A generalized model for the transport properties of dressed quantum Hall systems

Kosala Herath, and Malin Premaratne
Advanced Computing and Simulation Laboratory(AχL),
Department of Electrical and Computer Systems Engineering,
Monash University, Clayton, Victoria 3800, Australia
(Dated: August 9, 2021)

A generalized mathematical model for predicting the transport properties of a quantum system exposed to a stationary magnetic field and a high intensity electromagnetic field is presented. The new formulation, which applies to two-dimensional(2D) dressed quantum Hall systems, is based on Landau quantization theory and Floquet-Drude conductivity approach. We model our system as a two-dimensional electron gas (2DEG) that interacts with two external fields. To analyze the strong light coupling with the 2DEG, we employ the Floquet theory as a nonperturbative procedure. Moreover, the Floquet-Fermi golden rule is adopted to explore the impurity scattering effects on charge transport in disordered quantum Hall systems. We derive fully analytical expressions to describe longitudinal components in the conductivity tensor in dressed quantum Hall systems. Subsequently, we demonstrate that the conductivity characteristics of quantum Hall systems can be manipulated using a strong external light. Our results align with well-established and experimentally verified theoretical descriptions for undressed systems while providing a more generalized analysis on the conductivity characterstics in quantum Hall systems. Thus, our model can be applied to accurately interpret the usage of external strong radiation as a tool in nanoscale quantum devices.

I. INTRODUCTION

Manipulating light-matter interactions in the quantum regime paved the path for an astonishing number of useful technologies in the last century. Quantum optics, which study these interactions, have drawn research attention to the disciplines of optoelectronics [1–3], sensing [4–6], energy harvesting [7, 8], quantum computing [9–11], bio-information [12, 13], and many other specialities of recent technologies [14]. The studies on quantum optics of nanostructures were generally centered on metamaterials [15, 16], quantum plasmonic effects [17, 18], lasers and amplifiers [19, 20], and quantum cavity physics [21, 22]. However, in recent years, one of the foremost aim of examining nanostructures under external radiation was understanding their electron transport characteristics [23–30].

Better understanding the fundamental mechanisms of charge transport can allow us to invent novel nanoelectronic devices and optimize their performance [31]. Most recent studies on the subject have considered the driving field as a perturbation field [26, 27]. However, this assumption breaks down for systems under high-intensity illuminations [30, 32]. Modeling an electromagnetic field under a perturbative formalism involves expanding the interaction terms in powers of the field intensity. At high intensities, the higher order terms influence the physics more strongly and the basis of the perturbative treatment begins to break down. In these instances, a more accurate treatment needs to adopt. Thus, we treat the interacting fermion system and the radiation as one combined quantum system, namely dressed system [27, 33, 34]. Here the applied high-intensity electromagnetic field identify as the dressing field.

Theoretical analyses on the transport properties of dressed fermion systems were recently reported in Refs. [25, 27, 30]. Furthermore, in Ref. [30] a general expression for conductivity in a dressed system has been derived in a fully closed analytical form. In their study, a novel type of Green's functions, namely four-times Green's functions were used to derive the Floquet-Drude conductivity formula. This opened the path to explore and exploit the charge transport attributes of nanostructures under an intense dressing field.

Quantum Hall effect [35] observed in two-dimensional (2D) fermion systems at low temperatures under strong stationary magnetic fields manifest remarkable magneto-transport behaviors. Transport properties of these systems have recently attracted both theoretical [36–42] and experimental [43–45] interest. Endo *et al.* [42] presented the calculations of longitudinal and transverse conductivity tensor components and their relationship in a quantum Hall system. These theoretical calculations align better with experimental observations compared to previous studies.

In contrast, more interesting phenomena can be observed by simultaneously applying a dressing field to a quantum Hall system already under a non-oscillating magnetic field. Whilst there exist several leading theories for calculating conductivity tensor elements in quantum Hall systems [37, 41, 42], they have not been utilized to describe the optical manipulation of charge transport. Recently, Dini *et al.* [29] have investigated the one-directional conductivity behavior of dressed quantum Hall systems. However, they have not adopted the state-of-the-art model to describe the conductivity in a quantum Hall system. In their study, they used the conductivity models from Refs. [37, 41], and as mentioned in Endo *et al.* [42], those models predict a semi-elliptical broadening agaist Fermi level for each Landau levels and provide less agreement with the empirical results.

In the present analysis, we present a robust mathematical model for a dressed two-dimensional electron gas (2DEG) subject to another nonoscillating magnetic field. A stationary magnetic field is applied perpendicularly across the surface of the 2DEG system. This causes the orbital motion of the electrons to be quantized, and a discrete energy spectrum with Landau splitting is observed [46]. In this study, we explicitly calculate the longitudinal components $(\sigma^{xx}, \sigma^{yy})$ of the conductivity tensor in a periodically driven quantum Hall system by developing a generalized analytical description using the Floquet-Drude conductivity [30]. Finally, we demonstrate that our generalized model reproduces the results of the state-of-the-art conductivity model in Ref. [42], which was developed for the more specific case of quantum Hall systems without the external dressing field. Moreover, we find that the optical field can be used as a mechanism to regulate transport behavior in numerous 2D nanostructures which can serve as a basis for many useful nanoelectronic devices. We believe that our theoretical analysis and visual depictions of numerical results will lead to a better understanding of manipulating charge transport. Moreover, this will inspire advanced developments in nanoscale quantum devices.

The paper is organized as follows. In Sec. II, we introduce our dressed quantum Hall system and the exact wave function solutions for the given configuration. Sec. III, provides the Floquet theory interpretation of these wave functions. We introduce Floquet-Fermi golden rule for a quantum Hall system in Sec. IV, and use it in Sec. V to derive analytical expressions for longitudinal components of conductivity. The derived theoritical model is further analyzed numerically using empirical system parameters and compared with previous studies in Sec. VI. In Sec. VII, we summarize our analysis and present our conlusions.

II. SCHRODINGER PROBLEM FOR A DRESSED QUANTUM HALL SYSTEM

Our system consists of a 2DEG placed on the xy-plane of the three-dimensional coordinate space. In our analysis, the 2DEG is subjected to a nonoscillating magnetic field $\mathbf{B} = (0, 0, B)^{\mathrm{T}}$ which is pointed towards the z axis. In addition, a linearly polarized strong light is applied perpendicular to the 2DEG surface. We specially select the frequency of the dressing field ω to be in the off-ersonant regime such that the field behaves as a purely dressing field. Furthermore, without limiting the generality we choose y-polorized electric field $\mathbf{E} = (0, E\sin(\omega t), 0)^{\mathrm{T}}$ for the linearly polarized dressing field (Fig. 1). Here B and E represent the amplitudes of the stationary magnetic field and oscillating electric field respectively.

Using Landau gauge for the stationary magnetic field, we can represent it as a vector potential $\mathbf{A}_s = (-By, 0, 0)^{\mathrm{T}}$. Furthermore, we model the dynamic dressing field in the Coulomb gauge as $\mathbf{A}_d(t) = (0, [E/\omega]\cos(\omega t), 0)^{\mathrm{T}}$. These vector potentials are coupled to the momentum of 2DEG as kinetic momentum [47, 48]. Thus, our system can be represented with a time-dependent Hamiltonian

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[\hat{\mathbf{p}} - e\big[\mathbf{A}_s + \mathbf{A}_d(t)\big]\Big]^2, \tag{1}$$

where m_e is the effective electron mass, e is the magnitude of the electron charge, and $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y, 0)^{\mathrm{T}}$ represents the canonical momentum operator for 2DEG with electron momentum $(p_x, p_y, 0)^{\mathrm{T}}$. The exact solutions for the time-dependent Schrödinger equation $i\hbar\ \mathrm{d}\psi/\mathrm{d}t = \hat{H}_e(t)\psi$ were already derived in Refs. [29, 49, 50]. Here we present them as a set of wave functions defined by two quantum numbers (n, m)

$$\begin{aligned}\psi_{n,m}&(x,y,t)\\ &= \frac{1}{\sqrt{L_x}}\chi_n\big(y - y_0 - \zeta(t)\big)\\ &\times \exp\bigg(\frac{i}{\hbar}\Big[-\epsilon_n t + p_x x + \frac{eE[y-y_0]}{\omega}\cos(\omega t)\\ &+ m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \int_0^t dt' L(\zeta, \dot{\zeta}, t')\Big]\bigg),\end{aligned} \tag{2}$$

where $n \in \mathbb{Z}_0^+$ and $m \in \mathbb{Z}$. Here L_x and L_y are dimensions of the 2DEG surface, and $\hbar$ is the reduced Planck constant. The center of the cyclotron orbit on the y-axis is given by $y_0 = -p_x/eB$ with $p_x = 2\pi\hbar m/L_x$. Moreover, χ_n are well known eigenstate solutions for the Schrödinger equation of the stationary quantum harmonic oscillator

$$\chi_n(y) = \frac{\sqrt{\kappa}}{\sqrt{2^n n!}} e^{-\kappa^2 y^2/2}\mathcal{H}_n(\kappa y), \tag{3}$$

with eigenvalues $\epsilon_n = \hbar\omega_0(n+1/2)$ where $\kappa = \sqrt{m_e\omega_0/\hbar}$, $\mathcal{H}_n(\cdot)$ is the n-th Hermite polynomial, and the cyclotron frequency $\omega_0 = eB/m_e$. The path shift of the driven classical oscillator $\zeta(t)$ is given by

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t), \tag{4}$$

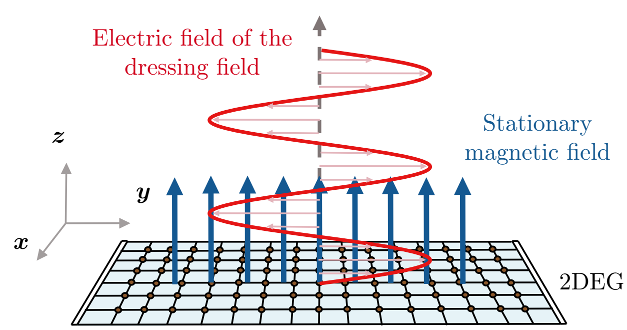

FIG. 1. Our 2DEG system only confined in the (x, y) plane while both stationary magnetic field $\mathbf{B}$ and dressing field (with y-polarized electric field $\mathbf{E}$) are applied perpendicular to the surface of 2DEG.

while the Lagrangian of the driven classical oscillator $L(\zeta, \dot{\zeta}, t)$ can be identified as

$$L(\zeta, \dot{\zeta}, t) = \frac{1}{2} m_e \dot{\zeta}^2(t) - \frac{1}{2} m_e \omega_0^2 \zeta^2(t) + eE\zeta(t)\sin(\omega t). \tag{5}$$

For details of the full derivation refer to Appendix A. The exponential phase shifts in Eq. (2) represent the influence of the stationary magnetic field and dressing field on the electron behavior of our system. Therefore, we can observe that the magneto-transport characteristics of 2DEG can be renormalized by a nonoscillating magnetic field along with a dressing field.

III. FLOQUET THEORY PERSPECTIVE

Symmetry conditions often gives useful insights into the behaviors of physical quantum systems. For instance, in the famous Bloch analysis of electrons in quantum systems introduces a mathematical explanation for quantum systems occupying a discrete translational symmetry in configuration space. Similarly, Floquet theory gives a mathematical formalism that can be used for translational symmetry in time rather than in space [32, 51, 52]. The Floquet-Drude conductivity theory was emplyoed recently by Wackerl *et al.* [30] as a method to analyze the transport properties of quantum systems exposed to strong radiation. In their work, they have presented more accurate results than the former theoretical descriptions for the conductivity of nanoscale systems in the presence of a dressing field. Therefore, we apply the Floquet-Drude conductivity theory to analyze our 2DEG system which is subjected to both a stationary magnetic field and a dressing field.

First, we need to identify the *quasienergies* and time periodic *Floquet modes* [32] for the wave functions given in Eq. (2). By factorizing the wave function into a linearly time dependent part and a periodic time dependent part, we present the quasienergies as

$$\varepsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right) - \Delta_\varepsilon, \tag{6}$$

which only depends on a single quantum number n. The Floquet modes can then be recognized as

$$\begin{aligned}\phi_{n,m}(x, y, t) =& \frac{1}{\sqrt{L_x}} \chi_n\big(y - y_0 - \zeta(t)\big) \\ &\times \exp\left(\frac{i}{\hbar}\left[p_x x + \frac{eE[y - y_0]}{\omega}\cos(\omega t)\right.\right. \\ &\left.\left. + m_e \dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \xi\right]\right),\end{aligned} \tag{7}$$

with

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2 - \omega^2)}, \tag{8}$$

and

$$\xi = \frac{(eE)^2\big(3\omega^2 - \omega_0^2\big)}{8m_e\omega(\omega_0^2 - \omega^2)^2}\sin(2\omega t). \tag{9}$$

It is important to note that these Floquet modes are time-periodic ($T = 2\pi/\omega$) functions. At resonance $\omega = \omega_0$, the energy levels occupy a continuous spectrum and the quasienergy formalism is no longer valid [53]. Therefore, in this work we choose a dressing field frequency obeying the condition $\omega \neq \omega_0$.

Performing the Fourier transform over the confined 2D space, we obtain the momentum space (k_x, k_y) representation of Floquet modes

$$\begin{aligned}\phi_{n,m}(k_x, k_y, t) =& \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \\ &\times \exp\left(i\xi - ik_y[d\sin(\omega t) + y_0]\right),\end{aligned} \tag{10}$$

where

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_n\left(\frac{k}{\kappa}\right). \tag{11}$$

Here we have introduced new parameters

$$b = \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)}, \tag{12}$$

and

$$d = \frac{eE}{m_e(\omega_0^2 - \omega^2)}. \tag{13}$$

For a detailed derivation, refer to Appendix B. Using Floquet theory, the wave functions derived in Eq. (2) can be written as Floquet states in momentum space as

$$\psi_{n,m}(k_x, k_y, t) = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)\phi_{n,m}(k_x, k_y, t). \tag{14}$$

IV. INVERSE SCATTERING TIME ANALYSIS

The Floquet-Fermi golden rule was proposed in Ref. [30] as an approach to analyze the transport properties of dressed quantum systems with impurities. However, this theory has not been applied for a dressed quantum Hall system in the previous studies. In this analysis, we use Floquet-Fermi golden rule to identify the effects induced by impurities on the magneto-transport properties. With the help of $t - t'$ formalism [30, 32, 54–56] and applying Floquet states derived in Eq. (14), we can derive an expression for (l, l')-th element of the inverse scattering time matrix for the N-th Landau level as

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll'} = \frac{\varrho^2}{eB}\delta(\varepsilon-\varepsilon_N) \times \int_{-\infty}^{\infty} dk_1 \left[J_l\left(\frac{b\hbar}{eB}[k_x-k_1]\right) J_{l'}\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times \left| \int_{-\infty}^{\infty} dk_2\, \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2 \right], \quad (15)$$

where $\varrho = \eta_{imp}L_x[V_{imp}/eB]^{1/2}$, ε is a given energy value, $J_l(\cdot)$ are Bessel functions of the first kind with l-th integer order, and ε_N is the energy of the N-th Landau level. A more detailed derivation is given in Appendix C. We modeled the effect caused by impurities in the considered system as a single perturbation potential. Since random impurities in a disordered metal offers a better approximation for experimental conditions, we assumed that our perturbation potential is formed by a group of randomly distributed impurities. Thus, the total scattering potential in the 2DEG has been represented as a sum of uncorrelated single impurity potentials $\upsilon(\mathbf{r})$. Here η_{imp} is the number of impurities in a unit area, $V_{imp} = \langle |V_{k'_x,k_x}|^2\rangle_{imp}$ with $V_{k'_x,k_x} = \langle k'_x|\upsilon(x)|k_x\rangle$, and $\langle x|k_x\rangle = e^{-ik_xx}$. Moreover in this analysis, $\langle\cdot\rangle_{imp}$ represents the average over the impurity disorder.

Next, we analyze the contribution of the inverse scattering time matrix elements on the transport properties of our system. Since the disorder in the system can not singificantly alter the eigenenergy values of the undressed system [30], we can neglect the contribution of all off-diagonal elements in the inverse scattering time matrix. Subsequenty we consider only the central ($l = l' = 0$) diagonal element of the inverse scattering time matrix which has the largest contribution to the transport characteristics. Along with this assumption, we introduce a new parameter as the scattering-induced broadening of the N-th Landau level [29, 42]

$$\Gamma_N^{00}(\varepsilon,k_x) = \hbar\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{00}, \quad (16)$$

which simplifies to

$$\Gamma_N^{00}(\varepsilon,k_x) = \frac{\varrho^2}{eB}\delta(\varepsilon-\varepsilon_N) \times \int_{-\infty}^{\infty} dk_1 \left[J_0^2\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times \left| \int_{-\infty}^{\infty} dk_2\, \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2 \right]. \quad (17)$$

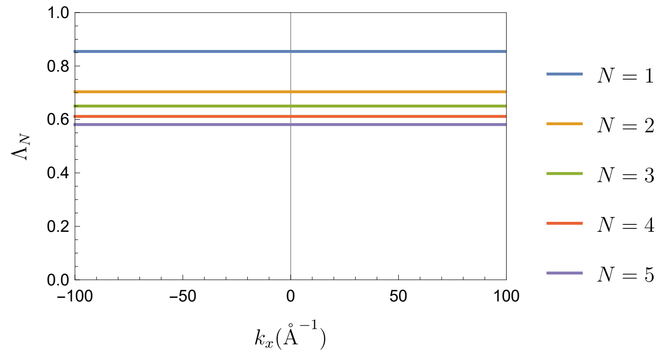

FIG. 2. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0,1,2,3,4$) against x-directional momentum value k_x in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2\times10^{12}$ rads^{-1} and intensity $I = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

In addition, for a scattering scenario taking place wihtin the same Landau level, we are able to present the delta distribution of the energy by the interpretation [29]

$$\delta(\varepsilon-\varepsilon_N) \approx \frac{1}{\pi\Gamma_N^{00}(\varepsilon,k_x)}. \quad (18)$$

Then we write the central element of inverse scattering time matrix in the more compact form

$$\Gamma_N^{00}(\varepsilon,k_x) = \varrho\left[\int_{-\infty}^{\infty} dk_1\; J_0^2(\lambda_1[k_x-k_1]) \times \left|\int_{-\infty}^{\infty} dk_2\; \tilde{\chi}_N(\lambda_2k_2)\tilde{\chi}_N(\lambda_2[k_1-k_2-k_x])\right|^2\right]^{-\frac{1}{2}}, \quad (19)$$

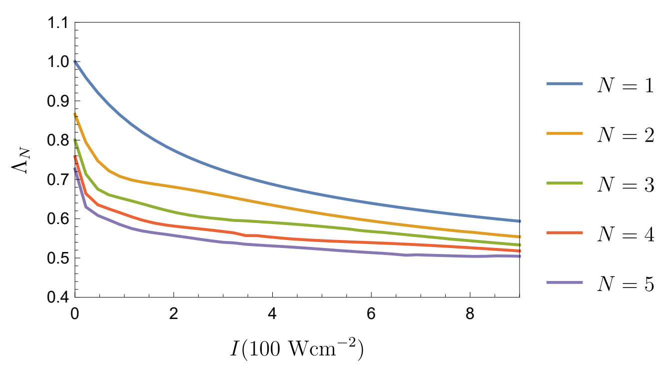

FIG. 3. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0,1,2,3,4$) against dressing field intensity I, in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2\times10^{12}$ rads^{-1}. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

where $\lambda_1 \equiv \hbar b/eB$ and $\lambda_2 \equiv \hbar\kappa/eB$. To analyze the effects of the dressing field on the scattering-induced broadening, we introduce the normalized N-th Landau level scattering-induced broadening as

$$\Lambda_N(k_x) \equiv \frac{\Gamma_N^{00}(\varepsilon, k_x)}{\Gamma_{N=0}^{00}(\varepsilon, k_x)\big|_{E=0}}, \tag{20}$$

which can be evaluated with

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty} dk_1\; J_0^2(\lambda_1[k_x - k_1])\left|\int_{-\infty}^{\infty} dk_2\; \tilde{\chi}_N(\lambda_2 k_2)\tilde{\chi}_N(\lambda_2[k_1 - k_2 - k_x])\right|^2}{\int_{-\infty}^{\infty} dk_1\; \left|\int_{-\infty}^{\infty} dk_2\; \tilde{\chi}_0(\lambda_2 k_2)\tilde{\chi}_0(\lambda_2[k_1 - k_2 - k_x])\right|^2}\right]^{1/2}. \tag{21}$$

Normalized energy band broadening against x-directional momentum component k_x for different Landau levels ($N = 0, 1, 2, 3, 4$) has been calculated for GaAs-based quantum well and the results are depicted in Fig. (2) and Fig. (3). To make a comparison, we have selected the experiment parameters to match with analysis in Ref. [42]. In that study, the authors have assumed that effective mass of the electron in GaAs-based quantum well system is $m_e \approx 0.07\tilde{m}_e$ where $\tilde{m}_e$ is mass of the electron [30, 42, 57]. In addition, they used the broadening of the undressed 0-th Landau level Γ_0 as 0.24 me$V$. Therefore, in our calculations, we assumed that the natural least Landau level broadening also has this value: $\Gamma_{N=0}^{00}|_{E=0} = 0.24$ meV. Here, we observe that the normalized energy broadening value for each Landau level is independent of the x-directional momentum k_x value and we are able to manipulate it by the dressing field. When the dressing field's intensity increases, the energy broadening is reduced, which leads to changes in the transport properties of the dressed quantum Hall system. To analyze these adjustments in detail, we derive a analytical expression for the conductivity of a dressed quantum Hall system in the next section.

V. FLOQUET-DRUDE CONDUCTIVITY IN QUANTUM HALL SYSTEMS

A general theory for the conductivity of a dressed system with the disorder averaging was reported by Wackerl *et al.* [30, 58]. This theory, the general x-directional longitudinal DC-limit conductivity has been characterized as

$$\begin{aligned}\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\Bigg[\left(-\frac{\partial f}{\partial\varepsilon}\right)\\ \times \operatorname{tr}\left[j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))\right]\Bigg],\end{aligned} \tag{22}$$

where j_0^x and $\mathbf{G}^{r,a}(\varepsilon)$ are the x-directional electric current operator matrix elements' 0-th Fourier component and the white noise disorder averaged Floquet Green function matrix [30, 58] respectively defined against the Floquet modes of the considering system. Here we have assumed that only 0-th Fourier component of the current operator is contributing to the conductivity. In addition, A is the area of the considered two-dimensional system, f is the partial distribution function, and Π is a function that can be chosen such that

$$\Pi - \frac{\hbar\omega}{2} \leq \varepsilon_N < \Pi + \frac{\hbar\omega}{2}. \tag{23}$$

Here ε_N are quasienergies of all relevant Floquet states, and $tr[\cdot]$ is the trace of the considering operator.

Next, we restrict our analysis into off-resonant regime $\omega\tau_0 \gg 1$), where τ_0 is the scattering time of the undriven system. Thus, the x-directional longitudinal conductivity given in Eq. (22) can be expanded using only the central entry Fourier components ($l = l' = 0$) of Floquet modes $|\phi_{n,m}\rangle \equiv |n, k_x\rangle$ as

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\langle n, k_x|j_0^x(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon) - \mathbf{G}_0^a(\varepsilon))|n, k_x\rangle\,, \tag{24}$$

where V_{k_x} is the volume of considering x-directional momentum space. Next, we evaluate the above expression as follows

$$\begin{aligned}\sigma^{xx} =& \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}^4}\sum_{k_x}\sum_{n}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3}\\ &\times\langle n, k_x|j_0^x|n_1, k_{x1}\rangle\,\langle n_1, k_{x1}|(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))|n_2, k_{x2}\rangle\,\langle n_2, k_{x2}|j_0^x|n_3, k_{x3}\rangle\,\langle n_3, k_{x3}|(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))|n, k_x\rangle\,.\end{aligned} \tag{25}$$

We can diagonalize the impurity averaged Green's functions using a unitary transformation ($\mathbf{T} \equiv |n, k_x\rangle$) as mentioned in Refs. [30, 58, 59]. Thus, we evaluate the matrix elements of the difference between retarded and advanced Green's functions as

$$\langle n_1, k_{x1}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))\mathbf{T}|n_2, k_{x2}\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n_1}\big)^2 + [\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right], \tag{26}$$

and

$$\langle n_3, k_{x3}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon) - \mathbf{G}^a(\varepsilon))\mathbf{T}|n, k_x\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n}\big)^2 + [\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right]. \tag{27}$$

Here we introduced the retarded self-energy matrix $\mathbf{\Sigma}^r$ which is the sum of all irreducible diagrams [30, 58]. Applying the matrix elements of the electric current operator in Landau levels and expressions from Eq. (26) and Eq. (27) back into Eq. (25) we obtain

$$\begin{aligned}\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\sum_{n_1,n_2}&\\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1} + \sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n_1}\big)^2 + [\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right]&\\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n,n_2+1} + \sqrt{\frac{n_2}{2}}\delta_{n,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n}\big)^2 + [\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right]&,\end{aligned} \tag{28}$$

For the full derivation of electric current operators in quantum Hall system refer to Appendix D. After the expansion, the only non-zero term would be

$$\begin{aligned}\sigma^{xx} = \frac{-1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)&\\ \times\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n+1}}}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n+1}\big)^2 + \Big[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})_{\varepsilon_{n+1}}\Big]^2}\right]\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n}}}{\big(\frac{1}{\hbar}\varepsilon - \frac{1}{\hbar}\varepsilon_{n}\big)^2 + \Big[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})_{\varepsilon_{n}}\Big]^2}\right]&.\end{aligned} \tag{29}$$

The inverse scattering time matrix is equal to the diagonalized contrast of the retarded and advanced self-energy [30, 58]. In addition, on the diagonal the contrast of the retarded and advanced Green's function can be represented with the imaginary component of the retarded self-energy [30, 58]. Subsequenty, we can identify the following property

$$\left(\frac{1}{\tau(\varepsilon, k_x)}\right)^{ll} = -2\mathrm{Im}\big[\mathbf{T}^\dagger\mathbf{\Sigma}^r(\varepsilon, k_x)\mathbf{T}\big]^{ll}. \tag{30}$$

Aterwards, considering only the central element ($l = 0$) of the inverse scattering time matrix, we can restructure the derived conductivity expression in 29 as follows

$$\sigma^{xx} = \frac{1}{\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\Pi-\hbar\omega/2}^{\Pi+\hbar\omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)\left[\frac{\tilde{\Gamma}(\varepsilon_{n+1})}{(\varepsilon_F - \varepsilon_{n+1})^2 + \tilde{\Gamma}^2(\varepsilon_{n+1})}\right]\left[\frac{\tilde{\Gamma}(\varepsilon_{n})}{(\varepsilon_F - \varepsilon_{n})^2 + \tilde{\Gamma}^2(\varepsilon_{n})}\right], \tag{31}$$

with $\tilde{\Gamma}(\varepsilon_n, k_x) \equiv (\hbar/2\tau(\varepsilon_n, k_x))^{00}$. Since we already identified that the inverse scattering time matrix's central element is independent of k_x value, we can drop the k_x-dependent in the $\tilde{\Gamma}(\varepsilon_n, k_x)$ terms and get the sum over all available momentum space in x direction. However, by considering the condition that the center of the cyclotron orbit y_0 must physically lie within the considered system, we can identify that

$$-\frac{m_e\omega_0 Ly}{2\hbar} \le k_x \le \frac{m_e\omega_0 Ly}{2\hbar}. \tag{32}$$

We use the Fermi-Dirac distribution as our partial distribution function (f) for our system

$$f(\varepsilon) = \frac{1}{[\exp(\varepsilon - \varepsilon_F)/k_B T] + 1}, \tag{33}$$

where k_B is theBoltzmann constant, T is the absolute temperature and ε_F is the Fermi energy of the system. Considering the above distribution for extremely low temperature conditions, we can use the following approximation

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F). \tag{34}$$

Moreover, let $\Pi = \varepsilon_F$ and the derived expression in 31 leads to

$$\sigma^{xx} = \frac{e^2}{\hbar}\frac{1}{\pi A}\sum_n \frac{(n+1)}{\gamma_n \gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right], \tag{35}$$

where $X_F \equiv (\varepsilon_F/\hbar\omega_0 - 1/2)$ and $\gamma_n \equiv \tilde{\Gamma}(\varepsilon_n)/\hbar\omega_0$. Following the same steps as above derivation, we can derive the longitudinal conductivity in y-direction by applying the electric current operator for y-direction derived in Appendix D

$$\sigma^{yy} = \frac{e^2}{\hbar}\frac{1}{\pi A}\frac{1}{e^2 B^2}\sum_n \frac{(n+1)}{\gamma_n \gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right]. \tag{36}$$

VI. MANIPULATE CONDUCTIVITY IN QUANTUM HALL SYSTEMS

To identify the longtitudinal conductivity characteristics of quantum Hall system under external dressing field, first we derive an expression for normalized longitudinal conductivity as a function of Fermi energy X_F and intensity of the dressing field I. Here we derive a normalized x-directional conductivity of dressed quantum Hall system using the natural conductivity of the least Landau level

$$\tilde{\sigma}^{xx} = \sigma_0 \sum_n \frac{(n+1)}{0.0037\Lambda_n \Lambda_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F-n-1}{0.06\Lambda_n}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{0.06\Lambda_{n+1}}\right)^2}\right], \tag{37}$$

with $\sigma^0 = e^2/\pi\hbar A$.

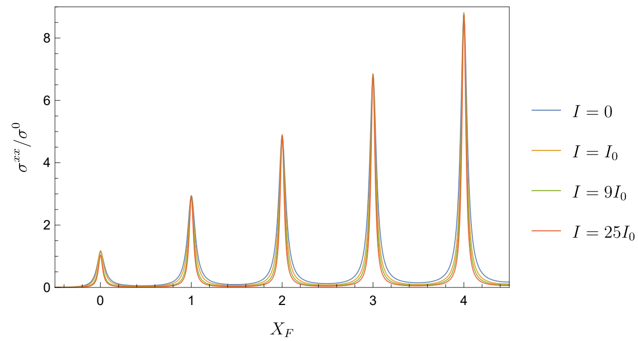

FIG. 4. Normalized longitudinal conductivity $\tilde{\sigma}^{xx}$ against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2 \times 10^{12}$ rads^{-1} and $I_0 = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

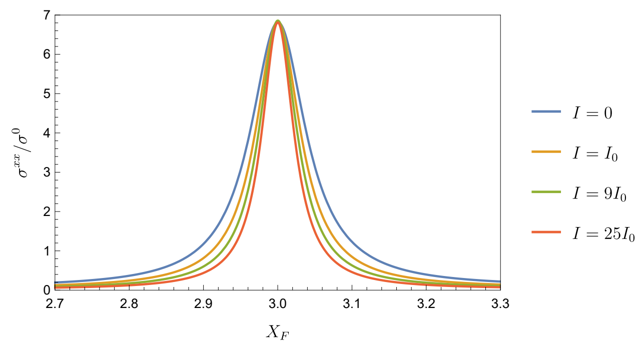

FIG. 5. 3rd Landau level's normalized longitudinal conductivity $\tilde{\sigma}^{xx}$ against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well under a nonoscillating magnetic field with $B = 1.2$ T, dressing field with frequency of $\omega = 2 \times 10^{12}$ rads^{-1} and $I_0 = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$

.

As illustrated in Fig. 4 and 5 we can manipulate the normalized longitudinal conductivity $\tilde{\sigma}^{xx}$ with the applied dressing field's intensity and the Fermi level X_F of the considered system. For a given dressing field intensity, the longitudinal conductivity vary with the Fermi level of the system showing sharp peaks at each Landau energy level. In quantum Hall systems, electrons are restricted to bear only the Landau energies. Thus, the conductivity reduces significantly when the Fermi level is not aligned with any of the Landau levels. In contrast, near each Landau level, the conductivity achieve excessive values compared to other areas. Moreover, as illustrates in Fig. 4 the peak value of the normalized longitudinal conductivity on each Landau level gets increase with the Landau level number.

On the other hand, considering the effects of the applied dressing field on the longitudinal conductivity of

2DEG, we can identify that the dressing field has sharpen the conductivity peaks. When we increase the intensity level of the dressing field, the conductivity regions get more weaken as illustrate in the 5. However, the peak value of the conductivity at each Landau level has the same value as the undressed system. This demonstrates our ability to tune the width of the conductivity regions in these quantum Hall systems with the help of a dressing field.

These characteristics align well with the outcomes demonstrated by Dini *et al.* [29]. As the authors remarked in that work, since the Fermi level of the system can be changed with the applied gate voltage of the material this can be utilized as a 2D switch for nanoscale optoelectronics applications. Controlling the external dressing field's intensity, allows us to fine-tune this switching mechanism for an optimized performance. However, we can distinguish that the shapes and behavior of the conductivity regions illustrated in Fig. 4 and 5 are generally incompatible with the results reported in Ref. [29]. This is due to the selection of the conventional longitudinal conductivity theory of 2DEG from Refs. [37, 41]. The semi-elliptical conductivity regions illustrated in Refs. [29, 37, 41], have less consistency with the experimentally observed data for Landau levels [42]. In our study of the transport properties of quantum Hall systems, we developed the conductivity expression starting from the Floquet-Drude conductivity [30] and we achieved outcomes that align with the results depicted in Ref. [42]. The theory on the conductivity of quantum Hall systems in Ref. [42] provides an excellent agreement with the experimently observed results in GaAs/AlGaAs 2DEG for the low magnetic field range. However, they have not considered the tunability that can be achieved with a dressing field. In this analysis, we account both of the magnetic and dressing field effects that can affect the transport properties of 2DEG, leading to a more generalized theory. Thus in this study, we were able to demonstrate that using Floquet-Drude conductivity method, one can derive a more generalized mathematical model which fits better with experiment for the charge transport properties of quantum Hall systems.

VII. CONCLUSIONS

In this paper, we introduced a generalized mathematical model for prediciting charge transport properties in a 2DEG under a nonoscillating magnetic field and a high intensity light. Under the uniform magnetic field, the charged particles can only settle in discrete energy values which leads to the Landau quantization. We modeled the behavior of electrons in Landau levels under the dressing field utilizing the Floquet-Drude conductivity method. We assumed the impurities in the material as a Gaussian random scattering potential. Finally, we derived expressions for x-directional and y-directional longitudinal components of electric conductivity tensor for the considered system.

Our derived analytical expressions disclosed that the transport characteristics of the dressed quantum Hall system can be controlled by the applied dressing field's intensity. Using detailed numerical calculations with empirical system parameters, we further analyzed the manipulation of conductivity components using the dressing field. We found that the graphical illustrations that we obtained from these numerical calculatiuons are capable of producing the same behavior as experiments on quantum Hall systems in the absence of a dressing field. Furthermore, we identified that by increasing the intensity of the radiation, the conductivity regions near the Landau levels can be squeezed. Despite this behavior being identified in previous works, their results did not coincide with the more accurate description of conductivity components in undressed quantum Hall systems. However, our generalized analysis on conductivity of dressed quantum Hall systems provide a well-suited description for these specific quantum Hall systems.

In summary, the primary purpose of this study was to broaden the modern descriptions on transport properties of dressed quantum Hall systems. Moreover, our detailed theoretical analysis showed that the recently introduced Floquet-Drude conductivity model can be adopted to extend the models that were used to describe the transport characteristics in quantum Hall systems. Owing the ability to control the conductivity regions, high intensity external illumination may be used as a trigger for two-dimensional quantum switching devices which are employed as the building blocks of next generation nanoelectronic devices. As a concluding remark, we believe that our findings of this paper can be used towards understanding the 2D opto-electronic nano transistors, enhancing their performance, and inventing novel appliances.

ACKNOWLEDGMENTS

K.H wish to acknowledge the members of AχL at Monash University for their encouragement and support and specially T.N. Perera and R.T.Wijesekara for insightful discussions. The work of K.H is supported by the Monash University Institute of Graduate Research.

Appendix A: Wave function solutions for a dressed quantum Hall system

The derivation of the solutions for the time dependent Schrödinger equation with our system's Hamiltonian (Eq. 1) is quite similar to that followed in Refs. [29, 49]. We start with expanding the Hamiltonian for two-dimensional case

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[[\hat{p}_x + eBy]^2 + \left[\hat{p}_y - \frac{eE}{\omega}\cos(\omega t)\right]^2\right]. \tag{A1}$$

Since $\left[\hat{H}_e(t), \hat{p}_x\right] = 0$, both of these operators share same eigenfunctions ${L_x}^{-1/2}\exp\left(\frac{ip_x x}{\hbar}\right)$ where $p_x = 2\pi\hbar m/L_x$ with $m \in \mathbb{Z}$. Thus, we re-arrange the Hamiltonian using the definition of canonical momentum in y-direction and this leads to

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[[p_x + eBy]^2 + \left[-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right]^2\right]. \tag{A2}$$

Subsequenty we define the *center of the cyclotron orbit* on the y-axis $y_0 = -p_x/eB$ and the *cyclotron frequency* $\omega_0 = eB/m_e$. This leads to a new arrangement of the Hamiltonian

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}\tilde{y}^2 + \frac{1}{2m_e}\Bigg[-\hbar^2\frac{\partial^2}{\partial\tilde{y}^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial\tilde{y}} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\Bigg], \tag{A3}$$

where we used a variable substitution $\tilde{y} = (y - y_0)$. Furthermore, we assume that the wave function solutions for the time-dependent Schrödinger equation of considered quantum system

$$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = \hat{H}_e(t)\psi, \tag{A4}$$

can be presented by the following form

$$\psi_m(x,\tilde{y},t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieE\tilde{y}}{\hbar\omega}\cos(\omega t)\right)\vartheta(\tilde{y},t), \tag{A5}$$

where $\vartheta(\tilde{y},t)$ is a function that satisfy the property

$$\left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 - eE\tilde{y}\sin(\omega t) - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} - i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]\vartheta(\tilde{y},t) = 0. \tag{A6}$$

If we turn off the dressing field ($E = 0$), this equation leads to the Schrödinger equation with the simple harmonic oscillator Hamiltonian

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y},t)}{\mathrm{d}t} = \left[\frac{\hat{p}_{\tilde{y}}^2}{2m_e} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2\right]\vartheta(\tilde{y},t). \tag{A7}$$

Thus, we can identify $S(t) \equiv eE\sin(\omega t)$ term as an external force act on the harmonic oscillator, and we can solve this as a forced harmonic oscillator in $\tilde{y}$ axis

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y},t)}{\mathrm{d}t} = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2 - \tilde{y}S(t)\right]\vartheta(\tilde{y},t). \tag{A8}$$

This system is exactly solvable, and we can solve the equation using the methods explained by Husimi [49]. We introduce a time dependent shifted coordinate $y' = \tilde{y} - \zeta(t)$ and perform the following unitary transformation

$$\vartheta(y',t) = \exp\left(\frac{im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y',t). \tag{A9}$$

This leads to

$$i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Bigg[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 + \left[m_e\ddot{\zeta} + m_e\omega_0^2\zeta - S(t)\right]y' + \left[-\frac{1}{2}m_e\dot{\zeta}^2 + \frac{1}{2}m_e\omega_0^2\zeta^2 - \zeta S(t)\right]\Bigg]\varphi(y',t). \tag{A10}$$

Subsequenty, we can restrict $\zeta(t)$ function such that

$$m_e\ddot{\zeta} + m_e\omega_0^2\zeta = S(t), \tag{A11}$$

and that simply our previous expression as

$$i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Bigg[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 - L(\zeta,\dot{\zeta},t)\Bigg]\varphi(y',t). \tag{A12}$$

Here

$$L(\zeta,\dot{\zeta},t) = \frac{1}{2}m_e\dot{\zeta}^2 - \frac{1}{2}m_e\omega_0^2\zeta^2 + \zeta S(t), \tag{A13}$$

is the Lagrangian of a classical driven oscillator. To proceed further, another unitary transform can be introduced as follows

$$\varphi(y',t) = \exp\left(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\right)\chi(y',t), \tag{A14}$$

and subtitling Eq. (A14) back in Eq. (A12) yields

$$i\hbar\frac{\partial}{\partial t}\chi(y',t) = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2\right]\chi(y',t). \tag{A15}$$

This is the well-known Schrödinger equation of the quantum harmonic oscillator. This allows us to identify the well known eigenfunctions [60, 61]

$$\chi_n(y) = \frac{\sqrt{\kappa}}{\sqrt{2^n n!}}e^{-\kappa^2 y^2/2}\mathcal{H}_n(\kappa y), \tag{A16}$$

with eigenvalues

$$\epsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right) \text{ for } n \in \mathbb{Z}_0^+. \tag{A17}$$

Here, $\kappa = \sqrt{m_e\omega_0/\hbar}$, and $\mathcal{H}_n$ are the Hermite polynomials. Thus, we can identify the solutions for Eq. (A8) as

$$\vartheta_n(\tilde{y},t) = \chi_n(\tilde{y} - \zeta(t))\exp\Bigg(\frac{i}{\hbar}\Bigg[-\epsilon_n t + m_e\dot{\zeta}(t)\left[\tilde{y} - \zeta(t)\right] + \int_0^t dt' L(\zeta,\dot{\zeta},t')\Bigg]\Bigg). \tag{A18}$$

Since $\chi_n(x)$ functions forms a complete set, any general solution $\vartheta_(\tilde{y},t)$ can be presented with the help of the solutions derived in Eq. (A18).

Finally, we consider our scenario where we assumed that $S(t) = eE\sin(\omega t)$, and we derive the solution for Eq. (A11) as

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t). \tag{A19}$$

Subtitling solutions in Eq. (A18) back in Eq. (A5), we obtain a set of wave functions with two different quantum number (n,m) that satisfy the time dependent Schrödinger equation Eq. (A4) as follows

$$\begin{aligned}\psi_{n,m}(x,y,t) = &\frac{1}{\sqrt{L_x}}\chi_n\left(y - y_0 - \zeta(t)\right)\exp\Bigg(\frac{i}{\hbar}\Bigg[-\epsilon_n t \\ &+ p_x x + \frac{eE[y-y_0]}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] \\ &+ \int_0^t dt' L(\zeta,\dot{\zeta},t')\Bigg]\Bigg).\end{aligned} \tag{A20}$$

Appendix B: Floquet modes and quasienergies

1. Position space representation

First we define the time integral of the Lagrangian of the classical oscillator given in Eq. (5), over a period $T = 2\pi/\omega$ as

$$\Delta_\varepsilon = \frac{1}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t'). \tag{B1}$$

Additionally, performing this integral, we can obtain a more simplified result

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2 - \omega^2)}. \tag{B2}$$

Next, we define another parameter

$$\xi = \int_0^t dt'\ L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t, \tag{B3}$$

and after simplifying, we can identify

$$\xi = \frac{(eE)^2\left(3\omega^2 - \omega_0^2\right)}{8m_e\omega(\omega_0^2 - \omega^2)^2}\sin(2\omega t), \tag{B4}$$

which is a periodic function in time. Using these parameters, we can factorize the wave function given in Eq. (2) as linearly time dependent part and periodic time dependent part as follows

$$\begin{aligned}\psi_\alpha(x,y,t) = &\exp\left(\frac{i}{\hbar}[-\epsilon_n t + \Delta_\varepsilon t]\right)\frac{1}{\sqrt{L_x}}\chi_n\left(y - y_0 - \zeta(t)\right) \\ &\times\exp\Bigg(\frac{i}{\hbar}\Bigg[p_x x + \frac{eEy}{\omega}\cos(\omega t) \\ &+ m_e\dot{\zeta}(t)\big[y - \zeta(t)\big] + \int_0^t dt' L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t\Bigg]\Bigg).\end{aligned} \tag{B5}$$

This leads to separate the linear time dependent phase component as the quasienergies

$$\varepsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right) - \Delta_\varepsilon, \tag{B6}$$

while rest of the components as time-periodic Floquet modes

$$\begin{aligned}\phi_{n,m}(x,y,t) \equiv &\frac{1}{\sqrt{L_x}}\chi_n\left(y - y_0 - \zeta(t)\right)\exp\Bigg(\frac{i}{\hbar}\Bigg[p_x x \\ &+ \frac{eE[y-y_0]}{\omega}\cos(\omega t) \\ &+ m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \xi\Bigg]\Bigg).\end{aligned} \tag{B7}$$

2. Momentum space representation

We perform continuous Fourier transform over the considering confined space $A = L_x L_y$ on the Floquet modes given in Eq. (7) to realize the Floquet modes in momentum space

$$\begin{aligned}&\phi_{n,m}(k_x,k_y,t) \\ &= \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\Big[m_e\dot{\zeta}(t)\zeta(t) - \xi\Big]\right) \\ &\times\int_{-L_y/2}^{L_y/2} dy\ \exp(-i[k_y - \gamma(t)]y)\chi_n[y - \mu(t)] \\ &\times\frac{1}{\sqrt{L_x}}\int_{-L_x/2}^{L_x/2} dx\ \exp(-ik_x x)\exp\left(\frac{ip_x}{\hbar}x\right).\end{aligned} \tag{B8}$$

Here we used new two parameters

$$\mu(t) = \frac{eE\sin(\omega t)}{m_e(\omega_0^2 - \omega^2)} + y_0, \tag{B9}$$

and

$$\gamma(t) = \frac{eE\omega_0^2\cos(\omega t)}{\hbar\omega(\omega_0^2 - \omega^2)}. \tag{B10}$$

Subsequenty, using the Fourier transform identity [48]

$$\int_{L_x} dx\ \exp\left(-ik_x x + \frac{ip_x}{\hbar}x\right) = L_x\delta_{k_x,\frac{p_x}{\hbar}}, \tag{B11}$$

we can derive

$$\phi_{n,m}(k_x,k_y,t) = \Phi_{n,m}(k_y,t)\delta_{k_x,\frac{p_x}{\hbar}} \times \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\left[m_e\dot{\zeta}(t)\zeta(t)-\xi\right]\right), \tag{B12}$$

where we can define $\Phi_{n,m}(k_y,t)$ as

$$\Phi_{n,m}(k_y,t) = \sqrt{L_x}\int_{-L_y/2}^{L_y/2} dy\ \chi_n[y-\mu(t)] \times \exp(-i[k_y-\gamma(t)]y). \tag{B13}$$

Substituting $k'_y = k_y - \gamma(t)$ with $y' = y - \mu(t)$, and assuming that the size of the considered 2DEG sample in y-direction is considerably large ($L_y \to \infty$), we can obtain

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\int_{-\infty}^{\infty} dy'\ \chi_n(y')\exp\left(-ik'_yy'\right). \tag{B14}$$

Moreover, we can identify that the above integral represents the Fourier transform of $\{\chi_n\}$ functions. In addition, using the symmetric conditions of the Fourier transform for Gauss-Hermite functions $\theta_n(x)$ [62]

$$\mathcal{FT}[\theta_n(\kappa x),x,k] = \frac{i^n}{|\kappa|}\theta_n(k/\kappa), \tag{B15}$$

we can simply the Eq. (B14) as

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\tilde{\chi}_n\left(k'_y\right), \tag{B16}$$

with

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_\alpha\left(\frac{k}{\kappa}\right). \tag{B17}$$

Finally, substitute Eq. (B16) back into Eq. (B12) and this leads to

$$\phi_{n,m}(k_x,k_y,t) = \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \times \exp\left(i\xi - ik_y\left[d\sin(\omega t)+\frac{\hbar k_x}{eB}\right]\right), \tag{B18}$$

where

$$b \equiv \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2-\omega^2)}, \tag{B19}$$

and

$$d \equiv \frac{eE}{m_e(\omega_0^2-\omega^2)}. \tag{B20}$$

It is necessary to notice that k_x is quantized with $k_x = 2\pi m/L_x\ ,\ m\in\mathbb{Z}$.

Appendix C: Floquet Fermi golden rule for dressed quntum Hall systems

The Floquet Fermi golden rule derivation for our quantum Hall system with the help of $t-t'$ formalism is given here in detail. The t-t'-Floquet states [30, 32]

$$|\psi_{n,m}(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)|\phi_{n,m}(t')\rangle\,, \tag{C1}$$

are derived by separating the aperiodic and periodic components of Eq. (14). Additionally, these fulfill the t-t'-Schrödinger equation [30, 32]

$$i\hbar\frac{\partial}{\partial t}|\psi_{n,m}(t,t')\rangle = H_F(t')|\psi_{n,m}(t,t')\rangle\,, \tag{C2}$$

where *Floquet Hamiltonian* defined as

$$H_F(t') \equiv H_e(t') - i\hbar\frac{\partial}{\partial t'}. \tag{C3}$$

Next we can identify the time evolution operator corresponding to the t-t'-Schrödinger equation

$$U_F(t,t_0;t') = \exp\left(-\frac{i}{\hbar}H_F(t')[t-t_0]\right), \tag{C4}$$

and the advantage of t-t' formalism lies on this time evolution operator which avoids any time ordering operators [30].

For our scenario, consider a time-independent total perturbation $V(\mathbf{r})$ which has been turned on at the reference time $t = t_0$, then the t-t'-Schrödinger equation becomes

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n,m}(t,t')\rangle = [H_F(t')+V(\mathbf{r})]|\Psi_{n,m}(t,t')\rangle\,, \tag{C5}$$

and this introduce a new wave function solution $\Psi_{n,m}$ for the system with the given total perturbation. If $t \le t_0$, both solutions of the Schrödinger equations (Eq. (C2) and Eq. (C5)) coincide

$$|\psi_{n,m}(t,t')\rangle = |\Psi_{n,m}(t,t')\rangle \quad \text{when} \quad t \le t_0. \tag{C6}$$

Now move into the interaction picture representation [47, 48] of the t-t'-Floquet state

$$|\Psi_{n,m}(t,t')\rangle_I = U_0^\dagger(t,t_0;t')|\Psi_{n,m}(t,t')\rangle\,, \tag{C7}$$

and due to time independence, the perturbation in the interaction picture has the same form as Schrödinger picture representation

$$V_I(\mathbf{r}) = U_0^\dagger(t,t_0;t')V(\mathbf{r})U_0(t,t_0;t') = V(\mathbf{r}). \tag{C8}$$

This drive us to the t-t'-Schrödinger equation representation in the interaction picture

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n,m}(t,t')\rangle_I = V_I(\mathbf{r})|\Psi_{n,m}(t,t')\rangle_I\,, \tag{C9}$$

with the recursive solution [47, 48]

$$\begin{aligned}|\Psi_{n,m}(t,t')\rangle_I = &|\Psi_{n,m}(t_0,t')\rangle_I \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; V_I(\mathbf{r})\,|\Psi_{n,m}(t_1,t')\rangle_I\,.\end{aligned} \tag{C10}$$

Iterating the solution only up to the first order (Born approximation) we obtain

$$\begin{aligned}|\Psi_{n,m}(t,t')\rangle_I \approx &|\psi_{n,m}(t_0,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; V_I(\mathbf{r})\,|\psi_{n,m}(t_0,t')\rangle\,.\end{aligned} \tag{C11}$$

In addition, since our Floquet states create a basis, we can represent any solution using these Floquet states:

$$|\Psi_\alpha(t,t')\rangle = \sum_\beta a_{\alpha,\beta}(t,t')\,|\psi_\beta(t,t')\rangle\,, \tag{C12}$$

where we used a single notation to represent two quantum numbers; $\alpha \equiv (n_\alpha, m_\alpha)$ and $\beta \equiv (n_\beta, m_\beta)$. Then we can identify the *scattering amplitude* as $a_{\alpha,\beta}(t,t') = \langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle$ and this can evaluate with

$$\begin{aligned}a_{\alpha,\beta}(t,t') = &\langle\psi_\beta(t,t')|\psi_\alpha(t,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,.\end{aligned} \tag{C13}$$

Since the t-t'-Floquet states are orthonormal and assuming $t_0 = 0$ and $\alpha \neq \beta$ this leads to

$$a_{\alpha,\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{C14}$$

Now consider a scattering phenomenon from a t-t'-Floquet state $|\psi_\beta(t,t')\rangle$ into a distinct t-t'-Floquet state $|\Psi_\alpha(t,t')\rangle$ that occupied with a constant quansienergy ε (Fig. 6):

$$\begin{aligned}|\psi_\beta(t,t')\rangle = &\exp\left(-\frac{i}{\hbar}\varepsilon_\beta t\right)|\phi_\beta(t')\rangle \\ &\xRightarrow{\text{scattering}} |\Psi_\alpha(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon t\right)|\Phi_\alpha(t')\rangle\,.\end{aligned} \tag{C15}$$

It is important to remember that a state of this considering system can be represented by two independent quantum numbers: n represents the landau level and m represents the quantized momentum in x-direction. The scattering amplitude for this scattering scenario can be calculated using the equation derived in Eq. (C14)

$$a_{\alpha\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\; e^{\frac{i}{\hbar}(\varepsilon_\beta-\varepsilon)t_1}\,\langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle\,, \tag{C16}$$

and assuming for a long time $t \to \infty$, we can turn this integral into a delta distribution

$$a_{\alpha\beta}(t') = -2\pi i\delta(\varepsilon_\beta - \varepsilon)Q, \tag{C17}$$

where $Q \equiv \langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle$ and using completeness properties we can re-write this as

$$Q = \sum_{\mathbf{k}}\sum_{\mathbf{k}'}\langle\phi_\beta(t')|\mathbf{k}'\rangle\;\langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle\;\langle\mathbf{k}|\phi_\alpha(t')\rangle\,, \tag{C18}$$

and separating x and y directional momentum we can derive (we already assumed that $L_y \to \infty$)

$$Q = \sum_{k_x}\sum_{k'_x}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\; V_{\mathbf{k}',\mathbf{k}}\phi^\dagger_\beta(\mathbf{k}',t')\phi_\alpha(\mathbf{k},t'), \tag{C19}$$

with $V_{\mathbf{k}',\mathbf{k}} \equiv \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle$.

Since we are presenting the the perturbation potential $V(\mathbf{r})$ by using a group of randomly distributed impurities, we take into account N_{imp} number of identical single impurity potentials distributed at randomly but in fixed positions $\mathbf{r}_i$. Then scattering potential $V(\mathbf{r})$ can be identified as the sum of uncorrelated single impurity potentials $\upsilon(\mathbf{r})$:

$$V(\mathbf{r}) \equiv \sum_{i=1}^{N_{imp}} \upsilon(\mathbf{r}-\mathbf{r}_i). \tag{C20}$$

Next we model the perturbation $V(\mathbf{r})$ as a Gaussian random potential where one can choose the zero of energy such that the potential is zero on average. This model is characterized by [63]

$$\langle\upsilon(\mathbf{r})\rangle_{imp} = 0, \tag{C21a}$$

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon(\mathbf{r}-\mathbf{r}'), \tag{C21b}$$

where $\langle\cdot\rangle_{imp}$ denoted the average over realizations of the impurity disorder and $\Upsilon(\mathbf{r}-\mathbf{r}')$ is any decaying function depends only on $\mathbf{r}-\mathbf{r}'$. In addition, this model assumes that $\upsilon(\mathbf{r}-\mathbf{r}')$ only depends on the position difference $|\mathbf{r}-\mathbf{r}'|$, and it decays with a characteristic length r_c. Since this study considers the case where the wavelength of radiation or scattering electrons is much greater than r_c, it is a good approximation to make two-point correlation function to be

$$\langle\upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon^2_{imp}\delta(\mathbf{r}-\mathbf{r}'), \tag{C22}$$

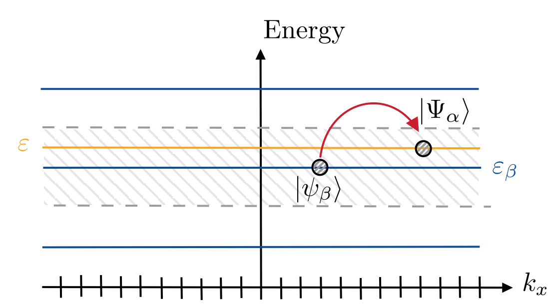

FIG. 6. Scattering from $|\psi_\beta(t,t')\rangle$ to constant energy state $|\Psi_\alpha(t,t')\rangle$ due to scattering potential created by impurities.

where Υ_{imp} is strength of the delta potential and a random potential $V(\mathbf{r})$ with this property is called white noise [63]. Then we can approximately model the total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i). \tag{C23}$$

Then we can calculate $V_{\mathbf{k}',\mathbf{k}}$ using this assumption as follows

$$\begin{aligned}
V_{\mathbf{k}',\mathbf{k}} &= \left\langle \mathbf{k}' \middle| \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i) \middle| \mathbf{k} \right\rangle && \text{(C24a)}\\
&= \sum_{i=1}^{N_{imp}} \int_{-\infty}^{\infty} dy \left[\frac{1}{\sqrt{L_x L_y}} e^{ik'_y y}\delta(y-y_i) \right.\\
&\quad \left. \times \frac{1}{\sqrt{L_x L_y}} e^{-ik_y y} \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle \right] && \text{(C24b)}\\
&= \sum_{i=1}^{N_{imp}} \frac{1}{L_x L_y} e^{i(k'_y-k_y)y} \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle . && \text{(C24c)}
\end{aligned}$$

Since $v(\mathbf{r})$ in momentum space is a constant value, each impurity produce same impurity potential for every x-directional momentum pairs and assuming the total number of scatters N_{imp} is microscopically large, we can derive

$$\begin{aligned}
V_{\mathbf{k}',\mathbf{k}} &= V_{k'_x,k_x} \frac{N_{imp}}{L_y L_x} \int_{-\infty}^{\infty} dy_i\, e^{i\left(k'_y-k_y\right)y_i} && \text{(C25a)}\\
&= \eta_{imp} V_{k'_x,k_x} \delta(k'_y-k_y), && \text{(C25b)}
\end{aligned}$$

where

$$V_{k'_x,k_x} \equiv \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle , \tag{C26}$$

is a constant value for every i impurity and η_{imp} is number of impurities in a unit area. It is important to notice that $\langle x|k_x\rangle = e^{-ik_x x}$.

Now using Eq. (10) and Eq. (C25b) on Eq. (C19), we obtain (with changing variable $t' \to t$)

$$\begin{aligned}
Q = \sum_{k_x}\sum_{k'_x} &\eta_{imp}V_{k'_x,k_x} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\ \delta(k'_y-k_y)\\
&\times \sqrt{L_x}\exp(ik'_y[d\sin(\omega t)+y'_0])\tilde{\chi}_{n_\beta}(k'_y-b\cos(\omega t))\\
&\times \sqrt{L_x}\exp(-ik_y[d\sin(\omega t)+y_0])\tilde{\chi}_{n_\alpha}(k_y-b\cos(\omega t)),
\end{aligned} \tag{C27}$$

and this can simplify as

$$Q = \sum_{k_x}\sum_{k'_x} \eta_{imp} L_x V_{k'_x,k_x} I, \tag{C28}$$

with

$$I = \int_{-\infty}^{\infty} dk_y\ \tilde{\chi}_{n_\beta}(k_y-b\cos(\omega t))\tilde{\chi}_{n_\alpha}(k_y-b\cos(\omega t)) \times \exp(-ik_y[y_0-y'_0]). \tag{C29}$$

To avoid the energy exchange from the dressing field and electrons in Landau levels, the applied radiation must be a purely dressing field. Therefore, in this study we assume that the dressing field only can renormalize the the probability of electron scattering inside a same Landau energy level ($n_\alpha = n_\beta = N$). This transform the Eq. (C29) to

$$I = \int_{-\infty}^{\infty} dk_y\ \tilde{\chi}_N^2(k_y-b\cos(\omega t))\exp(-ik_y[y_0-y'_0]). \tag{C30}$$

Using Fourier transform of Gauss-Hermite functions [62] and convolution theorem [64, 65] we can derive

$$\begin{aligned}
I = 2\pi &\exp(b[y'_0-y_0]\cos(\omega t))\\
&\times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0-y'_0-y).
\end{aligned} \tag{C31}$$

Therefore, finally the scattering amplitude derived in Eq. (C17) can be evaluated for given $k_x = 2\pi m_\alpha/L_x$ and $k'_x = 2\pi m_\beta/L_x$ as

$$\begin{aligned}
a_{\alpha\beta}(k_x,k'_x,t) = &-2\pi i\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N-\varepsilon)\\
&\times \exp(b[y'_0-y_0]\cos(\omega t))\\
&\times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0-y'_0-y).
\end{aligned} \tag{C32}$$

Since this scattering amplitude is time-periodic we can write this as a Fourier series expansion

$$a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} a^l_{\alpha\beta}(k_x,k'_x)e^{-il\omega t}. \tag{C33}$$

In addition, using Jacobi-Anger expansion [66, 67]

$$e^{iz\cos(\theta)} = \sum_{l=-\infty}^{\infty} i^l J_l(z)e^{-il\theta}, \tag{C34}$$

where $J_l(\cdot)$ are Bessel functions of the first kind with l-th integer order, and we can re-write the Eq. (C32) as follows

$$\begin{aligned}
a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} &-2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N-\varepsilon)\\
&\times J_l(b[y'_0-y_0])\\
&\times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0-y'_0-y)e^{-il\omega t},
\end{aligned} \tag{C35}$$

and then the Fourier series component can be identified as

$$\begin{aligned}
a^l_{\alpha\beta}(k_x,k'_x) = &-2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x}\\
&\times \delta(\varepsilon_N-\varepsilon)J_l(b[y'_0-y_0])\\
&\times \int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0-y'_0-y).
\end{aligned} \tag{C36}$$

Now define *transition probability matrix* as

$$(A_{\alpha\beta})^{l,l'} \equiv a^l_{\alpha\beta}\left[a^{l'}_{\alpha\beta}\right]^*, \tag{C37}$$

and this becomes

$$\begin{aligned}(A_{\alpha\beta})^{l,l'}(k_x,k'_x) =& [2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2\delta^2(\varepsilon_N-\varepsilon)\\ &\times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0])\\ &\times\left|\int_{-\infty}^{\infty}dy\;\chi_{n_\beta}(y)\chi_{n_\beta}(y_0-y'_0-y)\right|^2.\end{aligned} \tag{C38}$$

Then describing the square of the delta distribution using following interpretation [25, 29]

$$\delta^2(\varepsilon)=\delta(\varepsilon)\delta(0)=\frac{\delta(\varepsilon)}{2\pi\hbar}\int_{-t/2}^{t/2}e^{i0\times t'/\hbar}dt' \;=\frac{\delta(\varepsilon)t}{2\pi\hbar}, \tag{C39}$$

and executing the time derivation operation on each matrix element we receive the *transition amplitude matrix* elements:

$$\begin{aligned}\Gamma^{ll'}_{\alpha\beta}(k_x,k'_x) =& \frac{2\pi\eta^2_{imp}L^2_x}{\hbar}|V_{k'_x,k_x}|^2\delta(\varepsilon_\beta-\varepsilon)\\ &\times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0])\\ &\times\left|\int_{-\infty}^{\infty}dy\;\chi_N(y)\chi_N(y_0-y'_0-y)\right|^2.\end{aligned} \tag{C40}$$

An impurity average of white noise potential allows to identify $\langle|V_{k'_x,k_x}|^2\rangle_{imp}=V_{imp}$. Furthermore, the inverse scattering time matrix can be identified as the sum of all available momentum over the impurity averaged transition probability matrix element [30, 58]

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta}=\frac{1}{L_x}\sum_{k'_x}\langle\Gamma^{ll'}_{\alpha\beta}(k'_x,k_x)\rangle_{imp}, \tag{C41}$$

and applying the 1-dimensional momentum continuum limit $\sum_{k'_x}\longrightarrow L_x/2\pi\int dk'_x$ and this leads to

$$\begin{aligned}&\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta}\\ &=\frac{2\pi\eta^2_{imp}L^2_x}{\hbar}\frac{V_{imp}}{2\pi}\delta(\varepsilon_\beta-\varepsilon)\\ &=\int_{-\infty}^{\infty}dk'_x J_l\left(\frac{b\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k'_x]\right)\\ &\times\left|\int_{-\infty}^{\infty}dy\;\chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x-k_x]-y\right)\right|^2.\end{aligned} \tag{C42}$$

Using substitution $k'_x=k_1$ and $y=\hbar k_2/eB$ finally we can derive our expression for the inverse scattering time matrix for N-th Landau level as follows

$$\begin{aligned}&\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{N}\\ &=\frac{\eta^2_{imp}L^2_x\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\\ &\times\int_{-\infty}^{\infty}dk_1 J_l\left(\frac{b\hbar}{eB}[k_x-k_1]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k_1]\right)\\ &\times\left|\int_{-\infty}^{\infty}dk_2\;\chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2.\end{aligned} \tag{C43}$$

Appendix D: Current operators in dressed Landau levels

In this section we are hoping to derive the current density operator for N-th Landau level. We already found the exact solution for our time dependent Hamiltonian Eq. (1) and we identified them as Floquet states in Eq. (14). The Floquet modes derived in Eq. (10) can be represented as states using quantum number for the simplicity of notation as follows

$$|\phi_{n,m}\rangle\equiv|n,k_x\rangle\,. \tag{D1}$$

Using above complete set of eigenstates of Floquet Hamiltonian Eq. (C3) [30, 32, 52] we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'}\equiv\langle n,k_x|\,\hat{\mathbf{j}}\,|n',k'_x\rangle\,, \tag{D2}$$

and the particle current operator for our system [47, 48] can be identified as

$$\hat{\mathbf{j}}=\frac{1}{\tilde{m}}(\hat{\mathbf{p}}-e[\mathbf{A}_s+\mathbf{A}_d(t)]), \tag{D3}$$

where $\tilde{m}$ is mass of the considering particle.

First consider the conductivity in x-direction, and we can identify that x-directional current operator as

$$\hat{j}_x=\frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial x}+eBy\right). \tag{D4}$$

Now calculate the matrix elements of x-directional current operator in Floquet mode basis

$$(j_x)_{nm,n'm'}=\langle n,k_x|\,\frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial x}+eBy\right)|n',k'_x\rangle\,. \tag{D5}$$

Then evaluate these using Floquet modes derived in Eq. (7) and obtain

$$\begin{aligned}(j_x)_{nm,n'm'}=&\frac{1}{\tilde{m}}\delta_{k_x,k'_x}\int dy\,\Big[[\hbar k'_x+eBy]\\ &\times\chi_n\big(y-y_0-\zeta(t)\big)\chi_{n'}\big(y-y_0-\zeta(t)\big)\Big].\end{aligned} \tag{D6}$$

Then let $(y - y_0 - \zeta(t)) = \bar{y}$ and we can derive

$$(j_x)_{nm,n'm'} = \frac{1}{\tilde{m}}\delta_{k_x,k'_x}\int d\bar{y}\,\Big[[\hbar k'_x + eB\bar{y} - \hbar k'_x + eB\zeta(t)] \times \chi_n(\bar{y})\chi_{n'}(\bar{y})\Big]. \tag{D7}$$

Using following integral identities of Floquet modes which are made up with Gauss-Hermite functions [68, 69]

$$\int dy\; \chi_n(y)\chi_{n'}(y) = \delta_{n',n}, \tag{D8a}$$

$$\int dy\; y\chi_n(y)\chi_{n'}(y) = \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right), \tag{D8b}$$

we simplify the matrix elements of x-directional current operator to

$$(j_x)_{nm,n'm'} = \frac{1}{\tilde{m}}\delta_{k_x,k'_x}eB \times \left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \zeta(t)\delta_{n',n}\right]. \tag{D9}$$

Due to high complexity of extract solution, in this study we only consider the constant contribution. Therefore, we evaluate the $s = 0$ component of the Fourier series with

$$(j^x_{s=0})_{nm,n'm'} = \frac{eB}{\tilde{m}}\delta_{k_x,k'_x} \times \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D10}$$

For electric current operator we can apply the electron's charge and effective mass and this leads to

$$\left(j^x_{s=0}\right)^{electron}_{nm,n'm'} = \frac{e^2B}{m_e}\delta_{k_x,k'_x} \times \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D11}$$

Next we consider the transverse conductivity in y-direction, and we can identify that y-directional current operator as

$$\hat{j}_y = \frac{1}{\tilde{m}}\left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right). \tag{D12}$$

Then the matrix elements of y-directional current operator in Floquet mode basis are derived as

$$(j_y)_{nm,n'm'} = \langle n, k_x|\,\frac{-1}{\tilde{m}}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n', k'_x\rangle\,. \tag{D13}$$

After following the same steps done for x-directional current operator, we can derive the $s = 0$ component of matrix elements for y-directional electric current operator as

$$\left(j^y_{s=0}\right)^{electron}_{nm,n'm'} = \frac{ie\hbar}{m_e}\delta_{k_x,k'_x} \times \left[\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right]. \tag{D14}$$

[1] Y. Liu, N. O. Weiss, X. Duan, H. C. Cheng, Y. Huang, and X. Duan, Nat. Rev. Mater. **1**, 16042 (2016).

[2] R. T. Wijesekara, S. D. Gunapala, M. I. Stockman, and M. Premaratne, Phys. Rev. B **101**, 245402 (2020).

[3] L. Tao, Z. Chen, Z. Li, J. Wang, X. Xu, and J.-B. Xu, InfoMat **3**, 36 (2020).

[4] D. Rodrigo, O. Limaj, D. Janner, D. Etezadi, F. J. G. De Abajo, V. Pruneri, and H. Altug, Science **349**, 165 (2015).

[5] S. Pirandola, B. R. Bardhan, T. Gehring, C. Weedbrook, and S. Lloyd, Nat. Photoics **12**, 724 (2018).

[6] H. Hapuarachchi, S. Mallawaarachchi, H. T. Hattori, W. Zhu, and M. Premaratne, J. Phys. Condens. Matter. **30**, 054006 (2018).

[7] M. Yuan, M. Liu, and E. H. Sargent, Nature Energy **1**, 16016 (2016).

[8] B. Sun, O. Ouellette, F. P. G. de Arquer, O. Voznyy, Y. Kim, M. Wei, A. H. Proppe, M. I. Saidaminov, J. Xu, M. Liu, *et al.*, Nat. Commun. **9**, 4003 (2018).

[9] J. Huh, G. G. Guerreschi, B. Peropadre, J. R. McClean, and A. Aspuru-Guzik, Nat. Photonics **9**, 615 (2015).

[10] S. Slussarenko and G. J. Pryde, Appl. Phys. Rev. **6**, 041303 (2019).

[11] U. L. Andersen, Nature **591**, 40 (2021).

[12] A. Marais, B. Adams, A. K. Ringsmuth, M. Ferretti, J. M. Gruber, R. Hendrikx, M. Schuld, S. L. Smith, I. Sinayskiy, T. P. Krüger, *et al.*, J. R. Soc. Interface **15**, 20180640 (2018).

[13] F. Bian, L. Sun, L. Cai, Y. Wang, and Y. Zhao, Proc. Natl. Acad. Sci. **117**, 22736 (2020).

[14] N. Rivera and I. Kaminer, Nat. Rev. Phys. **2**, 538 (2020).

[15] V. M. Shalaev, Nat. Photonics **1**, 41 (2007).

[16] K. J. Si, D. Sikdar, Y. Chen, F. Eftekhari, Z. Xu, Y. Tang, W. Xiong, P. Guo, S. Zhang, Y. Lu, *et al.*, ACS Nano **8**, 11086 (2014).

[17] H. Hapuarachchi, S. D. Gunapala, and M. Premaratne, J. Phys. Condens. Matter. **31**, 325301 (2019).

[18] T. Perera, S. D. Gunapala, M. I. Stockman, and M. Premaratne, J. Phys. Chem. C **124**, 27694 (2020).

[19] Y. Zhang, R. E. Russo, and S. S. Mao, Appl. Phys. Lett.

87, 043106 (2005).
[20] W. W. Chow and F. Jahnke, Prog. Quantum. Electron. **37**, 109 (2013).
[21] M. Tsang, Phys. Rev. A **81**, 063837 (2010).
[22] A. Devi, S. D. Gunapala, M. I. Stockman, and M. Premaratne, Phys. Rev. A **102**, 013701 (2020).
[23] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B **84**, 235108 (2011).
[24] Y. Zhou and M. W. Wu, Phys.Rev.B **83**, 245436 (2011).
[25] O. V. Kibis, Europhys. Lett. **107**, 57003 (2014).
[26] A. A. Pervishko, O. V. Kibis, S. Morina, and I. A. Shelykh, Phys. Rev. B **92**, 205403 (2015).
[27] S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. Shelykh, Phys. Rev. B **91**, 155312 (2015).
[28] H. Dehghani, T. Oka, and A. Mitra, Phys. Rev. B **91**, 155422 (2015).
[29] K. Dini, O. V. Kibis, and I. A. Shelykh, Phys. Rev. B **93**, 235411 (2016).
[30] M. Wackerl, P. Wenk, and J. Schliemann, Phys. Rev. B **101**, 184204 (2020).
[31] M. Premaratne and G. P. Agrawal, *Theoretical Foundations of Nanoscale Quantum Devices* (Cambridge University Press, 2021).
[32] M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998).
[33] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom—Photon Interactions* (John Wiley & Sons, Ltd, 1998).
[34] M. Scully and M. Zubairy, *Introduction to Quantum Optics* (Cambridge University Press, 2001).
[35] S. Girvin and R. Prange, *The quantum Hall effect* (Springer-Verlag, New York, 1990).
[36] T. Ando, Y. Matsumoto, Y. Uemura, M. Kobayashi, and K. F. Komatsubara, J. Phys. Soc. Jpn. **32**, 859 (1972).
[37] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[38] T. Ando, J. Phys. Soc. Jpn. **36**, 1521 (1974).
[39] T. Ando, J. Phys. Soc. Jpn. **37**, 622 (1974).
[40] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1233 (1974).
[41] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[42] A. Endo, N. Hatano, H. Nakamura, and R. Shirasaki, J. Phys.: Condens. Matter **21**, 345803 (2009).
[43] A. Allerman, W. Xu, N. Hauser, and C. Jagadish, J. Appl. Phys. **77**, 2052 (1995).
[44] B. Tieke, R. Fletcher, U. Zeitler, A. Geim, M. Henini, and J. Maan, Phys. Rev. Lett. **78**, 4621 (1997).
[45] W. Pan, J. Xia, H. Stormer, D. Tsui, C. Vicente, E. Adams, N. Sullivan, L. Pfeiffer, K. Baldwin, and K. West, Phys. Rev. Lett. **95**, 066808 (2005).
[46] L. Landau, Z. Physik **64**, 629 (1930).
[47] G. D. Mahan, *Many-particle physics* (Springer Science & Business Media, New York, 2000).
[48] H. Bruus and K. Flensberg, *Many-body quantum theory in condensed matter physics: an introduction* (Oxford University Press, New York, 2004).
[49] K. Husimi, Prog. Theor. Phys. **9**, 381 (1953).
[50] T. Dittrich, P. Hänggi, G. L. Ingold, B. Kramer, G. Schön, and W. Zwerger, *Quantum Transport and Dissipation* (Wiley-VCH, Weinheim, 1998).
[51] G. Floquet, Scientific annals of the École Normale Supérieure **12**, 47 (1883).
[52] M. Holthaus, J. Phys. B: At., Mol. Opt. Phys. **49**, 013001 (2015).
[53] V. Popov and A. Perelomov, J. Exp. Theor. Phys. **30**, 910 (1970).
[54] H. Sambe, Phys. Rev. A **7**, 2203 (1975).
[55] U. Peskin, R. Kosloff, and N. Moiseyev, J. Chem. Phys. **100**, 8849 (1994).
[56] S. C. Althorpe, D. J. Kouri, D. K. Hoffman, and N. Moiseyev, Chem. Phys. **217**, 289 (1997).
[57] R. Winkler, *Spin-orbit Coupling Effects in Two-Dimensional Electron and Hole Systems* (Springer, Berlin, 2003).
[58] M. Wackerl, *Transport in Periodically Driven Systems*, Ph.D. thesis, University of Regensburg (2020).
[59] N. Tsuji, T. Oka, and H. Aoki, Phys. Rev. B **78**, 235124 (2008).
[60] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics* (Cambridge University Press, 2018).
[61] R. Shankar, *Principles of Quantum Mechanics* (Springer, New York, 1994).
[62] E. Celeghini, M. Gadella, and M. A. del Olmo, Symmetry **13**, 853 (2021).
[63] E. Akkermans and G. Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambridge University Press, 2010).
[64] G. Arfken, *Mathematical Methods for Physicists* (Academic Press, Inc. , San Diego, 1985).
[65] R. Bracewell, *The Fourier Transform and Its Applications* (McGraw-Hill, New York, 1978).
[66] A. Cuyt, V. Petersen, B. Verdonk, H. Waadeland, and W. Jones, *Handbook of Continued Fractions for Special Functions* (Springer, Dordrecht, 2008).
[67] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover, New York, 1964).
[68] V.Vedenyapin, A. Sinitsyn, and E. Dulov, *Kinetic Boltzmann, Vlasov and Related Equations* (Elsevier, Amsterdam, 2011).
[69] G. Szegö, *Orthogonal Polynomials* (American Mathematical Society, New York, 1959).