The extraction of work from quantum coherence

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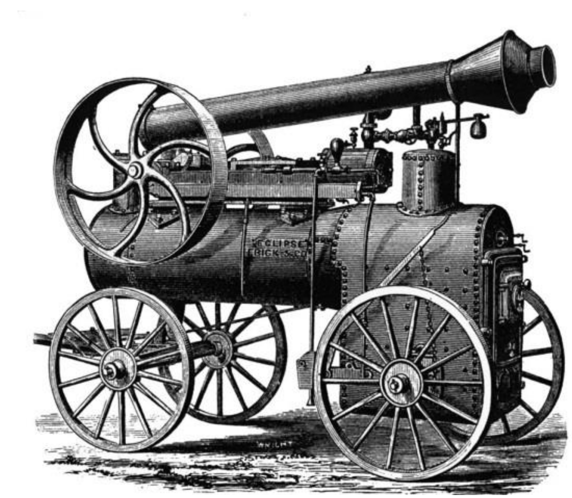


The concept of free energy

The Kelvin–Planck statement of the second law of thermodynamics

It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.

Thermodynamic free energy: $F = U - TS$

Using density matrix formalism: $F(\rho) = \mathrm{Tr}(\rho H) + kT\,\mathrm{Tr}(\rho \ln \rho)$

For thermal equilibrium state: $F(\gamma) = -kT\ \ln Z$

(where: $\gamma = \frac{e^{-\beta H}}{Z} = \sum_n \frac{e^{-\beta E_n}}{Z} |E_n\rangle\langle E_n|, \quad Z = \sum_n e^{-\beta E_n}$)

The maximum amount of extractable work from a system in a state ρ:

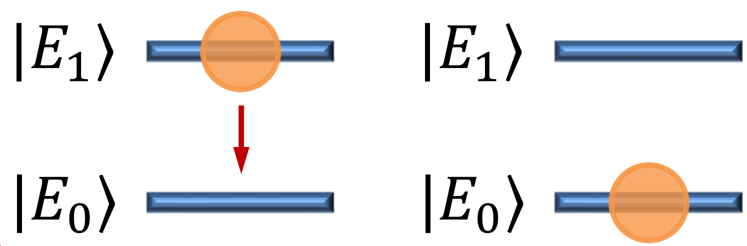
$$\Delta F(\rho) \coloneqq F(\rho) - F(\gamma) = \mathrm{Tr}(\rho H) + kT\big(\ln Z + \mathrm{Tr}(\rho \ln \rho)\big)$$

The concept of free energy

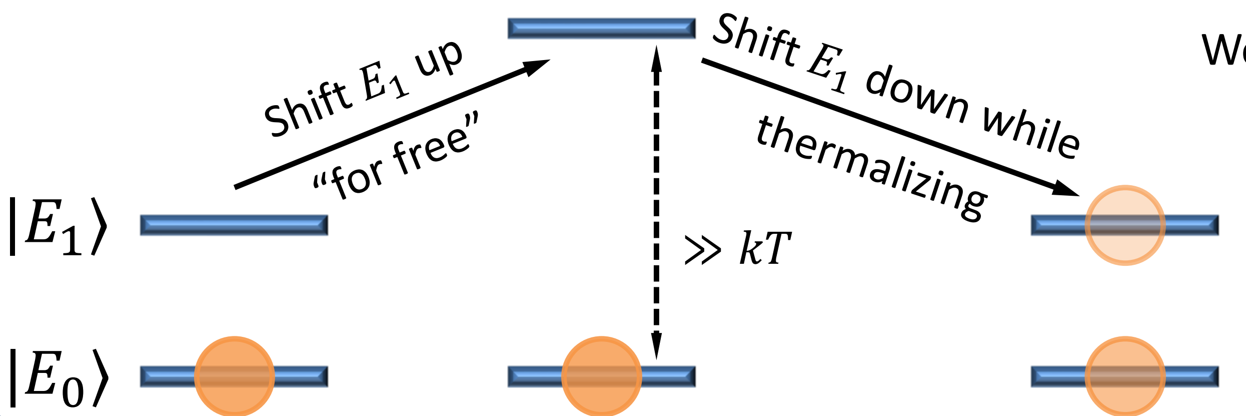
Example: Work extraction from a two-level system in an excited state:

$$\Delta F(|E_1\rangle\langle E_1|) = E_1 - E_0 + kT \ln Z$$

"Energetic" component



"Information" component



Work extracted in the process:

$$W = \int_{\infty}^{E_1 - E_0} \frac{e^{-\beta E}}{1 + e^{-\beta E}} dE$$

$$= kT \ln Z$$

Problem

Where is this work stored? In the classical control field? If so, how do we use it?

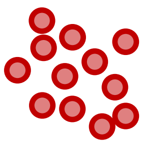
M. Frenzel, D. Jennings, T. Rudolph, Phys. Rev. E **90**, 052136 (2014)

Quantum description

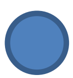
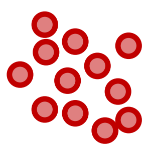
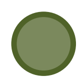
Given

 System in a state ρ described by Hamiltonian $H = \sum_n E_n |E_n\rangle\langle E_n|$

And a general thermal bath

 Environment described by an arbitrary Hamiltonian H_E prepared in a thermal state γ_E

We can couple them through an energy-preserving unitary U:

$Tr_E\big(U($  $\otimes$  $)U^\dagger\big) =$ 

$$[U, H + H_E] = 0$$

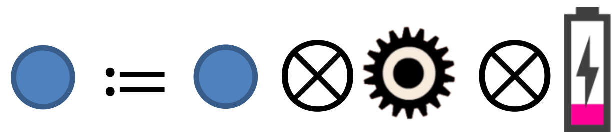
"Encoding" 1st Law

Formal definition of **thermal operations**:

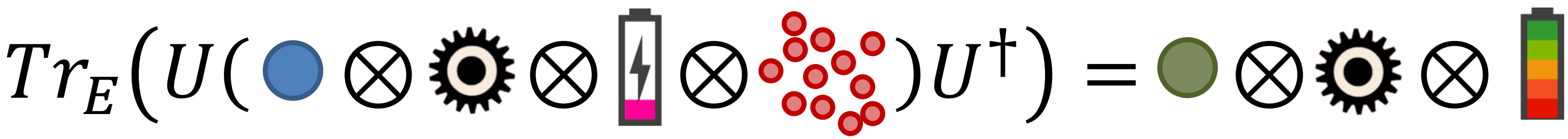
$$\mathcal{E}_T(\rho) = Tr_E\big(U(\rho \otimes \gamma_E)U^\dagger\big)$$

Quantum description

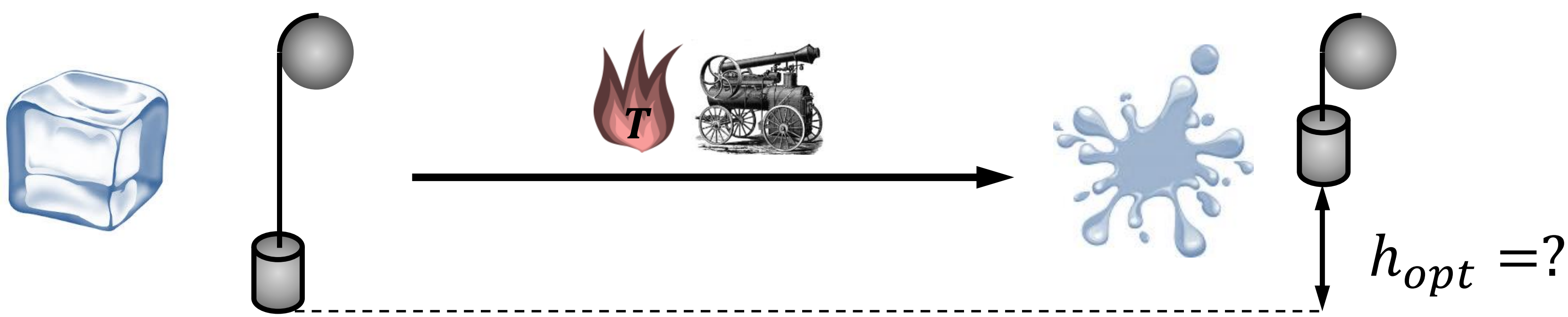
In order to study work extraction process we explicitly model the work storage system (battery) and any ancillary systems used in the process:



And consider a thermal operation on the joint system:



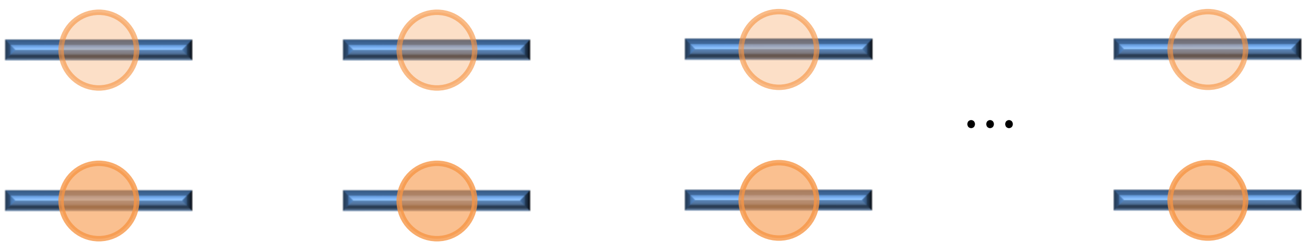
This way we can study questions like:



Work extraction from incoherent states

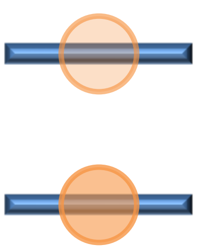
Recovering the classical result as the averaged extractable work per copy when $N \to \infty$:

$$\langle \mathrm{W}(\rho) \rangle = \Delta F(\rho)$$



F. Brandão *et al.*, Proc. Natl. Acad. Sci. U.S.A. **112** 3275 (2015)

Advent of single-shot thermodynamics



Probability $1-\varepsilon$ → Extract guaranteed amount of work $\Delta F^{\varepsilon}(\rho)$

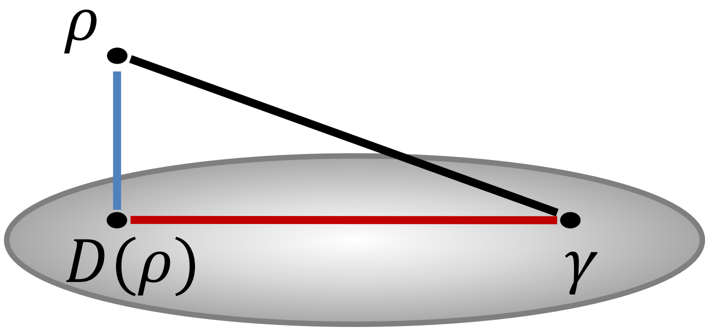
Probability ε → Protocol fails

J. Åberg, Nat. Commun. **4** 1925 (2013)

States with coherence - work-locking

$$\Delta F(\rho) = \Delta F(D(\rho)) + A(\rho)$$

$kT\ S(D(\rho)||\gamma)$ $\quad$ $kT\ S(\rho||D(\rho))$



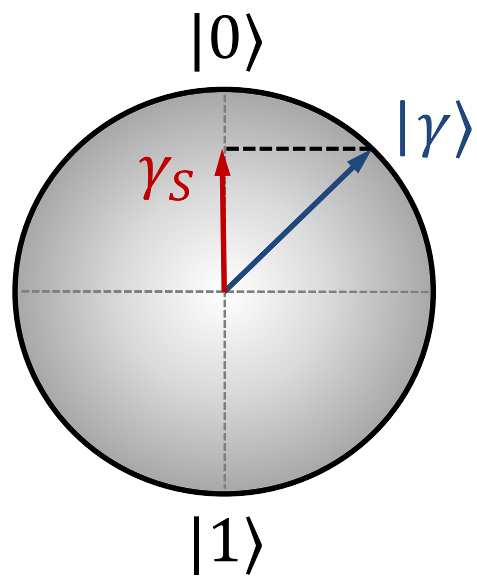
Where $D(\cdot)$ is a dephasing superoperator:

$$D(\rho) = \sum_n |E_n\rangle\langle E_n|\,\rho\,|E_n\rangle\langle E_n|$$

Coherence part of free energy is locked!

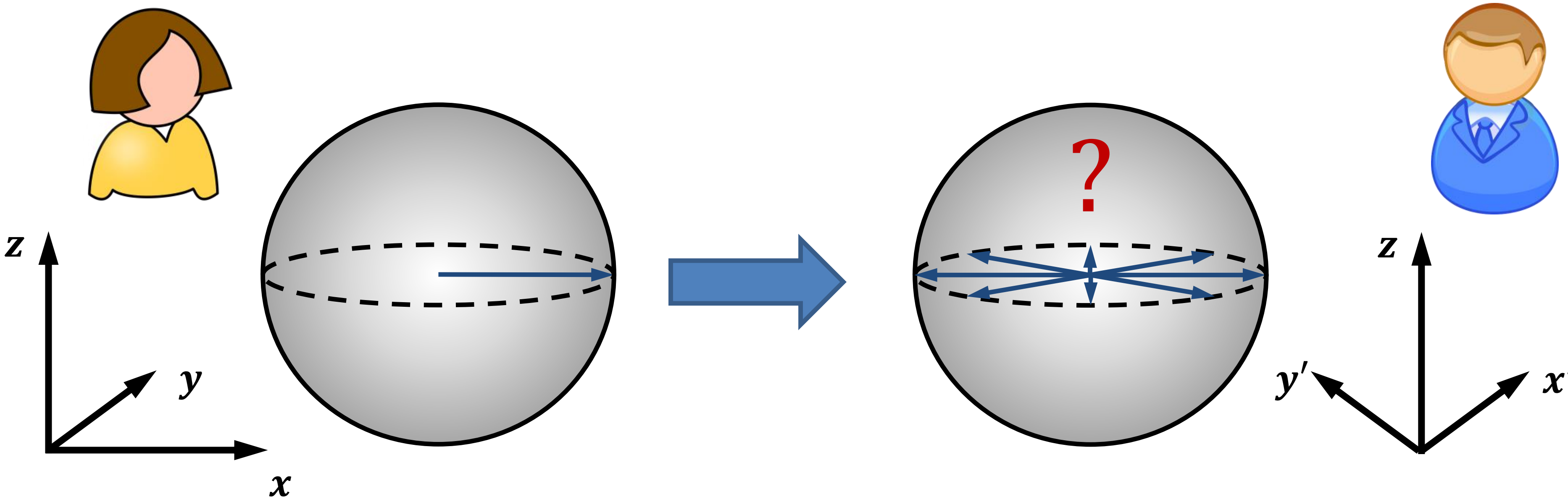
$$\rho \to W \Leftrightarrow D(\rho) \to W$$

E.g. The amount of work that can be extracted from pure qubit state $|\gamma\rangle$ is zero.



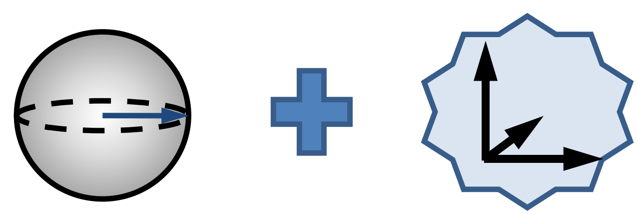
M. Lostaglio, D. Jennings, T. Rudolph
Nat. Commun. **6** 6383 (2015)

Why is coherence locked?



Problem:
In classical case simply measure the system.
In quantum case no information without disturbance.

Solution:
Send ancillary system that encodes the reference frame.



Another problem:
Reference frame is also a quantum system.
Again: no information without disturbance.

S. Bartlett, T. Rudolph, R. Spekkens,
Rev. Mod. Phys. **79** 555 (2007)

Why is coherence locked?

Thermal operations are time-translation symmetric:

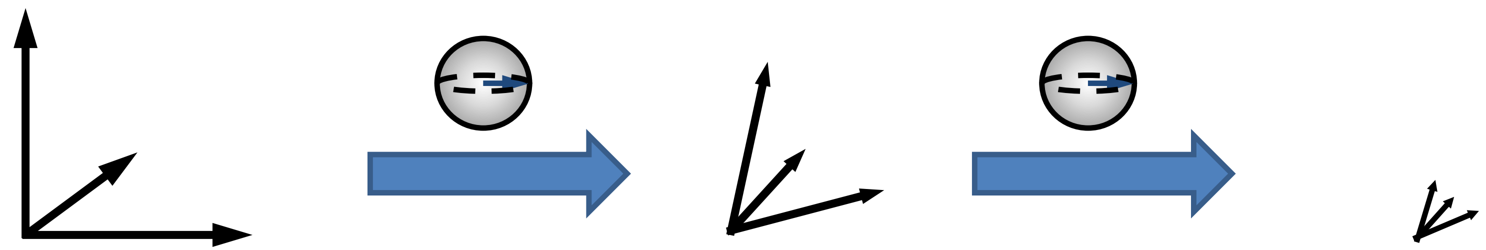
$$\mathcal{E}_T\left(e^{-iHt}\rho e^{iHt}\right) = e^{-iHt}\mathcal{E}_T(\rho)e^{iHt}$$

No reference frame = average over the free evolution: $\rho \rightarrow D(\rho)$

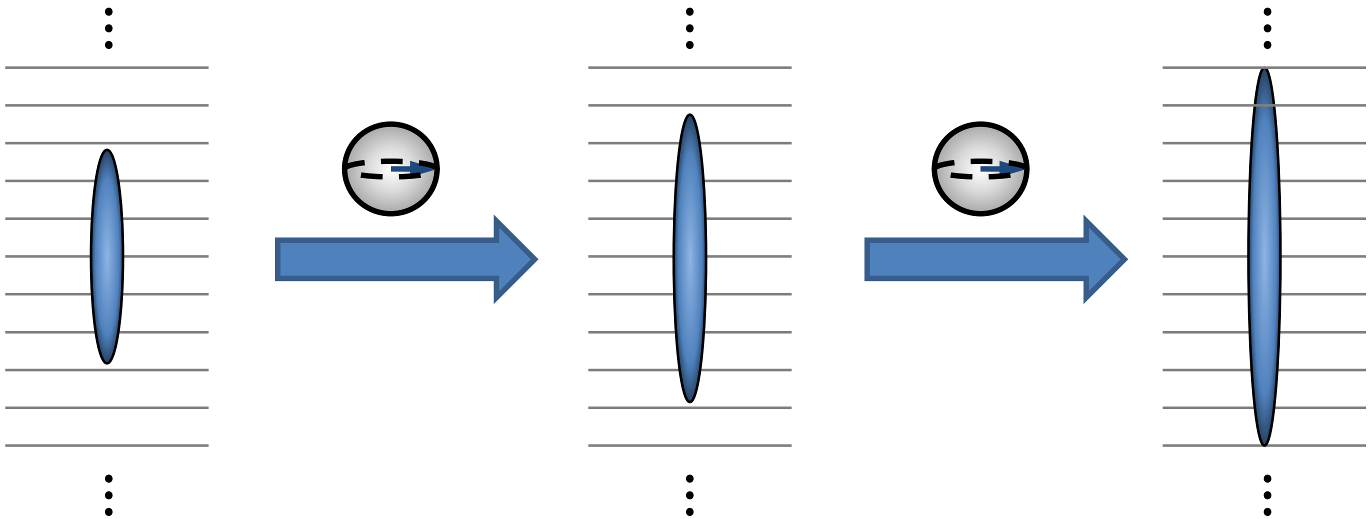
Reference = ancillary system in a state with coherence

Example: Single-mode bosonic field $H_R = \sum_n n(E_1 - E_0)\,|n\rangle\langle n|$ in a coherent state $|\alpha\rangle$ or a uniform superposition of energy eigenstates $|\psi_L\rangle \propto \sum_{n=0}^{L} |n\rangle$.

Using a reference frame one can access the information encoded in coherences and therefore extract more work than $\Delta F(D(\rho))$, but:



Coherence catalysis?



The reference gets disturbed: $\rho_R \to \rho_R{}' \to \rho_R''$

But its “quality” Δ stays constant: $\langle\Delta(\rho_R)\rangle = \langle\Delta(\rho_R{}')\rangle = \langle\Delta(\rho_R{}'')\rangle$

Problems:

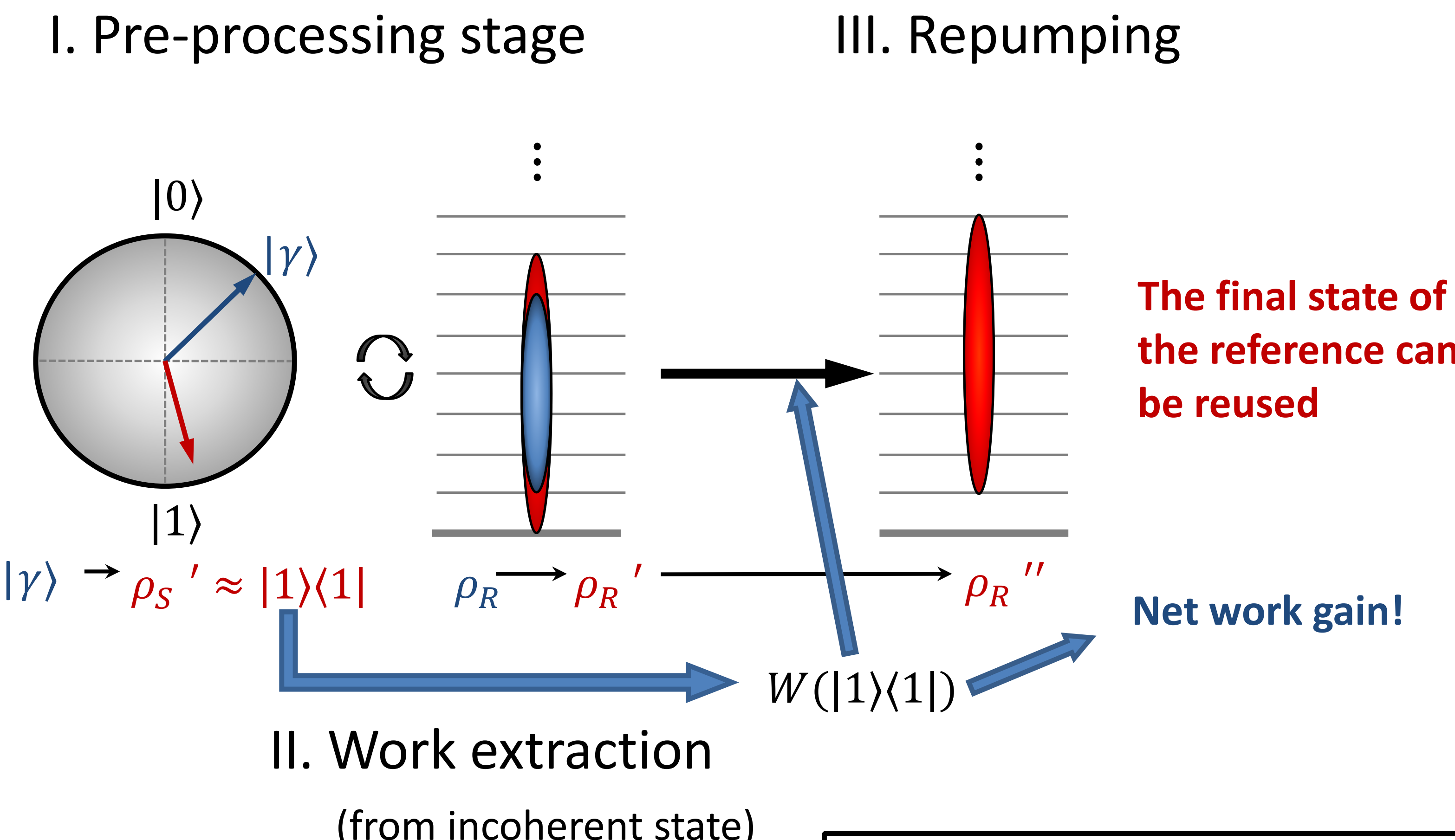
1. Unphysical Hamiltonian – no ground state.
2. Reference itself is an infinite reservoir of free energy – simply lower its state as long you want.

J. Åberg, Phys. Rev. Lett. **113** 150402 (2014)

Unlocking work with a repeatable resource

Solution:

Use a single-mode bosonic field (a laser): $H_R = \sum_n n(E_1 - E_0)\,|n\rangle\langle n|$



K. Korzekwa, M. Lostaglio, J. Oppenheim, D. Jennings
New J. Phys. **18**, 023045 (2016)

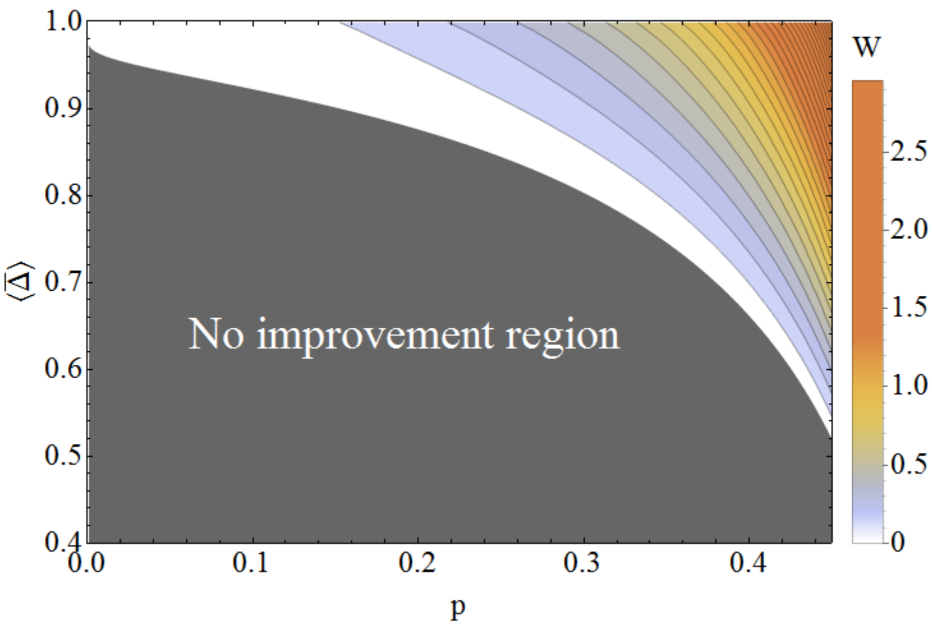
Results

In the limit of a unbounded reference (strong laser field) all work can be extracted from coherence, without deteriorating the reference (the laser field):

$$W(\rho) \to \Delta F(\rho), \qquad \langle \Delta(\rho_R) \rangle \to \langle \Delta(\rho_R{}') \rangle$$

However, even a bounded reference can unlock some work from coherence without being deteriorated:

$$\Delta F(D(\rho)) < W(\rho) < \Delta F(\rho), \qquad \langle \Delta(\rho_R) \rangle = \langle \Delta(\rho_R{}') \rangle$$



$\langle \Delta \rangle$ - quality of the reference

$\langle \Delta \rangle = 1 \Leftrightarrow$ unbounded coherence

$\langle \Delta \rangle = 0 \Leftrightarrow$ no coherence

***p* – thermal occupation of excited state**

$p = 0 \Leftrightarrow T = 0 \qquad p = \frac{1}{2} \Leftrightarrow T = \infty$

Conclusions

- In the presence of a heat bath only “speakable” information can be converted into work.
- Coherence in the energy eigenbasis forms “unspeakable information”; conversion into work requires a reference frame, e.g. a laser in a coherent state.
- Coherence resources of a reference frame should be used in a repeatable way.
- Unbounded reference – all the coherence can be repeatably converted into work.
- Finite reference – part of the coherence can be repeatably converted into work.

Thank you!