Metamodeling of Structural Systems with Parametric Uncertainty Subject to Stochastic Excitation

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Talk Outline

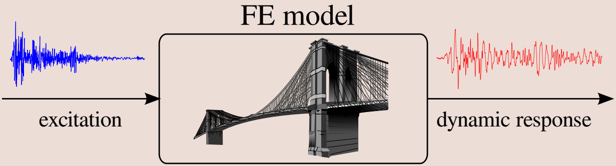
1. Introduction
2. PC-NARX metamodels
3. Numerical Case Study
4. Concluding Remarks & Outlook

Background & motivation

Simulation models are extensively used in civil engineering practice. Such models allow the user to

- understand structural system performance,
- predict structural behavior,
- diagnose damage,
- optimize design, etc

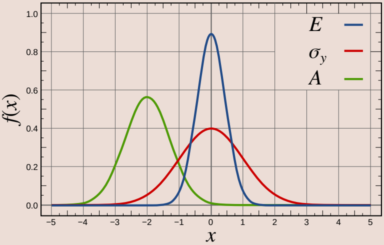
without "threatening" the integrity of the structure. Yet, in a lot of cases realistic excitation is either unrealizable or too costly.



The simulation of dynamic response through FE models requires excessive computational resources particularly for complex, large structures.

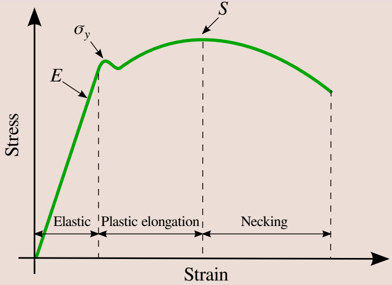
Problem characteristics

Structural system is characterized by parameter uncertainty



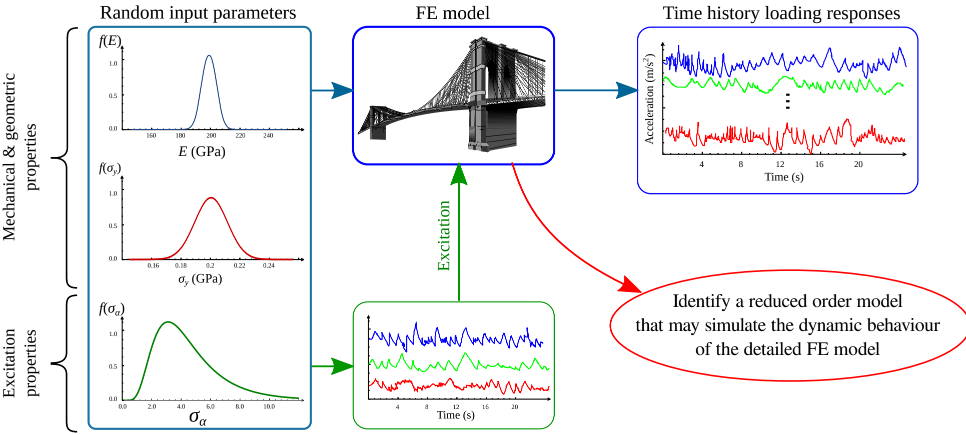
The behaviour of the modelled structure has to be examined for a range of structural characteristics.

Nonlinearities are taken into account



The impact of different types of excitation (of different magnitude and/or spectral content) should also be examined.

The metamodeling problem



Problem definition

Consider a structural system represented by a numerical model $\mathscr{M}$ characterized by uncertain input parameters $\xi = [\xi_1, \xi_2, \ldots, \xi_M]^{\mathsf{T}}$ with known joint pdf $f(\xi)$. The dynamic response of $\mathscr{M}$ to a given input excitation $x[t, \xi]$ will also be a random variable:

$$y[t,\xi] = \mathscr{M}(x[1,\xi], x[2,\xi], \ldots, x[t,\xi], \xi), \quad t = 1, 2, \ldots, T$$

A metamodel $\tilde{\mathscr{M}}$ which must be able to predict and/or simulate the detailed numerical model results in a computationally inexpensive way and with sufficient accuracy is sought.

Objectives of the study

- Development of a metamodeling method based on PC-NARX models
- Introduction of PC-NARX identification methods for both prediction and simulation purposes.
- The validation of PC-NARX metamodeling method through its application to the case of a five-storey shear frame model subjected to dynamic excitation leading to nonlinear response.

Polynomial Chaos Nonlinear ARX (PC-NARX) models

$$\boxed{y[t] = \sum_{i=1}^{n_\theta} \theta_i(\xi) \cdot g_i(z[t]) + e[t]}$$

random parameters $\theta_i(\xi)$ describe the uncertainty propagation. They may be expanded on a PC basis orthogonal to the pdf of the random input variables ξ

$$\theta_i(\xi) = \sum_{j=1}^{p} \theta_{i,j} \cdot \phi_{d(j)}(\xi)$$

$g_i(\cdot)$: nonlinear function operators that reflect the nonlinear structural dynamics

$z[t] = \big[y[t-1], \ldots, y[t-n_a], x[t], \ldots, x[t-n_b]\big]^{\mathsf{T}}$: regression vector
n_a, n_b: maximum output and input time lags
$e[t]$: residual sequence
$\theta_{i,j}$: unknown deterministic coefficients of projection
$d(j)$: multi-indices of the multivariate polynomial basis

n_θ: number of nonlinear regression terms
σ_e^2: residual sequence variance
$\phi_{d(j)}$: basis functions orthonormal w.r.t. the joint pdf of ξ

PC-NARX parameter estimation

- coefficients of projection $\theta_{i,j}$

PC-NARX structure selection

- select nonlinear functions $g_i(z[t])$ (polynomial, wavelet, radial basis functions, and so on)
- select PC functional subspace

Estimation of a PC-NARX model for purposes of prediction

Consider K simulations conducted with

$\xi_k = [\xi_{k,1}, \xi_{k,2}, \ldots, \xi_{k,M}]^{\mathsf{T}}$: random input parameter vector realizations

$x_k^T = \{x_k[1,\xi_k], x_k[2,\xi_k], \ldots, x_k[T,\xi_k]\}$: set of input excitation signals ($k = 1, 2, \ldots, K$)

$\Downarrow$

$$y_k^T = \{y_k[1,\xi_k], y_k[2,\xi_k], \ldots, y_k[T,\xi_k]\}$$

corresponding set of the full scale numerical model dynamic responses

(assumed to also follow a PC-NARX model)

$\Downarrow$

Estimation of the coefficients of projection $\theta = [\, a_{1,1}, \ldots, a_{n_a,p} b_{0,1}, \ldots, b_{n_b,p} \,]^{\mathsf{T}}$ based on the minimization of the Prediction Error criterion:

$$\widehat{\theta} = \arg\min_{\theta} \left\{ \sum_{k=1}^{K} \sum_{t=1}^{T} (y_k[t] - \hat{y}_k[t|t-1])^2 \right\} = \arg\min_{\theta} \left\{ \sum_{k=1}^{K} \sum_{t=1}^{T} e_k^2[t] \right\}$$

$\hat{y}_k[t|t-1]$: PC-ARX model's one-step-ahead prediction

$\Downarrow$

Ordinary Least Squares (OLS) estimator: $\boxed{\widehat{\theta} = \left(\Phi^{\mathsf{T}}(\xi) \cdot \Phi(\xi)\right)^{-1} \cdot \left(\Phi(\xi)^{\mathsf{T}} \cdot Y\right)}$

$\Phi(\xi)$: regression matrix $\quad$ Y : pooled response signal vector

Estimation of a PC-ARX model for purposes of simulation

The simulated response of a given PC-NARX metamodel may be obtained recursively as:

$$\boxed{\bar{y}_k[t] = \sum_{i=1}^{n_\theta} \theta_i(\xi_k) \cdot g_i(\bar{z}[t]), \quad t = 1, 2, \ldots, T}$$

with given initial conditions $\{\bar{y}_k[1-n_a], \ldots, \bar{y}_k[0]\}$ and $\{x_k[1-n_b], \ldots, x_k[0]\}$

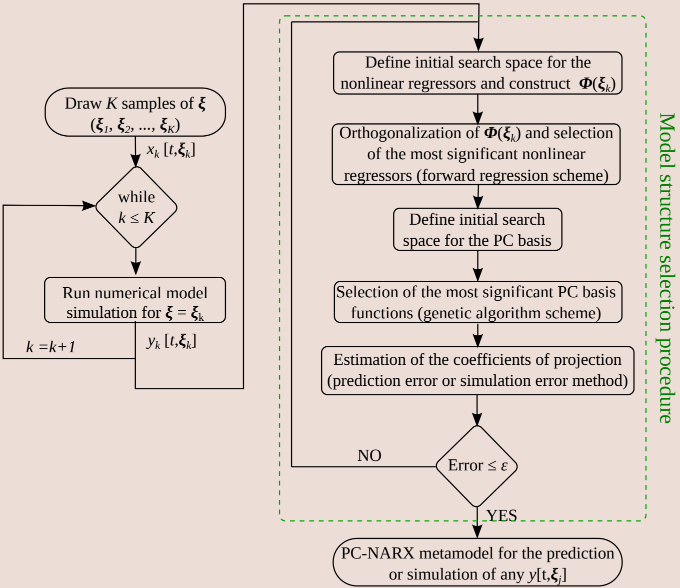
Estimation of the model coefficients of projection θ based on the minimization of the Simulation Error criterion:

$$\widehat{\theta}_s = \arg\min_{\theta_s} \left\{ \sum_{k=1}^{K} \sum_{t=1}^{T} (y_k[t] - \bar{y}_k[t])^2 \right\} = \arg\min_{\theta_s} \left\{ \sum_{k=1}^{K} \sum_{t=1}^{T} \varepsilon_k^2[t] \right\}$$

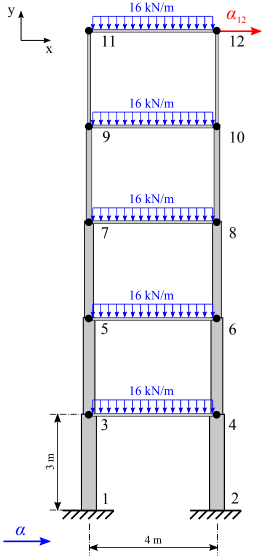
$$\Downarrow$$

Iterative nonlinear optimization methods

Flowchart of the complete identification scheme



A five-storey shear frame model



Mechanical & geometric properties

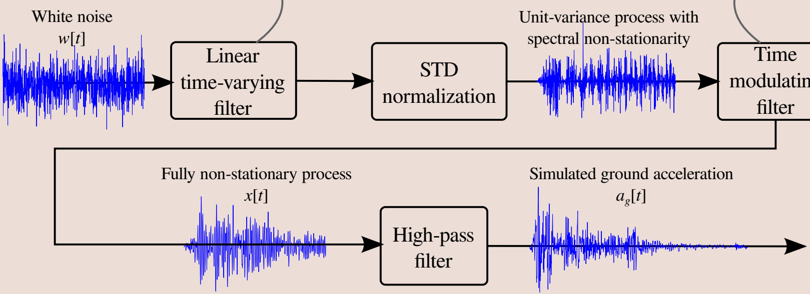
Geometric		Mechanical	
Cross-sectional area	cm^2		
1st storey columns	900	Poisson ratio	0.29
2nd storey columns	625	Density (kg/m^3)	7850
3rd storey columns	400	Yield stress (MPa)	200
4th storey columns	225	Tangent modulus (GPa)	10
5th storey columns	100		
Horizontal beams	100		

Parametric modelling of earthquake accelerograms

[S. Rezaeian & A.D. Kiureghian 2010]

$$h[t-\tau,\tau] = \frac{\omega_f(\tau)}{\sqrt{1-\zeta^2}} \exp[-\zeta \cdot \omega_f(\tau)(t-\tau)] \times \sin[\omega_f(\tau)\sqrt{1-\zeta^2}(t-\tau)]$$

$$q(t,\boldsymbol{\alpha}) = \alpha_1 t^{\alpha_2 - 1} \exp(\alpha_3 t)$$

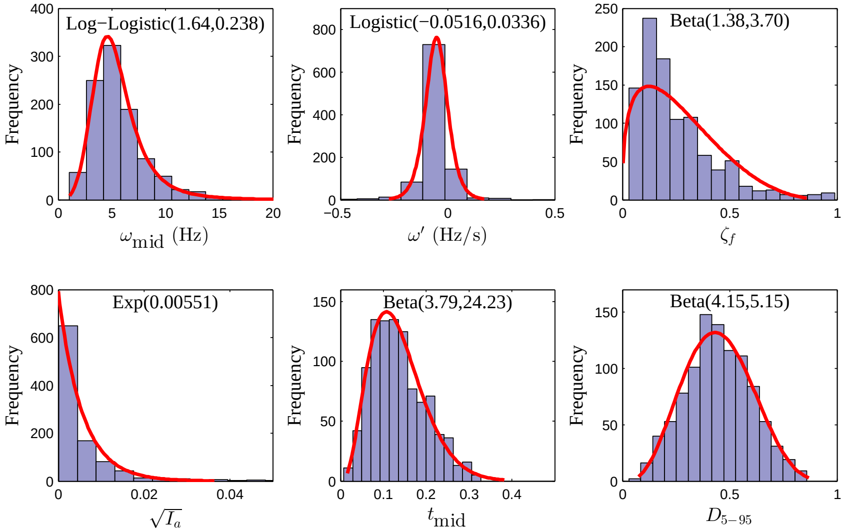


ζ_f: damping ratio of the filter
$\omega_f(\tau) = \omega_{\text{mid}} + \omega'(\tau - t_{\text{mid}})$: filter's frequency
ω_{mid}: filter frequency at t_{mid}
ω': rate of change of the filter frequency

α_i's: parameters of the modulating gamma function. Directly estimated from:
I_a: Arias intensity
D_{5-95} : effective duration of the motion
t_{mid}: time at which 45% of the Arias intensity is reached

Modelling of the PEER database accelerograms

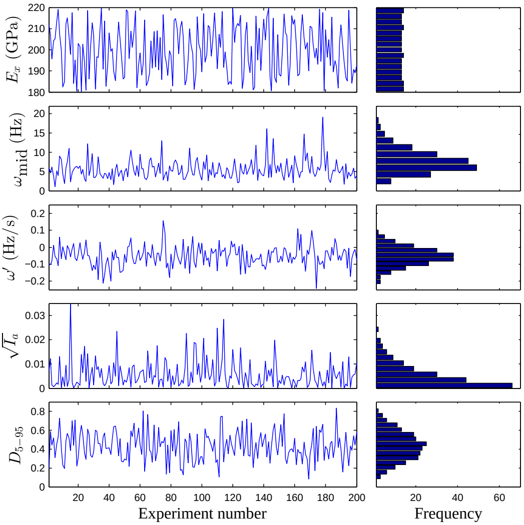
(results from the 1000 accelerograms with the best fit)



Input random vector realizations for the 200 simulations conducted

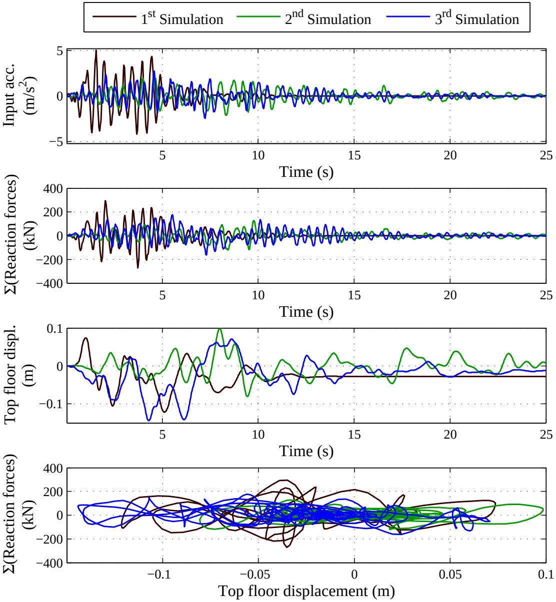
Random input variables

Variable	Distribution	pdf parameters
E (GPa)	Uniform	$\min = 180$ $\max = 220$
ω_{mid} (Hz)	Log-Logistic	$\alpha = 1.64$ $\beta = 0.238$
ω' (Hz/s)	Logistic	$\mu = -0.0516$ $\sigma = 0.0336$
$\sqrt{I_a}$	Exponential	$\mu = 0.00551$
D_{5-95}	Beta	$\alpha = 4.15$ $\beta = 5.15$



Input-output data

Simulation time	$T_{\mathrm{sim}} = 25$ (s)
Sampling frequency	$f_s = 40$ (Hz)
# of samples	$T = 1000$
# of experiments	$K = 200$



PC-NARX identification results

Nonlinear regressors:

Initial search space:

$g_i(z[t]) = z_{j1}^{\ell_1}[t] \cdot z_{j2}^{\ell_2}[t]$ with $\ell_1, \ell_2 = 0, \ldots 3$, $\ell_1 + \ell_2 \leq 3$

$z[t] = [\ y[t-1], \ldots, y[t-10], x[t], x[t-1], \ldots, x[t-10]\ ]^{\mathsf{T}}$

Finally selected terms:

$y[t-1], \ldots, y[t-10], x[t], x[t-1], \ldots, x[t-10],$

$y[t-1] \cdot y^2[t-2], \ldots, y[t-1] \cdot y^2[t-10],$

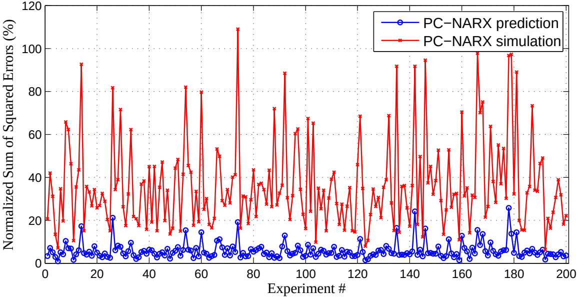
$y^2[t-1] \cdot y[t-2], \ldots, y^2[t-1] \cdot y[t-10],$

$y^3[t-1], y^3[t-2], y^3[t-3].$

Multi-indices of the selected PC basis functions

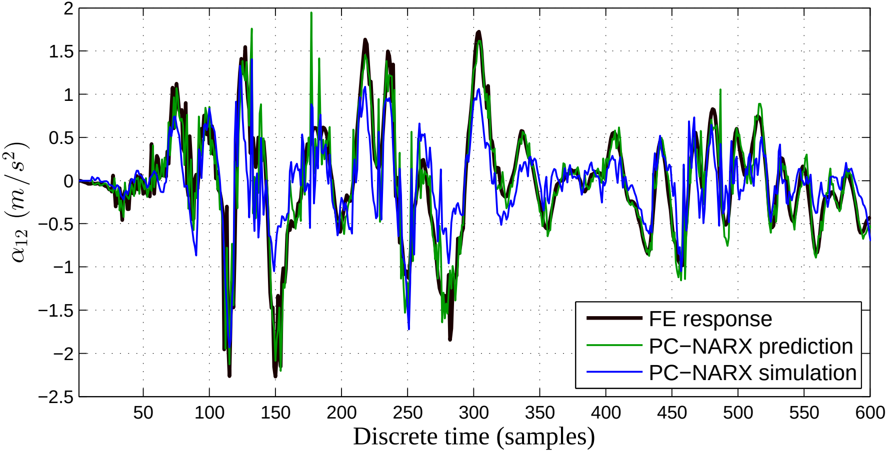
	E	ω_{mid}	ω'	I_a	D_{5-95}
$d(1)$	0	0	0	0	0
$d(2)$	1	0	0	0	0
$d(3)$	0	1	0	0	0
$d(4)$	0	0	0	0	1
$d(5)$	1	0	0	1	0
$d(6)$	0	1	0	0	1
$d(7)$	0	2	0	0	0
$d(8)$	1	2	0	0	0
$d(9)$	0	3	0	0	0

PC-NARX based prediction and simulation errors



Validation based on a real earthquake ground motion acceleration excitation: FE vs PC-NARX metamodel

(El Centro earthquake time history loading)



Normalized residual sum of squares: prediction 13.15%, simulation 38.10%

Concluding Remarks

- Stochastic metamodels of low order that are capable of accurately approximating FE models are developed.
- The metamodeling method is based on Nonlinear ARX models and Polynomial Chaos basis expansion.
- The numerical results show good prediction and simulation accuracy of the dynamic response of the model.
- The proposed methodology may be adapted as an approximative low cost surrogate for a number of purposes such as vibration control, SHM, model updating and others.

The PC basis functions $\phi_{d(j)}$ are orthonormal with respect to the joint probability density function of ξ:

$$E[\phi_\alpha(\xi), \phi_\beta(\xi)] = \delta_{\alpha,\beta} = \begin{cases} 1 & \text{for } \alpha = \beta \\ 0 & \text{otherwise} \end{cases}$$

PDF	Support	Polynomials
Normal (Gaussian)	$(-\infty, \infty)$	Hermite
Uniform	$[-1, 1]$	Legendre
Gamma	(0,1)	Laguerre
Chebyshev	(-1, 1)	Chebyshev
Beta	(-1, 1)	Jacobi