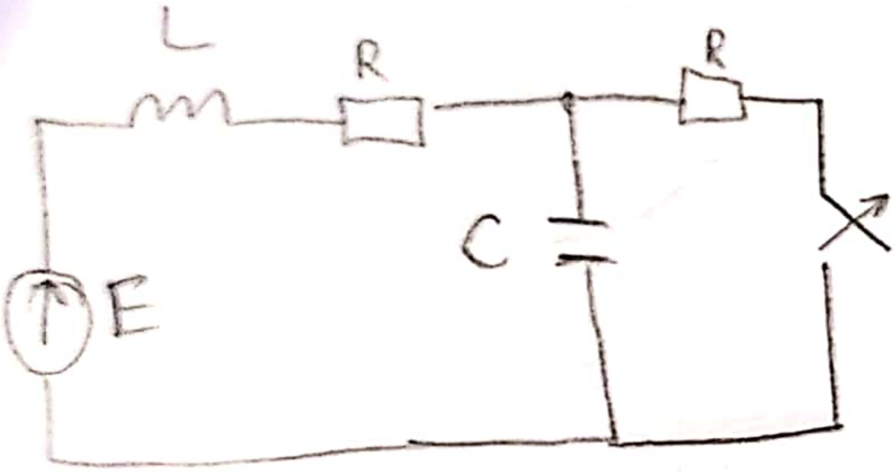
Задача 7.



$R = 6\ Ом$

$C = 20\cdot10^{-6}\,F = 2\cdot10^{-5}\,F$

$L = 100\cdot10^{-6}\,H = 10^{-4}\,H$

$E = 48\,V$

$u_C^{(t)} = u_{пр} + u_{св} = E + A_1 e^{p_1 t} + A_2 e^{p_2 t}$ $\quad i_L(0) = \frac{E}{2R}$

$u_C(0) = u_C(0-) = \frac{ER}{2R} = \frac{E}{2}$, $u_C(\infty) = E$,

хар. ур-е: $R + j\omega L + \frac{1}{j\omega C} = 0 \mid p = j\omega \Rightarrow p^2LC + pRC + 1 = 0$

$$p_{1,2} = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC} = \frac{-6\cdot2\cdot10^{-5} \pm \sqrt{36\cdot4\cdot10^{-10} - 4\cdot10^{-4}\cdot2\cdot10^{-5}}}{2\cdot10^{-4}\cdot2\cdot10^{-5}} = \frac{-12\pm8}{4\cdot10^{-4}}$$

$\underline{p_1 = -5\cdot10^4}$ $\quad \underline{p_2 = -10^4}$

$$\begin{cases} u_c(0) = E + A_1 + A_2 = \frac{E}{2} \\ u_c'(0) = \frac{I_c}{C} = \frac{E}{2RC} = p_1 A_1 + p_2 A_2 \end{cases} \Rightarrow \begin{cases} A_1 = \frac{E\left(\frac{1}{RC}+p_2\right)}{2(p_1-p_2)} \\ A_2 = -\frac{E}{2}\left(1 + \frac{\frac{1}{RC}+p_2}{p_1-p_2}\right) \end{cases}$$

$$u_c(t) = E\left(1 + \frac{1\cdot\left(\frac{1}{RC}+p_2\right)\cdot e^{p_1 t}}{2(p_1-p_2)} - \frac{1}{2}\left(1 + \frac{\frac{1}{RC}+p_2}{p_1-p_2}\right)e^{p_2 t}\right) =$$

$$\cancel{= \left[\frac{\frac{1}{RC}+p_2}{p_1-p_2} = -\frac{11}{36}\right] = E\left(1 - \frac{11}{72}e^{-5\cdot 10^4 t} - \frac{1}{2} + \frac{11}{72}e^{10^4 t}\right) =}$$

$$\cancel{= \frac{E}{2} + \frac{11 E}{72}\left(e^{10^4 t} - e^{-5\cdot 10^4 t}\right)}$$

$$= \left[\frac{\frac{1}{RC}+p_2}{p_1-p_2} = \frac{1}{24}\right] = E + \frac{E}{48}e^{-5\cdot 10^4 t} - \frac{E\cdot 25}{48}e^{-10^4 t} =$$

$$= \boxed{48 + e^{-5\cdot 10^4 t} - 25 e^{-10^4 t}}$$

$$i_L = i_{\text{пр}} + i_{\text{св}} \qquad i_{\text{пр}} = 0$$

$$i_L = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

$$\begin{cases} i_L(0) = A_1 + A_2 = \frac{E}{2R} \\ L\frac{dI_L(0)}{dt} + I(0)R + u_c(0) = E \end{cases} \Rightarrow$$

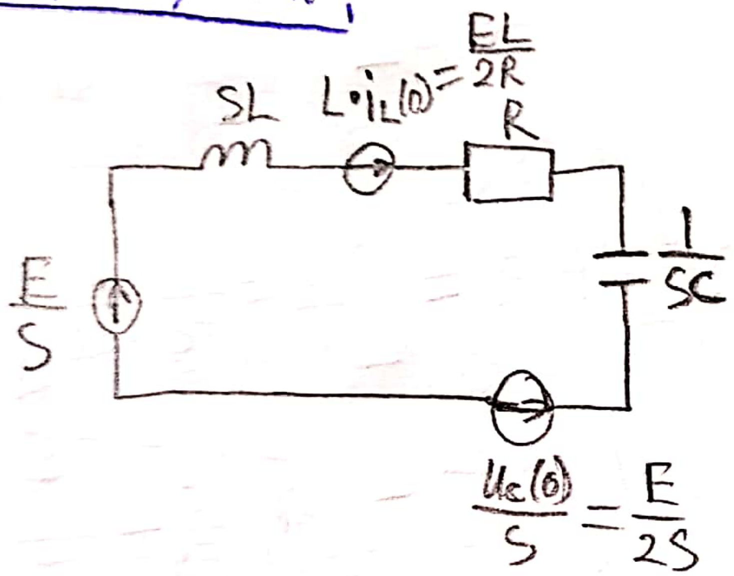
$$\Rightarrow \begin{cases} A_1 + A_2 = \frac{E}{2R} \\ L(A_1 p_1 + A_2 p_2) + R(A_1 + A_2) = \frac{E}{2} \end{cases} \Rightarrow \begin{cases} A_1 = \frac{-E p_2}{2R(p_1-p_2)} \\ A_2 = \frac{E}{2R}\left(1 + \frac{p_2}{p_1-p_2}\right) \end{cases}$$

$$A_1 = \left[\frac{p_2}{p_1-p_2} = \frac{1}{4}\right] = -\frac{48}{12 \cdot 4} = -1$$

$$A_2 = \frac{48 \cdot 5}{12 \cdot 4} = 5$$

$$i_L(t) = \boxed{-e^{-5 \cdot 10^4 t} + 5e^{-10^4 t}}$$

Операторный:



$$I(s) = \frac{\frac{E}{s} + \frac{EL}{2R} - \frac{E}{2s}}{R + sL + \frac{1}{sC}} = \frac{E\left(\frac{1}{2s} + \frac{L}{2R}\right) sC}{s^2LC + sRC + 1} =$$

$$= E\left(\frac{\frac{\cancel{C}}{2}}{L\cancel{C}(s-s_1)(s-s_2)} + \frac{\frac{sL\cancel{C}}{2R}}{(s-s_1)(s-s_2)L\cancel{C}}\right) = \frac{E}{2L}\left(\frac{1 + \frac{sL}{R}}{(s-s_1)(s-s_2)}\right) =$$

$$= \frac{E}{2L}\left(\frac{1}{(s-s_1)(s-s_2)} + \frac{L}{R} \cdot \overbrace{\frac{s}{(s-s_1)(s-s_2)}}^{K}\right) =$$

$$K = \frac{A}{s-s_1} + \frac{B}{s-s_2} \Rightarrow s = (A+B)s - (As_2 + Bs_1) \Rightarrow \begin{cases} A = -\frac{s_1}{s_2 - s_1} \\ B = \frac{s_2}{s_2 - s_1} \end{cases}$$

Користувалась

$$\ominus\ \frac{E}{2L}\left(\frac{1}{(s-s_1)(s-s_2)}+\frac{L}{R}\left(\frac{s_2}{(s_2-s_1)(s-s_2)}-\frac{s_1}{(s_2-s_1)(s-s_1)}\right)\right)\doteqdot$$

$$\doteqdot\frac{E}{2L}\left(\frac{e^{s_1t}-e^{s_2t}}{s_1-s_2}+\frac{L}{R}\left(\frac{s_2}{s_2-s_1}\cdot e^{s_2t}-\frac{s_1}{s_2-s_1}\cdot e^{s_1t}\right)\right)=$$

$$=\frac{E}{2L}\left(\frac{e^{s_1t}-e^{s_2t}}{s_1-s_2}\right)+\frac{E}{2R}\left(\frac{s_2e^{s_2t}}{s_2-s_1}-\frac{s_1e^{s_1t}}{s_2-s_1}\right)=$$

$$=\left[s_1=-5\cdot10^4,\ s_2=-10^4\right]=E\left(\left(\frac{-1\left(e^{-5\cdot10^4t}-e^{-10^4t}\right)}{2\cdot10^{-4}\cdot4\cdot10^4}\right)+\right.$$

$$\left.+\frac{1}{12}\left(\frac{-\cancel{10^4}e^{-10^4t}+5\cdot\cancel{10^4}e^{-5\cdot10^4t}}{4\cdot\cancel{10^4}}\right)\right)=\frac{E}{48}\left(5e^{-5\cdot10^4t}-e^{-10^4t}\right)-$$

$$-\frac{E}{8}\left(e^{-5\cdot10^4t}-e^{-10^4t}\right)=5e^{-5\cdot10^4t}-e^{-10^4t}-6e^{-5\cdot10^4t}+6e^{-10^4t}=$$

$$=\boxed{5e^{-10^4t}-e^{-5\cdot10^4t}}$$

$$i_L(t)=5e^{-10^4t}-e^{-5\cdot10^4t}$$

$$U_C(s) = \frac{I(s)}{sC} + \frac{U_C(0)}{s} = \frac{I(s)}{sC} + \frac{E}{2s} =$$

$$= \frac{E}{2LC}\left(\underbrace{\frac{1}{s(s-s_1)(s-s_2)}}_{\shortparallel} + \frac{L}{R}\cdot\frac{1}{(s-s_1)(s-s_2)}\right) + \frac{E}{2s} \doteq$$

к оригиналу

$$\frac{1}{(s-s_1)(3s_1^2-2(s_1^2+s_1s_2)+s_1s_2)} + \frac{1}{(s-s_2)(3s_2^2-2(s_2^2+s_1s_2)+s_1s_2)} + \frac{1}{s(s_1s_2)}$$

$$\doteq \frac{E}{2LC}\left(\frac{e^{s_1t}}{3s_1^2-2(s_1^2+s_1s_2)+s_1s_2} + \frac{e^{s_2t}}{3s_2^2-2(s_2^2+s_1s_2)+s_1s_2} + \frac{1}{s_1s_2}\right.$$

$$\left.+ \frac{L}{R(s_1-s_2)}e^{s_1t} - \frac{L}{R(s_1-s_2)}e^{s_2t}\right) + \frac{E}{2} =$$

$$= \boxed{48 + e^{-5\cdot10^4 t} - 25e^{-10^4 t}}$$

$U_C(t)$ //

~~Сопротивление~~ Цепь перейдёт в колеб. режим, если

∃ пара компл.-сопр. корней ~~функции~~ хар.ур-я, т.е. $D<0$

$$R^2C^2 - 4LC < 0 \Rightarrow R^2 < \frac{4L}{C} \Rightarrow \underbrace{-2\sqrt{\frac{L}{C}}}_{R<0 \text{ не бывает}} < R < 2\sqrt{\frac{L}{C}} = 2\sqrt{5}$$

$R \in [0; 2\sqrt{5}]\ \Omega$