Work extraction from quantum coherence

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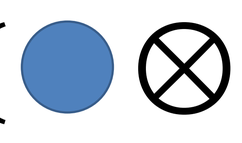
1. Thermodynamic setting

System  Environment 

Arbitrary state: ρ_S
Hamiltonian: H_S

Thermal State: $\gamma_E \propto e^{-\beta H_E}$
Hamiltonian: H_E

Joint energy-conserving unitary evolution

$U($  $)U^\dagger$

$[U, H_S + H_E] = 0$

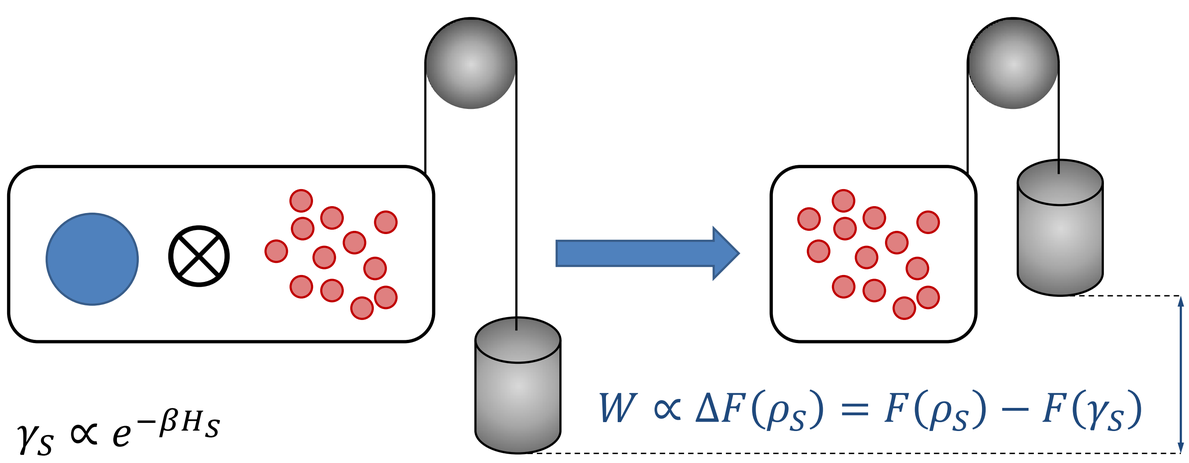
Hence the evolution of the system is described by *thermal operations*:

$$\mathcal{E}_T(\rho_S) = Tr_E\left(U(\rho_S \otimes \gamma_E)U^\dagger\right),$$

$$\mathcal{E}_T\left(e^{-iH_S t}\rho_S \rho e^{iH_S t}\right) = e^{-iH_S t}\mathcal{E}_T(\rho_S)e^{iH_S t}$$

that form a subset of time-translation symmetric operations:

2. Work extraction & work-locking



$\gamma_S \propto e^{-\beta H_S}$

$F(\rho) = Tr(\rho H_S) - kTS(\rho)$

$W \propto \Delta F(\rho_S) = F(\rho_S) - F(\gamma_S)$

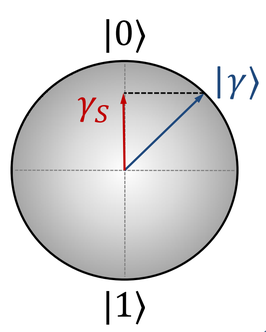
Work ∝ Free energy difference

Coherence part of free energy is locked!

$\rho_S \to W \Leftrightarrow D(\rho_S) \to W$

$$D(\rho_S) = \sum_{n,m} |n\rangle\langle n|\rho_S|n\rangle\langle n|$$

E.g. The amount of work that can be extracted from pure qubit state $|\gamma\rangle$ is zero.

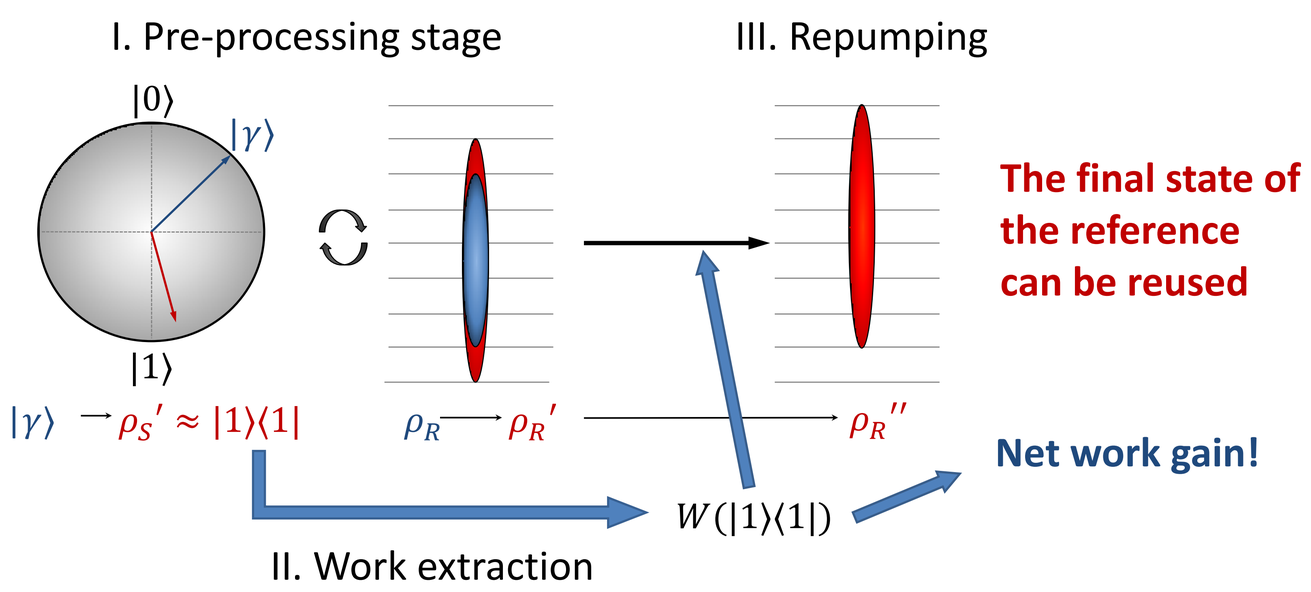


3. Unlocking work with a repeatable resource

Idea: Introduce an ancillary ladder system (reference) with coherence that can be reused infinitely many times

$$H_R = \sum_n \hbar\omega_0 n\,|n\rangle\langle n|$$

E.g. Single-mode bosonic field in a coherent state $|\alpha\rangle$ or a uniform superposition of energy eigenstates $|\psi_L\rangle \propto \sum_{n=0}^{L}|n\rangle$.



The final state of the reference can be reused

Net work gain!

4. Results

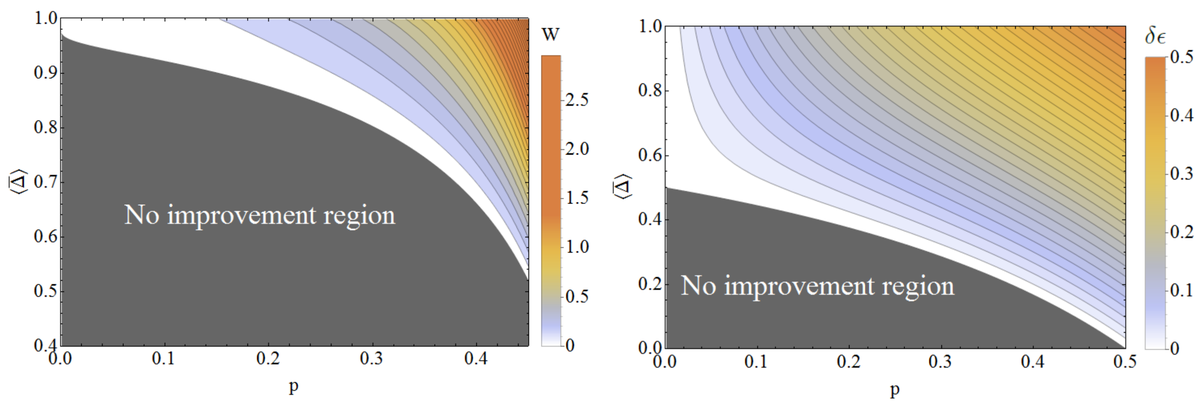
- In the limit of a classical (unbounded) reference (properly defined size $N \to \infty$) all work can be extracted from coherence:

$$W(\rho_S) \to \Delta F(\rho_S)$$

- Using a bounded reference the amount of work that can be extracted is strictly smaller than the expected free energy difference:

$$W(\rho_S) < \Delta F(\rho_S)$$

- Explicit protocols for bounded reference frames (described by quality parameter $\langle\bar{\Delta}\rangle$) extracting work from coherence in single-shot and asymptotic scenarios.



Work extraction from a qubit state $|\gamma\rangle = \sqrt{1-p}|0\rangle + \sqrt{p}|1\rangle$
(where p is the thermal occupation of excited state).
Left: Asymptotic scenario.
Right: Single-shot scenario ($\delta\varepsilon$ - the decrease in failure probability)