Magnetic propeties of a two dimentional electron gas strongly coupled to lights

Kosala Herath

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1 Floquet Fermi Goldern Rule

In this section we are going to derive the Floquet Fermi goldern rule for above derived quantum Floquet states using $t - t'$ formalism.

The Floquet states (**??**) fullfills the $t - t'$ Schrödinger equation [*Ref:myReport] as follows

$$i\hbar \frac{\partial}{\partial t} |\psi_\alpha(t, t')\rangle = H_F(t') |\psi_\alpha(t, t')\rangle \tag{1.1}$$

where Floquet Hamiltonian given by

$$H_F(t') \equiv H_e(t) - i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \tag{1.2}$$

and

$$|\psi_\alpha(t, t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon_\alpha t\right) |\Phi_\alpha(t')\rangle \tag{1.3}$$

Now for the Eq. (1.1) corresponding time evolution operator satisfy the Schrödinger equation

$$U_0(t, t_0; t') = \exp\left(-\frac{i}{\hbar} H_F(t')[t - t_0]\right) \tag{1.4}$$

Consider a time-independent total perturbation $V(\mathbf{r})$ switched on at the reference time $t- = t_0$, then Schrödinger equation becomes

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle = [H_F(t') + V(\mathbf{r})] |\Psi_\alpha(t, t')\rangle \tag{1.5}$$

and when $t \leq t_0$ both solutions of the Schrödinger equation coincide

$$|\psi_\alpha(t, t')\rangle = |\Psi_\alpha(t, t')\rangle \quad \text{when} \quad t \leq t_0 \tag{1.6}$$

Now, we can introduce the interaction picture representation of the $t - t'$ Floquet state as

$$|\Psi_\alpha(t, t')\rangle_I = U_0^\dagger(t, t_0; t') |\Psi_\alpha(t, t')\rangle \tag{1.7}$$

and the perturbation in the interaction picture will be

$$V_I(\mathbf{r}) = U_0^\dagger(t, t_0; t') V(\mathbf{r}) U_0(t, t_0; t') = V(\mathbf{r}). \tag{1.8}$$

This leads to the Schrödinger eqution in the interction picture

$$i\hbar \frac{\partial}{\partial t} |\Psi_\alpha(t, t')\rangle_I = V_I(\mathbf{r}) |\Psi_\alpha(t, t')\rangle_I \tag{1.9}$$

with the recursive solution

$$|\Psi_\alpha(t, t')\rangle_I = |\Psi_\alpha(t_0, t')\rangle_I + \frac{1}{i\hbar} \int_{t_0}^{t} dt_1\, V_I(\mathbf{r}) |\Psi_\alpha(t_1, t')\rangle_I \tag{1.10}$$

Iterating the solution only upto first order (Born approximation) this leads to

$$|\Psi_\alpha(t,t')\rangle_I \approx |\psi_\alpha(t_0,t')\rangle + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; V_I(\mathbf{r})\,|\psi_\alpha(t_0,t')\rangle \tag{1.11}$$

and multiply it by $\langle\psi_\beta(t_0,t')|$ and we will get

$$\langle\psi_\beta(t_0,t')|\Psi_\alpha(t,t')\rangle_I = \langle\psi_\beta(t_0,t')|\psi_\alpha(t_0,t')\rangle + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\;\; \langle\psi_\beta(t_0,t')|\,V_I(\mathbf{r})\,|\psi_\alpha(t_0,t')\rangle\,. \tag{1.12}$$

Then introdusing unitory operator U_0 we can re-write this as

$$\begin{aligned}\langle\psi_\beta(t_0,t')|U_0^\dagger(t,t_0;t')|\Psi_\alpha(t,t')\rangle = &\;\langle\psi_\beta(t_0,t')|U_0^\dagger(t,t_0;t')U_0(t,t_0;t')|\psi_\alpha(t_0,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\;\; \langle\psi_\beta(t_0,t')|\,U_0^\dagger(t_1,t_0;t')V(\mathbf{r})U_0(t_1,t_0;t')\,|\psi_\alpha(t_0,t')\rangle\end{aligned} \tag{1.13}$$

and this can be simplied as

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = \langle\psi_\beta(t,t')|\psi_\alpha(t,t')\rangle + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\;\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{1.14}$$

Since our $t-t'$ Floquet states are orthonormal [*Ref:myReport- t-t' formalism] we can derive that

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = \delta_{\alpha\beta}\exp(i\omega[t'-t]) + \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\;\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{1.15}$$

Now, set $t_0 = 0$ and for a case $\alpha \neq \beta$ this will simplied to

$$\langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = -\frac{i}{\hbar}\int_{0}^{t} dt_1\;\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{1.16}$$

In addition, since our Floquet states create a basis for composite space we can represent any solution using our Floquet states

$$|\Psi_\alpha(t,t')\rangle = \sum_\beta a_{\alpha\beta}(t,t')\,|\psi_\beta(t,t')\rangle\,. \tag{1.17}$$

Therefore we can derive a equation for this *scattering amplitude* as

$$a_{\alpha\beta}(t,t') = \langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle = -\frac{i}{\hbar}\int_{0}^{t} dt_1\;\; \langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{1.18}$$

Now lets assume a scattering event from a $t-t'$ Floquet state $|\psi_\beta(t,t')\rangle$ into another $t-t'$ Floquet state $|\Psi_\alpha(t,t')\rangle$ with constant quansienergy ε given as follows

$$|\Psi_\alpha(t,t')\rangle = \exp\!\left(-\frac{i}{\hbar}\varepsilon t\right)|\Phi_\alpha(t')\rangle \tag{1.19}$$

Now consider a scattering event

$$\psi_\beta(\mathbf{k}',t,t') = \exp\!\left(-\frac{i}{\hbar}\varepsilon_\beta t\right)\Phi_\beta(\mathbf{k}',t') \Rightarrow \Psi_\alpha(\mathbf{k},t,t') = \exp\!\left(-\frac{i}{\hbar}\varepsilon t\right)\Phi_\alpha(\mathbf{k},t') \tag{1.20}$$

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