

HW 7 XG BOOST Konstanty Uoniewin

SCENARIO A

$$\lambda^* = \underset{\lambda}{\operatorname{argmin}} \sum_{i=1}^n L(y_i, \lambda) \quad \sigma(z) = \frac{1}{1+e^{-z}}$$

$$L(y, \lambda) = -y \log(\sigma(\lambda)) - (1-y) \log(1-\sigma(\lambda))$$

$$L(y, \lambda) = -y \log\left(\frac{1}{1+e^{-\lambda}}\right) - (1-y) \log\left(\frac{e^{-\lambda}}{1+e^{-\lambda}}\right)$$

$$= -y (\log 1 - \log(1+e^{-\lambda})) - (1-y) (\log e^{-\lambda} - \log(1+e^{-\lambda}))$$

$$= -y (0 - \log(1+e^{-\lambda})) - (1-y) (-\lambda - \log(1+e^{-\lambda}))$$

$$= y \log(1+e^{-\lambda}) + (y-1) (-\lambda - \log(1+e^{-\lambda}))$$

$$\frac{\partial L}{\partial \lambda} = \frac{y}{1+e^{-\lambda}} \cdot (-e^{-\lambda}) + (y-1) \left(-1 - \frac{1}{1+e^{-\lambda}} \cdot (-e^{-\lambda}) \right)$$

$$= y \frac{-e^{-\lambda}}{1+e^{-\lambda}} + (y-1) \left(\frac{e^{-\lambda}}{1+e^{-\lambda}} - 1 \right) =$$

$$= -y - \frac{e^{-\lambda}}{1+e^{-\lambda}} + 1 = 0 \Rightarrow 1-y = \frac{1}{1+e^{-\lambda}}$$

$$e^{-\lambda} + 1 = \frac{1}{y} / \log \quad -\lambda = \log\left(\frac{1}{y} - 1\right)$$

$$\lambda^* = -\log\left(\frac{1-y}{y}\right) = \log\left(\frac{y}{1-y}\right)$$

$$f(\lambda) = \sum_{i=1}^n L(y_i, \lambda) = \sum_{i=1}^n \left[-y_i \log(\sigma(\lambda)) - (1-y_i) \log(1-\sigma(\lambda)) \right]$$

$$= -\log(\sigma(\lambda)) \cdot \sum y_i - \log(1-\sigma(\lambda)) \cdot \sum (1-y_i)$$

$$= -\log(\sigma(\lambda)) m - \log(1-\sigma(\lambda)) k$$

... take sum

$$\lambda^* = \log\left(\frac{\frac{m}{n}}{1-\frac{m}{n}}\right) = \log\left(\frac{m}{k}\right)$$

SCENARIO 1B

$$\sigma'(x) = \sigma(x)(1-\sigma(x))$$

$$f(\lambda) = \sum_{i=1}^n L(y_i, f_i + \lambda) =$$

$$= \sum_{i=1}^n \left[-y_i \log(\sigma(f_i + \lambda)) - (1-y_i) \log(1-\sigma(f_i + \lambda)) \right]$$

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