Magnetic propeties of a two dimentional electron gas strongly coupled to lights

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1 Floquet theory

Since we describe the lifetime of an electron in certain Landau level using conventianal perturbation theeory, now we can apply the Floquet theory to identify the difference of these methods.
First we need to identify the *quasienergies* and periodic *Floquet modes* for derived wavefunctions (**??**) for a 2DEG system with both stationary magnetic field and strong dressing filed.
Let's consider the following paramter which is lineraly increasing in time

$$\Delta_E t \equiv \frac{t}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t') \tag{1.1}$$

where we can calculate this using Eq. (**??**) and (**??**) as follows

$$\begin{aligned}\Delta_E t = \frac{t}{T}\int_0^T dt'\ \frac{1}{2}m_e\frac{(eE\omega)^2}{m_e^2(\omega_0^2-\omega^2)^2}\cos^2(\omega t') - \frac{1}{2}m_e\omega_0^2\frac{(eE)^2}{m_e^2(\omega_0^2-\omega^2)^2}\sin^2(\omega t')\\ + \frac{eE}{m_e(\omega_0^2-\omega^2)}\sin(\omega t')eE\sin(\omega t')\end{aligned} \tag{1.2}$$

$$\begin{aligned}\Delta_E t = \frac{t\omega}{2\pi}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\int_0^T dt'\ \cos^2(\omega t') - \omega_0^2\int_0^T dt'\ \sin^2(\omega t')\\ + 2(\omega_0^2-\omega^2)\int_0^T dt'\ \sin^2(\omega t')\Bigg]\end{aligned} \tag{1.3}$$

$$\Delta_E t = \frac{t\omega}{2\pi}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\left[\omega^2\frac{\pi}{\omega} - \omega_0^2\frac{\pi}{\omega} + 2(\omega_0^2-\omega^2)\frac{\pi}{\omega}\right] \tag{1.4}$$

$$\Delta_E t = \frac{t\omega}{2}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}(\omega_0^2-\omega^2) = \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)}t \tag{1.5}$$

Since this is the continuous increasing part of the Laggrangian integral in Eq. (**??**) we can make this as 2ω periodic function as follows

$$\Lambda \equiv \int_0^t dt'\ L(\zeta,\dot{\zeta},t') - \frac{t}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t') \tag{1.6}$$

which can be proved as follows. First consider the first term of the Λ

$$\begin{aligned}\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\int_0^t dt'\ \cos^2(\omega t') - \omega_0^2\int_0^t dt'\ \sin^2(\omega t')\\ + 2(\omega_0^2-\omega^2)\int_0^t dt'\ \sin^2(\omega t')\Bigg]\end{aligned} \tag{1.7}$$

$$\begin{aligned}\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\left[\frac{t}{2}+\frac{\sin(2\omega t)}{4\omega}\right] - \omega_0^2\left[\frac{t}{2}-\frac{\sin(2\omega t)}{4\omega}\right]\\ + 2(\omega_0^2-\omega^2)\left[\frac{t}{2}-\frac{\sin(2\omega t)}{4\omega}\right]\Bigg]\end{aligned} \tag{1.8}$$

$$\int_0^t dt'\, L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\frac{t}{2}\left[\omega^2-\omega_0^2+2\omega_0^2-2\omega^2\right] + \frac{\sin(2\omega t)}{4\omega}\left[\omega^2+\omega_0^2-2\omega_0^2+2\omega^2\right]\Bigg] \tag{1.9}$$

$$\int_0^t dt'\, L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)}t + \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) \tag{1.10}$$

then using Eq.(1.5) we can write this as

$$\int_0^t dt'\, L(\zeta,\dot{\zeta},t') = \Delta_E t + \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t). \tag{1.11}$$

Now we can express

$$\Lambda = \Delta_E t + \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) - \Delta_E t = \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) \tag{1.12}$$

which is a periodic function in time with 2ω frequency.
Now using this parmaters we can factorize the wavefunction (**??**) as linearly time dependend part and periodic time dependend part as follows

$$\begin{aligned}\psi_n(x,y,t) =& \exp\left(\frac{i}{\hbar}[-E_n t+\Delta_E t]\right)\frac{1}{\sqrt{L_x}}\chi_n\big(y-\zeta(t)\big)\\ &\times\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eEy}{\omega}\cos(\omega t) + m_e\dot{\zeta(t)}\big[y-\zeta(t)\big] + \int_0^t dt' L(\zeta,\dot{\zeta},t') - \Delta_E t\right]\right)\end{aligned} \tag{1.13}$$

where we can identify(let $n\to\alpha$) the *quasienergies* as

$$\varepsilon_\alpha \equiv \hbar\omega_0\left(\alpha+\frac{1}{2}\right) - \Delta_E \quad \text{where} \quad \alpha = 0,1,2,... \tag{1.14}$$

and *Floquet modes* as

$$\Phi_\alpha(x,y,t) \equiv \frac{1}{\sqrt{L_x}}\chi_\alpha\big(y-\zeta(t)\big)\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eEy}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y-\zeta(t)\big] + \Lambda\right]\right) \tag{1.15}$$

with

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2-\omega^2)}\sin(\omega t) \quad \text{and} \quad \dot{\zeta}(t) = \frac{eE\omega}{m_e(\omega_0^2-\omega^2)}\cos(\omega t) \tag{1.16}$$

where *Floquet modes* are time-periodic functions that also create a complete orthonormal set. ■

Therefore the solutions (Floquet states) for the periodic Hamiltonian (**??**) can be writting in position space as

$$\psi_\alpha(x,y,t) = \exp\left(-\frac{i}{\hbar}\varepsilon_\alpha t\right)\Phi_\alpha(x,y,t) \tag{1.17}$$

where

$$\varepsilon_\alpha \equiv \left(\frac{eB\hbar}{m_e}\right)\left(\alpha+\frac{1}{2}\right) - \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)} \quad \text{where} \quad \alpha = 0,1,2,... \tag{1.18}$$

and

$$\begin{aligned}\Phi_\alpha(x,y,t) \equiv& \frac{1}{\sqrt{L_x}}\chi_\alpha\left(y - \frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\\ &\times\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eEy}{\omega}\cos(\omega t) + \frac{eE\omega y}{(\omega_0^2-\omega^2)}\cos(\omega t)\right]\right)\\ &\times\exp\left(\frac{i}{\hbar}\left[-\frac{(eE)^2\omega}{2m_e(\omega^2-\omega_0^2)^2}\sin(2\omega t) + \frac{(eE)^2\big(3\omega_0^2-\omega^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t)\right]\right)\end{aligned} \tag{1.19}$$

Now we can write this by more simplying and considering spacial dependencies and using previous subtituting done in Eq. (**??**) we can derive that

$$\Phi_\alpha(x,y,t) \equiv \frac{1}{\sqrt{L_x}}\chi_\alpha\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{ip_x}{\hbar}x\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right](y-y_0)\right) \times \exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right) \tag{1.20}$$

Now we can transform this solution in spacial variable into the momentum space using Fourier trasform over the considering space.

$$\begin{aligned}\Phi_\alpha(k_x,k_y,t) = {} & \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dy\ \exp(-ik_y y)\left[\frac{1}{\sqrt{L_x}}\chi_\alpha\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right)\right] \\ & \times \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\ \exp(-ik_x x)\left[\exp\left(\frac{ip_x}{\hbar}x\right)\right] \\ & \times \exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right) \times \exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\end{aligned} \tag{1.21}$$

Then this can be re-write as follows

$$\Phi_\alpha(k_x,k_y,t) = \exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\delta\left(k_x-\frac{p_x}{\hbar}\right)\Theta(k_y,t) \tag{1.22}$$

where we used

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\ \exp\left(-ik_x x+\frac{ip_x}{\hbar}x\right) = \sqrt{2\pi}\delta\left(k_x-\frac{p_x}{\hbar}\right) \tag{1.23}$$

and

$$\Theta(k_y,t) \equiv \int_{-\infty}^{\infty} dy\ \exp(-ik_y y)\left[\frac{1}{\sqrt{L_x}}\chi_\alpha\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right)\right] \tag{1.24}$$

and this can be simplied as

$$\Theta(k_y,t) = \frac{1}{\sqrt{L_x}}\int_{-\infty}^{\infty} dy\ \chi_\alpha\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(-ik_y y+\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right). \tag{1.25}$$

Then by defining

$$\mu(t) \equiv \frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)} + y_0 \tag{1.26}$$

and

$$\gamma(t) \equiv \frac{eE\omega_0^2\cos(\omega t)}{\hbar\omega(\omega_0^2-\omega^2)} \tag{1.27}$$

we can re-write this by neglecting time dependencies as

$$\Theta(k_y,t) = \frac{1}{\sqrt{L_x}}\int_{-\infty}^{\infty} dy\ \chi_\alpha(y-\mu)\exp(-i(k_y-\gamma)y). \tag{1.28}$$

We can subtitute following variables

$$k_y{}' = k_y-\gamma \quad \text{and} \quad y' = y-\mu \tag{1.29}$$

and this leads to

$$\Theta(k_y{}',t) = \frac{e^{-ik_y{}'\mu}}{\sqrt{L_x}}\int_{-\infty}^{\infty} dy'\ \chi_\alpha(y')\exp\left(-ik_y{}'y'\right). \tag{1.30}$$

We know that $\{\chi_\alpha\}$ are well-known harmonic eigenfunctions as given in the Eq. (**??**). Due to the symmetric condition the Fourier transform of these functions are same with the position eigen functions by changing

$$y \to k_y \quad \text{and} \quad \frac{m\omega_0}{\hbar} \to \frac{\hbar}{m\omega_0} \tag{1.31}$$

Therefore

$$\Theta({k_y}', t) = \frac{e^{-i{k_y}'\mu}}{\sqrt{L_x}} \tilde{\chi}_\alpha\big({k_y}'\big) \tag{1.32}$$

where

$$\tilde{\chi}_\alpha\big({k_y}'\big) = \frac{1}{\sqrt{2^n \alpha!}} \cdot \left(\frac{\hbar}{\pi m_e \omega_0}\right)^{1/4} \cdot e^{-\frac{\hbar}{2m_e\omega_0}({k_y}')^2} \cdot \mathcal{H}_\alpha\left(\sqrt{\frac{\hbar}{m_e\omega_0}} {k_y}'\right) \tag{1.33}$$

Using Eq. (1.32) and Eq. (1.22) we can derive that

$$\begin{aligned}\Phi_\alpha(k_x, k_y, t) = \exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right] y_0\right) \exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\big(\omega_0^2+\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\delta\Big(k_x - \frac{p_x}{\hbar}\Big) \\ \times \frac{e^{-i(k_y-\gamma)\mu}}{\sqrt{L_x}} \tilde{\chi}_\alpha(k_y-\gamma)\end{aligned} \tag{1.34}$$

and this can be re-write subtituting μ and γ values as follows

$$\begin{aligned}\Phi_\alpha(k_x, k_y, t) = \exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right] y_0\right) \exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\big(\omega_0^2+\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\delta\Big(k_x - \frac{p_x}{\hbar}\Big) \\ \times \exp\left(-ik_y\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp(-ik_y y_0) \\ \times \exp\left(i\frac{1}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right] y_0\right)\frac{1}{\sqrt{L_x}}\tilde{\chi}_\alpha(k_y-\gamma)\end{aligned} \tag{1.35}$$

and

$$\begin{aligned}\Phi_\alpha(k_x, k_y, t) = \exp\left(\frac{i}{\hbar}\left[\frac{(eE)^2\big(3\omega_0^2-\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\delta\Big(k_x - \frac{p_x}{\hbar}\Big) \\ \times \exp\left(-ik_y\left[\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)} + y_0\right]\right)\frac{1}{\sqrt{L_x}}\tilde{\chi}_\alpha(k_y-\gamma)\end{aligned} \tag{1.36}$$

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