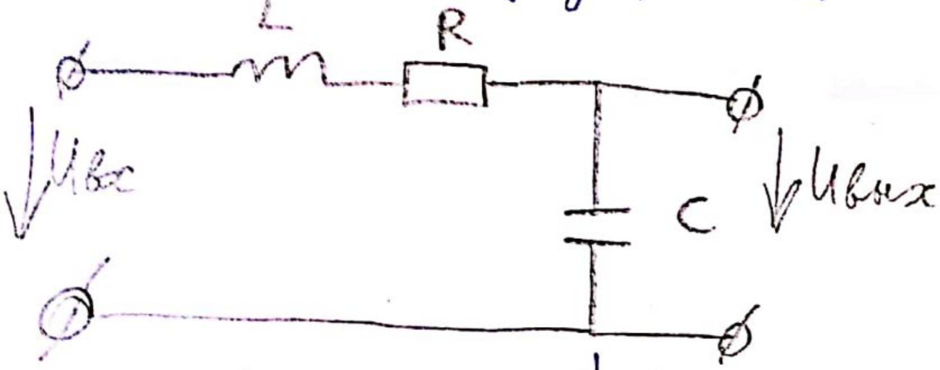
Типовая задача 0.



перед-ая ф-я

$$W(s)=\frac{U_{вых}(s)}{U_{вх}(s)}=\frac{\frac{1}{sC}}{sL+R+\frac{1}{sC}}=\frac{1}{s^2LC+sRC+1}$$

$$s_{1,2}=\frac{-RC\pm\sqrt{R^2C^2-4LC}}{2LC}$$

вр-ая хар-ка: $w(t)=L^{-1}\{W(s)\}=L^{-1}\left\{\frac{1}{LC(s-s_1)(s-s_2)}\right\}=$

$$=\frac{1(t)}{LC(s_1-s_2)}\left(e^{s_1t}-e^{s_2t}\right)$$

переходная: $h(t)=L^{-1}\{W(s)\cdot\frac{1}{s}\}=L^{-1}\left\{\frac{1}{LCs(s-s_1)(s-s_2)}\right\}=$

$$=\frac{1(t)}{LC(s_1-s_2)}\left(\frac{e^{s_1t}-1}{s_1}-\frac{e^{s_2t}-1}{s_2}\right)$$

Реакции:

1) с пом. передат. ф-и.

$$U_{вх}(t)=U_m\left(1(t)-1(t-t_1)\right)$$

$U_{вх}(s)=\frac{U_m}{s}\left(1-e^{-st_1}\right)$; $U_{вых}(s)=W(s)\cdot U_{вх}(s)=\frac{U_m}{LC}\cdot\frac{1-e^{-st_1}}{s(s-1)(s-s_2)}$

$$U_{вых}(t)=L^{-1}\{U_{вых}(s)\}=L^{-1}\left\{\frac{U_m}{LC}\left(\frac{1}{s(s-s_1)(s-s_2)}-\frac{e^{-st_1}}{s(s-1)(s-s_2)}\right)\right\}=$$

$$=U_m\left(h(t)-h(t-t_1)\right)$$

2) Интеграл наложения

$$u_{вых}(t) = \int_0^t u_{вх}(\tau)\, w(t-\tau)\, d\tau = \int_0^t U_m \left(1(\tau) - 1(\tau - t_1)\right) \cdot$$

$$\cdot \frac{1(t-\tau)}{LC(s_1-s_2)} \left(e^{s_1(t-\tau)} - e^{s_2(t-\tau)}\right) d\tau = \frac{U_m}{LC(s_1-s_2)} \int_0^t \left(1(\tau) - 1(\tau - t_1)\right) \cdot$$

$$\cdot \left(e^{s_1(t-\tau)} - e^{s_2(t-\tau)}\right) d\tau = \frac{U_m}{LC(s_1-s_2)} \left(\int_0^t \left(e^{s_1(t-\tau)} - e^{s_2(t-\tau)}\right) d\tau -\right.$$

$$\left. - \int_{t_1}^t \left(e^{s_1(t-\tau)} - e^{s_2(t-\tau)}\right) d\tau \right) = \frac{U_m}{LC(s_1-s_2)} \left(\left(\frac{e^{s_2(t-\tau)}}{s_2} - \frac{e^{s_1(t-\tau)}}{s_1} \right)\Bigg|_0^t - \right.$$

$$- \left(\frac{e^{s_2(t-\tau)}}{s_2} - \frac{e^{s_1(t-\tau)}}{s_1} \right)\Bigg|_{t_1}^t = \frac{U_m}{LC(s_1-s_2)} \left(\left(\frac{1}{s_2} - \frac{1}{s_1} - \frac{e^{s_2 t}}{s_2} + \frac{e^{s_1 t}}{s_1} \right) \cdot 1(t) - \right.$$

$$\left. - \left(\frac{1}{s_2} - \frac{1}{s_1} - \frac{e^{s_2(t-t_1)}}{s_2} + \frac{e^{s_1(t-t_1)}}{s_1} \right) \cdot 1(t-t_1) \right) =$$

$$= U_m \left(\frac{1(t)}{LC(s_1-s_2)} \left(\frac{e^{s_1 t} - 1}{s_1} - \frac{e^{s_2 t} - 1}{s_2} \right) - \frac{1(t-t_1)}{LC(s_1-s_2)} \left(\frac{e^{s_1(t-t_1)} - 1}{s_1} - \frac{e^{s_2(t-t_1)} - 1}{s_2} \right) \right) =$$

$$= U_m \left(h(t) - h(t-t_1) \right)$$

3) Интеграл Дюамеля

$$u_{вых}(t) = u_{вх}(0)\cdot h(t) + \int_0^t u'_{вх}(\tau)\, h(t-\tau)\, d\tau \ominus$$

$$u_{вх}(0) = U_m;\ u'_{вх} = U_m(\delta(t) - \delta(t-t_1))$$

$$\ominus U_m h(t) + U_m \int_0^t (\delta(t) - \delta(t-t_1))\, h(t-\tau)\, d\tau =$$

опр. только в t_1

$$= U_m h(t) + U_m\Big(\int_0^t \delta(t)\, \underbrace{h(t-t)}_{=0} d\tau - \int_0^t \delta(t-t_1)\, h(t-\tau)\, d\tau\Big) =$$

$$= U_m (h(t) - h(t-t_1))$$

4) На периодическую посл-ть

$$T = 2t_1 \qquad u_{вх}(t) = U_m \sum_{n=0}^{\infty} (1(t-2nt_1) - 1(\underbrace{t - t_1 - 2n t_1}_{= t - t_1(1+2n)}))$$

$$u_{вых}(t) = U_m \sum_{n=0}^{\infty} (h(t-2nt_1) - h(t - t_1(1+2n)))$$

5) На гарм-й сигнал

$$u_{вх}(t) = U_m \sin(\omega_0 t)\cdot 1(t) \risingdotseq U_m \frac{\omega_0}{S^2+\omega_0^2}$$

$$u_{вых}(S) = W(S)\cdot u_{вх}(S) = \frac{U_m \omega_0}{(S^2+\omega_0^2)(\underbrace{S^2 LC + SRC + 1}_{LC(S-S_1)(S-S_2)})} = U_{пр} + \underbrace{U_{св}}_{= Ae^{S_1 t} + Be^{S_2 t}} =$$

$$= K \sin(\omega_0 t + \varphi) + Ae^{S_1 t} + Be^{S_2 t}$$

$$K(j\omega_0) = \frac{1}{-\omega_0^2 LC + \omega_0 RC j + 1} = \frac{1}{\underset{a}{(1-\omega_0^2 LC)} + j\underset{b}{\omega_0 RC}} =$$

$$= \underbrace{\frac{1}{\sqrt{1-2\omega_0^2 LC + \omega_0^4 L^2 C^2 + \omega_0^2 R^2 C^2}}}_{K} e^{-j\,\mathrm{arctg}\frac{\omega_0 RC}{1-\omega_0^2 LC}}$$

$$\varphi = -\frac{\omega_0 RC}{1-\omega_0^2 LC}$$

$$F_1(s) = U_m \omega_0; \quad F_2(s) = (s^2+\omega_0^2)(s^2 LC + sRC + 1) =$$

$$= s^4 LC + s^3 RC + s^2(LC\omega_0^2 + 1) + s\omega_0^2 RC + \omega_0^2$$

$$F_2'(s) = 4s^3 LC + 3s^2 RC + 2s(LC\omega_0^2 + 1) + \omega_0^2 RC$$

$$A = \frac{F_1(s_1)}{F_2'(s_1)} = \frac{U_m \omega_0}{4s_1^3 LC + 3s_1^2 RC + 2s_1(LC\omega_0^2+1) + \omega_0^2 RC} \overset{1}{=}$$

$$B = \frac{F_1(s_2)}{F_2'(s_2)} = \frac{U_m \omega_0}{4s_2^3 LC + 3s_2^2 RC + 2s_2(LC\omega_0^2+1) + \omega_0^2 RC} \overset{2}{=}$$

$$\overset{1}{=} \cancel{\frac{U_m \omega_0}{4LC}} \frac{\frac{U_m \omega_0}{LC}}{(s_1^2+\omega_0^2)(s_1-s_2)} = \frac{U_m \omega_0}{LC(s_1^2+\omega_0^2)(s_1-s_2)}$$

$$\overset{2}{=} \frac{U_m \omega_0}{LC(s_2^2+\omega_0^2)(s_2-s_1)}$$

$$U_{вых}(t) = \left(1 + \omega_0^2 C(\omega_0^2 L^2 C + R^2 C - 2L)\right) \sin\left(\omega_0 t - \frac{\omega_0 RC}{1-\omega_0^2 LC}\right) + \frac{U_m \omega_0}{LC(s_1-s_2)}\left(\frac{e^{s_1 t}}{s_1^2+\omega_0^2} - \frac{e^{s_2 t}}{s_2^2+\omega_0^2}\right)$$