1 General

run.sh

```
g++ -g -O2 -std=gnu++17 -static prog.cpp
./a.exe
```

test.sh

```
# compile and test all *.in and *.ans
g++ -g -O2 -std=gnu++17 -static prog.cpp
for i in *.in; do
  f=${i%.in}
  ./a.exe < $i > "$f.out"
  diff -b -q "$f.ans" "$f.out"
done
```

Header

```
// use better compiler options
#pragma GCC optimize("Ofast","unroll-loops")
#pragma GCC target("avx2,fma")
// include everything
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <sys/resource.h>
// namespaces
using namespace std;
using namespace __gnu_cxx; // rope
using namespace __gnu_pbds; // tree/trie
// common defines
#define fastio ios_base::sync_with_stdio(0);cin.tie(0);
#define nostacklim rlimit RZ;getrlimit(3,&RZ);RZ.rlim_cur=-1;setrlimit(3,&RZ);
#define DEBUG(v) cout<<"DEBUG: "<<#v<<" = "<<v<<'\n';
#define ll long long
#define ull unsigned ll
#define i128 __int128
#define u128 unsigned i128
#define ld long double
// global variables
mt19937 rng((uint32_t)chrono::steady_clock::now().time_since_epoch().count());
```

Fast IO

```
// _unlocked is faster, but not universally supported (windows, codeforces)
#define inchar() getchar/*_unlocked*/()
#define outchar(x) putchar/*_unlocked*/(x)
void read(unsigned int& n) {
  char c; n = 0;
  while ((c=inchar())!=' '&&c!='\n')
    n = n * 10 + c - '0';
}
void read(int& n) {
  char c; n = 0; int s = 1;
  if ((c=inchar())=='-') s = -1;
  else n = c - '0';
  while ((c=inchar())!=' '&&c!='\n')
    n = n * 10 + c - '0';
  n *= s;
}
void read(ld& n) {
  char c; n = 0;
  ld m = 0, o = 1; bool d = false; int s = 1;
  if ((c=inchar())=='-') s = -1;
  else if (c == '.') d = true;
  else n = c - '0';
  while ((c=inchar())!=' '&&c!='\n') {
    if (c == '.') d = true;
    else if (d) { m=m*10+c-'0'; o*=0.1; }
    else n = n * 10 + c - '0';
  }
  n = s * (n + m * o);
}
void read(double& n) {
  ld m; read(m); n = m;
}
void read(float& n) {
  ld m; read(m); n = m;
}
void read(string& s) {
  char c; s = "";
  while((c=inchar())!=' '&&c!='\n')
    s += c;
}
bool readline(string& s) {
  char c; s = "";
  while(c=inchar()) {
    if (c == '\n') return true;
    if (c == EOF) return false;
    s += c;
  }
  return false;
}
void print(unsigned int n) {
  if (n / 10) print(n / 10);
  outchar(n % 10 + '0');
}
void print(int n) {
  if (n < 0) { outchar('-'); n*=-1; }
  print((unsigned int)n);
}
```

2 Algorithms

Min/Max Subarray

```
// max - compare = a < b, reset = a < 0
// min - compare = a > b, reset = a > 0
// returns {sum, {start, end}}
pair<int, pair<int, int>> ContiguousSubarray(int* a, int size, bool(*compare)(int, int), bool(*reset)(int), int defbest = 0) {
  int best = defbest, cur = 0, start = 0, end = 0, s = 0;
  for (int i = 0; i < size; i++) {
    cur += a[i];
    if ((*compare)(best, cur)) { best = cur; start = s; end = i; }
    if ((*reset)(cur)) { cur = 0; s = i + 1; }
  }
  return {best, {start, end}};
}
```

Quickselect

```
#define QSNE -999999
int partition(int arr[], int l, int r)
{
  int x = arr[r], i = l;
  for (int j = l; j <= r - 1; j++)
    if (arr[j] <= x)
      swap(arr[i++], arr[j]);
  swap(arr[i], arr[r]);
  return i;
}
// find k'th smallest element in unsorted array, only if all distinct
int qselect(int arr[], int l, int r, int k)
{
  if (!(k > 0 && k <= r - l + 1)) return QSNE;
  swap(arr[l + rng() % (r-l+1)], arr[r]);
  int pos = partition(arr, l, r);
  if (pos-l==k-1) return arr[pos];
  if (pos-l>k-1) return qselect(arr,l,pos-1,k);
  return qselect(arr, pos+1, r, k-pos+l-1);
}
// TODO: compare against std::nth_element()
```

Saddleback Search

```
// search for v in 2d array arr[x][y], sorted on both axis
pair<int, int> saddleback_search(int** arr, int x, int y, int v) {
  int i = x-1, j = 0;
  while (i >= 0 && j < y) {
    if (arr[i][j] == v) return {i, j};
    (arr[i][j] > v)? i--: j++;
  }
  return {-1, -1};
}
```

Ternary Search

```
// < max, > min, or any other unimodal func
#define TERNCOMP(a,b) (a)<(b)
int ternsearch(int a, int b, int (*f)(int)) {
  while (b-a > 4) {
    int m = (a+b)/2;
    if (TERNCOMP((*f)(m), (*f)(m+1))) a = m;
    else b = m+1;
  }
  for (int i = a+1; i <= b; i++)
    if (TERNCOMP((*f)(a), (*f)(i)))
      a = i;
  return a;
}
#define TERNPREC 0.000001
double ternsearch(double a, double b, double (*f)(double)) {
  while (b-a > TERNPREC * 4) {
    double m = (a+b)/2;
    if (TERNCOMP((*f)(m), (*f)(m + TERNPREC))) a = m;
    else b = m + TERNPREC;
  }
  for (double i = a + TERNPREC; i <= b; i += TERNPREC)
    if (TERNCOMP((*f)(a), (*f)(i)))
      a = i;
  return a;
}
```

3 Data Structures

Fenwick Tree

```
// Fenwick tree, array of cumulative sums - O(log n) updates, O(log n) gets
struct Fenwick {
  int n; ll* tree;
  void update(int i, int val) {
    ++i;
    while (i <= n) {
      tree[i] += val;
      i += i & (-i);
    }
  }
  Fenwick(int size) {
    n = size;
    tree = new ll[n+1];
    for (int i = 1; i <= n; i++)
      tree[i] = 0;
  }
  Fenwick(int* arr, int size) : Fenwick(size) {
    for (int i = 0; i < n; i++)
      update(i, arr[i]);
  }
  ~Fenwick() { delete[] tree; }
  ll operator[](int i) {
    if (i < 0 || i > n) return 0;
    ll sum = 0;
    ++i;
    while (i>0) {
      sum += tree[i];
      i -= i & (-i);
    }
    return sum;
  }
  ll getRange(int a, int b) { return operator[](b) - operator[](a-1); }
};
```

Hashtable

```
// similar to unordered_map, but faster
struct chash {
  const uint64_t C = (ll)(2e18 * M_PI) + 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
int main() {
  gp_hash_table<ll,int,chash> hashtable({},{},{},{},{1<<16});
  for (int i = 0; i < 100; i++)
    hashtable[i] = 200+i;
  if (hashtable.find(10) != hashtable.end())
    cout << hashtable[10];
}
```

Ordered Set

```
typedef tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update> ordered_set;
int main()
{
  ordered_set o_set;
  o_set.insert(5); o_set.insert(1); o_set.insert(3);
  // get second smallest element
  cout << *(o_set.find_by_order(1)) << '\n';
  // number of elements less than k=4
  cout << o_set.order_of_key(4) << '\n';
}
```

Rope

```
// O(log n) insert, delete, concatenate
int main() {
  // generate rope
  rope<int> v;
  for (int i = 0; i < 100; i++)
    v.push_back(i);
  // move range to front
  rope<int> copy = v.substr(10, 10);
  v.erase(10, 10);
  v.insert(copy.mutable_begin(), copy);
  // print elements of rope
  for (auto it : v)
    cout << it << " ";
}
```

Segment Tree

```
//max(a,b), min(a,b), a+b, a*b, gcd(a,b), a^b
struct SegmentTree {
  typedef int T;
  static constexpr T UNIT = INT_MIN;
  T f(T a, T b) {
    if (a == UNIT) return b;
    if (b == UNIT) return a;
    return max(a,b);
  }
  int n; vector<T> s;
  SegmentTree(int n, T def=UNIT) : s(2*n, def), n(n) {}
  SegmentTree(vector<T> arr) : SegmentTree(arr.size()) {
    for (int i=0;i<arr.size();i++) update(i,arr[i]);
  }
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos*2+1]);
  }
  T query(int b, int e) { // query [b, e)
    T ra = UNIT, rb = UNIT;
    for (b+=n, e+=n; b<e; b/=2, e/=2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);
  }
  T get(int p) { return query(p, p+1); }
};
```

Trie

```cpp
typedef trie<string, null_type, trie_string_access_traits<>,
 pat_trie_tag, trie_prefix_search_node_update> trie_type;
int main() {
 // generate trie
 trie_type trie;
 for (int i = 0; i < 20; i++)
  trie.insert(to_string(i)); // true if new, false if old
 // print things with prefix "1"
 auto range = trie.prefix_range("1");
 for (auto it = range.first; it != range.second; it++)
  cout << *it << " ";
}
```

4 String

Aho Corasick

```cpp
// range of alphabet for automata to consider
// MAXC = 26, OFFC = 'a' if only lowercase
const int MAXC = 256;
const int OFFC = 0;
struct aho_corasick {
 struct state
 {
  set<pair<int, int>> out;
  int fail; vector<int> go;
  state() : fail(-1), go(MAXC, -1) {}
 };
 vector<state> s;
 int id = 0;
 aho_corasick(string* arr, int size) : s(1) {
  for (int i = 0; i < size; i++) {
   int cur = 0;
   for (int c : arr[i]) {
    if (s[cur].go[c-OFFC] == -1) {
     s[cur].go[c-OFFC] = s.size();
     s.push_back(state());
    }
    cur = s[cur].go[c-OFFC];
   }
   s[cur].out.insert({arr[i].size(), id++});
  }
  for (int c = 0; c < MAXC; c++)
   if (s[0].go[c] == -1)
    s[0].go[c] = 0;
  queue<int> sq;
  for (int c = 0; c < MAXC; c++) {
   if (s[0].go[c] != 0) {
    s[s[0].go[c]].fail = 0;
    sq.push(s[0].go[c]);
   }
  }
  while (sq.size()) {
   int e = sq.front(); sq.pop();
   for (int c = 0; c < MAXC; c++) {
    if (s[e].go[c] != -1) {
     int failure = s[e].fail;
     while (s[failure].go[c] == -1)
      failure = s[failure].fail;
     failure = s[failure].go[c];
     s[s[e].go[c]].fail = failure;
     for (auto length : s[failure].out)
      s[s[e].go[c]].out.insert(length);
     sq.push(s[e].go[c]);
    }
   }
  }
 }
 // list of {start pos, pattern id}
 vector<pair<int, int>> search(string text)
 {
  vector<pair<int, int>> toret;
  int cur = 0;
  for (int i = 0; i < text.size(); i++) {
   while (s[cur].go[text[i]-OFFC] == -1)
    cur = s[cur].fail;
   cur = s[cur].go[text[i]-OFFC];
   if (s[cur].out.size())
    for (auto end : s[cur].out)
     toret.push_back({i - end.first + 1, end.second});
  }
  return toret;
 }
};
```

Boyer Moore

```cpp
struct defint { int i = -1; };
vector<int> boyermoore(string txt, string pat) {
 vector<int> toret; unordered_map<char, defint> badchar;
 int m = pat.size(), n = txt.size();
 for (int i = 0; i < m; i++) badchar[pat[i]].i = i;
 int s = 0;
 while (s <= n - m) {
  int j = m - 1;
  while (j >= 0 && pat[j] == txt[s + j]) j--;
  if (j < 0) {
   toret.push_back(s);
   s += (s + m < n) ? m - badchar[txt[s + m]].i : 1;
  } else
   s += max(1, j - badchar[txt[s + j]].i);
 }
 return toret;
}
```

English Conversion

```cpp
const string ones[] = {"", "one", "two", "three", "four", "five", "six", "seven", "eight", "nine"};
const string teens[] ={"ten", "eleven", "twelve", "thirteen", "fourteen", "fifteen", "sixteen", "seventeen", "eighteen", "nineteen"};
const string tens[] = {"twenty", "thirty", "forty", "fifty", "sixty", "seventy", "eighty", "ninety"};
const string mags[] = {"thousand", "million", "billion", "trillion", "quadrillion", "quintillion", "sextillion", "septillion"};
string convert(int num, int carry) {
 if (num < 0) return "negative " + convert(-num, 0);
 if (num < 10) return ones[num];
 if (num < 20) return teens[num % 10];
 if (num < 100) return tens[(num / 10) - 2] + (num%10==0?"":" ") + ones[num % 10];
 if (num < 1000) return ones[num / 100] + (num/100==0?"":" ") + "hundred" + (num%100==0?"":" ") + convert(num % 100, 0);
 return convert(num / 1000, carry + 1) + " " + mags[carry] + " " + convert(num % 1000, 0);
}
string convert(int num) {
 return (num == 0) ? "zero" : convert(num, 0);
}
```

Knuth Morris Pratt

```cpp
vector<int> kmp(string txt, string pat) {
  vector<int> toret;
 int m = txt.length(), n = pat.length();
 int next[n + 1];
 for (int i = 0; i < n + 1; i++)
  next[i] = 0;
 for (int i = 1; i < n; i++) {
  int j = next[i + 1];
  while (j > 0 && pat[j] != pat[i])
   j = next[j];
  if (j > 0 || pat[j] == pat[i])
   next[i + 1] = j + 1;
 }
 for (int i = 0, j = 0; i < m; i++) {
  if (txt[i] == pat[j]) {
   if (++j == n)
    toret.push_back(i - j + 1);
  } else if (j > 0) {
   j = next[j];
   i--;
  }
 }
 return toret;
}
```

Longest Common Prefix

```cpp
string lcp(string* arr, int n) {
 if (n == 0) return "";
 sort(arr, arr + n);
 string r = ""; int v = 0;
 while (v < arr[0].length() && arr[0][v] == arr[n-1][v])
  r += arr[0][v++];
 return r;
}
```

Longest Common Subsequence

```cpp
string lcs(string a, string b) {
 int m = a.length(), n = b.length();
 int L[m+1][n+1];
 for (int i = 0; i <= m; i++) {
  for (int j = 0; j <= n; j++) {
   if (i == 0 || j == 0) L[i][j] = 0;
   else if (a[i-1] == b[j-1]) L[i][j] = L[i-1][j-1]+1;
   else L[i][j] = max(L[i-1][j], L[i][j-1]);
  }
 }
 // return L[m][n]; // length of lcs
 string out = "";
 int i = m - 1, j = n - 1;
 while (i >= 0 && j >= 0) {
  if (a[i] == b[j]) {
   out = a[i--] + out;
   j--;
  }
  else if (L[i][j+1] > L[i+1][j]) i--;
  else j--;
 }
 return out;
}
```

Longest Common Substring

```cpp
// l is array of palindrome length at that index
int manacher(string s, int* l) {
 int n = s.length() * 2;
 for (int i = 0, j = 0, k; i < n; i += k, j = max(j-k, 0)) {
  while (i >= j && i + j + 1 < n && s[(i-j)/2] == s[(i+j+1)/2]) j++;
  l[i] = j;
  for (k = 1; i >= k && j >= k && l[i-k] != j-k; k++)
   l[i+k] = min(l[i-k], j-k);
 }
 return *max_element(l, l + n);
}
```

Subsequence Count

```cpp
// "banana", "ban" >> 3 (ban, ba..n, b..an)
ull subsequences(string body, string subs) {
 int m = subs.length(), n = body.length();
 if (m > n) return 0;
 ull** arr = new ull*[m+1];
 for (int i = 0; i <= m; i++) arr[i] = new ull[n+1];
 for (int i = 1; i <= m; i++) arr[i][0] = 0;
 for (int i = 0; i <= n; i++) arr[0][i] = 1;
 for (int i = 1; i <= m; i++)
  for (int j = 1; j <= n; j++)
   arr[i][j] = arr[i][j-1] + ((body[j-1] == subs[i-1])? arr[i-1][j-1] : 0);
 return arr[m][n];
}
```

5 Math

Catalan Numbers

```cpp
ull* catalan = new ull[1000000];
void genCatalan(int n, int mod) {
 catalan[0] = catalan[1] = 1;
 for (int i = 2; i <= n; i++) {
  catalan[i] = 0;
  for (int j = i - 1; j >= 0; j--) {
   catalan[i] += (catalan[j] * catalan[i-j-1]) % mod;
   if (catalan[i] >= mod)
    catalan[i] -= mod;
  }
 }
}
// TODO: consider binomial coefficient method
```

Combinatorics (nCr, nPr)

```cpp
// can optimize by precomputing factorials, and fact[n]/fact[n-r]
ull nPr(ull n, ull r) {
 ull v = 1;
 for (ull i = n-r+1; i <= n; i++)
  v *= i;
 return v;
}
ull nPr(ull n, ull r, ull m) {
 ull v = 1;
 for (ull i = n-r+1; i <= n; i++)
  v = (v * i) % m;
 return v;
}
ull nCr(ull n, ull r) {
 long double v = 1;
 for (ull i = 1; i <= r; i++)
  v = v * (n-r+i) /i;
 return (ull)(v + 0.001);
}
// requires modulo math
// can optimize by precomputing mfac and minv-mfac
ull nCr(ull n, ull r, ull m) {
 return mfac(n, m) * minv(mfac(k, m), m) % m * minv(mfac(n-k, m), m) % m;
}
```

Chinese Remainder Theorem

```cpp
bool ecrt(ll* r, ll* m, int n, ll& re, ll& mo) {
 ll x, y, d; mo = m[0]; re = r[0];
 for (int i = 1; i < n; i++) {
  d = egcd(mo, m[i], x, y);
  if ((r[i] - re) % d != 0) return false;
  x = (r[i] - re) / d * x % (m[i] / d);
  re += x * mo;
  mo = mo / d * m[i];
  re %= mo;
 }
 re = (re + mo) % mo;
 return true;
}
```

Count Digit Occurences

```cpp
/*count(n,d) counts the number of occurences of a digit d in the range [0,n]*/
ll digit_count(ll n, ll d) {
 ll result = 0;
 while (n != 0) {
  result += ((n%10) == d ? 1 : 0);
  n /= 10;
 }
 return result;
}
ll count(ll n, ll d) {
 if (n < 10) return (d > 0 && n >= d);
 if ((n % 10) != 9) return digit_count(n, d) + count(n-1, d);
 return 10*count(n/10, d) + (n/10) + (d > 0);
}
```

Discrete Logarithm

```
unordered_map<int, int> dlogc;
int discretelog(int a, int b, int m) {
  dlogc.clear();
  ll n = sqrt(m)+1, an = 1;
  for (int i = 0; i < n; i++)
    an = (an * a) % m;
  ll c = an;
  for (int i = 1; i <= n; i++) {
    if (!dlogc.count(c)) dlogc[c] = i;
    c = (c * an) % m;
  }
  c = b;
  for (int i = 0; i <= n; i++) {
    if (dlogc.count(c)) return (dlogc[c] * n - i + m - 1) % (m-1);
    c = (c * a) % m;
  }
  return -1;
}
```

Euler Phi / Totient

```
int phi(int n) {
  int r = n;
  for (int i = 2; i * i <= n; i++) {
    if (n % i == 0) r -= r / i;
    while (n % i == 0) n /= i;
  }
  if (n > 1) r -= r / n;
  return r;
}
#define n 100000
ll phi[n+1];
void computeTotient() {
  for (int i=1; i<=n; i++) phi[i] = i;
  for (int p=2; p<=n; p++) {
    if (phi[p] == p) {
      phi[p] = p-1;
      for (int i = 2*p; i<=n; i += p) phi[i] = (phi[i]/p) * (p-1);
    }
  }
}
```

Factorials

```
// digits in factorial
#define kamenetsky(n) (floor((n * log10(n / M_E)) + (log10(2 * M_PI * n) / 2.0)) + 1)
// approximation of factorial
#define stirling(n) ((n == 1) ? 1 : sqrt(2 * M_PI * n) * pow(n / M_E, n))
// natural log of factorial
#define lfactorial(n) (lgamma(n+1))
```

Prime Factorization

```
// do not call directly
ll pollard_rho(ll n, ll s) {
  ll x, y;
  x = y = rand() % (n - 1) + 1;
  int head = 1, tail = 2;
  while (true) {
    x = mult(x, x, n);
    x = (x + s) % n;
    if (x == y) return n;
    ll d = __gcd(max(x - y, y - x), n);
    if (1 < d && d < n) return d;
    if (++head == tail) y = x, tail <<= 1;
  }
}
// call for prime factors
void factorize(ll n, vector<ll> &divisor) {
  if (n == 1) return;
  if (isPrime(n)) divisor.push_back(n);
  else {
    ll d = n;
    while (d >= n) d = pollard_rho(n, rand() % (n - 1) + 1);
    factorize(n / d, divisor);
    factorize(d, divisor);
  }
}
```

Farey Fractions

```
// generate 0 <= a/b <= 1 ordered, b <= n
// farey(4) = 0/1 1/4 1/3 1/2 2/3 3/4 1/1
// length is sum of phi(i) for i = 1 to n
vector<pair<int, int>> farey(int n) {
  int h = 0, k = 1, x = 1, y = 0, r;
  vector<pair<int, int>> v;
  do {
    v.push_back({h, k});
    r = (n-y)/k;
    y += r*k; x += r*h;
    swap(x,h); swap(y,k);
    x = -x; y = -y;
  } while (k > 1);
  v.push_back({1, 1});
  return v;
}
```

Fast Fourier Transform

```
#define cd complex<double>
const double PI = acos(-1);
void fft(vector<cd>& a, bool invert) {
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j & bit; bit >>= 1) j ^= bit;
    j ^= bit;
    if (i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len) {
      cd w(1);
      for (int j = 0; j < len / 2; j++) {
        cd u = a[i+j], v = a[i+j+len/2] * w;
        a[i+j] = u + v;
        a[i+j+len/2] = u - v;
        w *= wlen;
      }
    }
  }
  if (invert)
    for (auto& x : a)
      x /= n;
}
vector<int> fftmult(vector<int> const& a, vector<int> const& b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1 << (32 - __builtin_clz(a.size() + b.size() - 1));
  fa.resize(n); fb.resize(n);
  fft(fa, false); fft(fb, false);
  for (int i = 0; i < n; i++) fa[i] *= fb[i];
  fft(fa, true);
  vector<int> toret(n);
  for (int i = 0; i < n; i++) toret[i] = round(fa[i].real());
  return toret;
}
```

Greatest Common Denominator

```
ll egcd(ll a, ll b, ll& x, ll& y) {
  if (b == 0) { x = 1; y = 0; return a; }
  ll gcd = egcd(b, a % b, x, y);
  x -= a / b * y;
  swap(x, y);
  return gcd;
}
```

Josephus Problem

```
// 0-indexed, arbitrary k
int josephus(int n, int k) {
  if (n == 1) return 0;
  if (k == 1) return n-1;
  if (k > n) return (josephus(n-1,k)+k)%n;
  int res = josephus(n-n/k,k)-n%k;
  return res + ((res<0)?n:res/(k-1));
}
// fast case if k=2, traditional josephus
int josephus(int n) {
  return 2*(n-(1<<(32-__builtin_clz(n)-1)));
}
```

Least Common Multiple

```
#define lcm(a,b) ((a*b)/__gcd(a,b))
```

Modulo Operations

```
#define MOD 1000000007
#define madd(a,b,m) (a+b-((a+b-m>=0)?m:0))
#define mult(a,b,m) ((ull)a*b%m)
#define msub(a,b,m) (a-b+((a<b)?m:0))
ll mpow(ll b, ll e, ll m) {
  ll x = 1;
  while (e > 0) {
    if (e % 2) x = (x * b) % m;
    b = (b * b) % m;
    e /= 2;
  }
  return x % m;
}
ull mfac(ull n, ull m) {
  ull f = 1;
  for (int i = n; i > 1; i--)
    f = (f * i) % m;
  return f;
}
// if m is not guaranteed to be prime
ll minv(ll b, ll m) {
  ll x = 0, y = 0;
  if (egcd(b, m, x, y) != 1) return -1;
  return (x % m + m) % m;
}
ll mdiv_compmod(int a, int b, int m) {
  if (__gcd(b, m) != 1) return -1;
  return mult(a, minv(b, m), m);
}
// if m is prime (like 10^9+7)
ll mdiv_primemod (int a, int b, int m) {
  return mult(a, mpow(b, m-2, m), m);
}
```

Miller-Rabin Primality Test

```
// Miller-Rabin primality test - O(10 log^3 n)
bool isPrime(ull n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  ull s = n - 1;
  while (s % 2 == 0) s /= 2;
  for (int i = 0; i < 10; i++) {
    ull temp = s;
    ull a = rand() % (n - 1) + 1;
    ull mod = mpow(a, temp, n);
    while (temp!=n-1&&mod!=1&&mod!=n-1) {
      mod = mult(mod, mod, n);
      temp *= 2;
    }
    if (mod!=n-1&&temp%2==0) return false;
  }
  return true;
}
```

Sieve of Eratosthenes

```
bitset<100000001> sieve;
// generate sieve - O(n log n)
void genSieve(int n) {
  sieve[0] = sieve[1] = 1;
  for (ull i = 3; i * i < n; i += 2)
    if (!sieve[i])
      for (ull j = i * 3; j <= n; j += i * 2)
        sieve[j] = 1;
}
// query sieve after it's generated - O(1)
bool querySieve(int n) {
  return n == 2 || (n % 2 != 0 && !sieve[n]);
}
```

Simpson's / Approximate Integrals

```
// integrate f from a to b, k iterations
// error <= (b-a)/18.0 * M * ((b-a)/2k)^4
// where M = max(abs(f````(x))) for x in [a,b]
// "f" is a function "double func(double x)"
double Simpsons(double a, double b, int k, double (*f)(double)) {
  double dx = (b-a)/(2.0*k), t = 0;
  for (int i = 0; i < k; i++)
    t += ((i==0)?1:2)*(*f)(a+2*i*dx) + 4 * (*f)(a+(2*i+1)*dx);
  return (t + (*f)(b)) * (b-a) / 6.0 / k;
}
```

Common Equations Solvers

```
// ax^2 + bx + c = 0, find x
vector<double> solveEq(double a, double b, double c) {
  vector<double> r;
  double z = b * b - 4 * a * c;
  if (z == 0)
    r.push_back(-b/(2*a));
  else if (z > 0) {
    r.push_back((sqrt(z)-b)/(2*a));
    r.push_back((sqrt(z)+b)/(2*a));
  }
  return r;
}
// ax^3 + bx^2 + cx + d = 0, find x
vector<double> solveEq(double a, double b, double c, double d) {
  vector<double> res;
  long double a1 = b/a, a2 = c/a, a3 = d/a;
  long double q = (a1*a1 - 3*a2)/9.0, sq = -2*sqrt(q);
  long double r = (2*a1*a1*a1 - 9*a1*a2 + 27*a3)/54.0;
  long double z = r*r-q*q*q, theta;
  if (z <= 0) {
    theta = acos(r/sqrt(q*q*q));
    res.push_back(sq*cos(theta/3.0) - a1/3.0);
    res.push_back(sq*cos((theta+2.0*PI)/3.0) - a1/3.0);
    res.push_back(sq*cos((theta+4.0*PI)/3.0) - a1/3.0);
  }
  else {
    res.push_back(pow(sqrt(z)+fabs(r), 1/3.0));
    res[0] = (res[0] + q / res[0]) * ((r<0)?1:-1) - a1 / 3.0;
  }
  return res;
}
// linear diophantine equation ax + by = c, find x and y
// infinite solutions of form x+k*b/g, y-k*a/g
bool solveEq(ll a, ll b, ll c, ll &x, ll &y, ll &g) {
  g = egcd(abs(a), abs(b), x, y);
  if (c % g) return false;
  x *= c / g * ((a < 0) ? -1 : 1);
  y *= c / g * ((b < 0) ? -1 : 1);
  return true;
}
// m = # equations, n = # variables, a[m][n+1] = coefficient matrix
// a[i][0]x + a[i][1]y + ... + a[i][n]z = a[i][n+1]
const double eps = 1e-7;
bool zero(double a) { return (a < eps) && (a > -eps); }
vector<double> solveEq(double **a, int m, int n) {
  int cur = 0;
  for (int i = 0; i < n; i++) {
    for (int j = cur; j < m; j++) {
      if (!zero(a[j][i])) {
        if (j != cur) swap(a[j], a[cur]);
        for (int sat = 0; sat < m; sat++) {
```

```
        if (sat == cur) continue;
        double num = a[sat][i] / a[cur][i];
        for (int sot = 0; sot <= n; sot++)
          a[sat][sot] -= a[cur][sot] * num;
      }
      cur++;
      break;
    }
  }
 }
 for (int j = cur; j < m; j++)
  if (!zero(a[j][n])) return vector<double>();
 vector<double> ans(n,0);
 for (int i = 0, sat = 0; i < n; i++)
  if (sat < m && !zero(a[sat][i]))
   ans[i] = a[sat][n] / a[sat++][i];
 return ans;
}
```

6 Graph

```
struct edge {
 int u,v,w;
 edge (int u,int v,int w) : u(u),v(v),w(w) {}
 edge () : u(0), v(0), w(0) {}
};
bool operator < (const edge &e1, const edge &e2) { return e1.w < e2.w; }
bool operator > (const edge &e1, const edge &e2) { return e1.w > e2.w; }
struct subset { int p, rank; };
```

Eulerian Path

```
#define edge_list vector<edge>
#define adj_sets vector<set<int>>
struct EulerPathGraph {
 adj_sets graph; // actually indexes incident edges
 edge_list edges; int n; vector<int> indeg;
 EulerPathGraph(int n): n(n) {
  indeg = *(new vector<int>(n,0));
  graph = *(new adj_sets(n, set<int>()));
 }
 void add_edge(int u, int v) {
  graph[u].insert(edges.size());
  indeg[v]++;
  edges.push_back(edge(u,v,0));
 }
 bool eulerian_path(vector<int> &circuit) {
  if(edges.size()==0) return false;
  stack<int> st;
  int a[] = {-1, -1};
  for(int v=0;v<n;v++) {
   if(indeg[v]!=graph[v].size()) {
    bool b = indeg[v] > graph[v].size();
    if (abs(((int)indeg[v])-((int)graph[v].size())) > 1) return false;
    if (a[b] != -1) return false;
    a[b] = v;
   }
  }
  int s = (a[0]!=-1 && a[1]!=-1 ? a[0] : (a[0]==-1 && a[1]==-1 ? edges[0].u : -1));
  if(s==-1) return false;
  while(!st.empty() || !graph[s].empty()) {
   if (graph[s].empty()) {
    circuit.push_back(s); s = st.top(); st.pop(); }
   else {
    int w = edges[*graph[s].begin()].v;
    graph[s].erase(graph[s].begin());
    st.push(s); s = w;
   }
  }
  circuit.push_back(s);
  return circuit.size()-1==edges.size();
 }
};
```

Minimum Spanning Tree

```
// returns vector of edges in the mst
// graph[i] = vector of edges incident to vertex i
// places total weight of the mst in &total
// if returned vector has size != n-1, there is no MST
vector<edge> mst(vector<vector<edge>> graph, ll &total) {
 total = 0;
 priority_queue<edge, vector<edge>, greater<edge>> pq;
 vector<edge> MST;
 bitset<20001> marked; // change size as needed
 marked[0] = 1;
 for (edge ep : graph[0]) pq.push(ep);
 while(MST.size()!=graph.size()-1 && pq.size()!=0) {
  edge e = pq.top(); pq.pop();
  int u = e.u, v = e.v, w = e.w;
  if(marked[u] && marked[v]) continue;
  else if(marked[u]) swap(u, v);
  for(edge ep : graph[u]) pq.push(ep);
  marked[u] = 1;
  MST.push_back(e);
  total += e.w;
 }
 return MST;
}
```

Union Find

```
int uf_find(subset* s, int i) {
 if (s[i].p != i) s[i].p = uf_find(s, s[i].p);
 return s[i].p;
}
void uf_union(subset* s, int x, int y) {
 int xp = uf_find(s, x), yp = uf_find(s, y);
 if (s[xp].rank > s[yp].rank) s[yp].p = xp;
 else if (s[xp].rank < s[yp].rank) s[xp].p = yp;
 else { s[yp].p = xp; s[xp].rank++; }
}
```

7 2D Geometry

```
#define point complex<double>
#define EPS 0.0000001
#define sq(a) ((a)*(a))
#define cb(a) ((a)*(a)*(a))
double dot(point a, point b) { return real(conj(a)*b); }
double cross(point a, point b) { return imag(conj(a)*b); }
struct line { point a, b; };
struct circle { point c; double r; };
struct segment { point a, b; };
struct triangle { point a, b, c; };
struct rectangle { point tl, br; };
struct convex_polygon {
 vector<point> points;
 convex_polygon(vector<point> points) : points(points) {}
 convex_polygon(triangle a) {
  points.push_back(a.a); points.push_back(a.b); points.push_back(a.c);
 };
 convex_polygon(rectangle a) {
  points.push_back(a.tl); points.push_back({real(a.tl), imag(a.br)});
  points.push_back(a.br); points.push_back({real(a.br), imag(a.tl)});
 }
};
struct polygon {
 vector<point> points;
 polygon(vector<point> points) : points(points) {}
 polygon(triangle a) {
  points.push_back(a.a); points.push_back(a.b); points.push_back(a.c);
 }
 polygon(rectangle a) {
  points.push_back(a.tl); points.push_back({real(a.tl), imag(a.br)});
  points.push_back(a.br); points.push_back({real(a.br), imag(a.tl)});
 }
 polygon(convex_polygon a) {
  for (point v : a.points)
   points.push_back(v);
 }
};
// triangle methods
double area_heron(double a, double b, double c) {
 if (a < b) swap(a, b);
 if (a < c) swap(a, c);
 if (b < c) swap(b, c);
 if (a > b + c) return -1;
 return sqrt((a+b+c)*(c-a+b)*(c+a-b)*(a+b-c)/16.0);
}
// segment methods
double lengthsq(segment a) { return sq(real(a.a) - real(a.b)) + sq(imag(a.a) - imag(a.b)); }
double length(segment a) { return sqrt(lengthsq(a)); }
// circle methods
double circumference(circle a) { return 2 * a.r * M_PI; }
double area(circle a) { return sq(a.r) * M_PI; }
// rectangle methods
double width(rectangle a) { return abs(real(a.br) - real(a.tl)); }
double height(rectangle a) { return abs(imag(a.br) - real(a.tl)); }
double diagonal(rectangle a) { return sqrt(sq(width(a)) + sq(height(a))); }
double area(rectangle a) { return width(a) * height(a); }
double perimeter(rectangle a) { return 2 * (width(a) + height(a)); }
// check if `a` fit's inside `b`
// swap equalities to exclude tight fits
bool doesFitInside(rectangle a, rectangle b) {
 int x = width(a), w = width(b), y = height(a), h = height(b);
 if (x > y) swap(x, y);
 if (w > h) swap(w, h);
 if (w < x) return false;
 if (y <= h) return true;
 double a=sq(y)-sq(x), b=x*h-y*w, c=x*w-y*h;
 return sq(a) <= sq(b) + sq(c);
}
// polygon methods
// get both area and centroid
pair<double, point> area(polygon a) {
 int n = a.points.size();
 double area = 0;
 point c(0, 0);
 for 9int i = n - 1, j = 0; j < n; i = j++) {
  double a = cross(a.points[i], a.points[j]) / 2;
  area += a;
  c += (a.points[i] + a.points[j]) * (a / 3);
 }
 c /= area;
 return {c, area};
}
```

Intersection

```
// -1 coincide, 0 parallel, 1 intersection
int intersection(line a, line b, point& p) {
 if (abs(cross(a.b - a.a, b.b - b.a)) > EPS) {
  p = cross(b.a - a.a, b.b - a.b) / cross(a.b - a.a, b.b - b.a) * (b - a) + a;
  return 1;
 }
 if (abs(cross(a.b - a.a, a.b - b.a)) > EPS) return 0;
 return -1;
}
// area of intersection
double intersection(circle a, circle b) {
 double d = abs(a.c - b.c);
 if (d <= b.r - a.r) return area(a);
 if (d <= a.r - b.r) return area(b);
 if (d >= a.r + b.r) return 0;
 double alpha = acos((sq(a.r) + sq(d) - sq(b.r)) / (2 * a.r * d));
 double beta = acos((sq(b.r) + sq(d) - sq(a.r)) / (2 * b.r * d));
 return sq(a.r) * (alpha - 0.5 * sin(2 * alpha)) + sq(b.r) * (beta - 0.5 * sin(2 * beta));
}
// -1 outside, 0 inside, 1 tangent, 2 intersection
int intersection(circle a, circle b, vector<point>& inter) {
 double d2 = norm(b.c - a.c), rS = a.r + b.r, rD = a.r - b.r;
 if (d2 > sq(rS)) return -1;
 if (d2 < sq(rD)) return 0;
 double ca = 0.5 * (1 + rS * rD / d2);
 point z = point(ca, sqrt(sq(a.r) / d2 - sq(ca)));
 inter.push_back(a.c + (b.c - a.c) * z);
 if (abs(imag(z)) > EPS) inter.push_back(a.c + (b.c - a.c) * conj(z));
 return inter.size();
}
// points of intersection
vector<point> intersection(line a, circle c) {
 vector<point> inter;
 c.c -= a.a;
 a.b -= a.a;
 point m = a.b * real(c.c / a.b);
 double d2 = norm(m - c.c);
 if (d2 > sq(c.r)) return 0;
 double l = sqrt((sq(c.r) - d2) / norm(a.b));
 inter.push_back(a.a + m + l * a.b);
 if (abs(l) > EPS) inter.push_back(a.a + m - l * a.b);
 return inter;
}
// area of intersection
double intersection(rectangle a, rectangle b) {
 double x1 = max(real(a.tl), real(b.tl)), y1 = max(imag(a.tl), imag(b.tl));
 double x2 = min(real(a.br), real(b.br)), y2 = min(imag(a.br), imag(b.br));
 return (x2 <= x1 || y2 <= y1) ? 0 : (x2-x1)*(y2-y1);
}
```

Convex Hull

```
bool cmp(point a, point b) {
 if (abs(real(a) - real(b)) > EPS) return real(a) < real(b);
 if (abs(imag(a) - imag(b)) > EPS) return imag(a) < imag(b);
```

```cpp
  return false;
}
convex_polygon convexhull(polygon a) {
  sort(a.points.begin(), a.points.end(), cmp);
  vector<point> lower, upper;
  for (int i = 0; i < a.points.size(); i++) {
    while (lower.size() >= 2 &&
      cross(lower.back() - lower[lower.size() - 2], a.points[i] - lower.back()) < EPS)
      lower.pop_back();
    while (upper.size() >= 2 &&
      cross(upper.back() - upper[upper.size() - 2], a.points[i] - upper.back()) > -EPS)
      upper.pop_back();
    lower.push_back(a.points[i]);
    upper.push_back(a.points[i]);
  }
  lower.insert(lower.end(), upper.rbegin() + 1, upper.rend());
  return convex_polygon(lower);
}
```

8 3D Geometry

```cpp
struct point3d {
  double x, y, z;
  point3d operator+(point3d a) const { return {x+a.x, y+a.y, z+a.z}; }
  point3d operator*(double a) const { return {x*a, y*a, z*a}; }
  point3d operator-() const { return {-x, -y, -z}; }
  point3d operator-(point3d a) const { return *this + -a; }
  point3d operator/(double a) const { return *this * (1/a); }
  double norm() { return x*x + y*y + z*z; }
  double abs() { return sqrt(norm()); }
  point3d normalize() { return *this / this->abs(); }
};
double dot(point3d a, point3d b) { return a.x*b.x + a.y*b.y + a.z*b.z; }
point3d cross(point3d a, point3d b) { return {a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x}; }
struct line3d { point3d a, b; };
struct plane { double a, b, c, d; } // a*x + b*y + c*z + d = 0
struct sphere { point3d c; double r; };
#define sq(a) ((a)*(a))
#define cb(a) ((a)*(a)*(a))
double surface(circle a) { return 4 * sq(a.r) * M_PI; }
double volume(circle a) { return 4.0/3.0 * cb(a.r) * M_PI; }
```

9 Optimization

Snoob

```cpp
// SameNumberOfOneBits, next permutation
int snoob(int a) {
  int b = a & -a, c = a + b;
  return c | ((a ^ c) >> 2) / b;
}
// example usage
int main() {
  char l1[] = {'1', '2', '3', '4', '5'};
  char l2[] = {'a', 'b', 'c', 'd'};
  int d1 = 5, d2 = 4;
  // prints 12345abcd, 1234a5bcd, ...
  int min = (1<<d1)-1, max = min << d2;
  for (int i = min; i <= max; i = snoob(i)) {
    int p1 = 0, p2 = 0, v = i;
    while (p1 < d1 || p2 < d2) {
      cout << ((v & 1) ? l1[p1++] : l2[p2++]);
      v /= 2;
    }
    cout << '\n';
  }
}
```

Powers

```cpp
bool isPowerOf2(ll a) {
  return a > 0 && !(a & a-1);
}
bool isPowerOf3(ll a) {
  return a>0&&!(12157665459056928801ull%a);
}
bool isPower(ll a, ll b) {
  double x = log(a) / log(b);
  return abs(x-round(x)) < 0.00000000001;
}
```

10 Additional

Limits

int	± 2147483647	$\pm 2^{31}-1$	10^9
uint	4294967295	$2^{32}-1$	10^9
ll	± 9223372036854775807	$\pm 2^{63}-1$	10^{18}
ull	18446744073709551615	$2^{64}-1$	10^{19}
i128	$\pm 170141183460469231\ldots$	$\pm 2^{127}-1$	10^{38}
u128	$340282366920938463\ldots$	$2^{128}-1$	10^{38}

Complexity classes input size (per second):

$O(n^n)$ or $O(n!)$	$n \leq 10$
$O(2^n)$	$n \leq 20$
$O(n^3)$	$n \leq 500$
$O(n^2)$	$n \leq 5000$
$O(n \log n)$ or $O(n)$	$n \leq 10^6$