November 15, 2021

Ashot Melikyan,
Associate Editor,
Physical Review B.

Dear Professor Melikyan,

Thank you very much for your effort in managing the review process of our manuscript. We are also thankful for the reviewer comments, and believe the second version of the manuscript we submit herewith has been significantly improved by the constructive criticism we received. Additionally, the subsequent sections of this document discuss the reviewer comments and our responses to them.

Please note that in the following sections, the statements in **blue** are the comments of the reviewers. Our responses are shown in black letters, and the modifications we have done to the manuscript are given in red.

General changes to the manuscript

reviewers`

We have made minor changes in language and presentation to improve clarity, and to match the rest of the manuscript better to the changes done to address the reviewer's comments.

- Section I - seventh paragraph (page 2):
  In Sec. VII, we discuss the physical significance of our theoretical results and their possible employments in future nanoelectronic devices. Finally, we summarize our findings and present our conclusions in Sec. VIII.
- Section VI - fourth paragraph (page 8):
  Considering the effects of the applied dressing field on the longitudinal conductivity of 2DEG, we can identify that the dressing field has sharpened the conductivity peaks. the
- Section VIII - first paragraph (page 10):
  Finally, we derived analytical expressions for the diagonal components of electric conductivity tensor concerning the 2DEG quantum Hall system under low temperatures.

Response to the comments of Reviewer 1

We would like to thank the reviewer for bringing the deficiencies of our manuscript to our attention and providing constructive feedback to improve the quality of our work. We have considered all of your suggestions seriously and revised our manuscript as described below.

We agree that our work has a theoretical bias, and the reader may benefit from some application perspective from our theoretical results to characterize or design nanoelectronic devices. Therefore, we have discussed the physical significance of our theoretical results and their possible use in optimizing nanoelectronic device performance. First, we added a detailed comparison between our theoretical results and experiment observation when no radiation is present. We also added a new section (Section VII) on this very aspect.

Comm ... **ewed towards a purely mathe-matic** ... **on to realistic two-dimensional electr** ... **sults can be applied to under-stand** ... **es and can be used to optimize devic** ... **will have a minimal impact on the c** ...

~~We str ... ults and their application in current nanoel ... e made a discussion on the physical signific ... the optimization of nanoelectronic device ... theoritical results and experiment observ ... ction VII) to incorporate the above discuss ... below,~~

- Section VI - third paragraph (page 8):
  By comparing the theoretical [1–7] and experimental [7–13] studies on the magnetoresistance of 2DEG

of

when no radiation is present against

analyzed

quantum Hall systems ~~under zero radiation with~~ our results, we can identify that longitudinal conductivity oscillations in Fig. 4 are a repetition of the Shubnikov–de Haas(SdH) oscillations. As observed ... mental work ~~done by~~ Caviglia *et al.* [13], the SdH oscillations period only depends on ... cular component of the applied magnetic field. Therefore, we can identify t... ~~done in low-temperatures [7, 9–13] with a different type of 2DEG have been examined~~ ... ~~cillations against the applied magnetic field amplitude.~~ The magnetic field's perpendicular ... ~~mplitude defines the cyclotron frequency (ω_0)~~ of the ~~considering~~ system. Therefore we can achieve the same oscillatory behavior for longitudinal conductivity in 2DEG quantum Hall system by changing the gate voltage. The ~~change of the~~ gate voltage applied to the system ~~will modify~~ the Fermi level of the system. Since the Landau level energy is only dependent on the cyclotron frequency, this ~~will generate the same SdH oscillations against gate voltage~~. This oscillatory behavior with applied gate voltage has been observed in an experiment done by Wakabayashi *et al.* [8] in an inversion layer on a silicon surface at low temperature. By comparing those ~~empirical~~ observations with our results ~~given~~ in Fig. 4, we can identify that our oscillations also show ~~are also shown~~ the same characteristic behavior with $\hbar\omega_0$ periodicity against Fermi level value. Furthermore, when we increase the gate voltage, the Fermi level ~~of the system gets increased~~. This will locate to the Fermi level on higher-order Landau levels. As illustrated in Fig. 4, this will result in a higher conductivity peak value at the higher-order Landau levels. This same behavio gate voltage changes generates SdH oscillations. ...perimental observations presented in Ref. [8] ~~as well.~~ ~~However, in our analysis, we have analytically explained~~ the controllability of these conductivity regions using a dressing field.

The cyclotron frequency \omega_0 of the system depends on the ....

modifies

rises.

; i.e.

- Section VII (page 9):

this makes no sense: perpendicular to what? propagation direction, or something else, please rewrite

I do not understand this sentence. rewrite

VII. PHYSICAL SIGNIFI... OUTCOMES

With the realization of 2DEGs in Si-MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measurements on such systems to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new ... condensed matter physics that continues to produce phenomenal advancements in electronic ... The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus ...d to integer values of the conductivity quantum $(e^2/\hbar)$ in the off-diagonal conductivity, with ...eously peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening in low-temperatures is the disord... The broaden sequence of delta functions of density of states implies the osc... experime... longitudinal conductivity which is known as S...) oscillati...

In our work, we provide analytical results describing

can describe the experimentally observed

Strikingly, we show in Fig. 5 that

Our theoretical analysis on longitudinal conductivity behavior of dressed quantum Hall system developed by considering low-temperature limit with gaussian impurity broadening assumptions. As illustrated in Fig. 4, we ~~were~~ also able to demonstrate the same SdH oscillations ~~as experimental~~ results ~~observed~~ in Refs.[7, 8] through our model. Under the undressed condition, our results overlap with the conductivity measurement for quantum Hall systems [7]. ~~However, from our results given in Fig. 5, we demonstrate that~~ we can manipulate the broadening of these conductivity peaks using an external dressing field. In low-temperature ... broadening of these conductivity peaks is impurity scattering and using an e... can suppress the scattering which results in shrinkage of both the scattering-in... e longitudinal conductivity peaks.

, we can exert significant control over the conductivity externally.

Research on novel states of matter has driv... nt-day nanoelectronic devices. In particular, controllable manipulation of mat... a gate electric field has revolutionized the development of material science and technology [18, 19]. The charge carrier concentration of a ~~considering~~ system is an imperative parameter that defines ~~the conductivity properties of the system.~~ We can manipulate ~~that~~ using the electrostatic field~~-effect mechanism~~ ~~and it is an ideal tool to control the conductivity in some specific systems~~. A 2DEG under static magnetic field with quantum Hall

As

the charge carrier concentration

of

an

its conductivity properties.

We also observe that

illustrates how

controls

a

studies exists on

effects is … t the gate electric field ~~can be used to manipulate the~~ conductivity. A considerable number of researches ~~have been performed~~ using different types of 2D field-effect transistors (FETs) in magnetic fields to study the charge transport in the quantum limit [8, 20–22]. ~~In the study done by~~ Yang *et al.* [20], the ~~authors have observed~~ quantized Hall plateaus and Shubnikov–de Haas oscillations for longitudinal conductivity against gate voltage in black phosph… static magnetic fields in low-temperatures. Since the Fermi level of the system can be … applied gate voltage, this ~~behavior can be easily mapped into our results given in Fig. 4. However, the specialty of our outcomes is we owned the capability of manipulating~~ the broadening of the conductivity regions using an external dressing field. Although Yang *et al.* [20], achieved broadening in longitudinal conductivity peaks by changing the temperature in a low range, in this study we presented a general ~~mathematical description to manipulate~~ longitudinal conductivity broadening using ~~only~~ a high-intensity electromagnetic field.

theory on manipulating

study shows

our Fig. 4 depicts this result.

high-intensity electromagnetic

…e realization of the underlying mechanism of 2D FETs in the quantum realm pr… in next-generation nanoelectronic applications. In a particular application that uses the switching mechanism of … FETs wi… , we can achieve high and low output conductivities by changing the input gate voltage. As a result of manipulating the broadening of conductivity regions, we can shrink the broadening of conductivity peaks around Landau levels using a ~~high-intensity~~ dressing field. This will enhance the sensitivity of FETs which provides the ability to …erve narrow changes in gate voltage. Furthermore, adopting the mechanism presented in Ref. [23], … can ~~utilize our manipulation of~~ conductivity peaks into very sensitive, narrowband high-frequency …iation detectors. ~~Based on these ready-to-use nanoelectronic devices and their feasible optimization, … believe that our mathematical description offers the potential to realize advanc… …ices.~~ Furthermore, this theoretical model will help to develop simulation tools th… quantum effects in magnetotransport properties of 2D nanostructures.

limit

manipulate

In particular, our analysis shows that we can manipulate

We envision these advances in nanoelectronics from our theoretical model and knowledge generated from the insight it provides to the underlying dependencies.

We envision these advances in nanoelectronics from our theoretical advances and its eminating applications.

… Moreover, this current research direction has a significant overla… … and theoretical studies of quantum Hall systems that started with … …nce states in high mobility systems [Zudov et al, Phys. Rev. B 64, … …hys. Rev. Lett. 92, 146801 (2004)] and gave rise to theoretical … [Durst et a… …1, 086803 (2003), Dmitriev et al. … …005), Dmit… … B 80, 165327 (2009)]. The pres… …ect to vari… …a discussed earlier in the literatu… Hall systems.

work

We agree with the reviewer that it is useful to have a discussion

comparing

~~We thank the reviewer for p… …essity of comparison between~~ our theoretical model and previous discussions on tra… …ntum Hall systems. Among the mentioned studies we can identify that experimental researches [24–27] are specifically aimed at the unusual oscillations of the magnetoresistance induced by the microwave (millimeter-wave) radiation in 2DEG quantum Hall systems. These oscillations are known as *microwave-induced resistance oscillations* (MIROs). To describe these behaviors Durst *et al.* [28] introduced a simple theoretical model assuming that the experimentally observed oscillations are a consequence of photoexcited disorder-scattered electrons. However, later a novel model was proposed in Refs. [29–31] considering the changes of the electron distribution function done by the micro… …eneralized models [29–31] have succ… …havior of MIROs in experimentally relevant temperatures which were missed in the previous model [28]. Furthermore, we can recognize that the underlying mechanism of all these models [28–31] is microwave photon absorption by an electron in the considering system. ~~However~~, in our theoretical model, we take into account higher frequencies than microwaves as the dressing field. Therefore we can identify ~~major~~ dissimilaritie… our system ~~under the analysis~~ and MIROs systems [24–31]. ~~We listed these dissimilarities as follo… comparison~~.

at

They are:

and quantify signficant

- The ~~mentioned~~ experiments on magnetoresistance oscillations on 2DEG quantum Hall systems [24–27] were performed in the microwave frequency range (30 – 150 GHz). This leads to building the theoretical models presented in Refs. [28, 29] by assuming that these oscillations are caused by photoexcited …ctrons. Since these models consider on relevant frequency range (microwave radiation), it will allow us to acknowledge the photon absorption by electrons. In contrast to that, our consideration is only

small case the

work reported in

that

associated

consider

in our analysis.

small case the

electromagnetic

focused on the systems with high-frequency range dressing fields which will not be assoc[illegible] [illegible]oton absorption.

- The applied microwave radiation power on the MIROs experiments [25, 26] are varies around the $1\,\mathrm{mW\,cm^{-2}}$ range. Ho[illegible] take the dressing field as a high-intensity field, where we can not address t[illegible]rturbation ~~to our considering system~~. This leads us to use recognize new [illegible] with conventional Landau levels. As we mentioned in our results, we used h[illegible]ds [illegible]e $100\,\mathrm{W\,cm^{-2}}$ range in our numerical calculations.

what do you mean by "use recognize" rewrite sentence

influence of

dressing high-intensity electromagnetic field

- These fascinating MIROs are only observed under the weak magnetic fields ($B < 0.2\,\mathrm{T}$) in the e[illegible]ments performed in Refs. [24–27]. In this range of we[illegible]n only observe M[illegible] ~~and~~ Shubnikov–de Haas (SdH) oscillations reveal only on high-intensity magnetic fields. In comp[illegible] to our analysis, we are interested in the SdH oscillations and manipulation of [illegible] Ther[illegible] aim at the effects induced by high-intensity magne[illegible]

,

small case in

leads to two distinguishable magnetotransport scenarios.

pure

- In our work, we analyzed the 2DEG q[illegible]h a high-intensity [illegible] does not ~~contribute to a energy~~ exchange between the high-frequency dressing [illegible] Therefor[illegible] applied electromagnetic radiation as a purely [illegible] are two possible absorption mechanisms in a 2DEG quantum Hall system namely electron transitions between distinct Landau levels and electron transitions between distinct states of the same broadened Landau level. To avoid these absorptions we have tuned the dressing field into a high-frequency range in our system under the analysis. Furthermore, due to the high-intensity dressing f[illegible] enti[illegible]e conventional 2DEG quantum Hall system will be restructured. We have add[illegible]ns through the Floquet theory. However, the MIROs models [28, 29] are based on low-frequency cases where the system is able to absorb low-frequency photons from the field.

,

scenarios

Based on the above discussion,

as

~~Examining the above-mentioned points~~, we can identify that the high-frequency and low-frequency illumination of 2DEG quantum Hall systems ~~leads to two particular natures of magnetotransport properties~~. ~~However, we identify that it is crucial to acknowledge these well-defined findings in MIROs and their relationship to our considered system. We have added this clarification in the manuscript.~~

;

- Section I - fifth paragraph (page 1):
  Recently, experimental research on the effects of microwave illumination of 2DEG systems revealed microwave-induced resistance oscillations (MIRO) under weak magnetic fields [24–27]. This inspired investigations on theoretical description of MIROs, and several semiclassical and q[illegible] equation formalisms have been proposed [illegible]ng mechanism of [illegible] These ~~analytical~~ formalisms provide a prope[illegible] the experimental observations of MIROs. However, these experimental and theoretical works have been linked to photon absorption from low-frequency (microwave) fields. In contrast to that, high-frequency external illumination on a 2DEG quantum Hall system [illegible] purely dressing (nonabsorbable) field. The impacts [illegible]y dressing field on magnetotransport properties of 2DEG quantum Hall system need to be described by a non absorption ~~mechanism~~ and it has escaped the researchers' attention before. Lately, Dini *et al.* [32] have investigated the one-directional conductivity behavior of dressed quantum Hall system ~~under~~ high-frequency field.

pure

influence of

subjected to a

,

- Section VI - sixth paragraph (page 9):
  Lately, several experimental [24–27] and theoretical [28–31] studies have uncovered various remarkable magnetotransport properties induced by microwave radiation on 2DEG quantum Hall systems. However, these experiments and theoretical models only examined the behavior of MIROs in 2DEG systems which are based on photon absorption from the applied low-frequency field. It is important to state that the difference between our SdH oscillations and MIROs [24–27] by considering the frequency range of the applied electromagnetic field. In our case, the applied dressing field is in the off-resonant range and that will only change the broadening of Landau levels ~~and~~ avoid any contribution towards the photon absorptions. This will clearly describe the crucial difference between low-frequency illumination and high-frequency illumination effects on 2DEG quantum systems. Therefore our theoretical analysis can help to fill the gap in knowledge of the high-frequency dressing field effects on 2DEG quantum Hall systems.

,

, but

it

This is a valuable comment.

Comment 3 - The manuscript will also provide more impact if it demonstrates how the new results can help to improve the future development of nanoelectronic devices. After these questions are addressed, the manuscript will be suitable for publication in Physical Review B. Otherwise, it will fit better to a more mathematically oriented journal.

~~We thank the reviewer for pointing out the importance of these facts that will help to elevate the value of our work.~~ As we mentioned in comment 1, considering the importance of discussing the physical significance of our results, we added a new section (Section VII) to address ~~these facts.~~

this comment.

included

- Section VII (page 8):

This section - please refer to my previous edits as it is a repeat.

VII. PHYSICAL SIGNIFICANCE OF T[illegible]

With the realization of 2DEGs in Si-MOSFETs (Metal[illegible]eld Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measu[illegible] to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new realm in condensed matter physics that continues to produce phenomenal advancements in electronic systems. The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus quantized to integer values of the conductivity quantum $(e^2/\hbar)$ in the off-diagonal conductivity, with simultaneously peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening in low-temperatures is the disorders in materials [16, 17]. The broaden sequence of delta functions of density of states implies the oscillating behavior in the experimental measurements of longitudinal conductivity which is known as Shubnikov–de Haas (SdH) oscillations. [7, 8].

Our theoretical analysis on longitudinal conductivity behavior of dressed quantum Hall system developed by considering low-temperature limit with gaussian impurity broadening assumptions. As illustrated in Fig. 4, we were also able to demonstrate the same SdH oscillations as experimental results observed in Refs. [7, 8] through our model. Under the undressed condition, our results overlap with the conductivity measurement for quantum Hall systems [7]. However, from our results given in Fig. 5, we demonstrate that we can manipulate the broadening of these conductivity peaks using an external dressing field. In low-temperatures, the principal cause of broadening of these conductivity peaks is impurity scattering and using an external dressing field we can suppress the scattering which results in shrinkage of both the scattering-induced broadening and the longitudinal conductivity peaks.

Research on novel states of matter has driven the evolution of present-day nanoelectronic devices. In particular, controllable manipulation of material properties through a gate electric field has revolutionized the development of material science and technology [18, 19]. The charge carrier concentration of a considering system is an imperative parameter that defines the conductivity properties of the system. We can manipulate that using the electrostatic field-effect mechanism and it is an ideal tool to control the conductivity in some specific systems. A 2DEG under static magnetic field with quantum Hall effects is an excellent example that the gate electric field can be used to manipulate the conductivity. A considerable number of researches have been performed using different types of 2D field-effect transistors (FETs) in magnetic fields to study the charge transport in the quantum limit [8, 20–22]. In the study done by Yang *et al.* [20], the authors have observed quantized Hall plateaus and Shubnikov–de Haas oscillations for longitudinal conductivity against gate voltage in black phosphorus FET under static magnetic fields in low-temperatures. Since the Fermi level of the system can be altered with the applied gate voltage, this behavior can be easily mapped into our results given in Fig. 4. However, the specialty of our outcomes is we owned the capability of manipulating the broadening of the conductivity regions using an external dressing field. Although Yang *et al.* [20], achieved broadening in longitudinal conductivity peaks by changing the temperature in a low range, in this study we presented a general mathematical description to manipulate longitudinal conductivity broadening using only a high-intensity electromagnetic field.

The realization of the underlying mechanism of 2D FETs in the quantum realm promises its potential in next-generation nanoelectronic applications. In a particular application that uses the switching mechanism of the above-discussed FETs with quantum Hall effects, we can achieve high and low output conductivities by changing the input gate voltage. As a result of manipulating the broadening of conductivity regions, we can shrink the broadening of conductivity peaks around Landau levels using a high-intensity dressing field. This will enhance the sensitivity of FETs which provides the ability to observe narrow changes in gate voltage. Furthermore, adopting the mechanism presented in Ref. [23], we can utilize our manipulation of conductivity peaks into very sensitive, narrowband high-frequency radiation detectors. Based on these ready-to-use nanoelectronic devices and their feasible optimization, we believe that our mathematical description offers the potential to realize advanced nanoelectronic devices. Furthermore, this theoretical model will help to develop simulation tools that will design the quantum effects in magnetotransport properties of 2D nanostructures.

modelled

Comment 4 - The quantum Quantum Hall effect requires high mobility samples. In these [illegible]s, the structure of the disorder is usually complicated and combines both short-length potentials of impurities and [illegible] homogeneities. The interplay of these comp[illegible] [illegible]t the transport properties of 2DEGs. What [illegible] [illegible]present manuscript and hidden in the notations for Vimp? What ar[illegible]ty of eq. (15)?

here again, we ilucitat

However, here we again elucidate our disorder model and approximations made to derive the Eq. [15] for clarity and to

We thank the reviewer for raising [illegible] our previous manuscript, we have presented the detailed derivation of Eq. [15] [illegible] disorder under Appendix C. ~~However we will discuss our selection of disorder m~~[illegible]~~de to derive the Eq. [15] here as well.~~

I do not understand this sentence, please rewrite. It has no meaning now.

We modeled the effect caused by impurities in the considered system as a single perturbation potential. [illegible]g the electric properties for a specific impurity configuration is a rather formidable task and is not [illegible]l in this work since it is unlikely to have exactly the evaluated impurity configuration , . Therefore, in this study, we consider the statistically averaged properties of 2DEG over impurity [illegible]tions. In addition, we consider only our considering static disorder corresponds to the situation [illegible] the electrons scatter elastically. First, we adapt the Edwards model [33] to represent [illegible] assume [illegible]stributed impurities over the considering system and we approximate this into a Gaussian [illegible] noise.

precisely

,

Since we are presenting the perturbation potential $V(\mathbf{r})$ by a group of randomly localized impurities, we ~~take into account~~ N_{imp} number of identical single impurity potentials distributed randomly but in fixed positions $\mathbf{r}_i$. Thus, we can describe the scattering potential $V(\mathbf{r})$ as the sum of uncorrelated single impurity potentials $v(\mathbf{r})$

Kosala, V(r) is the scattering potential. So, what is perturbation V(r) here means

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} v \quad [illegible] \tag{1}$$

Furthermore, we model the perturbation $V(\mathbf{r})$ as a Gau[illegible] [illegible]here one [illegible] which solely zero of energy such that the potential is zero on average. [illegible] characterized by the [illegible] equations [illegible]

, and

it is reasonable to approximate its

$$[illegible] \tag{2a}$$

$$[illegible]\mathbf{r}-\mathbf{r}'), \tag{2b}$$

where $\langle\cdot\rangle_{imp}$ represents the average over the impurity disorder and $\Upsilon(\mathbf{r}-\mathbf{r}')$ is any decaying function depends only on $\mathbf{r}-\mathbf{r}'$. ~~In addition, this model assumes that $v(\mathbf{r}-\mathbf{r}')$ only depends on the magnitude of the position difference $|\mathbf{r}-\mathbf{r}'|$, and it~~ decays with a characteristic length r_c. Since this study considers the case where the wavelength of radiation or scattering electron is much greater than r_c, ~~it is a better approximation to make a~~ two-point correlation function be by

$$\langle v(\mathbf{r})v(\mathbf{r}')\rangle_{imp} = \Upsilon^2_{imp}\delta(\mathbf{r}-\mathbf{r}'), \tag{3}$$

where Υ^2_{imp} is a constant. A random potential $V(\mathbf{r})$ with this property is called white noise [33]. Then we

positive

of a

magneto-transport

write

Using this expression, we can evaluate

Furthermore, considering the properties of a Gaussian white noise impurity distribution [33.34], we assume Vimp is a constant in the momentum space.

ca[...] [...]e total scattering potential as

$$V(\mathbf{r}) = \sum_{i=1}^{N_{imp}} \Upsilon_{imp}\delta(\mathbf{r} - \mathbf{r}_i).$$

~~Then we calculate~~ the Floquet-Fermi golden rule for a dressed quantum Hall system usin[...] ~~impurity potential. In the derivation, we have defined the V_{imp} value which is constant in~~ ~~It is a property of Gaussian white noise impurity distribution [33, 34].~~ However, through[...] we use only the first-order contribution (Born approximation) of the impurity potentia[...] steps for the derivation of the Floquet-Fermi golden rule for a dressed quantum Hall system are included in

results

intriguing

Such a choice enables us

Since previous studies on Floquet-Drude conductivity [34], and magnetotransport pr[...] [7] and dressed [32] quantum Hall systems have used this particular Gaussian white noise potential, we also selected this pertucular impurity model to describe our system. ~~This opens a path~~ to compare our analytical outcomes with these previous models[...] in the comment, consideration of other impurity disorder models and their impa[...]ort properties of dresse[...] [...]em would be an interesting future resea[...]

. We do not consider a simple impurity distribution here as

revised

distribution

Since we have only mentioned the validity condi[...]e Eq. [15] in Appendix C and not in the main text of the previous manuscript, we [...] conditions to the main text of the new manuscript.

have modelled

represent a measured

- Section IV - first paragraph (page 4):
  We ~~modeled~~ the effect caused by impurities ~~in the considered system~~ as a single short-range perturbation potential. Analyzing the electric properties for a specific impurity configuration is a rather formidable task ~~and is not examined in this work since~~ it is unlikely ~~to have exactly the evaluated~~ impurity configuration in an experiment. Therefore, in this study, we consider the statistically averaged properties of 2DEG over impurity configurations. ~~Furthermore, we assumed that our pe[...]~~ ~~potential is formed by a group of randomly distributed impurities under the Edwards impurity model~~ ~~[33, 34].~~ ~~Thus, we represented~~ the total scattering potential in the 2DEG as a sum of uncorrelated single impurity potentials $v(\mathbf{r})$. ~~Then we~~ approximate the impurity potential into a Gaussian white noise [33, 34]. ~~Here~~ η_{imp} is the number of impurities in a unit area, $V_{imp} = \langle |V_{k'_x,k_x}|^2\rangle_{imp}$ with $V_{k'_x,k_x} = \langle k'_x|v(x)|k_x\rangle$, and $\langle x|k_x\rangle = e^{-ik_x x}$. Moreover, in this analysis, $\langle\cdot\rangle_{imp}$ represents the average over the impurity disorder. In this derivation, we only considered the first order (the Born approximation) contribution from the impurity potential.

as

Furthermore, we have assumed that a group of randomly distributed impurities forms our perturbation potential under the Edwards impurity model [33, 34].

This enables us to

Essentially, all these assumptions means that we can write

where

[...]nt 5 - If continuous illumination is applied to an el[...] [...]tem will heat [...]tely. The [...]is stabilized when [...] and electron- [...] scattering mechanisms are included. What are [...]idity for eqs. [...]? The deviations of the distribution function from i[...] [...]n effects that can significantly overcome the equilibrium contributions, s[...] [...]s. Rev. B 80, 165327 (2009). However, to capture these effects, a treatme[...] [...]ude formalism is insufficient and a complete quantum kinetic equation ha[...] [...]n the authors argue why their results still present some interest to the [...] [...]ey potentially disregard more significant contributions?

This is a good, valid question. First, we discuss the validity of the Eq. [33] and Eq. [34]. ~~In our analysis,~~ ~~we have~~ derived the longitudinal conductivity of a dressed quantum Hall system without any partial distribution function specification until Eq. [31]. In contrast to the MIROs analysis [29–31], we consider the dressed quantum Hall system under the off-resonant condition, ~~where the absorption of photons are avoided.~~ Therefore, we can select the Fermi-Dirac distribution as our particle distribution function ~~on the general~~ ~~expression given in~~ Eq. [31]

we consider

operates at

for

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \varepsilon_F)/k_BT] + 1}. \tag{5}$$

Here, k_B is the Boltzmann constant, T is the absolute temperature and ε_F is the Fermi energy of the system. Then ~~we selected~~ a special scenario where ~~we assumed that~~ the system ~~is under~~ a low-temperature limit or

Our analysis

,

but photon absorption does not happen.

i.e.,

[illegible]ition $k_B T \ll \varepsilon_F$. Since previous theoretical studies [7, 32, 34] on magnetotransport properties of [illegible]uantum Hall systems were derived under this assumption, we selected this co[illegible] comparison with their outcomes. In addition, as mentioned in previous experi[illegible] SdH oscillations [11, 25, 26] in 2DEG undressed quantum Hall systems with increasing temperature, we can identify the decaying behavior of SdH oscillations. Therefore, if we need to maintain a good oscillatory behavior we need to work in low temperatures. Since in our analysis, we study the manipulation of SdH oscillations using a dressing field, it is reasonable ~~to extend~~ our general derivation in given Eq. [31] ~~into a simple expression through a~~ low-temperature limit. ~~Then we can assume that~~ the derivative of the Fermi-Dirac distribution is sharply peaked around the Fermi energy. ~~This~~ can be represented by the delta function [7]

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F). \tag{6}$$

Applying these adequate conditions, we can simplify our derivation into single expression that can be compared with previous studies [7, 32, 34] on magnetotransport in dressed quantum Hall systems[illegible] added clarification on the above assumptions in the m[illegible]

- Section V - second paragraph (page 7):
  ~~Since~~ we consider the dressed quantum Hall system under ~~the~~ off-resonant condition, the absorption of photons are avoided. Therefore, we select the Fermi-Dirac distribution as our partial dist[illegible] [illegible]nction (f) for our system

$$[illegible] = \frac{1}{\exp[(\varepsilon - \varepsilon_F)/k_B T] + 1},$$

[illegible] k_B is the Boltzmann constant, T is the absolute temperature and ε_F is the Fermi energy of the [illegible]. ~~Under the low-temperature conditions~~ ($k_B T \ll \varepsilon_F$), ~~we can assume that~~ the derivative of the [illegible] Dirac distribution is sharply peaked around the Fermi energy. ~~This~~ can be represented by the [illegible]unction [7]

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F). \tag{34}$$

[illegible]ue ~~that when we consider the experimental observations of a quantum Hall system a strong~~ [illegible]d can increase ~~the temperature of the sample~~. However, as mentioned in a previous study by Dini *et al.* [32] and Kibis *et al.* [35], ~~we can~~ maintain [illegible] [illegible]ving ~~narrow pulses of a strong dressing field to observe the discussed phenomena~~.

Although we have extended our general derivation given in Eq. [31] into special consideration for comparison, we can improve our theoretical model into different types of particle distribution functions. As presented in the previous work done by Dmitriev *et al.* [30], a well-defined analytical discussion needs to address the changes in distribution functions in our derived general longitudinal conductivity formula and this [illegible] an noteworthy extension to our study.

[illegible]ent 6 - The authors presented the results for the conductivity te[illegible] components. They seem to be identical, apart of a dimensional factor $(eB)^2$ The equality of these two components is expected for isotropic systems. Does the polarization of the electromagnetic field break the isotropy? Do the two components of the conductivity remain equal even for an arbitrary direction of a linearly polarized field? Is there a non-Hall contribution to the xy-component?

We express our gratitude towards the reviewer for pointing out this inadvertent oversight do[illegible] component of the current operator calculation, and this has led to an unexpected normalizat[illegible] the yy-component of conductivity expression. Since we have only analyzed the normalized x[illegible] of the conductivity tensor, ~~the remaining analysis is not affected by this change~~. ~~After this change, we can~~ predict that both xx-component and yy-component show the same conductivity behaviors. We added these amendments to the current operator calculations done in Appendix D and the longitudinal conductivity calculation under Section V.

- Appendix D (page 16):

we also adopted the same conditions as it enables us to compare results with those studies.
simplify
magneto-transport
Under these assumptions,
,
operate
generic
, and
where photon absorption does not happen.
for
this
can
At low-temperatures, i.e. k_
Yes, indeed, experiments on quantum hall systems subject to a strong dressing field show elevated temperatures.
,
approximated
, and
demonstrated
low system temperature
a
studies
a
,
limiting the dressing field to narrow pulses.
it is possible to
With this correction,
for subsequent analysis, this oversight has no impact on the analysis reported in the manuscsript.
used
.
observe
the
This

Appendix D: Current operators for a dressed quantum Hall system

In this section, we derive the current density operator for the N-th Landau level in a dressed quantum Hall system. We already found the exact solution for the time-dependent Schrödinger equation with the Hamiltonian give in Eq. (1) and we identified them as the Floquet states in Eq. (14). For the simplicity of notation, we can represent the Floquet modes derived in Eq. (10) as quantum states using their corresponding quantum numbers as follows

$$|\phi_{n,m}\rangle = |n, k_x\rangle \,. \tag{D1}$$

Using this complete set of quantum states [34, 36, 37], we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'} = \langle n, k_x| \, \hat{\mathbf{j}} \, |n', k'_x\rangle \,. \tag{D2}$$

Next, we can identify the particle current operator for our system [38, 39] as

$$\hat{\mathbf{j}} = \frac{1}{\widetilde{m}} \left\{ \hat{\mathbf{p}} - e\left[\mathbf{A}_s + \mathbf{A}_d(t)\right] \right\} , \tag{D3}$$

where $\widetilde{m}$ is the mass of the considering particle.

First, we consider the x-directional particle current operator component, and we can identify that as

$$\hat{j}_x = \frac{1}{\widetilde{m}} \left(-i\hbar \frac{\partial}{\partial x} + eBy \right) . \tag{D4}$$

Next, we calculate the matrix elements of x-directional current operator against our Floquet mode basis

$$(j_x)_{nm,n'm'} = \langle n, k_x| \, \frac{1}{\widetilde{m}} \left(-i\hbar \frac{\partial}{\partial x} + eBy \right) |n', k'_x\rangle \,, \tag{D5}$$

and we evaluate these using the Floquet modes derived in Eq. (7) as follows

,

$$\begin{aligned} (j_x)_{nm,n'm'} =& \frac{1}{\widetilde{m}} \delta_{k_x,k'_x} \int \left(\hbar k'_x + eBy\right) \\ & \times \chi_n\left(y - y_0 - \zeta(t)\right) \chi_{n'}\left(y - y_0 - \zeta(t)\right) dy. \end{aligned} \tag{D6}$$

Let $[y - y_0 - \zeta(t)] = \bar{y}$ and we can obtain

of

$$(j_x)_{nm,n'm'} = \frac{1}{\widetilde{m}} \delta_{k_x,k'_x} \int \left[\hbar k'_x + eB\bar{y} - \hbar k'_x + eB\zeta(t)\right] \chi_n(\bar{y})\chi_{n'}(\bar{y}) d\bar{y}. \tag{D7}$$

Using the following integral identities of the Floquet modes which are made up with Gauss-Hermite functions [40, 41]

$$\int \chi_n(y)\chi_{n'}(y)dy = \delta_{n',n}, \tag{D8}$$

that

$$\int y\chi_n(y)\chi_{n'}(y)dy = \frac{1}{\kappa} \left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right) , \tag{D9}$$

we simplify the matrix elements of x-directional current operator to obtain

$$(j_x)_{nm,n'm'} = \frac{eB}{\widetilde{m}\kappa} \delta_{k_x,k'_x} \left[\left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right) + \zeta(t)\delta_{n',n} \right] . \tag{D10}$$

Due to high complexity of extract solution, in this study we only consider the constant contribution. Therefore, we can identify the 0-th component of the Fourier series as

$$(j_x)_{nm,n'm'} = \frac{eB}{\widetilde{m}\kappa} \delta_{k_x,k'_x} \left(\sqrt{\frac{n+1}{2}} \delta_{n',n+1} + \sqrt{\frac{n}{2}} \delta_{n',n-1} \right) . \tag{D11}$$

what constant.
Need to qualify.

To calculate the substitute

~~For a~~ electric current operator, we ~~can apply the~~ electron's charge and ~~the~~ effective mass ~~of the electron~~ to the above derived equation. This leads to

$$\left(j^x_{s=0}\right)^{electron}_{nm,n'm'} = \frac{e\hbar}{m_e l_0}\delta_{k_x,k'_x}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right). \tag{D12}$$

where $l_0 = \sqrt{\hbar/eB}$ is the magnetic length.

Moreover, we can identify the y-directional current operator component as

$$\hat{j}_y = \frac{1}{\widetilde{m}}\left(-i\hbar\frac{\partial}{\partial y} - \frac{eE}{\omega}\cos(\omega t)\right). \tag{D13}$$

Using this operator, we can represent the matrix elements of y-directional current operator in Floquet mode basis as

$$(j_y)_{nm,n'm'} = \langle n,k_x|\,\frac{-1}{\widetilde{m}}\left(i\hbar\frac{\partial}{\partial y} + \frac{eE}{\omega}\cos(\omega t)\right)|n',k'_x\rangle\,. \tag{D14}$$

After following the same steps done for the x-directional current operator, and recursion relation of the first derivative of Gauss-Hermite functions

$$\frac{\partial\chi_n(y)}{\partial y} = \kappa\left[-\sqrt{\frac{n+1}{2}}\chi_{n+1}(y) + \sqrt{\frac{n}{2}}\chi_{n-1}(y)\right], \tag{D15}$$

we can identify the 0-th component of matrix elements for y-directional electric current operator as

$$\left(j^y_{s=0}\right)^{electron}_{nm,n'm'} = \frac{ie\hbar\kappa}{m_e}\delta_{k_x,k'_x}\left(\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right). \tag{D16}$$

$$\left(j^y_{s=0}\right)^{electron}_{nm,n'm'} = \frac{ie\hbar}{m_e l_0}\delta_{k_x,k'_x}\left(\sqrt{\frac{n}{2}}\delta_{n',n-1} - \sqrt{\frac{n+1}{2}}\delta_{n',n+1}\right). \tag{D17}$$

- Section V - second paragraph (page 7):
  Moreover, let $\Pi = \varepsilon_F$ and the derived expression in Eq. (31) leads to

$$\sigma^{xx} = \frac{e^2 l_0^2}{\pi\hbar A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1+\left(\frac{X_F-n-1}{\gamma_{n+1}}\right)^2}\right]\left[\frac{1}{1+\left(\frac{X_F-n}{\gamma_n}\right)^2}\right], \tag{35}$$

where $X_F = [\varepsilon_F/(\hbar\omega_0) - 1/2]$, $\gamma_n = \widetilde{\Gamma}(\varepsilon_n)\big/(\hbar\omega_0)$, and $l_0 = \sqrt{\hbar/eB}$. Following the same steps as above derivation, we can derive the longitudinal conductivity in the y-direction by applying the electric current operator for y-direction derived in Appendix D

$$\sigma^{yy} = \frac{e^2 l_0^2}{\pi\hbar A}\sum_n \frac{(n+1)}{\gamma_n\gamma_{n+1}}\left[\frac{1}{1+\left(\frac{X_F-\cdots}{\gamma_{n}\cdots}\right)^2}\right]\left[\frac{1}{\cdots}\right]. \tag{36}$$

Suppose In that case, we could , which

~~If~~ we applied an arbitrary directional polarized dressing field, ~~we can~~ derive an expression for the longitudinal conductivity components by considering dressing field contribution for each direction (x and y) with the same steps presented in our analysis. This would change the amplitude of the considering dressing field (E) in each direction. ~~Now~~ there will be a new component in parallel to the considering conductivity component ~~and this~~ will modify the time-dependent Hamiltonian given in Eq. [2]. Then, we need to solve the Schrödinger equation with the modified Hamiltonian to find the new Floquet modes for the new system. These will define the effect on the inverse scattering time matrix components. As mentioned in Ref. [34] and our work, applied strong dressing field tends to change the quantum state of the considered system and creates novel states called Floquet states. The properties of these states depend on the characteristics of the applied dressing field. Therefore, the polarization method also changes the behavior of the Floquet states. These polarization-dependent conductivity behaviors in 2DEG systems can be found [illegible]nd in their work authors have illustrated the conductivity behavior with circular and linear polarized dressing fields. However, in a dressed quantum Hall system with a y-directional linear polarized field, we [illegible] same longitudinal conductivity behaviors in diagonal components of the conductivity tensor.

tensor

mainly focus

For analyzing our system, we mainly employ the Floquet-Drude conductivity ~~formula~~ introduced by Wackerl *et al.* [34]. T~~he Floquet-Drude conductivity formula was only developed for the diagonal components of the conductivity tensor~~ ~~of the considering system~~. ~~In this analysis~~ we were only focused on SdH oscillations in longitudinal conductivity components we analyzed the dressing field ~~effect~~ on two diagonal components ~~of the conductivity tensor~~. Therefore we are unable to predict the off-diagonal component behavior of the system ~~through our analysis~~ ~~and we need a novel formalism to handle these types of behaviors of dressed 2DEG quantum Hall systems~~.

diagonal components of the

As

.

we limit our analysis to

want

improved

Response to the comments of Reviewer 2

We would like to thank the reviewer for his/her insightful comments on our work. To address your comments, we had to add new materials and reorganize our text. As a result, we have ~~been able to~~ considerably improve our discussion on the underlying assumptions and corresponding physical significance of our analysis. ~~We hence believe that both the presentation and scientific content of this manuscript are of a much higher quality than our previous submission~~. We hope our response and the corresponding changes we incorporated into the manuscript would be sufficient to clarify the issues raised.

will

Comment 1 - **The paper is very low on comparison with experiments, for which there is a large amou… for the zero radiation case. I would like to see detailed analysis of how … with available experimental data. Without this addition, physical relev… calculations is questionable.**

In the revised manuscript, we have included (see Section VI)

We th… ting out the importance of this fact. ~~To overcome this shortage of discussion we made a detailed analysis of~~ our results against the undressed 2DEG quantum Hall system behavior. ~~We have inserted this discussion into our manuscript under Section VI.~~

corresponds to

- Section VI - third paragraph (page 8):
  By comparing the theoretical [1–7] and experimental [7–13] studies on the magnetoresistance of 2DEG … systems under zero radiation with our … that longitudinal … tivity oscillations in Fig. 4 ~~are a repetition of~~ the Sh… scillations. As … in the experimental work ~~done by~~ Caviglia *et al.* [… period only dep… the perpendicular component of the applied magne… can identify tha… experiments done in low-temperatures [7, 9–13] with a different type of 2DEG have been examined … gainst the applied magnetic field amplitude. The magnetic field's perpendicular component amplitude defines the cyclotron frequency (ω_0) of the ~~considering~~ system. Therefore we can achieve the same oscillatory behavior for longitudinal conductivity in 2DEG quantum Hall system by char… ~~he change of the gate voltage applied to the system will modify~~ the Fermi leve… e Landau level energy is only dependent on the cyclotron frequency, this will … oscillations against gate voltage. T… with applied gate… ed in an experiment done by Wakabayashi *et al.* [8] in an inversion layer on a… emperature. By comparing these empirical observations with our results ~~given~~ in Fig. 4, we can identify that our oscillations ~~are~~ also ~~shown~~ the same characteristic behavior with $\hbar\omega_0$ periodicity against Fermi level value. Furthermore, when we increase the gate voltage, the Fermi level of the system ~~gets increased~~. This will locate the Fermi level on higher-order Landau levels. As illustrated in Fig. 4, this will result in a hig… lue at the higher-order Landau levels. This same behavior was also observed … servations presented in Ref. [8] ~~as well~~. However, in our analysis, we have ar… controllability of these conductivity regions using a dressing field.

of

I do not understand this sentence. please rewrite.

Kosala, please remove this word "considering" from everywhere

because it modifies

Kosala, do not use this "given" like this. "in Fig. 4" has the same meaning.

show

increases. Kosala, why use this combination "gets increased" it is a passive sentence not suited for technical writing.

Kosala, this "as well" has no added value!

Comment 2 - **In the presence of disorder, Anderson … ologically protected edge modes are … ons when calcu… oefficients. How are these factors taken into account?**

we ommitted a

~~We agree with the reviewer that~~ the discussion on the Anderson localization and topologically protected edge modes ~~were missing from the previous manuscript~~. ~~We would like to mention to the reviewer that while deriving~~ Eq. [31], we have assumed that the applied static magnetic field is placed in a lower intensity … e can neglect several effects known to appear in a 2DEG quantum Hall system. Under this … an neglect the localization in the tails of Landau level peaks, the formation of the edge states, … ectron interactions, and the spin splitting as well. As mentioned in the work ~~done by~~ Endo *et* … ~~he given~~ magnetic field range, the undressed magnetotransport properties of GaAs/AlGaAs 2DEG quantum Hall system provide an excellent agreement between the experimental observations and theory that was developed by neglecting the above-mentioned effects. ~~We also followed the same theoretical derivation with given assumptions but with an additional dressing field~~. Our results for the undressed system ~~give a adequate agreement~~ with the results from Ref. [7], while we provide a better description to manipulate the width of the Landau level conductivity peaks. We have added this clarification in the manuscript.

Our work is very similar to theirs with an additional dressing field.

agress

within the

of interest,

as we neglected them in our work.

in

of

This is not an overly restrictive assumption and others have used similar assumption in their work e.g. see [7].

we limit

to a range where we can neglect effects such as

I - sixth paragraph (page 2):
portant to note that, ~~in this analysis~~, ~~we consider~~ the applied magnetic field ~~at a range where~~ lerson localization, the formation of the edge states, the spin splitting, and the electron-electron ions ~~can be neglected~~. ~~The use of this assumption can also be found in the study done by Endo~~ *et al.* [7].

modelled

of

,

- Section VIII - first paragraph (page 10):
We assumed the impurities in the material as a Gaussian random scattering potential ~~and~~ the applied magnetic field ~~within an order where~~ we can neglect effects of the Anderson localization, the formation of the edge states, the spin splitting, and the electron-electron interactions.

to a range where

,

Comment 3 - **Can the authors reproduce the quantized Hall conductance of filled Landau levels presence of disorder? Please dis**

restricted

for the analysis reported here.

cannot

~~yzing our system,~~ we mainly employ the Floquet-Drude conductivity formula introdu *et al.* [34]. The Floquet-Drude conductivity formula was only developed for the diagonal components of the conductivity tensor of the considering system. In this analysis we were only focused on SdH oscillations in longitudinal conductivity components we analyzed the dressing field effect on two diagonal components of the conductivity tensor through the Floquet-Drude conductivity formula. Therefore we ~~are unable to~~ predict the off-diagonal component behavior of the system ~~through our analysis and we need a novel formalism to handle these types of behaviors of dressed 2DEG quantum Hall systems~~. However, we can use the already established formalism introduced by Endo *et al.* [7] to predict the behavior of off-diagonal components of the conductivity tensor. As mentioned in Ref. [7], this theory on the relation between longitudinal and transverse conductivities highly match with the experiment observations

i.e.

agree

$$\frac{\mathrm{d}\widetilde{\sigma}^{xy}(X_F, I)}{\mathrm{d}B} \approx \pi\mu\frac{\hbar\omega_0}{\varepsilon_F}[\widetilde{\sigma}^{xx}(X_F, I)]^2. \tag{1}$$

,

reported

By examining the connection between longitudinal and transverse conductivities in the 2DEG quantum Hall system, we can expect the same Hall conductivity behavior in a dressed quantum Hall system. However, dressing field we can squeeze the conductivity peaks on Landau levels. ~~This will squeeze~~ th of longitudinal conductivity peaks placed near the Landau levels. This will imply sudden Hall conductivity transitions between one plateau to another near a Landau level. As ~~empirical observation shown~~ in Refs. [15, 43], we can predict a staircase behavior for quantum Hall conductivity agaist the gate voltage. With a external dressing field we will be able to get rapid transitions than the undressed system. However, to derive cal expression for the quantum Hall conductivity through Floquet-Drude conductivity needs to be reformulated from the begin and it will be the subject of our future study.

also

an

,

It

begining.

against

Comment 4 - **The way some previous works are g. Example from page 8: "Despite this behavior being identified in previous works, their results did not coincide with the more accurate description of conductivity components in undressed quantum Hall systems." This is not informative. Where, specifically, did results from earlier works fall short of reality, which have been better addressed in this work?**

~~We would like to thank the reviewer for pointing out the deficiencies in our manuscript and we have considered your recommendations seriously~~. We made the following changes in language and presentation to improve the clarity of our manuscript.

of

have

- Section VIII (page 8):
Despite this behavior being identified in previous works ~~done by~~ Dini *et al.* [32], ~~their results do not coincide with~~ the experimental observations of longitudinal conductivity components in undressed quantum Hall systems presented in Ref. [7]. This is ~~due to the fact that~~ the authors of Ref. [32] has ~~been~~ used the conventinal expression of longitudinal conductivity from Refs. [2, 6] ~~and this theory yields a~~ semi-elliptical broadening, ~~which~~ significantly deviates by the experimentally observed Landau levels [7]. However, our generalized analysis on the conductivity of dressed quantum Hall systems provide a well-suited description for empirically observed behaviors of undressed quantum Hall systems as well.

Kosala, rewrite this sentence. I cannot do it due to not knowing the experimental result.

- ction III - first paragraph (page 3):
their study [34], the authors have presented more accurate results than the former theoretical

, which considers

and

they cannot fully account for

in

beacuse

descriptions [42, 44] for the conductivity of nanoscale systems in the presence of a dressing field. Therefore, we apply the Floquet-Drude conductivity theory to analyze our 2DEG system which is subjected to both a stationary magnetic field and a dressing field.

- Section I - fifth paragraph (page 2):
  The authors of Ref. [32] used the conductivity models from Refs. [2, 6], and as mentioned in Endo *et al.* [7], those models predict a semi-elliptical broadening against Fermi level for each Landau level and provide less agreement with the empirical results.

- Section IV - third paragraph (page 5):
  In the study presented in Ref. [7], the authors have assumed that the effective mass of the electron in GaAs-based quantum well system is $m_e \approx 0.07\widetilde{m}_e$ where $\widetilde{m}_e$ is the mass of the electron [7, 34, 45].

Comment 5 - Adding some physical insight into the remarkable observation of radiation-induced narrowing of lineshapes (Figs 4, 5) will help elevate this work.

We agree with the reviewer that a discussion on our theoretical results and their physical significance in modern nanoelectronic devices is a vital requirement. We have added a new Section VII to overcome the above-mentioned deficiency of our manuscript. The total content of the section is given below,

- Section VII (page 8):

Refer to my previous edit on this section.

VII. PHYSICAL SIG… THE OUTCOMES

With the realization of 2DEGs in Si-MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) [14], Klitzing *et al.* [15] made the first transport measurements on such systems to reveal the quantum Hall effect. The empirical discovery of these unusual properties marked the beginning of a whole new realm in condensed matter physics that continues to produce phenomenal advancements in electronic systems. The quantum Hall effects in a 2DEG under a static magnetic field are described by plateaus quantized to integer values of the conductivity quantum ($e^2/\hbar$) in the off-diagonal conductivity, with simultaneously peaks at inter-plateau transition for the diagonal conductivity [7]. This is due to the applied magnetic field and it changes the energy spectrum of 2DEG dramatically. The magnetic field causes the density of states in 2DEG to split up into a sequence of delta functions, separated by an energy $\hbar\omega_0$, with ω_0 the cyclotron frequency which depends on the applied magnetic field. However, experimental results demonstrate that these Landau levels are broadened and the main source of these broadening in low-temperatures is the disorders in materials [16, 17]. The broaden sequence of delta functions of density of states implies the oscillating behavior in the experimental measurements of longitudinal conductivity which is known as Shubnikov–de Haas (SdH) oscillations. [7, 8].

Our theoretical analysis on longitudinal conductivity behavior of dressed quantum Hall system developed by considering low-temperature limit with gaussian impurity broadening assumptions. As illustrated in Fig. 4, we were also able to demonstrate the same SdH oscillations as experimental results observed in Refs. [7, 8] through our model. Under the undressed condition, our results overlap with the conductivity measurement for quantum Hall systems [7]. However, from our results given in Fig. 5, we demonstrate that we can manipulate the broadening of these conductivity peaks using an external dressing field. In low-temperatures, the principal cause of broadening of these conductivity peaks is impurity scattering and using an external dressing field we can suppress the scattering which results in shrinkage of both the scattering-induced broadening and the longitudinal conductivity peaks.

Research on novel states of matter has driven the evolution of present-day nanoelectronic devices. In particular, controllable manipulation of material properties through a gate electric field has revolutionized the development of material science and technology [18, 19]. The charge carrier concentration of a considering system is an imperative parameter that defines the conductivity properties of the system. We can manipulate that using the electrostatic field-effect mechanism and it is an ideal tool to control the conductivity in some specific systems. A 2DEG under static magnetic field with quantum Hall effects is an excellent example that the gate electric field can be used to manipulate the conductivity.

A considerable number of researches have been performed using different types of 2D field-effect transistors (FETs) in magnetic fields to study the charge transport in the quantum limit [8, 20–22]. In the study done by Yang *et al.* [20], the authors have observed quantized Hall plateaus and Shubnikov–de Haas oscillations for longitudinal conductivity against gate voltage in black phosphorus FET under static magnetic fields in low-temperatures. Since the Fermi level of the system can be altered with the applied gate voltage, this behavior can be easily mapped into our results given in Fig. 4. However, the specialty of our outcomes is we owned the capability of manipulating the broadening of the conductivity regions using an external dressing field. Although Yang *et al.* [20], achieved broadening in longitudinal conductivity peaks by changing the temperature in a low range, in this study we presented a general mathematical description to manipulate longitudinal conductivity broadening using only a high-intensity electromagnetic field.

The realization of the underlying mechanism of 2D FETs in the quantum realm promises its potential in next-generation nanoelectronic applications. In a particular application that uses the switching mechanism of the above-discussed FETs with quantum Hall effects, we can achieve high and low output conductivities by changing the input gate voltage. As a result of manipulating the broadening of conductivity regions, we can shrink the broadening of conductivity peaks around Landau levels using a high-intensity dressing field. This will enhance the sensitivity of FETs which provides the ability to observe narrow changes in gate voltage. Furthermore, adopting the mechanism presented in Ref. [23], we can utilize our manipulation of conductivity peaks into very sensitive, narrowband high-frequency radiation detectors. Based on these ready-to-use nanoelectronic devices and their feasible optimization, we believe that our mathematical description offers the potential to realize advanced nanoelectronic devices. Furthermore, this theoretical model will help to develop simulation tools that will design the quantum effects in magnetotransport properties of 2D nanostructures.

References

[1] T. Ando, Y. Matsumoto, Y. Uemura, M. Kobayashi, and K. F. Komatsubara, J. Phys. Soc. Jpn. **32**, 859 (1972).

[2] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).

[3] T. Ando, J. Phys. Soc. Jpn. **36**, 1521 (1974).

[4] T. Ando, J. Phys. Soc. Jpn. **37**, 622 (1974).

[5] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1233 (1974).

[6] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[7] A. Endo, N. Hatano, H. Nakamura, and R. Shirasaki, J. Phys. Condens. Matter **21**, 345803 (2009).

[8] J. Wakabayashi and S. Kawaji, J. Phys. Soc. Jpn. **44**, 1839 (1978).

[9] Y. Ochiai, M. Mizuno, M. Kawabe, K. Ishibashi, Y. Aoyagi, K. Gamo, and S. Namba, Jpn. J. Appl. Phys. **29**, L739 (1990).

[10] F. Mancoff, L. Zielinski, C. Marcus, K. Campman, and A. Gossard, Phy. Rev. B **53**, R7599 (1996).

[11] Y. G. Arapov, G. Alshanskii, G. Harus, V. Neverov, N. Shelushinina, M. Yakunin, and O. Kuznetsov, Nanotechnology **13**, 86 (2002).

[12] B. Grbić, C. Ellenberger, T. Ihn, K. Ensslin, D. Reuter, and A. Wieck, App. Phy.Lett. **85**, 2277 (2004).

[13] A. Caviglia, S. Gariglio, C. Cancellieri, B. Sacépé, A. Fete, N. Reyren, M. Gabay, A. Morpurgo, and J.-M. Triscone, Phy. Rev. Lett. **105**, 236802 (2010).

[14] A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Lett. **16**, 901 (1966).

[15] K. V. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[16] T. Ando and Y. Murayama, J. Phys. Soc. Jpn. **54**, 1519 (1985).

[17] O. E. Dial, R. C. Ashoori, L. N. Pfeiffer, and K. W. West, Nature **448**, 176 (2007).

[18] C. H. Ahn, J. M. Triscone, and J. Mannhart, Nature **424**, 1015 (2003).

[19] Y. Deng, Y. Yu, Y. Song, J. Zhang, N. Z. Wang, Z. Sun, Y. Yi, Y. Z. Wu, S. Wu, J. Zhu, *et al.*, Nature **563**, 94 (2018).

[20] F. Yang, Z. Zhang, N. Z. Wang, G. J. Ye, W. Lou, X. Zhou, K. Watanabe, T. Taniguchi, K. Chang, X. H. Chen, *et al.*, Nano Lett. **18**, 6611 (2018).

[21] G. Long, X. Chen, S. Xu, and N. Wang, in *Hybrid Nanomaterials-Flexible Electronics Materials* (IntechOpen, 2020).

[22] L. Li, Y. Yu, G. J. Ye, Q. Ge, X. Ou, H. Wu, D. Feng, X. H. Chen, and Y. Zhang, Nat. Nanotechnol. **9**, 372 (2014).

[23] K. Hirakawa, K. Yamanaka, Y. Kawaguchi, M. Endo, M. Saeki, and S. Komiyama, Phy. Rev. B **63**, 085320 (2001).

[24] M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, Phy. Rev. B **64**, 201311 (2001).

[25] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, Nature **420**, 646 (2002).

[26] M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, Phy. Rev. Lett. **90**, 046807 (2003).

[27] R. G. Mani, J. H. Smet, K. Von Klitzing, V. Narayanamurti, W. Johnson, and V. Umansky, Phy.Rev. Lett. **92**, 146801 (2004).

[28] A. C. Durst, S. Sachdev, N. Read, and S. M. Girvin, Phys. Rev. Lett. **91**, 086803 (2003).

[29] I. A. Dmitriev, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003).

[30] I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. B **71**, 115316 (2005).

[31] I. A. Dmitriev, M. Khodas, A. D. Mirlin, D. G. Polyakov, and M. G. Vavilov, Phys. Rev. B **80**, 165327 (2009).

[32] K. Dini, O. V. Kibis, and I. A. Shelykh, Phys. Rev. B **93**, 235411 (2016).

[33] E. Akkermans and G. Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambridge University Press, 2010).

[34] M. Wackerl, P. Wenk, and J. Schliemann, Phys. Rev. B **101**, 184204 (2020).

[35] O. V. Kibis, H. Sigurdsson, and I. A. Shelykh, Phy. Rev. B **91**, 235308 (2015).

[36] M. Holthaus, J. Phys. B: At., Mol. Opt. Phys. **49**, 013001 (2015).

[37] M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998).

[38] G. D. Mahan, *Many-particle physics* (Springer Science & Business Media, New York, 2000).

[39] H. Bruus and K. Flensberg, *Many-body quantum theory in condensed matter physics: an introduction* (Oxford University Press, New York, 2004).

[40] V.Vedenyapin, A. Sinitsyn, and E. Dulov, *Kinetic Boltzmann, Vlasov and Related Equations* (Elsevier, Amsterdam, 2011).

[41] G. Szegö, *Orthogonal Polynomials* (American Mathematical Society, New York, 1959).

[42] S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. Shelykh, Phys. Rev. B **91**, 155312 (2015).

[43] V. Gusynin, V. Miransky, S. Sharapov, and I. Shovkovy, Phy. Rev. B **74**, 195429 (2006).

[44] A. A. Pervishko, O. V. Kibis, S. Morina, and I. A. Shelykh, Phys. Rev. B **92**, 205403 (2015).

[45] R. Winkler, *Spin-orbit Coupling Effects in Two-Dimensional Electron and Hole Systems* (Springer, Berlin, 2003).

Sincerely yours,

Malin Premaratne
(Corresponding Author)
malin.premaratne@monash.edu