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1 General

test.sh

```
# compile and test all *.in and *.ans
g++ -g -O2 -std=gnu++17 -static prog.cpp
for i in *.in; do
    f=${i%.in}
    ./a.exe < $i > "$f.out"
    diff -b -q "$f.ans" "$f.out"
done
```

Header

```
// use better compiler options
#pragma GCC optimize("Ofast", "unroll-loops")
#pragma GCC target("avx2, fma")
```

```
// include everything
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <sys/resource.h>
```

```
// namespaces
using namespace std;
using namespace __gnu_cxx; // rope
using namespace __gnu_pbds; // tree/trie
```

```
// common defines
#define fastio
↳ ios_base::sync_with_stdio(0); cin.tie(0);
#define nostacklim rlimit
↳ RZ; getrlimit(3, &RZ); RZ.rlim_cur = 1; setrlimit(3, &RZ);
#define DEBUG(v) cout<<"DEBUG: "<<#v<<" = "<<v<<"\n";
#define ll long long
#define ull unsigned ll
#define i128 __int128
#define u128 unsigned i128
#define ld long double
```

```
// global variables
mt19937 rng((uint32_t) chrono::steady_clock::now().time_since_epoch().count());
```

Fast IO

```
void readn(unsigned int& n) {
    char c; n = 0;
    while ((c=getchar_unlocked())!=' ' && c!='\n')
        n = n * 10 + c - '0';
}
```

```
void readn(int& n) {
    char c; n = 0; int s = 1;
    if ((c=getchar_unlocked())=='-') s = -1;
    else n = c - '0';
    while ((c=getchar_unlocked())!=' ' && c!='\n')
        n = n * 10 + c - '0';
    n *= s;
}
```

```
void readn(ld& n) {
    char c; n = 0;
    ld m = 0, o = 1; bool d = false; int s = 1;
    if ((c=getchar_unlocked())=='-') s = -1;
    else if (c == '.') d = true;
    else n = c - '0';
    while ((c=getchar_unlocked())!=' ' && c!='\n')
        {
            if (c == '.') d = true;
            else if (d) { m=m*10+c-'0'; o*=0.1; }
            else n = n * 10 + c - '0';
        }
    n = s * (n + m * o);
}
```

```
void readn(double& n) {
    ld m; readn(m); n = m;
}

void readn(float& n) {
    ld m; readn(m); n = m;
}
```

```
void readn(string& s) {
    char c; s = "";
    while((c=getchar_unlocked())!=' ' && c!='\n')
        s += c;
}
```

```
bool readline(string& s) {
    char c; s = "";
    while(c=getchar_unlocked()) {
        if (c == '\n') return true;
        if (c == EOF) return false;
        s += c;
    }
    return false;
}
```

```
void printn(unsigned int n) {
    if (n / 10) printn(n / 10);
    putchar_unlocked(n % 10 + '0');
}
```

```
void printn(int n) {
```

```
.if (n < 0) { putchar_unlocked('-'); n*=-1; }
printn((unsigned int)n);
}
```

2 Algorithms

Min/Max Subarray

```
// max - compare = a < b, reset = a < 0
// min - compare = a > b, reset = a > 0
// returns {sum, {start, end}}
pair<int, pair<int, int>>
↳ ContiguousSubarray(int* a, int size,
↳ bool(*compare)(int, int),
↳ bool(*reset)(int), int defbest = 0) {
    int best = defbest, cur = 0, start = 0, end = 0, s = 0;
    for (int i = 0; i < size; i++) {
        cur += a[i];
        if ((*compare)(best, cur)) { best = cur;
        ↳ start = s; end = i; }
        if ((*reset)(cur)) { cur = 0; s = i + 1; }
    }
    return {best, {start, end}};
}
```

Quickselect

```
#define QSNE -999999
```

```
int partition(int arr[], int l, int r)
{
    int x = arr[r], i = l;
    for (int j = l; j <= r - 1; j++)
        if (arr[j] <= x)
            swap(arr[i++], arr[j]);
    swap(arr[i], arr[r]);
    return i;
}
```

```
// find k'th smallest element in unsorted
↳ array, only if all distinct
int qselect(int arr[], int l, int r, int k)
{
    if (!(k > 0 && k <= r - l + 1)) return QSNE;
    swap(arr[l + rng() % (r-l+1)], arr[r]);
    int pos = partition(arr, l, r);
    if (pos-l==k-1) return arr[pos];
    if (pos-l>k-1) return qselect(arr, l, pos-1, k);
    return qselect(arr, pos+1, r, k-pos+1-1);
}
```

```
// TODO: compare against std::nth_element()
```

Saddleback Search

```
// search for v in 2d array arr[x][y], sorted
↳ on both axis
pair<int, int> saddleback_search(int** arr,
↳ int x, int y, int v) {
    int i = x-1, j = 0;
    while (i >= 0 && j < y) {
```

```
.if (arr[i][j] == v) return {i, j};
. (arr[i][j] > v)? i--: j++;
}
return {-1, -1};
}
```

Ternary Search

```
// < max, > min, or any other unimodal func
#define TERNCOMP(a,b) (a)<(b)
int ternsearch(int a, int b, int (*f)(int)) {
    while (b-a > 4) {
        int m = (a+b)/2;
        if (TERNCOMP((*f)(m), (*f)(m+1))) a = m;
        else b = m+1;
    }
    for (int i = a+1; i <= b; i++)
        if (TERNCOMP((*f)(a), (*f)(i)))
            a = i;
    return a;
}
```

```
#define TERNPREC 0.000001
double ternsearch(double a, double b, double
↳ (*f)(double)) {
    while (b-a > TERNPREC * 4) {
        double m = (a+b)/2;
        if (TERNCOMP((*f)(m), (*f)(m + TERNPREC))) a
        ↳ = m;
        else b = m + TERNPREC;
    }
    for (double i = a + TERNPREC; i <= b; i +=
    ↳ TERNPREC)
        if (TERNCOMP((*f)(a), (*f)(i)))
            a = i;
    return a;
}
```

3 Data Structures

Fenwick Tree

```
// Fenwick tree, array of cumulative sums -
↳ O(log n) updates, O(log n) gets
```

```
struct Fenwick {
    int n; ll* tree;

    void update(int i, int val) {
        ++i;
        while (i <= n) {
            tree[i] += val;
            i += i & (-i);
        }
    }
};
```

```
Fenwick(int size) {
    n = size;
    tree = new ll[n+1];
    for (int i = 1; i <= n; i++)
        tree[i] = 0;
```

```

}
Fenwick(int* arr, int size) : Fenwick(size) {
    for (int i = 0; i < n; i++)
        update(i, arr[i]);
}
~Fenwick() { delete[] tree; }

ll operator[](int i) {
    if (i < 0 || i > n) return 0;
    ll sum = 0;
    ++i;
    while (i>0) {
        sum += tree[i];
        i -= i & (-i);
    }
    return sum;
}

ll getRange(int a, int b) { return
    ↪ operator[] (b) - operator[] (a-1); }
};

```

Hashtable

```

// similar to unordered_map, but faster
struct chash {
    const uint64_t C = (1ll)(2e18 * M_PI) + 71;
    ll operator()(ll x) const { return
    ↪ __builtin_bswap64(x*C); }
};

int main() {
    gp_hash_table<ll,int,chash>
    ↪ hashtable({}, {}, {}, {}, {1<16});
    for (int i = 0; i < 100; i++)
        hashtable[i] = 200+i;
    if (hashtable.find(10) != hashtable.end())
        cout << hashtable[10];
}

```

Ordered Set

```

typedef tree<int,null_type,less<int>,rb_tree<
    ↪ _tag,tree_order_statistics_node_update>
    ↪ ordered_set;

```

```

int main()
{
    ordered_set o_set;
    o_set.insert(5); o_set.insert(1);
    ↪ o_set.insert(3);
}

```

```

// get second smallest element
cout << *(o_set.find_by_order(1)) << '\n';

```

```

// number of elements less than k=4
cout << o_set.order_of_key(4) << '\n';
}

```

Rope

```

// O(log n) insert, delete, concatenate
int main() {

```

```

// generate rope
rope<int> v;
for (int i = 0; i < 100; i++)
    v.push_back(i);

// move range to front
rope<int> copy = v.substr(10, 10);
v.erase(10, 10);
v.insert(copy.mutable_begin(), copy);

// print elements of rope
for (auto it : v)
    cout << it << " ";
}

```

Segment Tree

```

//max(a,b), min(a,b), a+b, a*b, gcd(a,b), a^b
struct SegmentTree {
    typedef int T;
    static constexpr T UNIT = INT_MIN;
    T f(T a, T b) {
        if (a == UNIT) return b;
        if (b == UNIT) return a;
        return max(a,b);
    }
    int n; vector<T> s;
    SegmentTree(int n, T def=UNIT) : s(2*n, def),
    ↪ n(n) {}
    SegmentTree(vector<T> arr) :
    ↪ SegmentTree(arr.size()) {
        for (int i=0;i<arr.size();i++)
            update(i,arr[i]);
    }
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos*2+1]);
    }
    T query(int b, int e) { // query [b, e)
        T ra = UNIT, rb = UNIT;
        for (b+=n, e+=n; b<e; b/=2, e/=2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb);
        }
        return f(ra, rb);
    }
    T get(int p) { return query(p, p+1); }
};

```

Trie

```

typedef trie<string, null_type,
    ↪ trie_string_access_traits<>,
    ↪ pat_trie_tag,
    ↪ trie_prefix_search_node_update> trie_type;

```

```

int main() {
    // generate trie
    trie_type trie;
    for (int i = 0; i < 20; i++)

```

```

    trie.insert(to_string(i)); // true if new,
    ↪ false if old
    // print things with prefix "1"
    auto range = trie.prefix_range("1");
    for (auto it = range.first; it !=
    ↪ range.second; it++)
        cout << *it << " ";
}

```

4 String

Aho Corasick

```

// range of alphabet for automata to consider
// MAXC = 26, OFFC = 'a' if only lowercase
const int MAXC = 256;
const int OFFC = 0;

```

```

struct aho_corasick {
    struct state
    {
        set<pair<int, int> out;
        int fail; vector<int> go;
        state() : fail(-1), go(MAXC, -1) {}
    };
    vector<state> s;
    int id = 0;

    aho_corasick(string* arr, int size) : s(1) {
        for (int i = 0; i < size; i++) {
            int cur = 0;
            for (int c : arr[i]) {
                if (s[cur].go[c-OFFC] == -1) {
                    s[cur].go[c-OFFC] = s.size();
                    s.push_back(state());
                }
                cur = s[cur].go[c-OFFC];
            }
            s[cur].out.insert({arr[i].size(), id++});
        }

        for (int c = 0; c < MAXC; c++)
            if (s[0].go[c] == -1)
                s[0].go[c] = 0;
        queue<int> sq;
        for (int c = 0; c < MAXC; c++) {
            if (s[0].go[c] != 0) {
                s[s[0].go[c]].fail = 0;
                sq.push(s[0].go[c]);
            }
            while (sq.size()) {
                int e = sq.front(); sq.pop();
                for (int c = 0; c < MAXC; c++) {
                    if (s[e].go[c] != -1) {
                        int failure = s[e].fail;
                        while (s[failure].go[c] == -1)

```

```

                    failure = s[failure].fail;
                    failure = s[failure].go[c];
                    s[s[e].go[c]].fail = failure;
                    for (auto length : s[failure].out)
                        s[s[e].go[c]].out.insert(length);
                    sq.push(s[e].go[c]);
                }
            }
        }

        // list of {start pos, pattern id}
        vector<pair<int, int> search(string text)
        {
            vector<pair<int, int> toret;
            int cur = 0;

            for (int i = 0; i < text.size(); i++) {
                while (s[cur].go[text[i]-OFFC] == -1)
                    cur = s[cur].fail;
                cur = s[cur].go[text[i]-OFFC];

                if (s[cur].out.size())
                    for (auto end : s[cur].out)
                        toret.push_back({i - end.first + 1,
                        ↪ end.second});
            }
            return toret;
        }
};

```

Boyer Moore

```

struct defint { int i = -1; };
vector<int> boyermoore(string txt, string pat)
    ↪ {
    vector<int> toret; unordered_map<char,
    ↪ defint> badchar;
    int m = pat.size(), n = txt.size();
    for (int i = 0; i < m; i++) badchar[pat[i]].i
    ↪ = i;
    int s = 0;
    while (s <= n - m) {
        int j = m - 1;
        while (j >= 0 && pat[j] == txt[s + j]) j--;
        if (j < 0) {
            toret.push_back(s);
            s += (s + m < n) ? m - badchar[txt[s +
            ↪ m]].i : 1;
        } else
            s += max(1, j - badchar[txt[s + j]].i);
        }
    return toret;
}

```

English Conversion

```

const string ones[] = {"", "one", "two",
↳ "three", "four", "five", "six", "seven",
↳ "eight", "nine"};
const string teens[] = {"ten", "eleven",
↳ "twelve", "thirteen", "fourteen",
↳ "fifteen", "sixteen", "seventeen",
↳ "eighteen", "nineteen"};
const string tens[] = {"twenty", "thirty",
↳ "forty", "fifty", "sixty", "seventy",
↳ "eighty", "ninety"};
const string mags[] = {"thousand", "million",
↳ "billion", "trillion", "quadrillion",
↳ "quintillion", "sextillion",
↳ "septillion"};
string convert(int num, int carry) {
    if (num < 0) return "negative " +
↳ convert(-num, 0);
    if (num < 10) return ones[num];
    if (num < 20) return teens[num % 10];
    if (num < 100) return tens[(num / 10) - 2] +
↳ (num%10==0?"":" " ) + ones[num % 10];
    if (num < 1000) return ones[num / 100] +
↳ (num/100==0?"":" " ) + "hundred" +
↳ (num%100==0?"":" " ) + convert(num % 100,
↳ 0);
    return convert(num / 1000, carry + 1) + " " +
↳ mags[carry] + " " + convert(num % 1000,
↳ 0);
}
string convert(int num) {
    return (num == 0) ? "zero" : convert(num, 0);
}

```

Knuth Morris Pratt

```

vector<int> kmp(string txt, string pat) {
    vector<int> toret;
    int m = txt.length(), n = pat.length();

```

```

    int next[n + 1];
    for (int i = 0; i < n + 1; i++)
        next[i] = 0;

```

```

    for (int i = 1; i < n; i++) {
        int j = next[i + 1];
        while (j > 0 && pat[j] != pat[i])
            j = next[j];
        if (j > 0 || pat[j] == pat[i])
            next[i + 1] = j + 1;
    }

```

```

    for (int i = 0, j = 0; i < m; i++) {
        if (txt[i] == pat[j]) {
            if (++j == n)
                toret.push_back(i - j + 1);
        } else if (j > 0) {
            j = next[j];
        }
        i--;
    }
}

```

```

    }
    return toret;
}

```

Longest Common Prefix

```

string lcp(string* arr, int n) {
    if (n == 0) return "";
    sort(arr, arr + n);
    string r = ""; int v = 0;
    while (v < arr[0].length() && arr[0][v] ==
↳ arr[n-1][v])
        r += arr[0][v++];
    return r;
}

```

Longest Common Subsequence

```

string lcs(string a, string b) {
    int m = a.length(), n = b.length();

    int L[m+1][n+1];
    for (int i = 0; i <= m; i++) {
        for (int j = 0; j <= n; j++) {
            if (i == 0 || j == 0) L[i][j] = 0;
            else if (a[i-1] == b[j-1]) L[i][j] =
↳ L[i-1][j-1] + 1;
            else L[i][j] = max(L[i-1][j], L[i][j-1]);
        }
    }
    // return L[m][n]; // length of lcs
}

```

```

    string out = "";
    int i = m - 1, j = n - 1;
    while (i >= 0 && j >= 0) {
        if (a[i] == b[j]) {
            out = a[i] + out;
            j--;
        }
        else if (L[i][j+1] > L[i+1][j]) i--;
        else j--;
    }
    return out;
}

```

Longest Common Substring

```

// l is array of palindrome length at that
↳ index
int manacher(string s, int* l) {
    int n = s.length() * 2;
    for (int i = 0, j = 0, k; i < n; i += k, j =
↳ max(j-k, 0)) {
        while (i >= j && i + j + 1 < n && s[(i-j)/2]
↳ == s[(i+j+1)/2]) j++;
        l[i] = j;
        for (k = 1; i >= k && j >= k && l[i-k] !=
↳ j-k; k++)
            l[i+k] = min(l[i-k], j-k);
    }
    return *max_element(l, l + n);
}

```

Subsequence Count

```

// "banana", "ban" » 3 (ban, ba..n, b..an)
ull subsequences(string body, string subs) {
    int m = subs.length(), n = body.length();
    if (m > n) return 0;
    ull** arr = new ull*[m+1];
    for (int i = 0; i <= m; i++) arr[i] = new
↳ ull[n+1];
    for (int i = 1; i <= m; i++) arr[i][0] = 0;
    for (int i = 0; i <= n; i++) arr[0][i] = 1;
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            arr[i][j] = arr[i][j-1] + ((body[j-1] ==
↳ subs[i-1])? arr[i-1][j-1] : 0);
    return arr[m][n];
}

```

5 Math

Catalan Numbers

```

ull* catalan = new ull[1000000];

void genCatalan(int n, int mod) {
    catalan[0] = catalan[1] = 1;
    for (int i = 2; i <= n; i++) {
        catalan[i] = 0;
        for (int j = i - 1; j >= 0; j--) {
            catalan[i] += (catalan[j] * catalan[i-j-1])
↳ % mod;
        }
        if (catalan[i] >= mod)
            catalan[i] -= mod;
    }
}

```

// TODO: consider binomial coefficient method

Combinatorics (nCr, nPr)

```

// can optimize by precomputing factorials, and
↳ fact[n]/fact[n-r]
ull nPr(ull n, ull r) {
    ull v = 1;
    for (ull i = n-r+1; i <= n; i++)
        v *= i;
    return v;
}

```

```

ull nPr(ull n, ull r, ull m) {
    ull v = 1;
    for (ull i = n-r+1; i <= n; i++)
        v = (v * i) % m;
    return v;
}

```

```

ull nCr(ull n, ull r) {
    long double v = 1;
    for (ull i = 1; i <= r; i++)
        v = v * (n-r+i) / i;
}

```

```

    return (ull)(v + 0.001);
}

```

```

// requires modulo math
// can optimize by precomputing mfac and
↳ minv-mfac
ull nCr(ull n, ull r, ull m) {
    return mfac(n, m) * minv(mfac(k, m), m) % m *
↳ minv(mfac(n-k, m), m) % m;
}

```

Chinese Remainder Theorem

```

bool ecrt(ll* r, ll* m, int n, ll& re, ll& mo)
↳ {
    ll x, y, d; mo = m[0]; re = r[0];
    for (int i = 1; i < n; i++) {
        d = egcd(mo, m[i], x, y);
        if ((r[i] - re) % d != 0) return false;
        x = (r[i] - re) / d * x % (m[i] / d);
        re += x * mo;
        mo = mo / d * m[i];
        re %= mo;
    }
    re = (re + mo) % mo;
    return true;
}

```

Count Digit Occurences

```

/*count(n,d) counts the number of occurences of
↳ a digit d in the range [0,n]*/
ll digit_count(ll n, ll d) {
    ll result = 0;
    while (n != 0) {
        result += ((n%10) == d ? 1 : 0);
        n /= 10;
    }
    return result;
}

ll count(ll n, ll d) {
    if (n < 10) return (d > 0 && n >= d);
    if ((n % 10) != 9) return digit_count(n, d) +
↳ count(n-1, d);
    return 10*count(n/10, d) + (n/10) + (d > 0);
}

```

Discrete Logarithm

```

unordered_map<int, int> dlogc;
int discretelog(int a, int b, int m) {
    dlogc.clear();
    ll n = sqrt(m)+1, an = 1;
    for (int i = 0; i < n; i++)
        an = (an * a) % m;
    ;
    ll c = an;
}

```

```

for (int i = 1; i <= n; i++) {
    if (!dlogc.count(c)) dlogc[c] = i;
    c = (c * an) % m;
}

c = b;
for (int i = 0; i <= n; i++) {
    if (dlogc.count(c)) return (dlogc[c] * n - i
↪ + m - 1) % (m-1);
    c = (c * a) % m;
}
return -1;
}

```

Euler Phi / Totient

```

int phi(int n) {
    int r = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) r -= r / i;
        while (n % i == 0) n /= i;
    }
    if (n > 1) r -= r / n;
    return r;
}

```

```

#define n 100000
ll phi[n+1];
void computeTotient() {
    for (int i=1; i<=n; i++) phi[i] = i;
    for (int p=2; p<=n; p++) {
        if (phi[p] == p) {
            phi[p] = p-1;
            for (int i = 2*p; i<=n; i += p) phi[i] =
↪ (phi[i]/p) * (p-1);
        }
    }
}

```

Factorials

```

// digits in factorial
#define kamenetsky(n) (floor((n * log10(n /
↪ M_E)) + (log10(2 * M_PI * n) / 2.0)) + 1)

```

```

// approximation of factorial
#define stirling(n) ((n == 1) ? 1 : sqrt(2 *
↪ M_PI * n) * pow(n / M_E, n))

```

```

// natural log of factorial
#define lfactorial(n) (lgamma(n+1))

```

Prime Factorization

```

// do not call directly
ll pollard_rho(ll n, ll s) {
    ll x, y;
    x = y = rand() % (n - 1) + 1;
    int head = 1, tail = 2;
    while (true) {
        x = mult(x, x, n);
        x = (x + s) % n;

```

```

        if (x == y) return n;
        ll d = __gcd(max(x - y, y - x), n);
        if (1 < d && d < n) return d;
        if (++head == tail) y = x, tail <= 1;
    }
}

// call for prime factors
void factorize(ll n, vector<ll> &divisor) {
    if (n == 1) return;
    if (isPrime(n)) divisor.push_back(n);
    else {
        ll d = n;
        while (d >= n) d = pollard_rho(n, rand() %
↪ (n - 1) + 1);
        factorize(n / d, divisor);
        factorize(d, divisor);
    }
}

```

Farey Fractions

```

// generate 0 <= a/b <= 1 ordered, b <= n
// farey(4) = 0/1 1/4 1/3 1/2 2/3 3/4 1/1
// length is sum of phi(i) for i = 1 to n

```

```

vector<pair<int, int>> farey(int n) {
    int h = 0, k = 1, x = 1, y = 0, r;
    vector<pair<int, int>> v;
    do {
        v.push_back({h, k});
        r = (n-y)/k;
        y += r*k; x += r*h;
        swap(x,h); swap(y,k);
        x = -x; y = -y;
    } while (k > 1);
    v.push_back({1, 1});
    return v;
}

```

Fast Fourier Transform

```

#define cd complex<double>
const double PI = acos(-1);

```

```

void fft(vector<cd>& a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >= 1) j ^= bit;
        j ^= bit;

        if (i < j) swap(a[i], a[j]);
    }

```

```

    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 :
↪ 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {

```

```

            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }
}

```

```

if (invert)
    for (auto& x : a)
        x /= n;
}

```

```

vector<int> fftmult(vector<int> const& a,
↪ vector<int> const& b) {
    vector<cd> fa(a.begin(), a.end()),
    ↪ fb(b.begin(), b.end());
    int n = 1 << (32 - __builtin_clz(a.size() +
↪ b.size() - 1));
    fa.resize(n); fb.resize(n);

```

```

    fft(fa, false); fft(fb, false);
    for (int i = 0; i < n; i++) fa[i] *= fb[i];
    fft(fa, true);

    vector<int> toret(n);
    for (int i = 0; i < n; i++) toret[i] =
↪ round(fa[i].real());
    return toret;
}

```

Greatest Common Denominator

```

ll egcd(ll a, ll b, ll& x, ll& y) {
    if (b == 0) { x = 1; y = 0; return a; }
    ll gcd = egcd(b, a % b, x, y);
    x -= a / b * y;
    swap(x, y);
    return gcd;
}

```

Josephus Problem

```

// 0-indexed, arbitrary k
int josephus(int n, int k) {
    if (n == 1) return 0;
    if (k == 1) return n-1;
    if (k > n) return (josephus(n-1,k)+k)%n;
    int res = josephus(n-n/k,k)-n%k;
    return res + ((res<0)?n:res/(k-1));
}

```

```

// fast case if k=2, traditional josephus
int josephus(int n) {
    return 2*(n-(1<=(32-__builtin_clz(n)-1)));
}

```

Least Common Multiple

```

#define lcm(a,b) ((a*b)/__gcd(a,b))

```

Modulo Operations

```

#define MOD 1000000007
#define madd(a,b,m) (a+b-((a+b-m)>=0)?m:0))
#define mult(a,b,m) ((ull)a*b%m)
#define msub(a,b,m) (a-b+((a<b)?m:0))

```

```

ll mpow(ll b, ll e, ll m) {
    ll x = 1;
    while (e > 0) {
        if (e % 2) x = (x * b) % m;
        b = (b * b) % m;
        e /= 2;
    }
    return x % m;
}

```

```

ull mfac(ull n, ull m) {
    ull f = 1;
    for (int i = n; i > 1; i-)
        f = (f * i) % m;
    return f;
}

```

```

// if m is not guaranteed to be prime
ll minv(ll b, ll m) {
    ll x = 0, y = 0;
    if (egcd(b, m, x, y) != 1) return -1;
    return (x % m + m) % m;
}

```

```

ll mdiv_compmod(int a, int b, int m) {
    if (__gcd(b, m) != 1) return -1;
    return mult(a, minv(b, m), m);
}

```

```

// if m is prime (like 10^9+7)
ll mdiv_primemod (int a, int b, int m) {
    return mult(a, mpow(b, m-2, m), m);
}

```

Miller-Rabin Primality Test

```

// Miller-Rabin primality test - O(10 log^3 n)
bool isPrime(ull n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    ull s = n - 1;
    while (s % 2 == 0) s /= 2;
    for (int i = 0; i < 10; i++) {
        ull temp = s;
        ull a = rand() % (n - 1) + 1;
        ull mod = mpow(a, temp, n);
        while (temp!=n-1&&mod!=1&&mod!=n-1) {
            mod = mult(mod, mod, n);
            temp *= 2;

```



```

..}
..if (mod!=n-1&&temp%2==0) return false;
..}
..return true;
}

```

Sieve of Eratosthenes

```
bitset<100000001> sieve;
```

```

// generate sieve - O(n log n)
void genSieve(int n) {
    sieve[0] = sieve[1] = 1;
    for (ull i = 3; i * i < n; i += 2)
        if (!sieve[i])
            for (ull j = i * 3; j <= n; j += i * 2)
                sieve[j] = 1;
}

```

```

// query sieve after it's generated - O(1)
bool querySieve(int n) {
    return n == 2 || (n % 2 != 0 && !sieve[n]);
}

```

Simpson's / Approximate Integrals

```

// integrate f from a to b, k iterations
// error <= (b-a)/18.0 * M * ((b-a)/2k)^4
// where M = max(abs(f''''(x))) for x in [a,b]
// "f" is a function "double func(double x)"
double Simpsons(double a, double b, int k,
    ⇨ double (*f)(double)) {
    double dx = (b-a)/(2.0*k), t = 0;
    for (int i = 0; i < k; i++)
        t += ((i==0)?1:2)*(*f)(a+2*i*dx) + 4 *
            ⇨ (*f)(a+(2*i+1)*dx);
    return (t + (*f)(b)) * (b-a) / 6.0 / k;
}

```

Common Equations Solvers

```

// ax^2 + bx + c = 0, find x
vector<double> solveEq(double a, double b,
    ⇨ double c) {
    vector<double> r;
    double z = b * b - 4 * a * c;
    if (z == 0)
        r.push_back(-b/(2*a));
    else if (z > 0) {
        r.push_back((sqrt(z)-b)/(2*a));
        r.push_back((sqrt(z)+b)/(2*a));
    }
    return r;
}

```

```

// ax^3 + bx^2 + cx + d = 0, find x
vector<double> solveEq(double a, double b,
    ⇨ double c, double d) {
    vector<double> res;
    long double a1 = b/a, a2 = c/a, a3 = d/a;
    long double q = (a1*a1 - 3*a2)/9.0, sq =
        ⇨ -2*sqrt(q);

```

```

    long double r = (2*a1*a1*a1 - 9*a1*a2 +
        ⇨ 27*a3)/54.0;
    long double z = r*r-q*q*q, theta;
    if (z <= 0) {
        theta = acos(r/sqrt(q*q*q));
        res.push_back(sq*cos(theta/3.0) - a1/3.0);
        res.push_back(sq*cos((theta+2.0*PI)/3.0) -
            ⇨ a1/3.0);
        res.push_back(sq*cos((theta+4.0*PI)/3.0) -
            ⇨ a1/3.0);
    }
    else {
        res.push_back(pow(sqrt(z)+fabs(r), 1/3.0));
        res[0] = (res[0] + q / res[0]) *
            ⇨ ((r<0)?1:-1) - a1 / 3.0;
    }
    return res;
}

```

```

// m = # equations, n = # variables, a[m][n+1]
⇨ = coefficient matrix
// a[i][0]x + a[i][1]y + ... + a[i][n]z =
⇨ a[i][n+1]
const double eps = 1e-7;
bool zero(double a) { return (a < eps) && (a >
    ⇨ -eps); }
vector<double> solveEq(double **a, int m, int
    ⇨ n) {
    int cur = 0;
    for (int i = 0; i < n; i++) {
        for (int j = cur; j < m; j++) {
            if (!zero(a[j][i])) {
                if (j != cur) swap(a[j], a[cur]);
                for (int sat = 0; sat < m; sat++) {
                    if (sat == cur) continue;
                    double num = a[sat][i] / a[cur][i];
                    for (int sot = 0; sot <= n; sot++)
                        a[sat][sot] -= a[cur][sot] * num;
                }
                cur++;
                break;
            }
        }
        for (int j = cur; j < m; j++)
            if (!zero(a[j][n])) return vector<double>();
        vector<double> ans(n,0);
        for (int i = 0, sat = 0; i < n; i++)
            if (sat < m && !zero(a[sat][i]))
                ans[i] = a[sat][n] / a[sat++][i];
        return ans;
    }
}

```

6 Graph

```

struct edge {
    int u,v,w;
    edge (int u,int v,int w) : u(u),v(v),w(w) {}
    edge () : u(0), v(0), w(0) {}

```

```

};
bool operator < (const edge &e1, const edge
    ⇨ &e2) { return e1.w < e2.w; }
bool operator > (const edge &e1, const edge
    ⇨ &e2) { return e1.w > e2.w; }

struct subset { int p, rank; };

```

Eulerian Path

```

#define edge_list vector<edge>
#define adj_sets vector<set<int>

struct EulerPathGraph {
    adj_sets graph; // actually indexes incident
    ⇨ edges
    edge_list edges; int n; vector<int> indeg;
    EulerPathGraph(int n): n(n) {
        indeg = *(new vector<int>(n,0));
        graph = *(new adj_sets(n, set<int>()));
    }

    void add_edge(int u, int v) {
        graph[u].insert(edges.size());
        indeg[v]++;
        edges.push_back(edge(u,v,0));
    }

    bool eulerian_path(vector<int> &circuit) {
        if(edges.size()==0) return false;
        stack<int> st;
        int a[] = {-1, -1};
        for(int v=0;v<n;v++) {
            if(indeg[v]!=graph[v].size()) {
                bool b = indeg[v] > graph[v].size();
                if (abs(((int)indeg[v])-((int)graph[v]
                    ⇨ .size())) > 1) return
                    ⇨ false;
                if (a[b] != -1) return false;
                a[b] = v;
            }
        }
        int s = (a[0]!=-1 && a[1]!=-1 ? a[0] :
            ⇨ (a[0]==-1 && a[1]==-1 ? edges[0].u : -1));
        if(s==1) return false;
        while(!st.empty() || !graph[s].empty()) {
            if (graph[s].empty()) {
                circuit.push_back(s); s = st.top();
                ⇨ st.pop(); }
            else {
                int w = edges[*graph[s].begin()].v;
                graph[s].erase(graph[s].begin());
                ⇨ st.push(s); s = w;
            }
        }
        circuit.push_back(s);
        return circuit.size()-1==edges.size();
    }
};

```

Minimum Spanning Tree

```

// returns vector of edges in the mst
// graph[i] = vector of edges incident to
    ⇨ vertex i
// places total weight of the mst in &total
// if returned vector has size != n-1, there is
    ⇨ no MST
vector<edge> mst(vector<vector<edge>> graph, ll
    ⇨ &total) {
    total = 0;
    priority_queue<edge, vector<edge>,
        ⇨ greater<edge>> pq;
    vector<edge> MST;
    bitset<20001> marked; // change size as
        ⇨ needed
    marked[0] = 1;
    for (edge ep : graph[0]) pq.push(ep);
    while(MST.size()!=graph.size()-1 &&
        ⇨ pq.size()!=0) {
        edge e = pq.top(); pq.pop();
        int u = e.u, v = e.v, w = e.w;
        if(marked[u] && marked[v]) continue;
        else if(marked[u]) swap(u, v);
        for(edge ep : graph[u]) pq.push(ep);
        marked[u] = 1;
        MST.push_back(e);
        total += e.w;
    }
    return MST;
}

```

Union Find

```

int uf_find(subset* s, int i) {
    if (s[i].p != i) s[i].p = uf_find(s, s[i].p);
    return s[i].p;
}

```

```

void uf_union(subset* s, int x, int y) {
    int xp = uf_find(s, x), yp = uf_find(s, y);
    if (s[xp].rank > s[yp].rank) s[yp].p = xp;
    else if (s[xp].rank < s[yp].rank) s[xp].p =
        ⇨ yp;
    else { s[yp].p = xp; s[xp].rank++; }
}

```

7 2D Geometry

```

#define point complex<double>
double dot(point a, point b) { return
    ⇨ real(conj(a)*b); }
double cross(point a, point b) { return
    ⇨ imag(conj(a)*b); }

```

```

struct line { point a, b; };
struct circle { point c; double r; };
struct triangle { point a, b, c; };

```

```

struct rectangle { point tl, br; };

struct convex_polygon {
    vector<point> points;
    convex_polygon(triangle a) {
        points.push_back(a.a);
        points.push_back(a.b);
        points.push_back(a.c);
    };
    convex_polygon(rectangle a) {
        points.push_back(a.tl);
        points.push_back({real(a.tl),
            imag(a.br)});
        points.push_back(a.br);
        points.push_back({real(a.br),
            imag(a.tl)});
    }
};

#define sq(a) ((a)*(a))
double circumference(circle a) { return 2 *
    a.r * M_PI; }
double area(circle a) { return sq(a.r) * M_PI;
    };
double intersection(circle a, circle b) {
    double d = abs(a.c - b.c);
    if (d <= b.r - a.r) return area(a);
    if (d <= a.r - b.r) return area(b);
    if (d >= a.r + b.r) return 0;
    double alpha = acos((sq(a.r) + sq(d) -
        sq(b.r)) / (2 * a.r * d));
    double beta = acos((sq(b.r) + sq(d) -
        sq(a.r)) / (2 * b.r * d));
    return sq(a.r) * (alpha - 0.5 * sin(2 *
        alpha)) + sq(b.r) * (beta - 0.5 * sin(2 *
        beta));
}

double intersection(rectangle a, rectangle b)
    {
    double x1 = max(real(a.tl), real(b.tl)), y1 =
        max(imag(a.tl), imag(b.tl));
    double x2 = min(real(a.br), real(b.br)), y2 =
        min(imag(a.br), imag(b.br));
    return (x2 <= x1 || y2 <= y1) ? 0 :
        (x2-x1)*(y2-y1);
}

```

8 3D Geometry

```

struct point3d {
    double x, y, z;
    point3d operator+(point3d a) const { return
        {x+a.x, y+a.y, z+a.z}; }
    point3d operator*(double a) const { return
        {x*a, y*a, z*a}; }
    point3d operator-() const { return {-x, -y,
        -z}; }

```

```

    point3d operator-(point3d a) const { return
        *this + -a; }
    point3d operator/(double a) const { return
        *this * (1/a); }
    double norm() { return x*x + y*y + z*z; }
    double abs() { return sqrt(norm()); }
    point3d normalize() { return *this /
        this->abs(); }
};

```

```

double dot(point3d a, point3d b) { return
    a.x*b.x + a.y*b.y + a.z*b.z; }
point3d cross(point3d a, point3d b) { return
    {a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z,
    a.x*b.y - a.y*b.x}; }

```

```

struct line3d { point3d a, b; };
struct plane { double a, b, c, d; } // a*x +
    b*y + c*z + d = 0
struct sphere { point3d c; double r; };

```

```

#define sq(a) ((a)*(a))
#define cb(a) ((a)*(a)*(a))
double surface(circle a) { return 4 * sq(a.r)
    * M_PI; }
double volume(circle a) { return 4.0/3.0 *
    cb(a.r) * M_PI; }

```

9 Optimization

```

// SameNumberOfOneBits, next permutation
int snoob(int a) {
    int b = a & -a, c = a + b;
    return c | ((a ^ c) >> 2) / b;
}

```