CS 332 Fall 2025

Project #1

Koshi Harashima, Ben Cole

Due Date: 10/8, 2025

Problem 1.

Method

For each value $v \in \{10, \dots, 100\}$ and bid b, the winning probability is

$$P_{\text{win}}(b) = P(\text{opp} < b) + \tfrac{1}{2}P(\text{opp} = b), \qquad EU_{\text{exact}}(v, b) = (v - b)P_{\text{win}}(b).$$

In the exact calculation, the opponent bid distribution is the empirical class distribution: a value is chosen uniformly from $\{10, \dots, 100\}$ and a random classmate's bid for that value is drawn; ties are broken uniformly at random. The Monte Carlo (MC) estimate samples 20,000 such random opponent bids and averages realized utilities. The optimal bid $b^*(v)$ maximizes $(v - b)P_{\text{win}}(b)$.

Results

Regret is defined as the difference between the optimal expected utility and your own expected utility at each value.

Ben's case

value	my bid	win prob	EU exact	EU MC	opt bid	opt EU	regret
10	9	0.091	0.091	0.088	5	0.334	0.243
20	15	0.216	1.080	1.075	11	1.740	0.660
30	20	0.291	2.909	2.865	11	3.694	0.785
40	25	0.375	5.625	5.569	21	6.529	0.904
50	30	0.455	9.091	9.089	21	9.984	0.893
60	30	0.455	13.636	13.633	31	14.713	1.076
70	35	0.532	18.614	18.648	31	19.804	1.190
80	40	0.607	24.273	24.406	41	25.462	1.189
90	45	0.677	30.477	30.651	41	32.007	1.530
100	50	0.745	37.273	37.403	51	38.675	1.403

Average regret (opt EU - my EU): **0.98**.

Koshi's case

value	my bid	win prob	EU exact	EU MC	opt bid	opt EU	regret
10	9	0.091	0.091	0.088	5	0.334	0.243
20	19	0.252	0.252	0.250	11	1.740	1.487
30	29	0.418	0.418	0.417	11	3.694	3.276
40	39	0.577	0.577	0.580	21	6.529	5.952
50	49	0.716	0.716	0.718	21	9.984	9.268
60	59	0.805	0.805	0.806	31	14.713	13.908
70	69	0.855	0.855	0.856	31	19.804	18.949
80	79	0.905	0.905	0.904	41	25.462	24.557
90	89	0.950	0.950	0.951	41	32.007	31.057
100	99	0.991	0.991	0.991	51	38.675	37.685

Average regret (opt EU - my EU): **14.63**.

Takeaways

- Exact and Monte Carlo expected utilities closely align, confirming simulation accuracy.
- Optimal bids shade below value and rise smoothly with v, matching first-price auction theory.
- Ben's bids were close to optimal (avg regret was about 1 util); Koshi's near-truthful bids overpaid and reduced utility.
- A good strategy is to choose, for each value v, the b that maximizes $(v - b)\,\Pr[\text{opp} < b]$ using the empirical opponent distribution.

Problem 2.

Method

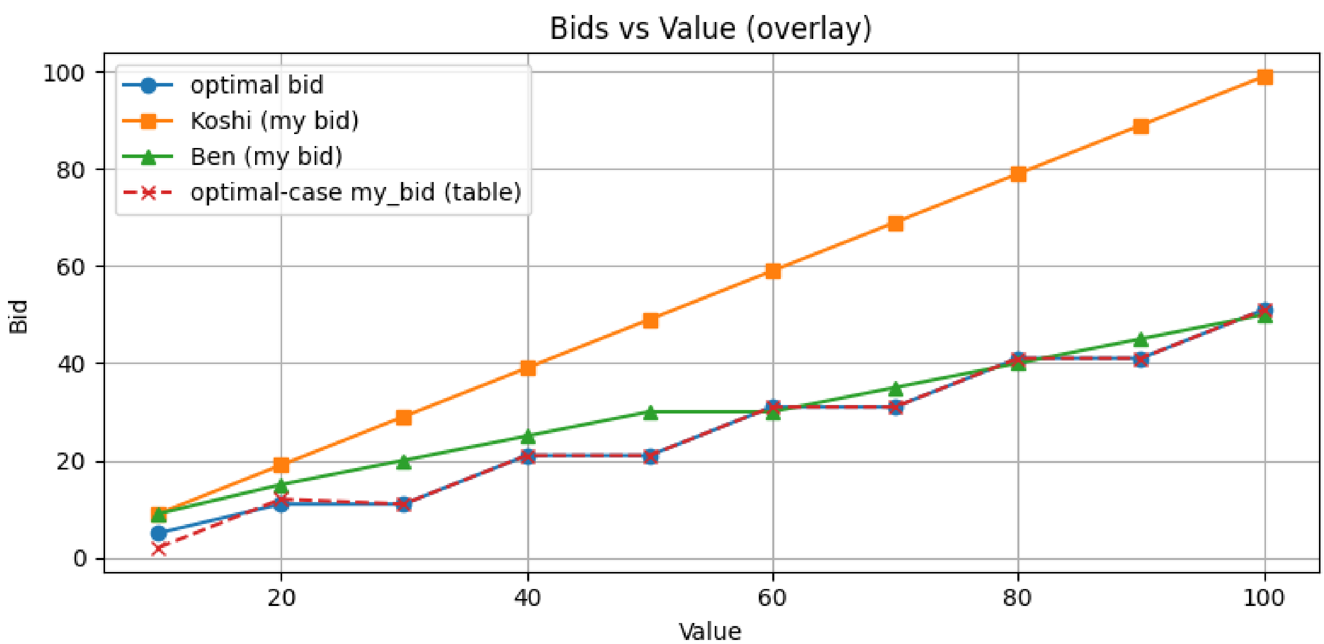
- **(1) Data-driven bid:** From `bid_data.csv`, estimate the win CDF $F_v(b) = \Pr(B_{\text{opp}} \le b \mid v)$ and choose $\hat{b}^*(v) = \arg\max_{0 \le b \le v} (v - b)\,\hat{F}_v(b)$.
- **(2) Bid in Theory:** Two-bidder first-price: assume opponent bids linearly $b_{\text{opp}} = \alpha X$ and choose b to maximize $U(b \mid v) = (v - b)\,\Pr(\text{win at } b)$

Result

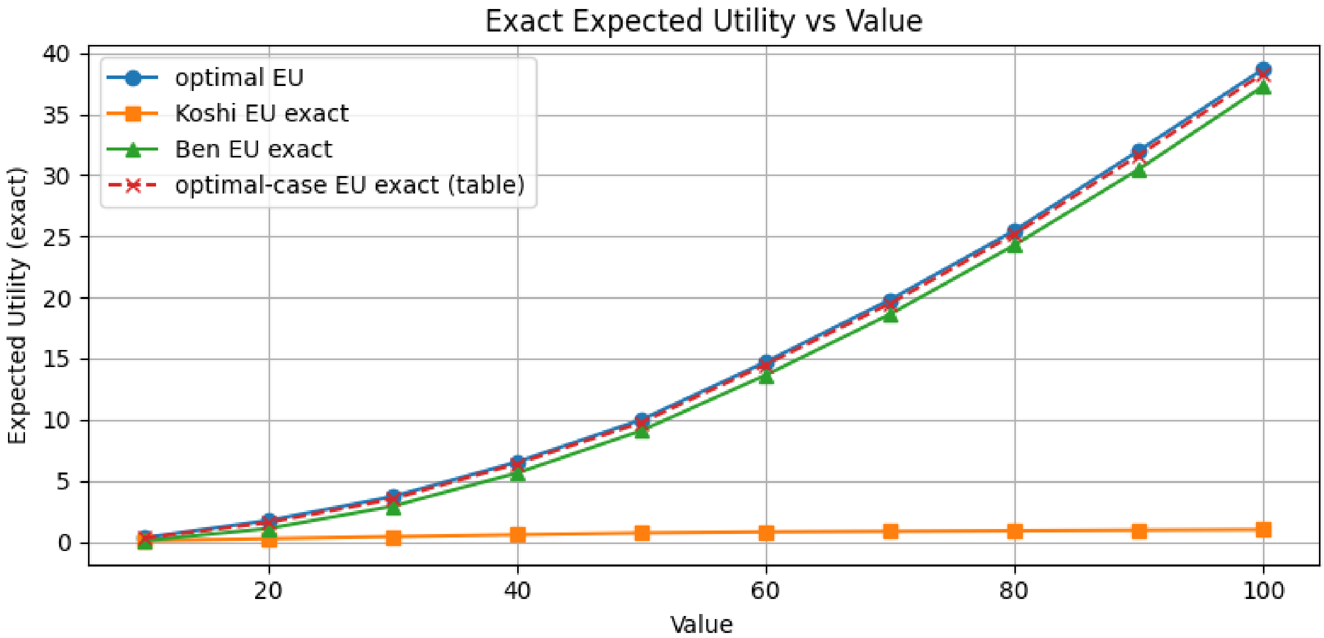
- **(1) Data-driven bid:** More samples $\Rightarrow$ more accurate $\hat{F}_v$ and higher expected profit than with few samples.
- **(2) Bid in Theory:** The optimal bid is $v/2$, consistent with the optima recovered from the sample.

Takeaways

- **More simulation means more accurate results.** (With many runs, simulated profit matches the exact calculation.)
- **Bid below your value; about half is a good rule of thumb.** (With two similar bidders, the math says bid $\approx v/2$.)



(a) Bid vs. value



(b) Expected utility vs. value