Wrocław University of Technology
Institute of Physics

Imperial College London

Controlled Quantum Dynamics

Quantum state transfer via spin chains

Kamil Korzekwa

Controlled Quantum Dynamics Centre Doctoral Training, Imperial College, London, UK

Paweł Machnikowski

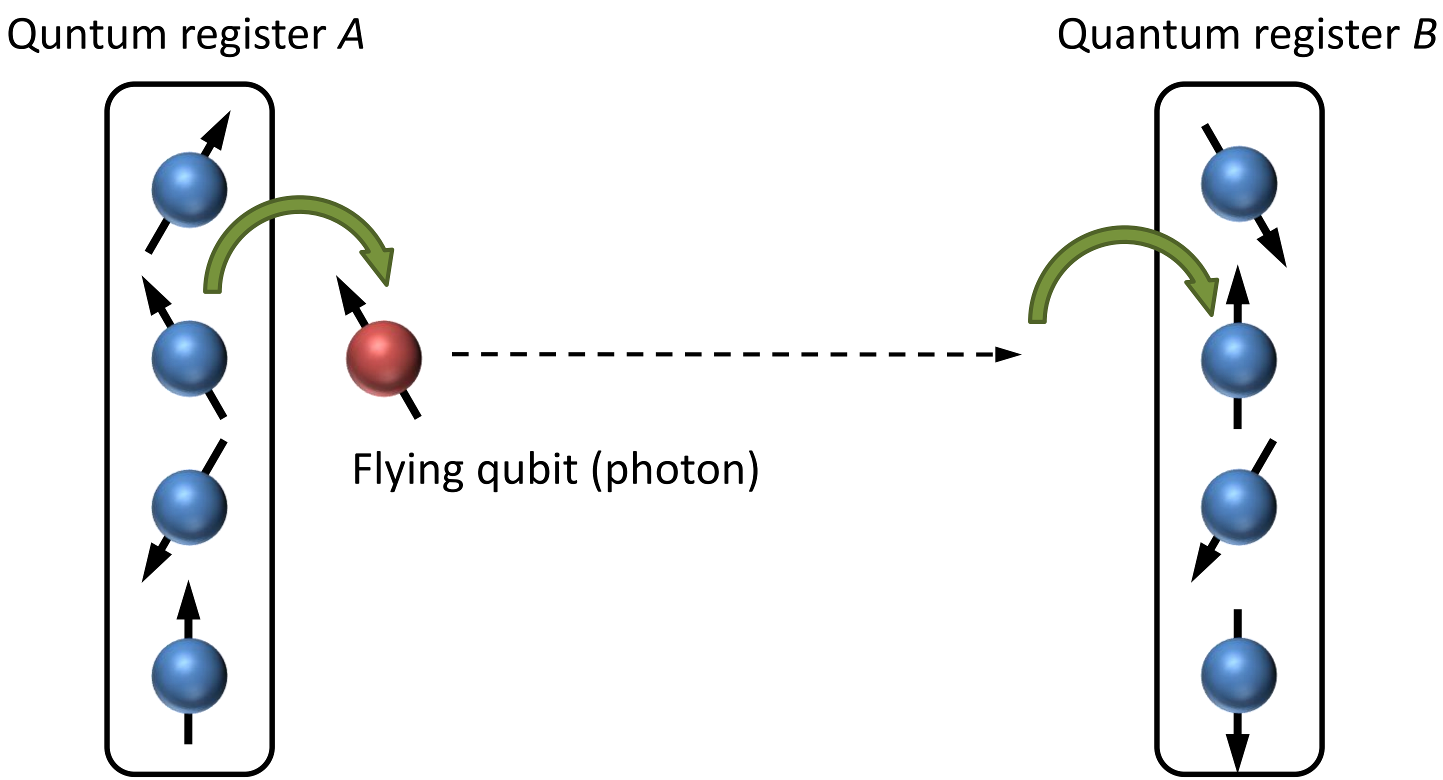
Institute of Physics, Wrocław University of Technology, Wrocław, Poland

Contents

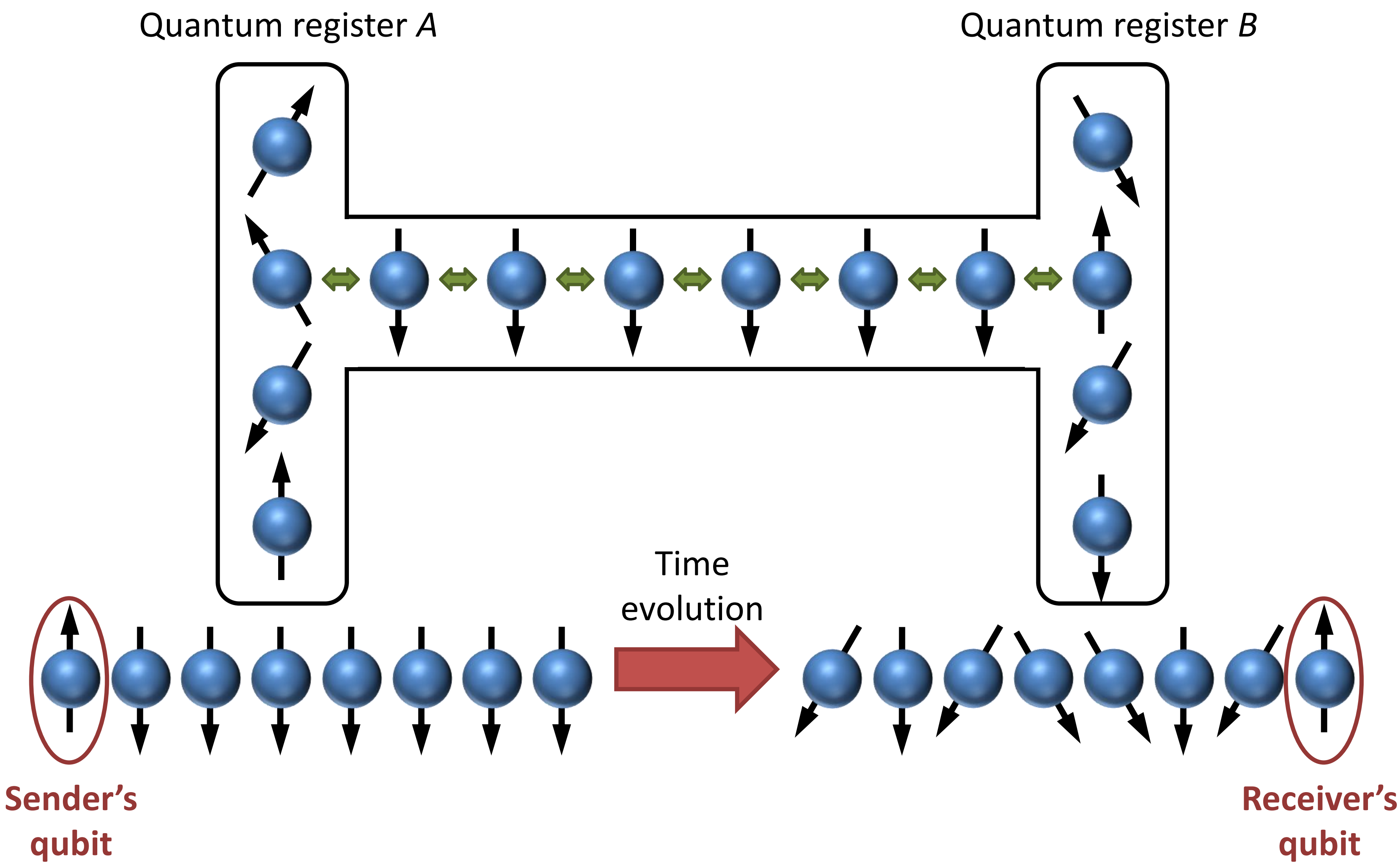
Motivation

DiVincenzo's criteria for Quantum Computer Networkability:

- The ability to interconvert stationary and flying qubits.
- The ability to faithfully transmit flying qubits between specific locations.



Motivation


Quantum register *A*
Quantum register *B*
Time
evolution
Sender's
qubit
Receiver's
qubit

Goals

Resilience to imperfections

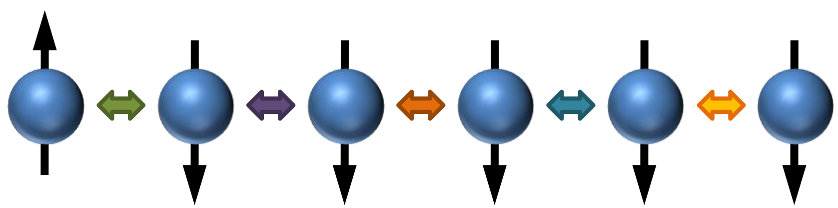
Limited control

Experimental feasibility

Transfer on demand

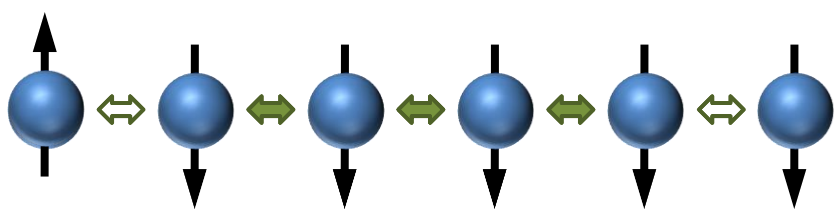
Examples of proposed protocols

Engineered couplings



Number of controlled parameters scale with the size of the system

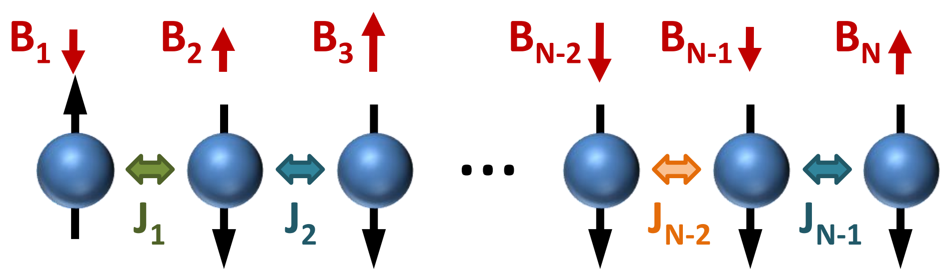
Weak coupling of terminal spins



Very fragile to perturbations

System

$$H=\sum_{l=1}^{N-1} J_l\left(|l\rangle\langle l+1|+\mathrm{h.c.}\right)+\sum_{l=1}^{N} B_l|l\rangle\langle l| = \begin{pmatrix} B_1 & J_1 & 0 & \dots & 0 & 0 \\ J_1 & B_2 & J_2 & \dots & 0 & 0 \\ 0 & J_2 & B_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & B_{N-1} & J_{N-1} \\ 0 & 0 & 0 & \dots & J_{N-1} & B_N \end{pmatrix}$$



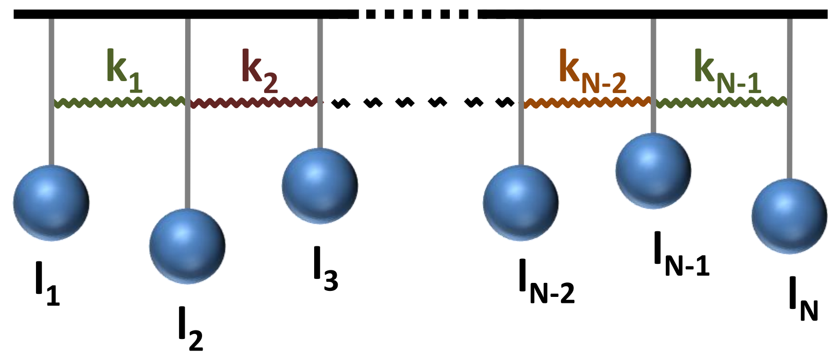
Question: How to find the dynamics of this system?
Answer: Standard procedure:

- Find Hamiltonian eigenvectors and eigenvalues $|\Psi_n\rangle, E_n$
- Decompose initial state in the basis of these eigenstates $|\Psi(0)\rangle=\sum_n c_n|\Psi_n\rangle$
- Evolve according to $|\Psi(0)\rangle=\sum_n c_n e^{-iE_n t/\hbar}|\Psi_n\rangle$

Classical analogue

System of N coupled pendulums:

$$\begin{aligned}
\ddot{\Theta}_1 &= -\frac{g}{l_1}\Theta_1 - \frac{k_1}{m}(\Theta_1 - \Theta_2) \\
\ddot{\Theta}_2 &= -\frac{g}{l_2}\Theta_2 + \frac{k_1}{m}(\Theta_1 - \Theta_2) - \frac{k_2}{m}(\Theta_2 - \Theta_3) \\
&\vdots \\
\ddot{\Theta}_N &= -\frac{g}{l_N}\Theta_N - \frac{k_{N-1}}{m}(\Theta_N - \Theta_{N-1})
\end{aligned}$$



Look for normal mode soultions of the form $\Theta_n = A_n e^{i\omega t}$:

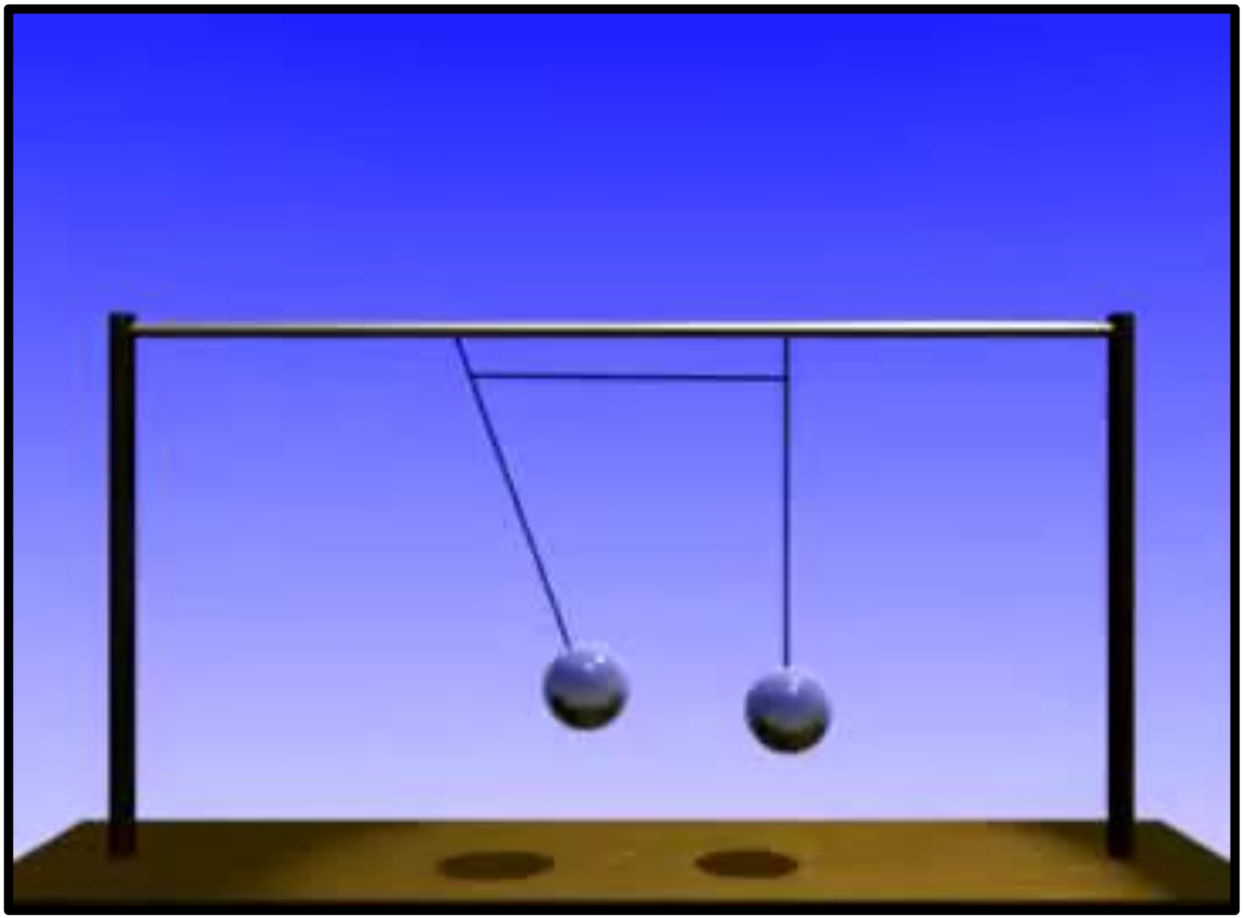
$$\begin{pmatrix}
B_1 & J_1 & 0 & \dots & 0 & 0 \\
J_1 & B_2 & J_2 & \dots & 0 & 0 \\
0 & J_2 & B_3 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \dots & B_{N-1} & J_{N-1} \\
0 & 0 & 0 & \dots & J_{N-1} & B_N
\end{pmatrix}
\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{N-1} \\ A_N \end{pmatrix}
= \omega^2 \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{N-1} \\ A_N \end{pmatrix}$$

Eigenvalue-eigenvector problem with the same matrix as spin chain Hamiltonian

Where: $B_n = \frac{g}{l_n} + \frac{k_{n-1} + k_n}{m}$ and $J_n = -\frac{k_n}{m}$

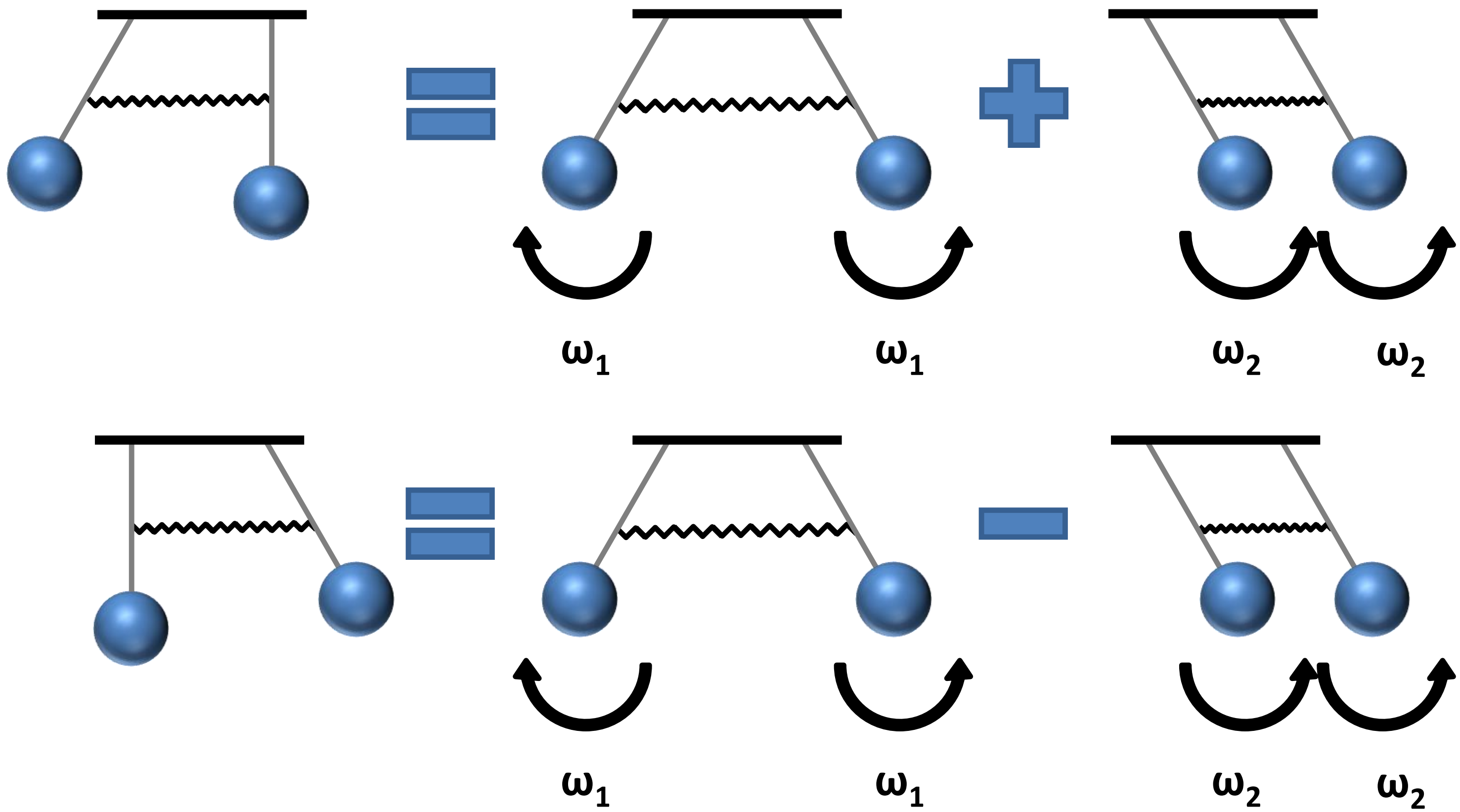
Classical analogue – simplest example

Phenomenum: Energy transfer (beats)



Classical analogue – simplest example

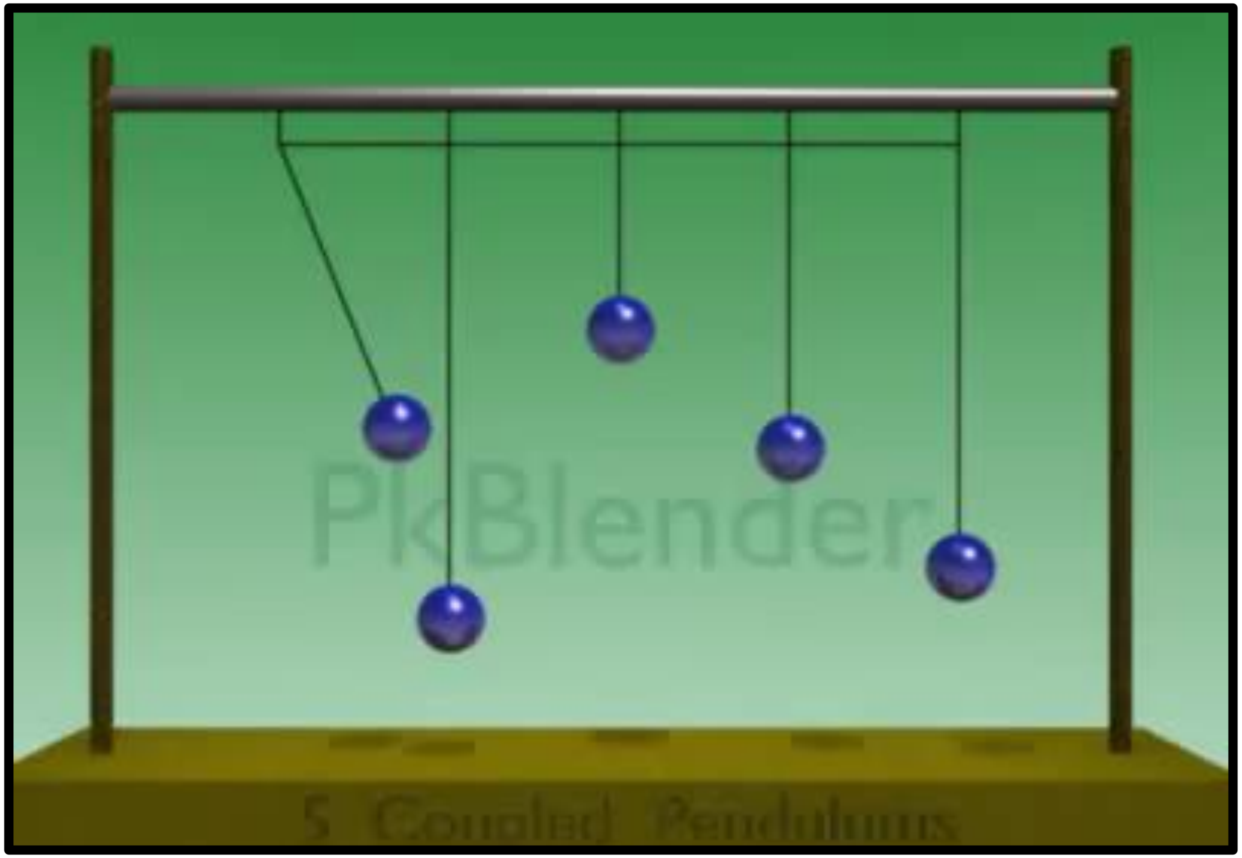
Explanation:



Relative phase of π between the modes after: $T = \dfrac{\pi}{\omega_1 - \omega_2}$

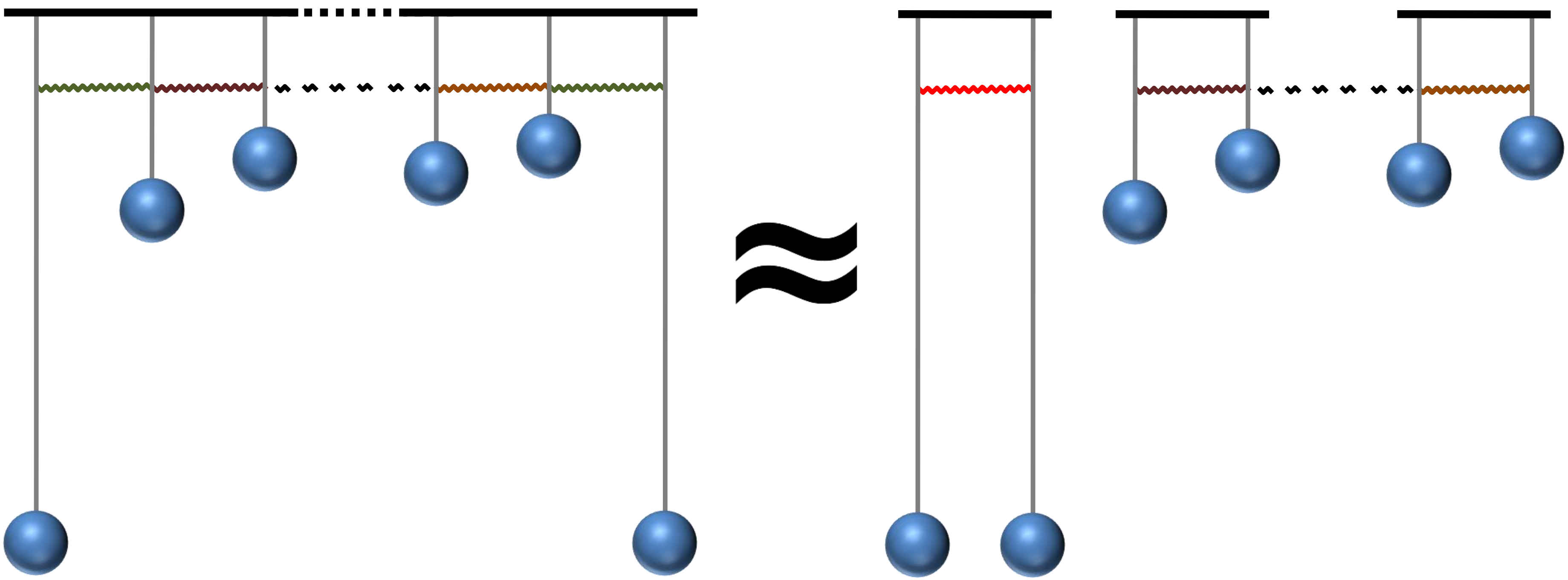
Classical analogue – importance of resonance

Phenomenum: Resonant energy transfer



Classical analogue – importance of resonance

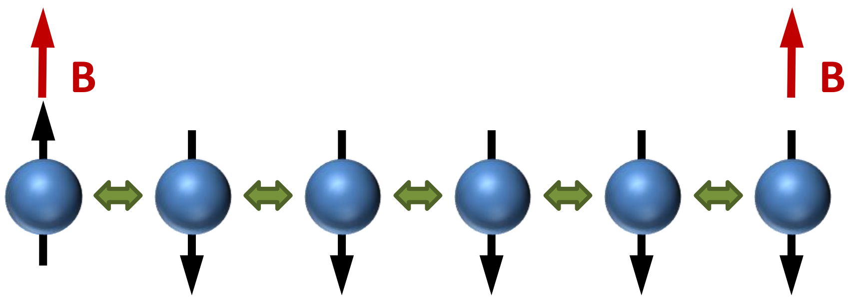
Explanation:

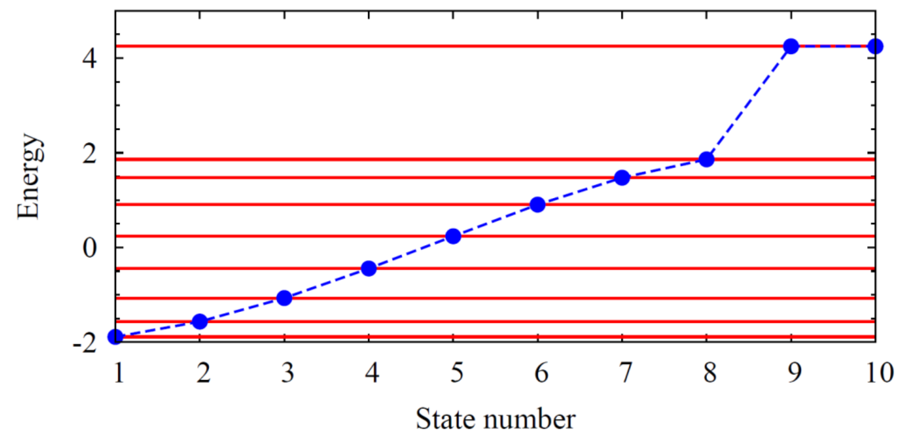


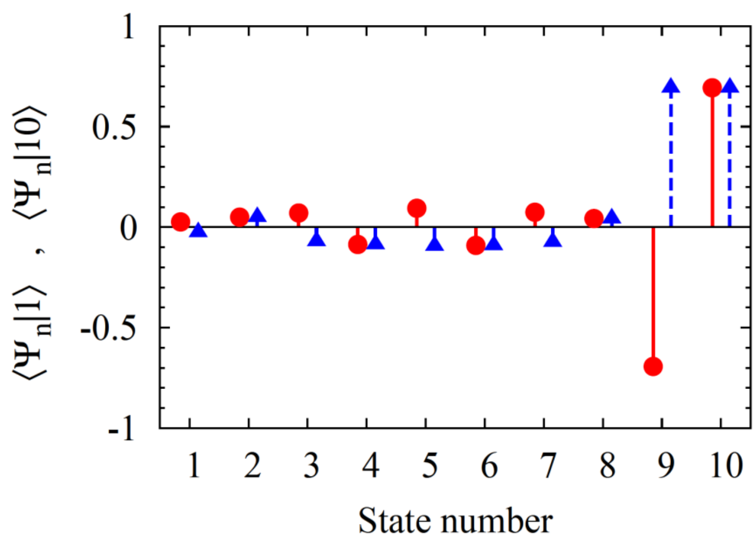
Normal modes are mainly composed of systems with similiar frequencies (energies). The subsystem whose components are in resonance and which is energetically separated from the rest of the system can be considered as independent (the system then only influences the effective coupling between components of the subsystem).

Energy separation of terminal spins

First let's consider uniform spin chain:







For **B** large enough we approximately get:

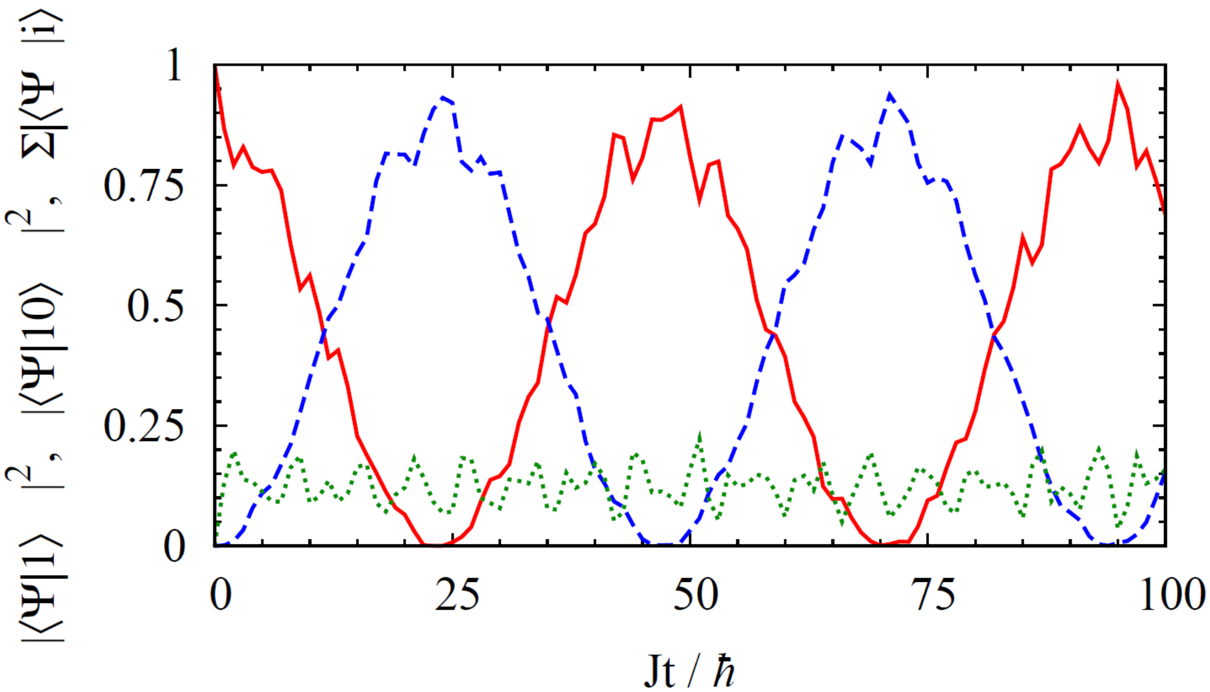
Spin inverted at 1st site $|1\rangle = \dfrac{|\Psi_N\rangle + |\Psi_{N-1}\rangle}{\sqrt{2}}$

Spin inverted at Nth site $|N\rangle = \dfrac{|\Psi_N\rangle - |\Psi_{N-1}\rangle}{\sqrt{2}}$

Parameters: N=10, J=1, B=4

Energy separation of terminal spins

Rabi oscillations achieved:



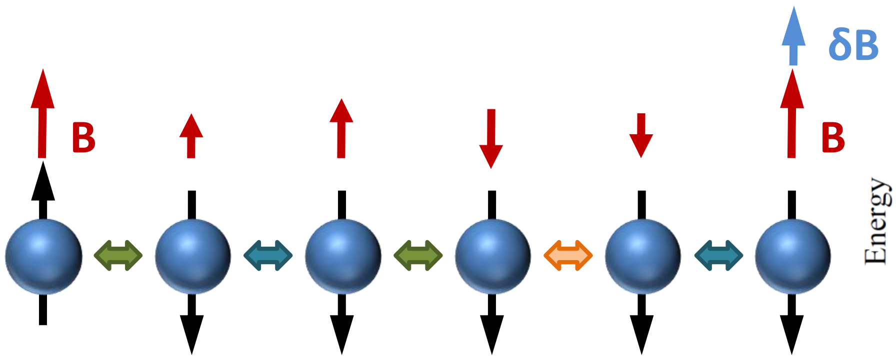
Limited control

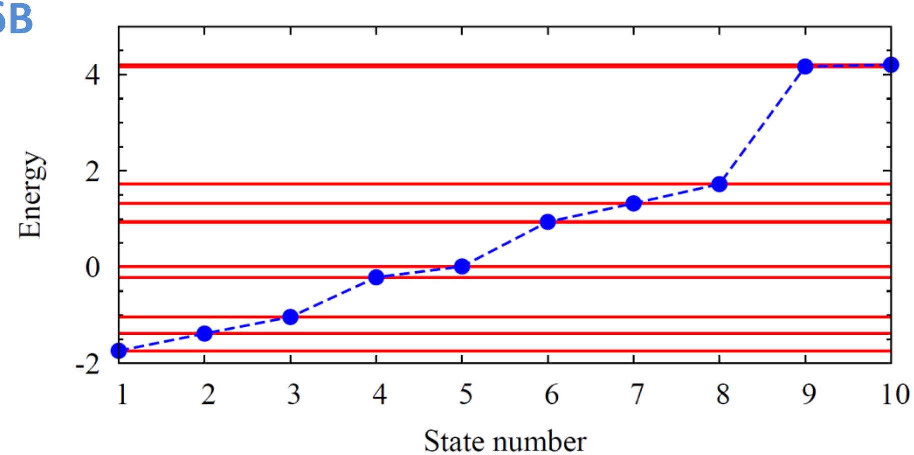
Question: Does it also work for systems with imperfections, i.e. non-uniform couplings and energies?
Answer: Yes, but…

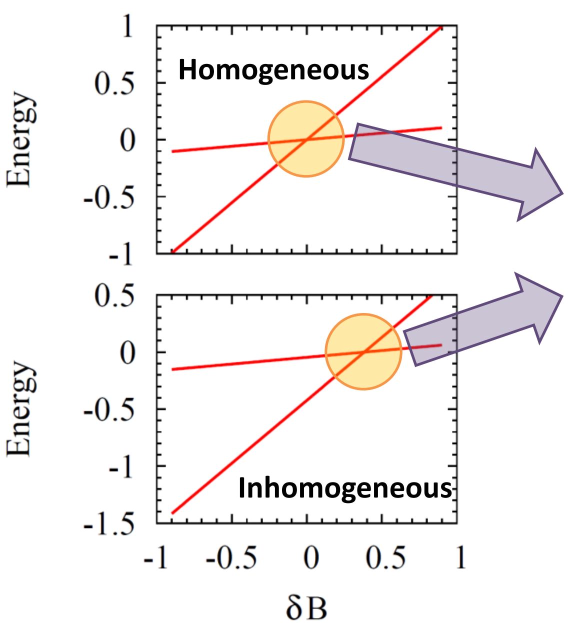
Parameters: N=10, J=1, B=4

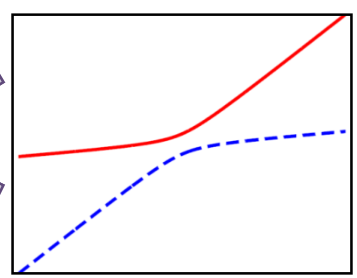
Influence of imperfections

For non-uniform spin chain use assymetric energy shift:









Transfer time:

$$\tau_f = \pi\hbar/(2V)$$

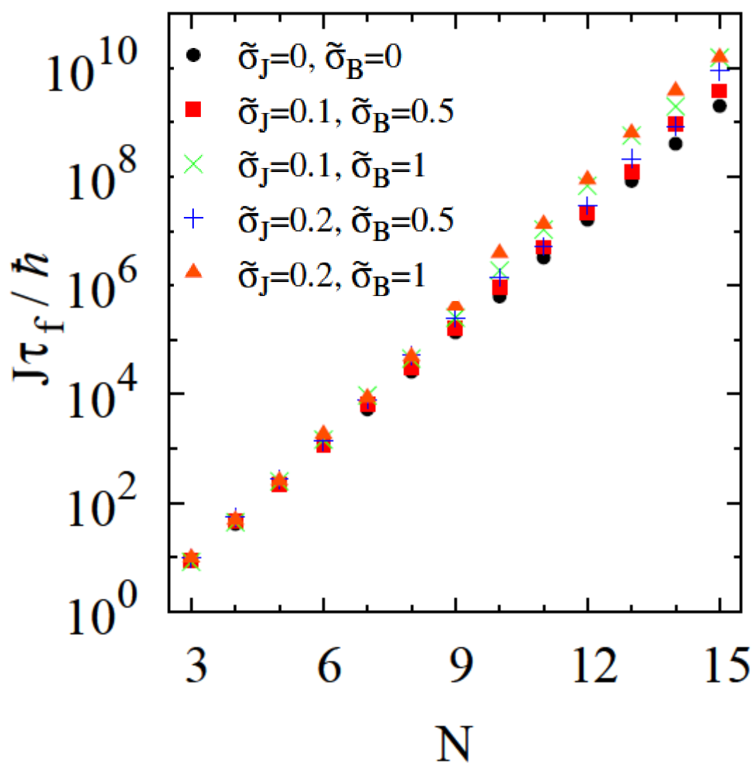
Inhomogeneity influence mainly states delocalized in the central part of the chain

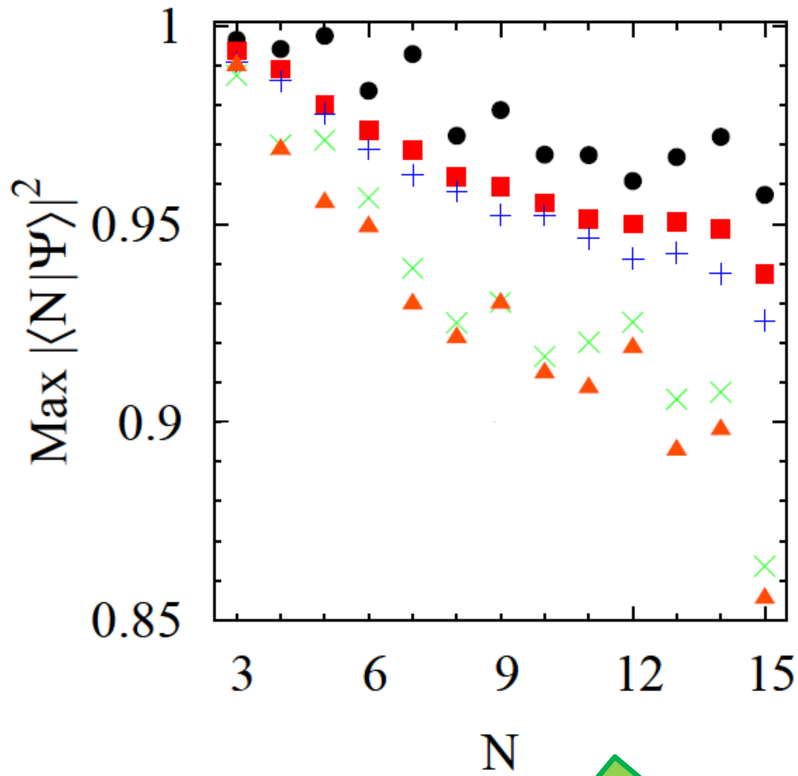
It shifts the resonance between 2 highest energy eigenstates, but only slightly affect its width

Parameters: N=10, J=1, B=4, σ_J=0.1, σ_B=0.5

Influence of imperfections

Transfer time and fidelity only slightly affected by imperfections:





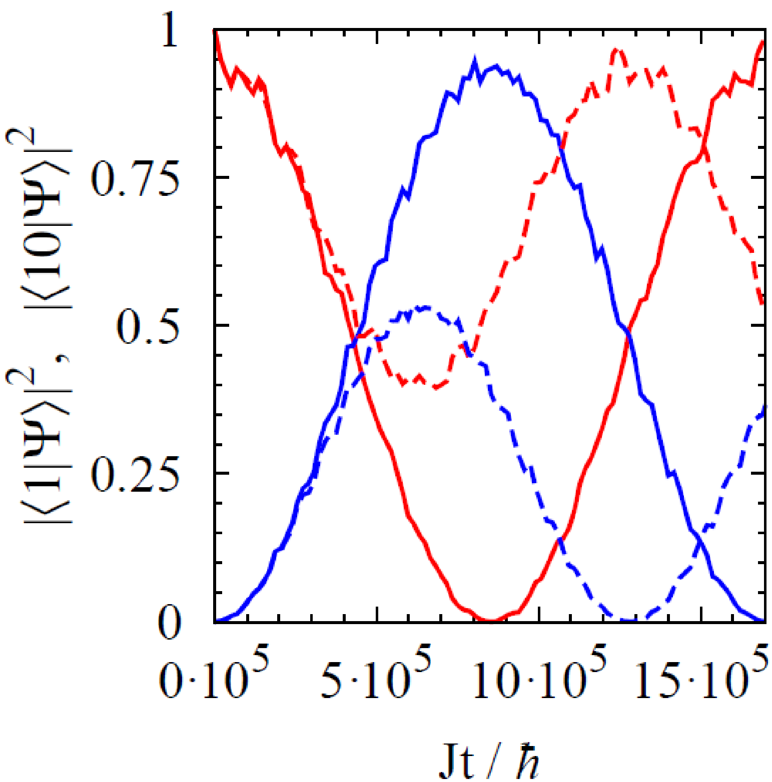
Resilience to imperfections

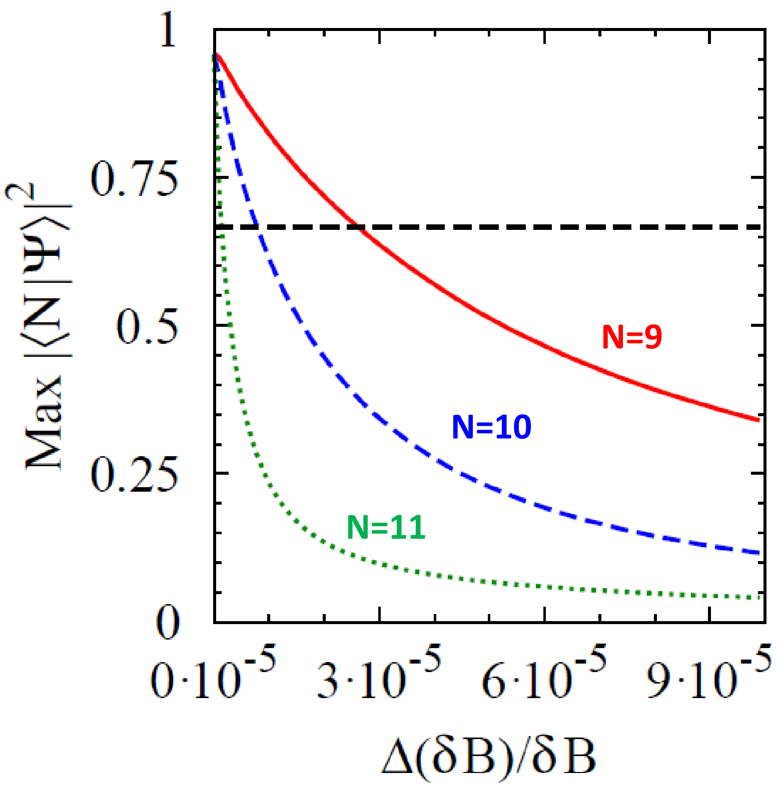
Question: Is it experimentally feasible to control **δB**?
Answer: Well…

Parameters: J=1, B=5

Experimental feasibility

Obtained fidelity is very sensitive to small variations of compensating energy shift:





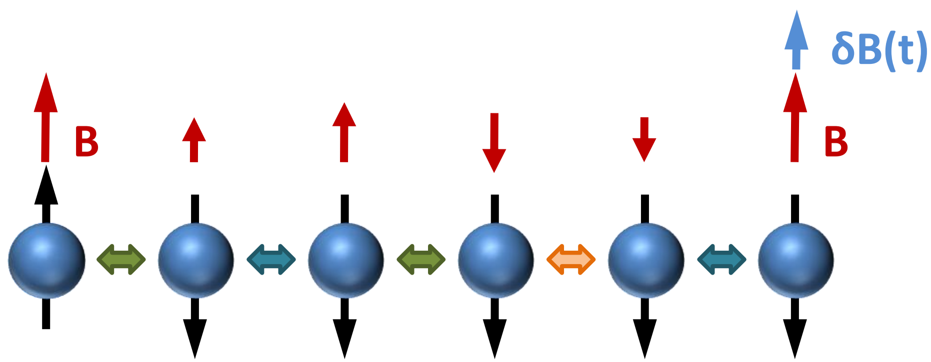
Experimental feasibility

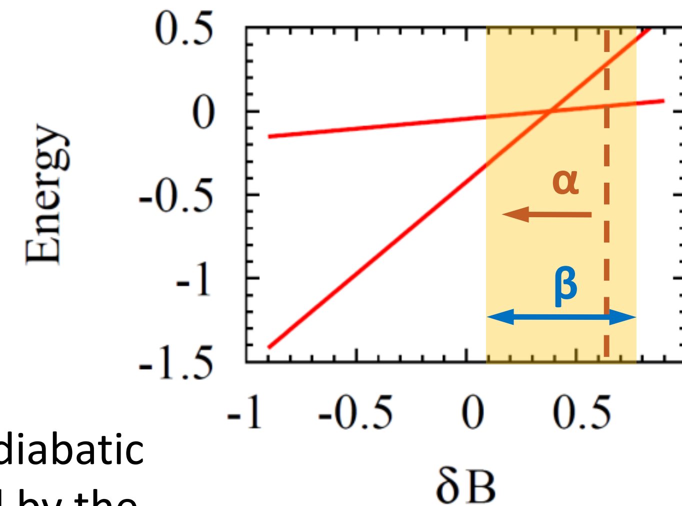
But…

Parameters: N=10, J=1, B=5, σ_J=0.1, σ_B=0.5

Adiabatic protocol

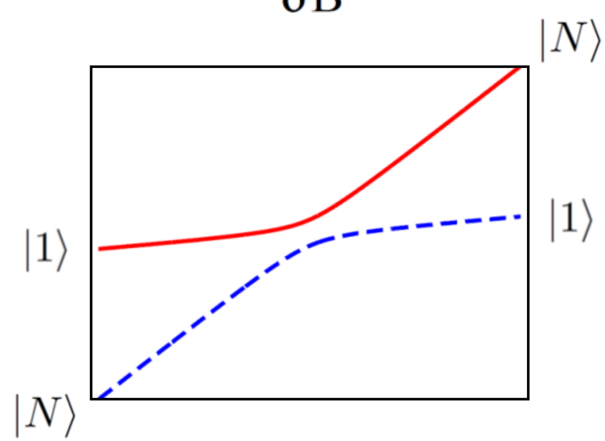
Use slowly varying assymetric energy shift to sweep states through resonance:





For finite energy shift sweep $\alpha = dB/dt$ nonadiabatic tranistion is possible. Its probability is described by the Landau-Zener formula:

$$P_{\rm na} = \exp\left(-\frac{2\pi}{\hbar}\frac{|V|^2}{\alpha}\right)$$



Fidelity and transfer time are given by:

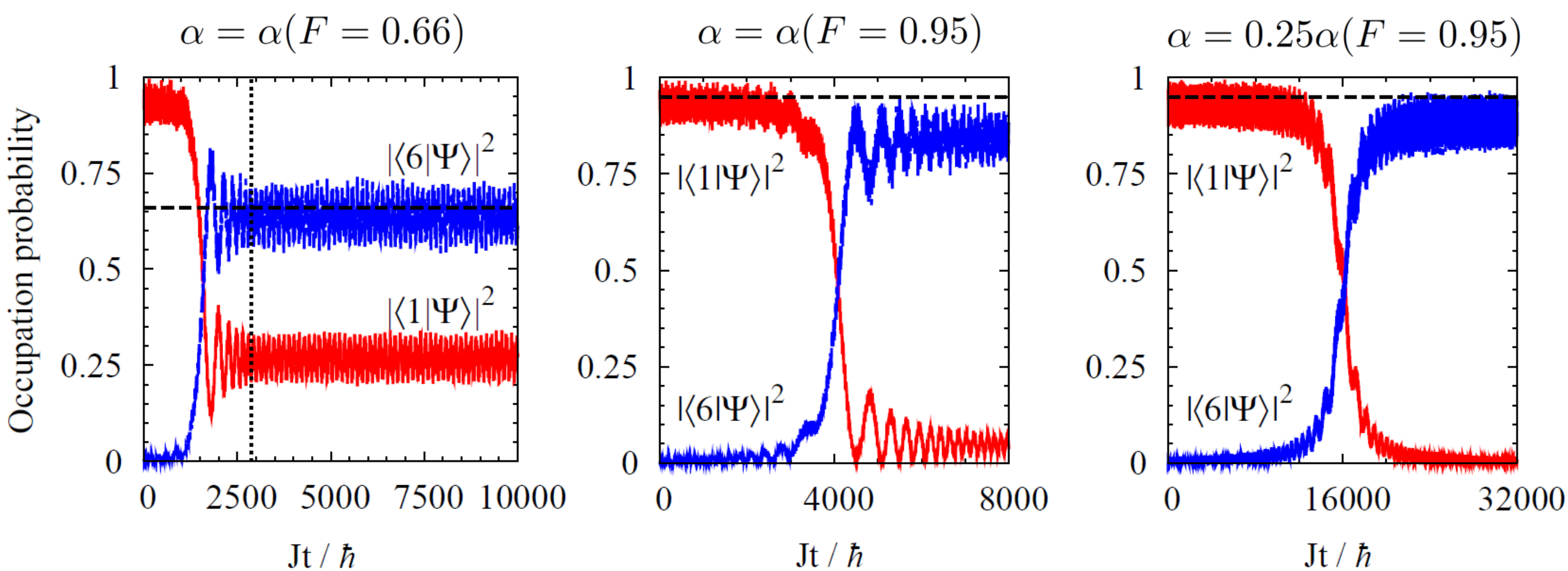
$$F = 1 - P_{\rm na}$$

$$\tau_{\rm a} = -[\hbar\beta/(\pi V)]\ln(1-F)$$

$$\tau_{\rm a}/\tau_{\rm f} = -2\beta/\pi^2\ln(1-F)$$

Adiabatic protocol

Transfer with fidelity consistent with Landau-Zener formula:



Experimental feasibility ✓

Transfer on demand ✓

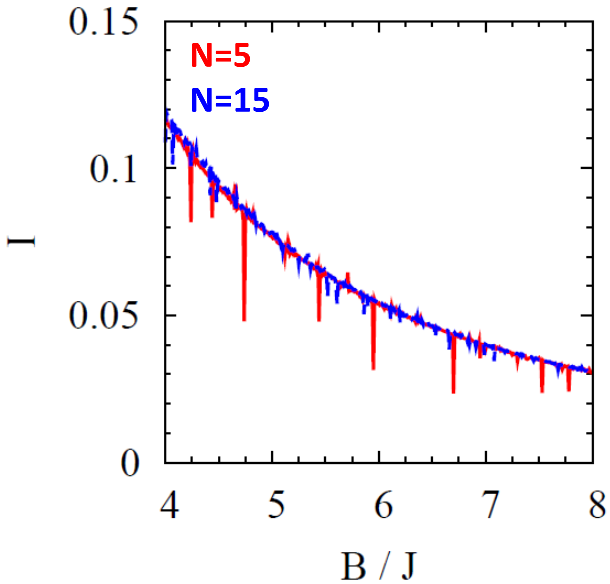
Question: What limits perfect transfer?

Parameters: $N=10$, $J=1$, $B=5$, $\sigma_J=0.1$, $\sigma_B=0.5$, $\beta=20$

Leakage of quantum information

Trace distance measure of information available in the central part of the chain:

$$I = \frac{1}{2}\,\mathrm{Tr}\,|\rho_1 - \rho_0| = 1 - |c_1|^2 - |c_N|^2$$



Almost independent of N

$\tilde{\sigma}_J$ \ $\tilde{\sigma}_B$	0	0.5	1
0	7.65%	8.26%	10.65%
0.1	7.80%	7.89%	11.12%
0.2	8.30%	8.23%	9.80%

Only slightly dependent on inhomogeneity parameters

Question: Is this important from the point of view of security against eavesdropping?

Eavesdropping

Von Neumann entropy measure of information available in the subsystem of spins described by the set {i}:

$$S(\rho_{\{i\}}) = -p \ln p - (1-p)\ln(1-p) \qquad \text{where:} \quad p = \frac{1}{2}\sum_{j\in\{i\}} |c_j|^2$$

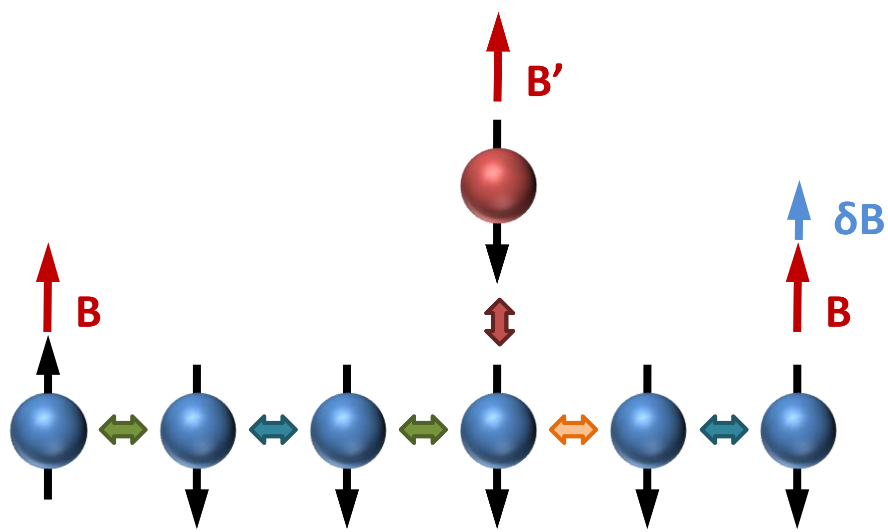
The rate of information transfer to the receiver's qubit:

$$\frac{dS(\rho_N)}{dt} = -i_{N-1}\, g(|c_N|^2) \qquad \text{where:} \quad i_l = \frac{J_l}{i\hbar}\left(c_l^* c_{l+1} - c_{l+1}^* c_l\right) \qquad g(x) = \frac{1}{2}\ln\left(\frac{2-x}{x}\right)$$

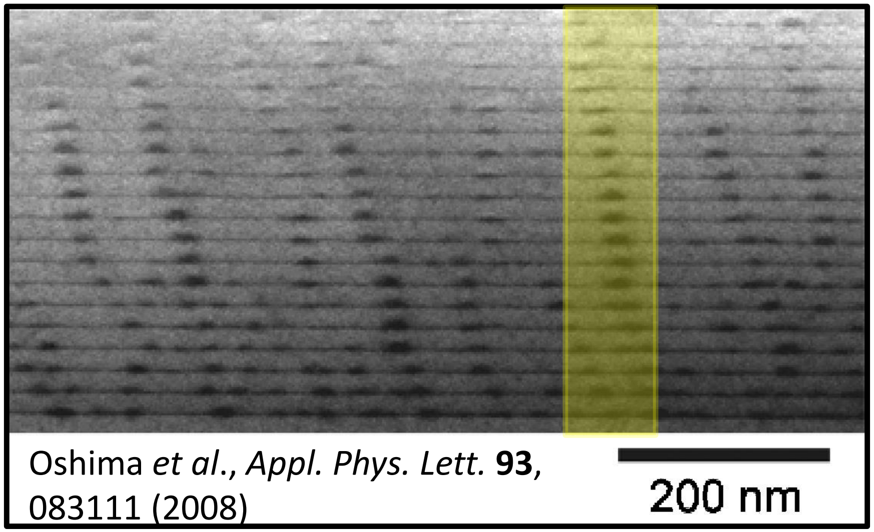
The less information is available in the central part of the chain:

•The slower the information will be transferred to receiver

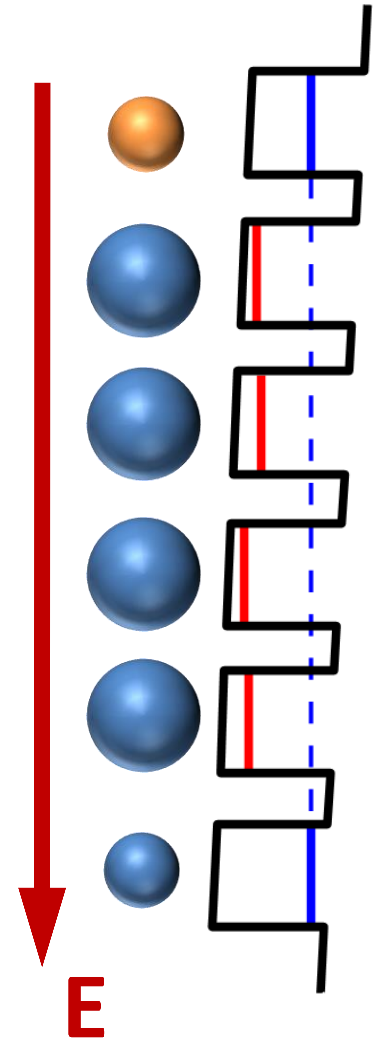
• The longer it will take the eavesdropper to intercept the information



Exemplary experimental realization – quantum dots chain



Oshima *et al.*, *Appl. Phys. Lett.* **93**, 083111 (2008)

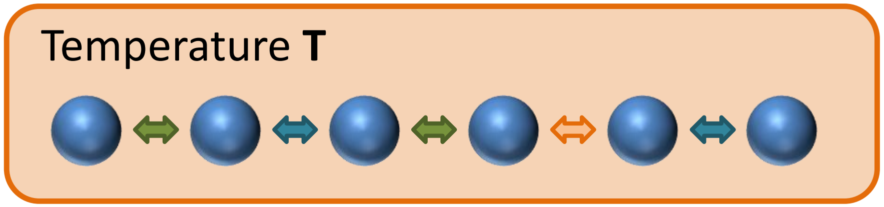


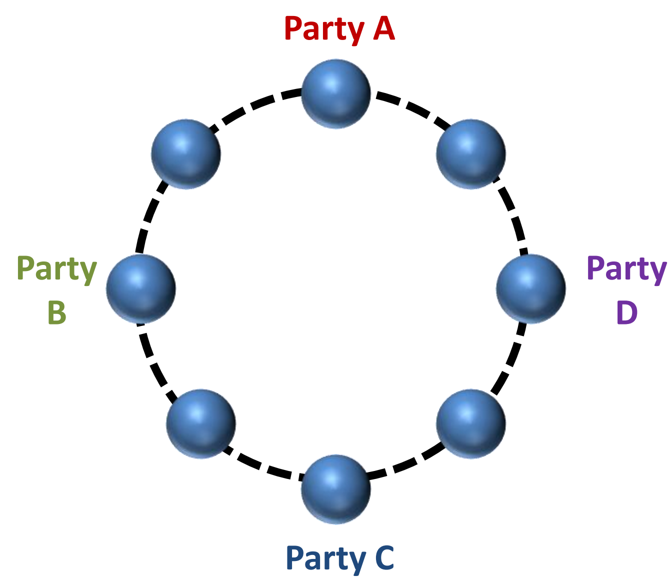
Spin chain	Quantum dots chain
Spin XY coupling	Tunneling coupling
Energetic separation of terminal states	Controlling size of terminal dots (smaller dots – higher energies)
Compensating magnetic field δB	Compensating electric field E along quantum dots chain

K. Korzekwa and P. Machnikowski, *Acta Phys. Pol. A* **120**, 859 (2011)

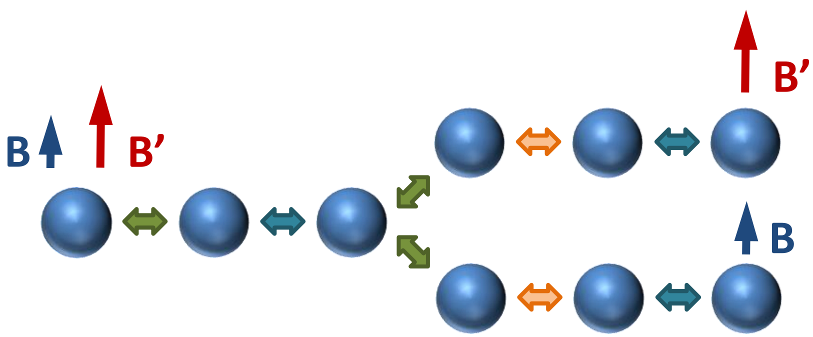
Paths to follow in the future

- Modelling realistic perturbations - thermal environment

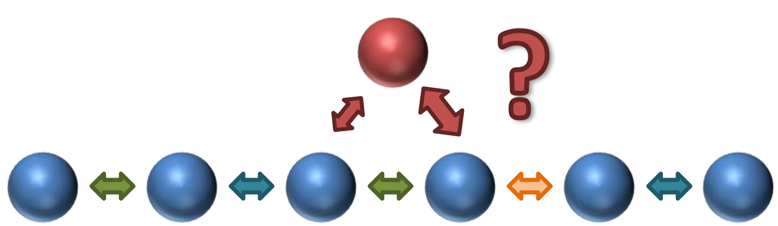


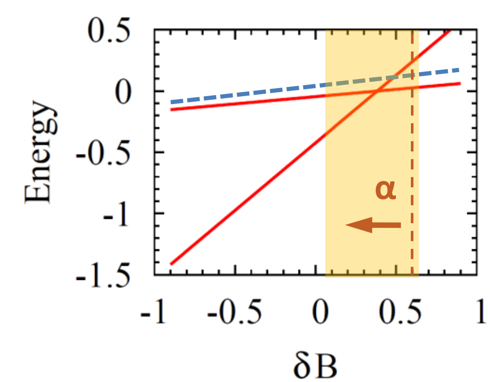


- Dual rail & multiple energy channels



- Optimal eavesdropping and security protocols





Conclusions

- **Protocol with limited control and resilience to imperfections**

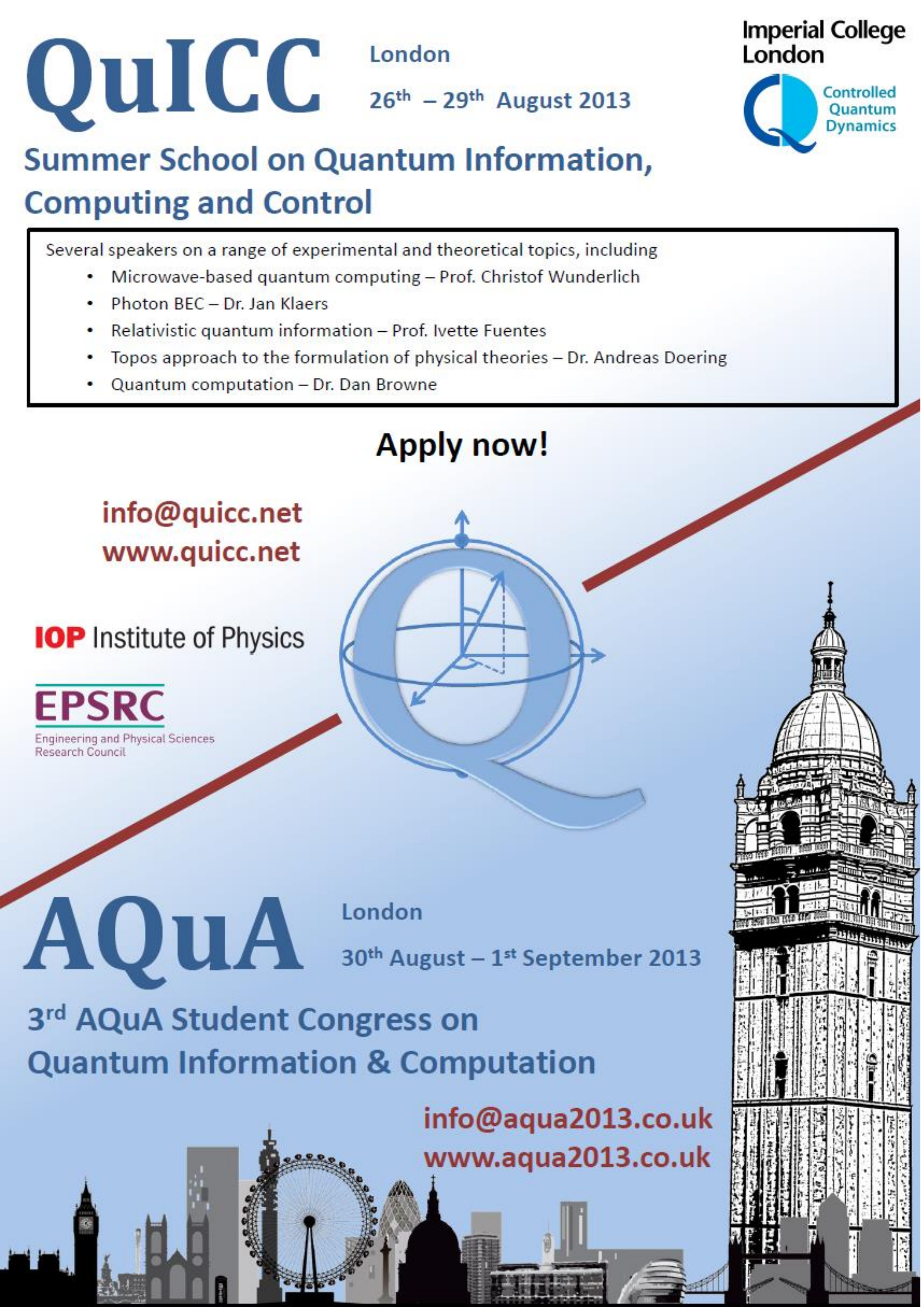
  The negative impact of the inhomogeneity caused both by the local magnetic fields and disorder in exchange couplings can be overcome by separating terminal states with external magnetic field that brings them to resonance.

- **Adiabatic variation – experimentally feasible and on demand**

  Adiabatic transfer in a spin chain can be well described within an approximate model of a two-level system and the transfer time can be obtained by using Landau-Zener formula for a given desired fidelity.

- **Small amount of information in the central part of the chain**

  Controllable trade-off between security (necessary time to intercept information by eavesdropper) and transfer speed.



Speakers already confirmed:

- Prof. Ivette Fuentes (University of Nottingham, UK) - **Relativistic quantum information**
- Prof. Christof Wunderlich (Universität Siegen, Germany) - **Microwave-based quantum gates**
- Prof. Ed Hinds (Imperial College London, UK) - **Single-photon sources and the EDM experiment**
- Dr. Andreas Doering (University of Oxford, UK) - **Topos approach to the formulation of physical theories**
- Dr. Jan Klaers (Universität Bonn, Germany) - **Photon BEC**
- Dr. Dan Browne (University College London, UK) - **Measurement-based quantum computing and Bell inequalities**

Thank You!