A generalized model for the transport properties of dressed quantum Hall systems

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A generalized mathematical model for predicting the transport properties of a quantum system exposed to a stationary magnetic field and a high intensity electromagnetic field is presented. The new formulation, which applies to two-dimensional(2D) dressed quantum Hall systems, is based on Landau quantization theory and Floquet-Drude conductivity approach. We model our system as a two-dimensional electron gas (2DEG) that interacts with two external fields. To analyze the strong light coupling with the 2DEG, we employ the Floquet theory as a nonperturbative procedure. Moreover, the Floquet-Fermi golden rule is adopted to explore the impurity scattering effects on charge transport in disordered quantum Hall systems. We derive fully analytical expressions to describe longitudinal components in the conductivity tensor in dressed quantum Hall systems. Subsequently, we demonstrate that the conductivity characteristics of quantum Hall systems can be manipulated using a strong external light. Our results align with well-established and experimentally verified theoretical descriptions for undressed systems while providing a more generalized analysis on the conductivity characterstics in quantum Hall systems. Thus, our model can be applied to accurately interpret the usage of external strong radiation as a tool in nanoscale quantum devices.

I. INTRODUCTION

Manipulating light-matter interactions in the quantum regime paved the path for an astonishing number of useful technologies in the last century. Quantum optics, which study these interactions, have drawn research attention to the disciplines of optoelectronics [1–3], sensing [4–6], energy harvesting [7, 8], quantum computing [9–11], bio-information [12, 13], and many other specialities of recent technologies [14]. The studies on quantum optics of nanostructures were generally centered on metamaterials [15, 16], quantum plasmonic effects [17, 18], lasers and amplifiers [19, 20], and quantum cavity physics [21, 22]. However, in recent years, one of the foremost aim of examining nanostructures under external radiation was understanding their electron transport characteristics [23–30].

Better understanding the fundamental mechanisms of charge transport can allow us to invent novel nanoelectronic devices and optimize their performance [31]. Most recent studies on the subject have considered the driving field as a perturbation field [26, 27]. However, this assumption breaks down for systems under high-intensity illuminations [30, 32]. Modeling an electromagnetic field under a perturbative formalism involves expanding the interaction terms in powers of the field intensity. At high intensities, the higher order terms influence the physics more strongly and the basis of the perturbative treatment begins to break down. In these instances, a more accurate treatment needs to adopt. Thus, we treat the interacting fermion system and the radiation as one combined quantum system, namely dressed system [27, 33, 34]. Here the applied high-intensity electromagnetic field identify as the dressing field.

Theoretical analyses on the transport properties of dressed fermion systems were recently reported in Refs. [25, 27, 30]. Furthermore, in Ref. [30] a general expression for conductivity in a dressed system has been derived in a fully closed analytical form. In their study, a novel type of Green's functions, namely four-times Green's functions were used to derive the Floquet-Drude conductivity formula. This opened the path to explore and exploit the charge transport attributes of nanostructures under an intense dressing field.

Quantum Hall effect [35] observed in two-dimensional fermion systems at low temperatures under strong stationary magnetic fields manifest remarkable magneto-transport behaviors. Transport properties of these systems have recently attracted both theoretical [36–42] and experimental [43–45] interest. Endo *et al.* [42] presented the calculations of longitudinal and transverse conductivity tensor components and their relationship in a quantum Hall system. These theoretical calculations align better with experimental observations compared to previous studies.

In contrast, more interesting phenomena can be observed by simultaneously applying a dressing field to a quantum Hall system already under a non-oscillating magnetic field. Whilst there exist several leading theories for calculating conductivity tensor elements in quantum Hall systems [37, 41, 42], they have not been utilized to describe the optical manipulation of charge transport. Recently, Dini *et al.* [29] have investigated the one-directional conductivity behavior of dressed quantum Hall systems. However, they have not adopted the state-of-the-art model to describe the conductivity in a quantum Hall system. In their study, they used the conductivity models from Refs. [37, 41], and as mentioned in Endo *et al.* [42], those models predict a semi-elliptical broadening agaist Fermi level for each Landau levels and provide less agreement with the empirical results.

In the present analysis, we present a robust mathematical model for a dressed two-dimensional electron gas (2DEG) subject to another nonoscillating magnetic field. A stationary magnetic field is applied perpendicularly across the surface of the 2DEG system. This causes the orbital motion of the electrons to be quantized, and a discrete energy spectrum with Landau splitting is observed [46]. In this study, we explicitly calculate the longitudinal components (σ^{xx}, σ^{yy}) of the conductivity tensor in a periodically driven quantum Hall system by developing a generalized analytical description using the Floquet-Drude conductivity [30]. Finally, we demonstrate that our generalized model reproduces the results of the state-of-the-art conductivity model in Ref. [42], which was developed for the more specific case of quantum Hall systems without the external dressing field. Moreover, we find that the optical field can be used as a mechanism to regulate transport behavior in numerous two-dimensional nanostructures which can serve as a basis for many useful nanoelectronic devices. We believe that our theoretical analysis and visual depictions of numerical results will lead to a better understanding of manipulating charge transport. Moreover, this will inspire advanced developments in nanoscale quantum devices.

The paper is organized as follows. In Sec. II, we introduce our dressed quantum Hall system and the exact wave function solutions for the given configuration. Sec. III, provides the Floquet theory interpretation of these wave functions. We introduce Floquet-Fermi golden rule for a quantum Hall system in Sec. IV, and use it in Sec. V to derive analytical expressions for longitudinal components of conductivity. The derived theoritical model is further analyzed numerically using empirical system parameters and compared with previous studies in Sec. VI. In Sec. VII, we summarize our analysis and present our conlusions.

II. SCHRODINGER PROBLEM FOR A DRESSED QUANTUM HALL SYSTEM

Our system consists of a 2DEG placed on the xy-plane of the three-dimensional coordinate space. In our analysis, the 2DEG is subjected to a nonoscillating magnetic field $\mathbf{B} = (0, 0, B)^{\mathrm{T}}$ which is pointed towards the z axis. In addition, a linearly polarized strong light is applied perpendicular to the 2DEG surface. We specially select the frequency of the dressing field ω to be in the off-ersonant regime such that the field behaves as a purely dressing field. Furthermore, without limiting the generality we choose y-polorized electric field $\mathbf{E} = (0, E\sin(\omega t), 0)^{\mathrm{T}}$ for the linearly polarized dressing field (Fig. 1). Here B and E represent the amplitudes of the stationary magnetic field and oscillating electric field respectively.

Using Landau gauge for the stationary magnetic field, we can represent it as a vector potential $\mathbf{A}_s = (-By, 0, 0)^{\mathrm{T}}$. Furthermore, we model the dynamic dressing field in the Coulomb gauge as $\mathbf{A}_d(t) = (0, [E/\omega]\cos(\omega t), 0)^{\mathrm{T}}$. These vector potentials are coupled to the momentum of 2DEG as kinetic momentum [47, 48]. Thus, our system can be represented with a time-dependent Hamiltonian

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[\hat{\mathbf{p}} - e\big[\mathbf{A}_s + \mathbf{A}_d(t)\big]\Big]^2, \tag{1}$$

where m_e is the effective electron mass, e is the magnitude of the electron charge, and $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y, 0)^{\mathrm{T}}$ represents the canonical momentum operator for 2DEG with electron momentum $(p_x, p_y, 0)^{\mathrm{T}}$. The exact solutions for the time-dependent Schrödinger equation $i\hbar\ \mathrm{d}\psi/\mathrm{d}t = \hat{H}_e(t)\psi$ were already derived in Refs. [29, 49, 50]. Here we present them as a set of wave functions defined by two quantum numbers (n, m)

$$\begin{aligned}\psi_{n,m}&(x,y,t)\\ &= \frac{1}{\sqrt{L_x}}\chi_n\big(y - y_0 - \zeta(t)\big)\\ &\times \exp\bigg(\frac{i}{\hbar}\bigg[-\epsilon_n t + p_x x + \frac{eE[y-y_0]}{\omega}\cos(\omega t)\\ &+ m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \int_0^t dt' L(\zeta, \dot{\zeta}, t')\bigg]\bigg),\end{aligned} \tag{2}$$

where $n \in \mathbb{Z}_0^+$ and $m \in \mathbb{Z}$. Here L_x and L_y are dimensions of the 2DEG surface, and $\hbar$ is the reduced Planck constant. The center of the cyclotron orbit on the y-axis is given by $y_0 = -p_x/eB$ with $p_x = 2\pi\hbar m/L_x$. Moreover, χ_n are well known eigenstate solutions for the Schrödinger equation of the stationary quantum harmonic oscillator

$$\chi_n(y) = \frac{\sqrt{\kappa}}{\sqrt{2^n n!}} e^{-\kappa^2 y^2/2}\mathcal{H}_n(\kappa y) \quad \text{with} \quad \kappa = \sqrt{\frac{m_e\omega_0}{\hbar}}, \tag{3}$$

with eigenvalues $\epsilon_n = \hbar\omega_0(n + 1/2)$ where $\omega_0 = eB/m_e$ being the cyclotron frequency and $\mathcal{H}_n(\cdot)$ is the n-th Hermite polynomial. The path shift of the driven classical oscillator $\zeta(t)$ is given by

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t), \tag{4}$$

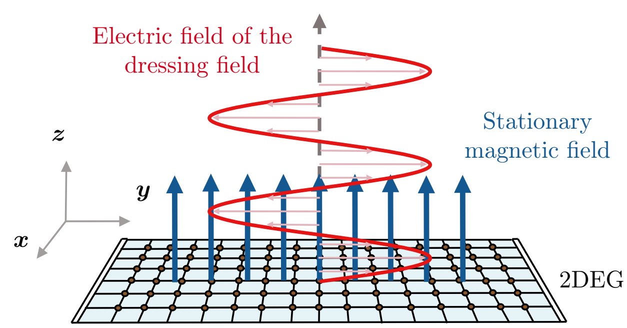

FIG. 1. Our 2DEG system only confined in the (x, y) plane while both stationary magnetic field $\mathbf{B}$ and dressing field (with y-polarized electric field $\mathbf{E}$) are applied perpendicular to the surface of 2DEG.

while the Lagrangian of the driven classical oscillator $L(\zeta, \dot{\zeta}, t)$ can be identified as

$$L(\zeta, \dot{\zeta}, t) = \frac{1}{2} m_e \dot{\zeta}^2(t) - \frac{1}{2} m_e \omega_0^2 \zeta^2(t) + eE\zeta(t)\sin(\omega t). \tag{5}$$

For details of the full derivation refer to Appendix A. The exponential phase shifts in Eq. (2) represent the influence of the stationary magnetic field and dressing field on the electron behavior of our system. Therefore, we can observe that the magneto-transport characteristics of 2DEG can be renormalized by a nonoscillating magnetic field along with a dressing field.

III. FLOQUET THEORY PERSPECTIVE

The general interpretations of physical quantum systems are mostly derived using symmetry conditions. The Famous Bloch analysis of electrons in quantum systems introduces a mathematical explanation for quantum systems occupying a discrete translational symmetry in configuration space. Floquet theory gives a mathematical formalism that can be used for translational symmetry in time rather than in space [32, 51, 52]. The Floquet-Drude conductivity theory was introduced recently by Wackerl *et al.* [30] as a method to analyze the transport properties of quantum systems exposed to strong radiation. In their work, they have presented more accurate results than the former theoretical descriptions for the conductivity of nanoscale systems in the presence of a dressing field. Therefore, we apply the Floquet-Drude conductivity theory to analyze our 2DEG system which is subjected to both a stationary magnetic field and a dressing field.

First, we need to identify the *quasienergies* and time periodic *Floquet modes* [32] for the wave functions given in Eq. (2). By factorizing the wave function into a linearly time dependent part and a periodic time dependent part, we present the quasienergies as

$$\varepsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right) - \Delta_\varepsilon, \tag{6}$$

which only depend~~s~~ on a single quantum number n. The Floquet modes can then be recognized as

$$\begin{aligned}\phi_{n,m}(x, y, t) =& \frac{1}{\sqrt{L_x}} \chi_n\big(y - y_0 - \zeta(t)\big) \\ &\times \exp\Bigg(\frac{i}{\hbar}\Bigg[p_x x + \frac{eE[y - y_0]}{\omega}\cos(\omega t) \\ &+ m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \xi\Bigg]\Bigg),\end{aligned} \tag{7}$$

with

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2 - \omega^2)}, \tag{8}$$

and

$$\xi = \frac{(eE)^2\big(3\omega^2 - \omega_0^2\big)}{8m_e\omega(\omega_0^2 - \omega^2)^2}\sin(2\omega t). \tag{9}$$

It is important to note that these Floquet modes are time-periodic ($T = 2\pi/\omega$) functions. At resonance $\omega = \omega_0$, the energy levels occupy a continuous spectrum and the discrete quasienergies are no longer valid [53]. Therefore, in this work we choose a dressing field frequency obeying the condition $\omega \neq \omega_0$.

Performing the Fourier transform over the confined two-dimensional space, we obtain the momentum space (k_x, k_y) representation of Floquet modes

$$\begin{aligned}\phi_{n,m}(k_x, k_y, t) =& \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \\ &\times \exp\left(i\xi - ik_y[d\sin(\omega t) + y_0]\right),\end{aligned} \tag{10}$$

where

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_n\left(\frac{k}{\kappa}\right). \tag{11}$$

Here we used new parameters

$$b = \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)}, \tag{12}$$

and

$$d = \frac{eE}{m_e(\omega_0^2 - \omega^2)}. \tag{13}$$

For a detailed derivation, ~~please~~ refer to Appendix B. ~~Now~~ using Floquet theory, the wave functions derived in Eq. (2) can be written as Floquet states in momentum space

$$\psi_{n,m}(k_x, k_y, t) = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)\phi_{n,m}(k_x, k_y, t). \tag{14}$$

IV. INVERSE SCATTERING TIME ANALYSIS

The Floquet-Fermi golden rule was proposed as an approach to analyze the transport properties of dressed quantum systems with impurities in Ref. [30]. However, this theory has not been applied for a dressed quantum Hall system in the previous studies. In this analysis, we use Floquet-Fermi golden rule to identify the effects induced by impurities on the magneto-transport properties. With the help of $t - t'$ formalism [30, 32, 54–56] and applying Floquet states derived in Eq. (14), we can derive an expression for (l, l')-th element of the inverse scattering time matrix for the N-th Landau level as

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll'} = \frac{\varrho^2}{eB}\delta(\varepsilon-\varepsilon_N) \times \int_{-\infty}^{\infty} dk_1 \left[J_l\left(\frac{b\hbar}{eB}[k_x-k_1]\right) J_{l'}\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times \left| \int_{-\infty}^{\infty} dk_2\, \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2 \right], \tag{15}$$

where $\varrho = \eta_{imp}L_x[V_{imp}/eB]^{1/2}$, ε is a given energy value, $J_l(\cdot)$ are Bessel functions of the first kind with l-th integer order, and ε_N is the energy of N-th Landau level. Detailed derivation is given in Appendix C. We modeled the effect cause by impurities in the considering system by a single perturbation potential. Since random impurities in a disordered metal is a better approximation for experimental results, we assumed that our perturbation potential is formed by a group of randomly distributed impurities. Thus, the total scattering potential in 2DEG has been represented as a sum of uncorrelated single impurity potentials $\upsilon(\mathbf{r})$. Here η_{imp} is the number of impurities in a unit area, $V_{imp} = \langle |V_{k'_x,k_x}|^2\rangle_{imp}$ with $V_{k'_x,k_x} = \langle k'_x|\upsilon(x)|k_x\rangle$, and $\langle x|k_x\rangle = e^{-ik_x x}$. Moreover in this analysis, $\langle\cdot\rangle_{imp}$ represents the average over the impurity disorder.

Next we analyze the contribution of the inverse scattering time matrix elements on the transport properties of our system. Since the disorder in the system can not alter the eigenenergy values of the undressed system [30], we can neglect the contribution of all off-diagonal elements in the inverse scattering time matrix. ~~Then~~ we consider only the central ($l = l' = 0$) diagonal element of the inverse scattering time matrix which has the largest contribution to the transport characteristics. Along with this assumption, we introduce a new parameter as the scattering-induced broadening of the N-th Landau level [29, 42]

$$\Gamma_N^{00}(\varepsilon,k_x) = \hbar\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{00}, \tag{16}$$

and this leads to

$$\Gamma_N^{00}(\varepsilon,k_x) = \frac{\varrho^2}{eB}\delta(\varepsilon-\varepsilon_N) \times \int_{-\infty}^{\infty} dk_1 \left[J_0^2\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times \left| \int_{-\infty}^{\infty} dk_2\, \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2 \right]. \tag{17}$$

In addition, for a scenario of scattering take place inside the same Landau level, we are able to present the delta distribution of the energy using the following interpretation [29]

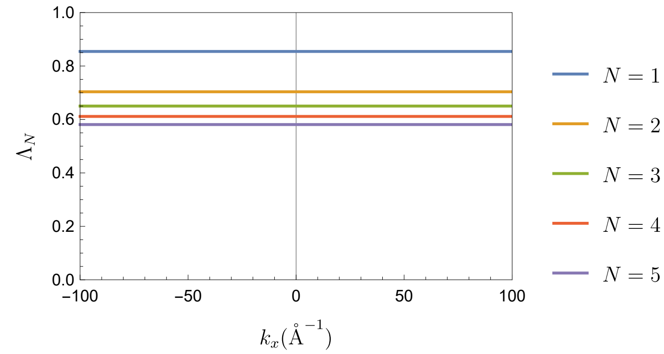

FIG. 2. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0, 1, 2, 3, 4$) against x-directional momentum value k_x in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2\times10^{12}$ rads^{-1} and intensity $I = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

$$\delta(\varepsilon-\varepsilon_N) \approx \frac{1}{\pi\Gamma_N^{00}(\varepsilon,k_x)}. \tag{18}$$

Then we can write the central element of inverse scattering time matrix in more compact form

$$\Gamma_N^{00}(\varepsilon,k_x) = \varrho\left[\int_{-\infty}^{\infty} dk_1\, J_0^2(\lambda_1[k_x-k_1]) \times \left|\int_{-\infty}^{\infty} dk_2\, \tilde{\chi}_N(\lambda_2 k_2)\tilde{\chi}_N(\lambda_2[k_1-k_2-k_x])\right|^2\right]^{-\frac{1}{2}}, \tag{19}$$

where $\lambda_1 \equiv \hbar b/eB$ and $\lambda_2 \equiv \hbar\kappa/eB$. To analyze the effect done by the dressing field on the scattering-induced

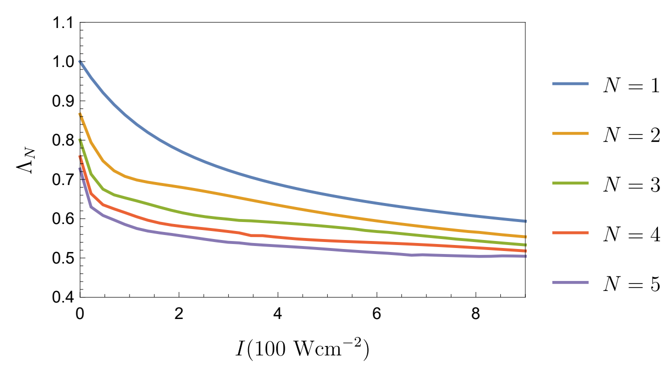

FIG. 3. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0, 1, 2, 3, 4$) against dressing field intensity I, in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2\times10^{12}$ rads^{-1}. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

broadening, we can introduce normalized N-th Landau level scattering-induced broadening as

$$\Lambda_N(k_x) \equiv \frac{\Gamma^{00}_N(\varepsilon, k_x)}{\Gamma^{00}_{N=0}(\varepsilon, k_x)\big|_{E=0}}, \tag{20}$$

and this leads to

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} dk_1\; J_0^2(\lambda_1[k_x - k_1])\left|\int_{-\infty}^{\infty} dk_2\; \tilde{\chi}_N(\lambda_2 k_2)\tilde{\chi}_N(\lambda_2[k_1 - k_2 - k_x])\right|^2}{\int_{-\infty}^{\infty} dk_1\; \left|\int_{-\infty}^{\infty} dk_2\; \tilde{\chi}_0(\lambda_2 k_2)\tilde{\chi}_0(\lambda_2[k_1 - k_2 - k_x])\right|^2}\right]^{1/2}. \tag{21}$$

Normalized energy band broadening against x-directional momentum component k_x for different Landau levels ($N = 0, 1, 2, 3, 4$) has been calculated for GaAs-based quantum well and results are depicted in Fig. (2) and Fig. (3). To make a comparison, we have selected the experiment parameters to match with analysis ~~done~~ in Ref. [42]. In their study, they have assumed that effective mass of the electron in GaAs-based quantum well system is $m_e \approx 0.07\tilde{m}_e$ where $\tilde{m}_e$ is mass of the electron [30, 42, 57]. In addition, they used the broadening of the natural (without a dressing field) 0-th Landau level Γ_0 as 0.24 me$V$. Therefore, in our calculations, we assumed that the natural least Landau level broadening also has this value: $\Gamma^{00}_{N=0}|_{E=0} = 0.24$ meV. Here we can identify that the normalized energy broadening value for each Landau level is independent of the x-directional momentum k_x value and we are able to manipulate it by the dressing field. When the dressing field's intensity increase, the energy broadening gets reduced and this make adjustments in transport properties. In the next section, we are going to derive a analytical expression for the conductivity in dressed quantum Hall systems to analyze these adjustments.

V. FLOQUET-DRUDE CONDUCTIVITY IN QUANTUM HALL SYSTEMS

A general theory for the conductivity in dressed systems with the disorder averaging was reported by Wackerl *et al.* [30, 58]. ~~With~~in this theory, the general x-directional longitudinal DC limit conductivity has been discribed by

$$\begin{aligned}\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\Bigg[\left(-\frac{\partial f}{\partial\varepsilon}\right) \\ \times\, \mathrm{tr}\left[j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))\right]\Bigg],\end{aligned} \tag{22}$$

where j_0^x and $\mathbf{G}^{r,a}(\varepsilon)$ are x-directional electric current operator matrix elements' $s = 0$ ~~the~~ Fourier component (see Appendix D) and white noise disorder averaged Floquet Green function matrix [30, 58] respectively defined against the Floquet modes of the considering system. Here we have assumed that only $s = 0$ Fourier component of the current operator is contributing to the conductivity. In addition, A is the area of the considering two-dimensional system, partial distribution function is given by f and Π is a function that can be chosen such that

$$\Pi - \frac{\hbar\omega}{2} \leq \varepsilon_N < \Pi + \frac{\hbar\omega}{2}, \tag{23}$$

where ε_N are quasienergies of all considering Floquet states. Moreover, $tr[\cdot]$ is the trace of operator.

Then Eq. (22) can be expanded in off resonant regime ($\omega\tau_0 \gg 1$), where τ_0 is the scattering time of the undriven system, using only central entry Fourier components ($l = l' = 0$) of Floquet modes $|\phi_{n,m}\rangle \equiv |n, k_x\rangle$

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\langle n, k_x| j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon)) j_0^x(\mathbf{G}_0^r(\varepsilon) - \mathbf{G}_0^a(\varepsilon))|n, k_x\rangle\,, \tag{24}$$

where V_{k_x} is the volume of considering x-directional momentum space. Then this can evaluate as follows

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}^4}\sum_{k_x}\sum_{n}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3} \\ \times \langle n,k_x|j_0^x|n_1,k_{x1}\rangle \langle n_1,k_{x1}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n_2,k_{x2}\rangle \langle n_2,k_{x2}|j_0^x|n_3,k_{x3}\rangle \langle n_3,k_{x3}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n,k_x\rangle . \tag{25}$$

Since we can diagonalize the impurity averaged Green's functions using unitary transformation ($\mathbf{T} \equiv |n,k_x\rangle$) mentioned in Refs. [30, 58, 59]. Thus, we evaluate the matrix element of the difference between retarded and advanced Green's functions as

$$\langle n_1,k_{x1}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n_2,k_{x2}\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})]^2}\right], \tag{26}$$

and

$$\langle n_3,k_{x3}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n,k_x\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})]^2}\right]. \tag{27}$$

Here we used the retarded self-energy matrix $\boldsymbol{\Sigma}^r$ which is the sum of all irreducible diagrams [30, 58]. Applying the matrix elements of the electric current operator in Landau levels (see Appendix D) and results form Eq. (26) and Eq. (27) back into Eq. (25) we ~~can~~ obtain

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\sum_{n_1,n_2} \\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})]^2}\right] \\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})]^2}\right], \tag{28}$$

and after the expansion, the only remaining non-zero term would be

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1) \\ \times\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n+1}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})_{\varepsilon_{n+1}}\Big]^2}\right]\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\boldsymbol{\Sigma}^r\mathbf{T})_{\varepsilon_{n}}\Big]^2}\right]. \tag{29}$$

Since the inverse scattering time matrix being equal to the diagonalized contrast of the retarded and advanced self-energy, and on the diagonal the contrast of the retarded and advanced Green's function can be represented with the imaginary component of the retarded self-energy [30, 58], we can identify the following property

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll} = -2\mathrm{Im}\big[\mathbf{T}^\dagger\boldsymbol{\Sigma}^r(\varepsilon,k_x)\mathbf{T}\big]^{ll}. \tag{30}$$

~~Then~~ using central element ($l=0$) of the inverse scattering time matrix, we can restructure the derived conductivity expression as ~~follows~~

$$\sigma^{xx} = \frac{1}{\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)\left[\frac{\tilde{\Gamma}(\varepsilon_{n+1})}{(\varepsilon_F-\varepsilon_{n+1})^2+\tilde{\Gamma}^2(\varepsilon_{n+1})}\right]\left[\frac{\tilde{\Gamma}(\varepsilon_{n})}{(\varepsilon_F-\varepsilon_{n})^2+\tilde{\Gamma}^2(\varepsilon_{n})}\right], \tag{31}$$

where $\tilde{\Gamma}(\varepsilon_n,k_x) \equiv (\hbar/2\tau(\varepsilon_n,k_x))^{00}$. We already identified that the inverse scattering time matrix's central element is independent of k_x value and we can get the sum over all available momentum space in x direction. However, by considering the condition that the center of the cyclotron orbit y_0 must physically lie within the consider~~ing~~ system $-L_y/2 < y_0 < L_y/2$, we can identify that

$$-\frac{m_e\omega_0 Ly}{2\hbar} \leq k_x \leq \frac{m_e\omega_0 Ly}{2\hbar}. \tag{32}$$

~~Then~~ we use the Fermi-Dirac distribution as our partial distribution function (f) for our system

$$f(\varepsilon) = \frac{1}{[\exp(\varepsilon - \varepsilon_F)/k_B T] + 1}, \tag{33}$$

where k_B is Boltzmann constant, T is absolute temperature and ε_F is Fermi energy of the system. Considering the above distribution with extremely low temperature conditions, we can approximate

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F), \tag{34}$$

~~and~~ by letting $\Pi = \varepsilon_F$, the expression for conductivity leads to

$$\sigma^{xx} = \frac{e^2}{\hbar}\frac{1}{\pi A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right], \tag{35}$$

where $X_F \equiv (\varepsilon_F/\hbar\omega_0 - 1/2)$ and $\gamma_n \equiv \tilde{\Gamma}(\varepsilon_n)/\hbar\omega_0$. Same as above derivation, we can derive the longitudinal conductivity in y-direction by using the current operator derived in Appendix D

$$\sigma^{yy} = \frac{e^2}{\hbar}\frac{1}{\pi A}\frac{1}{e^2B^2}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right]. \tag{36}$$

VI. MANIPULATE CONDUCTIVITY IN QUANTUM HALL SYSTEMS

To identify the characteristics of the longitudinal conductivity of the quantum Hall systems with external dressing field, first we ~~can~~ derive an expression for a normalized longitudinal conductivity as a function of Fermi energy X_F and intensity of the dressing field I. Here we have the normalized x-directional conductivity using the natural conductivity of the least Landau level

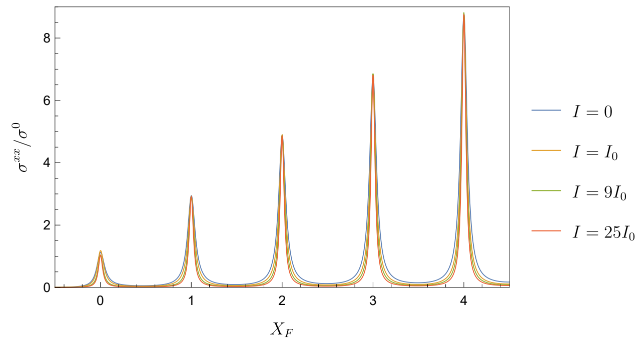

FIG. 4. Normalized longitudinal conductivity σ_{xx} against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2\times10^{12}$ rads^{-1} and $I_0 = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

$$\frac{\sigma^{xx}}{\sigma^0} = \sum_n \frac{(n+1)}{0.0037\Lambda_n\Lambda_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{0.06\Lambda_n}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{0.06\Lambda_{n+1}}\right)^2}\right], \tag{37}$$

where $\sigma^0 = (e^2/\pi\hbar A)$. We use this expression to illustrate the changes that can be done to the longitudinal conductivity in 2DEG using an external dressing field. As given in Fig. 4 and 5 we can manipulate the longitudinal conductivity σ_{xx} using an external dressing field's intensity and the Fermi level X_F of the considering system. For a given dressing field intensity, the longitudinal conductivity vary against the Fermi level of the system by creating sharp peaks at each Landau level energy values. Since electrons are restricted to have only Landau ~~level~~ energies, the conductivity gets very low values when the Fermi level is not aligned with any of the Landau level energy values. In contrast, on each Landau level, the conductivity can achieve very high values compared to other areas and as illustrates in Fig. 4 the peak value of longitudinal conductivity on each Landau level gets increase with the Landau level number.

Considering the effect of the external dressing field on the longitudinal conductivity of 2DEG, we can identify that high intensities shrink the conductivity regions near Landau levels. However, the peak value of the conductivity at each Landau level has the same value as the undressed system. This demonstrates that we are able to tune the width of the regions of conductivity in these quantum Hall systems with the help of a dressing field. These characteristics are aligned with results demonstrated by Dini *et al.* [29] and as they remarked since the Fermi level of the system can be changed with the applied gate voltage of the material this can be used

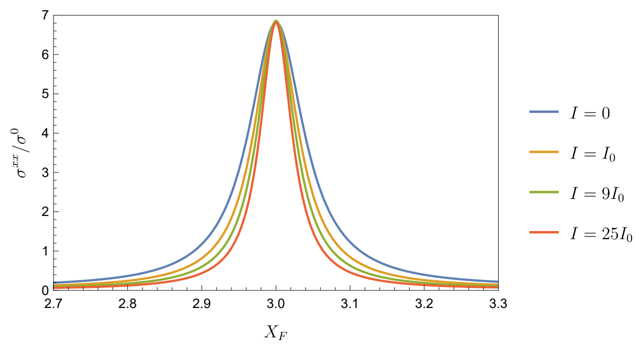

FIG. 5. 3rd Landau level's normalized longitudinal conductivity σ_{xx} against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2 \times 10^{12}$ rads^{-1} and $I_0 = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$

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as a 2D switch for nanoscale optoelectronics applications. Controlling the external dressing field we are able to fine-tune the switching mechanism for optimized performance. Furthermore, we can distinguish that the shapes and behavior of the conductivity regions illustrated in Fig. 4 and 5 are generally incompatible with the results reported in Ref. [29]. This is due to the selection of the conventional longitudinal conductivity theory of 2DEG from Refs. [37, 41]. The semi-elliptical conductivity regions illustrated in Refs. [29, 37, 41], have less consistency with the experimentally observed Landau levels ~~representation~~ [42]. In our study on the transport properties of quantum Hall systems, we developed the conductivity expression starting from Floquet-Drude conductivity [30] and our results are much more aligned with the result mentioned in Ref. [42]. The description of conductivity of quantum Hall systems demonstrated in Ref. [42] has an excellent agreement between the theory and experiment obtained in a GaAs/AlGaAs 2DES for the low magnetic field range. However, they have not considered the tunability that can be achieved with the external strong dressing field. In this analysis, we account for both magnetic and dressing field effects that can be applied to the transport properties of 2DEG, and we have presented a more generalized theory. As a concluding remark, in this study, we were able to demonstrate that using Floquet-Drude conductivity method, one can derive a more experimental fitting and generalized mathematical model for the charge transport properties of quantum Hall systems.

VII. CONCLUSIONS

In this analysis, we introduced a generalized mathematical model for charge transport properties in a 2DEG under a nonoscillating magnetic field and a high intensity light. Under the uniform magnetic field, the charged particles can only settle in discrete energy values and this leads to the Landau quantization. We modeled the behavior of electrons in Landau levels under the dressing field utilizing the Floquet-Drude conductivity method by considering impurities in the material as Gaussian random scattering potential. Finally, we derived expressions for x-directional and y-directional longitudinal components of electric conductivity tensor for the considered system.

Our derived analytical expressions disclosed that the transport characteristics of the dressed quantum Hall system can be controlled by the applied dressing field's intensity. Using detailed numerical calculations with empirical system parameters, we further analyzed the manipulation of conductivity components using the dressing field. We found that the graphical illustrations we gained are capable of produce the same behavior as the experiment ~~conductivity~~ found in quantum Hall systems without a dressing field. Furthermore, we identified that by regulating the intensity of the radiation, the conductivity regions near the Landau levels can be squeezed. Despite, this behavior has been identified in previous works, their results are not coinciding with the more accurate description of conductivity components in undressed quantum Hall systems. However, our generalized analysis of conductivity in dressed quantum Hall systems provide a well-suited description for these special quantum Hall systems.

In summary, the primary purpose of this study was to broaden the modern descriptions of transport properties in dressed quantum Hall systems. Moreover, our detailed theoretical analysis showed that the recently introduced Floquet-Drude conductivity model can be adopted to extend the models that were used to describe the transport characteristics in quantum Hall systems. ~~Due to~~ owing the ability to control the conductivity regions, high intensity external illumination can be used as a trigger for two-dimensional quantum switching devices which are employed as the building blocks of next generation nanoelectronic devices. Finally, we identified that our findings of this paper can be used towards understanding nanoscale quantum devices, enhancing their performance, and inventing novel appliances.

ACKNOWLEDGMENTS

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Appendix A: Wave function solutions of dressed Landau levels

The deriving process of solutions for time dependent Schrödinger equation with the Hamiltonian of our system (Eq. 1) quite similar to that followed in Refs. [29, 49]. We start with expanding the Hamiltonian for two-dimensional case

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[\left[\hat{p}_x + eBy\right]^2 + \left[\hat{p}_y - \frac{eE}{\omega}\cos(\omega t)\right]^2\right], \tag{A1}$$

and since $\left[\hat{H}_e(t), \hat{p}_x\right] = 0$, both operators share same (simultaneous) eigenfunctions $\frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar}\right)$ with $p_x = 2\pi\hbar m/L_x$, $m \in \mathbb{Z}$. Therefore, we re-arrange the Hamiltonian using definition of canonical momentum in y-direction to derive

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[\left[p_x + eBy\right]^2 + \left[-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right]^2\right]. \tag{A2}$$

We now define the *center of the cyclotron orbit* on the y-axis $y_0 \equiv -p_x/eB$ and the *cyclotron frequency* $\omega_0 \equiv eB/m_e$. This leads to a new arrangement of the Hamiltonian

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}\tilde{y}^2 + \frac{1}{2m_e}\left[-\hbar^2\frac{\partial^2}{\partial\tilde{y}^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial\tilde{y}} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\right], \tag{A3}$$

where we used a variable substitution $\tilde{y} = (y - y_0)$. Now we assume that the wave function solutions for the time-dependent Schrödinger equation of considering quantum system

$$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = \hat{H}_e(t)\psi, \tag{A4}$$

can present by the following form

$$\psi_m(x,\tilde{y},t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieE\tilde{y}}{\hbar\omega}\cos(\omega t)\right)\vartheta(\tilde{y},t), \tag{A5}$$

where $\vartheta(\tilde{y},t)$ is a function that satisfy the following property

$$\left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 - eE\tilde{y}\sin(\omega t) - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} - i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]\vartheta(\tilde{y},t) = 0. \tag{A6}$$

If we turn off the strong dressing field ($E = 0$), this equation leads to simple harmonic oscillator Hamiltonian

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y},t)}{\mathrm{d}t} = \left[\frac{\hat{p}_{\tilde{y}}^2}{2m_e} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2\right]\vartheta(\tilde{y},t). \tag{A7}$$

It is important to notice that we can identify the $S(t) \equiv eE\sin(\omega t)$ part as an external force act on the harmonic oscillator, and we can solve this as a forced harmonic oscillator in $\tilde{y}$ axis

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y},t)}{\mathrm{d}t} = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2 - \tilde{y}S(t)\right]\vartheta(\tilde{y},t). \tag{A8}$$

This system can be exactly solvable, and we can solve the equation using the methods explained by Husimi [49] as follows. We introduce a time dependent shifted coordinate $y' = \tilde{y} - \zeta(t)$ and perform following unitary transformation

$$\vartheta(y',t) = \exp\left(\frac{im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y',t), \tag{A9}$$

and this yields

$$\begin{aligned}i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[&-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 \\ &+ \Big[m_e\ddot{\zeta} + m_e\omega_0^2\zeta - S(t)\Big]y' \\ &+ \Big[-\frac{1}{2}m_e\dot{\zeta}^2 + \frac{1}{2}m_e\omega_0^2\zeta^2 - \zeta S(t)\Big]\Big]\varphi(y',t).\end{aligned} \tag{A10}$$

Then we can restrict our $\zeta(t)$ function such that

$$m_e\ddot{\zeta} + m_e\omega_0^2\zeta = S(t), \tag{A11}$$

and that leads to

$$\begin{aligned}i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[&-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 \\ &- L(\zeta,\dot{\zeta},t)\Big]\varphi(y',t),\end{aligned} \tag{A12}$$

where

$$L(\zeta,\dot{\zeta},t) = \frac{1}{2}m_e\dot{\zeta}^2 - \frac{1}{2}m_e\omega_0^2\zeta^2 + \zeta S(t), \tag{A13}$$

is the Lagrangian of a classical driven oscillator. To proceed further, another unitary transform can be introduced as follows

$$\varphi(y',t) = \exp\left(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\right)\chi(y',t), \tag{A14}$$

and subtitling Eq. (A14) back in Eq. (A12) yields

$$i\hbar\frac{\partial}{\partial t}\chi(y',t) = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2\right]\chi(y',t). \tag{A15}$$

This is the well known Schrödinger equation of the quantum harmonic oscillator. This allows us to identify with the well known eigenfunctions [60, 61]

$$\chi_n(y) = \frac{\sqrt{\kappa}}{\sqrt{2^n n!}}e^{-\kappa^2y^2/2}\mathcal{H}_n(\kappa y) \quad \text{with} \quad \kappa = \sqrt{\frac{m_e\omega_0}{\hbar}}, \tag{A16}$$

which are propositional to the Hermite polynomials $\mathcal{H}_n$, with eigenvalues

$$\epsilon_n = \hbar\omega_0\left(n+\frac{1}{2}\right)\ ,\ n\in\mathbb{Z}_0^+. \tag{A17}$$

Therefore, we can identify the solutions of Eq. (A8) as

$$\vartheta_n(\tilde{y},t) = \chi_n(\tilde{y}-\zeta(t))\exp\left(\frac{i}{\hbar}\left[-\epsilon_n t + m_e\dot{\zeta}(t)\left[\tilde{y}-\zeta(t)\right] + \int_0^t dt' L(\zeta,\dot{\zeta},t')\right]\right). \tag{A18}$$

Since set $\{\chi_n(x)\}$ functions forms a complete set, any general solution $\vartheta_(\tilde{y},t)$ can be presented with the help of solutions derived in Eq. (A18).

Finally, we consider our scenario where we assumed that $S(t) = eE\sin(\omega t)$, and we can derive the solution for Eq. (A11):

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2-\omega^2)}\sin(\omega t). \tag{A19}$$

Subtitling solutions in Eq. (A18) back in Eq. (A5), we can obtain the set of wave functions with two different quantum number (n,m) that satisfy the time dependent Schrödinger equation Eq. (A4) as follows

$$\psi_{n,m}(x,y,t) = \frac{1}{\sqrt{L_x}}\chi_n\left(y-y_0-\zeta(t)\right)\exp\left(\frac{i}{\hbar}\left[-\epsilon_n t + p_x x + \frac{eE[y-y_0]}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\left[y-y_0-\zeta(t)\right] + \int_0^t dt' L(\zeta,\dot{\zeta},t')\right]\right). \tag{A20}$$

Appendix B: Floquet modes and quasienergies

1. Position space representation

First define the time integral of Lagrangian of the classical oscillator given in Eq. (5), over a $T = 2\pi/\omega$ period as

$$\Delta_\varepsilon = \frac{1}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t'), \tag{B1}$$

and after performing the integral using Eq. (4), we can obtain more simplified result:

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)}. \tag{B2}$$

Next define another parameter

$$\xi = \int_0^t dt'\ L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t, \tag{B3}$$

and after simplifying, this leads to

$$\xi = \frac{(eE)^2\left(3\omega^2-\omega_0^2\right)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t), \tag{B4}$$

which is a periodic function in time with 2ω frequency. Now using these parameters we can factorize the wave function Eq. (2) as linearly time dependent part and periodic time dependent part as follows

$$\psi_\alpha(x,y,t) = \exp\left(\frac{i}{\hbar}[-\epsilon_n t + \Delta_\varepsilon t]\right)\frac{1}{\sqrt{L_x}}\chi_n\left(y-y_0-\zeta(t)\right) \times\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eEy}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\left[y-\zeta(t)\right] + \int_0^t dt' L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t\right]\right), \tag{B5}$$

and this leads to separate linear time dependent phase component as the quasienergies

$$\varepsilon_n = \hbar\omega_0\left(n+\frac{1}{2}\right) - \Delta_\varepsilon, \tag{B6}$$

while rest of the components as time-periodic Floquet modes

$$\phi_{n,m}(x,y,t) \equiv \frac{1}{\sqrt{L_x}}\chi_n\left(y-y_0-\zeta(t)\right)\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eE[y-y_0]}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\left[y-y_0-\zeta(t)\right] + \xi\right]\right). \tag{B7}$$

2. Momentum space representation

To write the Floquet modes in momentum space, we perform continuous Fourier transform over the considering confined space $A = L_x L_y$ on the Floquet modes given in Eq. (7):

$$\begin{aligned}\phi_{n,m}&(k_x,k_y,t)\\ &= \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\left[m_e\dot{\zeta}(t)\zeta(t)-\xi\right]\right)\\ &\times\int_{-L_y/2}^{L_y/2} dy\ \exp(-i[k_y-\gamma(t)]y)\chi_n[y-\mu(t)]\\ &\times\frac{1}{\sqrt{L_x}}\int_{-L_x/2}^{L_x/2} dx\ \exp(-ik_x x)\exp\left(\frac{ip_x}{\hbar}x\right),\end{aligned} \tag{B8}$$

where

$$\mu(t) = \frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)} + y_0 \quad , \quad \gamma(t) = \frac{eE\omega_0^2\cos(\omega t)}{\hbar\omega(\omega_0^2-\omega^2)}. \tag{B9}$$

Next using the Fourier transform identity [48]

$$\int_{L_x} dx\ \exp\left(-ik_x x + \frac{ip_x}{\hbar}x\right) = L_x\delta_{k_x,\frac{p_x}{\hbar}}, \tag{B10}$$

we can derive

$$\phi_{n,m}(k_x,k_y,t) = \Phi_{n,m}(k_y,t)\delta_{k_x,\frac{p_x}{\hbar}} \times \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\left[m_e\dot{\zeta}(t)\zeta(t)-\xi\right]\right). \tag{B11}$$

Here we defined $\Phi_{n,m}(k_y,t)$ as

$$\Phi_{n,m}(k_y,t) = \sqrt{L_x}\int_{-L_y/2}^{L_y/2} dy\ \chi_n[y-\mu(t)] \times \exp(-i[k_y-\gamma(t)]y). \tag{B12}$$

Substituting $k'_y = k_y - \gamma(t)$ and $y' = y - \mu(t)$ and assuming that size of the 2DEG sample in y-direction is considerably large ($L_y \to \infty$), we can obtain

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\int_{-\infty}^{\infty} dy'\ \chi_n(y')\exp\left(-ik'_y y'\right). \tag{B13}$$

We can identify that the integral represents the Fourier transform of $\{\chi_n\}$ functions and using the symmetric conditions [62] for the Fourier transform of Gauss-Hermite functions $\theta_n(x)$:

$$\mathcal{FT}[\theta_n(\kappa x),x,k] = \frac{i^n}{|\kappa|}\theta_n(k/\kappa), \tag{B14}$$

Eq. (B13) can be simplified as

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\tilde{\chi}_n\left(k'_y\right), \tag{B15}$$

with

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_\alpha\left(\frac{k}{\kappa}\right). \tag{B16}$$

Substitute Eq. (B15) back in Eq. (B11) and this leads to

$$\phi_{n,m}(k_x,k_y,t) = \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \times \exp\left(i\xi - ik_y\left[d\sin(\omega t) + \frac{\hbar k_x}{eB}\right]\right), \tag{B17}$$

where

$$b \equiv \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2-\omega^2)} \quad d \equiv \frac{eE}{m_e(\omega_0^2-\omega^2)}, \tag{B18}$$

and it is necessary to notice that k_x is quantized with $k_x = 2\pi m/L_x\ ,\ m \in \mathbb{Z}$.

Appendix C: Floquet Fermi golden rule for dressed quntum Hall systems

The Floquet Fermi golden rule derivation for our quantum Hall system with the help of $t-t'$ formalism is given here in detail. The t-t'-Floquet states [30, 32]

$$|\psi_{n,m}(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)|\phi_{n,m}(t')\rangle\,, \tag{C1}$$

are derived by separating the aperiodic and periodic components of Eq. (14). Additionally, these fulfill the t-t'-Schrödinger equation [30, 32]

$$i\hbar\frac{\partial}{\partial t}|\psi_{n,m}(t,t')\rangle = H_F(t')|\psi_{n,m}(t,t')\rangle\,, \tag{C2}$$

where *Floquet Hamiltonian* defined as

$$H_F(t') \equiv H_e(t') - i\hbar\frac{\partial}{\partial t'}. \tag{C3}$$

Next we can identify the time evolution operator corresponding to the t-t'-Schrödinger equation

$$U_F(t,t_0;t') = \exp\left(-\frac{i}{\hbar}H_F(t')[t-t_0]\right), \tag{C4}$$

and the advantage of t-t' formalism lies on this time evolution operator which avoids any time ordering operators [30].

For our scenario, consider a time-independent total perturbation $V(\mathbf{r})$ which has been turned on at the reference time $t = t_0$, then the t-t'-Schrödinger equation becomes

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n,m}(t,t')\rangle = [H_F(t') + V(\mathbf{r})]|\Psi_{n,m}(t,t')\rangle\,, \tag{C5}$$

and this introduce a new wave function solution $\Psi_{n,m}$ for the system with the given total perturbation. If $t \le t_0$, both solutions of the Schrödinger equations (Eq. (C2) and Eq. (C5)) coincide

$$|\psi_{n,m}(t,t')\rangle = |\Psi_{n,m}(t,t')\rangle \quad \text{when} \quad t \le t_0. \tag{C6}$$

Now move into the interaction picture representation [47, 48] of the t-t'-Floquet state

$$|\Psi_{n,m}(t,t')\rangle_I = U_0^\dagger(t,t_0;t')|\Psi_{n,m}(t,t')\rangle\,, \tag{C7}$$

and due to time independence, the perturbation in the interaction picture has the same form as Schrödinger picture representation

$$V_I(\mathbf{r}) = U_0^\dagger(t,t_0;t')V(\mathbf{r})U_0(t,t_0;t') = V(\mathbf{r}). \tag{C8}$$

This drive us to the t-t'-Schrödinger equation representation in the interaction picture

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n,m}(t,t')\rangle_I = V_I(\mathbf{r})|\Psi_{n,m}(t,t')\rangle_I\,, \tag{C9}$$

with the recursive solution [47, 48]

$$|\Psi_{n,m}(t,t')\rangle_I = |\Psi_{n,m}(t_0,t')\rangle_I + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ V_I(\mathbf{r})|\Psi_{n,m}(t_1,t')\rangle_I\,. \tag{C10}$$

Iterating the solution only up to the first order (Born approximation) we obtain

$$|\Psi_{n,m}(t,t')\rangle_I \approx |\psi_{n,m}(t_0,t')\rangle + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ V_I(\mathbf{r})|\psi_{n,m}(t_0,t')\rangle\,. \tag{C11}$$

In addition, since our Floquet states create a basis, we can represent any solution using these Floquet states:

$$|\Psi_\alpha(t,t')\rangle = \sum_\beta a_{\alpha,\beta}(t,t')\,|\psi_\beta(t,t')\rangle\,, \tag{C12}$$

where we used a single notation to represent two quantum numbers; $\alpha \equiv (n_\alpha, m_\alpha)$ and $\beta \equiv (n_\beta, m_\beta)$. Then we can identify the *scattering amplitude* as $a_{\alpha,\beta}(t,t') = \langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle$ and this can evaluate with

$$\begin{aligned} a_{\alpha,\beta}(t,t') =& \langle\psi_\beta(t,t')|\psi_\alpha(t,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \end{aligned} \tag{C13}$$

Since the t-t'-Floquet states are orthonormal and assuming $t_0 = 0$ and $\alpha \neq \beta$ this leads to

$$a_{\alpha,\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{C14}$$

Now consider a scattering phenomenon from a t-t'-Floquet state $|\psi_\beta(t,t')\rangle$ into a distinct t-t'-Floquet state $|\Psi_\alpha(t,t')\rangle$ that occupied with a constant quansienergy ε (Fig. 6):

$$\begin{aligned} |\psi_\beta(t,t')\rangle =& \exp\left(-\frac{i}{\hbar}\varepsilon_\beta t\right)|\phi_\beta(t')\rangle \\ &\xRightarrow{\text{scattering}} |\Psi_\alpha(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon t\right)|\Phi_\alpha(t')\rangle\,. \end{aligned} \tag{C15}$$

It is important to remember that a state of this considering system can be represented by two independent quantum numbers: n represents the landau level and m represents the quantized momentum in x-direction. The scattering amplitude for this scattering scenario can be calculated using the equation derived in Eq. (C14)

$$a_{\alpha\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\ e^{\frac{i}{\hbar}(\varepsilon_\beta-\varepsilon)t_1}\,\langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle\,, \tag{C16}$$

and assuming for a long time $t \to \infty$, we can turn this integral into a delta distribution

$$a_{\alpha\beta}(t') = -2\pi i\delta(\varepsilon_\beta - \varepsilon)Q, \tag{C17}$$

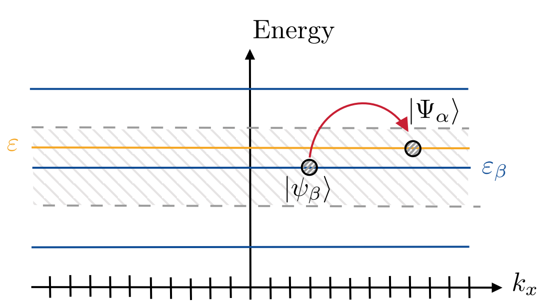

FIG. 6. Scattering from $|\psi_\beta(t,t')\rangle$ to constant energy state $|\Psi_\alpha(t,t')\rangle$ due to scattering potential created by impurities.

where $Q \equiv \langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle$ and using completeness properties we can re-write this as

$$Q = \sum_{\mathbf{k}}\sum_{\mathbf{k}'}\langle\phi_\beta(t')|\mathbf{k}'\rangle\ \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle\ \langle\mathbf{k}|\phi_\alpha(t')\rangle\,, \tag{C18}$$

and separating x and y directional momentum we can derive (we already assumed that $L_y \to \infty$)

$$Q = \sum_{k_x}\sum_{k'_x}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\ V_{\mathbf{k}',\mathbf{k}}\phi^\dagger_\beta(\mathbf{k}',t')\phi_\alpha(\mathbf{k},t'), \tag{C19}$$

with $V_{\mathbf{k}',\mathbf{k}} \equiv \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle$.

Since we are presenting the the perturbation potential $V(\mathbf{r})$ by using a group of randomly distributed impurities, we take into account N_{imp} number of identical single impurity potentials distributed at randomly but in fixed positions $\mathbf{r}_i$. Then scattering potential $V(\mathbf{r})$ can be identified as the sum of uncorrelated single impurity potentials $\upsilon(\mathbf{r})$:

$$V(\mathbf{r}) \equiv \sum_{i=1}^{N_{imp}}\upsilon(\mathbf{r}-\mathbf{r}_i). \tag{C20}$$

Next we model the perturbation $V(\mathbf{r})$ as a Gaussian random potential where one can choose the zero of energy such that the potential is zero on average. This model is characterized by [63]

$$\langle\upsilon(\mathbf{r})\rangle_{imp} = 0, \tag{C21a}$$

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon(\mathbf{r}-\mathbf{r}'), \tag{C21b}$$

where $\langle\cdot\rangle_{imp}$ denoted the average over realizations of the impurity disorder and $\Upsilon(\mathbf{r}-\mathbf{r}')$ is any decaying function depends only on $\mathbf{r}-\mathbf{r}'$. In addition, this model assumes that $\upsilon(\mathbf{r}-\mathbf{r}')$ only depends on the position difference $|\mathbf{r}-\mathbf{r}'|$, and it decays with a characteristic length r_c. Since this study considers the case where the wavelength of radiation or scattering electrons is much greater than r_c, it is a good approximation to make two-point correlation function to be

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon^2_{imp}\delta(\mathbf{r}-\mathbf{r}'), \tag{C22}$$

where Υ_{imp} is strength of the delta potential and a random potential $V(\mathbf{r})$ with this property is called white noise [63]. Then we can approximately model the total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}}\Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i). \tag{C23}$$

Then we can calculate $V_{\mathbf{k}',\mathbf{k}}$ using this assumption as follows

$$V_{\mathbf{k}',\mathbf{k}} = \left\langle \mathbf{k}' \middle| \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i) \middle| \mathbf{k} \right\rangle \tag{C24a}$$

$$= \sum_{i=1}^{N_{imp}} \int_{-\infty}^{\infty} dy \left[\frac{1}{\sqrt{L_x L_y}} e^{ik'_y y}\delta(y-y_i) \right.$$

$$\left. \times \frac{1}{\sqrt{L_x L_y}} e^{-ik_y y} \langle k'_x | \Upsilon_{imp}\delta(x-x_i)|k_x\rangle \right] \tag{C24b}$$

$$= \sum_{i=1}^{N_{imp}} \frac{1}{L_x L_y} e^{i(k'_y-k_y)y} \langle k'_x | \Upsilon_{imp}\delta(x-x_i)|k_x\rangle . \tag{C24c}$$

Since $v(\mathbf{r})$ in momentum space is a constant value, each impurity produce same impurity potential for every x-directional momentum pairs and assuming the total number of scatters N_{imp} is microscopically large, we can derive

$$V_{\mathbf{k}',\mathbf{k}} = V_{k'_x,k_x} \frac{N_{imp}}{L_y L_x} \int_{-\infty}^{\infty} dy_i \; e^{i\left(k'_y - k_y\right)y_i} \tag{C25a}$$

$$= \eta_{imp} V_{k'_x,k_x} \delta(k'_y - k_y), \tag{C25b}$$

where

$$V_{k'_x,k_x} \equiv \langle k'_x | \Upsilon_{imp}\delta(x-x_i)|k_x\rangle , \tag{C26}$$

is a constant value for every i impurity and η_{imp} is number of impurities in a unit area. It is important to notice that $\langle x|k_x\rangle = e^{-ik_x x}$.

Now using Eq. (10) and Eq. (C25b) on Eq. (C19), we obtain (with changing variable $t' \to t$)

$$Q = \sum_{k_x}\sum_{k'_x} \eta_{imp} V_{k'_x,k_x} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y \; \delta(k'_y - k_y)$$
$$\times \sqrt{L_x}\exp(ik'_y[d\sin(\omega t) + y'_0])\tilde{\chi}_{n_\beta}(k'_y - b\cos(\omega t))$$
$$\times \sqrt{L_x}\exp(-ik_y[d\sin(\omega t) + y_0])\tilde{\chi}_{n_\alpha}(k_y - b\cos(\omega t)), \tag{C27}$$

and this can simplify as

$$Q = \sum_{k_x}\sum_{k'_x} \eta_{imp} L_x V_{k'_x,k_x} I, \tag{C28}$$

with

$$I = \int_{-\infty}^{\infty} dk_y \; \tilde{\chi}_{n_\beta}(k_y - b\cos(\omega t))\tilde{\chi}_{n_\alpha}(k_y - b\cos(\omega t))$$
$$\times \exp(-ik_y[y_0 - y'_0]). \tag{C29}$$

To avoid the energy exchange from the dressing field and electrons in Landau levels, the applied radiation must be a purely dressing field. Therefore, in this study we assume that the dressing field only can renormalize the the probability of electron scattering inside a same Landau energy level ($n_\alpha = n_\beta = N$). This transform the Eq. (C29) to

$$I = \int_{-\infty}^{\infty} dk_y \; \tilde{\chi}^2_N(k_y - b\cos(\omega t))\exp(-ik_y[y_0 - y'_0]). \tag{C30}$$

Using Fourier transform of Gauss-Hermite functions [62] and convolution theorem [64, 65] we can derive

$$I = 2\pi\exp(b[y'_0 - y_0]\cos(\omega t))$$
$$\times \int_{-\infty}^{\infty} dy \; \chi_N(y)\chi_N(y_0 - y'_0 - y). \tag{C31}$$

Therefore, finally the scattering amplitude derived in Eq. (C17) can be evaluated for given $k_x = 2\pi m_\alpha/L_x$ and $k'_x = 2\pi m_\beta/L_x$ as

$$a_{\alpha\beta}(k_x,k'_x,t) = -2\pi i\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N - \varepsilon)$$
$$\times \exp(b[y'_0 - y_0]\cos(\omega t))$$
$$\times \int_{-\infty}^{\infty} dy \; \chi_N(y)\chi_N(y_0 - y'_0 - y). \tag{C32}$$

Since this scattering amplitude is time-periodic we can write this as a Fourier series expansion

$$a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} a^l_{\alpha\beta}(k_x,k'_x)e^{-il\omega t}. \tag{C33}$$

In addition, using Jacobi-Anger expansion [66, 67]

$$e^{iz\cos(\theta)} = \sum_{l=-\infty}^{\infty} i^l J_l(z)e^{-il\theta}, \tag{C34}$$

where $J_l(\cdot)$ are Bessel functions of the first kind with l-th integer order, and we can re-write the Eq. (C32) as follows

$$a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} -2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N - \varepsilon)$$
$$\times J_l(b[y'_0 - y_0])$$
$$\times \int_{-\infty}^{\infty} dy \; \chi_N(y)\chi_N(y_0 - y'_0 - y)e^{-il\omega t}, \tag{C35}$$

and then the Fourier series component can be identified as

$$a^l_{\alpha\beta}(k_x,k'_x) = -2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x}$$
$$\times \delta(\varepsilon_N - \varepsilon)J_l(b[y'_0 - y_0])$$
$$\times \int_{-\infty}^{\infty} dy \; \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y). \tag{C36}$$

Now define *transition probability matrix* as

$$(A_{\alpha\beta})^{l,l'} \equiv a^l_{\alpha\beta}\left[a^{l'}_{\alpha\beta}\right]^*, \tag{C37}$$

and this becomes

$$(A_{\alpha\beta})^{l,l'}(k_x,k'_x) =[2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2\delta^2(\varepsilon_N-\varepsilon) \times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0]) \times\left|\int_{-\infty}^{\infty}dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0-y'_0-y)\right|^2. \tag{C38}$$

Then describing the square of the delta distribution using following interpretation [25, 29]

$$\delta^2(\varepsilon)=\delta(\varepsilon)\delta(0)=\frac{\delta(\varepsilon)}{2\pi\hbar}\int_{-t/2}^{t/2}e^{i0\times t'/\hbar}dt' \ =\frac{\delta(\varepsilon)t}{2\pi\hbar}, \tag{C39}$$

and executing the time derivation operation on each matrix element we receive the *transition amplitude matrix* elements:

$$\Gamma^{ll'}_{\alpha\beta}(k_x,k'_x) =\frac{2\pi\eta^2_{imp}L^2_x}{\hbar}|V_{k'_x,k_x}|^2\delta(\varepsilon_\beta-\varepsilon) \times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0]) \times\left|\int_{-\infty}^{\infty}dy\ \chi_N(y)\chi_N(y_0-y'_0-y)\right|^2. \tag{C40}$$

An impurity average of white noise potential allows to identify $\langle|V_{k'_x,k_x}|^2\rangle_{imp}=V_{imp}$. Furthermore, the inverse scattering time matrix can be identified as the sum of all available momentum over the impurity averaged transition probability matrix element [30, 58]

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta}=\frac{1}{L_x}\sum_{k'_x}\langle\Gamma^{ll'}_{\alpha\beta}(k'_x,k_x)\rangle_{imp}\,, \tag{C41}$$

and applying the 1-dimensional momentum continuum limit $\sum_{k'_x}\longrightarrow L_x/2\pi\int dk'_x$ and this leads to

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} =\frac{2\pi\eta^2_{imp}L^2_x}{\hbar}\frac{V_{imp}}{2\pi}\delta(\varepsilon_\beta-\varepsilon) =\int_{-\infty}^{\infty}dk'_xJ_l\left(\frac{b\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k'_x]\right) \times\left|\int_{-\infty}^{\infty}dy\ \chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x-k_x]-y\right)\right|^2. \tag{C42}$$

Using substitution $k'_x=k_1$ and $y=\hbar k_2/eB$ finally we can derive our expression for the inverse scattering time matrix for N-th Landau level as follows

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{N} =\frac{\eta^2_{imp}L^2_x\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N) \times\int_{-\infty}^{\infty}dk_1J_l\left(\frac{b\hbar}{eB}[k_x-k_1]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times\left|\int_{-\infty}^{\infty}dk_2\ \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2. \tag{C43}$$

Appendix D: Current operators in dressed Landau levels

In this section we are hoping to derive the current density operator for N-th Landau level. We already found the exact solution for our time dependent Hamiltonian Eq. (1) and we identified them as Floquet states in Eq. (14). The Floquet modes derived in Eq. (10) can be represented as states using quantum number for the simplicity of notation as follows

$$|\phi_{n,m}\rangle\equiv|n,k_x\rangle\,. \tag{D1}$$

Using above complete set of eigenstates of Floquet Hamiltonian Eq. (C3) [30, 32, 52] we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'}\equiv\langle n,k_x|\,\hat{\mathbf{j}}\,|n',k'_x\rangle\,, \tag{D2}$$

and the particle current operator for our system [47, 48] can be identified as

$$\hat{\mathbf{j}}=\frac{1}{\tilde{m}}(\hat{\mathbf{p}}-e[\mathbf{A}_s+\mathbf{A}_d(t)]), \tag{D3}$$

where $\tilde{m}$ is mass of the considering particle.

First consider the conductivity in x-direction, and we can identify that x-directional current operator as

$$\hat{j}_x=\frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial x}+eBy\right). \tag{D4}$$

Now calculate the matrix elements of x-directional current operator in Floquet mode basis

$$(j_x)_{nm,n'm'}=\langle n,k_x|\,\frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial x}+eBy\right)|n',k'_x\rangle\,. \tag{D5}$$

Then evaluate these using Floquet modes derived in Eq. (7) and obtain

$$(j_x)_{nm,n'm'} =\frac{1}{\tilde{m}}\delta_{k_x,k'_x}\int dy\,\Big[[\hbar k'_x+eBy] \times\chi_n\big(y-y_0-\zeta(t)\big)\chi_{n'}\big(y-y_0-\zeta(t)\big)\Big]. \tag{D6}$$

Then let $(y - y_0 - \zeta(t)) = \bar{y}$ and we can derive

$$(j_x)_{nm,n'm'} = \frac{1}{\tilde{m}}\delta_{k_x,k'_x}\int d\bar{y}\,\Big[[\hbar k'_x + eB\bar{y} - \hbar k'_x + eB\zeta(t)] \times \chi_n(\bar{y})\chi_{n'}(\bar{y})\Big]. \tag{D7}$$

Using following integral identities of Floquet modes which are made up with Gauss-Hermite functions [68, 69]

$$\int dy\ \chi_n(y)\chi_{n'}(y) = \delta_{n',n}, \tag{D8a}$$

$$\int dy\ y\chi_n(y)\chi_{n'}(y) = \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right), \tag{D8b}$$

we simplify the matrix elements of x-directional current operator to

$$(j_x)_{nm,n'm'} = \frac{1}{\tilde{m}}\delta_{k_x,k'_x}eB \times \left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \zeta(t)\delta_{n',n}\right]. \tag{D9}$$

Due to high complexity of extract solution, in this study we only consider the constant contribution. Therefore, we evaluate the $s = 0$ component of the Fourier series with

$$(j^x_{s=0})_{nm,n'm'} = \frac{eB}{\tilde{m}}\delta_{k_x,k'_x} \times \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D10}$$

For electric current operator we can apply the electron's charge and effective mass and this leads to

$$\left(j^x_{s=0}\right)^{electron}_{nm,n'm'} = \frac{e^2B}{m_e}\delta_{k_x,k'_x} \times \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D11}$$

Next we consider the transverse conductivity in y-direction, and we can identify that y-directional current operator as

$$\hat{j}_y = \frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right). \tag{D12}$$

Then the matrix elements of y-directional current operator in Floquet mode basis are derived as

$$(j_y)_{nm,n'm'} = \langle n, k_x|\,\frac{-1}{\tilde{m}}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n', k'_x\rangle\,. \tag{D13}$$

After following the same steps done for x-directional current operator, we can derive the $s = 0$ component of matrix elements for y-directional electric current operator as

$$\left(j^y_{s=0}\right)^{electron}_{nm,n'm'} = \frac{ie\hbar}{m_e}\delta_{k_x,k'_x} \times \left[\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right]. \tag{D14}$$

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