November 19, 2021

Ashot Melikyan,
Associate Editor,
Physical Review B.

Dear Professor Melikyan,

Thank you very much for your effort in managing the review process of our manuscript. We are also thankful for the reviewer comments, and believe the second version of the manuscript we submit herewith has been significantly improved by the constructive criticism we received. Additionally, the subsequent sections of this document discuss ~~the various comments and our response to them.~~

We have made some minor changes in language and presentation to improve the clarity and organization of the manuscript.

Please note that in ~~the following~~ ~~the reviewers' comments are shown in blue and our~~ responses are shown ~~in~~ red.

General changes to the manuscript

~~We have made minor changes in language and presentation to improve clarity, and to match the rest of the manuscript better to the changes done to address the reviewers' comments.~~

use

- Section I - seventh paragraph (page 2):
  In Sec. VII, we discuss the physical significance of our theoretical results and their possible employments in future nanoelectronic devices. Finally, we summarize our findings and present our conclusions in Sec. VIII.

- Section VI - fourth paragraph (page 8):
  Considering the effects of the applied dressing field on the longitudinal conductivity of 2DEG, we can identify that the dressing field has sharpened the conductivity peaks.

- Section VIII - first paragraph (page 10):
  Finally, we derived analytical expressions for the diagonal components of the electric conductivity tensor concerning the 2DEG quantum Hall system operates at low temperatures.

Response to the comments of Reviewer 1

We would like to thank the reviewer for bringing the deficiencies of our manuscript to our attention and providing constructive feedback to improve the quality of our work. We have considered all of your suggestions seriously and revised our manuscript as described below.

Comment 1 - My concern is that the manuscript is heavily skewed towards a purely mathematical formulation of the problem. It has a minimal connection to realistic two-dimensional electron systems. The manuscript does not discuss how the results can be applied to understanding mechanisms of charge transport in nanoelectronic devices and can be used to optimize device performance. Without such discussion, the manuscript will have a minimal impact on the community working on developing nanoelectronics.

We agree that our work has a theoretical bias, and the reader may benefit from some application perspective from our theoretical results to characterize or design nanoelectronic devices. Therefore, we have discussed the physical significance of our theoretical results and their possible use in optimizing nanoelectronic device performance. First, we added a detailed comparison between our theoretical results and experiment observations when no radiation is present. We also added a new section (Section VII) on this very aspect.

- Section VI - third paragraph (page 8):
  By comparing the theoretical [1–7] and experimental [7–13] studies on the magnetoresistance of 2DEG quantum Hall systems when no radiation is present against our results, we can identify that longitudinal conductivity oscillations in Fig. 4 are a repetition of the Shubnikov–de Haas(SdH) oscillations. As

observed in the experimental work of Caviglia *et al.* [13], the period of SdH oscillations depends only on the perpendicular component of the magnetic field to the plane of the 2DEG. Therefore, we can identify that many experiments on different types of 2DEGs at low-temperatures [7, 9–13] have analyzed these SdH oscillations against the applied magnetic field's amplitude. The cyclotron frequency (ω_0) of the system depends on the magnetic field's perpendicular component to the plane of the 2DEG. The applied gate voltage to the system modifies the Fermi level of the system. Since the Landau level energy is only dependent on the cyclotron frequency, this gate voltage variation under a constant magnetic field amplitude generates the same SdH oscillations. This oscillatory behavior against applied gate voltage has been observed in an experiment done by Wakabayashi *et al.* [8] in an inversion layer on a silicon surface at low temperature. By comparing these observations against our results in Fig. 4, we can identify that our oscillations also show the same characteristic behavior; i.e $\hbar\omega_0$ periodic SdH oscillations against the Fermi energy. Furthermore, when we increase the gate voltage, the Fermi level rises. As illustrated in Fig. 4, this will result in a higher conductivity peak value at the higher-order Landau levels. This same behavior was also observed in the experimental observations presented in Ref. [8]. In our work, we provide analytical results describing the controllability of these conductivity regions using a dressing field.

- Section VII (page 9):

VII. PHYSICAL SIGNIFICANCE OF THE OUTCOMES

With the realization of 2DEGs in Si-MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measurements on such systems to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new realm in condensed matter physics that continues to produce phenomenal advancements in electronic systems. The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus quantized to integer values of the conductivity quantum ($e^2/\hbar$) in the transverse or Hall conductivity, with simultaneous peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening at low temperatures is the disorders in materials [16, 17]. The broaden sequence of delta functions of the density of states implies the oscillating behavior in the experimental measurements of longitudinal conductivity which is known as SdH oscillations [7, 8].

Our theoretical analysis on longitudinal conductivity behavior of dressed quantum Hall system was developed by considering low-temperature limit with gaussian impurity broadening assumptions. As illustrated in Fig. 4, we can describe the experimentally observed SdH oscillation results in Refs.[7, 8] through our model. Under the undressed condition, our results overlap with the conductivity measurement of quantum Hall systems [7]. Strikingly, we show in Fig. 5 that we can manipulate the broadening of these conductivity peaks using an external dressing field. At low temperatures, the principal cause of broadening of these conductivity peaks is impurity-induced scattering. Using an external dressing field, we can suppress the impurity-induced scattering and this results a reduction of both the scattering-induced broadening and the longitudinal conductivity peaks.

Research on novel states of matter has driven the evolution of present-day nanoelectronic devices. In particular, controllable manipulation of material properties through a gate electric field has revolutionized the development of material science and technology [18, 19]. The charge carrier concentration of a system is an imperative parameter that defines its conductivity properties. As we can manipulate the charge carrier concentration using an electrostatic field-effect, we exert significant control over the conductivity externally. A 2DEG under static magnetic field with quantum Hall effects is an excellent example that illustrates how the gate electric field controls conductivity. We also observed that a considerable number of studies exist on charge transport in the quantum limit that employed different types of 2D field-effect transistors (FETs) subjected to magnetic fields [8, 20–22]. Yang *et al.* [20] study shows quantized Hall plateaus and SdH oscillations for longitudinal conductivity against gate voltage

in black phosphorus FET under static magnetic fields at low temperatures. Since the Fermi level of a system can be altered with the applied gate voltage, our Fig. 4 depicts this result. In particular, our analysis shows that we can manipulate the broadening of the conductivity regions using an external dressing field. Although Yang *et al.* [20], achieved broadening in longitudinal conductivity peaks by changing the temperature in a low range, in this study we presented a general theory on manipulating longitudinal conductivity broadening using a high-intensity electromagnetic field.

The realization of the underlying mechanism of 2D FETs in the quantum realm promises its potential in next-generation nanoelectronic applications. In a particular application that uses the switching mechanism of the above-discussed FETs with quantum Hall effects, we can achieve high and low output conductivities by changing the input gate voltage. As a result of manipulating the broadening of conductivity regions, we can limit the broadening of conductivity peaks around Landau levels using a dressing high-intensity electromagnetic field. This will enhance the sensitivity of FETs which provides the ability to observe narrow changes in gate voltage. Furthermore, adopting the mechanism presented in Ref. [23], we can manipulate conductivity peaks into very sensitive, narrowband high-frequency radiation detectors. We envision these advances in nanoelectronics from our theoretical model and knowledge generated from the insight it provides to the underlying dependencies. Furthermore, this theoretical model will aid in the development of simulation tools that will design the quantum effects in magneto-transport properties of 2D nanostructures.

Comment 2 - Moreover, this current research direction has a significant overlap with previous experimental and theoretical studies of quantum Hall systems that started with the observation of zero-resistance states in high mobility systems [Zudov et al, Phys. Rev. B 64, 201311 (2001), Mani et al, Phys. Rev. Lett. 92, 146801 (2004)] and gave rise to theoretical models for the phenomenon [Durst et al, Phys. Rev. Lett. 91, 086803 (2003), Dmitriev et al., Phys. Rev. B 71, 115316 (2005), Dmitriev et al, Phys. Rev. B 80, 165327 (2009)]. The present manuscript needs to connect to various known phenomena discussed earlier in the literature on quantum Hall systems.

We agree with the reviewer that it is useful to have a discussion comparing our theoretical model and previous work on the transport properties of quantum Hall systems. Among the mentioned studies we can identify that experimental work reported in Refs. [24–27] are specifically aimed at the unusual oscillations of the magnetoresistance induced by the microwave (millimeter-wave) radiation in 2DEG quantum Hall systems. These oscillations are known as *microwave-induced resistance oscillations* (MIROs). To describe these behaviors Durst *et al.* [28] introduced a simple theoretical model assuming that the experimentally observed oscillations are a consequence of photoexcited disorder-scattered electrons. However, later Refs. [29–31] proposed a novel model considering the changes were made on the electron distribution function by the microwave field. These more generalized models [29–31] have successfully described the behavior of MIROs at experimentally relevant temperatures that were missed in the previous model [28]. Furthermore, we can recognize that the underlying mechanism of all these models [28–31] is microwave photon absorption by an electron in the associated system. In our theoretical model, we take into account higher frequencies than microwaves as the dressing electromagnetic field. Therefore we can identify and quantify significant dissimilarities between our system and MIRO systems [24–31]. They are:

- the experiments on magnetoresistance oscillations on 2DEG quantum Hall systems [24–27] were performed in the microwave frequency range $(30 - 150\,\mathrm{GHz})$. This leads to building the theoretical models presented in Refs. [28, 29] by assuming that these oscillations are caused by photoexcited electrons. Since these models consider on a relevant frequency range (microwave radiation), it will allow us to acknowledge the photon absorption by electrons. In contrast to that, our consideration is only focused on the systems with high-frequency range dressing fields which will not be associated with any photon absorption.

- the applied microwave radiation power on the MIRO experiments [25, 26] varies around the $1\,\mathrm{mW\,cm^{-2}}$ range. However, in our analysis, we take the dressing field as a high-intensity electromagnetic field, where we cannot consider the dressing field as a perturbation in our analysis. This leads us to recognize the formation of Floquet states together with conventional Landau levels. As we mentioned in our results, we used dressing high-intensity electromagnetic fields of magnitude around the $100\,\mathrm{W\,cm^{-2}}$

range in our numerical calculations.

- these fascinating MIROs are only observed under the influence of weak magnetic fields ($B < 0.2\,\text{T}$) in experiments performed in Refs. [24–27]. In this range of weak magnetic fields, we can only observe MIROs as Shubnikov–de Haas (SdH) oscillations reveal only on high-intensity magnetic fields. In comparison to our analysis, we are interested in the SdH oscillations and manipulation of their characteristics. Therefore in our analysis, we aim at the effects induced by higher intensity magnetic fields ($B \sim 1\,\text{T}$) than the MIRO experiments.

- in our work, we analyzed the 2DEG quantum Hall system with a dressing high-intensity electromagnetic field which does not contribute to energy exchange between the high-frequency dressing field and electrons. Therefore we have assumed the applied electromagnetic radiation as a pure dressing field. There are two possible absorption mechanisms in a 2DEG quantum Hall system; namely electron transitions between distinct Landau levels and electron transitions between distinct states of the same broadened Landau level. To avoid these absorptions, we have tuned the dressing field into a high-frequency range in our system under the analysis. Furthermore, due to the dressing high-intensity electromagnetic field, the entire electronic states of the conventional 2DEG quantum Hall system will be restructured. We have addressed these modifications through the Floquet theory. However, the MIRO models [28, 29] are based on low-frequency cases where the system can absorb low-frequency photons from the field.

Based on the above discussion, we can identify that the high-frequency and low-frequency illumination of 2DEG quantum Hall systems leads to two distinguishable magneto-transport scenarios.

- Section I - fifth paragraph (page 1):
Recently, experimental research on the effects of microwave illumination of 2DEG systems revealed microwave-induced resistance oscillations (MIROs) under weak magnetic fields [24–27]. This inspired investigations on the theoretical description of MIROs, and several semiclassical and quantum kinetic equation formalisms have been proposed to address the underlying mechanism of MIROs [28–31]. These formalisms provide a proper explanation for the experimental observations of MIROs. However, these experimental and theoretical works have been linked to photon absorption from low-frequency (microwave) electromagnetic fields. In contrast to that, high-frequency external illumination on a 2DEG quantum Hall system needs to be studied as a pure dressing (nonabsorbable) field. The influence induced by a pure dressing field on magneto-transport properties of 2DEG quantum Hall system needs to be described by a non-absorption mechanism, and it has escaped the researchers' attention before. Lately, Dini *et al.* [32] have investigated the one-directional conductivity behavior of dressed quantum Hall system subjected to a high-frequency field.

- Section VI - sixth paragraph (page 9):
Lately, several experimental [24–27] and theoretical [28–31] studies have uncovered various remarkable magneto-transport properties induced by microwave radiation on 2DEG quantum Hall systems. However, these experiments and theoretical models only analyzed the behavior of MIROs in 2DEG systems which are based on photon absorption from the applied low-frequency field. It is important to state that the difference between our SdH oscillations and MIROs [24–27] by considering the frequency range of the applied electromagnetic field. In our case, the applied dressing field is in the off-resonant range, and it will only change the broadening of Landau levels, but avoid any contribution towards the photon absorptions. This will clearly describe the crucial difference between low-frequency illumination and high-frequency illumination effects on 2DEG quantum Hall systems. Therefore, our theoretical analysis can help to fill the gap in knowledge of the high-frequency dressing field effects on 2DEG quantum Hall systems.

Comment 3 - The manuscript will also provide more impact if it demonstrates how the new results can help to improve the future development of nanoelectronic devices. After these questions are addressed, the manuscript will be suitable for publication in Physical Review B. Otherwise, it will fit better to a more mathematically oriented journal.

This is a valid comment. As we mentioned in comment 1, considering the importance of discussing the physical significance of our results, we included a new section (Section VII) to address this comment.

- Section VII (page 8):

VII. PHYSICAL SIGNIFICANCE OF THE OUTCOMES

With the realization of 2DEGs in Si-MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measurements on such systems to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new realm in condensed matter physics that continues to produce phenomenal advancements in electronic systems. The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus quantized to integer values of the conductivity quantum ($e^2/\hbar$) in the off-diagonal conductivity, with simultaneous peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening at low temperatures is the disorders in materials [16, 17]. The broaden sequence of delta functions of the density of states implies the oscillating behavior in the experimental measurements of longitudinal conductivity which is known as SdH oscillations [7, 8].

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theoretical model will aid in the development of simulation tools that will design the quantum effects in magneto-transport properties of 2D nanostructures.

Comment 4 - The quantum Quantum Hall effect requires high mobility samples. In these samples, the structure of the disorder is usually complicated and combines both short-length potentials of impurities and long-length electrostatic inhomogeneities. The interplay of these components of disorder opens exciting questions about the transport properties of 2DEGs. What is the structure of disorder considered in the present manuscript and hidden in the notations for Vimp? What are the conditions for validity of eq. (15)?

We thank the reviewer for raising this important question. In our previous manuscript, we have presented the detailed derivation of Eq. [15] with a discussion on models of disorder under Appendix C. However, here we again elucidate our disorder model and approximations made to derive the Eq. [15] for clarity and to respond to this question.

We modeled the effect caused by impurities in the considered system as a single perturbation potential. Analyzing the electric properties for a specific impurity distribution is a rather formidable task. We do not consider a specific impurity distribution here as it is unlikely to represent a measured impurity configuration in an experiment. Therefore, in this study, we consider the statistically averaged properties of 2DEG over impurity configurations. We start our analysis by considering a static disorder that corresponds only to elastic electron scattering processes. We adapt the Edwards model [33] to represent the randomly distributed impurities over the associated system and we approximate this into a Gaussian white noise.

Since we are presenting the perturbation potential $V(\mathbf{r})$ by a group of randomly localized impurities, we assume N_{imp} number of identical single impurity potentials distributed randomly but in fixed positions $\mathbf{r}_i$. Thus, we can describe the perturbation potential $V(\mathbf{r})$ as the sum of uncorrelated single impurity potentials $v(\mathbf{r})$,

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} v(\mathbf{r} - \mathbf{r}_i). \tag{1}$$

Furthermore, we approximate this perturbation potential as a Gaussian random potential where one can choose the zero of energy such that the potential is zero on average. This model is characterized by the following two equations [33]

$$\langle v(\mathbf{r}) \rangle_{imp} = 0, \tag{2a}$$

$$\langle v(\mathbf{r}) v(\mathbf{r}') \rangle_{imp} = \Upsilon(\mathbf{r} - \mathbf{r}'), \tag{2b}$$

where $\langle \cdot \rangle_{imp}$ represents the average over the impurity disorder and $\Upsilon(\mathbf{r} - \mathbf{r}')$ is any decaying function that solely depends only on $\mathbf{r} - \mathbf{r}'$, and decays with a characteristic length r_c. Since this study considers the case where the wavelength of radiation or a scattering electron is much greater than r_c, it is reasonable to approximate its two-point correlation function by

$$\langle v(\mathbf{r}) v(\mathbf{r}') \rangle_{imp} = \Upsilon_{imp}^2 \delta(\mathbf{r} - \mathbf{r}'), \tag{3}$$

where Υ_{imp}^2 is a positive constant. A random potential $V(\mathbf{r})$ with this property is called white noise [33]. Then we can approximately write the total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} \Upsilon_{imp} \delta(\mathbf{r} - \mathbf{r}_i). \tag{4}$$

Using this expression, we can evaluate the Floquet-Fermi golden rule for a dressed quantum Hall system. Furthermore, considering the properties of a Gaussian white noise impurity distribution [33, 34], we assume V_{imp} is a constant in the momentum space. Here $V_{imp} = \langle |V_{k'_x, k_x}|^2 \rangle_{imp}$ with $V_{k'_x, k_x} = \langle k'_x | v(x) | k_x \rangle$, and $\langle x | k_x \rangle = e^{-ik_x x}$. All the detailed steps for the derivation of the Floquet-Fermi golden rule for a dressed quantum Hall system are included in Appendix C. However, throughout the derivation, we use only the first-order contribution (Born approximation) of the impurity potential.

Since previous studies on Floquet-Drude conductivity [34], and magneto-transport properties in undressed [7] and dressed [32] quantum Hall systems have used this particular Gaussian white noise potential, we also selected this particular impurity model to describe our system. Such a choice enables us to compare our analytical results with these previous models. As you have mentioned in the comment, consideration of other impurity disorder models and their impact on the magneto-transport properties of dressed quantum Hall system would be an intriguing future research possibility.

Since we have only mentioned the validity conditions we used to derive the Eq. [15] in Appendix C and not in the main text of the previous manuscript, we have added these validity conditions to the main text of the revised manuscript.

- Section IV - first paragraph (page 4):
  We have modeled the effect caused by impurities as a single short-range perturbation potential. Analyzing the electric properties for a specific impurity distribution is a rather formidable task. We do not consider a specific impurity distribution here as it is unlikely to represent a measured impurity configuration in an experiment. Therefore, in this study, we consider the statistically averaged properties of 2DEG over impurity configurations. Furthermore, we have assumed that a group of randomly distributed impurities forms our perturbation potential under the Edwards impurity model [33, 34]. Essentially, all these assumptions mean that we can write the total scattering potential in the 2DEG as a sum of uncorrelated single impurity potentials $v(\mathbf{r})$. This enables us to approximate the impurity potential as a Gaussian white noise [33, 34]. Here η_{imp} is the number of impurities in a unit area, $V_{imp} = \langle |V_{k'_x, k_x}|^2 \rangle_{imp}$ with $V_{k'_x, k_x} = \langle k'_x|v(x)|k_x\rangle$, and $\langle x|k_x\rangle = e^{-ik_x x}$. Moreover, in this analysis, $\langle \cdot \rangle_{imp}$ represents the average over the impurity disorder. In this derivation, we only considered the first order (the Born approximation) contribution from the impurity potential.

Comment 5 - If continuous illumination is applied to an electron system, the system will heat indefinitely. The distribution function is stabilized when the electron-electron and electron-phonon scattering mechanisms are included. What are the conditions of validity for eqs. (33-34)? The deviations of the distribution function from its equilibrium result in effects that can significantly overcome the equilibrium contributions, see Dmitriev et al, Phys. Rev. B 80, 165327 (2009). However, to capture these effects, a treatment within Floquet-Drude formalism is insufficient and a complete quantum kinetic equation has to be analyzed. Can the authors argue why their results still present some interest to the community even if they potentially disregard more significant contributions?

This is a good, valid question. First, we discuss the validity of the Eq. [33] and Eq. [34]. Our analysis derived the longitudinal conductivity of a dressed quantum Hall system without any partial distribution function specification until Eq. [31]. In contrast to the MIRO analysis [29–31], we consider the dressed quantum Hall system under the off-resonant condition, where the photon absorption does not happen. Therefore, we can select the Fermi-Dirac distribution as our particle distribution function for Eq. [31]

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \varepsilon_F)/k_B T] + 1}. \tag{5}$$

Here, k_B is the Boltzmann constant, T is the absolute temperature, and ε_F is the Fermi energy of the system. Then we consider a special scenario where the system operates at a low-temperature limit, i.e., the condition $k_B T \ll \varepsilon_F$. Since previous theoretical studies [7, 32, 34] on magneto-transport properties of 2DEG quantum Hall systems were derived under this assumption, we also adopted the same conditions as it enables us to compare results with those studies. In addition, as mentioned in previous experimental studies on SdH oscillations [11, 25, 26] in 2DEG undressed quantum Hall systems with increasing temperature, we can identify the decaying behavior of SdH oscillations. Therefore, if we need to maintain a good oscillatory behavior, we need to operate in low temperatures. Since in our analysis, we study the manipulation of SdH oscillations using a dressing field, it is reasonable to simplify our general derivation given in Eq. [31] for low-temperature limit. Under these assumptions, the derivative of the Fermi-Dirac distribution is sharply peaked around the Fermi energy, and can be represented by the delta function [7]

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F). \tag{6}$$

Applying these adequate conditions, we can simplify our derivation into a single expression that can be compared with previous studies [7, 32, 34] on magneto-transport in dressed quantum Hall systems. We have added clarification on the above assumptions in the revised manuscript.

V - second paragraph (page 7):

Consider a dressed quantum Hall system under off-resonant condition, where photon absorption cannot happen. Therefore, we can select the Fermi-Dirac distribution as our partial distribution (f) for our system

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \varepsilon_F)/k_B T] + 1}, \tag{33}$$

where k_B is the Boltzmann constant, T is the absolute temperature, and ε_F is the Fermi energy of the system. At low-temperatures, i.e., $k_B T \ll \varepsilon_F$, the derivative of this distribution is sharply peaked around the Fermi energy, and can be approximated by a delta function [7]

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F). $$

Yes, indeed, experiments on quantum hall systems subject to a strong dressing field show elevated temperatures. However, as demonstrated in previous studies by Dini *et al.* [32] and Kibis *et al.* [35], it is possible to maintain low system temperature by limiting the dressing field to narrow pulses.

Although we have extended our general derivation given in Eq. [31] into special consideration for comparison, we can improve our theoretical model into different types of particle distribution functions. As presented in the previous work of Dmitriev *et al.* [30], a well-defined analytical discussion needs to address the changes in distribution functions in our derived general longitudinal conductivity formula, and this would be a noteworthy extension to our study.

Comment 6 - The authors presented the results for the conductivity tensor's xx- and yy-components. They seem to be identical, apart of a dimensional factor $(eB)^2$ in the denominator. The equality of these two components is expected for isotropic systems. Does the polarization of the electromagnetic field break the isotropy? Do the two components of the conductivity remain equal even for an arbitrary direction of a linearly polarized field? Is there a non-Hall contribution to the xy-component?

We express our gratitude towards the reviewer for pointing out this inadvertent oversight done in the yy-component of the current operator calculation. This has led to an unexpected normalization factor in the yy-component of the conductivity expression. Since we have only used the normalized xx-component of the conductivity tensor for subsequent analysis, this oversight has no impact on the analysis reported in the manuscript. With this correction, we observe that both xx-component and yy-component show the same conductivity behaviors. We added these amendments to the current operator calculations done in Appendix D and the longitudinal conductivity calculation under Section V.

- Appendix D (page 16):

Appendix D: Current operators for a dressed quantum Hall system

In this section, we derive the current density operator for the N-th Landau level in a dressed quantum Hall system. We already found the exact solution for the time-dependent Schrödinger equation with the Hamiltonian give in Eq. (1) and we identified them as the Floquet states in Eq. (14). For the simplicity of notation, we can represent the Floquet modes derived in Eq. (10) as quantum states using their corresponding quantum numbers as follows

$$|\phi_{n,m}\rangle = |n, k_x\rangle. \tag{D1}$$

Using this complete set of quantum states [34, 36, 37], we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'} = \langle n, k_x | \, \hat{\mathbf{j}} \, |n', k'_x\rangle. \tag{D2}$$

Next, we can identify the particle current operator for our system [38, 39] as

$$\hat{\mathbf{j}} = \frac{1}{\widetilde{m}} \left\{ \hat{\mathbf{p}} - e \left[\mathbf{A}_s + \mathbf{A}_d(t) \right] \right\}, \tag{D3}$$

where $\widetilde{m}$ is the mass of the associated particle.

First, we consider the x-directional particle current operator component, and we can identify that as

$$\hat{j}_x = \frac{1}{\widetilde{m}} \left(-i\hbar \frac{\partial}{\partial x} + eBy \right). \tag{D4}$$

Next, we calculate the matrix elements of x-directional current operator against our Floquet mode basis

$$(j_x)_{nm,n'm'} = \langle n, k_x | \frac{1}{\widetilde{m}} \left(-i\hbar \frac{\partial}{\partial x} + eBy \right) | n', k_x' \rangle, \tag{D5}$$

and we evaluate these using the Floquet modes derived in Eq. (7) as follows

$$(j_x)_{nm,n'm'} = \frac{1}{\widetilde{m}} \delta_{k_x,k_x'} \int (\hbar k_x' + eBy) \\ \times \chi_n \left(y - y_0 - \zeta(t) \right) \chi_{n'} \left(y - y_0 - \zeta(t) \right) dy. \tag{D6}$$

Let $[y - y_0 - \zeta(t)] = \bar{y}$, and we can obtain

$$(j_x)_{nm,n'm'} = \frac{1}{\widetilde{m}} \delta_{k_x,k_x'} \int \left[\hbar k_x' + eB\bar{y} - \hbar k_x' + eB\zeta(t) \right] \chi_n(\bar{y}) \chi_{n'}(\bar{y}) d\bar{y}. \tag{D7}$$

Using the following integral identities of the Floquet modes that are made of Gauss-Hermite functions [40, 41]

$$\int \chi_n(y) \chi_{n'}(y) dy = \delta_{n',n}, \tag{D8}$$

$$\int y \chi_n(y) \chi_{n'}(y) dy = \frac{1}{\kappa} \left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right), \tag{D9}$$

we simplify the matrix elements of x-directional current operator to obtain

$$(j_x)_{nm,n'm'} = \frac{eB}{\widetilde{m}\kappa} \delta_{k_x,k_x'} \left[\left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right) + \zeta(t) \delta_{n',n} \right]. \tag{D10}$$

Due to high complexity of extract solution, in this study we only consider the constant contribution from the Fourier series components of the above expression. Therefore, we can identify the 0-th component of the Fourier series as

$$(j_x)_{nm,n'm'} = \frac{eB}{\widetilde{m}\kappa} \delta_{k_x,k_x'} \left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right). \tag{D11}$$

To calculate the electric current operator, we substitute the electron's charge and effective mass to the above derived equation. This leads to

$$\left(j_{s=0}^x \right)_{nm,n'm'}^{electron} = \frac{e\hbar}{m_e l_0} \delta_{k_x,k_x'} \left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right). \tag{D12}$$

where $l_0 = \sqrt{\hbar/eB}$ is the magnetic length.

Moreover, we can identify the y-directional current operator component as

$$\hat{j}_y = \frac{1}{\widetilde{m}} \left(-i\hbar \frac{\partial}{\partial y} - \frac{eE}{\omega} \cos(\omega t) \right). \tag{D13}$$

9

Using this operator, we can represent the matrix elements of y-directional current operator in Floquet mode basis as

$$(j_y)_{nm,n'm'} = \langle n,k_x| \frac{-1}{\widetilde{m}}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n',k_x'\rangle. \tag{D14}$$

After following the same steps done for the x-directional current operator, and recursion relation of the first derivative of Gauss-Hermite functions

$$\frac{\partial \chi_n(y)}{\partial y} = \kappa\left[-\sqrt{\frac{n+1}{2}}\chi_{n+1}(y) + \sqrt{\frac{n}{2}}\chi_{n-1}(y)\right], \tag{D15}$$

we can identify the 0-th component of matrix elements for y-directional electric current operator as

$$\left(j_{s=0}^y\right)_{nm,n'm'}^{electron} = \frac{ie\hbar\kappa}{m_e}\delta_{k_x,k_x'}\left(\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right). \tag{D16}$$

$$\left(j_{s=0}^y\right)_{nm,n'm'}^{electron} = \frac{ie\hbar}{m_e l_0}\delta_{k_x,k_x'}\left(\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right). \tag{D17}$$

- Section V - second paragraph (page 7):
  Moreover, let $\Pi = \varepsilon_F$ and the derived expression in Eq. (31) leads to

$$\sigma^{xx} = \frac{e^2 l_0^2}{\pi\hbar A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right], \tag{35}$$

where $X_F = [\varepsilon_F/(\hbar\omega_0) - 1/2]$, $\gamma_n = \widetilde{\Gamma}(\varepsilon_n)\big/(\hbar\omega_0)$, and $l_0 = \sqrt{\hbar/eB}$. Following the same steps as above derivation, we can derive the longitudinal conductivity in the y-direction by applying the electric current operator for y-direction derived in Appendix D

$$\sigma^{yy} = \frac{e^2 l_0^2}{\pi\hbar A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right]. \tag{36}$$

apply

of interest

Suppose we applied an arbitrary directional polarized dressing field. In that case, we could derive an expression for the longitudinal conductivity components by considering dressing field contribution for each direction (x and y) with the same steps presented in our analysis. This would change the amplitude of the associated dressing field (E) in each direction. There will be a new component in parallel to the ~~considering~~ conductivity component, which will modify the time-dependent Hamiltonian given in Eq. [2]. Then, we need to solve the Schrödinger equation with the modified Hamiltonian to find the new Floquet modes for the new system. These will define the effect on the inverse scattering time matrix components. As mentioned in Ref. [34] and our work, applied strong dressing field tends to change the quantum state of the considered system and creates novel states called Floquet states. The properties of these states depend on the characteristics of the applied dressing field. Therefore, the polarization method also changes the behavior of the Floquet states. These polarization-dependent conductivity behaviors in 2DEG systems can be found in references [34, 42] and in their work authors have illustrated the conductivity behavior with circular and linear polarized dressing fields. However, in a dressed quantum Hall system with a y-directional linear polarized field, we predict the same longitudinal conductivity behaviors in diagonal components of the conductivity tensor.

For analyzing our system, we mainly employ the diagonal components of the Floquet-Drude conductivity tensor introduced by Wackerl *et al.* [34]. As we mainly focus on SdH oscillations in longitudinal conductivity components, we limit our analysis to the dressing field influence on two diagonal components of the conductivity tensor. Therefore we are unable to predict the off-diagonal component behavior of the system.

Response to the comments of Reviewer 2

We want to thank the reviewer for his/her insightful comments on our work. To address your comments, we had to add new materials and reorganize our text. As a result, we have been able to considerably improve our discussion on the underlying assumptions and corresponding physical significance of our analysis. We hope our response and the corresponding changes we incorporated into the manuscript will be sufficient to clarify the issues raised.

Comment 1 - The paper is very low on comparison with experiments, for which there is a large amount of data available for the zero radiation case. I would like to see detailed analysis of how Figs 4 and 5 compare with available experimental data. Without this addition, physical relevance of such detailed calculations is questionable.

We thank the reviewer for pointing out the importance of this fact. In the revised manuscript, we have included a detailed analysis of our results against the undressed 2DEG quantum Hall system behavior.

- Section VI - third paragraph (page 8):
  By comparing the theoretical [1–7] and experimental [7–13] studies on the magnetoresistance of 2DEG quantum Hall systems when no radiation is present against our results, we can identify that longitudinal conductivity oscillations in Fig. 4 are a repetition of the Shubnikov–de Haas(SdH) oscillations. As observed in the experimental work of Caviglia *et al.* [13], the period of SdH oscillations depends only on the perpendicular component of the magnetic field to the plane of the 2DEG. Therefore, we can identify that many experiments on different types of 2DEGs at low-temperatures [7, 9–13] have analyzed these SdH oscillations against the applied magnetic field's amplitude. The cyclotron frequency (ω_0) of the system depends on the magnetic field's perpendicular component to the plane of the 2DEG. The applied gate voltage to the system modifies the Fermi level of the system. Since the Landau level energy is only dependent on the cyclotron frequency, this gate voltage variation under a constant magnetic field amplitude generates the same SdH oscillations. This oscillatory behavior against applied gate voltage has been observed in an experiment done by Wakabayashi *et al.* [8] in an inversion layer on a silicon surface at low temperature. By comparing these observations against our results in Fig. 4, we can identify that our oscillations also show the same characteristic behavior; i.e $\hbar\omega_0$ periodic SdH oscillations against the Fermi energy. Furthermore, when we increase the gate voltage, the Fermi level rises. As illustrated in Fig. 4, this will result in a higher conductivity peak value at the higher-order Landau levels. This same behavior was also observed in the experimental observations presented in Ref. [8]. In our work, we provide analytical results describing the controllability of these conductivity regions using a dressing field.

Comment 2 - In the presence of disorder, Anderson localization and topologically protected edge modes are dominant considerations when calculating DC transport coefficients. How are these factors taken into account?

We omitted a discussion on the Anderson localization and topologically protected edge modes as we neglected them in our work. While deriving Eq. [31], we have assumed that the applied static magnetic field is placed in a lower intensity range where we can neglect several effects known to appear in a 2DEG quantum Hall system. Under this condition, we can neglect the localization in the tails of Landau level peaks, the formation of the edge states, the electron-electron interactions, and the spin splitting as well. As mentioned in the work of Endo *et al.* [7], within the magnetic field range of interest, the undressed magneto-transport properties of GaAs/AlGaAs 2DEG quantum Hall system provide an excellent agreement between the experimental observations and theory that was developed by neglecting the above-mentioned effects. Our work is very similar to theirs with an additional dressing field. Our results for the undressed system agree with the results in Ref. [7], while we provide a better description to manipulate the width of the Landau level conductivity peaks. We have added this clarification in the manuscript.

and

- Section I - sixth paragraph (page 2):
  It is important to note that, we limit the applied magnetic field to a range where we can neglect effects such as Anderson localization, formation of the edge states, spin splitting, the electron-electron interactions. This is not an overly restrictive assumption and others have used the same assumption

These are not overly restrictive assumptions, and other have successfully used same assumptions in their work e.g. see xxxx et. al [7]

in their work, e.g. see *et al.* [7].

- Section VIII - first paragraph (page 10):
  We modeled the impurities in the material as a Gaussian random scattering potential, and restricted the applied magnetic field to a range where we can neglect effects of Anderson localization, formation of the edge states, spin splitting, and electron-electron interactions.

Comment 3 - Can the authors reproduce the quantized Hall conductance of filled Landau levels in the presence of disorder? Please discuss.

For analyzing our system, we mainly employ the diagonal components of the Floquet-Drude conductivity tensor introduced by Wackerl *et al.* [34]. As we mainly focus on SdH oscillations in longitudinal conductivity components, we limit our analysis to the dressing field influence on two diagonal components of the conductivity tensor. Therefore we cannot predict the off-diagonal component behavior of the system. However, we can use the already established formalism introduced by Endo *et al.* [7] to predict the behavior of off-diagonal components of the conductivity tensor. As mentioned in Ref. [7], this theory on the relation between longitudinal and transverse conductivities highly agree with experiment observations

$$\frac{\mathrm{d}\widetilde{\sigma}^{xy}(X_F, I)}{\mathrm{d}B} \approx \pi\mu\frac{\hbar\omega_0}{\varepsilon_F}[\widetilde{\sigma}^{xx}(X_F, I)]^2. \tag{1}$$

By examining the connection between longitudinal and transverse conductivities in the 2DEG quantum Hall system, we can expect the same Hall conductivity behavior in a dressed quantum Hall system. However, using the dressing field, we can squeeze the conductivity peaks on Landau levels, i.e., squeeze the region of longitudinal conductivity peaks placed near the Landau levels. This implies sudden Hall conductivity transitions between one plateau to another near a Landau level. As in Refs. [15, 43], we can also predict a staircase behavior of quantum Hall conductivity against the gate voltage. With an external dressing field, we will be able to get rapid transitions than the undressed system. However, to derive an analytical expression for the quantum Hall conductivity through Floquet-Drude conductivity needs to be reformulated from the beginning. It will be the subject of our future study.

Comment 4 - The way some previous works are cited is disappointing. Example from page 8: "Despite this behavior being identified in previous works, their results did not coincide with the more accurate description of conductivity components in undressed quantum Hall systems." This is not informative. Where, specifically, did results from earlier works fall short of reality, which have been better addressed in this work?

We made the following changes in language and presentation to improve the clarity of our manuscript.

- Section VIII (page 8):
  Despite this behavior being identified in the previous work of Dini *et al.* [32], they cannot fully account for the experimental observations of longitudinal conductivity components in undressed quantum Hall systems presented in Ref. [7]. The authors of Ref. [32] have used the conventional expression of longitudinal conductivity from Refs. [2, 6] and this theory yields a semi-elliptical broadening for the Landau levels. These predictions significantly deviate from the experimentally observed Landau level broadening [7]. However, our generalized analysis on the conductivity of dressed quantum Hall systems provides a well-suited description for empirically observed behaviors of undressed quantum Hall systems as well.

- Section III - first paragraph (page 3):
  In their study [34], the authors have presented more accurate results than the former theoretical descriptions [42, 44] for the conductivity of nanoscale systems in the presence of a dressing field.

- Section I - fifth paragraph (page 2):
  The authors of Ref. [32] used the conductivity models from Refs. [2, 6], and as mentioned in Endo *et al.* [7], those models predict a semi-elliptical broadening against Fermi level for each Landau level and provide less agreement with the empirical results.

- Section IV - third paragraph (page 5):
  In the study presented in Ref. [7], the authors have assumed that the effective mass of an electron in GaAs-based quantum well system is $m_e \approx 0.07\widetilde{m}_e$ where $\widetilde{m}_e$ is the mass of the electron [7, 34, 45].

Comment 5 - Adding some physical insight into the remarkable observation of radiation-induced narrowing of lineshapes (Figs 4, 5) will help elevate this work.

We agree with the reviewer that a discussion on our theoretical results and their physical significance in modern nanoelectronic devices is a vital requirement. We have added a new Section VII to overcome the above-mentioned deficiency of our manuscript. The total content of the section is given below,

- Section VII (page 8):

VII. PHYSICAL SIGNIFICANCE OF THE OUTCOMES

With the realization of 2DEGs in Si-MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measurements on such systems to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new realm in condensed matter physics that continues to produce phenomenal advancements in electronic systems. The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus quantized to integer values of the conductivity quantum ($e^2/\hbar$) in the off-diagonal conductivity, with simultaneous peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening at low temperatures is the disorders in materials [16, 17]. The broaden sequence of delta functions of the density of states implies the oscillating behavior in the experimental measurements of longitudinal conductivity which is known as SdH oscillations [7, 8].

Our theoretical analysis on longitudinal conductivity behavior of dressed quantum Hall system was developed by considering low-temperature limit with gaussian impurity broadening assumptions. As illustrated in Fig. 4, we can describe the experimentally observed SdH oscillation results in Refs.[7, 8] through our model. Under the undressed condition, our results overlap with the conductivity measurement of quantum Hall systems [7]. Strikingly, we show in Fig. 5 that we can manipulate the broadening of these conductivity peaks using an external dressing field. At low temperatures, the principal cause of broadening of these conductivity peaks is impurity-induced scattering. Using an external dressing field, we can suppress the impurity-induced scattering and this results a reduction of both the scattering-induced broadening and the longitudinal conductivity peaks.

Research on novel states of matter has driven the evolution of present-day nanoelectronic devices. In particular, controllable manipulation of material properties through a gate electric field has revolutionized the development of material science and technology [18, 19]. The charge carrier concentration of a system is an imperative parameter that defines its conductivity properties. As we can manipulate the charge carrier concentration using an electrostatic field-effect, we exert significant control over the conductivity externally. A 2DEG under static magnetic field with quantum Hall effects is an excellent example that illustrates how the gate electric field controls conductivity. We also observed that a considerable number of studies exist on charge transport in the quantum limit that employed different types of 2D field-effect transistors (FETs) subjected to magnetic fields [8, 20–22]. Yang *et al.* [20] study shows quantized Hall plateaus and SdH oscillations for longitudinal conductivity against gate voltage in black phosphorus FET under static magnetic fields at low temperatures. Since the Fermi level of a system can be altered with the applied gate voltage, our Fig. 4 depicts this result. In particular, our analysis shows that we can manipulate the broadening of the conductivity regions using an external dressing field. Although Yang *et al.* [20], achieved broadening in longitudinal conductivity peaks by changing the temperature in a low range, in this study we presented a general theory on manipulating longitudinal conductivity broadening using a high-intensity electromagnetic field.

The realization of the underlying mechanism of 2D FETs in the quantum realm promises its potential in next-generation nanoelectronic applications. In a particular application that uses the switching mechanism of the above-discussed FETs with quantum Hall effects, we can achieve high and low output conductivities by changing the input gate voltage. As a result of manipulating the broadening of conductivity regions, we can limit the broadening of conductivity peaks around Landau levels using a dressing high-intensity electromagnetic field. This will enhance the sensitivity of FETs which provides the ability to observe narrow changes in gate voltage. Furthermore, adopting the mechanism presented in Ref. [23], we can manipulate conductivity peaks into very sensitive, narrowband high-frequency radiation detectors. We envision these advances in nanoelectronics from our theoretical model and knowledge generated from the insight it provides to the underlying dependencies. Furthermore, this theoretical model will aid in the development of simulation tools that will design the quantum effects in magneto-transport properties of 2D nanostructures.

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Sincerely yours,

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