Magnetic propeties of a two dimentional electron gas strongly coupled to lights

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1 Current Operator in Landau Levels

Now consider about the current density operator for Nth Landau level. Since we have already found the extact solution for our time depenedent Hamiltonian and we have identify them as Floquet states with quesi energies. From these solutions we can identify the *Floquet modes* as given in Eq. (**??**) and using quantum numbers we can represent those states as follows

$$|\phi_\alpha\rangle = |\phi_{n,m}\rangle \equiv |n, k_x\rangle \quad \text{where} \quad k_x = m\frac{2\pi}{L_x} \tag{1.1}$$

Using above complete set of eigenstates of Floquet Hamiltonian we can represent the single particle current operator's matrix element as

$$(\mathbf{j})_{nm,n'm'} = \langle n, k_x| \,\hat{\mathbf{j}}\, |n', k'_x\rangle \tag{1.2}$$

where particle current operator for this system will be

$$\hat{\mathbf{j}} = \frac{1}{m}\Big(\hat{\mathbf{P}} - e[\mathbf{A}_s + \mathbf{A}_d(t)]\Big). \tag{1.3}$$

Howver, we are only consider the transverse conductivity in x direction we can identify that x directional current operator as

$$\hat{j}_x = \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right). \tag{1.4}$$

Now we can calculate the matrix elements of x directional current operator's matrix in Floquet mode basis as

$$(j_x)_{nm,n'm'} = \langle n, k_x| \,\hat{j}_x\, |n', k'_x\rangle = \langle n, k_x| \,\frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right) |n', k'_x\rangle \tag{1.5}$$

and we can evaluate these using Floquet modes derived in Eq.(**??**) as follows

$$\begin{aligned}(j_x)_{nm,n'm'} = &\int dx \int dy \,\frac{1}{\sqrt{L_x}}\chi_n\big(y - y_0 - \zeta(t)\big)\exp(-ik_x x) \\ &\times \frac{1}{m}\left(-i\hbar\frac{\partial}{\partial x} + eBy\right)\frac{1}{\sqrt{L_x}}\chi_{n'}\big(y - y_0 - \zeta(t)\big)\exp(ik'_x x)\end{aligned} \tag{1.6}$$

and this can be simplified as

$$\begin{aligned}(j_x)_{nm,n'm'} = &\frac{1}{mL_x}\int dx \exp(-i(k_x - k'_x)x) \int dy\, \chi_n\big(y - y_0 - \zeta(t)\big) \\ &\times (\hbar k'_x + eBy)\chi_{n'}\big(y - y_0 - \zeta(t)\big)\end{aligned} \tag{1.7}$$

and

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int dy\ (\hbar k'_x + eBy)\chi_n\big(y - y_0 - \zeta(t)\big)\chi_{n'}\big(y - y_0 - \zeta(t)\big). \tag{1.8}$$

Now let $y - y_0 - \zeta(t) = \bar{y}$ and we will get

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int d\bar{y}\ (\hbar k'_x + eB\bar{y} + eBy_0 + eB\zeta(t))\chi_n(\bar{y})\chi_{n'}(\bar{y}). \tag{1.9}$$

using definition of y_0 given in Eq. (**??**) this will be modiofied to

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}\int d\bar{y}\ (\hbar k'_x + eB\bar{y} - \hbar k'_x + eB\zeta(t))\chi_n(\bar{y})\chi_{n'}(\bar{y}) \tag{1.10}$$

and using integral indentities of Gauss-Hermite functions

$$\int dy\ \chi_n(y)\chi_{n'}(y) = \delta_{n',n} \tag{1.11}$$

$$\int dy\ y\chi_n(y)\chi_{n'}(y) = \left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{1.12}$$

this becomes

$$(j_x)_{nm,n'm'} = \frac{1}{m}\delta_{k_x,k'_x}eB\left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \zeta(t)\delta_{n',n}\right] \tag{1.13}$$

Due to complexity we can only consider the constant contribution and we allows only the one-cycle averaged current flow and then we can derive the $s = 0$ components of the Fourier series as

$$(j^x_{s=0})_{nm,n'm'} = \frac{1}{T}\int_0^T dt\ \frac{1}{m}\delta_{k_x,k'_x}eB\left[\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) + \frac{eE}{m(\omega_0^2 - \omega^2)}\sin(\omega t)\delta_{n',n}\right] \tag{1.14}$$

and this can be evaluate and get

$$(j^x_{s=0})_{nm,n'm'} = \frac{eB}{m}\delta_{k_x,k'_x}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{1.15}$$

For electric current operator we can introduce the electron's charge and effective mass

$$(j^x_{s=0})_{nm,n'm'} = \frac{e^2B}{m_e}\delta_{k_x,k'_x}\left(\sqrt{\frac{n+1}{2}}\delta_{n',n+1} + \sqrt{\frac{n}{2}}\delta_{n',n-1}\right) \tag{1.16}$$

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2 Floquet-Drude Conductivity in Quantum Hall Systems

The general expression for the conductivity [*Ref: Martin Wackerl Thesis 1.250] with the disorder averaging can be represent as follows

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\\ \times\,\mathrm{tr}\,[j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon)-\mathbf{G}_0^a(\varepsilon))].\end{aligned} \tag{2.1}$$

where j_0^x and $\mathbf{G}^{r,a}(\varepsilon)$ are x directional current operator matrix and white noise disorder averaged Green function matrix respectvely defined against to the *Floquet modes* of the system. Here we have assumed that only $s=0$ Fourier component of the current operator is contributing to the conductivity.
Now this can be expand in off resonant regime ($\omega\tau_0 \gg 1$)using only central entry Fourier components ($l=l'=0$) of *Floquet modes* mentioned in Eq. (1.1) as

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\\ \times\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\langle n,k_x|j_0^x(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))j_0^x(\mathbf{G}_0^r(\varepsilon)-\mathbf{G}_0^a(\varepsilon))|n,k_x\rangle\end{aligned} \tag{2.2}$$

and one can evaluate these matrix elements as follows

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\frac{1}{{L_x}^3}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3}\\ \times\,\langle n,k_x|j_0^x|n_1,k_{x1}\rangle\,\langle n_1,k_{x1}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n_2,k_{x2}\rangle\\ \times\,\langle n_2,k_{x2}|j_0^x|n_3,k_{x3}\rangle\,\langle n_3,k_{x3}|(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))|n,k_x\rangle\end{aligned} \tag{2.3}$$

Since we can diagonalize the impurity averaged Green's function using unitary trasnformation ($\mathbf{T}=|n,k_x\rangle$) [*Ref: Martin Wackerl - Paper] and we can evaluate the matrix element of differece between retarded and advanced Green's function as follows [*Ref: My report 2.535]

$$\langle n_1,k_{x1}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n_2,k_{x2}\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{2.4}$$

and

$$\langle n_3,k_{x3}|\mathbf{T}^\dagger(\mathbf{G}^r(\varepsilon)-\mathbf{G}^a(\varepsilon))\mathbf{T}|n,k_x\rangle = \left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{2.5}$$

Then appying the results we derived in previous section (2.21) we can calculate the conductivity

$$\begin{aligned}\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\frac{1}{{V_{k_x}}^3}\sum_{k_{x1},k_{x2},k_{x3}}\sum_{n_1,n_2,n_3}\\ \times\frac{e^2B}{m_e}\delta_{k_x,k_{x1}}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}\delta_{k_{x1},k_{x2}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]\\ \times\frac{e^2B}{m_e}\delta_{k_{x2},k_{x3}}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n_3,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n_3,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_3,n}\delta_{k_{x3},k_x}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]\end{aligned} \tag{2.6}$$

and this will be modified to

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\sum_{n_1,n_2}$$
$$\times\frac{e^2B}{m_e}\left(\sqrt{\frac{n+1}{2}}\delta_{n_1,n+1}+\sqrt{\frac{n}{2}}\delta_{n_1,n-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)\delta_{n_1,n_2}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n_1}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right]$$
$$\times\frac{e^2B}{m_e}\left(\sqrt{\frac{n_2+1}{2}}\delta_{n,n_2+1}+\sqrt{\frac{n_2}{2}}\delta_{n,n_2-1}\right)\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})]^2}\right] \tag{2.7}$$

and the only non-zero term would be

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{-1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\left(\frac{n+1}{2}\right)$$
$$\times\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)_{\varepsilon_{n+1}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})_{\varepsilon_{n+1}}\Big]^2}\right]\left[\frac{2i\mathrm{Im}\big(\mathbf{T}^\dagger\sum^r\mathbf{T}\big)_{\varepsilon_{n}}}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big[\mathrm{Im}(\mathbf{T}^\dagger\sum^r\mathbf{T})_{\varepsilon_{n}}\Big]^2}\right] \tag{2.8}$$

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Then using the following identity derived in [*Ref: My report 2.509]

$$\left(\frac{1}{\tau(\varepsilon,k_x)}\right)_{ll} = -2\mathrm{Im}\Big[\Big(\mathbf{T}^\dagger\sum^r\mathbf{T}\Big)_\varepsilon\Big]_{ll} \tag{2.9}$$

using central element of the inverse scattering time matrix we can modify our result as

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{1}{4\pi\hbar A}\frac{e^4B^2}{{m_e}^2}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\frac{1}{V_{k_x}}\sum_{k_x}\sum_{n}\left(\frac{n+1}{2}\right)$$
$$\times\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n+1},k_x)}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n+1},k_x)}\Big)^2}\right]\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n},k_x)}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n},k_x)}\Big)^2}\right] \tag{2.10}$$

We have identited that the inverse scattering time matrix's central element is not k_x dependent we can get the sum over all available momentum space in x direction. However by considering the condition that the center of the force of the oscillator y_0 must physically liw within the system $-L_y/2 < y_0 < L_y/2$, one can derive that

$$-\frac{m_e\omega_0 Ly}{2\hbar}\le k_x\le\frac{m_e\omega_0 Ly}{2\hbar} \tag{2.11}$$

and we can derive that

$$\frac{1}{V_{k_x}}\sum_{k_x} = \frac{m_e\omega_0 Ly}{\hbar V_{k_x}} = 1 \tag{2.12}$$

Thefore Eq. (2.10) modified to

$$\lim_{\omega\to 0}\mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2\omega_0^2}{4\pi\hbar A}\int_{\lambda-\hbar\Omega/2}^{\lambda+\hbar\Omega/2} d\varepsilon\Big(-\frac{\partial f}{\partial\varepsilon}\Big)\sum_{n}\left(\frac{n+1}{2}\right)$$
$$\times\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n+1})}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n+1}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n+1})}\Big)^2}\right]\left[\frac{\Big(\frac{1}{\tau(\varepsilon_{n})}\Big)}{\big(\frac{1}{\hbar}\varepsilon-\frac{1}{\hbar}\varepsilon_{n}\big)^2+\Big(\frac{1}{2\tau(\varepsilon_{n})}\Big)^2}\right] \tag{2.13}$$

Then using Fermi-Dirac distribution as our partical distribution function (f) for this system

$$f(\varepsilon) = \frac{1}{[\exp(\varepsilon-\varepsilon_F)/k_BT]+1} \tag{2.14}$$

where k_B is Botlzmann constant, T is absolute tempurature and ε_F is Fermi energy of the system. Using above distribution, for extreamly low tempuratures we can appromixate that

$$-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \approx \delta(\varepsilon - \varepsilon_F) \tag{2.15}$$

and this will mpre simplify our derivation of conductivity as

$$\lim_{\omega \to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2\omega_0^2}{4\pi\hbar A} \sum_n \left(\frac{n+1}{2}\right) \left[\frac{\left(\frac{1}{\tau(\varepsilon_{n+1})}\right)}{\left(\frac{1}{\hbar}\varepsilon_F - \frac{1}{\hbar}\varepsilon_{n+1}\right)^2 + \left(\frac{1}{2\tau(\varepsilon_{n+1})}\right)^2}\right] \left[\frac{\left(\frac{1}{\tau(\varepsilon_n)}\right)}{\left(\frac{1}{\hbar}\varepsilon_F - \frac{1}{\hbar}\varepsilon_n\right)^2 + \left(\frac{1}{2\tau(\varepsilon_n)}\right)^2}\right] \tag{2.16}$$

Now introduce a new paramter with a physical meaning od scattering-induced broading of the Landau level as follows

$$\Gamma_n \equiv \Gamma(\varepsilon_n) \equiv \left(\frac{\hbar}{2\tau(\varepsilon_n)}\right) \tag{2.17}$$

and then we can re-write Eq. (2.16) as follows

$$\lim_{\omega \to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2(\hbar\omega_0)^2}{\pi\hbar A} \sum_n \left(\frac{n+1}{2}\right) \left[\frac{\Gamma(\varepsilon_{n+1})}{(\varepsilon_F - \varepsilon_{n+1})^2 + \Gamma^2(\varepsilon_{n+1})}\right] \left[\frac{\Gamma(\varepsilon_n)}{(\varepsilon_F - \varepsilon_n)^2 + \Gamma^2(\varepsilon_n)}\right] \tag{2.18}$$

$$\lim_{\omega \to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2(\hbar\omega_0)^2}{\pi\hbar A} \sum_n \left(\frac{n+1}{2}\right) \left[\frac{\Gamma_{n+1}}{(\varepsilon_F - \varepsilon_{n+1})^2 + \Gamma_{n+1}^2}\right] \left[\frac{\Gamma_n}{(\varepsilon_F - \varepsilon_n)^2 + \Gamma_n^2}\right] \tag{2.19}$$

Now use new dimentionless paramters

$$X_F \equiv \frac{\varepsilon_F}{\hbar\omega_0} - \frac{1}{2} \tag{2.20}$$

and

$$\gamma_n \equiv \frac{\Gamma_n}{\hbar\omega_0}. \tag{2.21}$$

Therefore the Eq. (2.19) leads to

$$\lim_{\omega \to 0} \mathrm{Re}[\sigma^{xx}(0,\omega)] = \frac{e^2}{\hbar} \frac{1}{2\pi A} \sum_n (n+1) \left[\frac{\gamma_{n+1}}{(X_F - n - 1)^2 + \gamma_{n+1}^2}\right] \left[\frac{\gamma_n}{(X_F - n)^2 + \gamma_n^2}\right] \tag{2.22}$$

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