Magnetic propeties of a two dimentional electron gas strongly coupled to lights

K.Dini, O.V. Kibis and I.A. Shelykh

April 6, 2021

1 Schrödinger problem for Landau levels in dressed 2DEG

Our analysis is consider on 2 dimentional electronic gas which has distrubuted in (x, y) plane in configuration space. We are going to examine the properties of 2DEG with stationary magnetic field

$$\mathbf{B} = (0, 0, B)^T \tag{1.1}$$

which directed on z axis and a linearly y-polarized strong electomagnetic wave (dressing field) with electric field given by

$$\mathbf{E} = (0, E\sin(\omega t), 0)^T \tag{1.2}$$

which also propagate in z direction. Here B and E represent the amplitude of the stationary magnetic field and electric field of dressing field.

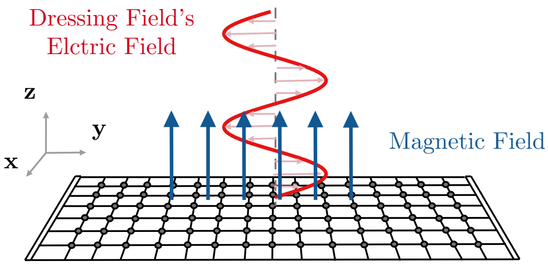

Figure 1: Stationary magnetic filed (blue color) and Strong EM wave (red color) applied to the 2DEG.

Using Landau gauge for the stationary magnetic field we can represent it using vector potential as

$$\mathbf{A}_s = (-By, 0, 0)^T \tag{1.3}$$

and choosing Coulomb gauge the dressing field can be present as the following vector potential

$$\mathbf{A}_d(t) = (0, [E/\omega]\cos(\omega t), 0)^T. \tag{1.4}$$

Now the Hamiltonian of an electron in 2DEG can be reads as

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[\hat{\mathbf{p}} - e\big(\mathbf{A}_s + \mathbf{A}_d(t)\big)\Big]^2 \tag{1.5}$$

where m_e is the effective mass of the electron and e is the magnitude (without considering the sign of the charge) of the electron charge. This can be simplified to

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(\hat{p}_x + eBy)\mathbf{e}_x + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))\mathbf{e}_y\Big]^2 \tag{1.6}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along x and y directions respectively. Moreover,

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(\hat{p}_x + eBy)^2 + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))^2\Big] \tag{1.7}$$

Since $[\hat{H}_e(t), \hat{p}_x] = 0$ both operators share same eigenvalue and eigen functions which are free electron wave functions. Therefore we can modify the Hamiltonian as follows

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[(p_x + eBy)^2 + (\hat{p}_y - \frac{eE}{\omega}\cos(\omega t))^2\Big]. \tag{1.8}$$

Using momentum operator definition

$$\hat{p}_y = -i\hbar\frac{\partial}{\partial y} \tag{1.9}$$

we can modify Eq. (1.8) as

$$\begin{aligned}\hat{H}_e(t) &= \frac{1}{2m_e}\Big[(p_x + eBy)^2 + \Big(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\Big)^2\Big] \\ &= \frac{1}{2m_e}\Big[(p_x + eBy)^2 + \Big(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\Big)^2\Big].\end{aligned} \tag{1.10}$$

Define the *center of the cyclotron orbit* along y axis as

$$y_0 \equiv \frac{-p_x}{eB} \tag{1.11}$$

and the *cyclotron frequency* as

$$\omega_0 \equiv \frac{eB}{m_e}. \tag{1.12}$$

Then the Hamiltonian will leads to

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y - y_0)^2 + \frac{1}{2m_e}\Big(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\Big)^2 \tag{1.13}$$

$$\begin{aligned}\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y - y_0)^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + i\hbar\frac{\partial}{\partial y}\Big[\frac{eE}{\omega}\cos(\omega t)\Big] \\ + \frac{i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big)\end{aligned} \tag{1.14}$$

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}(y - y_0)^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big). \tag{1.15}$$

Let

$$(y - y_0) \to y \tag{1.16}$$

and then this becomes

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}y^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big). \tag{1.17}$$

Now assume that the solution for the time-dependent schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}_e(t)\psi \tag{1.18}$$

can be represent by the following form

$$\psi(\mathbf{r}, t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieE(y - y_0)}{\hbar\omega}\cos(\omega t)\right)\phi(y - y_0, t). \tag{1.19}$$

Using the same subtution from Eq. (1.16) this becomes

$$\psi(x, y, t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieEy}{\hbar\omega}\cos(\omega t)\right)\phi(y, t). \tag{1.20}$$

Defining

$$\varphi(x, y, t) \equiv \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieEy}{\hbar\omega}\cos(\omega t)\right) \tag{1.21}$$

we can simply the the Eq. (1.20) as

$$\psi(x,y,t) = \varphi(x,y,t)\phi(y,t). \tag{1.22}$$

Let's subtitue Eq. (1.20) and Eq. (1.17) into Eq. (1.18) and we can observe that

$$\begin{aligned}\text{L.H.S} &= i\hbar\frac{\partial\psi}{\partial t} = i\hbar\Big(\frac{\partial\varphi}{\partial t}\phi + \frac{\partial\phi}{\partial t}\varphi\Big) = i\hbar\Big(\Big[\frac{-ieEy}{\hbar}\sin(\omega t)\Big]\varphi\phi + \varphi\frac{\partial\phi}{\partial t}\Big)\\ &= \big[eEy\sin(\omega t)\big]\varphi\phi + i\hbar\varphi\frac{\partial\phi}{\partial t}\end{aligned} \tag{1.23}$$

and

$$\begin{aligned}\text{R.H.S} &= \hat{H}_e(t)\psi\\ &= \Big[\frac{m_e\omega_0^2}{2}y^2 + \frac{1}{2m_e}\Big(-\hbar^2\frac{\partial^2}{\partial y^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial y} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Big)\Big]\varphi\phi\end{aligned} \tag{1.24}$$

where we will can calculate this part by part as follows:

$$\begin{aligned}\frac{-\hbar^2}{2m_e}\frac{\partial^2}{\partial y^2}(\varphi\phi) &= \frac{-\hbar^2}{2m_e}\frac{\partial}{\partial y}\Big[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\phi + \varphi\frac{\partial\phi}{\partial y}\Big]\\ &= \frac{-\hbar^2}{2m_e}\Big[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)^2\varphi\phi + \Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial y} + \Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial y} + \varphi\frac{\partial^2\phi}{\partial y^2}\Big]\\ &= \Big(\frac{e^2E^2}{2m_e\omega^2}\cos^2(\omega)t\Big)\varphi\phi - \Big(\frac{ieE\hbar}{m_e\omega}\cos(\omega)t\Big)\varphi\frac{\partial\phi}{\partial y} - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial y^2}\end{aligned} \tag{1.25}$$

and

$$\begin{aligned}\frac{2i\hbar eE}{2m_e\omega}\cos(\omega t)\frac{\partial}{\partial y}(\varphi\phi) &= \frac{i\hbar eE}{m_e\omega}\cos(\omega t)\Big[\Big(\frac{ieE}{\hbar\omega}\cos(\omega)t\Big)\varphi\phi + \varphi\frac{\partial\phi}{\partial y}\Big]\\ &= \Big(\frac{-e^2E^2}{m_e\omega^2}\cos(\omega)t\Big)\varphi\phi + \frac{i\hbar eE}{m_e\omega}\cos(\omega t)\varphi\frac{\partial\phi}{\partial y}.\end{aligned} \tag{1.26}$$

Therefore we can derive that

$$\text{R.H.S} = \Big[\frac{m_e\omega_0^2}{2}y^2 - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial y^2}\Big]\varphi\phi. \tag{1.27}$$

To satisfy the condition L.H.S=R.H.S we need to find a function $\phi(y,t)$ such that

$$\big[eEy\sin(\omega t)\big]\varphi\phi + i\hbar\varphi\frac{\partial\phi}{\partial t} = \Big[\frac{m_e\omega_0^2}{2}y^2 - \frac{\hbar^2}{2m_e}\varphi\frac{\partial^2\phi}{\partial y^2}\Big]\varphi\phi \tag{1.28}$$

which can be simplyfied as

$$\Big[\frac{m_e\omega_0^2}{2}y^2 - eEy\sin(\omega t) - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y^2} - i\hbar\frac{\partial}{\partial t}\Big]\phi(y,t) = 0 \tag{1.29}$$