

Magnetic propeties of a two dimentional electron gas strongly coupled to lights

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1 Floquet Fermi Goldern Rule

In this section we are going to derive the Floquet Fermi goldern rule for above derived quantum Floquet states using $t - t'$ formalism.

The Floquet states (??) fullfills the $t - t'$ Schrödinger equation [*Ref:myReport] as follows

$$i\hbar \frac{\partial}{\partial t} |\psi_\alpha(t, t')\rangle = H_F(t') |\psi_\alpha(t, t')\rangle \quad (1.1)$$

where Floquet Hamiltonian given by

$$H_F(t') \equiv H_e(t) - i\hbar \frac{d}{dt} \quad (1.2)$$

and

$$|\psi_\alpha(t, t')\rangle = \exp\left(-\frac{i}{\hbar} \varepsilon_\alpha t\right) |\Phi_\alpha(t')\rangle \quad (1.3)$$

Now for the Eq. (1.1) corresponding time evolution operator satisfy the Schrödinger equation

$$U_0(t, t_0; t') = \exp\left(-\frac{i}{\hbar} H_F(t') [t - t_0]\right) \quad (1.4)$$

Consider a time-independent total perturbation $V(\mathbf{r})$ switched on at the reference time $t- = t_0$, then Schrödinger equation becomes

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle = [H_F(t') + V(\mathbf{r})] |\Psi_\alpha(t, t')\rangle \quad (1.5)$$

and when $t \leq t_0$ both solutions of the Schrödinger equation coincide

$$|\psi_\alpha(t, t')\rangle = |\Psi_\alpha(t, t')\rangle \quad \text{when } t \leq t_0 \quad (1.6)$$

Now, we can introduce the interaction picture representation of the $t - t'$ Floquet state as

$$|\Psi_\alpha(t, t')\rangle_I = U_0^\dagger(t, t_0; t') |\Psi_\alpha(t, t')\rangle \quad (1.7)$$

and the perturbation in the interaction picture will be

$$V_I(\mathbf{r}) = U_0^\dagger(t, t_0; t') V(\mathbf{r}) U_0(t, t_0; t') = V(\mathbf{r}). \quad (1.8)$$

This leads to the Schrödinger equation in the interction picture

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle_I = V_I(\mathbf{r}) |\Psi_\alpha(t, t')\rangle_I \quad (1.9)$$

with the recursive solution

$$|\Psi_\alpha(t, t')\rangle_I = |\Psi_\alpha(t_0, t')\rangle_I + \frac{1}{i\hbar} \int_{t_0}^t dt_1 V_I(\mathbf{r}) |\Psi_\alpha(t_1, t')\rangle_I \quad (1.10)$$

Iterating the solution only upto first order (Born approximation) this leads to

$$|\Psi_\alpha(t, t')\rangle_I \approx |\psi_\alpha(t_0, t')\rangle + \frac{1}{i\hbar} \int_{t_0}^t dt_1 V_I(\mathbf{r}) |\psi_\alpha(t_0, t')\rangle \quad (1.11)$$

and multiply it by $\langle\psi_\beta(t_0, t')|$ and we will get

$$\langle\psi_\beta(t_0, t')|\Psi_\alpha(t, t')\rangle_I = \langle\psi_\beta(t_0, t')|\psi_\alpha(t_0, t')\rangle + \frac{1}{i\hbar} \int_{t_0}^t dt_1 \langle\psi_\beta(t_0, t')| V_I(\mathbf{r}) |\psi_\alpha(t_0, t')\rangle. \quad (1.12)$$

Then introducing unitary operator U_0 we can re-write this as

$$\begin{aligned} \langle\psi_\beta(t_0, t')|U_0^\dagger(t, t_0; t')|\Psi_\alpha(t, t')\rangle &= \langle\psi_\beta(t_0, t')|U_0^\dagger(t, t_0; t')U_0(t, t_0; t')|\psi_\alpha(t_0, t')\rangle \\ &+ \frac{1}{i\hbar} \int_{t_0}^t dt_1 \langle\psi_\beta(t_0, t')|U_0^\dagger(t_1, t_0; t')V(\mathbf{r})U_0(t_1, t_0; t')|\psi_\alpha(t_0, t')\rangle \end{aligned} \quad (1.13)$$

and this can be simplified as

$$\langle\psi_\beta(t, t')|\Psi_\alpha(t, t')\rangle = \langle\psi_\beta(t, t')|\psi_\alpha(t, t')\rangle + \frac{1}{i\hbar} \int_{t_0}^t dt_1 \langle\psi_\beta(t_1, t')| V(\mathbf{r}) |\psi_\alpha(t_1, t')\rangle. \quad (1.14)$$

Since our $t - t'$ Floquet states are orthonormal [*Ref:myReport- t-t' formalism] we can derive that

$$\langle\psi_\beta(t, t')|\Psi_\alpha(t, t')\rangle = \delta_{\alpha\beta} \exp(i\omega[t' - t]) + \frac{1}{i\hbar} \int_{t_0}^t dt_1 \langle\psi_\beta(t_1, t')| V(\mathbf{r}) |\psi_\alpha(t_1, t')\rangle. \quad (1.15)$$

Now, set $t_0 = 0$ and for a case $\alpha \neq \beta$ this will simplified to

$$\langle\psi_\beta(t, t')|\Psi_\alpha(t, t')\rangle = -\frac{i}{\hbar} \int_0^t dt_1 \langle\psi_\beta(t_1, t')| V(\mathbf{r}) |\psi_\alpha(t_1, t')\rangle. \quad (1.16)$$

In addition, since our Floquet states create a basis for composite space we can represent any solution using our Floquet states

$$|\Psi_\alpha(t, t')\rangle = \sum_{\beta} a_{\alpha\beta}(t, t') |\psi_\beta(t, t')\rangle. \quad (1.17)$$

Therefore we can derive a equation for this *scattering amplitude* as

$$a_{\alpha\beta}(t, t') = \langle\psi_\beta(t, t')|\Psi_\alpha(t, t')\rangle = -\frac{i}{\hbar} \int_0^t dt_1 \langle\psi_\beta(t_1, t')| V(\mathbf{r}) |\psi_\alpha(t_1, t')\rangle. \quad (1.18)$$

Now lets assume a scattering event from a $t - t'$ Floquet state $|\psi_\beta(t, t')\rangle$ into another $t - t'$ Floquet state $|\Psi_\alpha(t, t')\rangle$ with constant quansienenergy ε given as follows

$$|\Psi_\alpha(t, t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon t\right) |\Phi_\alpha(t')\rangle \quad (1.19)$$

Now consider a scattering event

$$\psi_\beta(\mathbf{k}', t, t') = \exp\left(-\frac{i}{\hbar}\varepsilon_\beta t\right) \Phi_\beta(\mathbf{k}', t') \longrightarrow \Psi_\alpha(\mathbf{k}, t, t') = \exp\left(-\frac{i}{\hbar}\varepsilon t\right) \Phi_\alpha(\mathbf{k}, t') \quad (1.20)$$

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