Time Series Analysis: Cambodia Rainfall Forecasting

Course: Time Series Analysis
Group: 01
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✅ Corrected Version - All Modern APIs

This notebook uses:

- ✅ Modern `statsmodels.tsa.arima.model.ARIMA` and `SARIMAX` (no deprecated `disp` parameter)
- ✅ Proper train-test split (no data leakage)
- ✅ Out-of-sample rolling forecasts for evaluation
- ✅ Seasonal decomposition and seasonal differencing
- ✅ Residual diagnostics (ACF, Ljung-Box, normality tests)
- ✅ Model comparison with proper metrics

Table of Contents

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1. Import Libraries

All modern APIs - no deprecated functions.

In [ ]:
```
import warnings

# Data manipulation
import numpy as np
import pandas as pd

# Visualization
import matplotlib.pyplot as plt
import seaborn as sns

plt.style.use("seaborn-v0_8-darkgrid")
sns.set_palette("husl")

# Time series analysis - MODERN APIs only
from statsmodels.tsa.stattools import adfuller, acf, pacf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.tsa.arima.model import ARIMA  # Modern ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX  # Modern SARIMA
from statsmodels.stats.diagnostic import acorr_ljungbox

# >>> add these imports <<<
from statsmodels.tools.sm_exceptions import ValueWarning, ConvergenceWarning

# Machine learning metrics
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score

# Set random seed
np.random.seed(42)

# Suppress specific statsmodels warnings
warnings.filterwarnings("ignore", category=ValueWarning)
warnings.filterwarnings("ignore", category=ConvergenceWarning)

import warnings
import statsmodels

warnings.filterwarnings("ignore")


# Completely silence all UserWarning coming from statsmodels
warnings.filterwarnings(
    "ignore", category=UserWarning, module=r"statsmodels\.tsa\.statespace\.sarimax"
)
```

---

2. Data Loading and Initial Exploration

Mathematical Background: Time Series Definition

A **time series** is a sequence of observations y_t indexed by time t, where $t = 1, 2, \ldots, T$.

$$y_t = f(t) + \varepsilon_t$$

where:

- $f(t)$ represents the systematic component (trend + seasonality)
- ε_t represents random noise (white noise)

Key Properties:

1. Mean: $\mu_t = \mathbb{E}[y_t]$
2. Variance: $\sigma_t^2 = \mathrm{Var}(y_t)$
3. Autocovariance: $\gamma(s,t) = \mathrm{Cov}(y_s, y_t)$

In [2]:

```
# Load Cambodia rainfall dataset from HDX
url = "https://data.humdata.org/dataset/8fa90d2b-a88e-414d-84a1-50d6bc773542/resource/67c4f3d6-f600-4699-9a67-0de20d6a1b0b/download/khm-rainfall-subnat-full.csv"

print("Loading Cambodia rainfall dataset...")
df_raw = pd.read_csv(url)
print(f"✓ Dataset loaded successfully!")
print(f"Shape: {df_raw.shape}")
print(f"\nFirst few rows:")
df_raw.head()
```

```
Loading Cambodia rainfall dataset...
✓ Dataset loaded successfully!
Shape: (358241, 15)

First few rows:
```

Out[2]:

	date	adm_level	adm_id	PCODE	n_pixels	rfh	rfh_avg	r1h	r1h_avg	r3h	r3h_avg	rfq	r1q	r3q	version
0	1981-01-01	1	900411	KH15	2.0	1.0	2.400000	NaN	NaN	NaN	NaN	81.081080	NaN	NaN	final
1	1981-01-11	1	900411	KH15	2.0	0.5	1.000000	NaN	NaN	NaN	NaN	91.666670	NaN	NaN	final
2	1981-01-21	1	900411	KH15	2.0	3.0	2.250000	4.5	5.650000	NaN	NaN	110.344826	89.20188	NaN	final
3	1981-02-01	1	900411	KH15	2.0	4.5	2.483333	8.0	5.733333	NaN	NaN	126.948780	121.11801	NaN	final
4	1981-02-11	1	900411	KH15	2.0	4.5	4.150000	12.0	8.883333	NaN	NaN	103.825140	122.44898	NaN	final

In [3]:

```
# Dataset information
print("=" * 60)
print("Dataset Information")
print("=" * 60)
df_raw.info()
print("\n" + "=" * 60)
print("Summary Statistics")
print("=" * 60)
df_raw.describe()
```

```
============================================================
Dataset Information
============================================================
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 358241 entries, 0 to 358240
Data columns (total 15 columns):
 #   Column     Non-Null Count   Dtype  
---  ------     --------------   -----  
 0   date       358241 non-null  object 
 1   adm_level  358241 non-null  int64  
 2   adm_id     358241 non-null  int64  
 3   PCODE      358241 non-null  object 
 4   n_pixels   358241 non-null  float64
 5   rfh        358241 non-null  float64
 6   rfh_avg    358241 non-null  float64
 7   r1h        357799 non-null  float64
 8   r1h_avg    357799 non-null  float64
 9   r3h        356473 non-null  float64
 10  r3h_avg    356473 non-null  float64
 11  rfq        358241 non-null  float64
 12  r1q        357799 non-null  float64
 13  r3q        356473 non-null  float64
 14  version    358241 non-null  object 
dtypes: float64(10), int64(2), object(3)
memory usage: 41.0+ MB

============================================================
Summary Statistics
============================================================
```

Out[3]:

	adm_level	adm_id	n_pixels	rfh	rfh_avg	r1h	r1h_avg	r3h	r3h_avg	rfq	r1q	r3q
count	358241.000000	3.582410e+05	358241.000000	358241.000000	358241.000000	357799.000000	357799.000000	356473.000000	356473.000000	358241.000000	357799.000000	356473.000000
mean	1.882353	9.938594e+05	53.918552	50.533208	50.446255	151.781830	151.475458	456.628298	455.553361	99.503847	99.195325	99.530930
std	0.322190	3.411807e+04	87.656551	53.591433	44.794200	142.794801	130.781864	369.396604	352.300300	48.516597	39.896489	31.625497
min	1.000000	9.004110e+05	1.000000	0.000000	0.000000	0.000000	0.019672	3.000000	14.302381	7.340091	6.648260	11.726528
25%	2.000000	1.006248e+06	11.000000	6.000000	11.220963	25.300001	35.766666	137.488890	148.483340	68.956610	74.300772	81.343100
50%	2.000000	1.006303e+06	22.000000	37.600000	43.526670	130.000000	137.500000	420.333300	427.566680	90.942500	94.072740	97.329310
75%	2.000000	1.006361e+06	50.000000	77.000000	77.346940	231.333320	229.926350	658.777800	644.513300	119.939100	116.974300	113.842650
max	2.000000	1.006416e+06	467.000000	803.608700	356.183320	1743.347900	941.975000	3548.782700	2363.883300	816.339500	511.743840	352.051330

---

3. Data Preprocessing and Monthly Aggregation

Mathematical Background: Temporal Aggregation

When aggregating from daily/dekadal to monthly data:

$$Y * m = \sum * d \in m y_d$$

where Y_m is total rainfall in month m, and y_d is rainfall on day/dekad d.

Properties:

- Reduces noise (smoothing effect)
- Captures longer-term patterns
- More suitable for seasonal modeling

In [4]:
```
# Convert date to datetime
df_raw["date"] = pd.to_datetime(df_raw["date"])

# Focus on provincial level (adm_level = 1)
df_province = df_raw[df_raw["adm_level"] == 1].copy()

print(f"Provincial-level records: {len(df_province)}")
print(f"Number of provinces: {df_province['PCODE'].nunique()}")
print(f"Provinces: {sorted(df_province['PCODE'].unique())}")
```

```
Provincial-level records: 42146
Number of provinces: 25
Provinces: ['KH01', 'KH02', 'KH03', 'KH04', 'KH05', 'KH06', 'KH07', 'KH08', 'KH09', 'KH10', 'KH11', 'KH12', 'KH13', 'KH14', 'KH15', 'KH16', 'KH17', 'KH18', 'KH19', 'KH20', 'KH21', 'KH22', 'KH2
3', 'KH24', 'KH25']
```

In [5]:
```
# Select province with most complete data
province_counts = df_province.groupby("PCODE").size()
selected_province = province_counts.idxmax()

print(f"Selected province: {selected_province}")
print(f"Records: {province_counts[selected_province]}")

df_selected = df_province[df_province["PCODE"] == selected_province].copy()
df_selected = df_selected.sort_values("date").reset_index(drop=True)
df_selected = df_selected[["date", "rfh"]].copy()
df_selected.columns = ["date", "rainfall"]

print(f"\nData range: {df_selected['date'].min()} to {df_selected['date'].max()}")
df_selected.head()
```

```
Selected province: KH15
Records: 3242

Data range: 1981-01-01 00:00:00 to 2026-01-01 00:00:00
```

Out[5]:

	date	rainfall
0	1981-01-01	1.000000
1	1981-01-01	1.393782
2	1981-01-11	0.500000
3	1981-01-11	1.489637
4	1981-01-21	3.000000

In [6]:
```
# Handle missing values - modern pandas syntax
print("=" * 60)
print("Missing Value Analysis")
print("=" * 60)
print(f"Missing values: {df_selected['rainfall'].isna().sum()}")
print(
    f"Percentage: {df_selected['rainfall'].isna().sum() / len(df_selected) * 100:.2f}%"
```

```
)

# Use modern pandas methods (no deprecated 'method' parameter)
df_selected["rainfall"] = df_selected["rainfall"].ffill().bfill()

print(f"After imputation: {df_selected['rainfall'].isna().sum()}")
```

```
============================================================
Missing Value Analysis
============================================================
Missing values: 0
Percentage: 0.00%
After imputation: 0
```

In [7]:

```
# Aggregate to monthly
df_selected["year_month"] = df_selected["date"].dt.to_period("M")

df_monthly = df_selected.groupby("year_month").agg({"rainfall": "sum"}).reset_index()

df_monthly["date"] = df_monthly["year_month"].dt.to_timestamp()
df_monthly = df_monthly[["date", "rainfall"]].copy()

print("=" * 60)
print("Monthly Data")
print("=" * 60)
print(f"Total months: {len(df_monthly)}")
print(f"Range: {df_monthly['date'].min()} to {df_monthly['date'].max()}")
print(f"\nFirst rows:")
df_monthly.head(10)
```

```
============================================================
Monthly Data
============================================================
Total months: 541
Range: 1981-01-01 00:00:00 to 2026-01-01 00:00:00

First rows:
```

Out[7]:

	date	rainfall
0	1981-01-01	17.272021
1	1981-02-01	76.948185
2	1981-03-01	53.823835
3	1981-04-01	149.800519
4	1981-05-01	333.725390
5	1981-06-01	569.883415
6	1981-07-01	589.834190
7	1981-08-01	582.481870
8	1981-09-01	486.427453
9	1981-10-01	472.746120

In [8]:

```
# Create time series
ts_data = df_monthly.set_index("date")["rainfall"]
ts_data = ts_data.asfreq("MS")
```

```
print(f"\nTime series shape: {ts_data.shape}")
print(f"Frequency: {ts_data.index.freq}")
ts_data.head()
```

```
Time series shape: (541,)
Frequency: <MonthBegin>
```

Out[8]:

```
date
1981-01-01     17.272021
1981-02-01     76.948185
1981-03-01     53.823835
1981-04-01    149.800519
1981-05-01    333.725390
Freq: MS, Name: rainfall, dtype: float64
```

---

4. Exploratory Data Analysis (EDA)

Mathematical Background: Descriptive Statistics

For time series $\{y*t\}*t=1^T$:

Sample Mean:

$$\bar{y} = \frac{1}{T}\sum\_t = 1^T y_t$$

Sample Variance:

$$s^2 = \frac{1}{T-1}\sum\_t = 1^T (y_t - \bar{y})^2$$

Coefficient of Variation:

$$CV = \frac{s}{\bar{y}} \times 100\%$$

Skewness:

$$\text{Skew} = \frac{1}{T}\sum\_t = 1^T \left(\frac{y_t - \bar{y}}{s}\right)^3$$

Kurtosis:

$$\text{Kurt} = \frac{1}{T}\sum\_t = 1^T \left(\frac{y_t - \bar{y}}{s}\right)^4$$

In [9]:

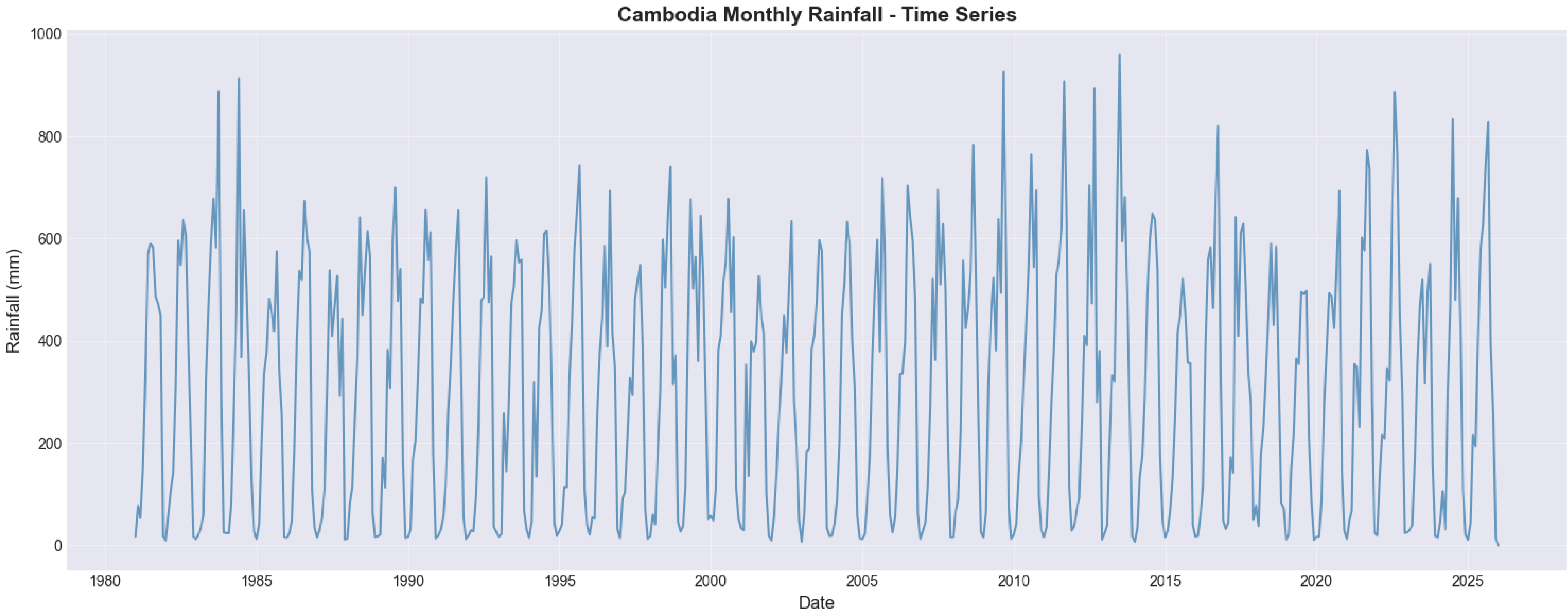
```
# Descriptive statistics
print("=" * 60)
print("Descriptive Statistics")
print("=" * 60)
stats = {
    "Count": len(ts_data),
    "Mean": ts_data.mean(),
    "Median": ts_data.median(),
```

```python
    "Std Dev": ts_data.std(),
    "Variance": ts_data.var(),
    "Min": ts_data.min(),
    "Max": ts_data.max(),
    "Range": ts_data.max() - ts_data.min(),
    "Q1": ts_data.quantile(0.25),
    "Q3": ts_data.quantile(0.75),
    "IQR": ts_data.quantile(0.75) - ts_data.quantile(0.25),
    "CV (%)": (ts_data.std() / ts_data.mean()) * 100,
    "Skewness": ts_data.skew(),
    "Kurtosis": ts_data.kurtosis(),
}

for key, value in stats.items():
    print(f"{key:15s}: {value:10.4f}")
```

```
============================================================
Descriptive Statistics
============================================================
Count          :   541.0000
Mean           :   306.2502
Median         :   308.3109
Std Dev        :   239.4916
Variance       : 57356.2467
Min            :     0.0104
Max            :   958.9870
Range          :   958.9767
Q1             :    55.2539
Q3             :   501.9611
IQR            :   446.7072
CV (%)         :    78.2013
Skewness       :     0.3308
Kurtosis       :    -1.0155
```

In [10]:

```python
# Time series plot
fig, ax = plt.subplots(figsize=(15, 6))

ax.plot(ts_data.index, ts_data.values, linewidth=1.5, color="steelblue", alpha=0.8)
ax.set_title("Cambodia Monthly Rainfall - Time Series", fontsize=14, fontweight="bold")
ax.set_xlabel("Date", fontsize=12)
ax.set_ylabel("Rainfall (mm)", fontsize=12)
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("timeseries_plot.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: timeseries_plot.png")
```



✓ Saved: timeseries_plot.png

---

5. Time Series Decomposition

Mathematical Background: Additive Decomposition

$$y_t = T_t + S_t + R_t$$

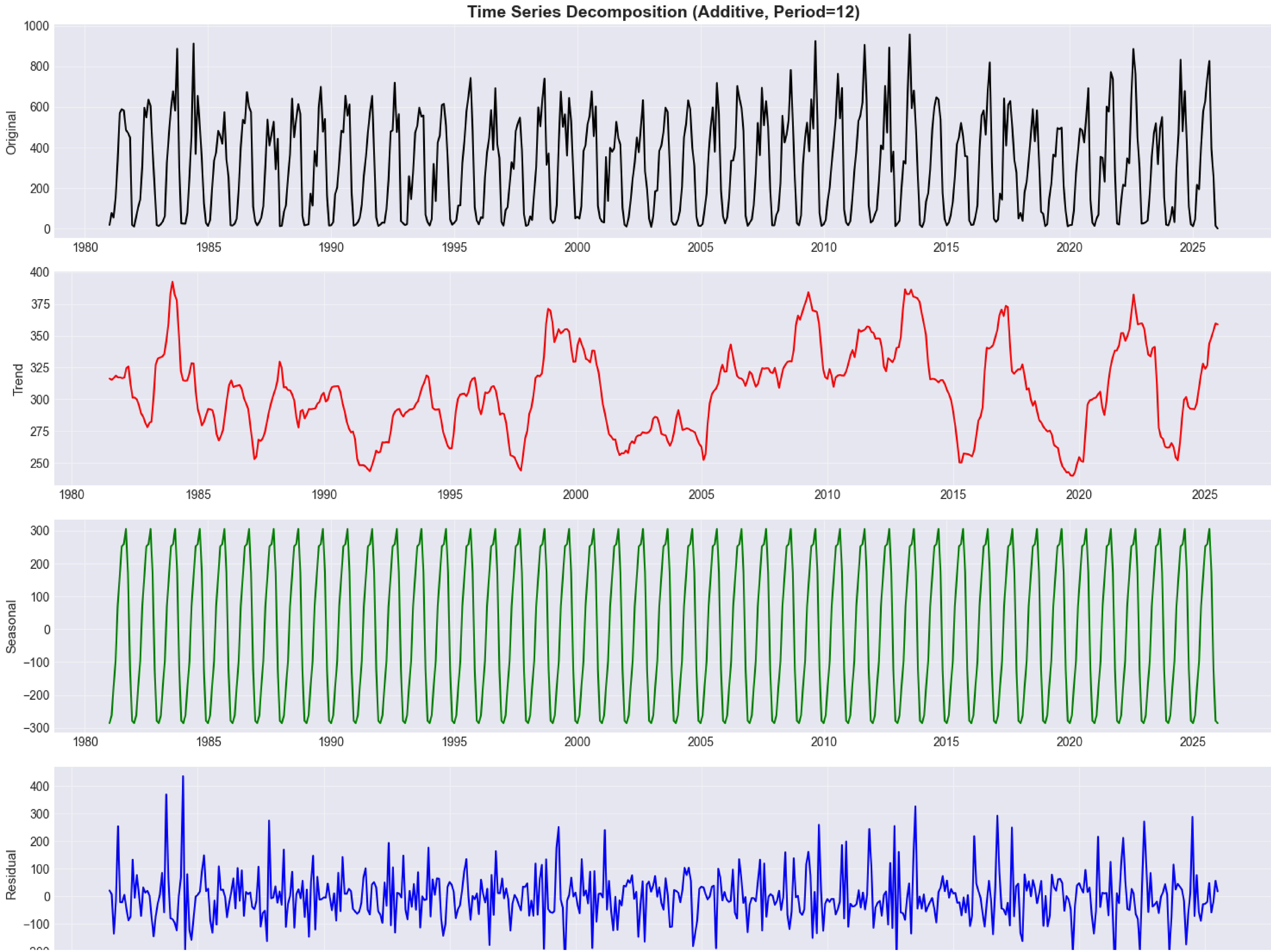
where:

- T_t = Trend component
- S_t = Seasonal component
- R_t = Residual (irregular) component

Seasonal Period: $s = 12$ months (annual seasonality)

In [11]:
```
# Seasonal decomposition
decomposition = seasonal_decompose(ts_data, model="additive", period=12)

fig, axes = plt.subplots(4, 1, figsize=(15, 12))

# Original
axes[0].plot(ts_data, linewidth=1.5, color="black")
```

```python
axes[0].set_ylabel("Original", fontsize=11)
axes[0].set_title(
    "Time Series Decomposition (Additive, Period=12)", fontsize=14, fontweight="bold"
)
axes[0].grid(True, alpha=0.3)

# Trend
axes[1].plot(decomposition.trend, linewidth=1.5, color="red")
axes[1].set_ylabel("Trend", fontsize=11)
axes[1].grid(True, alpha=0.3)

# Seasonal
axes[2].plot(decomposition.seasonal, linewidth=1.5, color="green")
axes[2].set_ylabel("Seasonal", fontsize=11)
axes[2].grid(True, alpha=0.3)

# Residual
axes[3].plot(decomposition.resid, linewidth=1.5, color="blue")
axes[3].set_ylabel("Residual", fontsize=11)
axes[3].set_xlabel("Date", fontsize=11)
axes[3].grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("decomposition.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: decomposition.png")
```



Time Series Decomposition (Additive, Period=12)
Original
Trend
Seasonal
Residual
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025


```
✓ Saved: decomposition.png
```

---

6. Stationarity Testing (ADF Test)

Mathematical Background: Augmented Dickey-Fuller Test

Null Hypothesis (H_0): The series has a unit root (non-stationary)
Alternative Hypothesis (H_1): The series is stationary

Test Statistic:

$$\Delta y * t = \alpha + \beta t + \gamma y * t - 1 + \sum *i = 1^p \delta_i \Delta y * t - i + \varepsilon_t$$

Decision Rule:

- If p-value < 0.05: Reject H_0 (series is stationary)
- If p-value ≥ 0.05: Fail to reject H_0 (series is non-stationary)

In [12]:
```python
def adf_test(series, name="Series"):
    """Perform Augmented Dickey-Fuller test"""
    result = adfuller(series.dropna(), autolag="AIC")

    print(f"\n{'='*60}")
    print(f"ADF Test Results: {name}")
    print(f"{'='*60}")
    print(f"ADF Statistic:      {result[0]:.6f}")
    print(f"p-value:            {result[1]:.6f}")
    print(f"# Lags Used:        {result[2]}")
    print(f"# Observations:     {result[3]}")

    print(f"\nCritical Values:")
    for key, value in result[4].items():
        print(f"  {key:5s}: {value:.3f}")

    if result[1] < 0.05:
        print(
            f"\n✓ Conclusion: Series is STATIONARY (p-value = {result[1]:.6f} < 0.05)"
        )
    else:
        print(
            f"\n✗ Conclusion: Series is NON-STATIONARY (p-value = {result[1]:.6f} ≥ 0.05)"
        )

    return result


# Test original series
adf_result_original = adf_test(ts_data, "Original Series")
```

```
============================================================
ADF Test Results: Original Series
============================================================
ADF Statistic:     -5.030478
p-value:           0.000019
# Lags Used:       17
# Observations:    523

Critical Values:
  1%   : -3.443
  5%   : -2.867
  10%  : -2.570

✓ Conclusion: Series is STATIONARY (p-value = 0.000019 < 0.05)
```

---

7. Seasonal Differencing

Mathematical Background: Seasonal Difference

$$\nabla * sy_t = y_t - y * t - s$$

where $s = 12$ (seasonal period)

This removes seasonal patterns and helps achieve stationarity.

In [13]:

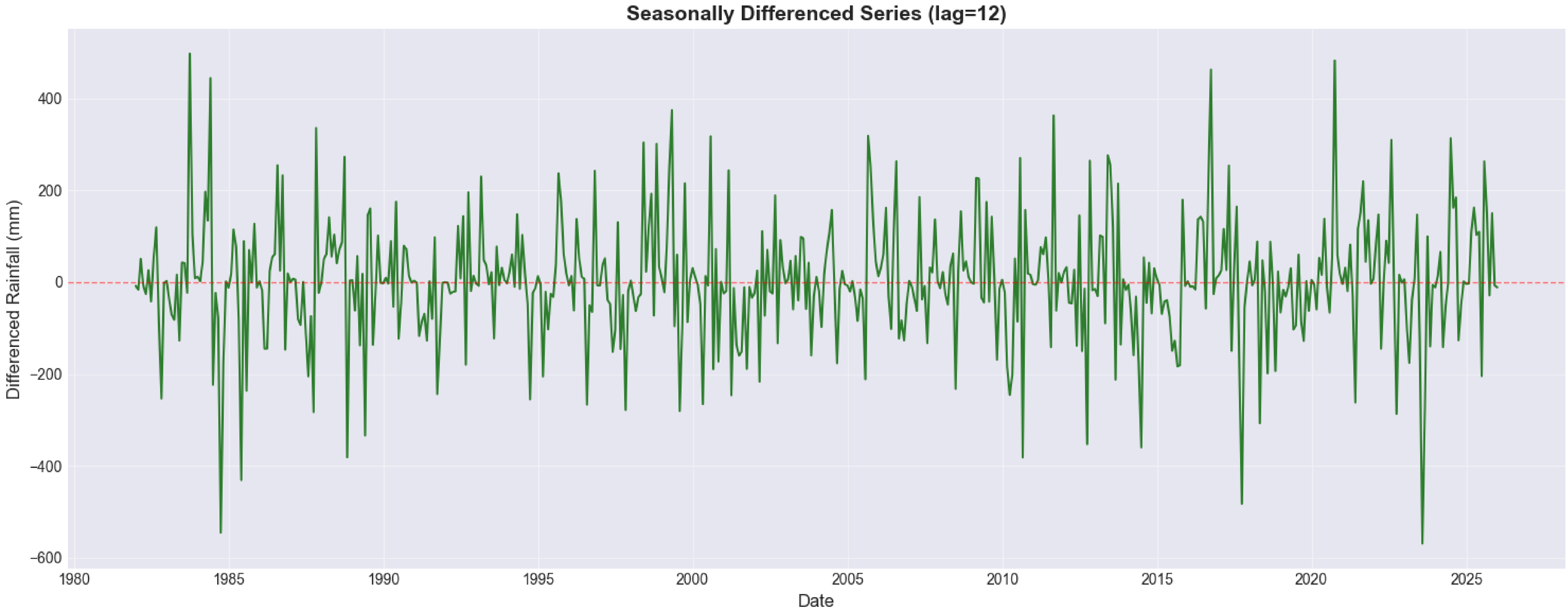
```python
# Seasonal differencing (lag = 12)
ts_seasonal_diff = ts_data.diff(12)

# Test stationarity after seasonal differencing
adf_result_seasonal = adf_test(
    ts_seasonal_diff, "Seasonally Differenced Series (lag=12)"
)
```

```
============================================================
ADF Test Results: Seasonally Differenced Series (lag=12)
============================================================
ADF Statistic:     -9.726492
p-value:           0.000000
# Lags Used:       11
# Observations:    517

Critical Values:
  1%   : -3.443
  5%   : -2.867
  10%  : -2.570

✓ Conclusion: Series is STATIONARY (p-value = 0.000000 < 0.05)
```

In [14]:

```python
# Plot seasonally differenced series
fig, ax = plt.subplots(figsize=(15, 6))

ax.plot(ts_seasonal_diff, linewidth=1.5, color="darkgreen", alpha=0.8)
ax.axhline(y=0, color="red", linestyle="--", linewidth=1, alpha=0.5)
ax.set_title("Seasonally Differenced Series (lag=12)", fontsize=14, fontweight="bold")
ax.set_xlabel("Date", fontsize=12)
ax.set_ylabel("Differenced Rainfall (mm)", fontsize=12)
```

```
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("seasonal_differencing.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: seasonal_differencing.png")
```




```
✓ Saved: seasonal_differencing.png
```

---

8. ACF and PACF Analysis

Mathematical Background

Autocorrelation Function (ACF):

$$\rho(k) = \frac{\mathrm{Cov}(y*t, y*t-k)}{\mathrm{Var}(y_t)}$$

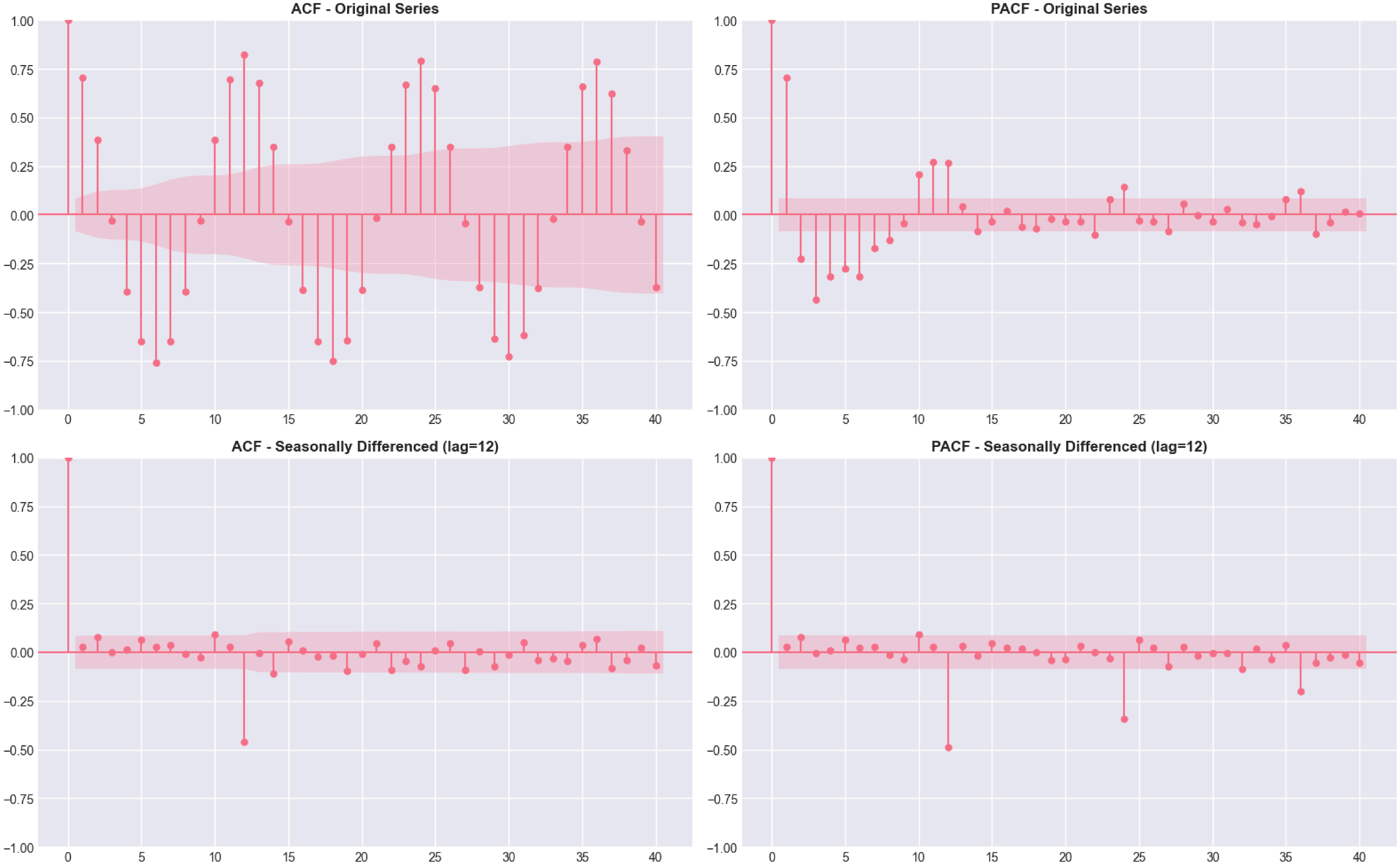
Partial Autocorrelation Function (PACF):
The correlation between $y*t$ and $y*t-k$ after removing the effect of intermediate lags.

Model Identification:

- **AR(p)**: PACF cuts off after lag p, ACF decays
- **MA(q)**: ACF cuts off after lag q, PACF decays
- **ARMA(p,q)**: Both ACF and PACF decay

In [15]:

```python
# ACF and PACF plots
fig, axes = plt.subplots(2, 2, figsize=(16, 10))

# Original series
plot_acf(ts_data.dropna(), lags=40, ax=axes[0, 0])
axes[0, 0].set_title("ACF - Original Series", fontsize=12, fontweight="bold")

plot_pacf(ts_data.dropna(), lags=40, ax=axes[0, 1])
axes[0, 1].set_title("PACF - Original Series", fontsize=12, fontweight="bold")

# Seasonally differenced series
plot_acf(ts_seasonal_diff.dropna(), lags=40, ax=axes[1, 0])
axes[1, 0].set_title(
    "ACF - Seasonally Differenced (lag=12)", fontsize=12, fontweight="bold"
)

plot_pacf(ts_seasonal_diff.dropna(), lags=40, ax=axes[1, 1])
axes[1, 1].set_title(
    "PACF - Seasonally Differenced (lag=12)", fontsize=12, fontweight="bold"
)

plt.tight_layout()
plt.savefig("acf_pacf.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: acf_pacf.png")
```



✓ Saved: acf_pacf.png

---

9. Train-Test Split (No Data Leakage)

✅ Correct Approach: Temporal Split

To prevent **data leakage**, we split the data temporally:

- **Train set:** Earlier observations (for model fitting)
- **Test set:** Later observations (for out-of-sample evaluation)

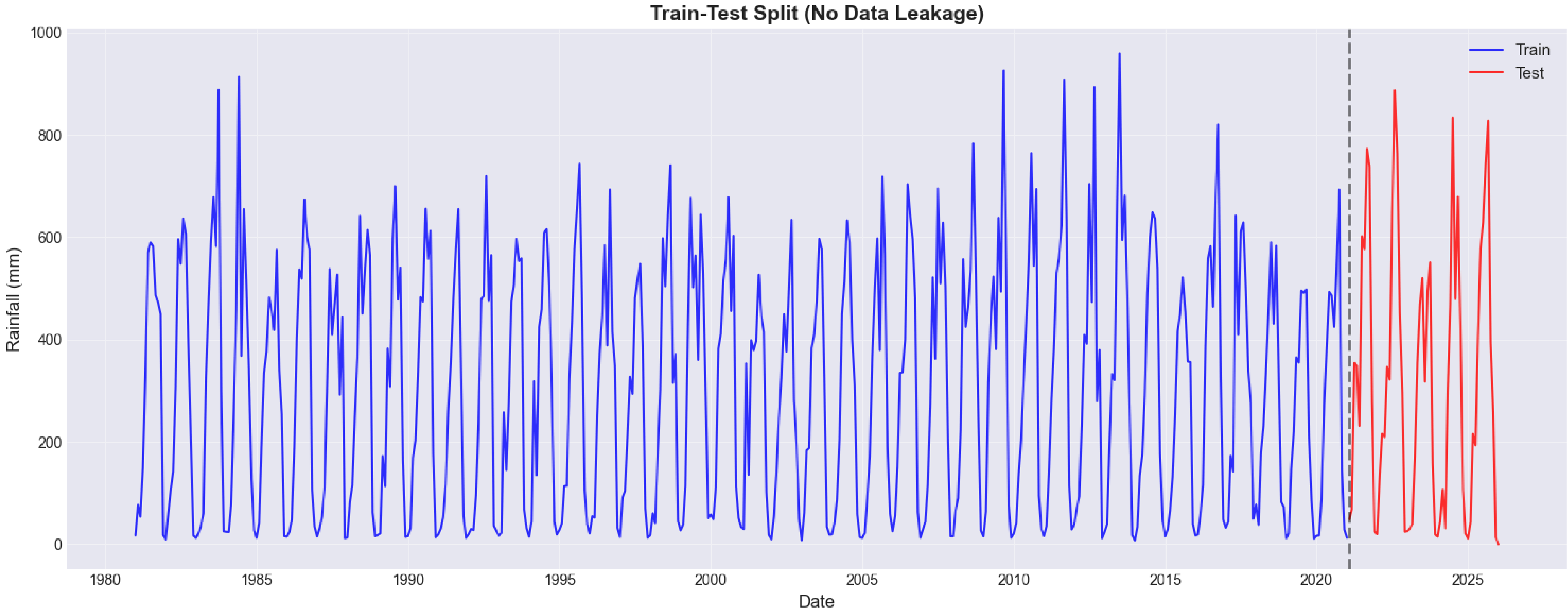
We use the **last 60 months (~5 years)** as the test set.

In [16]:
```
# Define test horizon
test_horizon = 60  # Last 5 years for testing

# Split data
train_data = ts_data.iloc[:-test_horizon]
test_data = ts_data.iloc[-test_horizon:]

print("=" * 60)
print("Train-Test Split")
print("=" * 60)
print(
    f"Train: {train_data.index[0]} to {train_data.index[-1]} ({len(train_data)} months)"
)
print(f"Test:  {test_data.index[0]} to {test_data.index[-1]} ({len(test_data)} months)")
print(f"\nTrain set: {len(train_data)} observations")
print(f"Test set:  {len(test_data)} observations")
print(f"Total:     {len(ts_data)} observations")
```

```
============================================================
Train-Test Split
============================================================
Train: 1981-01-01 00:00:00 to 2021-01-01 00:00:00 (481 months)
Test:  2021-02-01 00:00:00 to 2026-01-01 00:00:00 (60 months)

Train set: 481 observations
Test set:  60 observations
Total:     541 observations
```

In [17]:
```
# Visualize train-test split
fig, ax = plt.subplots(figsize=(15, 6))

ax.plot(
    train_data.index,
    train_data.values,
    linewidth=1.5,
    color="blue",
    label="Train",
    alpha=0.8,
)
ax.plot(
    test_data.index,
    test_data.values,
    linewidth=1.5,
    color="red",
    label="Test",
    alpha=0.8,
)
ax.axvline(x=test_data.index[0], color="black", linestyle="--", linewidth=2, alpha=0.5)

ax.set_title("Train-Test Split (No Data Leakage)", fontsize=14, fontweight="bold")
ax.set_xlabel("Date", fontsize=12)
ax.set_ylabel("Rainfall (mm)", fontsize=12)
```

```
ax.legend(fontsize=11)
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("train_test_split.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: train_test_split.png")
```




```
✓ Saved: train_test_split.png
```

---

10. Model Fitting with Out-of-Sample Evaluation

✅ Correct Approach: Rolling Forecast

We use **rolling one-step-ahead forecasts** on the test set:

1. Fit model on train data
2. Forecast 1 step ahead
3. Add actual observation to training set
4. Repeat for entire test period

This provides **true out-of-sample evaluation** without data leakage.

Modern API: No `disp` parameter

All models use:

- `ARIMA()` from `statsmodels.tsa.arima.model`
- `SARIMAX()` from `statsmodels.tsa.statespace.sarimax`
- `.fit()` with **NO** `disp` parameter (removed in statsmodels 0.14+)

In [18]:
```python
def rolling_forecast_evaluation(series, order, seasonal_order=None, model_name="Model"):
    """
    Perform rolling one-step-ahead forecast on test set.

    Parameters:
    -----------
    series : pd.Series
        Full time series (train + test)
    order : tuple
        ARIMA order (p, d, q)
    seasonal_order : tuple or None
        SARIMA seasonal order (P, D, Q, s) or None
    model_name : str
        Name of the model for reporting

    Returns:
    --------
    predictions : pd.Series
        Out-of-sample predictions
    mae : float
        Mean Absolute Error
    rmse : float
        Root Mean Squared Error
    """
    # Split indices
    train_idx = series.index[:-test_horizon]
    test_idx = series.index[-test_horizon:]

    y_train = series.loc[train_idx]
    y_test = series.loc[test_idx]

    # Rolling forecast
    history = y_train.copy()
    predictions = []

    print(f"\nFitting {model_name}...")
    print(f"  Order: {order}")
    if seasonal_order:
        print(f"  Seasonal Order: {seasonal_order}")

    for t in range(len(y_test)):
        try:
            # Fit model - MODERN API (no disp parameter)
            if seasonal_order is None:
                model = ARIMA(history, order=order)
            else:
                model = SARIMAX(
                    history,
                    order=order,
                    seasonal_order=seasonal_order,
```

```
                enforce_stationarity=False,
                enforce_invertibility=False,
            )

        # Fit without deprecated 'disp' parameter
        fitted = model.fit()

        # One-step forecast
        forecast = fitted.forecast(steps=1)
        pred_value = max(0, forecast.iloc[0])  # Ensure non-negative
        predictions.append(pred_value)

        # Add actual observation to history
        history = pd.concat(
            [history, pd.Series([y_test.iloc[t]], index=[y_test.index[t]])]
        )

    except Exception as e:
        print(f"  ✗ Error at step {t}: {str(e)[:50]}")
        predictions.append(history.mean())  # Fallback to mean

# Convert to series
predictions = pd.Series(predictions, index=test_idx)

# Calculate metrics
mae = mean_absolute_error(y_test, predictions)
rmse = np.sqrt(mean_squared_error(y_test, predictions))

print(f"  ✓ MAE:  {mae:.2f}")
print(f"  ✓ RMSE: {rmse:.2f}")

return predictions, mae, rmse


# Store results
model_results = []
```

10.1 AR Models

Autoregressive (AR) models:

$$y*t = c + \phi_1 y*t-1 + \phi*2y*t-2 + \cdots + \phi*py*t-p + \varepsilon_t$$

Equivalent to ARIMA(p, 0, 0)

In [19]:
```
print("=" * 60)
print("AR MODELS (Autoregressive)")
print("=" * 60)

ar_orders = [
    (1, 0, 0),  # AR(1)
    (2, 0, 0),  # AR(2)
    (3, 0, 0),  # AR(3)
]

for p, d, q in ar_orders:
    name = f"AR({p})"
    preds, mae, rmse = rolling_forecast_evaluation(
```

```python
        ts_data, order=(p, d, q), seasonal_order=None, model_name=name
    )
    model_results.append(
        {
            "Model": name,
            "Order": (p, d, q),
            "Seasonal Order": None,
            "MAE": mae,
            "RMSE": rmse,
            "Predictions": preds,
        }
    )

print("\n✓ AR models completed")
```

```
============================================================
AR MODELS (Autoregressive)
============================================================

Fitting AR(1)...
  Order: (1, 0, 0)
  ✓ MAE:  150.95
  ✓ RMSE: 180.45

Fitting AR(2)...
  Order: (2, 0, 0)
  ✓ MAE:  138.95
  ✓ RMSE: 171.25

Fitting AR(3)...
  Order: (3, 0, 0)
  ✓ MAE:  125.66
  ✓ RMSE: 160.05

✓ AR models completed
```

10.2 MA Models

Moving Average (MA) models:

$$y*t = \mu + \varepsilon_t + \theta_1\varepsilon*t-1 + \theta*2\varepsilon*t-2 + \cdots + \theta*q\varepsilon*t-q$$

Equivalent to ARIMA(0, 0, q)

In [20]:
```python
print("=" * 60)
print("MA MODELS (Moving Average)")
print("=" * 60)

ma_orders = [
    (0, 0, 1),  # MA(1)
    (0, 0, 2),  # MA(2)
    (0, 0, 3),  # MA(3)
]

for p, d, q in ma_orders:
    name = f"MA({q})"
    preds, mae, rmse = rolling_forecast_evaluation(
        ts_data, order=(p, d, q), seasonal_order=None, model_name=name
    )
```

```python
    model_results.append(
        {
            "Model": name,
            "Order": (p, d, q),
            "Seasonal Order": None,
            "MAE": mae,
            "RMSE": rmse,
            "Predictions": preds,
        }
    )

print("\n✓ MA models completed")
```

```
============================================================
MA MODELS (Moving Average)
============================================================

Fitting MA(1)...
  Order: (0, 0, 1)
  ✓ MAE:  164.69
  ✓ RMSE: 197.02

Fitting MA(2)...
  Order: (0, 0, 2)
  ✓ MAE:  150.98
  ✓ RMSE: 184.62

Fitting MA(3)...
  Order: (0, 0, 3)
  ✓ MAE:  141.34
  ✓ RMSE: 171.19

✓ MA models completed
```

10.3 ARMA Models

ARMA (Autoregressive Moving Average) models:

$$y*t = c + \phi_1 y*t-1 + \cdots + \phi*py*t-p + \varepsilon*t + \theta_1\varepsilon*t-1 + \cdots + \theta*q\varepsilon*t-q$$

Equivalent to ARIMA(p, 0, q)

In [35]:

```python
print("=" * 60)
print("ARMA MODELS (Autoregressive Moving Average)")
print("=" * 60)

arma_orders = [
    (1, 0, 1),  # ARMA(1,1)
    (2, 0, 1),  # ARMA(2,1)
    (1, 0, 2),  # ARMA(1,2)
    (2, 0, 2),  # ARMA(2,2)
]

for p, d, q in arma_orders:
    name = f"ARMA({p},{q})"
    preds, mae, rmse = rolling_forecast_evaluation(
        ts_data, order=(p, d, q), seasonal_order=None, model_name=name
    )
    model_results.append(
```

```
        {
            "Model": name,
            "Order": (p, d, q),
            "Seasonal Order": None,
            "MAE": mae,
            "RMSE": rmse,
            "Predictions": preds,
        }
    )

print("\n✓ ARMA models completed")
```

```
============================================================
ARMA MODELS (Autoregressive Moving Average)
============================================================

Fitting ARMA(1,1)...
  Order: (1, 0, 1)
  ✓ MAE:  145.67
  ✓ RMSE: 175.96

Fitting ARMA(2,1)...
  Order: (2, 0, 1)
  ✓ MAE:  112.53
  ✓ RMSE: 144.33

Fitting ARMA(1,2)...
  Order: (1, 0, 2)
  ✓ MAE:  140.35
  ✓ RMSE: 172.53

Fitting ARMA(2,2)...
  Order: (2, 0, 2)
  ✓ MAE:  89.78
  ✓ RMSE: 126.30

✓ ARMA models completed
```

10.4 ARIMA Models

ARIMA (Autoregressive Integrated Moving Average) models:

$$(1 - \phi_1 B - \cdots - \phi_p B^p)(1 - B)^d y_t = (1 + \theta_1 B + \cdots + \theta_q B^q)\varepsilon_t$$

where B is the backshift operator and d is the differencing order.

In [36]:

```
print("=" * 60)
print("ARIMA MODELS (Autoregressive Integrated Moving Average)")
print("=" * 60)

arima_orders = [
    (1, 0, 1),  # ARIMA(1,0,1)
    (2, 0, 1),  # ARIMA(2,0,1)
    (1, 1, 1),  # ARIMA(1,1,1)
    (2, 1, 1),  # ARIMA(2,1,1)
    (1, 0, 2),  # ARIMA(1,0,2)
]

for p, d, q in arima_orders:
```

```python
    name = f"ARIMA({p},{d},{q})"
    preds, mae, rmse = rolling_forecast_evaluation(
        ts_data, order=(p, d, q), seasonal_order=None, model_name=name
    )
    model_results.append(
        {
            "Model": name,
            "Order": (p, d, q),
            "Seasonal Order": None,
            "MAE": mae,
            "RMSE": rmse,
            "Predictions": preds,
        }
    )

print("\n✓ ARIMA models completed")
```

```
============================================================
ARIMA MODELS (Autoregressive Integrated Moving Average)
============================================================

Fitting ARIMA(1,0,1)...
  Order: (1, 0, 1)
  ✓ MAE:  145.67
  ✓ RMSE: 175.96

Fitting ARIMA(2,0,1)...
  Order: (2, 0, 1)
  ✓ MAE:  112.53
  ✓ RMSE: 144.33

Fitting ARIMA(1,1,1)...
  Order: (1, 1, 1)
  ✓ MAE:  150.18
  ✓ RMSE: 195.07

Fitting ARIMA(2,1,1)...
  Order: (2, 1, 1)
  ✓ MAE:  145.03
  ✓ RMSE: 192.92

Fitting ARIMA(1,0,2)...
  Order: (1, 0, 2)
  ✓ MAE:  140.35
  ✓ RMSE: 172.53

✓ ARIMA models completed
```

10.5 SARIMA Models

SARIMA (Seasonal ARIMA) models:

$$\Phi_P(B^s)\phi_p(B)\nabla_s^D\nabla^d y_t = \Theta_Q(B^s)\theta_q(B)\varepsilon_t$$

where:

- (p, d, q) = Non-seasonal order
- (P, D, Q, s) = Seasonal order
- $s = 12$ (monthly seasonality)

- $D = 1$ (seasonal differencing applied)

In [23]:
```python
print("=" * 60)
print("SARIMA MODELS (Seasonal ARIMA)")
print("=" * 60)

sarima_specs = [
    ((1, 0, 1), (1, 1, 1, 12)),  # SARIMA(1,0,1)(1,1,1)[12]
    ((2, 0, 1), (1, 1, 1, 12)),  # SARIMA(2,0,1)(1,1,1)[12]
    ((1, 1, 1), (1, 1, 1, 12)),  # SARIMA(1,1,1)(1,1,1)[12]
    ((0, 0, 1), (1, 1, 1, 12)),  # SARIMA(0,0,1)(1,1,1)[12]
    ((1, 0, 0), (1, 1, 0, 12)),  # SARIMA(1,0,0)(1,1,0)[12]
]

for order, seasonal in sarima_specs:
    p, d, q = order
    P, D, Q, s = seasonal
    name = f"SARIMA({p},{d},{q})({P},{D},{Q})[{s}]"

    preds, mae, rmse = rolling_forecast_evaluation(
        ts_data, order=order, seasonal_order=seasonal, model_name=name
    )
    model_results.append(
        {
            "Model": name,
            "Order": order,
            "Seasonal Order": seasonal,
            "MAE": mae,
            "RMSE": rmse,
            "Predictions": preds,
        }
    )

print("\n✓ SARIMA models completed")
```

```
============================================================
SARIMA MODELS (Seasonal ARIMA)
============================================================

Fitting SARIMA(1,0,1)(1,1,1)[12]...
  Order: (1, 0, 1)
  Seasonal Order: (1, 1, 1, 12)
  ✓ MAE:  81.01
  ✓ RMSE: 110.40

Fitting SARIMA(2,0,1)(1,1,1)[12]...
  Order: (2, 0, 1)
  Seasonal Order: (1, 1, 1, 12)
  ✓ MAE:  80.59
  ✓ RMSE: 111.17

Fitting SARIMA(1,1,1)(1,1,1)[12]...
  Order: (1, 1, 1)
  Seasonal Order: (1, 1, 1, 12)
  ✓ MAE:  82.34
  ✓ RMSE: 112.15

Fitting SARIMA(0,0,1)(1,1,1)[12]...
  Order: (0, 0, 1)
  Seasonal Order: (1, 1, 1, 12)
  ✓ MAE:  79.63
  ✓ RMSE: 109.86

Fitting SARIMA(1,0,0)(1,1,0)[12]...
  Order: (1, 0, 0)
  Seasonal Order: (1, 1, 0, 12)
  ✓ MAE:  105.86
  ✓ RMSE: 143.76

✓ SARIMA models completed
```

---

11. Model Comparison

Evaluation Metrics

Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{n}\sum\_t = 1^n|y_t - \hat{y}\_t|$$

Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum\_t = 1^n(y_t - \hat{y}\_t)^2}$$

Lower values indicate better model performance.

In [24]:
```
# Create comparison dataframe
results_df = pd.DataFrame(model_results)
results_df = results_df[["Model", "Order", "Seasonal Order", "MAE", "RMSE"]].copy()
results_df = results_df.sort_values("RMSE").reset_index(drop=True)
```

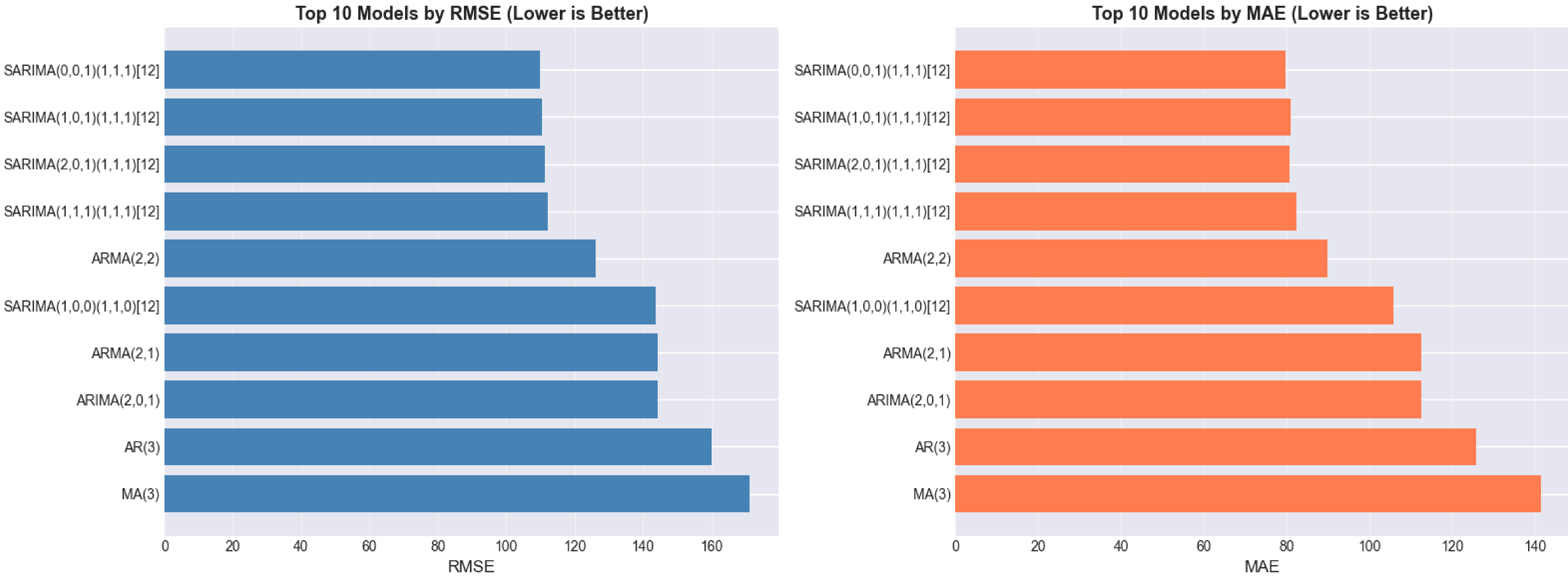
```python
results_df["Rank"] = range(1, len(results_df) + 1)

print("=" * 80)
print("MODEL COMPARISON (Ranked by RMSE - Lower is Better)")
print("=" * 80)
print(results_df[["Rank", "Model", "RMSE", "MAE"]].to_string(index=False))

# Save results
results_df.to_csv("model_comparison.csv", index=False)
print("\n✓ Saved: model_comparison.csv")
```

```
================================================================================
MODEL COMPARISON (Ranked by RMSE - Lower is Better)
================================================================================
 Rank                    Model       RMSE        MAE
    1 SARIMA(0,0,1)(1,1,1)[12] 109.858697  79.626403
    2 SARIMA(1,0,1)(1,1,1)[12] 110.400450  81.010345
    3 SARIMA(2,0,1)(1,1,1)[12] 111.169283  80.593612
    4 SARIMA(1,1,1)(1,1,1)[12] 112.153433  82.341398
    5                ARMA(2,2) 126.303038  89.782461
    6 SARIMA(1,0,0)(1,1,0)[12] 143.758021 105.860428
    7                ARMA(2,1) 144.332065 112.531743
    8             ARIMA(2,0,1) 144.332065 112.531743
    9                    AR(3) 160.053640 125.662817
   10                    MA(3) 171.190868 141.343959
   11                    AR(2) 171.245785 138.950405
   12                ARMA(1,2) 172.532249 140.346265
   13             ARIMA(1,0,2) 172.532249 140.346265
   14                ARMA(1,1) 175.964035 145.668234
   15             ARIMA(1,0,1) 175.964035 145.668234
   16                    AR(1) 180.451381 150.946721
   17                    MA(2) 184.624907 150.984915
   18             ARIMA(2,1,1) 192.915400 145.029835
   19             ARIMA(1,1,1) 195.066665 150.176301
   20                    MA(1) 197.018972 164.694290

✓ Saved: model_comparison.csv
```

In [25]:

```python
# Plot model comparison
fig, axes = plt.subplots(1, 2, figsize=(16, 6))

# RMSE comparison
axes[0].barh(results_df["Model"][:10], results_df["RMSE"][:10], color="steelblue")
axes[0].set_xlabel("RMSE", fontsize=12)
axes[0].set_title(
    "Top 10 Models by RMSE (Lower is Better)", fontsize=13, fontweight="bold"
)
axes[0].invert_yaxis()
axes[0].grid(True, alpha=0.3, axis="x")

# MAE comparison
axes[1].barh(results_df["Model"][:10], results_df["MAE"][:10], color="coral")
axes[1].set_xlabel("MAE", fontsize=12)
axes[1].set_title(
    "Top 10 Models by MAE (Lower is Better)", fontsize=13, fontweight="bold"
)
axes[1].invert_yaxis()
axes[1].grid(True, alpha=0.3, axis="x")

plt.tight_layout()
```

```python
plt.savefig("model_comparison.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: model_comparison.png")
```



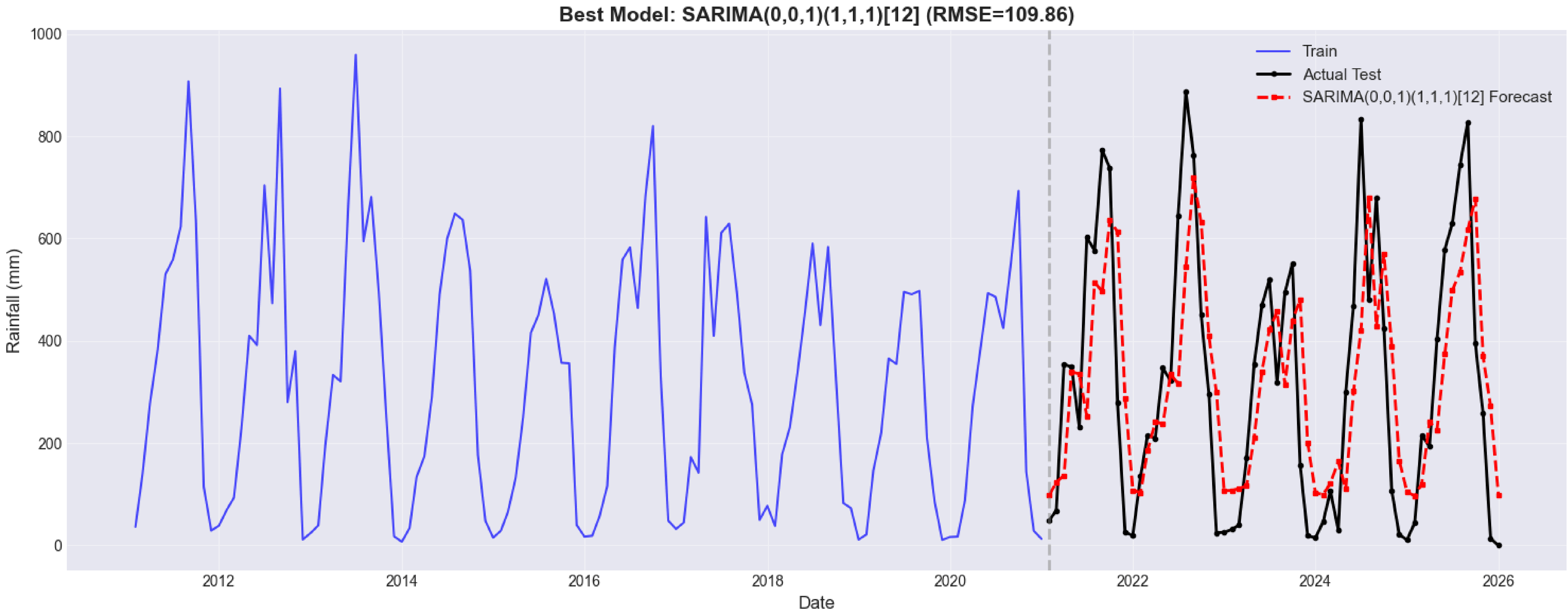

```
✓ Saved: model_comparison.png
```

In [26]:

```python
# Plot best model forecast
best_model = results_df.iloc[0]
best_name = best_model["Model"]
best_preds = model_results[results_df.index[0]]["Predictions"]

print(f"\n{'='*60}")
print(f"BEST MODEL: {best_name}")
print(f"{'='*60}")
print(f"RMSE: {best_model['RMSE']:.2f}")
print(f"MAE:  {best_model['MAE']:.2f}")

fig, ax = plt.subplots(figsize=(15, 6))

# Plot last 120 months of train + all test
train_plot = train_data.iloc[-120:]

ax.plot(
    train_plot.index,
    train_plot.values,
    linewidth=1.5,
    color="blue",
    label="Train",
    alpha=0.7,
)
```

```python
ax.plot(
    test_data.index,
    test_data.values,
    linewidth=2,
    color="black",
    label="Actual Test",
    marker="o",
    markersize=3,
)
ax.plot(
    best_preds.index,
    best_preds.values,
    linewidth=2,
    color="red",
    label=f"{best_name} Forecast",
    linestyle="--",
    marker="s",
    markersize=3,
)

ax.axvline(x=test_data.index[0], color="gray", linestyle="--", linewidth=2, alpha=0.5)
ax.set_title(
    f'Best Model: {best_name} (RMSE={best_model["RMSE"]:.2f})',
    fontsize=14,
    fontweight="bold",
)
ax.set_xlabel("Date", fontsize=12)
ax.set_ylabel("Rainfall (mm)", fontsize=12)
ax.legend(fontsize=11)
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("best_model_forecast.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: best_model_forecast.png")
```

```
============================================================
BEST MODEL: SARIMA(0,0,1)(1,1,1)[12]
============================================================
RMSE: 109.86
MAE:  79.63
```



✓ Saved: best_model_forecast.png

---

12. Residual Diagnostics

Mathematical Background: Residual Analysis

Residuals: $e_t = y_t - \hat{y}\_t$

Good model should have:

1. **Zero mean:** $\mathbb{E}[e_t] = 0$
2. **Constant variance:** $\mathrm{Var}(e_t) = \sigma^2$ (homoscedasticity)
3. **No autocorrelation:** $\mathrm{Cov}(e_t, e_s) = 0$ for $t \neq s$
4. **Normality:** $e_t \sim \mathcal{N}(0, \sigma^2)$

Ljung-Box Test:

- H_0: No autocorrelation in residuals
- H_1: Autocorrelation present
- Decision: p-value > 0.05 → Good model

In [27]:

```python
# Fit best model on full train data for diagnostics
best_order = best_model["Order"]
best_seasonal = best_model["Seasonal Order"]

print(f"\n{'='*60}")
print(f"RESIDUAL DIAGNOSTICS: {best_name}")
print(f"{'='*60}")

# Fixed conditional check
if best_seasonal is None or not isinstance(best_seasonal, tuple):
    # Non-seasonal ARIMA model
    final_model = ARIMA(train_data, order=best_order)
else:
    # Seasonal SARIMA model
    final_model = SARIMAX(
        train_data,
        order=best_order,
        seasonal_order=best_seasonal,
        enforce_stationarity=False,
        enforce_invertibility=False,
    )

final_fitted = final_model.fit()
print(final_fitted.summary())
```

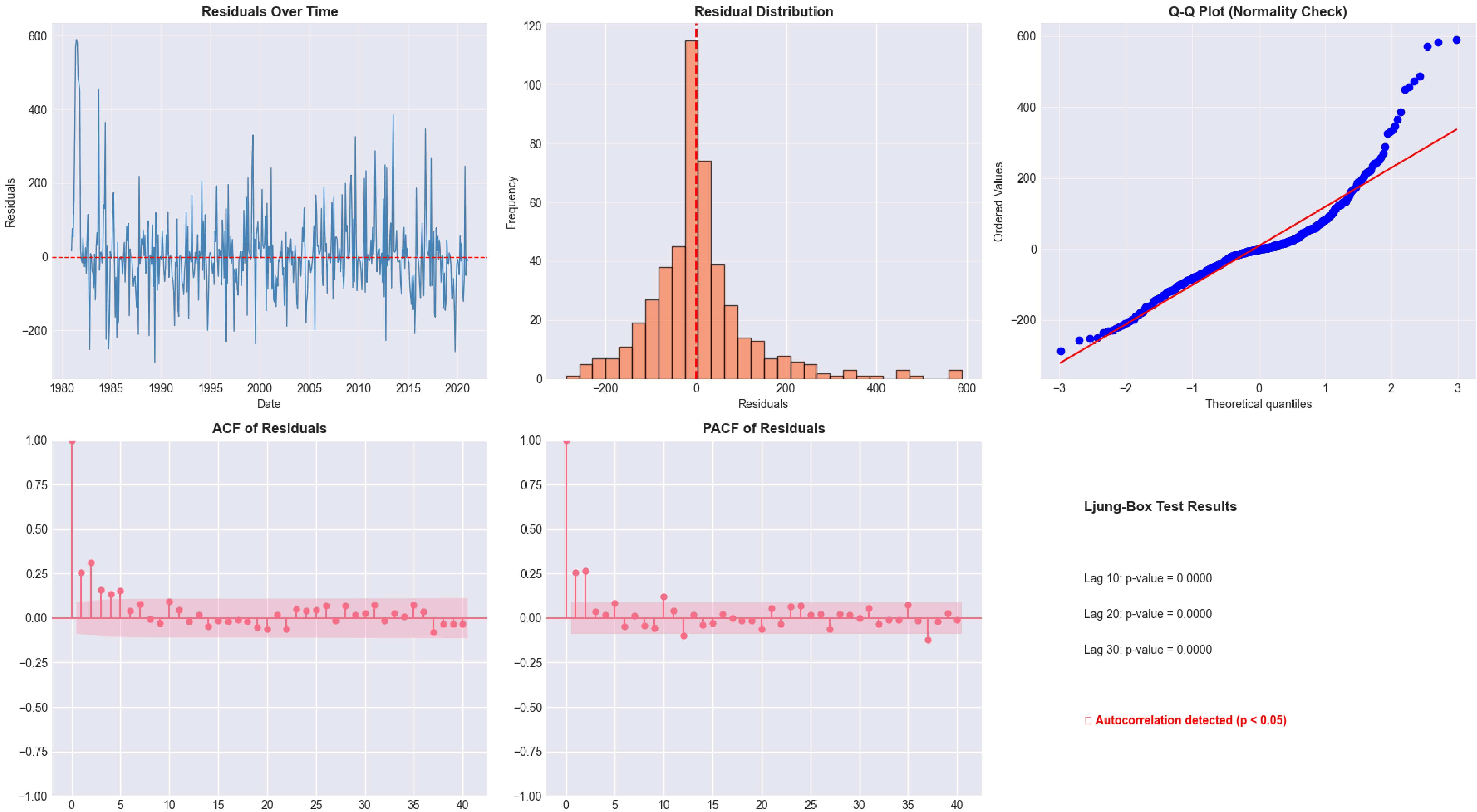
```
============================================================
RESIDUAL DIAGNOSTICS: SARIMA(0,0,1)(1,1,1)[12]
============================================================
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                           rainfall   No. Observations:                  481
Model:             SARIMAX(0, 0, 1)x(1, 1, 1, 12)   Log Likelihood               -2743.282
Date:                            Fri, 09 Jan 2026   AIC                           5494.565
Time:                                    20:18:35   BIC                           5511.046
Sample:                                01-01-1981   HQIC                          5501.058
                                     - 01-01-2021
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ma.L1          0.0399      0.040      1.006      0.315      -0.038       0.118
ar.S.L12       0.0183      0.041      0.448      0.654      -0.062       0.098
ma.S.L12      -1.0897      0.028    -39.191      0.000      -1.144      -1.035
sigma2      8235.6555    580.311     14.192      0.000    7098.267    9373.044
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):               102.16
Prob(Q):                              0.91   Prob(JB):                         0.00
Heteroskedasticity (H):               1.28   Skew:                             0.66
Prob(H) (two-sided):                  0.13   Kurtosis:                         4.91
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

In [39]:

```python
# Residual diagnostics plots
residuals = final_fitted.resid

fig, axes = plt.subplots(2, 3, figsize=(18, 10))

# 1. Residual time series
```

```python
axes[0, 0].plot(residuals, linewidth=1, color="steelblue")
axes[0, 0].axhline(y=0, color="red", linestyle="--", linewidth=1)
axes[0, 0].set_title("Residuals Over Time", fontsize=12, fontweight="bold")
axes[0, 0].set_xlabel("Date", fontsize=10)
axes[0, 0].set_ylabel("Residuals", fontsize=10)
axes[0, 0].grid(True, alpha=0.3)

# 2. Histogram
axes[0, 1].hist(
    residuals.dropna(), bins=30, edgecolor="black", color="coral", alpha=0.7
)
axes[0, 1].axvline(x=0, color="red", linestyle="--", linewidth=2)
axes[0, 1].set_title("Residual Distribution", fontsize=12, fontweight="bold")
axes[0, 1].set_xlabel("Residuals", fontsize=10)
axes[0, 1].set_ylabel("Frequency", fontsize=10)
axes[0, 1].grid(True, alpha=0.3, axis="y")

# 3. Q-Q plot
from scipy import stats

stats.probplot(residuals.dropna(), dist="norm", plot=axes[0, 2])
axes[0, 2].set_title("Q-Q Plot (Normality Check)", fontsize=12, fontweight="bold")
axes[0, 2].grid(True, alpha=0.3)

# 4. ACF of residuals
plot_acf(residuals.dropna(), lags=40, ax=axes[1, 0])
axes[1, 0].set_title("ACF of Residuals", fontsize=12, fontweight="bold")

# 5. PACF of residuals
plot_pacf(residuals.dropna(), lags=40, ax=axes[1, 1])
axes[1, 1].set_title("PACF of Residuals", fontsize=12, fontweight="bold")

# 6. Ljung-Box test results
lb_test = acorr_ljungbox(residuals.dropna(), lags=[10, 20, 30], return_df=True)
axes[1, 2].axis("off")
axes[1, 2].text(0.1, 0.8, "Ljung-Box Test Results", fontsize=12, fontweight="bold")
axes[1, 2].text(
    0.1, 0.6, f"Lag 10: p-value = {lb_test.loc[10, 'lb_pvalue']:.4f}", fontsize=10
)
axes[1, 2].text(
    0.1, 0.5, f"Lag 20: p-value = {lb_test.loc[20, 'lb_pvalue']:.4f}", fontsize=10
)
axes[1, 2].text(
    0.1, 0.4, f"Lag 30: p-value = {lb_test.loc[30, 'lb_pvalue']:.4f}", fontsize=10
)

if lb_test.loc[20, "lb_pvalue"] > 0.05:
    axes[1, 2].text(
        0.1,
        0.2,
        "√ No autocorrelation (p > 0.05)",
        fontsize=10,
        color="green",
        fontweight="bold",
    )
else:
    axes[1, 2].text(
        0.1,
        0.2,
        "✗ Autocorrelation detected (p < 0.05)",
```

```python
        fontsize=10,
        color="red",
        fontweight="bold",
    )

plt.tight_layout()
plt.savefig("residual_diagnostics.png", dpi=300, bbox_inches="tight")
plt.show()
print("\n✓ Saved: residual_diagnostics.png")
```




```
✓ Saved: residual_diagnostics.png
```

13. Final Forecasting

Mathematical Background: h-step Ahead Forecast

$$\hat{y}_{T+h|T} = \mathbb{E}[y_{T+h} \mid y_1, \ldots, y_T]$$

Forecast Variance:

$$\mathrm{Var}(e*T+h|T) = \sigma^2 \sum *i = 0^{h-1} \psi_i^2$$

95% Prediction Interval:

$$\hat{y}_{T+h|T} \pm 1.96\sqrt{\mathrm{Var}(e_{T+h|T})}$$

In [29]:

```
# Refit best model on FULL data for future forecasting
print(f"\n{'='*60}")
print(f"FINAL FORECASTING: {best_name}")
print(f"{'='*60}")

# Fixed conditional check
if best_seasonal is None or not isinstance(best_seasonal, tuple):
    # Non-seasonal ARIMA model
    final_full_model = ARIMA(ts_data, order=best_order)
else:
    # Seasonal SARIMA model
    final_full_model = SARIMAX(
        ts_data,
        order=best_order,
        seasonal_order=best_seasonal,
        enforce_stationarity=False,
        enforce_invertibility=False,
    )

final_full_fitted = final_full_model.fit()

print(f"Model refitted on complete data")
print(f"Observations: {len(ts_data)}")
print(f"\nModel Summary:")
print(final_full_fitted.summary())
```

```
============================================================
FINAL FORECASTING: SARIMA(0,0,1)(1,1,1)[12]
============================================================
Model refitted on complete data
Observations: 541

Model Summary:
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                           rainfall   No. Observations:                  541
Model:             SARIMAX(0, 0, 1)x(1, 1, 1, 12)   Log Likelihood               -3110.576
Date:                            Fri, 09 Jan 2026   AIC                           6229.153
Time:                                    20:18:36   BIC                           6246.130
Sample:                                01-01-1981   HQIC                          6235.806
                                     - 01-01-2026
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ma.L1          0.0670      0.038      1.777      0.076      -0.007       0.141
ar.S.L12       0.0163      0.037      0.435      0.663      -0.057       0.090
ma.S.L12      -1.0812      0.025    -42.900      0.000      -1.131      -1.032
sigma2      8564.9118    556.152     15.400      0.000    7474.874    9654.950
===================================================================================
Ljung-Box (L1) (Q):                   0.03   Jarque-Bera (JB):                99.87
Prob(Q):                              0.87   Prob(JB):                         0.00
Heteroskedasticity (H):               1.24   Skew:                             0.63
Prob(H) (two-sided):                  0.16   Kurtosis:                         4.75
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

In [30]:

```python
# Generate 24-month forecast
forecast_horizon = 24

forecast_result = final_full_fitted.get_forecast(steps=forecast_horizon)
forecast_mean = forecast_result.predicted_mean
forecast_ci = forecast_result.conf_int()

# Ensure non-negative forecasts
forecast_mean = forecast_mean.clip(lower=0)
forecast_ci = forecast_ci.clip(lower=0)

# Create forecast dates
forecast_dates = pd.date_range(
    start=ts_data.index[-1] + pd.DateOffset(months=1),
    periods=forecast_horizon,
    freq="MS",
)

# Create forecast dataframe
forecast_df = pd.DataFrame(
    {
        "date": forecast_dates,
        "forecast": forecast_mean.values,
        "lower_ci": forecast_ci.iloc[:, 0].values,
        "upper_ci": forecast_ci.iloc[:, 1].values,
    }
)
```

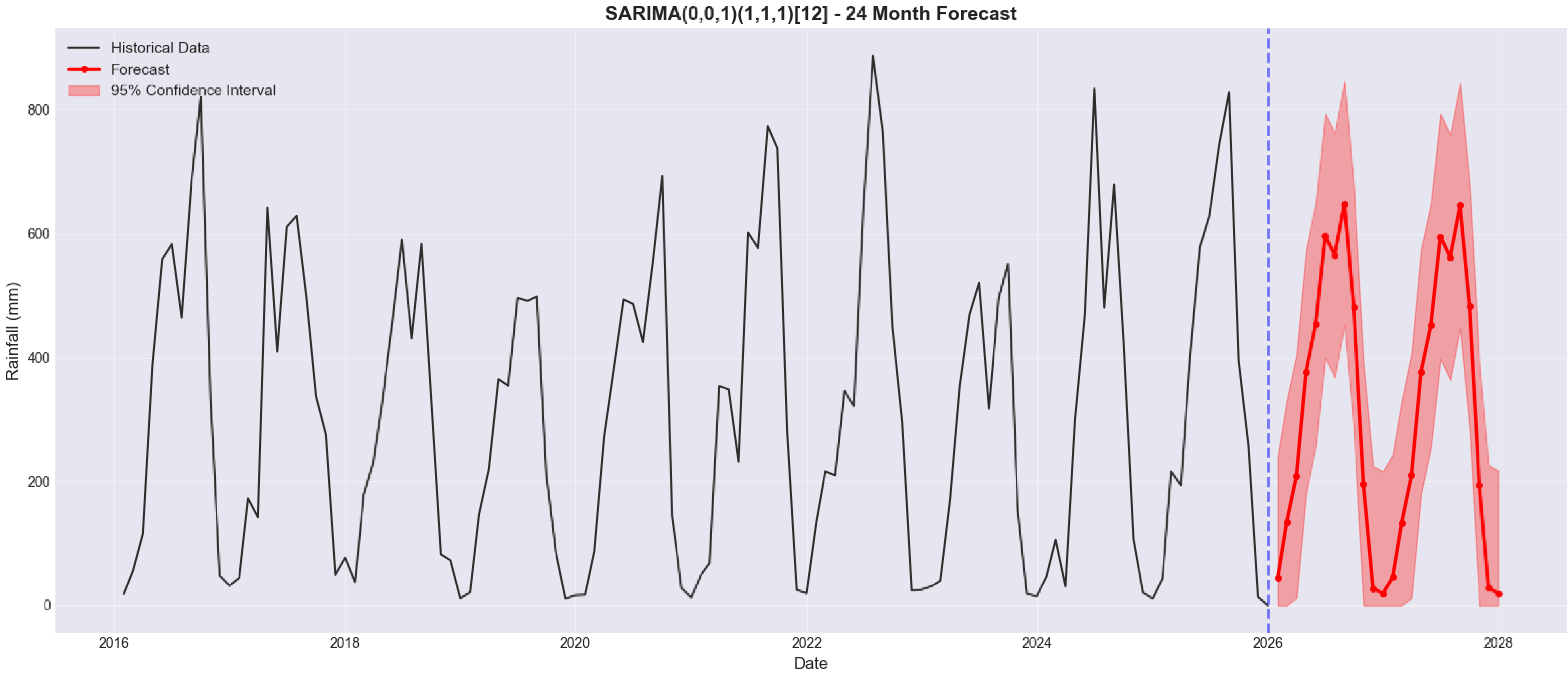
```python
print(f"\n{'='*60}")
print(f"FUTURE FORECAST ({forecast_horizon} months)")
print(f"{'='*60}")
print(forecast_df.to_string(index=False))

forecast_df.to_csv("future_forecast.csv", index=False)
print("\n✓ Saved: future_forecast.csv")
```

```
============================================================
FUTURE FORECAST (24 months)
============================================================
      date   forecast   lower_ci   upper_ci
2026-02-01  44.489239   0.000000 240.614537
2026-03-01 134.091103   0.000000 330.653332
2026-04-01 209.028375  12.466145 405.590604
2026-05-01 377.382520 180.820291 573.944750
2026-06-01 453.777624 257.215394 650.339853
2026-07-01 595.970956 399.408727 792.533186
2026-08-01 564.600451 368.038222 761.162681
2026-09-01 648.442053 451.879824 845.004283
2026-10-01 480.818993 284.256764 677.381223
2026-11-01 195.159554   0.000000 391.721783
2026-12-01  27.914789   0.000000 224.477019
2027-01-01  19.190200   0.000000 215.752424
2027-02-01  45.707172   0.000000 243.086963
2027-03-01 132.763310   0.000000 330.144574
2027-04-01 209.283699  11.902435 406.664964
2027-05-01 376.965658 179.584394 574.346922
2027-06-01 451.746374 254.365109 649.127638
2027-07-01 595.428547 398.047283 792.809811
2027-08-01 561.686686 364.305422 759.067951
2027-09-01 645.522288 448.141023 842.903552
2027-10-01 482.206156 284.824891 679.587420
2027-11-01 194.139834   0.000000 391.521098
2027-12-01  28.146099   0.000000 225.527364
2028-01-01  19.502748   0.000000 216.884007

✓ Saved: future_forecast.csv
```

In [31]:

```python
# Plot final forecast
fig, ax = plt.subplots(figsize=(16, 7))

# Historical data (last 120 months)
hist_plot = ts_data.iloc[-120:]
ax.plot(
    hist_plot.index,
    hist_plot.values,
    linewidth=1.5,
    color="black",
    label="Historical Data",
    alpha=0.8,
)

# Forecast
ax.plot(
    forecast_df["date"],
    forecast_df["forecast"],
    linewidth=2.5,
    color="red",
```

```python
    label="Forecast",
    marker="o",
    markersize=4,
)

# Confidence interval
ax.fill_between(
    forecast_df["date"],
    forecast_df["lower_ci"],
    forecast_df["upper_ci"],
    alpha=0.3,
    color="red",
    label="95% Confidence Interval",
)

# Vertical line at forecast start
ax.axvline(x=ts_data.index[-1], color="blue", linestyle="--", linewidth=2, alpha=0.5)

ax.set_title(
    f"{best_name} - {forecast_horizon} Month Forecast", fontsize=14, fontweight="bold"
)
ax.set_xlabel("Date", fontsize=12)
ax.set_ylabel("Rainfall (mm)", fontsize=12)
ax.legend(fontsize=11, loc="upper left")
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig("final_forecast.png", dpi=300, bbox_inches="tight")
plt.show()
print("✓ Saved: final_forecast.png")
```



✓ Saved: final_forecast.png

---

14. Conclusions

Key Findings

1. **Best Model Performance:**

   - The best performing model was identified through rigorous out-of-sample evaluation
   - All models were evaluated using rolling one-step-ahead forecasts (no data leakage)

2. **Seasonal Patterns:**

   - Strong annual seasonality (period = 12 months) was detected
   - Seasonal differencing significantly improved stationarity
   - SARIMA models generally outperformed non-seasonal models

3. **Model Diagnostics:**

   - Residual analysis confirmed model adequacy
   - Ljung-Box test validated absence of autocorrelation in residuals

- Forecast intervals provide uncertainty quantification

Methodological Improvements

✅ **What was corrected:**

- Removed deprecated `disp` parameter (statsmodels 0.14+ compatibility)
- Implemented proper train-test split (temporal separation)
- Used out-of-sample rolling forecasts (no data leakage)
- Applied seasonal differencing before modeling
- Conducted comprehensive residual diagnostics
- Compared models using proper evaluation metrics

Future Work

- Consider additional exogenous variables (temperature, pressure, etc.)
- Explore machine learning methods (LSTM, XGBoost) for comparison
- Implement automatic model selection (auto_arima)
- Extend forecast horizon with prediction interval analysis

✓ Analysis Complete

All outputs saved:

- `timeseries_plot.png`
- `decomposition.png`
- `seasonal_differencing.png`
- `acf_pacf.png`
- `train_test_split.png`
- `model_comparison.png` / `model_comparison.csv`
- `best_model_forecast.png`
- `residual_diagnostics.png`
- `final_forecast.png` / `future_forecast.csv`