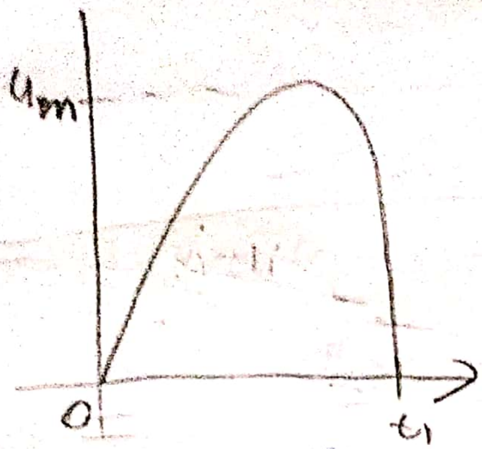
Задача 6.



Аналитическое выражение для импульса:

$$f(t)=U_m\left(\sin\omega t\cdot 1(t)-\sin\omega t\cdot 1(t-t_1)\right)$$

$$\omega=\frac{2\pi}{T},\ T=2t_1 \Rightarrow \omega=\frac{\pi}{t_1}$$

Изображение: $\left[\sin\omega t \doteqdot \frac{\omega}{s^2+\omega^2}\right]$

$$F(s)=U_m\left(\int_0^\infty e^{-st}\sin\omega t\,dt-\int_{t_1}^\infty e^{-st}\sin\omega t\,dt\right)=$$

$$=U_m\left(\int_0^\infty e^{-st}\sin\omega t\,dt-\int_0^\infty e^{-s(t+t_1)}\sin(\omega(t+t_1))\,dt\right)=$$

$$=U_m\left(\frac{\omega}{s^2+\omega^2}+e^{-st_1}\int_0^\infty e^{-st}\sin\omega t\,dt\right)=U_m\cdot\frac{\omega}{s^2+\omega^2}\left(1+e^{-st_1}\right)$$

Спектр:

$$S(j\omega)=U_m\cdot\frac{\pi t_1}{\pi^2-\omega^2t_1^2}\left(1+e^{-j\omega t_1}\right)=\frac{U_m\pi t_1}{\pi^2-\omega^2t_1^2}\left(1+\cos(\omega t_1)-j\sin(\omega t_1)\right)$$

$$a=\frac{U_m\pi t_1}{\pi^2-\omega^2t_1^2}\left(1+\cos(\omega t_1)\right),\quad b=-\frac{U_m\pi t_1\sin(\omega t_1)}{\pi^2-\omega^2t_1^2}$$

Амплитудный спектр: $A(\omega)=|S(j\omega)|=\sqrt{a^2+b^2}=$

$$=\underbrace{\frac{U_m\pi t_1}{|\pi^2-\omega^2t_1^2|}}_{A}\sqrt{1+2\cos(\omega t_1)+\cos^2(\omega t_1)+\sin^2(\omega t_1)}=A\sqrt{2(1+2\cos(\omega t_1))}=$$

$$=A\sqrt{4\cos^2\left(\frac{\omega t_1}{2}\right)}=2U_m\pi t_1\cdot\left|\frac{\cos\left(\frac{\omega t_1}{2}\right)}{\pi^2-\omega^2t_1^2}\right|$$

Фазовый спектр: $\varphi(\omega) = \operatorname{arctg}\frac{b}{a} = -\operatorname{arctg}\left(\frac{\sin(\omega t_1)}{1+\cos(\omega t_1)}\right) \ominus$

$= \left[\sin\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}},\ \cos\alpha = \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}\right] \ominus -\operatorname{arctg}\left(\operatorname{tg}\left(\frac{\omega t_1}{2}\right)\right) =$

$= -\frac{\omega t_1}{2}$

График амплитудного спектра:

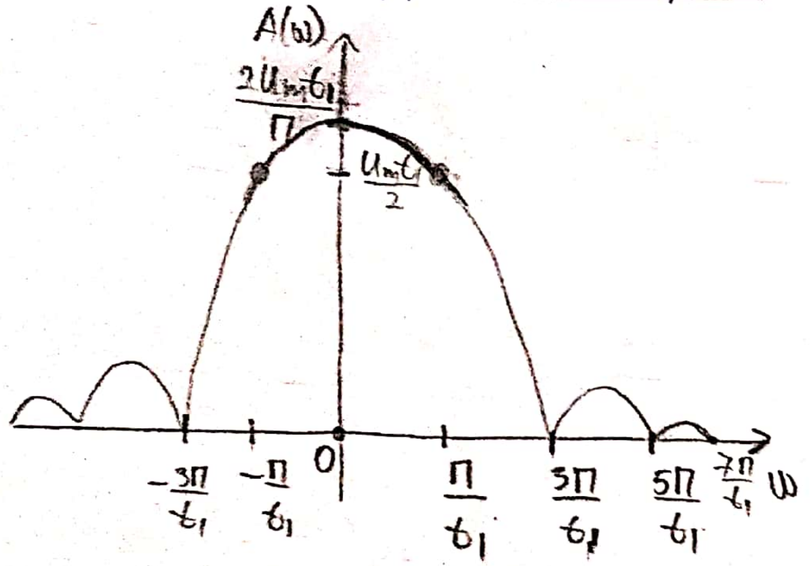
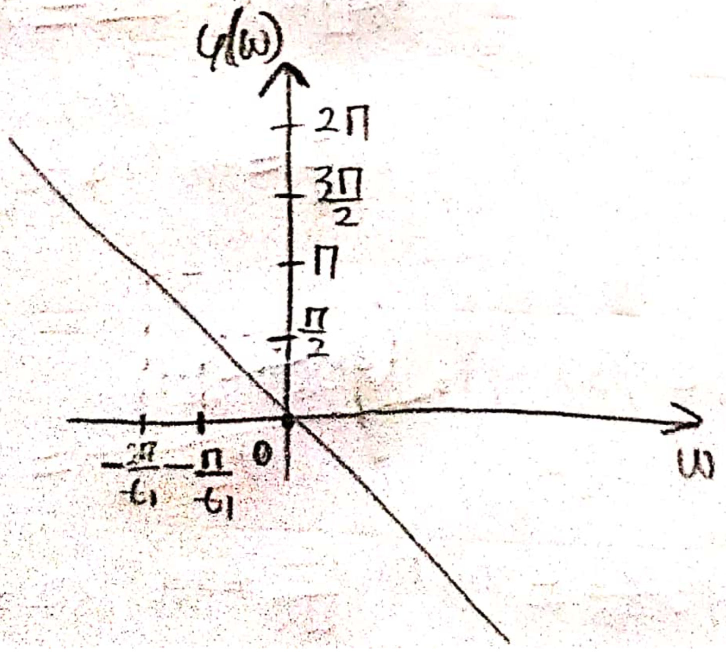


График фазового спектра:



Спектр. период-ой посл-ти:

1) $T = t_1$, $\omega_0 = \frac{2\pi}{t_1}$, $\dot{C}_k = \frac{\omega_0}{2\pi} S(k\omega_0)$

$$\dot{C}_k = \frac{U_m \pi \cancel{t_1}}{\cancel{t_1}\left(\pi^2 - \frac{4\pi^2 k^2 \cancel{t_1^2}}{\cancel{t_1^2}}\right)} \cdot \left(1 + \cos(2\pi k) - j\sin(2\pi k)\right) = \frac{2U_m}{\pi(1-4k^2)}$$

2) $T = 2t_1$, $\omega_0 = \frac{\pi}{t_1}$

$$\dot{C}_k = \frac{1}{2\cancel{t_1}} \cdot \frac{U_m \pi \cancel{t_1}}{\pi^2 - \pi^2 k^2}\left(1 + \cos(\pi k) - j\sin(\pi k)\right) = \frac{U_m}{2\pi(1-k^2)}\left(1 + (-1)^k\right)$$