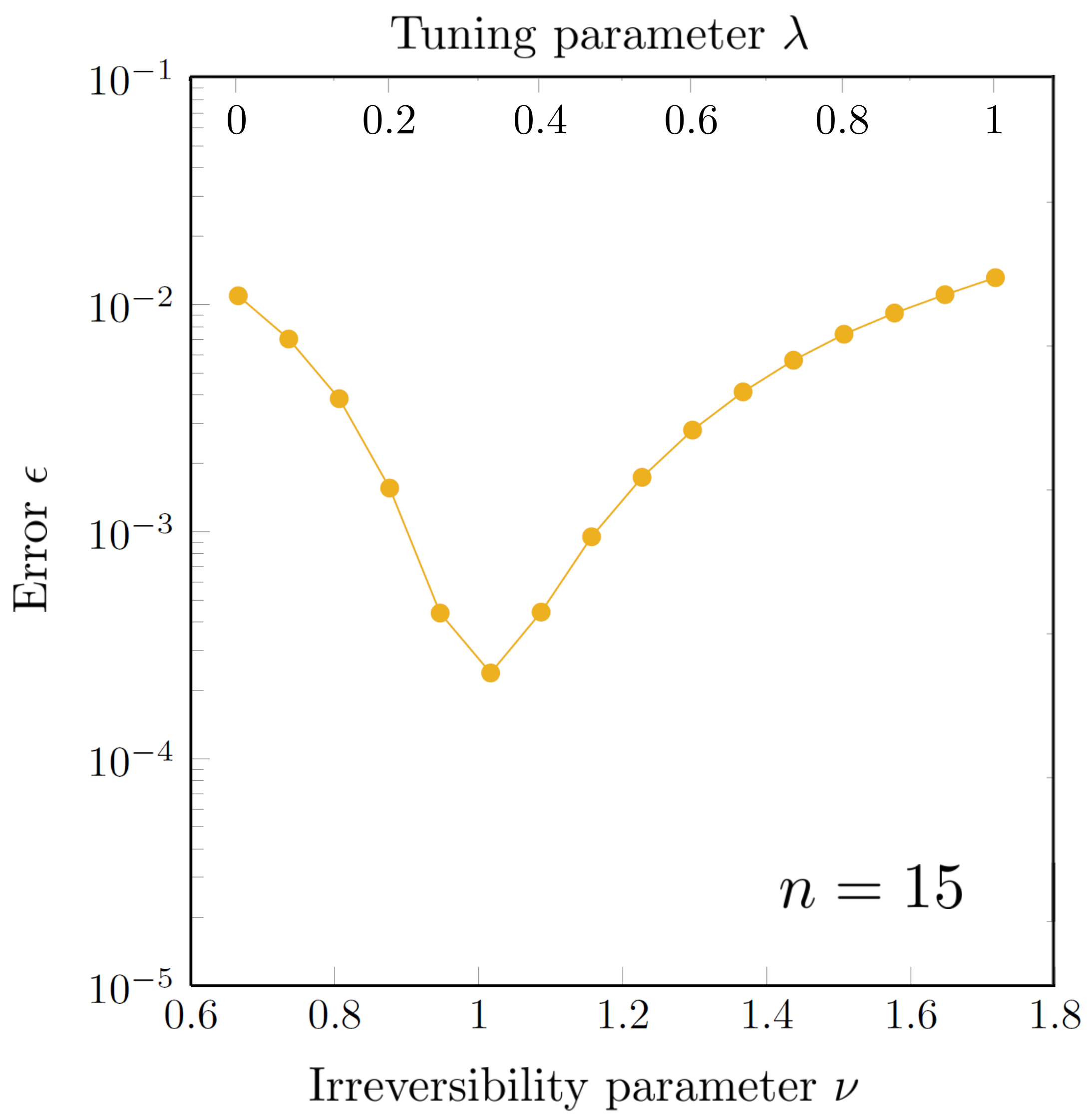

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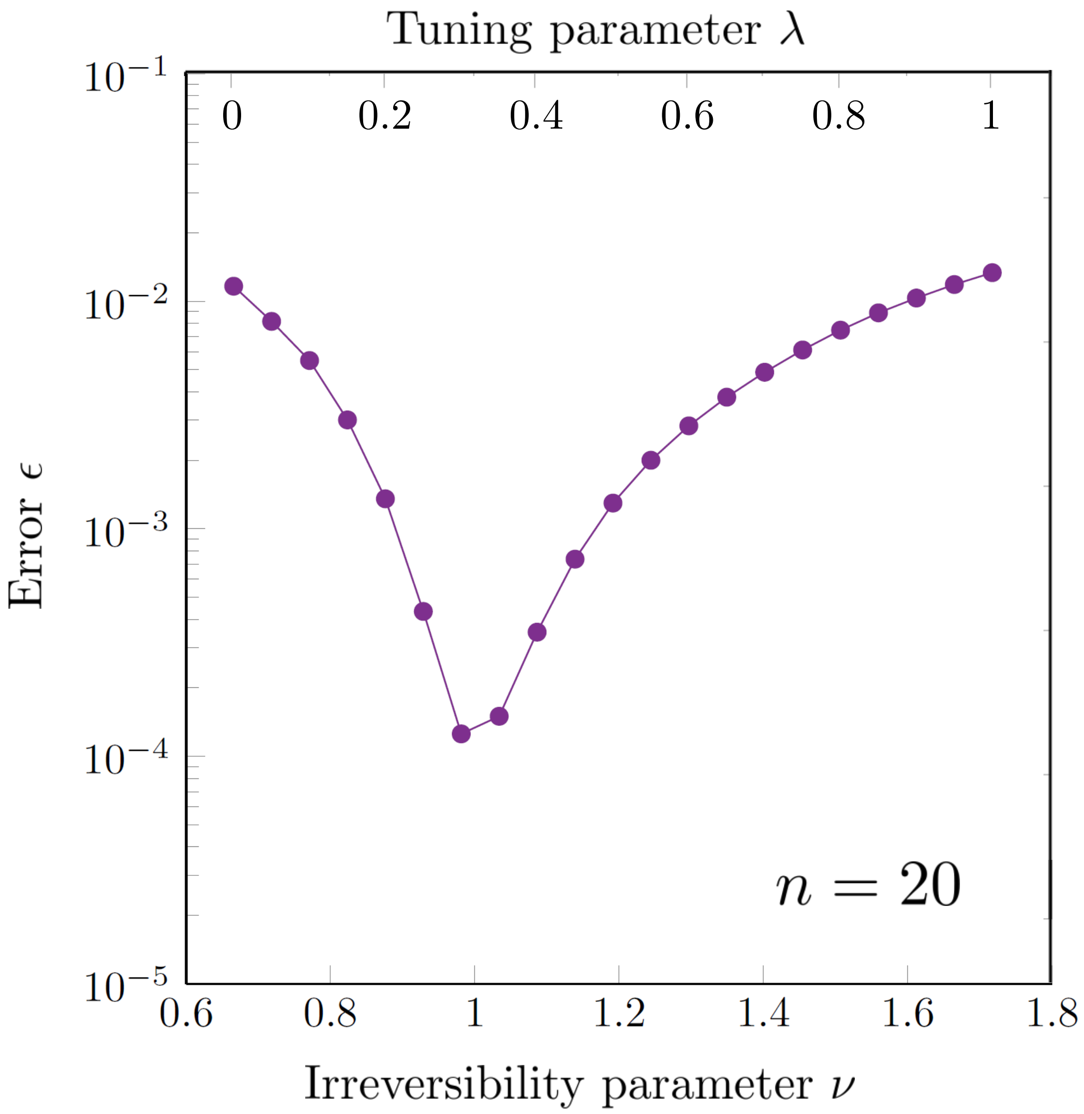

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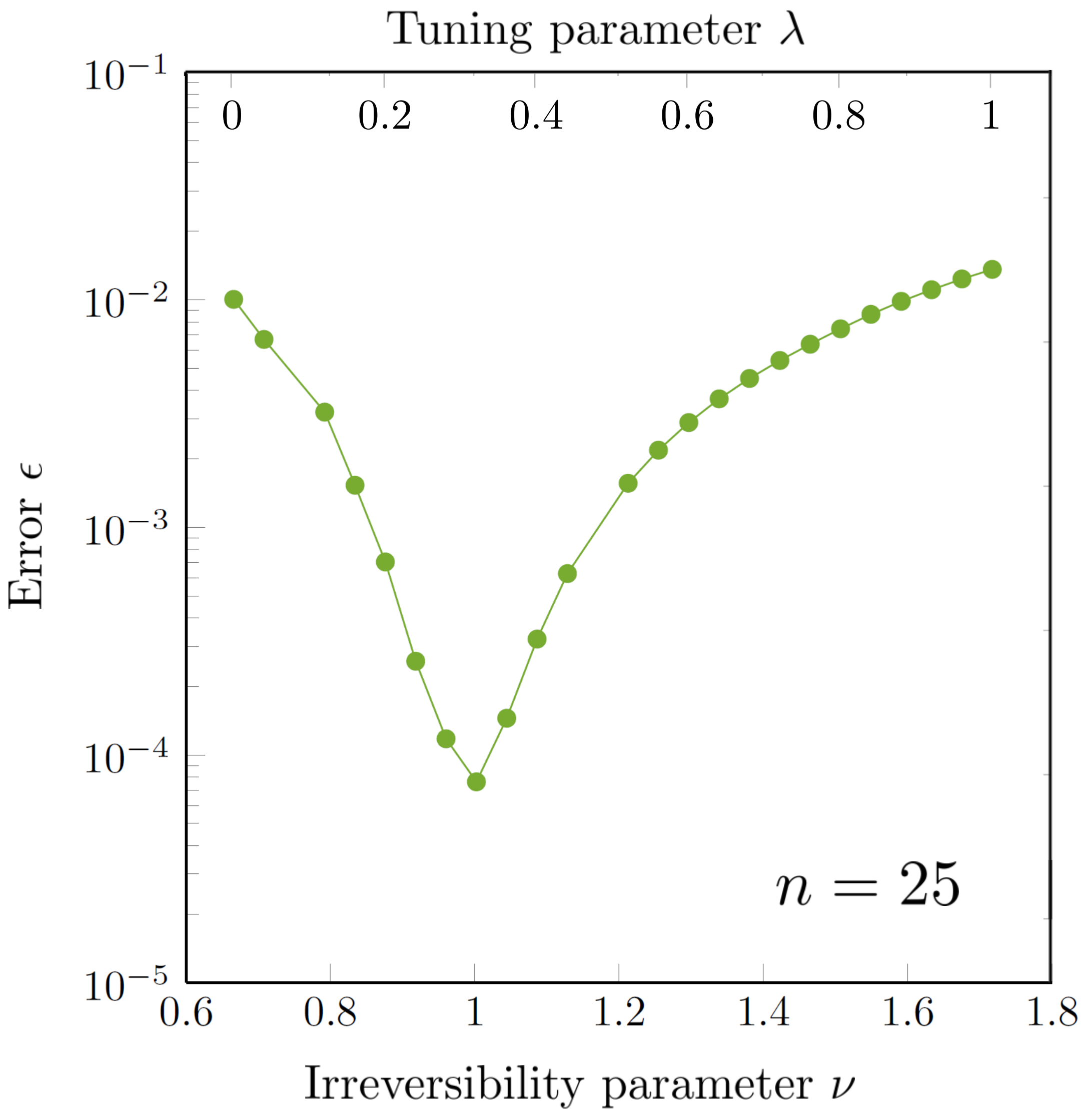

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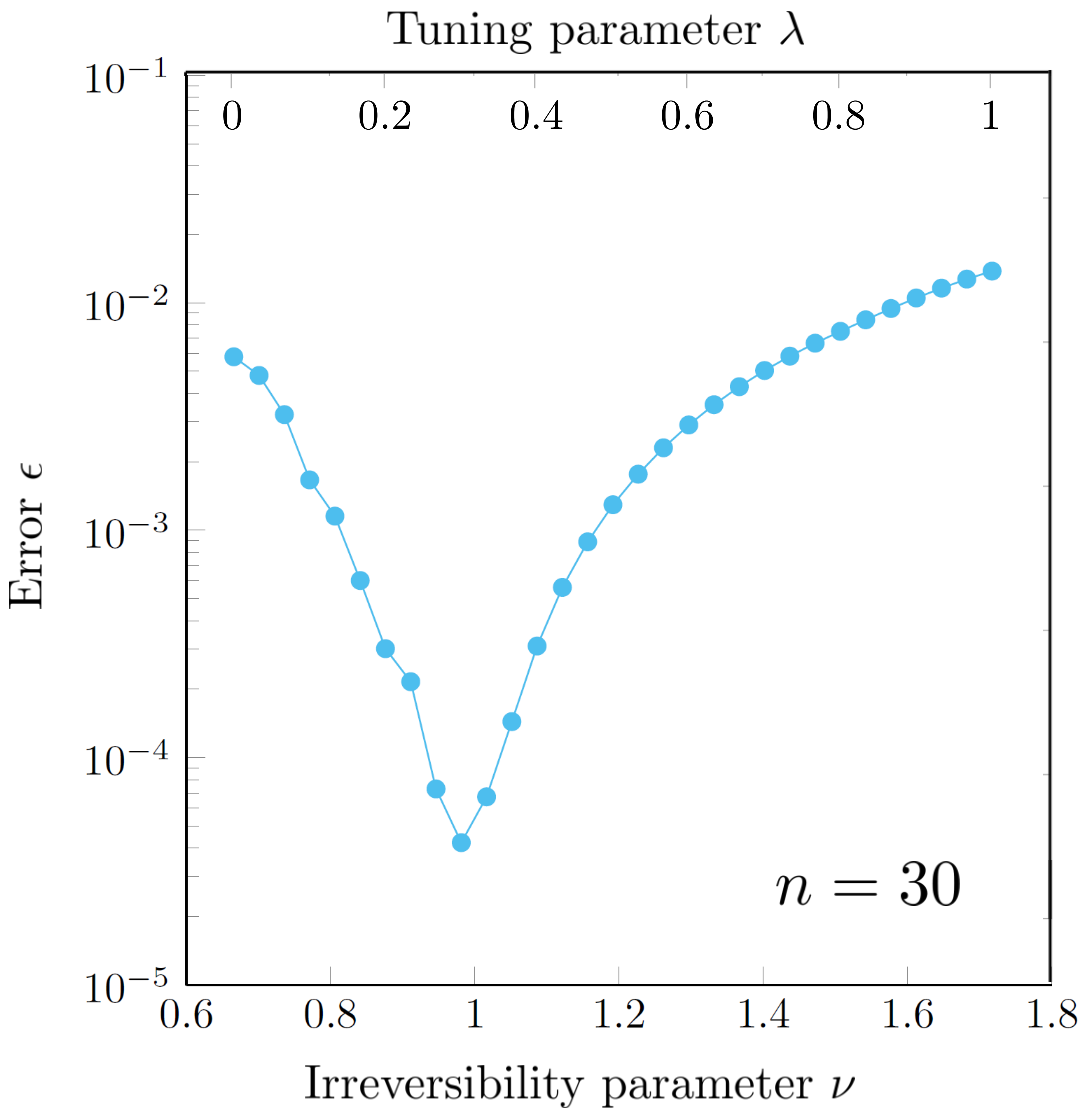

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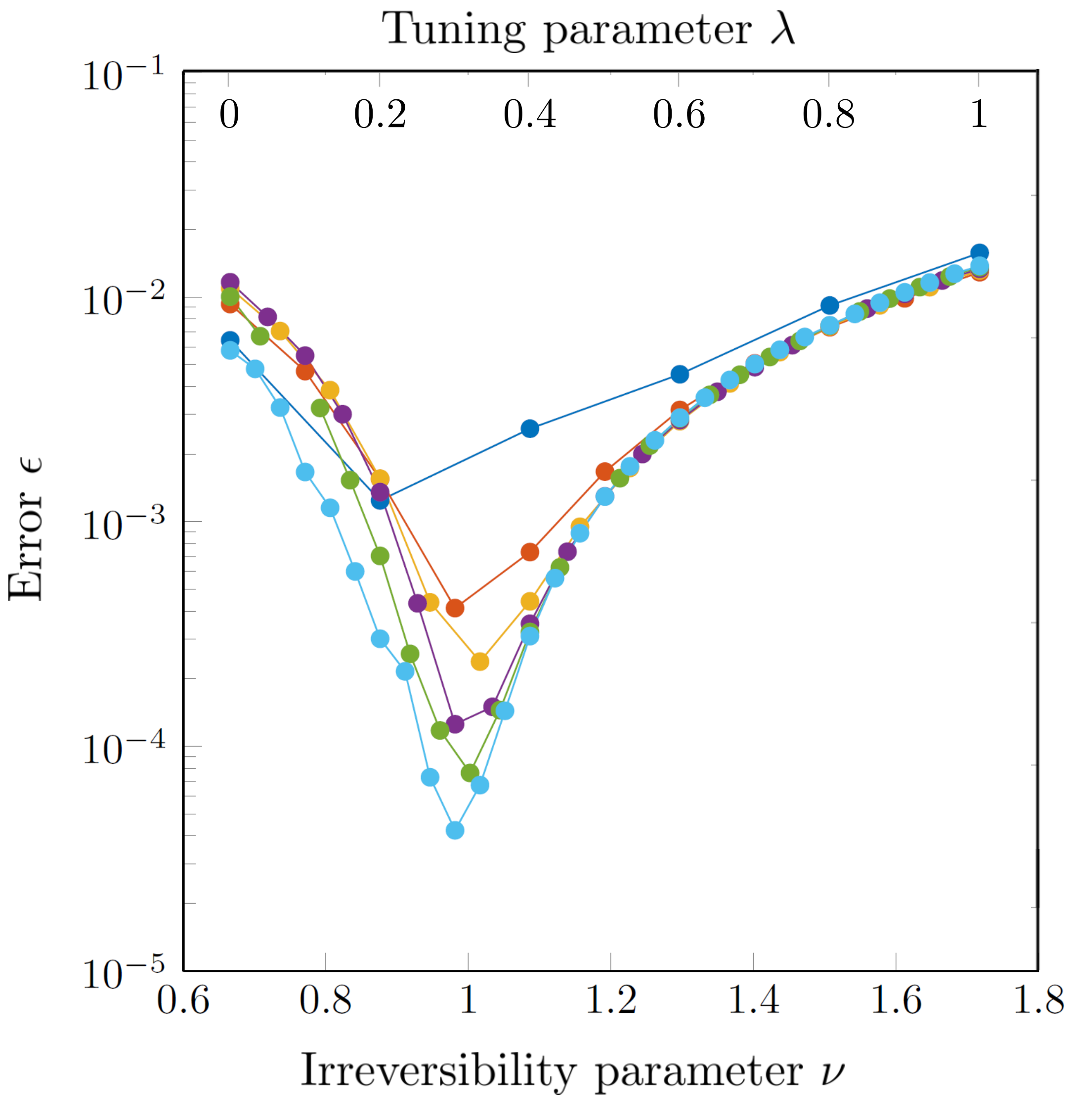

Outlook

- Apply the results to other thermodynamic problems involving finite-size baths (Landauer's erasure, fluctuation theorems, the third law of thermodynamics)
- Design experimental protocols employing the resonance phenomenon
- Extend finite-size analysis to other resource-theories (asymmetry, contextuality)
- Extend to general quantum states with coherence.
- Look for resonance phenomena in other quantum information processing tasks

Details:

Beyond the thermodynamic limit: finite-size corrections to state interconversion rates [arXiv:1711.01193]

Moderate deviation analysis of majorization-based resource interconversion [arXiv:1809.?????]

Avoiding irreversibility: engineering resonant conversions of quantum resources [arXiv:1809.?????]

Thank you!