# Magnetic propeties of a two dimentional electron gas strongly coupled to lights

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# 1 Inverse Scattering Time Analysis

We hace derived the inverse scattering time matrix element from previous section as follows

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll'} = \frac{4\pi^2 N_{imp}^2 V_{imp}}{eBL_x^2L_y^4}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dk'_x\int_{-\infty}^{\infty}d\bar{k}\; J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right) \times\chi_N^2\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N^2\left(\frac{\hbar}{eB}\left[\bar{k}-(k_x-k'_x)\right]\right). \tag{1.1}$$

THe disorder in the system is not supposed to change the eigenenergies of the bare system, hence all off0diogonal elements of the self-energy were neglected. Thesefore we can consider only the diagonal elements of the inverse scattering time matrix

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll} = \frac{4\pi^2 N_{imp}^2 V_{imp}}{eBL_x^2L_y^4}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dk'_x\int_{-\infty}^{\infty}d\bar{k}\; J_l^2\left(\frac{g\hbar}{eB}[k_x-k'_x]\right) \times\chi_N^2\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N^2\left(\frac{\hbar}{eB}\left[\bar{k}-(k_x-k'_x)\right]\right) \tag{1.2}$$

and introduce new variable as follows

$$y_1 = \frac{\hbar}{eB}[k'_x-k_x] \longrightarrow dk'_x = \frac{eB}{\hbar}dy_1 \tag{1.3}$$

and

$$y_2 = \frac{\hbar}{eB}\bar{k} \longrightarrow d\bar{k} = \frac{eB}{\hbar}dy_2 \tag{1.4}$$

then above equation modified to

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll} = \frac{4\pi^2 N_{imp}^2 V_{imp}eB}{\hbar^2 L_x^2L_y^4}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\; J_l^2(gy_1)\chi_N^2(y_2)\chi_N^2(y_1+y_2). \tag{1.5}$$

Lets consider how this expression change when we have turn off the dressing field ($E=0$). Thesefore the inverse scattering time becomes valid for only $l=0$

$$\begin{aligned}\left.\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{00}\right|_{E=0} &= \frac{4\pi^2 N_{imp}^2 V_{imp}eB}{\hbar^2 L_x^2L_y^4}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\; J_0^2(0)\chi_N^2(y_2)\chi_N^2(y_1+y_2)\\ &= \frac{4\pi^2 N_{imp}^2 V_{imp}eB}{\hbar^2 L_x^2L_y^4}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\; \chi_N^2(y_2)\chi_N^2(y_1+y_2)\end{aligned} \tag{1.6}$$

Thesefore we can analyze the behaviour of the inverse scattering time with $l=0$ central element of the matrix.

$$\Lambda_{00} \equiv \frac{(1/\tau)_N^{00}}{(1/\tau)_N^{00}\big|_{E=0}} \tag{1.7}$$

and this will be

$$\Lambda_{00}(k_x) = \frac{\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\; J_0^2(gy_1)\chi_N^2(y_2)\chi_N^2(y_1+y_2)}{\int_{-\infty}^{\infty}dy_1\int_{-\infty}^{\infty}dy_2\; \chi_N^2(y_2)\chi_N^2(y_1+y_2)} \tag{1.8}$$

where

$$g = \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2-\omega^2)} \tag{1.9}$$

and

$$\chi_N(y) = \frac{\sqrt{\mu}}{\sqrt{2^N N!\sqrt{\pi}}}\exp\left(-\frac{\mu^2y^2}{2}\right)\mathcal{H}_N(\mu y) \quad \text{with} \quad \mu\equiv\sqrt{\frac{m_e\omega_0}{\hbar}} \tag{1.10}$$

and this implies

$$\chi_N^2(y) = \frac{\mu}{2^N N!\sqrt{\pi}}\exp\left(-\mu^2y^2\right)\mathcal{H}_N^2(\mu y). \tag{1.11}$$

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