A generalized model for the transport properties of dressed quantum Hall systems

Kosala Herath, and Malin Premaratne
Advanced Computing and Simulation Laboratory(AχL),
Department of Electrical and Computer Systems Engineering,
Monash University, Clayton, Victoria 3800, Australia
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A generalized mathematical model for the transport properties of systems exposed to a stationary magnetic and a strong electromagnetic field is presented. The new formulation, which applies to the two-dimensional dressed quantum Hall systems, is based on Landau quantization theory and Floquet-Drude conductivity method. We model our system as a two-dimensional electron gas (2DEG) that interacts with two external fields. To incorporate the strong light coupling with the 2DEG, we utilize the Floquet theory to analyze the effect non perturbatively. Moreover, the Floquet Fermi "golden rule" is adopted to explore the scattering effects for Floquet states in disordered quantum Hall systems. Based on our fully analytical expression and particular graphical representations, we demonstrate that the characteristics of conductivities in two-dimensional quantum Hall systems can manipulate using a dressed field. The outcomes align with theoretical descriptions which are already well-suited with experimental results at the same time our theory provides a more generalized analysis on the properties of conductivity in quantum Hall systems. Thus, this model more realistically describes that how to use external strong radiation as a tool to utilize transport properties in various 2D nanostructures which serve as a basis for nano-optoelectronic devices.

I. INTRODUCTION

Interactions between light and matter have dragged research attention in the fields of optoelectronics, sensing, energy harvesting, quantum computing, bio-information, and in many branches of recent technologies. For many years, the foremost aims for examing the characteristics of dressed fermion systems were focused on the different types of atomic and molecular arrangements. These researches of extreme electron-light engagements introduced an astonishing scope of twentieth-century physics namely quantum optic physics.

On the other hand, in nanostructures that are applicable in electronic devices, the investigations with the help of quantum optic were centered on polaritonic and exciton influences on nanostructures and material characteristics of dressed electrons in two-dimensional(2D) materials and quantum wires. When considering the transport characteristics of dressed nanostructures, they are still expecting extensive analysis.

Therefore, transport properties of nanostructures exposed to a high intensity periodic electromagnetic fields have been explored theoretically in this study. The dressing field is analyzed non perturbatively using the Floquet theory whilst the probing field is examined perturbatively by applying the linear response method using the Kubo formula. The general Floquet-Drude conductivity has been derived in a fully closed analytical form in most recent research [1,2], introducing a novel type of Green's functions namely four-times Green's functions. As a consequence, the established formalism introduces a novel approach to manipulate the transport characteristics of nanostructures by an intense dressing field. From an empirical sense, this study applies directly to various nanostructures illuminated by a high-intensity electromagnetic field. In this research we have developed a robust mathematical model for dressed two-dimensional electron gas(2DEG) exposed to another stationary magnetic field and that will enable efficient manipulation of transport characteristics in nanoscale electronic devices.

When a stationary magentic field applied perpendicularly across the surface of 2DEG systems, the orbital motion of electrons becomes completely quantized and the energy spectrium becomes discrete by creating Landau levels. Such a singular system known as a quantum Hall system and in this study we explictly calculate the diagonal (σ_{xx}, σ_{yy}) components of the conductivity tensor in the periodically driven quantum Hall systems.

Although there are already exist a number of adavanced theories devoted to the calculation of conductivity tensor elements in a quantum Hall systems [3-5], they have not been applied to the optically manipulation the magneto-electric properties of the quantum Hall systems. However K. Dini et al. [6] have recently investigated the one directional conductivity behaviour of dressed quantum Hall systems, they have not used the state of art model to describe the conductivity in a quantum Hall system. In their study they used the conductivity models from T. Ando et al. [3,4] and as mentioned in A. Endo et al. investigation [5] those models are far less accurate representation of the experimantally observed Landau levels because they present a semi-elliptical broadening.

In this study we develop a genralized mathematical model to describe transport properties of dressed quantum Hall systems using Floquet-Drude conductivity [1,2]. In addition, we demonstrate that our generalized model is agreed with the state of art conductivity model [5] for specalized quantum Hall system which has been considered without the external dressing field. Therefore this theory describes that the dressing field can be used as a tool to utilize transport properties in various 2D nanostructures which serve as a basis for nano-optoelectonic devices.

II. SCHRÖDINGER PROBLEM FOR LANDAU LEVELS IN DRESSED 2DEG

Our system consist of a two-dimentional free electron gas (2DEG) confined in the (x, y) plane of the three-dimentional coordinate space. In our analysis, the 2DEG is subjected to a stationary magnetic field $\mathbf{B} = (0, 0, B)^{\mathrm{T}}$ which is pointed towards the z axis. In addition a linearly polorized strong light is applied perpendicular to the 2DEG plane and we specially tune the frequency of the field ω such that the optical field behaves as a purely dressing field (nonabsorbable). Without limiting the generality we can choose y-polorized electric field $\mathbf{E} = (0, E\sin(\omega t), 0)^{\mathrm{T}}$ for the dressing field configuration (Fig. 1). Here B and E represent the amplitude of the stationary magnetic field and oscillating electric field respectively.

Using Landau gauge for the stationary magnetic field, we can represent it using vector potential as $\mathbf{A}_s = (-By, 0, 0)^{\mathrm{T}}$ and choosing Coulomb gauge, the vector potential of the dynamic dressing radiation can be presented as $\mathbf{A}_d(t) = (0, [E/\omega]\cos(\omega t), 0)^{\mathrm{T}}$. These vector potentials are coupled to the momentum of 2DEG as kinetic momentum [1, 2] and this leads to the time-dependent Hamiltonian

$$\hat{H}_e(t) = \frac{1}{2m_e}\Big[\hat{\mathbf{p}} - e\big(\mathbf{A}_s + \mathbf{A}_d(t)\big)\Big]^2, \tag{1}$$

where m_e is the effective electron mass and e is the magnitude of the electron charge. $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y, 0)^{\mathrm{T}}$ represents the canonical momentum operator for 2DEG with electron momentums $p_{x,y}$. The exact solutions for the time-dependent Schrödinger equation $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\psi = \hat{H}_e(t)\psi$ was already given by Refs. [3–5] and we can present them as a set of wave functions defined by two quantum numbers (n, m)

$$\begin{aligned}\psi_{n,m}(x, y, t) = {}& \frac{1}{\sqrt{L_x}}\chi_n\left[y - y_0 - \zeta(t)\right]\exp\bigg(\frac{i}{\hbar}\Big[-\varepsilon_n t \\ &+ p_x x + \frac{eE(y-y_0)}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] \\ &+ \int_0^t dt' L(\zeta, \dot{\zeta}, t')\Big]\bigg),\end{aligned} \tag{2}$$

where $n \in \mathbb{Z}_0^+$ and $m \in \mathbb{Z}$; see Appendix A. Here $L_{x,y}$ are dimention of the 2DEG surface, $\hbar$ is the reduced Planck constant, and $y_0 = -p_x/eB$ is the center of the cyclotron orbit along y axis. χ_n are well known solutions for Schrödinger equation of a stationary quantum harmonic oscillartor

$$\chi_n(x) \equiv \frac{\sqrt{\kappa}}{\sqrt{2^n n!}}e^{-\kappa^2 x^2/2}\mathcal{H}_n(\kappa x) \quad \text{with} \quad \kappa = \sqrt{\frac{m_e\omega_0}{\hbar}}, \tag{3}$$

with eigenvalues given by $\varepsilon_n = \hbar\omega_0(n + 1/2)$ and $\omega_0 = eB/m_e$ is the cyclotron frequency. Each n value defines the energy(ε_n) of the respective Landau level. The path shift of the driven classical oscillator $\zeta(t)$ is given by

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t), \tag{4}$$

while the Lagrangian of the classical oscialltor $L(\zeta, \dot{\zeta}, t)$ can be idenfied as

$$L(\zeta, \dot{\zeta}, t) = \frac{1}{2}m_e\dot{\zeta}^2(t) - \frac{1}{2}m_e\omega_0^2\zeta^2(t) + eE\zeta(t)\sin(\omega t). \tag{5}$$

The exponential phase shifts in Eq. (2) represent the influence done by the stationary magnetic field and strong dressing field. Therefore we can accept that magneto-transport properties of 2DEG will be renormalized by the magnetic field as well as the dressing field.

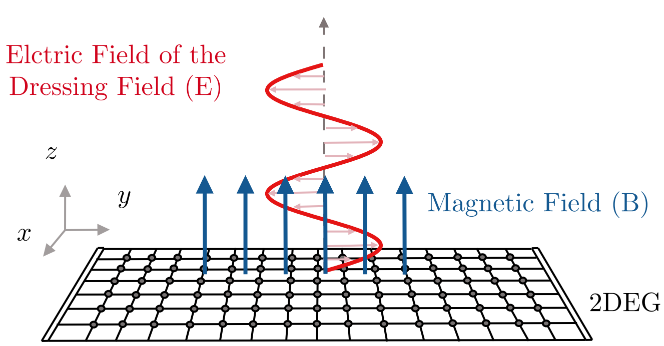

FIG. 1. Two dimentional eletron gas (2DEG) confined in the (x, y) plane while both stationary magnetic field $\mathbf{B}$ and strong dressing field with y-polorized electric field $\mathbf{E}$ are being applied perpendicular to the surface of 2DEG.

III. FLOQUET THEORY PERSPECTIVE

The general interpretations of physical systems are mostly derived using symmetry conditions in quantum theory. Famous Bloch analysis of electrons in quantum systems introduces a mathematical explanation of quantum systems occupying a discrete translational symmetry in configuration space. Floquet theory gives a mathematical formalism that can be used for translational symmetry in time rather than in space [6–8]. Examine the transport properties of systems exposed to strong radiation using the Floquet-Drude conductivity method introduced recently by M. Wackerl [9]. In their analysis they have presented more accurate results than previously existed theoretical descriptions for the conductivity in presence of a strong dressing field. Therefore, we are hoping to apply the Floquet-Drude conductivity method to analyse our 2DEG system which is subjected to both a staionary magnetic field and a strong dressing field.

First we need to identify the *quasienergies* and periodic *Floquet modes* for derived wavefunctions in Eq. (2). By factorizing the wavefunction into a linearly time dependent part and a peridic time dependend part, the quasienergies can be present as

$$\varepsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right) - \Delta_\varepsilon, \tag{6}$$

which is only depend on single quantum number (n) and Floquet modes are given by

$$\begin{aligned}\phi_{n,m}(x,y,t) =& \frac{1}{\sqrt{L_x}}\chi_n\left[y - y_0 - \zeta(t)\right]\exp\Bigg(\frac{i}{\hbar}\Bigg[p_x x \\ &+ \frac{eE(y-y_0)}{\omega}\cos(\omega t) \\ &+ m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \xi\Bigg]\Bigg),\end{aligned} \tag{7}$$

with

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2 - \omega^2)} \text{ and } \xi = \frac{(eE)^2\big(3\omega^2 - \omega_0^2\big)}{8m_e\omega(\omega_0^2 - \omega^2)^2}\sin(2\omega t). \tag{8}$$

It is important to notice that these Floquet modes are time-periodic ($T = 2\pi/\omega$) functions.

Then performing Fourier trasform over the confined two-dimentional space $A = L_x L_y$, we obtain the momentum space(k_x, k_y) representation of Floquet modes

$$\begin{aligned}\phi_{n,m}(k_x,k_y,t) =& \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \\ &\times \exp\left(i\xi - ik_y[d\sin(\omega t) + y_0]\right),\end{aligned} \tag{9}$$

where

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_n\left(\frac{k}{\kappa}\right) \tag{10}$$

and

$$b \equiv \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2 - \omega^2)} \quad d \equiv \frac{eE}{m_e(\omega_0^2 - \omega^2)}. \tag{11}$$

For details of the formalism the authors refer to Appendix B. Now using Floquet theory, the wave functions derived in Eq. (2) can be written in momentum space as Floquet states

$$\psi_{n,m}(k_x,k_y,t) = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)\phi_{n,m}(k_x,k_y,t). \tag{12}$$

IV. INVERSE SCATTERING TIME ANALYSIS

In Ref. [9] Floquet Fermi golden rule has been introduced as a method to analyse transport propeties in dressed quantum systems. However, this theory has not been applied for a dressed quantum Hall system and to identify magnetotransport properties in our system we use Floquet Fermi golden rule. With the help of $t - t'$ formalism [7, 9–12] and using Floquet states derived in Eq. (12) we can derive an expression for the inverse scattering time matrix ((l,l')th element) for the Nth Landau level, per a given energy ε and momentum k_x value for our considered quantum Hall system as

$$\begin{aligned}\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_N^{ll'} = \frac{\eta_{imp}^2 L_x^2\hbar V_{imp}}{(eB)^2}\delta(\varepsilon - \varepsilon_N)\int_{-\infty}^{\infty} dk_1 J_l\left(\frac{b\hbar}{eB}[k_x - k_1]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x - k_1]\right) \\ \times\left|\int_{-\infty}^{\infty} dk_2\, \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1 - k_x - k_2]\right)\right|^2,\end{aligned} \tag{13}$$

where $J_l(\cdot)$ are Bessel functions of the first kind with lth integer order and ε_N is the energy of Nth Landau level; see Appendix C. In this study, the perturbation potential is assumed to be formed by an ensemble of randomly distributed impurities, since random impurities in a disorded metal is a better approximation for experimental results. The total scattering potential in 2DEG is then given by the sum over uncorrelated single impurity potentials $\upsilon(\mathbf{r})$. Here η_{imp} is the number of impurities in a unit area and $V_{imp} = \left\langle |V_{k'_x,k_x}|^2\right\rangle$ with $V_{k'_x,k_x} = \langle k'_x|\upsilon(x)|k_x\rangle$ where $\langle x|k_x\rangle = e^{-ik_x x}/\sqrt{L_x}$.

Next we are going to analyse the contribution of the inverse scattering time matrix elements to the transport properies in 2DEG. The disorder in the system is not supposed to change the eigenenergies of the bare system [9], hence all off-diogonal elements of the self-energy were neglected and then we can consider only the central diagonal element ($l = l' = 0$) of the inverse scattering time matrix which has the largest contribution to the transport propeties

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{00}_{N} = \frac{\eta^2_{imp}L^2_x\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dk_1\; J_0^2\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times\left|\int_{-\infty}^{\infty}dk_2\;\chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2. \tag{14}$$

Introduce a new parameter with physical meaning of scattering-induced broadening of the Landau level as [5, 13]

$$\Gamma^{00}_N(\varepsilon,k_x)\equiv\hbar\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{00}_N, \tag{15}$$

and this leads to

$$\Gamma^{00}_N(\varepsilon,k_x) = \frac{\eta^2_{imp}L^2_x\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty}dk'_x\; J_0^2\left(\frac{b\hbar}{eB}[k_x-k'_x]\right) \times\left|\int_{-\infty}^{\infty}dk_2\;\chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k'_x-k_x-k_2]\right)\right|^2. \tag{16}$$

In addition, for the case of scattering within a same Landau level, one can present the delta distribution of the energy using the same physical interpretation [5] as follows

$$\delta(\varepsilon-\varepsilon_N)\approx\frac{1}{\pi\Gamma^{00}_N(\varepsilon,k_x)}, \tag{17}$$

and we write the central element of inverse scattering time matrix in more compact form

$$\Gamma^{00}_N(\varepsilon,k_x) = \varrho\left[\int_{-\infty}^{\infty}dk_1\; J_0^2(\lambda_1[k_x-k_1]) \times\left|\int_{-\infty}^{\infty}dk_2\;\tilde{\chi}_N(\lambda_2k_2)\tilde{\chi}_N(\lambda_2[k_1-k_2-k_x])\right|^2\right]^{-\frac{1}{2}}, \tag{18}$$

where $\varrho\equiv\eta_{imp}L_x[V_{imp}/eB]^{1/2}$, $\lambda_1\equiv\hbar b/eB$ and $\lambda_2\equiv\hbar\kappa/eB$. To analyze the contribution of dressing field on the scatering-induced broadening, normalized N-th Landau level scatering-induced broadening can be defined as

$$\Lambda_N(k_x)\equiv\frac{\Gamma^{00}_N(\varepsilon,k_x)}{\Gamma^{00}_{N=0}(\varepsilon,k_x)\big|_{E=0}}, \tag{19}$$

and this leads to

$$\Lambda_N(k_x) = \left[\frac{\int_{-\infty}^{\infty}dk_1\; J_0^2(\lambda_1[k_x-k_1])\left|\int_{-\infty}^{\infty}dk_2\;\tilde{\chi}_N(\lambda_2k_2)\tilde{\chi}_N(\lambda_2[k_1-k_2-k_x])\right|^2}{\int_{-\infty}^{\infty}dk_1\left|\int_{-\infty}^{\infty}dk_2\;\tilde{\chi}_0(\lambda_2k_2)\tilde{\chi}_0(\lambda_2[k_1-k_2-k_x])\right|^2}\right]^{1/2}. \tag{20}$$

Normalized energy band broadening against k_x for different Landau levels ($N=0,1,2,3,4$) has been calculated for GaAs-based quantum well and results are given in Fig. (2) and Fig. (4). To make comparison we have selected the experiment parameters to match with analysis done in Ref. [13]. In their study, they have assumed that the broadening of the natural(without a dressing field) 0-th Landau level Γ_0 is 0.24 me$V$. Therefore in our calculations, we assumed that the natural least Landau level broadening also has this value: $\Gamma^{00}_{N=0}|_{E=0}=0.24$ meV. Here we can identify that we can change the each Landau level normalized energy broadening value using apllied electromagnetic field. When the applied field's intensity increase the energy broadening gets reduced which make changes in conductivity of 2DEG.

V. FLOQUET-DRUDE CONDUCTIVITY IN QUANTUM HALL SYSTEMS

A general theory for the conductivity in dressed systems with the disorder averaging was introduced by M. Wackerl in Ref. [9, 14]. Within this theory, the general x-directional longitudinal DC limit conductivity can be

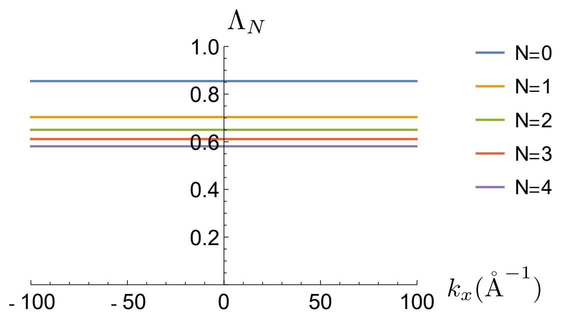

FIG. 2. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0, 1, 2, 3, 4$) against x-directional momentum value k_x in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, dressing field with frequency $\omega = 2 \times 10^{12}$ rads^{-1} and intensity $I = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 meV.

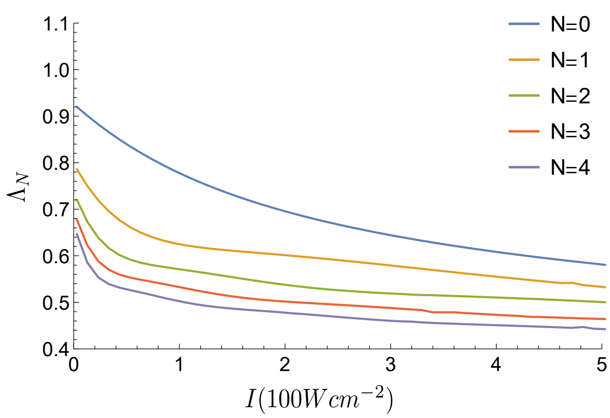

FIG. 3. The dependence of normalized scattering-induced broadening Λ_N for each Landau level ($N = 0, 1, 2, 3, 4$) against dressing field intensity I, in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, dressing field with frequency $\omega = 2 \times 10^{12}$ rads^{-1}. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 meV.

express with

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\omega/2}^{\lambda+\hbar\omega/2} d\varepsilon\Bigg[\Big(-\frac{\partial f}{\partial \varepsilon}\Big) \times \mathrm{tr}\left[j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\right]\Bigg]. \tag{21}$$

where j_0^x and $\mathbf{G}^{r,a}(\varepsilon)$ are x-directional electric current operator matrix elements' $s = 0$ th Fourier component (see Appendix D) and white noise disorder averaged Floquet Green function matrix [9, 14] respectvely defined against the Floquet modes of the considering system. Here we have assumed that only $s = 0$ th Fourier component of the current operator is contributing to the conductivity. In addition A is the area of the considering two-dimentionl system, partical distribution function is given by f and λ is a function that can be choose such that

$$\lambda - \frac{\hbar\omega}{2} \leq \varepsilon_N < \lambda + \frac{\hbar\omega}{2}, \tag{22}$$

where ε_N are quasienergies of all considering Floquet states.

Then Eq. (21) can be expanded in off resonant regime ($\omega\tau_0 \gg 1$), where τ_0 is the scattering time of the undriven system, using only central entry Fourier components ($l = l' = 0$) of Floquet modes $|\phi_{n,m}\rangle \equiv |n, k_x\rangle$

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial \varepsilon}\Big) \times \frac{1}{V_{k_x}}\sum_{k_x}\sum_{n} \langle n, k_x|j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon)-\mathbf{G}_0^a(\varepsilon))|n, k_x\rangle\,, \tag{23}$$

where V_{k_x} is the volume of considering x-directional momentum space. Then this can evalute by considering each matrix elements seperatly

$$\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial \varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\frac{1}{{V_x}^3}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3} \times \langle n, k_x|j_0^x|n_1, k_{x1}\rangle\,\langle n_1, k_{x1}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n_2, k_{x2}\rangle \times \langle n_2, k_{x2}|j_0^x|n_3, k_{x3}\rangle\,\langle n_3, k_{x3}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n, k_x\rangle\,. \tag{24}$$

Since we can diagonalize the impurity averaged Green's function using unitary trasnformation ($\mathbf{T} \equiv |n, k_x\rangle$) mentioned in Ref. [9, 14, 15] and we evaluate the matrix element of the differece between retarded and advanced Green's functions

$$\langle n_1, k_{x1}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n_2, k_{x2}\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right], \tag{25}$$

and

$$\langle n_3, k_{x3}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n, k_x\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right], \tag{26}$$

using the retared self-energy matrix $\mathbf{\Sigma}^r$ which is the sum over all irreducible diagrams [9, 14]. Applying the matrix elements of electric current operator in Landau levels (see Appendix D) and results form Eq. (25) and Eq. (26) back into Eq. (24) we obtain

$$\begin{aligned}\sigma^{xx} = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\sum_{n_1,n_2}&\\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)\delta_{n_1,n_2}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right]&\\ \times\frac{e^2B}{m_e}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})]^2}\right]&,\end{aligned} \tag{27}$$

and the only non-zero term would be

$$\begin{aligned}\frac{-1}{4\pi\hbar A}\sigma^{xx} = \frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)&\\ \times\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n+1}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})_{\varepsilon_{n+1}}\Big]^2}\right]\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T}\big)_{\varepsilon_{n}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\mathbf{\Sigma}^r\mathbf{T})_{\varepsilon_{n}}\Big]^2}\right]&.\end{aligned} \tag{28}$$

Since the inverse scattering time matrix being equal to the diagonalized differece of the retared and advanced self-energy and on the diagonal the differece of the retarded and advanced Green's function is equal to the imaginary part of the retared self-energy [9, 14] we can identify the following property

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll} = -2\mathrm{Im}\big[\mathbf{T}^\dagger\mathbf{\Sigma}^r(\varepsilon,k_x)\mathbf{T}\big]^{ll} \tag{29}$$

and using central element ($l=0$) of the inverse scattering time matrix we can modify the derived conductivity expression

$$\sigma^{xx} = \frac{1}{\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}(n+1)\left[\frac{\Gamma(\varepsilon_{n+1})}{\big(\varepsilon_F-\varepsilon_{n+1}\big)^2+\Gamma^2(\varepsilon_{n+1})}\right]\left[\frac{\Gamma(\varepsilon_{n})}{\big(\varepsilon_F-\varepsilon_{n}\big)^2+\Gamma^2(\varepsilon_{n})}\right], \tag{30}$$

with $\Gamma(\varepsilon_n,k_x)\equiv(\hbar/2\tau(\varepsilon_n,k_x))^{00}$. We already identited that the inverse scattering time matrix's central element is not k_x dependent we can get the sum over all available momentum space in x direction. However by considering the condition that the center of the force of the oscillator y_0 must physically lie within the considering system $-L_y/2<y_0<L_y/2$, we can identify

$$-\frac{m_e\omega_0 Ly}{2\hbar}\le k_x\le\frac{m_e\omega_0 Ly}{2\hbar}. \tag{31}$$

Then using Fermi-Dirac distribution as our partical distribution function (f) for this system

$$f(\varepsilon) = \frac{1}{[\exp(\varepsilon-\varepsilon_F)/k_BT]+1} \tag{32}$$

where k_B is Botlzmann constant, T is absolute tempurature and ε_F is Fermi energy of the system. Considering the above distribution with extreamly low tempurature conditions we can appromixate

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F) \tag{33}$$

and by letting $\lambda = \varepsilon_F$, the expression for conductivity leads to

$$\sigma^{xx} = \frac{e^2}{\hbar}\frac{1}{\pi A}\sum_n \frac{(n+1)}{\gamma_n \gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F - n - 1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F - n}{\gamma_n}\right)^2}\right], \tag{34}$$

where $X_F \equiv (\varepsilon_F/\hbar\omega_0 - 1/2)$ and $\gamma_n \equiv \Gamma(\varepsilon_n)/\hbar\omega_0$. Same as above derivation we can derive the transverse conductivity in y-direction by using the current operator derived in Appendix D

$$\sigma^{yy} = \frac{e^2}{\hbar}\frac{1}{\pi A}\frac{1}{e^2 B^2}\sum_n \frac{(n+1)}{\gamma_n \gamma_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F - n - 1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F - n}{\gamma_n}\right)^2}\right]. \tag{35}$$

VI. MANIPULATE CONDUCTIVITY IN QUANTUM HALL SYSTEM

To identify the characteristics of the transverse conductivity of the quantum Hall systems with external dressing field, first we can derive a expression for a normalized transverse conductivity as a function of fermi energy X_F and intensity of the dressing field I. Here we have normalized the conductivity using the natural conductivity of least Landau level.

$$\sigma^{xx}(X_F, I) = \sum_n \frac{(n+1)}{0.0037\Lambda_n \Lambda_{n+1}} \times \left[\frac{1}{1+\left(\frac{X_F - n - 1}{0.06\Lambda_n}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F - n}{0.06\Lambda_{n+1}}\right)^2}\right] \tag{36}$$

We use this expression to illustrate the changes that can be done to the transverse conductivity in 2DEG using external dressing field. As given in Fig. 4 and 5 we can manipulate the transverse conductivity σ_{xx} using external dressing field's intensity and the Fermi level X_F of the considering system. For a given dressing field intesity, the transverse conductivity vary against the Fermi level of the system by creating sharp peaks at each Landau level energy values. Since electrons are restricted to have only Landau level energies, the conductivity gets very low values when the Fermi level is not align with any of the Landau level energy values. In contrast, on

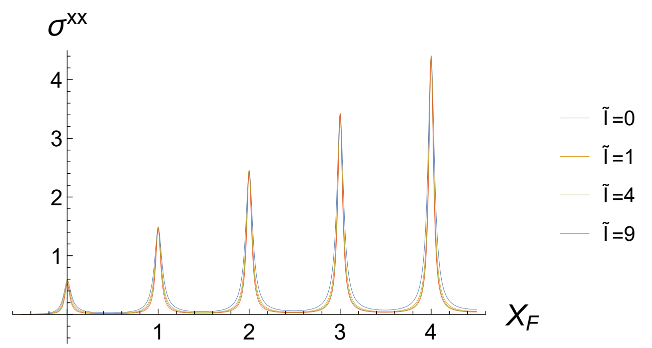

FIG. 4. Normalized transverse conductivity σ_{xx} against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, dressing field with frequency $\omega = 2 \times 10^{12}$ rads^{-1}. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$.

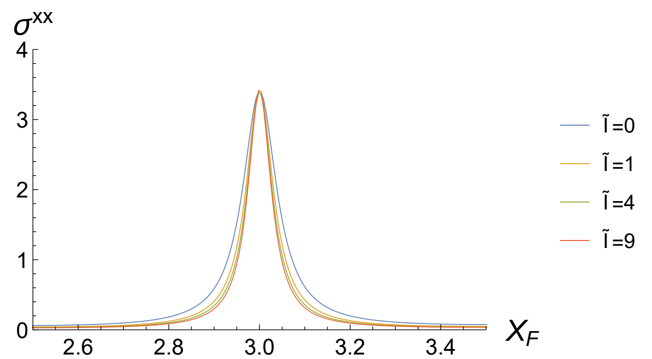

FIG. 5. 3rd Landau level's normalized transverse conductivity σ_{xx} against Fermi level X_F with different intensities I of the external dressing field in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, dressing field with frequency $\omega = 2 \times 10^{12}$ rads^{-1} and intensity $I = 100$ W/cm^2. In this calculation we have assumed that the natural broadening of 0-th Landau level Γ_0 is 0.24 me$V$

.

Landau levels conductivity can achieve very high values compared to other areas and as illustrates on Fig. 4 the peak value of transverse conductivity on each Landau level gets increase with the Landau level number.

Considering the effect of the external dressing field on transverse conductivity of 2DEG, we can identify that high intensities shrink the conductivity regions near Landau levels. However the peak value of the conductivity at each Landau level has the same value as the undressed system. This demostrate that we are able to tune the width of the regions of conductivity in these quantum Hall systems with the help of a dressing field. These characteristics are align with results demostrated by K. Dini et al.[5] and as they remarked since the Fermi level of the system can be change with the applied gate voltage of the material this can be used as a 2D switch for optoelectonic applications. Controlling the external dressing field we are able to fine-tune the switching mechanism for optimized performance. Furthermore we can distinguish that the shapes and behavoiur of the conductivity regions illustrated in Fig. 4 and 5 are generally incompatible with the results reported in Ref. [5]. This is due to the selection of the conventional transverse conductivity theory of 2DEG from Ref. [16, 17]. The semi-elliptical conductivity regions illustrated in Ref. [5, 16, 17], have less consistance with the experimentally observed Landau levels representation [13]. In our study on the transport properties of quantum Hall systems, we developed the conductivity expression starting from Floquet-Drude conductivity [9] and our results are much more align with the result represented by A. Endo et al. [13]. The description of conductivities of quantum Hall systems demostrated in Ref. [13] has excellent agreement between the theory and experiment obtained in a GaAs/AlGaAs 2DES for the low magnetic field range. However they have not consider about the tunability that can be achived with the external strong dressing field. In this analysis we account both magnetic and dressing field effects that can be applied into the transport properties of 2DEG and we have presented a more generalized theory. As a concluding remark, in this study we were able to demonstrate that using Floquet-Drude conductivity method one can derive a more experimental fitting and generalized mathematical model that describes the trasport properties of quantum Hall syatems.

VII. CONCLUSIONS

Considerable research effort in recent years has been de- voted to synthesizing materials whose thermal conductivity.

ACKNOWLEDGMENTS

We wish to acknowledge the support of the author community in using REVTEX, offering suggestions and encouragement, testing new versions, ...

Appendix A: Wave function solutions for Landau levels

The deriving process of solutions for Schrödinger equation with Hamiltonian of an electron in 2DEG (Eq. 1) quite similar to that followed in Refs. [3, 5]. We start with expanding the Hamiltonian for two-dimentional case

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[(\hat{p}_x + eBy)^2 + \left(\hat{p}_y - \frac{eE}{\omega}\cos(\omega t)\right)^2\right], \tag{A1}$$

and since $\left[\hat{H}_e(t), \hat{p}_x\right] = 0$ both operators share same (simultaneous) eigen functions $\frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar}\right)$ with $p_x = 2\pi\hbar m/L_x$, $m \in \mathbb{Z}$. Therefore we re-arrange the Hamiltonian using definition of canonical momentum in y-direction to derive

$$\hat{H}_e(t) = \frac{1}{2m_e}\left[(p_x + eBy)^2 + \left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right)^2\right]. \tag{A2}$$

We now define the *center of the cyclotron orbit* along y axis $y_0 \equiv -p_x/eB$ and the *cyclotron frequency* $\omega_0 \equiv eB/m_e$. This leads to a new arangement of the Hamiltonian

$$\hat{H}_e(t) = \frac{m_e\omega_0^2}{2}\tilde{y}^2 + \frac{1}{2m_e}\left[-\hbar^2\frac{\partial^2}{\partial\tilde{y}^2} + \frac{2i\hbar eE}{\omega}\cos(\omega t)\frac{\partial}{\partial\tilde{y}} + \frac{e^2E^2}{\omega^2}\cos^2(\omega t)\right], \tag{A3}$$

where we used a variable substitution $\tilde{y} = (y - y_0)$. Now we are assuming that the solutions for the time-dependent Schrödinger equation

$$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = \hat{H}_e(t)\psi, \tag{A4}$$

can present by the following form

$$\psi_m(x, \tilde{y}, t) = \frac{1}{\sqrt{L_x}}\exp\left(\frac{ip_x x}{\hbar} + \frac{ieE\tilde{y}}{\hbar\omega}\cos(\omega t)\right)\vartheta(\tilde{y}, t), \tag{A5}$$

where $\vartheta(\tilde{y}, t)$ is a function that need to be find to satisfy the following property

$$\left[\frac{m_e\omega_0^2}{2}\tilde{y}^2 - eE\tilde{y}\sin(\omega t) - \frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} - i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\right]\vartheta(\tilde{y}, t) = 0. \tag{A6}$$

If we turn off the strong dressing field ($E = 0$), this equation leads to simple harmonic oscillator Hamiltonian

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y}, t)}{\mathrm{d}t} = \left[\frac{\hat{p}_{\tilde{y}}^2}{2m_e} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2\right]\vartheta(\tilde{y}, t). \tag{A7}$$

It is important to notice that we can identify the $S(t) \equiv eE\sin(\omega t)$ part as a external force act on the harmonic oscillator and we can solve this as a forced harmonic oscillator in $\tilde{y}$ axis.

$$i\hbar\frac{\mathrm{d}\vartheta(\tilde{y}, t)}{\mathrm{d}t} = \left[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial\tilde{y}^2} + \frac{1}{2}m_e\omega_0^2\tilde{y}^2 - \tilde{y}S(t))\right]\vartheta(\tilde{y}, t). \tag{A8}$$

This system can be extacly solvable and we can solve the equation using the methods explained by Husimi [3] as follows. We introduce a time dependent shifted cordinate $y' = \tilde{y} - \zeta(t)$ and perform following unitary trasformation

$$\vartheta(y', t) = \exp\left(\frac{im_e\dot{\zeta}y'}{\hbar}\right)\varphi(y', t), \tag{A9}$$

and this yeilds

$$i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 + \Big[m_e\ddot{\zeta} + m_e\omega_0^2\zeta - S(t)\Big]y' + \Big[-\frac{1}{2}m_e\dot{\zeta}^2 + \frac{1}{2}m_e\omega_0^2\zeta^2 - \zeta S(t)\Big]\Big]\varphi(y',t). \tag{A10}$$

Then we can restrict our $\zeta(t)$ function such that

$$m_e\ddot{\zeta} + m_e\omega_0^2\zeta = S(t) \tag{A11}$$

and that leads to

$$i\hbar\frac{\partial\varphi(y',t)}{\partial t} = \Big[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 y'^2 - L(\zeta,\dot{\zeta},t)\Big]\varphi(y',t) \tag{A12}$$

where

$$L(\zeta,\dot{\zeta},t) \equiv \frac{1}{2}m_e\dot{\zeta}^2 - \frac{1}{2}m_e\omega_0^2\zeta^2 + \zeta S(t) \tag{A13}$$

is the largrangian of a classical driven oscillator. To proceed further, another unitary trasform can be introduced as follows

$$\varphi(y',t) = \exp\bigg(\frac{i}{\hbar}\int_0^t dt' L(\zeta,\dot{\zeta},t')\bigg)\chi(y',t), \tag{A14}$$

and subtiting Eq. (A14) back in Eq. (A12) yeilds

$$i\hbar\frac{\partial}{\partial t}\chi(y',t) = \bigg[-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y'^2} + \frac{1}{2}m_e\omega_0^2 {y'}^2\bigg]\chi(y',t). \tag{A15}$$

This is the well known Schrödinger equation of the quantum harmonic oscillator. This allows us to identify with the well-known eigenfucntions [18, 19]

$$\chi_n(y) \equiv \frac{\sqrt{\kappa}}{\sqrt{2^n n!}}e^{-\kappa^2y^2/2}\mathcal{H}_n(\kappa y) \quad \text{with} \quad \kappa = \sqrt{\frac{m_e\omega_0}{\hbar}}, \tag{A16}$$

which are propositional to the Hermite polynomials $\mathcal{H}_n$, with eigenvalues

$$\varepsilon_n = \hbar\omega_0\big(n + \frac{1}{2}\big)\ ,\ n \in \mathbb{Z}_0^+. \tag{A17}$$

Therefore we can identify the solutions of Eq. (A8) as

$$\vartheta_n(\tilde{y},t) = \chi_n(\tilde{y} - \zeta(t))\exp\bigg(\frac{i}{\hbar}\Big[-\varepsilon_n t + m_e\dot{\zeta(t)}\big(\tilde{y} - \zeta(t)\big) + \int_0^t dt' L(\zeta,\dot{\zeta},t')\Big]\bigg) \tag{A18}$$

The set $\{\chi_n(x)\}$ functions forms a complete set and thus any general solution $\vartheta_(\tilde{y},t)$ can be expaned in terms of the solutions given in Eq. (A18).

Finally we consider our scenario where we assumed that $S(t) = eE\sin(\omega t)$ and we can derive the solution for Eq. (A11)

$$\zeta(t) = \frac{eE}{m_e(\omega_0^2 - \omega^2)}\sin(\omega t). \tag{A19}$$

Subtiting solutions in Eq. (A18) back in Eq. (A5), we can obtain the set of wave functions with two different quantum number $(n{,}m)$ that satisfy the Schrödinger eqution Eq. (A4)

$$\psi_{n,m}(x,y,t) = \frac{1}{\sqrt{L_x}}\chi_n\left[y - y_0 - \zeta(t)\right]\exp\bigg(\frac{i}{\hbar}\Big[-\varepsilon_n t + p_x x + \frac{eE(y-y_0)}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y - y_0 - \zeta(t)\big] + \int_0^t dt' L(\zeta,\dot{\zeta},t')\Big]\bigg). \tag{A20}$$

Appendix B: Floquet modes and quasienergies

1. Position space representation

First define the time integral of Laggrangian of the classical oscillator given in Eq. (5), over a $T = 2\pi/\omega$ period as

$$\Delta_\varepsilon \equiv \frac{1}{T}\int_0^T dt'\ L(\zeta,\dot{\zeta},t'), \tag{B1}$$

and after performing the integral using Eq. (4), we can obtain more simplified result:

$$\Delta_\varepsilon = \frac{(eE)^2}{4m_e(\omega_0^2 - \omega^2)}. \tag{B2}$$

Next define another paramter

$$\xi \equiv \int_0^t dt'\ L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t, \tag{B3}$$

and after simplying, this leads to

$$\xi = \frac{(eE)^2\big(3\omega^2 - \omega_0^2\big)}{8m_e\omega(\omega_0^2 - \omega^2)^2}\sin(2\omega t), \tag{B4}$$

which is a periodic function in time with 2ω frequency. Now using these parmaters we can factorize the wavefunction Eq. (2) as linearly time dependend part and periodic time dependend part as follows

$$\psi_\alpha(x,y,t) = \exp\bigg(\frac{i}{\hbar}[-\varepsilon_n t + \Delta_\varepsilon t]\bigg)\frac{1}{\sqrt{L_x}}\chi_n\big(y - y_0 - \zeta(t)\big) \times \exp\bigg(\frac{i}{\hbar}\Big[p_x x + \frac{eEy}{\omega}\cos(\omega t) + m_e\dot{\zeta(t)}\big[y - \zeta(t)\big] + \int_0^t dt' L(\zeta,\dot{\zeta},t') - \Delta_\varepsilon t\Big]\bigg), \tag{B5}$$

and this leads to seperate linear time dependent phase component as the quasienergies

$$\varepsilon_n = \hbar\omega_0\bigg(n + \frac{1}{2}\bigg) - \Delta_\varepsilon \tag{B6}$$

while rest of the components as time-periodic Floquet modes

$$\phi_{n,m}(x,y,t) \equiv \frac{1}{\sqrt{L_x}}\chi_n\left[y-y_0-\zeta(t)\right]\exp\Bigg(\frac{i}{\hbar}\bigg[p_x x + \frac{eE(y-y_0)}{\omega}\cos(\omega t) + m_e\dot{\zeta}(t)\big[y-y_0-\zeta(t)\big]+\xi\bigg]\Bigg). \tag{B7}$$

2. Momentum space representation

To write the Floquet modes in momentum space, we perform continuous Fourier transform over the considering confined space $A = L_x L_y$ for Eq. (7)

$$\phi_{n,m}(k_x,k_y,t) = \int_{-L_y/2}^{L_y/2} dy\ \exp(-i[k_y-\gamma(t)]y)\chi_n[y-\mu(t)] \times \frac{1}{\sqrt{L_x}}\int_{-L_x/2}^{L_x/2} dx\ \exp(-ik_x x)\exp\left(\frac{ip_x}{\hbar}x\right) \times \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\left[m_e\dot{\zeta}(t)\zeta(t)-\xi\right]\right), \tag{B8}$$

where

$$\mu(t) \equiv \frac{eE\sin(\omega t)}{m_e(\omega_0^2-\omega^2)} + y_0 \quad , \quad \gamma(t) \equiv \frac{eE\omega_0^2\cos(\omega t)}{\hbar\omega(\omega_0^2-\omega^2)}. \tag{B9}$$

Next using the identity [2]

$$\int_{L_x} dx\ \exp\left(-ik_x x + \frac{ip_x}{\hbar}x\right) = L_x\delta_{k_x,\frac{p_x}{\hbar}}, \tag{B10}$$

we can derive

$$\phi_{n,m}(k_x,k_y,t) = \exp\left(\frac{-i\gamma(t)}{\hbar}y_0\right)\exp\left(\frac{-i}{\hbar}\left[m_e\dot{\zeta}(t)\zeta(t)-\xi\right]\right) \times \Phi_{n,m}(k_y,t)\delta_{k_x,\frac{p_x}{\hbar}}. \tag{B11}$$

Here we defined $\Phi_{n,m}(k_y,t)$ as

$$\Phi_{n,m}(k_y,t) \equiv \sqrt{L_x}\int_{-L_y/2}^{L_y/2} dy\ \chi_n[y-\mu(t)] \times \exp(-i[k_y-\gamma(t)]y). \tag{B12}$$

Subtituting $k'_y = k_y - \gamma(t)$ and $y' = y - \mu(t)$ and assuming that size of the 2DEG sample in y-direction is large ($L_y \to \infty$), we can obtain

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\int_{-\infty}^{\infty} dy'\ \chi_n(y')\exp\big(-ik'_y y'\big). \tag{B13}$$

We can identify that the integral represnts the Fourier transform of $\{\chi_n\}$ functions and using the symmetric conditions [20] for the Fourier transform of Gauss-Hermite functions $\theta_n(x)$:

$$\mathcal{FT}[\theta_n(\kappa x),x,k] = \frac{i^n}{|\kappa|}\theta_n(k/\kappa), \tag{B14}$$

Eq. (B13) can be simplified as

$$\Phi_{n,m}(k'_y,t) = \sqrt{L_x}e^{-ik'_y\mu}\tilde{\chi}_n\big(k'_y\big), \tag{B15}$$

with

$$\tilde{\chi}_n(k) = \frac{i^n}{\sqrt{2^n n!\sqrt{\pi}}}\left(\frac{1}{\kappa}\right)^{1/2} e^{-\frac{k^2}{2\kappa^2}}\mathcal{H}_\alpha\left(\frac{k}{\kappa}\right). \tag{B16}$$

Subtitute Eq. (B15) back in Eq. (B11) and this leads to

$$\phi_{n,m}(k_x,k_y,t) = \sqrt{L_x}\tilde{\chi}_n(k_y - b\cos(\omega t)) \times \exp\left(i\xi - ik_y\left[d\sin(\omega t) + \frac{\hbar k_x}{eB}\right]\right), \tag{B17}$$

where

$$b \equiv \frac{eE\omega_0^2}{\hbar\omega(\omega_0^2-\omega^2)} \qquad d \equiv \frac{eE}{m_e(\omega_0^2-\omega^2)}, \tag{B18}$$

and it is necessary to notice that k_x is quantized with $k_x = 2\pi m/L_x$, $m \in \mathbb{Z}$.

Appendix C: Floquet Fermi Golden Rule

The derivation of the Floquet Fermi golden rule for our quantum Hall system with the help of $t-t'$ formalism is given here in detail. The t-t'-Floquet states [7, 9]

$$|\psi_{n,m}(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon_n t\right)|\phi_{n,m}(t')\rangle\,. \tag{C1}$$

derived by seperating the aperiodic and periodic components of Eq. (12), fullfill the t-t'-Schrödinger equation [7, 9]

$$i\hbar\frac{\partial}{\partial t}|\psi_{n,m}(t,t')\rangle = H_F(t')\,|\psi_{n,m}(t,t')\rangle\,, \tag{C2}$$

where *Floquet Hamiltonian* defined as

$$H_F(t') \equiv H_e(t') - i\hbar\frac{\partial}{\partial t'}. \tag{C3}$$

Next we can identify the the time evolution operator corresponding to the t-t'-Schrödinger equation

$$U_F(t,t_0;t') = \exp\left(-\frac{i}{\hbar}H_F(t')[t-t_0]\right), \tag{C4}$$

and the advantage of t-t' formalism lies on this time evolution operator which avoids any time odering operatos [9].

For our scenario, consider a time-independent total perturbation $V(\mathbf{r})$ which has been switched on at the reference time $t = t_0$, then the t-t'-Schrödinger equation becomes

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n,m}(t,t')\rangle = [H_F(t') + V(\mathbf{r})]\,|\Psi_{n,m}(t,t')\rangle\,, \tag{C5}$$

by introducing new wave function $\Psi_{n,m}$ for the system with the given total perturabation. If $t \leq t_0$, both solutions of the Schrödinger equations (Eq. (C2) and Eq. (C5)) coincide

$$|\psi_{n,m}(t,t')\rangle = |\Psi_{n,m}(t,t')\rangle \quad \text{when} \quad t \leq t_0. \tag{C6}$$

Now move into the interaction picture representation [1, 2] of the t-t'-Floquet state

$$|\Psi_{n,m}(t,t')\rangle_I = U_0^\dagger(t,t_0;t')\,|\Psi_{n,m}(t,t')\rangle\,, \tag{C7}$$

and due to time independency, the perturbation in the interaction picture has the same form as Schrödinger picture

$$V_I(\mathbf{r}) = U_0^\dagger(t,t_0;t')V(\mathbf{r})U_0(t,t_0;t') = V(\mathbf{r}). \tag{C8}$$

This leads to the t-t'-Schrödinger eqution in the interction picture

$$i\hbar\frac{\partial}{\partial t}\,|\Psi_{n,m}(t,t')\rangle_I = V_I(\mathbf{r})\,|\Psi_{n,m}(t,t')\rangle_I\,, \tag{C9}$$

with the recursive solution [1, 2]

$$\begin{aligned}|\Psi_{n,m}(t,t')\rangle_I =& \,|\Psi_{n,m}(t_0,t')\rangle_I \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; V_I(\mathbf{r})\,|\Psi_{n,m}(t_1,t')\rangle_I\,.\end{aligned} \tag{C10}$$

Iterating the solution only upto the first order (Born approximation) we obtain

$$\begin{aligned}|\Psi_{n,m}(t,t')\rangle_I \approx& \,|\psi_{n,m}(t_0,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\; V_I(\mathbf{r})\,|\psi_{n,m}(t_0,t')\rangle\,.\end{aligned} \tag{C11}$$

In addition, since our Floquet states create a basis we can represent any solution using these Floquet states

$$|\Psi_\alpha(t,t')\rangle = \sum_\beta a_{\alpha,\beta}(t,t')\,|\psi_\beta(t,t')\rangle\,. \tag{C12}$$

where we used a single notation to represent two quantum numbers; $\alpha \equiv (n_\alpha, m_\alpha)$ and $\beta \equiv (n_\beta, m_\beta)$. Then we can identify the *scattering amplitude* as $a_{\alpha,\beta}(t,t') = \langle\psi_\beta(t,t')|\Psi_\alpha(t,t')\rangle$ and this can evaluate with

$$\begin{aligned}a_{\alpha,\beta}(t,t') =& \,\langle\psi_\beta(t,t')|\psi_\alpha(t,t')\rangle \\ &+ \frac{1}{i\hbar}\int_{t_0}^{t} dt_1\;\langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,.\end{aligned} \tag{C13}$$

Since the t-t'-Floquet states are orthonormal and assuming $t_0 = 0$ and $\alpha \neq \beta$ this leads to

$$a_{\alpha,\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\;\langle\psi_\beta(t_1,t')|\,V(\mathbf{r})\,|\psi_\alpha(t_1,t')\rangle\,. \tag{C14}$$

Now consider a scattering event from a t-t'-Floquet state $|\psi_\beta(t,t')\rangle$ into another t-t'-Floquet state $|\Psi_\alpha(t,t')\rangle$ with constant quansienergy ε:

$$\begin{aligned}|\psi_\beta(t,t')\rangle =& \exp\left(-\frac{i}{\hbar}\varepsilon_\beta t\right)|\phi_\beta(t')\rangle \\ &\longrightarrow |\Psi_\alpha(t,t')\rangle = \exp\left(-\frac{i}{\hbar}\varepsilon t\right)|\Phi_\alpha(t')\rangle\,.\end{aligned} \tag{C15}$$

It is important to remember that a state of this considering system can be represented by two independent quantum numbers: n represents the landau level and m represents the qunatized momentum in x-direction. The scattering amplitutde for this scattering scenario can be calculated using the equation derived in Eq. (C14)

$$a_{\alpha\beta}(t,t') = -\frac{i}{\hbar}\int_0^t dt_1\; e^{\frac{i}{\hbar}(\varepsilon_\beta-\varepsilon)t_1}\,\langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle\,, \tag{C16}$$

and assuiming for a long time $t \to \infty$, we can turn this integral into a delta distrubution

$$a_{\alpha\beta}(t') = -2\pi i\delta(\varepsilon_\beta - \varepsilon)Q, \tag{C17}$$

where $Q \equiv \langle\phi_\beta(t')|\,V(\mathbf{r})\,|\phi_\alpha(t')\rangle$ and using completeness properties we can re-write this as

$$Q = \sum_{\mathbf{k}}\sum_{\mathbf{k}'}\langle\phi_\beta(t')|\mathbf{k}'\rangle\;\langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle\;\langle\mathbf{k}|\phi_\alpha(t')\rangle\,, \tag{C18}$$

and seperating x and y directional momentums we can derive (we already assumed that $L_y \to \infty$)

$$Q = \sum_{k_x}\sum_{k'_x}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\; V_{\mathbf{k}',\mathbf{k}}\phi_\beta^\dagger(\mathbf{k}',t')\phi_\alpha(\mathbf{k},t'). \tag{C19}$$

with $V_{\mathbf{k}',\mathbf{k}} \equiv \langle\mathbf{k}'|V(\mathbf{r})|\mathbf{k}\rangle$.

Since the perturbation potential $V(\mathbf{r})$ is assumed to be formed by an ensemble of randomly distributed impurities, consider N_{imp} identical impurities positioned at randomly distributed but fixed positions $\mathbf{r}_i$. Then scattering potential $V(\mathbf{r})$ is given by the sum over uncorrelated single impurity potentials $\upsilon(\mathbf{r})$

$$V(\mathbf{r}) \equiv \sum_{i=1}^{N_{imp}}\upsilon(\mathbf{r}-\mathbf{r}_i). \tag{C20}$$

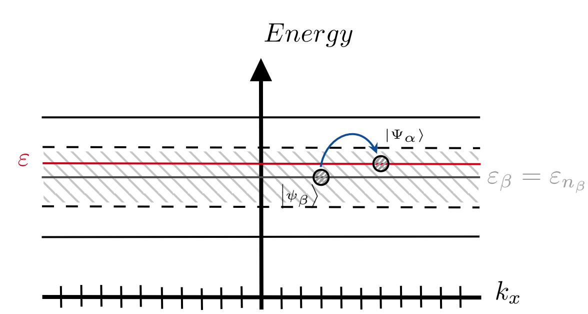

FIG. 6. Scattering from $|\psi_\beta(t,t')\rangle$ to constant energy state $|\Psi_\alpha(t,t')\rangle$ due to scattering potential created by impurities.

Next we model the perturbation $V(\mathbf{r})$ as a Gaussian random potential where one can choose the zero of enerrgy such that the potential is zero on average. This model is characterized by [21]

$$\langle \upsilon(\mathbf{r})\rangle_{imp} = 0 \tag{C21a}$$

$$\langle \upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon(\mathbf{r}-\mathbf{r}') \tag{C21b}$$

where $\langle\cdot\rangle_{imp}$ denoted the average over realizations of the impurity disorder and $\Upsilon(\mathbf{r}-\mathbf{r}')$ is any decaying function depends only on $\mathbf{r}-\mathbf{r}'$. In addition, this model assume that $\upsilon(\mathbf{r}-\mathbf{r}')$ only dependes on the position difference $|\mathbf{r}-\mathbf{r}'|$ and it decays with a characteristic leangth r_c. Since this study considers the case where the waveleagth of radiation or scattering electrons is much faster than r_c, it is a good approximation to make two-point correlation function to be

$$\langle \upsilon(\mathbf{r})\upsilon(\mathbf{r}')\rangle_{imp} = \Upsilon_{imp}^2\delta(\mathbf{r}-\mathbf{r}') \tag{C22}$$

where Υ_{imp} is stregth of the delta potential and a random potential $V(\mathbf{r})$ with this property is called white noise [21]. Then we can model approximately the total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i). \tag{C23}$$

Then we can calculate $V_{\mathbf{k}',\mathbf{k}}$ using this assumption as follows

$$V_{\mathbf{k}',\mathbf{k}} = \left\langle \mathbf{k}' \left| \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r}-\mathbf{r}_i) \right| \mathbf{k} \right\rangle \tag{C24a}$$

$$= \sum_{i=1}^{N_{imp}} \int_{-\infty}^{\infty} dy \left[\frac{1}{\sqrt{L_x L_y}} e^{ik'_y y}\delta(y-y_i) \times \frac{1}{\sqrt{L_x L_y}} e^{-ik_y y} \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle \right] \tag{C24b}$$

$$= \sum_{i=1}^{N_{imp}} \frac{1}{L_x L_y} e^{i(k'_y-k_y)y} \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle . \tag{C24c}$$

Since $\upsilon(\mathbf{r})$ in momentum space is a constant value, each impurity produce same impurity potential for every x-directional momentum pairs and assuming the total ncumber of scatterers N_{imp} is macroscopically large, we can derive

$$V_{\mathbf{k}',\mathbf{k}} = V_{k'_x,k_x} \frac{N_{imp}}{L_y L_x} \int_{-\infty}^{\infty} dy_i\ e^{i\left(k'_y - k_y\right)y_i} \tag{C25a}$$

$$= \eta_{imp} V_{k'_x,k_x} \delta(k'_y - k_y), \tag{C25b}$$

where

$$V_{k'_x,k_x} \equiv \langle k'_x|\Upsilon_{imp}\delta(x-x_i)|k_x\rangle . \tag{C26}$$

is a constant value for every i impurity and η_{imp} is number of impurities in a unit area. It is important to notice that $|k_x\rangle = e^{-ik_x x}$.

Now using Eq. (9) and Eq. (C25) on Eq. (C19), we obtain (with changing varable $t' \to t'$)

$$Q = \sum_{k_x}\sum_{k'_x} \eta_{imp} V_{k'_x,k_x} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_y dk'_y\ \delta(k'_y - k_y) \times \sqrt{L_x}\exp(ik'_y[d\sin(\omega t)+y'_0])\tilde{\chi}_{n_\beta}(k'_y - b\cos(\omega t)) \times \sqrt{L_x}\exp(-ik_y[d\sin(\omega t)+y_0])\tilde{\chi}_{n_\alpha}(k_y - b\cos(\omega t)), \tag{C27}$$

and this can simplify as

$$Q = \sum_{k_x}\sum_{k'_x} \eta_{imp} L_x V_{k'_x,k_x} I, \tag{C28}$$

with

$$I \equiv \int_{-\infty}^{\infty} dk_y\ \tilde{\chi}_{n_\beta}(k_y - b\cos(\omega t))\tilde{\chi}_{n_\alpha}(k_y - b\cos(\omega t)) \times \exp(-ik_y[y_0 - y'_0]). \tag{C29}$$

To avoid the energy exchange from external strong field and electrons, the applied radiation should be a purely dressing field. Therefore, the only effect of the dressing field on 2DEG is the renormalization of the probability of elastic electron scattering within the same Landau level ($n_\alpha = n_\beta = N$). This transform the Eq. (C29) to

$$I = \int_{-\infty}^{\infty} dk_y\ \tilde{\chi}_N^2(k_y - b\cos(\omega t))\exp(-ik_y[y_0 - y'_0]). \tag{C30}$$

Using Fourier transform of Gauss-Hermite functions [20] and convolution theorem [22, 23] we can derive

$$I = 2\pi\exp(b[y'_0 - y_0]\cos(\omega t)) \times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0 - y'_0 - y). \tag{C31}$$

Therefore finally the scattering amplitude derived in Eq. (C17) can be evaluated for given $k_x = 2\pi m_\alpha/L_x$ and $k'_x = 2\pi m_\beta/L_x$ as

$$a_{\alpha\beta}(k_x,k'_x,t) = -2\pi i\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N - \varepsilon) \times \exp(b[y'_0 - y_0]\cos(\omega t)) \times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0 - y'_0 - y), \tag{C32}$$

Since this scattering amplitude is time-periodic we can write this as a Fourier series expansion

$$a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} a^l_{\alpha\beta}(k_x,k'_x)e^{-il\omega t}. \tag{C33}$$

In addition, using Jacobi-Anger expansion [24, 25]

$$e^{iz\cos(\theta)} = \sum_{l=-\infty}^{\infty} i^l J_l(z)e^{-il\theta}, \tag{C34}$$

where $J_l(z)$ are Bessel functions of the first kind with l-th integer order and we can re-write the Eq. (C32) as folllows

$$a_{\alpha\beta}(k_x,k'_x,t) = \sum_{l=-\infty}^{\infty} -2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x}\delta(\varepsilon_N-\varepsilon) \times J_l(b[y'_0-y_0]) \times \int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0-y'_0-y)e^{-il\omega t} \tag{C35}$$

and then the Fourier series component can be identified as

$$a^l_{\alpha\beta}(k_x,k'_x) = -2\pi i^{l+1}\eta_{imp}L_x V_{k'_x,k_x} \times \delta(\varepsilon_N-\varepsilon)J_l(b[y'_0-y_0]) \times \int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0-y'_0-y). \tag{C36}$$

Now define *transition probability matrix*

$$(A_{\alpha\beta})_{l,l'} \equiv a^l_{\alpha\beta}\left[a^{l'}_{\alpha\beta}\right]^*, \tag{C37}$$

and this becomes

$$(A_{\alpha\beta})_{l,l'}(k_x,k'_x) = [2\pi\eta_{imp}L_x|V_{k'_x,k_x}|]^2\delta^2(\varepsilon_N-\varepsilon) \times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0]) \times \left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}(y_0-y'_0-y)\right|^2. \tag{C38}$$

Then desribing the square of the delta distribution using following procedure [5, 26]

$$\delta^2(\varepsilon) = \delta(\varepsilon)\delta(0) = \frac{\delta(\varepsilon)}{2\pi\hbar}\int_{-t/2}^{t/2} e^{i0\times t'/\hbar}dt' = \frac{\delta(\varepsilon)t}{2\pi\hbar}, \tag{C39}$$

and performing the time derivation of each matrix element yeild the *transition amplitude matrix*:

$$\Gamma^{ll'}_{\alpha\beta}(k_x,k'_x) = \frac{2\pi\eta^2_{imp}L^2_x}{\hbar}|V_{k'_x,k_x}|^2\delta(\varepsilon_\beta-\varepsilon) \times J_l(b[y'_0-y_0])J_{l'}(g[y'_0-y_0]) \times \left|\int_{-\infty}^{\infty} dy\ \chi_N(y)\chi_N(y_0-y'_0-y)\right|^2. \tag{C40}$$

An impurity average of white noise potential allows to identify $\left\langle |V_{k'_x,k_x}|^2\right\rangle = V_{imp}$ and the inverse scattering time matrix is the sum over all momentum over the transition probability matrix

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} \equiv \frac{1}{L_x}\sum_{k'_x}\langle\Gamma^{ll'}_{\alpha\beta}(k'_x,k_x)\rangle_{imp} \tag{C41}$$

and applying the 1-dimentional momentum continuum limit $\sum_{k'_x} \longrightarrow L_x/2\pi\int dk'_x$ and this leads to

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{\alpha\beta} = \frac{2\pi\eta^2_{imp}L^2_x}{\hbar}\frac{V_{imp}}{2\pi}\delta(\varepsilon_\beta-\varepsilon)\int_{-\infty}^{\infty} dk'_x J_l\left(\frac{b\hbar}{eB}[k_x-k'_x]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k'_x]\right) \times \left|\int_{-\infty}^{\infty} dy\ \chi_{n_\beta}(y)\chi_{n_\beta}\left(\frac{\hbar}{eB}[k'_x-k_x]-y\right)\right|^2. \tag{C42}$$

Using subtitution $k'_x = k_1$ and $y = \hbar k_2/eB$ finally we can derive our expression for the inverse scattering time matrix for N-th Landau level

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)^{ll'}_{N} = \frac{\eta^2_{imp}L^2_x\hbar V_{imp}}{(eB)^2}\delta(\varepsilon-\varepsilon_N)\int_{-\infty}^{\infty} dk_1 J_l\left(\frac{b\hbar}{eB}[k_x-k_1]\right)J_{l'}\left(\frac{b\hbar}{eB}[k_x-k_1]\right) \times \left|\int_{-\infty}^{\infty} dk_2\ \chi_N\left(\frac{\hbar}{eB}k_2\right)\chi_N\left(\frac{\hbar}{eB}[k_1-k_x-k_2]\right)\right|^2. \tag{C43}$$

Appendix D: Current operator in Landau levels

In this section we are hoping to derive the current density operator for N-th Landau level. We already found the extact solution for our time depenedent Hamiltonian Eq. (1) and we identified them as Floquet states with quesienergies Eq. (12). The Floquet modes derived in Eq. Eq. (9) can be represented as states using quantum number for the simplicity of notation as follows

$$|\phi_{n,m}\rangle \equiv |n,k_x\rangle \tag{D1}$$

Using above complete set of eigenstates of Floquet Hamiltonian Eq. (C3) [7–9] we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'} \equiv \langle n,k_x|\,\hat{\mathbf{j}}\,|n',k_x'\rangle\,, \tag{D2}$$

and particle current operator for our system [1, 2] by

$$\hat{\mathbf{j}} = \frac{1}{m}(\hat{\mathbf{p}} - e[\mathbf{A}_s + \mathbf{A}_d(t)]). \tag{D3}$$

where m is mass of the considering particle.

First consider the transverse conductivity in x-direction and we can identify that x-directional current operator as

$$\hat{j}_x = \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right). \tag{D4}$$

Now calculate the matrix elements of x-directional current operator in Floquet mode basis

$$(j_x)_{nm,n'm'} = \langle n,k_x|\,\frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right)|n',k_x'\rangle\,. \tag{D5}$$

Then evaluate these using Floquet modes derived in Eq. (7) and obtain

$$\begin{aligned}(j_x)_{nm,n'm'} =& \frac{1}{m}\delta_{k_x,k_x'}\int dy\,\Big[(\hbar k_x' + eBy)\\ &\times \chi_n\big(y-y_0-\zeta(t)\big)\chi_{n'}\big(y-y_0-\zeta(t)\big)\Big].\end{aligned} \tag{D6}$$

Then let $y - y_0 - \zeta(t) = \bar{y}$ and we can derive

$$\begin{aligned}(j_x)_{nm,n'm'} =& \frac{1}{m}\delta_{k_x,k_x'}\int d\bar{y}\,\Big[(\hbar k_x' + eB\bar{y} - \hbar k_x' + eB\zeta(t))\\ &\times \chi_n(\bar{y})\chi_{n'}(\bar{y})\Big].\end{aligned} \tag{D7}$$

Using following integral identities of Floquet modes which are made up with Gauss-Hermite functions [27, 28]

$$\int dy\;\chi_n(y)\chi_{n'}(y) = \delta_{n',n} \tag{D8a}$$

$$\int dy\;y\chi_n(y)\chi_{n'}(y) = \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right), \tag{D8b}$$

we simplfy the matrix elements of x-directional current operator to

$$\begin{aligned}(j_x)_{nm,n'm'} =& \frac{1}{m}\delta_{k_x,k_x'}eB\Bigg[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right)\\ &+ \zeta(t)\delta_{n',n}\Bigg]\end{aligned} \tag{D9}$$

Due to high complexity of extract solution, in this study we only consider the constant contribution. Therefore we evaluate the $s=0$ component of the Fourier series with

$$(j^x_{s=0})_{nm,n'm'} = \frac{eB}{m}\delta_{k_x,k_x'}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D10}$$

For electric current operator we can apply the electron's charge and effective mass and this leads to

$$\left(j^x_{s=0}\right)^{elctron}_{nm,n'm'} = \frac{e^2B}{m_e}\delta_{k_x,k_x'}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D11}$$

Next we consider the transverse conductivity in y-direction and we can identify that y-directional current operator as

$$\hat{j}_y = \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right). \tag{D12}$$

Then the matrix elements of y-directional current operator in Floquet mode basis are derived as

$$(j_y)_{nm,n'm'} = \langle n,k_x|\,\frac{-1}{m}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n',k_x'\rangle\,. \tag{D13}$$

After following the same steps done for x-directional current operator, we can derive the $s=0$ component of matrix elements for y-directional electric current operator

$$\left(j^y_{s=0}\right)^{elctron}_{nm,n'm'} = \frac{ie\hbar}{m_e}\delta_{k_x,k_x'}\left[\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right]. \tag{D14}$$

x

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