Floquet-Drude Conductivity in Dressed Quantum Hall Systems

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1 Schrödinger problem for Landau levels in dressed 2DEG

Our analysis start with considering 2 dimentional free electronic gas which has been distrubuted in confined (x, y) plane in configuration space.

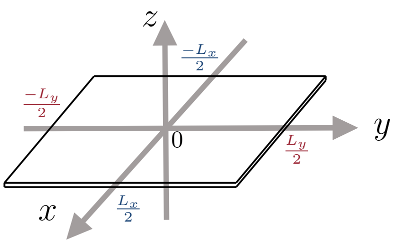

Figure 1: Confined 2DEG in configuration space with the size of $A = L_x L_y$.

We are going to examine the properties of 2DEG with stationary magnetic field

$$\mathbf{B} = (0, 0, B)^T \tag{1.1}$$

which directed on z axis and a linearly y-polarized strong electomagnetic wave (dressing field) with electric field given by

$$\mathbf{E} = (0, E\sin(\omega t), 0)^T \tag{1.2}$$

which also propagate in z direction. Here B and E represent the amplitude of the stationary magnetic field and electric field of dressing field.

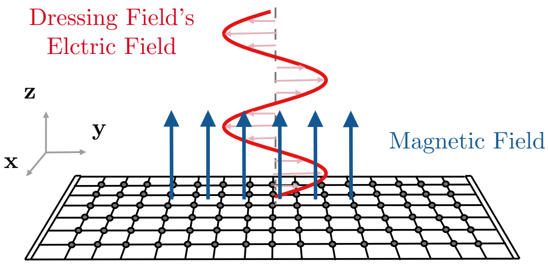

Figure 2: Stationary magnetic filed (blue color) and Strong EM wave (red color) applied to the 2DEG.

Using Landau gauge for the stationary magnetic field we can represent it using vector potential as

$$\mathbf{A}_s = (-By, 0, 0)^T \tag{1.3}$$

and choosing Coulomb gauge the dressing field can be present as the following vector potential

$$\mathbf{A}_d(t) = (0, [E/\omega]\cos(\omega t), 0)^T. \tag{1.4}$$

Now the Hamiltonian of an electron in 2DEG can be reads as

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[\hat{\mathbf{p}} - e\big(\mathbf{A}_s + \mathbf{A}_d(t)\big)\Big]^2 \tag{1.5}$$

where m_e is the effective mass of the electron and e is the magnitude (without considering the sign of the charge) of the electron charge. This can be simplified to

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(\hat{p}_x + eBy)\mathbf{e}_x + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))\mathbf{e}_y\Big]^2 \tag{1.6}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along x and y directions respectively. Moreover,

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(\hat{p}_x + eBy)^2 + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))^2\Big] \tag{1.7}$$

Since $[\hat{H}_e(t), \hat{p}_x] = 0$ both operators share same (simultaneous) eigen functions which are free electron wave functions $(\frac{1}{\sqrt{L_x}}\exp\big(\frac{ip_x x}{\hbar}\big))$. Therefore we can modify the Hamiltonian as follows

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(p_x + eBy)^2 + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))^2\Big]. \tag{1.8}$$

Using momentum operator definition

$$\hat{p}_y = -i\hbar\frac{\partial}{\partial y} \tag{1.9}$$

we can modify Eq. (1.8) as

$$\begin{aligned}\hat{H}_e(t) &= \frac{1}{2m_e}\Big[(p_x + eBy)^2 + \Big(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\Big)^2\Big] \\ &= \frac{1}{2m_e}\Big[(p_x + eBy)^2 + \Big(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\Big)^2\Big].\end{aligned} \tag{1.10}$$

Define the *center of the cyclotron orbit* along y axis as

$$y_0 \equiv \frac{-p_x}{eB} \tag{1.11}$$

and the *cyclotron frequency* as

$$\omega_0 \equiv \frac{eB}{m_e}. \tag{1.12}$$

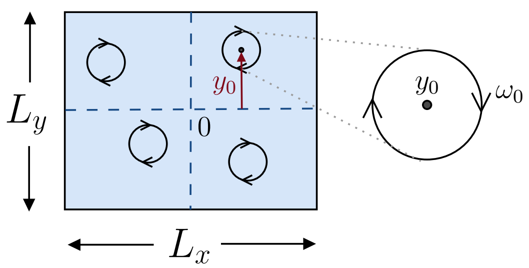

Figure 3: Paramters of the cyclotron orbits in the classical interpretation.

Then the Hamiltonian will leads to

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y - y_0)^2 + \frac{1}{2m_e}\Big(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\Big)^2 \tag{1.13}$$

$$\begin{aligned}\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y - y_0)^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + i\hbar\frac{\partial}{\partial y}\Big[\frac{eE}{\omega}\cos(\omega t)\Big] \\ + \frac{i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big)\end{aligned} \tag{1.14}$$

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y-y_0)^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big). \tag{1.15}$$

Let

$$\tilde{y} = (y-y_0) \longrightarrow dy = d\tilde{y} \tag{1.16}$$

and then this becomes

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}\tilde{y}^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial \tilde{y}^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial \tilde{y}} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big). \tag{1.17}$$

Now assume that the solution for the time-dependent schrödinger equation

$$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = \hat{H}_e(t)\psi \tag{1.18}$$

can be represent by the following form

$$\psi(\mathbf{r},t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_xx}{\hbar} + \frac{ieE(y-y_0)}{\hbar\omega}\cos(\omega t)\right)\phi(y-y_0,t). \tag{1.19}$$

Using the same subtution from Eq. (1.16) this becomes

$$\psi(x,\tilde{y},t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_xx}{\hbar} + \frac{ieE\tilde{y}}{\hbar\omega}\cos(\omega t)\right)\phi(\tilde{y},t). \tag{1.20}$$

Defining

$$\varphi(x,\tilde{y},t) \equiv \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_xx}{\hbar} + \frac{ieE\tilde{y}}{\hbar\omega}\cos(\omega t)\right) \tag{1.21}$$

we can simply the the Eq. (1.20) as

$$\psi(x,\tilde{y},t) = \varphi(x,\tilde{y},t)\phi(\tilde{y},t). \tag{1.22}$$

Let's subtitue Eq. (1.20) and Eq. (1.17) into Eq. (1.18) and we can observe that

$$\begin{aligned}\text{L.H.S} = i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} &= i\hbar\Big(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\phi + \frac{\mathrm{d}\phi}{\mathrm{d}t}\varphi\Big) = i\hbar\Big(\Big[\frac{-ieE\tilde{y}}{\hbar}\sin(\omega t)\Big]\varphi\phi + \varphi\frac{\mathrm{d}\phi}{\mathrm{d}t}\Big)\\ &= \big[eE\tilde{y}\sin(\omega t)\big]\varphi\phi + i\hbar\varphi\frac{\mathrm{d}\phi}{\mathrm{d}t}\end{aligned} \tag{1.23}$$

and

$$\begin{aligned}\text{R.H.S} &= \hat{H}_e(t)\psi\\ &= \left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial \tilde{y}^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial \tilde{y}} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big)\right]\varphi\phi\end{aligned} \tag{1.24}$$

where we will can calculate this part by part as follows:

$$\begin{aligned}\frac{-\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2}(\varphi\phi) &= \frac{-\hbar^2}{2m_e}\frac{\partial}{\partial\tilde{y}}\left[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\phi + \varphi\frac{\partial\phi}{\partial\tilde{y}}\right]\\ &= \frac{-\hbar^2}{2m_e}\left[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)^2\varphi\phi + \Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial\tilde{y}} + \Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial\tilde{y}} + \varphi\frac{\partial^2\phi}{\partial\tilde{y}^2}\right]\\ &= \Big(\frac{e^2E^2}{2m_e\omega^2}\cos^2(\omega)t\Big)\varphi\phi - \Big(\frac{ieE\hbar}{m_e\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial\tilde{y}} - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial\tilde{y}^2}\end{aligned} \tag{1.25}$$

and

$$\begin{aligned}\frac{2i\hbar eE}{2m_e\omega}\cos(\omega t)\frac{\partial}{\partial\tilde{y}}(\varphi\phi) &= \frac{i\hbar eE}{m_e\omega}\cos(\omega t)\left[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\phi + \varphi\frac{\partial\phi}{\partial\tilde{y}}\right]\\ &= \Big(\frac{-e^2E^2}{m_e\omega^2}\cos(\omega)t\Big)\varphi\phi + \frac{i\hbar eE}{m_e\omega}\cos(\omega t)\varphi\frac{\partial\phi}{\partial\tilde{y}}.\end{aligned} \tag{1.26}$$

Therefore we can derive that

$$\text{R.H.S} = \left[\frac{m_e\omega_0^2}{2}\tilde{y}^2\varphi\phi - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial\tilde{y}^2}\right]. \tag{1.27}$$

To satisfy the condition L.H.S=R.H.S we need to find a function $\phi(\tilde{y},t)$ such that

$$\big[eE\tilde{y}\sin(\omega t)\big]\varphi\phi + i\hbar\varphi\frac{\mathrm{d}\phi}{\mathrm{d}t} = \left[\frac{m_e\omega_0^2}{2}\tilde{y}^2\varphi\phi - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial\tilde{y}^2}\right] \tag{1.28}$$

by removing φ this can be simplyfied as

$$\left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 - eE\tilde{y}\sin(\omega t) - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} - i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]\phi(\tilde{y},t) = 0. \tag{1.29}$$

If we turn off the external dressing field, this equation leads to simple harmonic oscillator Hamiltonian as follows

$$\left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} - i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]\phi(\tilde{y},t) = 0 \tag{1.30}$$

$$i\hbar\frac{\mathrm{d}\phi(\tilde{y},t)}{\mathrm{d}t} = \left[\frac{\hat{p}_{\tilde{y}}^2}{2m_e} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2\right]\phi(\tilde{y},t). \tag{1.31}$$

Therefore we can identify the $S(t) \equiv eE\sin(\omega t)$ part as a external force act on the harmonic oscillator and we can solve this as a forced harmonic oscillator in $\tilde{y}$ axis.

$$i\hbar\frac{\mathrm{d}\phi(\tilde{y},t)}{\mathrm{d}t} = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2 - \tilde{y}S(t)]\right]\phi(\tilde{y},t). \tag{1.32}$$

This system can be extacly solvable and we can solve this equation using the methods explained by Husimi [*Ref:1] as follows.
First we can introduce the time dependent shifted corrdinte as

$$\tilde{y} \to y' = \tilde{y} - \zeta(t) \quad \Rightarrow \quad \tilde{y} = y' + \zeta(t) \tag{1.33}$$

and this implies that

$$\frac{\mathrm{d}\phi(y',t)}{\mathrm{d}t} = \frac{\partial\phi(y',t)}{\partial t} + \frac{\partial\phi(y',t)}{\partial y'}\frac{\partial y'}{\partial t} = \frac{\partial\phi(y',t)}{\partial t} - \dot{\zeta}(t)\frac{\partial\phi(y',t)}{\partial y'} \tag{1.34}$$

where $\dot{\zeta}(t) = \frac{\partial\zeta(t)}{\partial t}$. Therefore, Eq. (1.32) will be modified to

$$i\hbar\frac{\partial\phi(y',t)}{\partial t} = \left[i\hbar\dot{\zeta}\frac{\partial}{\partial y'} - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2(y'+\zeta)^2 - (y'+\zeta)S(t)\right]\phi(y',t). \tag{1.35}$$

Let's tranform the wave function using following unitary trasnform

$$\phi(y',t) = \exp\left(\frac{im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y',t) \tag{1.36}$$

and subtitte this into the Eq. (1.35) and we will get the following

$$\text{L.H.S} = \left[i\hbar\frac{\partial}{\partial t} - i\hbar\Big(\frac{im_e\ddot{\zeta}y'}{\hbar}\Big)\right]\exp\left(\frac{-im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y',t) \tag{1.37}$$

and

$$\begin{aligned}\text{R.H.S} = \Bigg[&i\hbar\dot{\zeta}\Big(\frac{im_e\dot{\zeta}}{\hbar}\Big) + i\hbar\dot{\zeta}\frac{\partial}{\partial y'}\\ &- \frac{\hbar^2}{2m_e}\Big[\Big(\frac{im_e\dot{\zeta}}{\hbar}\Big)^2 + \Big(\frac{2im_e\dot{\zeta}}{\hbar}\Big)\frac{\partial}{\partial y'} + \frac{\partial^2}{\partial y'^2}\Big]\\ &+ \frac{1}{2}m_e\omega_0^2y'^2 + \frac{1}{2}m_e\omega_0^2\zeta^2 + m_e\omega_0^2y'\zeta\\ &- y'S(t) - \zeta S(t)\Bigg]\exp\left(\frac{-im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y',t).\end{aligned} \tag{1.38}$$

Combining these two and removing exponential terms we can derive that

$$\begin{aligned} i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 + \Big[m_e\ddot{\zeta} + m_e\omega_0^2\zeta - S(t)\Big]y' \\ + \Big[-\frac{1}{2}m_e\dot{\zeta}^2 + \frac{1}{2}m_e\omega_0^2\zeta^2 - \zeta S(t)\Big]\Big]\varphi(y',t). \end{aligned} \tag{1.39}$$

Then we can restrict our $\zeta(t)$ function such that

$$m_e\ddot{\zeta} + m_e\omega_0^2\zeta = S(t) \tag{1.40}$$

and that leads to

$$i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 - L(\zeta,\dot{\zeta},t)\Big]\varphi(y',t) \tag{1.41}$$

where

$$L(\zeta,\dot{\zeta},t) \equiv \frac{1}{2}m_e\dot{\zeta}^2 - \frac{1}{2}m_e\omega_0^2\zeta^2 + \zeta S(t) \tag{1.42}$$

is the largrangian of a classical driven oscillator.

Now introduce new unitary transformation for the wavefunction as follows

$$\varphi(y',t) = \exp\Big(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\Big)\chi(y',t) \tag{1.43}$$

and subtite this into the Eq. (1.41) and gets

$$\begin{aligned} i\hbar\Big[\exp\Big(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\Big)\frac{\partial}{\partial t} + i\hbar L(\zeta,\dot{\zeta},t)\exp\Big(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\Big)\Big]\chi(y',t) \\ = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 - L(\zeta,\dot{\zeta},t)\Big]\exp\Big(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\Big)\chi(y',t) \end{aligned} \tag{1.44}$$

removing exponential terms finally we can derive that

$$i\hbar\frac{\partial}{\partial t}\chi(y',t) = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2\Big]\chi(y',t). \tag{1.45}$$

This is the well known Schrodinger equation of a stationary quantum harmonic oscillator. In terms of the eigenvalues

$$E_n = \hbar\omega_0(n+\frac{1}{2}) \tag{1.46}$$

of well-known harmonic eigenfucntions (using Gauss-Hermite functions ϑ)

$$\chi_n(x) \equiv \sqrt{\kappa}\vartheta(\kappa x) \quad \text{where} \quad \vartheta(x) = \frac{1}{\sqrt{2^n n!}}e^{-x^2/2}\mathcal{H}_n(x) \quad \text{with} \quad \kappa = \sqrt{\frac{m_e\omega_0}{\hbar}} \tag{1.47}$$

being propositional to the Hermite functions $\mathcal{H}_n$, the solutions of Eq. (1.32) can be represent as

$$\phi_n(\tilde{y},t) = \chi_n(\tilde{y}-\zeta(t))\exp\Big(\frac{i}{\hbar}\Big[-E_n t + m_e\dot{\zeta}(t)(\tilde{y}-\zeta(t)) + \int_0^t dt' L(\zeta,\dot{\zeta},t')\Big]\Big) \tag{1.48}$$

The set $\{\chi_n(x)\}$ forms a complete set and thus any general solution $\phi_(\tilde{y},t)$ can be expaned in terms of the solutions in Eq. (1.48).

Next we consider special case where we assumed

$$S(t) = eE\sin(\omega t) \tag{1.49}$$

and one can derive the Eq. (1.40) for $\zeta(t)$

$$m_e\ddot{\zeta} + m_e\omega_0^2\zeta = eE\sin(\omega t) \tag{1.50}$$

and using Green function method the solution can be write as

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t). \tag{1.51}$$

form this solutions we are able to derive the final solutions $\alpha = (n, m)$ where $n \in \mathbb{Z}_0^+ and m \in \mathbb{Z}$ are two quantum numbers that describe the state of the electron, can be present as

$$\begin{aligned}\psi_\alpha(x, \tilde{y}, t) =& \frac{1}{\sqrt{L_x}}\chi_n\big(\tilde{y} - \zeta(t)\big) \\ & \times \exp\left(\frac{i}{\hbar}\left[-E_n t + p_x x + \frac{eE\tilde{y}}{\omega}\cos(\omega t) + m_e\dot{\zeta(t)}\big[\tilde{y} - \zeta(t)\big] + \int_0^t dt' L(\zeta, \dot{\zeta}, t')\right]\right)\end{aligned} \tag{1.52}$$

and the exponential phase shifts represent the effect done by the stationary magnetic field and strong dressing field. In here p_x is qunatized with the quantum number m due to the spacial confinemnet in x direction.

$$p_x = m\frac{2\pi\hbar}{L_x} \quad , \quad m = 0, \pm 1, \pm 2, ... \tag{1.53}$$

Therefore we can assume that the magnetitranport properties of 2DEG will be renormalized by the magnetic field as well as the dressing field. ■

2 Floquet theory

Since we describe the lifetime of an electron in certain Landau level using conventianal perturbation theeory, now we can apply the Floquet theory to identify the difference of these methods.
First we need to identify the *quasienergies* and periodic *Floquet modes* for derived wavefunctions (1.52) for a 2DEG system with both stationary magnetic field and strong dressing filed.
Let's consider the following paramter which is lineraly increasing in time

$$\Delta_E t \equiv \frac{t}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t') \tag{2.1}$$

where we can calculate this using Eq. (1.42) and (1.51) as follows

$$\begin{aligned}\Delta_E t = \frac{t}{T}\int_0^T dt'\ \frac{1}{2}m_e\frac{(eE\omega)^2}{m_e^2(\omega_0^2-\omega^2)^2}\cos^2(\omega t') - \frac{1}{2}m_e\omega_0^2\frac{(eE)^2}{m_e^2(\omega_0^2-\omega^2)^2}\sin^2(\omega t')\\ + \frac{eE}{m_e(\omega_0^2-\omega^2)}\sin(\omega t')eE\sin(\omega t')\end{aligned} \tag{2.2}$$

$$\begin{aligned}\Delta_E t = \frac{t\omega}{2\pi}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\int_0^T dt'\ \cos^2(\omega t') - \omega_0^2\int_0^T dt'\ \sin^2(\omega t')\\ + 2(\omega_0^2-\omega^2)\int_0^T dt'\ \sin^2(\omega t')\Bigg]\end{aligned} \tag{2.3}$$

$$\Delta_E t = \frac{t\omega}{2\pi}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\left[\omega^2\frac{\pi}{\omega} - \omega_0^2\frac{\pi}{\omega} + 2(\omega_0^2-\omega^2)\frac{\pi}{\omega}\right] \tag{2.4}$$

$$\Delta_E t = \frac{t\omega}{2}\times\frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}(\omega_0^2-\omega^2) = \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)}t \tag{2.5}$$

Since this is the continuous increasing part of the Laggrangian integral in Eq. (1.52) we can make this as 2ω periodic function as follows

$$\Lambda \equiv \int_0^t dt'\ L(\zeta,\dot{\zeta},t') - \frac{t}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t') \tag{2.6}$$

which can be proved as follows. First consider the first term of the Λ

$$\begin{aligned}\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\int_0^t dt'\ \cos^2(\omega t') - \omega_0^2\int_0^t dt'\ \sin^2(\omega t')\\ + 2(\omega_0^2-\omega^2)\int_0^t dt'\ \sin^2(\omega t')\Bigg]\end{aligned} \tag{2.7}$$

$$\begin{aligned}\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\omega^2\left[\frac{t}{2}+\frac{\sin(2\omega t)}{4\omega}\right] - \omega_0^2\left[\frac{t}{2}-\frac{\sin(2\omega t)}{4\omega}\right]\\ + 2(\omega_0^2-\omega^2)\left[\frac{t}{2}-\frac{\sin(2\omega t)}{4\omega}\right]\Bigg]\end{aligned} \tag{2.8}$$

$$\begin{aligned}\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{2m_e(\omega_0^2-\omega^2)^2}\Bigg[\frac{t}{2}\left[\omega^2-\omega_0^2+2\omega_0^2-2\omega^2\right]\\ + \frac{\sin(2\omega t)}{4\omega}\left[\omega^2+\omega_0^2-2\omega_0^2+2\omega^2\right]\Bigg]\end{aligned} \tag{2.9}$$

$$\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)}t + \frac{(eE)^2\left(3\omega^2-\omega_0^2\right)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) \tag{2.10}$$

then using Eq.(2.5) we can write this as

$$\int_0^t dt'\ L(\zeta,\dot{\zeta},t') = \Delta_E t + \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t). \tag{2.11}$$

Now we can express

$$\Lambda = \Delta_E t + \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) - \Delta_E t = \frac{(eE)^2\big(3\omega^2-\omega_0^2\big)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t) \tag{2.12}$$

which is a periodic function in time with 2ω frequency.
Now using this parmaters we can factorize the wavefunction (1.52) as linearly time dependend part and periodic time dependend part as follows

$$\begin{aligned}\psi_\alpha(x,y,t) =& \exp\left(\frac{i}{\hbar}[-E_n t + \Delta_E t]\right)\frac{1}{\sqrt{L_x}}\chi_n\big(y-\zeta(t)\big) \\ &\times \exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eEy}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y-\zeta(t)\big] + \int_0^t dt' L(\zeta,\dot{\zeta},t') - \Delta_E t\right]\right)\end{aligned} \tag{2.13}$$

where we can identify(let $\alpha \to (n,m)$) the *quasienergies* as

$$\varepsilon_\alpha \equiv \varepsilon_n = \hbar\omega_0\left(n+\frac{1}{2}\right) - \Delta_E \quad \text{where} \quad n = 0,1,2,... \quad \text{for any given } m \tag{2.14}$$

which is only depend on one quantum number (n) and *Floquet modes* as

$$\phi_\alpha(x,\tilde{y},t) \equiv \frac{1}{\sqrt{L_x}}\chi_n\big(\tilde{y}-\zeta(t)\big)\exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eE\tilde{y}}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[\tilde{y}-\zeta(t)\big] + \Lambda\right]\right) \tag{2.15}$$

with

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2-\omega^2)}\sin(\omega t) \quad \text{and} \quad \dot{\zeta}(t) = \frac{eE\omega}{m_e(\omega_0^2-\omega^2)}\cos(\omega t) \tag{2.16}$$

where *Floquet modes* are time-periodic functions that also create a complete orthonormal set. ■

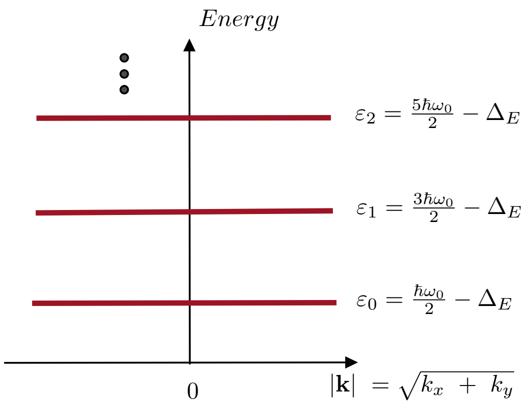

Figure 4: Quasienergies for each Landau levels against magnitude of momentum.

Therefore using Floquet theory, the solutions (Floquet states) for the periodic Hamiltonian (1.5) can be written in position space as

$$\psi_\alpha(x,\tilde{y},t) = \exp\left(-\frac{i}{\hbar}\varepsilon_\alpha t\right)\phi_\alpha(x,\tilde{y},t) \tag{2.17}$$

where

$$\varepsilon_\alpha \equiv \left(\frac{eB\hbar}{m_e}\right)\left(n+\frac{1}{2}\right) - \frac{(eE)^2}{4m_e(\omega_0^2-\omega^2)} \quad \text{where} \quad n = 0,1,2,... \tag{2.18}$$

and

$$\begin{aligned}\phi_\alpha(x,\tilde{y},t) \equiv & \frac{1}{\sqrt{L_x}}\chi_n\left(\tilde{y}-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\\ & \times\exp\left(\frac{i}{\hbar}\left[p_x x+\frac{eE\tilde{y}}{\omega}\cos(\omega t)+\frac{eE\omega\tilde{y}}{(\omega_0^2-\omega^2)}\cos(\omega t)\right]\right)\\ & \times\exp\left(\frac{i}{\hbar}\left[-\frac{(eE)^2\omega}{2m_e(\omega^2-\omega_0^2)^2}\sin(2\omega t)+\frac{(eE)^2\left(3\omega_0^2-\omega^2\right)}{8m_e\omega(\omega_0^2-\omega^2)^2}\sin(2\omega t)\right]\right)\end{aligned} \tag{2.19}$$

Now we can write this by more simplying and considering spacial dependencies and using previous subtituting done in Eq. (1.16) and now χ function depend on both quantum numbers because y_0 gives the p_x dependence and we can present as

$$\begin{aligned}\phi_\alpha(x,y,t) \equiv \frac{1}{\sqrt{L_x}}\chi_n\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{ip_x}{\hbar}x\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right](y-y_0)\right)\\ \times\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\end{aligned} \tag{2.20}$$

Now we can transform this solution in spacial variable into the momentum space using Fourier trasform over the considering confined space $A=L_xL_y$.

$$\begin{aligned}\phi_\alpha(k_x,k_y,t) = & \int_{-L_y/2}^{L_y/2} dy\ \exp(-ik_y y)\left[\frac{1}{\sqrt{L_x}}\chi_n\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right)\right]\\ & \times\int_{-L_x/2}^{L_x/2} dx\ \exp(-ik_x x)\left[\exp\left(\frac{ip_x}{\hbar}x\right)\right]\\ & \times\exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\times\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\end{aligned} \tag{2.21}$$

Then this can be re-write as follows

$$\phi_\alpha(k_x,k_y,t) = \exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\left(\omega_0^2+\omega^2\right)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\Theta_\alpha(k_y,t)\delta_{k_x,\frac{p_x}{\hbar}} \tag{2.22}$$

where we used

$$\int_{L_x} dx\ \exp\left(-ik_x x+\frac{ip_x}{\hbar}x\right) = L_x\delta_{k_x,\frac{p_x}{\hbar}} \tag{2.23}$$

and

$$\Theta_\alpha(k_y,t) \equiv \int_{-L_y/2}^{L_y/2} dy\ \exp(-ik_y y)\left[\sqrt{L_x}\chi_n\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right)\right] \tag{2.24}$$

and this can be simplied as

$$\Theta_\alpha(k_y,t) = \sqrt{L_x}\int_{-L_y/2}^{L_y/2} dy\ \chi_n\left(y-y_0-\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(-ik_y y+\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y\right). \tag{2.25}$$

Then by defining

$$\mu(t) \equiv \frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}+y_0 \tag{2.26}$$

and

$$\gamma(t) \equiv \frac{eE\omega_0^2\cos(\omega t)}{\hbar\omega(\omega_0^2-\omega^2)} \tag{2.27}$$

we can re-write this by neglecting time dependencies as

$$\Theta_\alpha(k_y,t)=\sqrt{L_x}\int_{-\infty}^{\infty}dy\;\chi_n(y-\mu)\exp(-i(k_y-\gamma)y). \tag{2.28}$$

We can subtitute following variables

$$k_y{}'=k_y-\gamma \quad \text{and} \quad y'=y-\mu \tag{2.29}$$

and for $L_y\to\infty$ this leads to

$$\Theta_\alpha(k_y{}',t)=\sqrt{L_x}e^{-ik_y{}'\mu}\int_{-\infty}^{\infty}dy'\;\chi_n(y')\exp\big(-ik_y{}'y'\big)=\sqrt{L_x}e^{-ik_y{}'\mu}\sqrt{\kappa}\int_{-\infty}^{\infty}dy'\;\vartheta_n(\kappa y')\exp\big(-ik_y{}'y'\big) \tag{2.30}$$

We know that $\{\chi_\alpha\}$ are well-known harmonic eigenfunctions(with Gauss-Hermite functions) as given in the Eq. (1.47). However, the equation in (2.30) represents the Fourier transform of the these Gauss-Hermite functions. Due to the symmetric condition [*Ref:E.Celeghini] the Fourier transform of these functions can be represent as

$$\mathcal{FT}[\vartheta_n(\kappa x),x,k]=\frac{i^n}{|\kappa|}\vartheta_n(k/\kappa) \tag{2.31}$$

Therefore

$$\Theta_\alpha(k_y{}',t)=\sqrt{L_x}e^{-ik_y{}'\mu}\times\frac{i^n}{\sqrt{\kappa}}\vartheta_n\left(\frac{k_y{}'}{\kappa}\right)=\sqrt{L_x}e^{-ik_y{}'\mu}\tilde{\chi}_n\big(k_y{}'\big) \tag{2.32}$$

where

$$\tilde{\chi}_n(k)=\frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2}e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_\alpha\left(\frac{k}{\kappa}\right). \tag{2.33}$$

Using Eq. (2.32) and Eq. (2.22) we can derive that

$$\begin{aligned}\phi_\alpha(k_y,t)=\exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\big(\omega_0^2+\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\\ \times\sqrt{L_x}e^{-i(k_y-\gamma)\mu}\tilde{\chi}_n(k_y-\gamma)\end{aligned} \tag{2.34}$$

where we included the k_x dependence into α quantum number using m value and this can be re-write subtituting μ and γ values as follows

$$\begin{aligned}\phi_\alpha(k_y,t)=\sqrt{L_x}\exp\left(\frac{-i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\exp\left(\frac{-i}{\hbar}\left[\frac{(eE)^2\big(\omega_0^2+\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\\ \times\exp\left(-ik_y\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp\left(\frac{i}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}\right)\exp(-ik_yy_0)\\ \times\exp\left(i\frac{1}{\hbar}\left[\frac{eE\omega_0^2\cos(\omega t)}{\omega(\omega_0^2-\omega^2)}\right]y_0\right)\tilde{\chi}_n(k_y-\gamma)\end{aligned} \tag{2.35}$$

and

$$\begin{aligned}\phi_\alpha(k_y,t)=\sqrt{L_x}\exp\left(\frac{i}{\hbar}\left[\frac{(eE)^2\big(3\omega_0^2-\omega^2\big)}{8\omega m_e(\omega_0^2-\omega^2)^2}\right]\sin(2\omega t)\right)\\ \times\exp\left(-ik_y\left[\frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)}+y_0\right]\right)\tilde{\chi}_n(k_y-\gamma).\end{aligned} \tag{2.36}$$

For notation convinient we can introduce few constant as follows

$$b\equiv\frac{(eE)^2\big(3\omega_0^2-\omega^2\big)}{8\hbar\omega m_e(\omega_0^2-\omega^2)^2} \tag{2.37}$$

and

$$d\equiv\frac{eE}{m_e(\omega_0^2-\omega^2)} \tag{2.38}$$

with

$$g \equiv \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)}. \tag{2.39}$$

Therefore we can write Eq. (2.36) as

$$\phi_\alpha(k_y, t) = \sqrt{L_x} e^{ib\sin(2\omega t)} e^{-ik_y[d\sin(\omega t)+y_0]} \tilde{\chi}_n(k_y - g\cos(\omega t)). \tag{2.40}$$

3 Floquet Fermi Goldern Rule

In this section we are going to derive the Floquet Fermi goldern rule for above derived quantum Floquet states using $t - t'$ formalism.

The Floquet states (2.17) fullfills the $t - t'$ Schrödinger equation [*Ref:myReport] as follows

$$i\hbar \frac{\partial}{\partial t} |\psi_\alpha(t, t')\rangle = H_F(t') |\psi_\alpha(t, t')\rangle \tag{3.1}$$

where Floquet Hamiltonian given by

$$H_F(t') \equiv H_e(t) - i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \tag{3.2}$$

and

$$|\psi_\alpha(t, t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon_\alpha t\right) |\phi_\alpha(t')\rangle \tag{3.3}$$

Now for the Eq. (3.1) corresponding time evolution operator satisfy the Schrödinger equation

$$U_0(t, t_0; t') = \exp\left(-\frac{i}{\hbar} H_F(t')[t - t_0]\right) \tag{3.4}$$

Consider a time-independent total perturbation $V(\mathbf{r})$ switched on at the reference time $t = t_0$, then Schrödinger equation becomes

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle = [H_F(t') + V(\mathbf{r})] |\Psi_\alpha(t, t')\rangle \tag{3.5}$$

and when $t \leq t_0$ both solutions of the Schrödinger equation coincide

$$|\psi_\alpha(t, t')\rangle = |\Psi_\alpha(t, t')\rangle \quad \text{when} \quad t \leq t_0 \tag{3.6}$$

Now, we can introduce the interaction picture representation of the $t - t'$ Floquet state as

$$|\Psi_\alpha(t, t')\rangle_I = U_0^\dagger(t, t_0; t') |\Psi_\alpha(t, t')\rangle \tag{3.7}$$

and the perturbation in the interaction picture will be

$$V_I(\mathbf{r}) = U_0^\dagger(t, t_0; t') V(\mathbf{r}) U_0(t, t_0; t') = V(\mathbf{r}). \tag{3.8}$$

This leads to the Schrödinger eqution in the interction picture

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle_I = V_I(\mathbf{r}) |\Psi_\alpha(t, t')\rangle_I \tag{3.9}$$

with the recursive solution

$$|\Psi_\alpha(t, t')\rangle_I = |\Psi_\alpha(t_0, t')\rangle_I + \frac{1}{i\hbar} \int_{t_0}^{t} dt_1 \; V_I(\mathbf{r}) |\Psi_\alpha(t_1, t')\rangle_I \tag{3.10}$$

Iterating the solution only upto first order (Born approximation) this leads to

$$|\Psi_\alpha(t, t')\rangle_I \approx |\psi_\alpha(t_0, t')\rangle + \frac{1}{i\hbar} \int_{t_0}^{t} dt_1 \; V_I(\mathbf{r}) |\psi_\alpha(t_0, t')\rangle \tag{3.11}$$

and multiply it by $\langle\psi_\beta(t_0, t')|$ and we will get

$$\langle\psi_\beta(t_0, t')|\Psi_\alpha(t, t')\rangle_I = \langle\psi_\beta(t_0, t')|\psi_\alpha(t_0, t')\rangle + \frac{1}{i\hbar} \int_{t_0}^{t} dt_1 \; \langle\psi_\beta(t_0, t')| V_I(\mathbf{r}) |\psi_\alpha(t_0, t')\rangle . \tag{3.12}$$

Then introdusing unitory operator U_0 we can re-write this as

$$\begin{aligned}\langle\psi_\beta(t_0,t')|U_0^\dagger(t,t_0;t')|\Psi_\alpha(t,t')\rangle = &\ \langle\psi_\beta(t_0,t')|U_0^\dagger(t,t_0;t')U_0(t,t_0;t')|\psi_\alpha(t_0,t')\rangle \\ &+\frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ \langle\psi_\beta(t_0,t')|\,U_0^\dagger(t_1,t_0;t')V(\mathbf{r})U_0(t_1,t_0;t')\,|\psi_\alpha(t_0,t')\rangle\end{aligned} \tag{3.13}$$

and this can be simplied as

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = \langle\psi_\beta(t,t')|\psi_\alpha(t,t')\rangle + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{3.14}$$

Since our $t-t'$ Floquet states are orthonormal [*Ref:myReport- t-t' formalism] we can derive that

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = \delta_{\alpha\beta}\exp(i\omega[t'-t]) + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{3.15}$$

Now, set $t_0=0$ and for a case $\alpha\neq\beta$ where we can represent $\alpha=(n_\alpha,m_\alpha)$ and $\beta=(n_\beta,m_\beta)$ and this will simplied to

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = -\frac{i}{\hbar}\int_{0}^{t} dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{3.16}$$

In addition, since our Floquet states create a basis for composite space we can represent any solution using our Floquet states

$$|\Psi_\alpha(t,t')\rangle = \sum_\beta a_{\alpha\beta}(t,t')\,|\psi_\beta(t,t')\rangle\,. \tag{3.17}$$

Therefore we can derive a equation for this *scattering amplitude* as

$$a_{\alpha\beta}(t,t') = \langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = -\frac{i}{\hbar}\int_{0}^{t} dt_1\ \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{3.18}$$

Now lets assume a scattering event from a $t-t'$ Floquet state $|\psi_\beta(t,t')\rangle$ into another $t-t'$ Floquet state $|\Psi_\alpha(t,t')\rangle$ with constant quansienergy ε given as follows

$$|\Psi_\alpha(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon t\right)|\Phi_\alpha(t')\rangle \tag{3.19}$$

Now consider a scattering event

$$\psi_\beta(\mathbf{k}',t,t') = \exp\left(-\frac{i}{\hbar}\varepsilon_\beta t\right)\phi_\beta(\mathbf{k}',t') \longrightarrow \Psi_\alpha(\mathbf{k},t,t') = \exp\left(-\frac{i}{\hbar}\varepsilon t\right)\Phi_\alpha(\mathbf{k},t') \tag{3.20}$$

Here we need to undestand a state of this considering system only be represented by two independent quantum numbers which are n energy eigen states and m quantum number which represents the qunatized momentum in x direction values. Lets calculate the scattering amplitutde of the above mentioned scattering scenario using the equation derived in (3.18).

$$\begin{aligned}a_{\alpha\beta}(\mathbf{k}',\mathbf{k},t,t') &= -\frac{i}{\hbar}\int_0^t dt_1\ \langle\psi_{\beta,\mathbf{k}'}(t_1,t')|\,V(\mathbf{r})\,|\psi_{\alpha,\mathbf{k}}(t_1,t')\rangle \\ &= -\frac{i}{\hbar}\int_0^t dt_1\ e^{\frac{i}{\hbar}(\varepsilon_\beta-\varepsilon)t_1}\,\langle\phi_{\beta,\mathbf{k}'}(t')|\,V(\mathbf{r})\,|\phi_{\alpha,\mathbf{k}}(t')\rangle\end{aligned} \tag{3.21}$$

Next assuimg this scenario for long time $t\to\infty$ we can turn this integral into a delta distrubution as follows

$$\begin{aligned}a_{\alpha\beta}(\mathbf{k}',\mathbf{k},t,t') &= -\frac{i}{\hbar}\lim_{t\to\infty}\left[\int_{-t/2}^{t/2} dt_1\ e^{\frac{i}{\hbar}(\varepsilon_\beta-\varepsilon)t_1}\,\langle\phi_{\beta,\mathbf{k}'}(t')|\,V(\mathbf{r})\,|\phi_{\alpha,\mathbf{k}}(t')\rangle\right] \\ &= -2\pi i\delta(\varepsilon_\beta-\varepsilon)\,\langle\phi_{\beta,\mathbf{k}'}(t')|\,V(\mathbf{r})\,|\phi_{\alpha,\mathbf{k}}(t')\rangle\end{aligned} \tag{3.22}$$

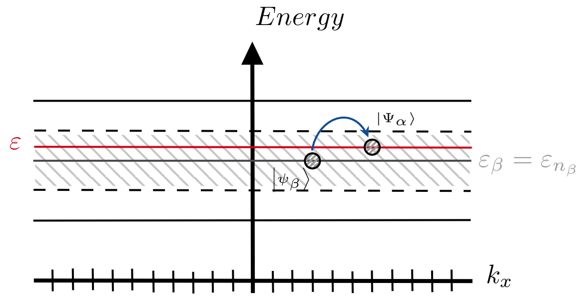

Figure 5: Scattering from $|\psi_\beta(t,t')\rangle$ to constant energy state $|\Psi_\alpha(t,t')\rangle$ due to scattering potential created by impurities.

Now lets consider about the inner product of the above derivation. Using completeness properties we can write that as follows

$$\begin{aligned} Q &\equiv \langle\phi_{\beta,\mathbf{k}'}(t')|\, V(\mathbf{r})\, |\phi_{\alpha,\mathbf{k}}(t')\rangle \\ &= \sum_{\mathbf{k}}\sum_{\mathbf{k}'} \langle\phi_{\beta,\mathbf{k}'}(t')|\mathbf{k}'\rangle \, \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle \, \langle\mathbf{k}|\phi_{\alpha,\mathbf{k}}(t')\rangle \end{aligned} \tag{3.23}$$

and seperating x and y directional momentums we can modify this as follows (Assuming $L_y \to \infty$) and since we assumed that $L_y \to \infty$

$$\begin{aligned} Q &\equiv \langle\phi_{\beta,\mathbf{k}'}(t')|\, V(\mathbf{r})\, |\phi_{\alpha,\mathbf{k}}(t')\rangle \\ &= \sum_{k_x}\sum_{k'_x} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y \; \phi_\beta(\mathbf{k}',t')\, \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle \, \phi_\alpha(\mathbf{k},t'). \end{aligned} \tag{3.24}$$

For a random white scattering potential we can represent the inner product of scattering potential with momentum as a constant value as

$$V_{\mathbf{k}',\mathbf{k}} \equiv \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle \,. \tag{3.25}$$

In this study, the perturbation potential is assumed to be formed by an ensemble of randomly distributed impurities, since randomimpurities iin a disorded metal is a better approximation for experimental results.
Consider N_{imp} identical impurities positioned at the randomly distributed but fixed potions $\mathbf{r}_i$. The elastic scattering potential $V(\mathbf{r})$ is then given by the sum over uncorrelated single impurity potentials $\upsilon(\mathbf{r})$

$$V(\mathbf{r}) \equiv \sum_{i=1}^{N_{imp}} \upsilon(\mathbf{r}-\mathbf{r}_i). \tag{3.26}$$

Now assume that the perturbation $V(\mathbf{r})$ is a Gaussian random potential where one can choose the zero of enerrgy such that the potential is zero on average. This model characterized by [*Ref: e.Akkermans G. Montambaux]

$$\langle\upsilon(\mathbf{r})\rangle_{imp} = 0 \tag{3.27}$$

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon(\mathbf{r}-\mathbf{r}') \tag{3.28}$$

where $\langle\cdot\rangle_{imp}$ denoted the average over realizations of the impurity disorder. In addition, this model assume that $\upsilon(\mathbf{r}-\mathbf{r}')$ only dependes on the position difference $|\mathbf{r}-\mathbf{r}'|$ and it decays with a characteristic leangth r_c. Since the study considers the case where the waveleagth of radiation or scattering electrons is much faster than r_c, it is good approximation to make two-point correlation function to be

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon^2_{imp}\delta(\mathbf{r}-\mathbf{r}') \tag{3.29}$$

and a random potential $V(\mathbf{r})$ with this property is called white noise [*Ref: e.Akkermans G. Montambaux]. Then we can choose approximately total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i). \tag{3.30}$$

Since $\langle \mathbf{r}|\mathbf{k}\rangle = \frac{1}{\sqrt{L_x L_y}}\exp(-i[k_x x + k_y y])$, we can calculate the Eq. (3.25) using this assumption as follows

$$\begin{aligned}
V_{\mathbf{k}',\mathbf{k}} &= \langle \mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle \\
&= \left\langle \mathbf{k}' \middle| \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i) \middle| \mathbf{k} \right\rangle \\
&= \left\langle \mathbf{k}' \middle| \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(x-x_i)\delta(y-y_i) \middle| \mathbf{k} \right\rangle \\
&= \sum_{i=1}^{N_{imp}} \int_{-\infty}^{\infty} dy\, \frac{1}{\sqrt{L_x L_y}} e^{ik'_y y}\delta(y-y_i)\frac{1}{\sqrt{L_x L_y}} e^{-ik_y y}\, \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle \\
&= \sum_{i=1}^{N_{imp}} \frac{1}{L_x L_y} e^{i(k'_y - k_y)y}\, \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle
\end{aligned} \tag{3.31}$$

Assuming the total umber of scatterers N_{imp} is macroscopically large and for each impurity will produce same impurity potential($V_{k'_x,k_x}$) (this is not sure and we need to prove this somehonw or we need to bring the $V_{\mathbf{k},\mathbf{k}'}$ without calculations)for every x-directional momentum pairs, we can achieve following expression

$$\begin{aligned}
V_{\mathbf{k}',\mathbf{k}} &= V_{k'_x,k_x}\frac{N_{imp}}{L_y L_x}\int_{-\infty}^{\infty} dy_i\; e^{i\left(k'_y - k_y\right)y_i} \\
&= \eta_{imp} V_{k'_x,k_x}\delta(k'_y - k_y)
\end{aligned} \tag{3.32}$$

where

$$V_{k'_x,k_x} \equiv \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle \tag{3.33}$$

is constant value for every i impurity and η_{imp} is number of impurities in a unit area. It is important to notice that $|k_x\rangle = e^{-ik_x x}$.

Therefore, using the Eq. (2.36), the Eq. (3.24) modified to (we can change varable $t' \to t$)

$$\begin{aligned}
Q = \sum_{k_x}\sum_{k'_x} &\eta_{imp} V_{k'_x,k_x}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\; \delta(k'_y - k_y) \\
&\times \sqrt{L_x}\exp(-ib\sin(2\omega t))\exp(ik'_y[d\sin(\omega t) + y'_0])\tilde{\chi}_{n_\beta}(k'_y - g\cos(\omega t)) \\
&\times \sqrt{L_x}\exp(ib\sin(2\omega t))\exp(-ik_y[d\sin(\omega t) + y_0])\tilde{\chi}_{n_\alpha}(k_y - g\cos(\omega t))
\end{aligned} \tag{3.34}$$

and we can simplify this as

$$\begin{aligned}
Q = \sum_{k_x}\sum_{k'_x} &\eta_{imp} L_x V_{k'_x,k_x}\int_{-\infty}^{\infty} dk_y \\
&\times \exp(ik_y y'_0)\tilde{\chi}_{n_\beta}(k_y - g\cos(\omega t))\exp(-ik_y y_0)\tilde{\chi}_{n_\alpha}(k_y - g\cos(\omega t))
\end{aligned} \tag{3.35}$$

and this can re-write as

$$Q = \sum_{k_x}\sum_{k'_x}\eta_{imp} L_x V_{k'_x,k_x} I \tag{3.36}$$

where

$$I \equiv \int_{-\infty}^{\infty} dk_y\; \tilde{\chi}_{n_\beta}(k_y - g\cos(\omega t))\tilde{\chi}_{n_\alpha}(k_y - g\cos(\omega t))\exp(-ik_y[y_0 - y'_0]). \tag{3.37}$$

To avoid the energy tramision from external high-frequency field and electrons in the system, the applied radiation should be purely dressing field. Therefore, the only effect of the dressing field on 2DEG is the renormalization of the probability of elastic electron scattering within same Landau level ($n_\alpha = n_\beta$). Therefore Eq. (3.37) can be modified to

$$I \equiv \int_{-\infty}^{\infty} dk_y \ \tilde{\chi}^2_{n_\beta}(k_y - g\cos(\omega t)) \exp(-ik_y[y_0 - {y'}_0]). \tag{3.38}$$

Lets consider about this integral and we can calculate it as using the following subtitution. Let

$$k_y - g\cos(\omega t) = \bar{k}_y \longrightarrow dk_y = d\bar{k}_y \tag{3.39}$$

and this leads to

$$I \equiv 2\pi \times \frac{1}{2\pi}\int_{-\infty}^{\infty} d\bar{k}_y \ \tilde{\chi}^2_{n_\alpha}\big(\bar{k}_y\big) \exp\big(-i\big(\bar{k}_y + g\cos(\omega t)\big)(y_0 - {y'}_0)\big). \tag{3.40}$$

Using Fourier transform of Gauss-Hermite functions and convolution theorem we can write this as

$$I \equiv 2\pi \exp(g[{y'}_0 - y_0]\cos(\omega t)) \int_{-\infty}^{\infty} dy \ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - {y'}_0 - y). \tag{3.41}$$

Therefore the scatterng amplitude (3.22) will modified to

$$a_{\alpha\beta}({k'}_x, k_x, t) = -2\pi i\delta(\varepsilon_\beta - \varepsilon)\sum_{k_x}\sum_{{k'}_x} \eta_{imp} L_x V_{{k'}_x, k_x} I \tag{3.42}$$

Consideirng qunatized momentum given in x direction derived in Eq. (1.53), we can identify the non-zero values for scattering amplitude using following conditions

$${k'}_x = \frac{p_{x_\beta}}{\hbar} = m'\frac{2\pi}{L_x} \quad \text{and} \quad k_x = \frac{p_{x_\alpha}}{\hbar} = m\frac{2\pi}{L_x}. \tag{3.43}$$

Then we can simplfied scattering amplitude for given ${k'}_x$ and k_x as

$$\begin{aligned} a_{\alpha\beta}({k'}_x, k_x, t) = -2\pi i\delta(\varepsilon_\beta - \varepsilon)\eta_{imp} L_x V_{{k'}_x, k_x} \exp(g[{y'}_0 - y_0]\cos(\omega t)) \\ \times \int_{-\infty}^{\infty} dy \ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - {y'}_0 - y) \end{aligned} \tag{3.44}$$

Since this scattering amplitude is time-periodic we can write this as a Fourier series expansion

$$a_{\alpha\beta}({k'}_x, k_x, t) = \sum_{l=-\infty}^{\infty} a^l_{\alpha\beta}({k'}_x, k_x) e^{-il\omega t}. \tag{3.45}$$

In addition, using Jacobi-Anger expansion

$$e^{iz\cos(\theta)} = \sum_{l=-\infty}^{\infty} i^l J_l(z) e^{-il\theta} \tag{3.46}$$

we can re-write the Eq.(3.44) as folllows

$$\begin{aligned} a_{\alpha\beta}({k'}_x, k_x, t) = -2\pi i\delta(\varepsilon_\beta - \varepsilon)\eta_{imp} L_x V_{{k'}_x, k_x} \sum_{l=-\infty}^{\infty} i^l J_l(g[{y'}_0 - y_0]) e^{-il\omega t} \\ \times \int_{-\infty}^{\infty} dy \ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - {y'}_0 - y) \end{aligned} \tag{3.47}$$

$$\begin{aligned} a_{\alpha\beta}({k'}_x, k_x, t) = \sum_{l=-\infty}^{\infty} -2\pi i^{l+1}\delta(\varepsilon_\beta - \varepsilon)\eta_{imp} L_x V_{{k'}_x, k_x} J_l(g[{y'}_0 - y_0]) \\ \times \int_{-\infty}^{\infty} dy \ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - {y'}_0 - y) e^{-il\omega t} \end{aligned} \tag{3.48}$$

Then we can identified the Fourier series component as

$$a^l_{\alpha\beta}(k'_x, k_x) = -2\pi i^{l+1}\delta(\varepsilon_\beta - \varepsilon)\eta_{imp}L_x V_{k'_x,k_x} J_l(g[y'_0 - y_0]) \int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y) \tag{3.49}$$

Now one can introduce the definition of the *transition probability matrix* as

$$(A_{\alpha\beta}(k'_x, k_x))_{l,l'} \equiv a^l_{\alpha\beta}(k'_x, k_x)\left[a^{l'}_{\alpha\beta}(k'_x, k_x)\right]^* \tag{3.50}$$

and this becomes

$$\begin{aligned}(A_{\alpha\beta}(k'_x, k_x))_{l,l'} &= [2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2 J_l(g[y'_0 - y_0])J_{l'}(g[y'_0 - y_0])\delta^2(\varepsilon_\beta - \varepsilon) \\ &\times \int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y) \int_{-\infty}^{\infty} d\bar{y}\ \chi_{n_\beta}(\bar{y})\chi_{n_\beta}(y_0 - y'_0 - \bar{y}).\end{aligned} \tag{3.51}$$

We can reduce these intragal into one variable and derive

$$\begin{aligned}(A_{\alpha\beta}(k'_x, k_x))_{l,l'} &= [2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2 J_l(g[y'_0 - y_0])J_{l'}(g[y'_0 - y_0])\delta^2(\varepsilon_\beta - \varepsilon) \\ &\times \left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y)\right|^2.\end{aligned} \tag{3.52}$$

Then desribing the square of the delta distribution using following procedure

$$\delta^2(\varepsilon) = \delta(\varepsilon)\delta(0) = \frac{\delta(\varepsilon)}{2\pi\hbar}\int_{-t/2}^{t/2} e^{i0\times t'/\hbar}dt' = \frac{\delta(\varepsilon)t}{2\pi\hbar} \tag{3.53}$$

one can modify our derivation in Eq. (3.51) as

$$\begin{aligned}(A_{\alpha\beta}(k'_x, k_x))_{l,l'} &= [2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2 J_l(g[y'_0 - y_0])J_{l'}(g[y'_0 - y_0])\delta(\varepsilon_\beta - \varepsilon)\frac{t}{2\pi\hbar} \\ &\times \left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y)\right|^2.\end{aligned} \tag{3.54}$$

Then performing thetime derivation of each matrix element yeild the *transition amplitude matrix* as folllows

$$\begin{aligned}\Gamma^{ll'}_{\alpha\beta}(k'_x, k_x) &\equiv \frac{d(A_{\alpha\beta}(k'_x, k_x))_{l,l'}}{dt} \\ &= \Lambda|V_{k'_x,k_x}|^2\delta(\varepsilon_\beta - \varepsilon)J_l(g[y'_0 - y_0])J_{l'}(g[y'_0 - y_0])\left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0 - y'_0 - y)\right|^2\end{aligned} \tag{3.55}$$

where

$$\Lambda \equiv \frac{2\pi\eta^2_{imp}L^2_x}{\hbar} \tag{3.56}$$

Now using defintion of y_0 given in Eq. (1.11) we can write that

$$y_0 - y'_0 = -\frac{p_{x_\alpha}}{eB} + \frac{p_{x_\beta}}{eB} = \frac{\hbar k'_x}{eB} - \frac{\hbar k_x}{eB} = \frac{\hbar}{eB}[k'_x - k_x] \tag{3.57}$$

and this leads Eq. (3.56) to

$$\begin{aligned}\Gamma^{ll'}_{\alpha\beta}(k'_x, k_x) &= \Lambda|V_{k'_x,k_x}|^2\delta(\varepsilon_\beta - \varepsilon)J_l\left(\frac{g\hbar}{eB}[k_x - k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x - k'_x]\right) \\ &\times \left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x - k_x] - y\right)\right|^2\end{aligned} \tag{3.58}$$

An impurity average of white noise potential allows to identify $\left\langle |V_{k'_x,k_x}|^2 \right\rangle = V_{imp}$ and the inverse scattering time matrix is the sum over all momentum over the transition probability matrix

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} \equiv \frac{1}{L_x}\sum_{k'_x} \langle \Gamma^{ll'}_{\alpha\beta}(k'_x,k_x)\rangle_{imp} \tag{3.59}$$

and this implies

$$\begin{aligned}\Gamma^{ll'}_{\alpha\beta}(k'_x,k_x) = \frac{\Lambda V_{imp}}{L_x}\sum_{k'_x}\delta(\varepsilon_\beta-\varepsilon)J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\\ \times\left|\int_{-\infty}^{\infty}dy\ \chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x-k_x]-y\right)\right|^2\end{aligned} \tag{3.60}$$

For the 1-dimentional case introduce the momentum continuum limit as folllows

$$\frac{1}{L_x}\sum_{k'_x} \longrightarrow \frac{1}{2\pi}\int dk'_x \tag{3.61}$$

and this leads to

$$\begin{aligned}\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} = \frac{\Lambda V_{imp}}{2\pi}\delta(\varepsilon_\beta-\varepsilon)\int_{-\infty}^{\infty}dk'_x J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\\ \times\left|\int_{-\infty}^{\infty}dy\ \chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x-k_x]-y\right)\right|^2\end{aligned} \tag{3.62}$$

Using following subtitution

$$y = \frac{\hbar\bar{k}}{eB} \longrightarrow dy = \frac{\hbar}{eB}d\bar{k} \tag{3.63}$$

we can modify above derivation as

$$\begin{aligned}\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} = \frac{\Lambda V_{imp}}{2\pi}\delta(\varepsilon_\beta-\varepsilon)\int_{-\infty}^{\infty}dk'_x J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\\ \times\left(\frac{\hbar}{eB}\right)^2\left|\int_{-\infty}^{\infty}d\bar{k}\ \chi_{n_\beta}\left(\frac{\hbar}{eB}\bar{k}\right)\chi_{n_\beta}\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2,\end{aligned} \tag{3.64}$$

and finally we can derive our expression for the *inverse scattering time matrix* for Nth Landau level (let $n_\alpha = n_\beta = N$)

$$\begin{aligned}\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{N} = \frac{\eta^2_{imp}L_x^2\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dk'_x J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\\ \times\left|\int_{-\infty}^{\infty}d\bar{k}\ \chi_{N}\left(\frac{\hbar}{eB}\bar{k}\right)\chi_{N}\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2.\end{aligned} \tag{3.65}$$

■

4 Inverse Scattering Time Analysis

We hace derived the inverse scattering time matrix element from previous section as follows

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll'} = \frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty} dk'_x J_l\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{g\hbar}{eB}[k_x-k'_x]\right) \times\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2. \tag{4.1}$$

The disorder in the system is not supposed to change the eigenenergies of the bare system, hence all off-diogonal elements of the self-energy were neglected. Thesefore we can consider only the central diagonal element ($l = l' = 0$) of the inverse scattering time matrix which has the largest contribution

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{00} = \frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty} dk'_x\; J_l^2\left(\frac{g\hbar}{eB}[k_x-k'_x]\right) \times\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2. \tag{4.2}$$

Now we can introduce a new parameter with physical meaning of scatterin-induced broadening of the Landau level as follows

$$\Gamma_N^{00}(\varepsilon,k_x) \equiv \hbar\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{00} \tag{4.3}$$

and this modify our previous expressing as

$$\Gamma_N^{00}(\varepsilon,k_x) = \frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty} dk'_x\; J_0^2\left(\frac{g\hbar}{eB}[k_x-k'_x]\right) \times\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2. \tag{4.4}$$

In addition, for the case of elastic scattering within the same Landau level, one can present the delta distribution of the energy using the same physical interpretation as follows

$$\delta(\varepsilon-\varepsilon_N) \approx \frac{1}{\pi\Gamma_N^{00}(\varepsilon,k_x)} \tag{4.5}$$

and this leads to

$$\left[\Gamma_N^{00}(\varepsilon,k_x)\right]^2 = \frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\int_{-\infty}^{\infty} dk'_x\; J_0^2\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2. \tag{4.6}$$

and

$$\Gamma_N^{00}(\varepsilon,k_x) = \left[\frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\int_{-\infty}^{\infty} dk'_x\; J_0^2\left(\frac{g\hbar}{eB}[k_x-k'_x]\right)\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N\left(\frac{\hbar}{eB}\bar{k}\right)\chi_N\left(\frac{\hbar}{eB}\left[k'_x-k_x-\bar{k}\right]\right)\right|^2\right]^{-1/2}. \tag{4.7}$$

This can be write in more compact form as follows

$$\Gamma_N^{00}(\varepsilon,k_x) = \left[\frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\int_{-\infty}^{\infty} dk'_x\; J_0^2(g\sigma[k_x-k'_x])\left|\int_{-\infty}^{\infty} d\bar{k}\,\chi_N(\sigma\bar{k})\chi_N(\sigma\left[k'_x-k_x-\bar{k}\right])\right|^2\right]^{-1/2} \tag{4.8}$$

where

$$g = \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2-\omega^2)} \qquad \sigma = \frac{\hbar}{eB} \tag{4.9}$$

and

$$\chi_N(x) = \frac{\sqrt{\kappa}}{\sqrt{2^N N!\sqrt{\pi}}} \exp\left(-\frac{\kappa^2 x^2}{2}\right)\mathcal{H}_N(\kappa x) \quad \text{with} \quad \kappa \equiv \sqrt{\frac{m_e\omega_0}{\hbar}}. \tag{4.10}$$

Using above definition we can identify Gauss-Hermite functions ($\tilde{\chi}_N$) and this can re-write as

$$\Gamma_N^{00}(\varepsilon, k_x) = \left[\frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\int_{-\infty}^{\infty} dk'_x \; J_0^2(g\sigma[k_x - k'_x])\left|\int_{-\infty}^{\infty} d\bar{k} \; \tilde{\chi}_N(\sigma\kappa\bar{k})\tilde{\chi}_N(\sigma\kappa[k'_x - k_x - \bar{k}])\right|^2\right]^{-1/2} \tag{4.11}$$

where

$$\tilde{\chi}_N(x) = \frac{1}{\sqrt{2^N N!\sqrt{\pi}}}\exp\left(-\frac{x^2}{2}\right)\mathcal{H}_N(x) \tag{4.12}$$

and this will be simplified to

$$\Gamma_N^{00}(\varepsilon, k_x) = \eta\left[\int_{-\infty}^{\infty} dk_1 \; J_0^2(\lambda_1[k_x - k_1])\left|\int_{-\infty}^{\infty} dk_2 \; \tilde{\chi}_N(\lambda_2 k_2)\tilde{\chi}_N(\lambda_2[k_1 - k_2 - k_x])\right|^2\right]^{-1/2} \tag{4.13}$$

where

$$\eta = \left[\frac{\eta_{imp}^2 L_x^2 \hbar V_{imp}}{(eB)^2}\right]^{1/2} \quad , \quad \lambda_1 = g\sigma \quad , \quad \lambda_2 = \sigma\kappa. \tag{4.14}$$

Now we can analyze the behaviour of the normalized N-th Landau level for broadening as follows

$$\Lambda_N(k_x) \equiv \frac{(1/\tau)_N^{00}}{(1/\tau)_0^{00}\big|_{E=0}} = \frac{\Gamma_N^{00}(\varepsilon, k_x)}{\Gamma_0^{00}(\varepsilon, k_x)\big|_{E=0}} \tag{4.15}$$

and this will be

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} dk_1 \; J_0^2(\lambda_1[k_x - k_1])\left|\int_{-\infty}^{\infty} dk_2 \; \tilde{\chi}_N(\lambda_2 k_2)\tilde{\chi}_N(\lambda_2[k_1 - k_2 - k_x])\right|^2}{\int_{-\infty}^{\infty} dk_1 \left|\int_{-\infty}^{\infty} dk_2 \; \tilde{\chi}_0(\lambda_2 k_2)\tilde{\chi}_0(\lambda_2[k_1 - k_2 - k_x])\right|^2}\right]^{1/2}. \tag{4.16}$$

Lets calculate these constants for GaAs-based quantum well with following given physical constants and system external paramters.

Physical constant name	**Symbol**	**Value in SI-units**
Electron charge	e	1.602×10^{-19} C
Electron mass	m	9.109×10^{-31} kg
Reduced Planck's constant	$\hbar$	1.054×10^{-34} kgm^{2}s^{-1}
Speed of light	c	2.998×10^{8} ms^{-1}
Vacuum permittivity	ε_0	8.854×10^{-12} C^{2}s^{2}kg^{-1}m^{-3}

Table 1: Physical constant values in SI-units

External paramter name	**Symbol**	**Value in SI-units**
Average intensity	I	$\tilde{I} \times 100$ W/cm^2 $= \tilde{I} \times 10^6$ W/m^2
Magnetic field	B	1.2 T
Driving frequency	ω	2×10^{12} rads^{-1}
Effective mass	m_e	$0.071 \times m = 6.467 \times 10^{-32}$ kg

Table 2: System external paramter values. ($\tilde{I}$ is a dimentionless value.)

Therfore we can calculate following values

$$\omega_0 = \frac{eB}{m_e} = 2.97265 \times 10^{12}\ \mathrm{s}^{-1} \tag{4.17}$$

$$\sigma = \frac{\hbar}{eB} = 5.4851 \times 10^{-16}\ \mathrm{m}^2 \tag{4.18}$$

$$E = \sqrt{\frac{2I}{c\varepsilon_0}} \tag{4.19}$$

$$g = \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)} = \frac{e\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)}\sqrt{\frac{2I}{c\varepsilon_0}} = 3.80958 \times 10^7 \times \sqrt{\tilde{I}}\ \mathrm{m}^{-1} \tag{4.20}$$

$$\kappa = \sqrt{\frac{m_e\omega_0}{\hbar}} = 4.2698 \times 10^7\ \mathrm{m}^{-1} \tag{4.21}$$

Since

$$\lambda_1 = g\sigma = 2.08959 \times 10^{-8} \times \sqrt{\tilde{I}}\ \mathrm{m} \quad \text{and} \quad \lambda_2 = \kappa\sigma = 2.34203 \times 10^{-8}\ \mathrm{m} \tag{4.22}$$

we can choose our integral dummy variables k_1, k_2 and momentum variable k_x are in one range as follows

$$k_x, k_1, k_2 \approx 10^8\ \mathrm{m}^{-1} \tag{4.23}$$

Using above values we can re-write the normalized energy broadening of the N-th Landau level as

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2\Big(2.090\sqrt{\tilde{I}} \times [k_x - k_1]\Big)\Big|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N(2.342 \times [k_1 - k_2 - k_x])\Big|^2}{\int_{-\infty}^{\infty} dk_1 \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0(2.342 \times [k_1 - k_2 - k_x])\right|^2}\right]^{1/2}. \tag{4.24}$$

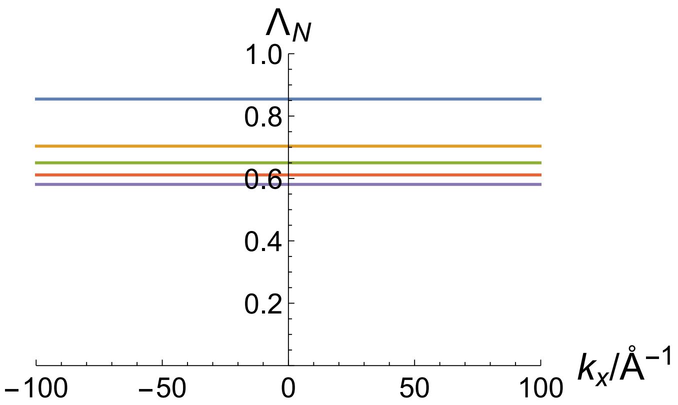

Figure 6: Normalized energy band broadening with k_x for different Landau levels ($N = 0, 1, 2, 3, 4$) for $\tilde{I} = 1$.

To check the variability of this expression with k_x value we check it with a constant intensity. Therefore let $\tilde{I} = 1$ and we can re-write above as

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2(2.090 \times [k_x - k_1])\Big|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N(2.342 \times [k_1 - k_2 - k_x])\Big|^2}{\int_{-\infty}^{\infty} dk_1 \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0(2.342 \times [k_1 - k_2 - k_x])\right|^2}\right]^{1/2}. \tag{4.25}$$

then introduce new variable

$$k_1 - k_x = \tilde{k} \quad \longrightarrow \quad dk_1 = d\tilde{k} \tag{4.26}$$

and since k_1 can vary between all the range we can modify our Eq. (4.25) as follows

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} d\tilde{k}\ J_0^2\Big(2.090 \times \tilde{k}\Big)\left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N\Big(2.342 \times \Big[\tilde{k} - k_2\Big]\Big)\right|^2}{\int_{-\infty}^{\infty} d\tilde{k}\ \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0\Big(2.342 \times \Big[\tilde{k} - k_2\Big]\Big)\right|^2} \right]^{1/2} \tag{4.27}$$

and letting $\tilde{k} = k_1$ we can find that this is do not depend on the value of k_x. Therefore

$$\Lambda_N = \left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2(2.090 \times k_1)\left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N(2.342 \times [k_1 - k_2])\right|^2}{\int_{-\infty}^{\infty} dk_1\ \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0(2.342 \times [k_1 - k_2])\right|^2} \right]^{1/2} \tag{4.28}$$

Now we can draw the values of Λ_N agaist k_x to compare the differese of each Landau level's normalized energy band broadening as given in Figure 6.
Now let's check how the Λ_N value change with the applied dressing field's intensity. Using following equation we can identify Λ_N dependency on dressing field's intensity as given in Figure 7.

$$\Lambda_N = \left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2\Big(2.090\sqrt{\tilde{I}} \times k_1\Big)\left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N(2.342 \times [k_1 - k_2])\right|^2}{\int_{-\infty}^{\infty} dk_1\ \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0(2.342 \times [k_1 - k_2])\right|^2} \right]^{1/2}. \tag{4.29}$$

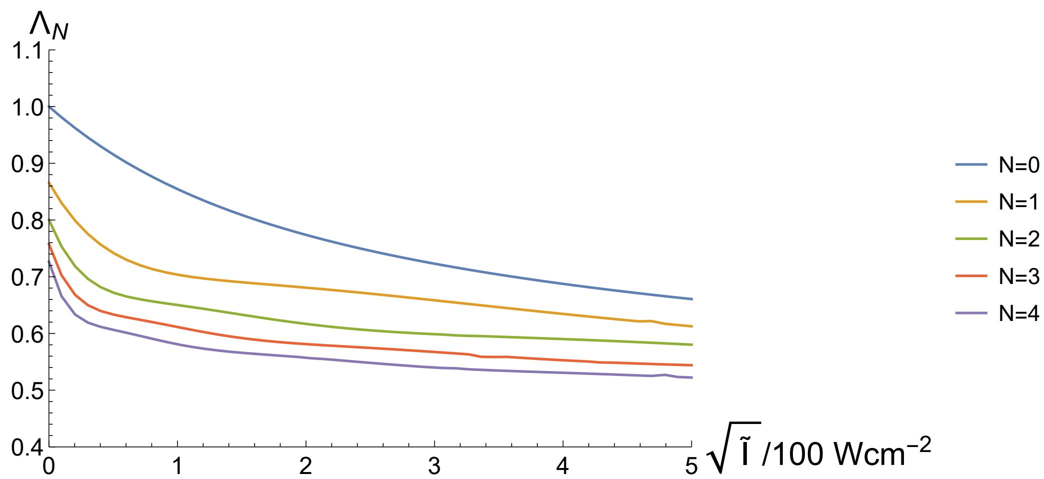

Figure 7: Normalized energy band broadening with k_x for different Landau levels ($N = 0, 1, 2, 3, 4$) for $\tilde{I} = 1$.

Here we can identify that we can change the each Landau level normalized energy broadening value using apllied electromagnetic field. When the applied field's intensity increase the energy broadening gets reduced which make changes in conductivity.
To make comparison we can select the experiment parameters from previous analysis on transverse conductivity in [*Ref: Akira Endo and Naomichi Hantno]. In this study we have assumed that the broadening of Landau levels are same and the value for Γ_N non-existing dressing field is 0.24 me$V$. Therefore in our study for the comparison we can assume that the least Landau level broadening has this value.

$$\Gamma_{N=0}^{00}\big|_{E=0} = 0.24 \text{ meV} \tag{4.30}$$

Then using the calculated normalized broadening relations for each Landu levels, we can evaluate the energy band broadening values as follows

$$\Gamma_N^{00} = \Lambda_N \times \Gamma_{N=0}^{00}\big|_{E=0}. \tag{4.31}$$

Therfore, the energy band broadening for Landau level with dressing field can be calculate as

$$\Gamma_N^{00} = \Lambda_N \times \Gamma_{N=0}^{00}\big|_{E=0}. \tag{4.32}$$

and

$$\Gamma_N^{00} = 0.24 \times \Lambda_N \text{ meV}. \tag{4.33}$$

$$\Gamma_N^{00}(\tilde{I}) = 0.24 \times \left[\frac{\int_{-\infty}^{\infty} dk_1 \; J_0^2\left(2.090\sqrt{\tilde{I}} \times k_1\right) \left| \int_{-\infty}^{\infty} dk_2 \; \tilde{\chi}_N(2.342 \times k_2)\tilde{\chi}_N(2.342 \times [k_1 - k_2]) \right|^2}{\int_{-\infty}^{\infty} dk_1 \left| \int_{-\infty}^{\infty} dk_2 \; \tilde{\chi}_0(2.342 \times k_2)\tilde{\chi}_0(2.342 \times [k_1 - k_2]) \right|^2} \right]^{1/2} \text{meV}. \tag{4.34}$$

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5 Current Operator in Landau Levels

Now consider about the current density operator for Nth Landau level. Since we have already found the extact solution for our time depenedent Hamiltonian and we have identify them as Floquet states with quesi energies. From these solutions we can identify the *Floquet modes* as given in Eq. (2.15) and using quantum numbers we can represent those states as follows

$$|\phi_\alpha\rangle = |\phi_{n,m}\rangle \equiv |n, k_x\rangle \quad \text{where} \quad k_x = m\frac{2\pi}{L_x} \tag{5.1}$$

Using above complete set of eigenstates of Floquet Hamiltonian we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'} = \langle n, k_x| \, \hat{\mathbf{j}} \, |n', k'_x\rangle \tag{5.2}$$

where particle current operator for this system will be

$$\hat{\mathbf{j}} = \frac{1}{m}\Big(\hat{\mathbf{P}} - e[\mathbf{A}_s + \mathbf{A}_d(t)]\Big). \tag{5.3}$$

Let's consider the transverse conductivity in x direction and we can identify that x directional current operator as

$$\hat{j}_x = \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right). \tag{5.4}$$

Now we can calculate the matrix elements of x directional current operator's matrix in Floquet mode basis as

$$(j_x)_{nm,n'm'} = \langle n, k_x| \, \hat{j}_x \, |n', k'_x\rangle = \langle n, k_x| \, \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right) |n', k'_x\rangle \tag{5.5}$$

and we can evaluate these using Floquet modes derived in Eq.(2.15) as follows

$$\begin{aligned}(j_x)_{nm,n'm'} = &\int dx \int dy \, \frac{1}{\sqrt{L_x}}\chi_n\big(y - y_0 - \zeta(t)\big)\exp(-ik_x x) \\ &\times \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right)\frac{1}{\sqrt{L_x}}\chi_{n'}\big(y - y_0 - \zeta(t)\big)\exp(ik'_x x)\end{aligned} \tag{5.6}$$

and this can be simplified as

$$\begin{aligned}(j_x)_{nm,n'm'} = &\frac{1}{mL_x}\int dx \exp(-i(k_x - k'_x)x) \int dy \; \chi_n\big(y - y_0 - \zeta(t)\big) \\ &\times (\hbar k'_x + eBy)\chi_{n'}\big(y - y_0 - \zeta(t)\big)\end{aligned} \tag{5.7}$$

and

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int dy \; (\hbar k'_x + eBy)\chi_n\big(y - y_0 - \zeta(t)\big)\chi_{n'}\big(y - y_0 - \zeta(t)\big). \tag{5.8}$$

Now let $y - y_0 - \zeta(t) = \bar{y}$ and we will get

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int d\bar{y} \; (\hbar k'_x + eB\bar{y} + eBy_0 + eB\zeta(t))\chi_n(\bar{y})\chi_{n'}(\bar{y}). \tag{5.9}$$

using definition of y_0 given in Eq. (1.11) this will be modiofied to

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int d\bar{y} \; (\hbar k'_x + eB\bar{y} - \hbar k'_x + eB\zeta(t))\chi_n(\bar{y})\chi_{n'}(\bar{y}) \tag{5.10}$$

and using integral indentities of Gauss-Hermite functions

$$\int dy \; \chi_n(y)\chi_{n'}(y) = \delta_{n',n} \tag{5.11}$$

$$\int dy\; y\chi_n(y)\chi_{n'}(y) = \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{5.12}$$

this becomes

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}eB\left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \zeta(t)\delta_{n',n}\right] \tag{5.13}$$

Due to complexity we can only consider the constant contribution and we allows only the one-cycle averaged current flow and then we can derive the $s = 0$ components of the Fourier series as

$$(j^x_{s=0})_{nm,n'm'} = \frac{1}{T}\int_0^T dt\; \frac{1}{m}\delta_{k_x,k'_x}eB\left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \frac{eE}{m(\omega_0^2-\omega^2)}\sin(\omega t)\delta_{n',n}\right] \tag{5.14}$$

and this can be evaluate and get

$$(j^x_{s=0})_{nm,n'm'} = \frac{eB}{m}\delta_{k_x,k'_x}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{5.15}$$

For electric current operator we can introduce the electron's charge and effective mass

$$(j^x_{s=0})_{nm,n'm'} = \frac{e^2B}{m_e}\delta_{k_x,k'_x}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{5.16}$$

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Next we can consider the transverse conductivity in y direction and we can identify that y directional current operator as

$$\hat{j}_y = \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right). \tag{5.17}$$

Now we can calculate the matrix elements of y directional current operator's matrix in Floquet mode basis as

$$(j_y)_{nm,n'm'} = \langle n,k_x|\,\hat{j}_y\,|n',k'_x\rangle = \langle n,k_x|\,\frac{-1}{m}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n',k'_x\rangle \tag{5.18}$$

and we can evaluate these using Floquet modes derived in Eq.(2.15) as follows

$$\begin{aligned}(j_y)_{nm,n'm'} &= \int dx\int dy\;\frac{1}{\sqrt{L_x}}\chi_n\big(y-y_0-\zeta(t)\big)\exp(-ik_xx)\\ &\times\frac{-1}{m}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)\frac{1}{\sqrt{L_x}}\chi_{n'}\big(y-y_0-\zeta(t)\big)\exp(ik'_xx)\end{aligned} \tag{5.19}$$

then introducing new variable

$$y - y_0 - \zeta(t) = \bar{y} \quad\longrightarrow\quad dy = d\bar{y} \tag{5.20}$$

and follwing identity [*Ref: Appendix A Hermite fucntions]

$$\frac{\partial\chi_n(y)}{\partial y} = -\sqrt{\frac{n+1}{2}}\chi_{n+1}(y) + \sqrt{\frac{n}{2}}\chi_{n-1}(y) \tag{5.21}$$

above expression can be simplified as previous case (let $y = \bar{y}$)

$$(j_y)_{nm,n'm'} = \frac{-1}{m}\delta_{k_x,k'_x}\int dy\;\chi_n(y)\left[-i\hbar\left(\sqrt{\frac{n'+1}{2}}\chi_{n'+1}(y) - \sqrt{\frac{n'}{2}}\chi_{n'-1}(y)\right) + \frac{eE}{\omega}\cos(\omega t)\chi_{n'}(y)\right]. \tag{5.22}$$

Then considering Gauss-Hermite integral indentities we can derive that

$$(j_y)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k_x'}\left[i\hbar\left(\sqrt{\frac{n'+1}{2}}\delta_{n,n'+1} - \sqrt{\frac{n'}{2}}\delta_{n,n'-1}\right) - \frac{eE}{\omega}\cos(\omega t)\delta_{n,n'}\right] \tag{5.23}$$

Due to complexity we can only consider the constant contribution and we allows only the one-cycle averaged current flow and then we can derive the $s = 0$ components of the Fourier series as

$$(j^y_{s=0})_{nm,n'm'} = \frac{1}{T}\int_0^T dt\ \frac{1}{m}\delta_{k_x,k_x'}\left[i\hbar\left(\sqrt{\frac{n'+1}{2}}\delta_{n,n'+1} - \sqrt{\frac{n'}{2}}\delta_{n,n'-1}\right) - \frac{eE}{\omega}\cos(\omega t)\delta_{n,n'}\right] \tag{5.24}$$

and this can be evaluate and get

$$(j^y_{s=0})_{nm,n'm'} = \frac{i\hbar}{m}\delta_{k_x,k_x'}\left[\sqrt{\frac{n'+1}{2}}\delta_{n,n'+1} - \sqrt{\frac{n'}{2}}\delta_{n,n'-1}\right] \tag{5.25}$$

and this can be re-write as

$$(j^y_{s=0})_{nm,n'm'} = \frac{i\hbar}{m}\delta_{k_x,k_x'}\left[\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right] \tag{5.26}$$

For electric current operator we can introduce the electron's charge and effective mass

$$(j^y_{s=0})_{nm,n'm'} = \frac{ie\hbar}{m}\delta_{k_x,k_x'}\left[\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right] \tag{5.27}$$

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6 Floquet-Drude Conductivity in Quantum Hall Systems

The general expression for the conductivity [*Ref: Martin Wackerl Thesis 1.250] with the disorder averaging can be represent as follows

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\\ \times\,\mathrm{tr}\,[j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon)-\mathbf{G}_0^a(\varepsilon))].\end{aligned} \tag{6.1}$$

where j_0^x and $\mathbf{G}^{r,a}(\varepsilon)$ are x directional current operator matrix and white noise disorder averaged Green function matrix respectvely defined against to the *Floquet modes* of the system. Here we have assumed that only $s=0$ Fourier component of the current operator is contributing to the conductivity.
Now this can be expand in off resonant regime ($\omega\tau_0 \gg 1$)using only central entry Fourier components ($l=l'=0$) of *Floquet modes* mentioned in Eq. (5.1) as

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\\ \times\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\langle n,k_x|j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon)-\mathbf{G}_0^a(\varepsilon))|n,k_x\rangle\end{aligned} \tag{6.2}$$

and one can evaluate these matrix elements as follows

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\frac{1}{{L_x}^3}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3}\\ \times\ \langle n,k_x|j_0^x|n_1,k_{x1}\rangle\ \langle n_1,k_{x1}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n_2,k_{x2}\rangle\\ \times\ \langle n_2,k_{x2}|j_0^x|n_3,k_{x3}\rangle\ \langle n_3,k_{x3}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n,k_x\rangle\end{aligned} \tag{6.3}$$

Since we can diagonalize the impurity averaged Green's function using unitary trasnformation ($\mathbf{T}=|n,k_x\rangle$) [*Ref: Martin Wackerl - Paper] and we can evaluate the matrix element of differece between retarded and advanced Green's function as follows [*Ref: My report 2.535]

$$\langle n_1,k_{x1}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n_2,k_{x2}\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{6.4}$$

and

$$\langle n_3,k_{x3}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n,k_x\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{6.5}$$

Then appying the results we derived in previous section (6.17) we can calculate the conductivity

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\frac{1}{{V_{k_x}}^3}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3}\\ \times\frac{e^2B}{m_e}\delta_{k_x,k_{x1}}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]\\ \times\frac{e^2B}{m_e}\delta_{k_{x2},k_{x3}}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n_3,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n_3,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]\end{aligned} \tag{6.6}$$

and this will be modified to

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\sum_{n_1,n_2}$$
$$\times\frac{e^2B}{m_e}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]$$
$$\times\frac{e^2B}{m_e}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{6.7}$$

and the only non-zero term would be

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)$$
$$\times\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)_{\varepsilon_{n+1}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})_{\varepsilon_{n+1}}\Big]^2}\right]\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)_{\varepsilon_{n}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})_{\varepsilon_{n}}\Big]^2}\right] \tag{6.8}$$

■

Then using the following identity derived in [*Ref: My report 2.509]

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_{ll} = -2\mathrm{Im}\Big[\Big(\mathbf{T}^\dagger\sum^r\mathbf{T}\Big)_\varepsilon\Big]_{ll} \tag{6.9}$$

using central element of the inverse scattering time matrix we can modify our result as

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)$$
$$\times\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n+1},k_x)}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n+1},k_x)}\Big)^2}\right]\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n},k_x)}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n},k_x)}\Big)^2}\right] \tag{6.10}$$

We have identited that the inverse scattering time matrix's central element is not k_x dependent we can get the sum over all available momentum space in x direction. However by considering the condition that the center of the force of the oscillator y_0 must physically liw within the system $-L_y/2 < y_0 < L_y/2$, one can derive that

$$-\frac{m_e\omega_0 Ly}{2\hbar}\leq k_x\leq\frac{m_e\omega_0 Ly}{2\hbar} \tag{6.11}$$

and we can derive that

$$\frac{1}{V_{k_x}}\sum_{k_x} = \frac{m_e\omega_0 Ly}{\hbar V_{k_x}} = 1 \tag{6.12}$$

Thefore Eq. (6.10) modified to

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2\omega_0^2}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\sum_{n}(n+1)$$
$$\times\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n+1})}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n+1})}\Big)^2}\right]\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n})}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n})}\Big)^2}\right] \tag{6.13}$$

Then using Fermi-Dirac distribution as our partical distribution function (f) for this system

$$f(\varepsilon) = \frac{1}{[\exp(\varepsilon-\varepsilon_F)/k_BT]+1} \tag{6.14}$$

where k_B is Botlzmann constant, T is absolute tempurature and ε_F is Fermi energy of the system. Using above distribution, for extreamly low tempuratures we can appromixate that

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F) \tag{6.15}$$

and this will mpre simplify our derivation of conductivity as

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2\omega_0^2}{4\pi\hbar A}\sum_n (n+1)\left[\frac{\left(\frac{1}{\tau(\varepsilon_{n+1})}\right)}{\left(\frac{1}{\hbar}\varepsilon_F - \frac{1}{\hbar}\varepsilon_{n+1}\right)^2 + \left(\frac{1}{2\tau(\varepsilon_{n+1})}\right)^2}\right]\left[\frac{\left(\frac{1}{\tau(\varepsilon_n)}\right)}{\left(\frac{1}{\hbar}\varepsilon_F - \frac{1}{\hbar}\varepsilon_n\right)^2 + \left(\frac{1}{2\tau(\varepsilon_n)}\right)^2}\right] \tag{6.16}$$

Now introduce a new paramter with a physical meaning od scattering-induced broading of the Landau level as follows

$$\Gamma_n \equiv \Gamma(\varepsilon_n) \equiv \left(\frac{\hbar}{2\tau(\varepsilon_n)}\right) \tag{6.17}$$

and then we can re-write Eq. (6.16) as follows

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2(\hbar\omega_0)^2}{\pi\hbar A}\sum_n (n+1)\left[\frac{\Gamma(\varepsilon_{n+1})}{(\varepsilon_F - \varepsilon_{n+1})^2 + \Gamma^2(\varepsilon_{n+1})}\right]\left[\frac{\Gamma(\varepsilon_n)}{(\varepsilon_F - \varepsilon_n)^2 + \Gamma^2(\varepsilon_n)}\right] \tag{6.18}$$

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2(\hbar\omega_0)^2}{\pi\hbar A}\sum_n (n+1)\left[\frac{\Gamma_{n+1}}{(\varepsilon_F - \varepsilon_{n+1})^2 + \Gamma_{n+1}^2}\right]\left[\frac{\Gamma_n}{(\varepsilon_F - \varepsilon_n)^2 + \Gamma_n^2}\right] \tag{6.19}$$

Now use new dimentionless paramters

$$X_F \equiv \frac{\varepsilon_F}{\hbar\omega_0} - \frac{1}{2} \tag{6.20}$$

and

$$\gamma_n \equiv \frac{\Gamma_n}{\hbar\omega_0}. \tag{6.21}$$

Therefore the Eq. (6.19) leads to

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2}{\hbar}\frac{1}{\pi A}\sum_n (n+1)\left[\frac{\gamma_{n+1}}{(X_F - n - 1)^2 + \gamma_{n+1}^2}\right]\left[\frac{\gamma_n}{(X_F - n)^2 + \gamma_n^2}\right] \tag{6.22}$$

and

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2}{\hbar}\frac{1}{\pi A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1 + \left(\frac{X_F - n - 1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1 + \left(\frac{X_F - n}{\gamma_n}\right)^2}\right] \tag{6.23}$$

■

Same as above derivation we can derive the transverse conductivity in y direction by using the current operator derived in Eq. (5.27) as follows

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{yy}(0,\omega)] = \frac{1}{4\pi\hbar A}\frac{e^2\hbar^2}{m^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial \varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_n -(n+1)$$
$$\times\left[\frac{2i\mathrm{Im}\left(\mathbf{T}^\dagger \sum^r \mathbf{T}\right)_{\varepsilon_{n+1}}}{\left(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n+1}\right)^2 + \left[\mathrm{Im}(\mathbf{T}^\dagger \sum^r \mathbf{T})_{\varepsilon_{n+1}}\right]^2}\right]\left[\frac{2i\mathrm{Im}\left(\mathbf{T}^\dagger \sum^r \mathbf{T}\right)_{\varepsilon_n}}{\left(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_n\right)^2 + \left[\mathrm{Im}(\mathbf{T}^\dagger \sum^r \mathbf{T})_{\varepsilon_n}\right]^2}\right] \tag{6.24}$$

and same as above derivation this can be simplified into

$$\lim_{\omega\to 0} \mathrm{Re}[\sigma^{yy}(0,\omega)] = \frac{e^2}{\hbar}\frac{1}{\pi A}\frac{1}{e^2B^2}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1 + \left(\frac{X_F - n - 1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1 + \left(\frac{X_F - n}{\gamma_n}\right)^2}\right] \tag{6.25}$$

■

7 Manipulate Conductivity in Quantum Hall System

Now using the relations derived in Eq. (4.34) and Eq. (??) we can derive that

$$\Gamma_n = \left(\frac{\hbar}{2\tau(\varepsilon_n)}\right) = \frac{1}{2}\Gamma_N^{00}(\tilde{I}) \tag{7.1}$$

and this will be

$$\Gamma_n(\tilde{I}) = 0.12\times\left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2\left(2.090\sqrt{\tilde{I}}\times k_1\right)\left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_N(2.342\times k_2)\tilde{\chi}_N(2.342\times[k_1-k_2])\right|^2}{\int_{-\infty}^{\infty} dk_1\ \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342\times k_2)\tilde{\chi}_0(2.342\times[k_1-k_2])\right|^2}\right]^{1/2} \text{meV} \tag{7.2}$$

In addition we can calculate the cyclotron energy as

$$\hbar\omega_0 = 1.95663 \text{ meV} \tag{7.3}$$

and

$$\gamma_n = \frac{\Gamma_n}{\hbar\omega_0} = 0.06133\times\Lambda_n(\tilde{I}) \approx 0.061\Lambda_n(\tilde{I}) \tag{7.4}$$

Now we can use this into our conductivity expression derived in Eq. (6.23) and present the normalized transverse conductivity as a function of fermi energy and intensity of the dressing field

$$\sigma^{xx}(X_F,\tilde{I}) = \sum_n \frac{(n+1)}{0.0037\Lambda_n\Lambda_{n+1}}\left[\frac{1}{1+\left(\frac{X_F-n-1}{0.06\Lambda_n}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{0.06\Lambda_{n+1}}\right)^2}\right] \tag{7.5}$$

where

$$\Lambda_n = \left[\frac{\int_{-\infty}^{\infty} dk_1\ J_0^2\left(2.090\sqrt{\tilde{I}}\times k_1\right)\left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_n(2.342\times k_2)\tilde{\chi}_n(2.342\times[k_1-k_2])\right|^2}{\int_{-\infty}^{\infty} dk_1\ \left|\int_{-\infty}^{\infty} dk_2\ \tilde{\chi}_0(2.342\times k_2)\tilde{\chi}_0(2.342\times[k_1-k_2])\right|^2}\right]^{1/2}. \tag{7.6}$$

■

Now using Eq. (7.6) we can calculate the normalized broadening of each Landau levels as given in Table 3.

$\tilde{I}$	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
0	1.0000	0.8660	0.8004	0.7578	0.7266	0.7021	0.6821	0.6653	0.6507	0.6380	0.6267
1	0.8546	0.7037	0.6502	0.6114	0.5810	0.5584	0.5416	0.5280	0.5160	0.5050	0.4948
4	0.6875	0.6345	0.5902	0.5528	0.5307	0.5118	0.4958	0.4840	0.4730	0.4629	0.4547
9	0.5936	0.5539	0.5333	0.5180	0.5006	0.4812	0.4685	0.4564	0.4469	0.4377	0.4305

Table 3: Λ_n values for each Landau level by changing applied dressing field intensity $(\tilde{I})$.

Using above values we can analyse the changes can be done to the transverse conductivity using apllied dressing field. As given in Figure 8 and 9 we can manipulate the conductivity using external dressing field. When the applied field's intensity increase the broadening of energy bands of Landau levels get reduced the conductivity also get decrease all the regions except the peak point of the energy band. Using this manipulation we can filter the conductivity which is change with the Fermi level. Since Fermi level can be change with the applied gate voltage of the material this can be used as a 2D switch for optoelectonic applications. Using the applied dressing field we can fine tune the switching mechanism.

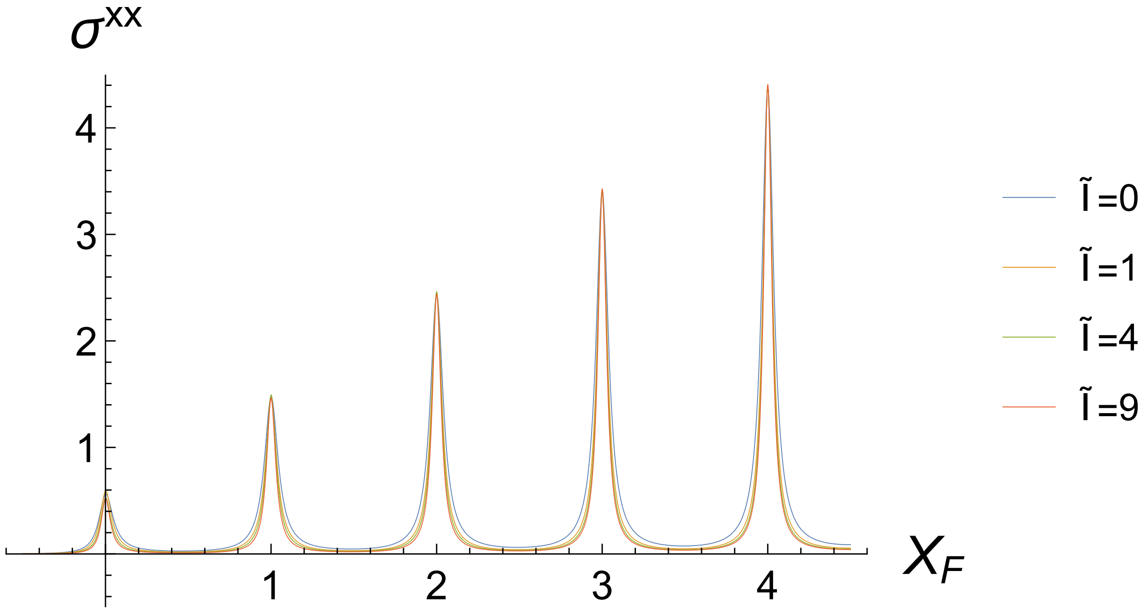

Figure 8: Normalized transverse conductivity against Fermi level (X_F) with different intensities ($\tilde{I}$) of dressing field.

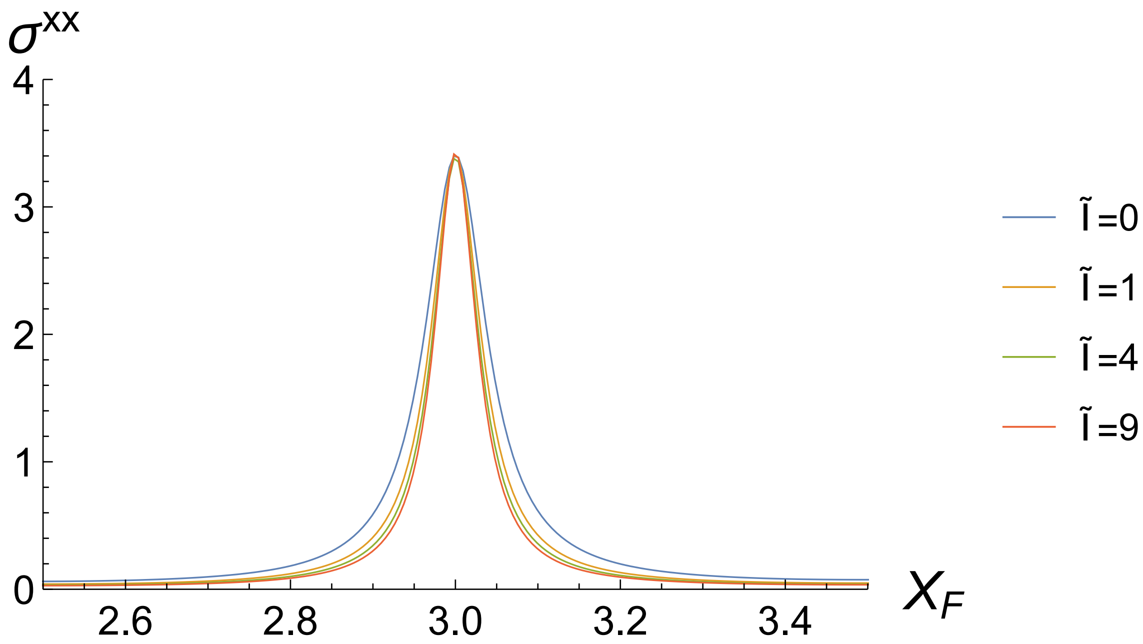

Figure 9: 3rd Landau level's normalized transverse conductivity against Fermi level (X_F) with different intensities ($\tilde{I}$) of dressing field.