Project 1

CS 332, Fall 2025

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Outline

1. Part 1
   - 1. winning probability and expected utility with your bids
   - 2. the optimal bids
   - 3. Better bid strategy

2. Part 2
   - optimal data-driven bid strategy
   - optimal bid in theory

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Part 1

In Part 1,

- **Methods:** Exact Estimation and Monte Carlo Estimation.
- **Results:** Koshi's strategy did not work well, while Ben's strategy worked pretty well.
- **Takeaways:**: Monte Carlo approximately becomes exact as the sample size increases. The better bid strategy is to bid conservatively.

Model

- **Setting:** single-item, first-price, two bidders (you vs. one opponent).
- **Private value:** $v \in \{10, 20, \ldots, 100\}$. You bid $b \in [0, v]$.
- **Payoff:** if win, $u = v - b$; else $u = 0$.
- **Winning probability:**

$$P_{\text{win}}(b) = P(\text{opp} < b) + \tfrac{1}{2} P(\text{opp} = b).$$

- **Objective:**

$$EU(v, b) = (v - b)\, P_{\text{win}}(b), \qquad b^*(v) = \arg\max_{0 \le b \le v} EU(v, b).$$

Calculation methods: Exact vs Monte Carlo

- **Empirical analysis:**
  pick $V \sim \mathrm{Unif}\{10, \ldots, 100\}$; sample opponent bid set $\{b_i\}_{i=1}^{n}$ for that V.
- **Exact estimate:**

$$\widehat{P}_{\mathrm{win}}(b) \;=\; \frac{\#\{b_i < b\} \;+\; \frac{1}{2}\#\{b_i = b\}}{n}$$

$$\mathrm{EU}_{\mathrm{exact}}(v, b) \;=\; (v - b)\,\widehat{P}_{\mathrm{win}}(b)$$

- **Monte Carlo estimator:**
  draw $B_{\mathrm{opp}}^{(t)}$ from empirical model for $t = 1, \ldots, T$,

$$\widehat{EU}_{\mathrm{MC}}(v, b) = \frac{1}{T}\sum_{t}\left(1\{b > B_{\mathrm{opp}}^{(t)}\} + \tfrac{1}{2}1\{b = B_{\mathrm{opp}}^{(t)}\}\right)(v - b).$$

  - use $T = 20{,}000$ for stable MC estimates.

Koshi's Case

1.Calculate your winning probability and expected utility with your bids submitted in Ex 1.2 for each of your values.

value	my bid	win prob	EU exact	EU MC	opt bid	opt EU	regret
10	9	0.091	0.091	0.088	5	0.334	0.243
20	19	0.252	0.252	0.250	11	1.740	1.487
30	29	0.418	0.418	0.417	11	3.694	3.276
40	39	0.577	0.577	0.580	21	6.529	5.952
50	49	0.716	0.716	0.718	21	9.984	9.268
60	59	0.805	0.805	0.806	31	14.713	13.908
70	69	0.855	0.855	0.856	31	19.804	18.949
80	79	0.905	0.905	0.904	41	25.462	24.557
90	89	0.950	0.950	0.951	41	32.007	31.057
100	99	0.991	0.991	0.991	51	38.675	37.685

I took a strategy in which I always bid $b = v - 1$ (one unit below valuation).
Average Regret :

Ben's Case

2. Calculate your winning probability and expected utility with your bids submitted in Ex 1.2 for each of your values.

value	my bid	win prob	EU exact	EU MC	opt bid	opt EU	regret
10	9	0.091	0.091	0.088	5	0.334	0.243
20	15	0.216	1.080	1.075	11	1.740	0.660
30	20	0.291	2.909	2.865	11	3.694	0.785
40	25	0.375	5.625	5.569	21	6.529	0.904
50	30	0.455	9.091	9.089	21	9.984	0.893
60	30	0.455	13.636	13.633	31	14.713	1.076
70	35	0.532	18.614	18.648	31	19.804	1.190
80	40	0.607	24.273	24.406	41	25.462	1.189
90	45	0.677	30.477	30.651	41	32.007	1.530
100	50	0.745	37.273	37.403	51	38.675	1.403

Average Regret :

Optimal-Bids

- Calculate the optimal bids values.

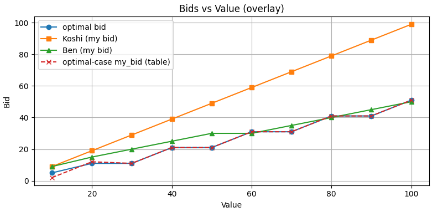
$$b^*(v) = \arg\max_{0 \le b \le v} (v - b)\, F(b)$$

- the answer is

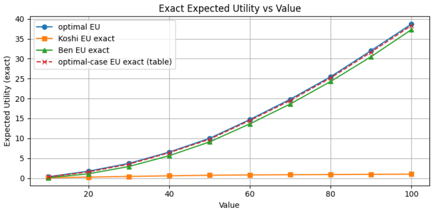
value	b_opt_exact	util_opt_exact	b_opt_mc	util_opt_mc
10	5.1	0.3341	1.6	0.3507
20	11.1	1.7395	11.1	1.8067
30	11.1	3.6941	11.1	3.8367
40	21.1	6.5291	21.6	6.6608
50	21.1	9.9836	21.6	10.2808
60	31.1	14.7127	31.3	14.9455
70	31.1	19.8036	31.3	20.1530
80	41.1	25.4618	41.0	25.8814
90	41.1	32.0073	41.0	32.5176
100	51.1	38.6755	41.0	39.1539

About Good Strategy

- Compare the utility you obtained to the optimal utility you could have obtained. Can you conclude anything about a good strategy in this auction?
  - from Koshi's case: we observe that it is not recommended to bid too close to your valuation.
  - From Ben's case: he bids conservatively, yet his bids are close to the optimal bids and show very good expected-utility performance.



(a) Bid vs. value



(b) Expected utility vs. value

Appendix: Regret

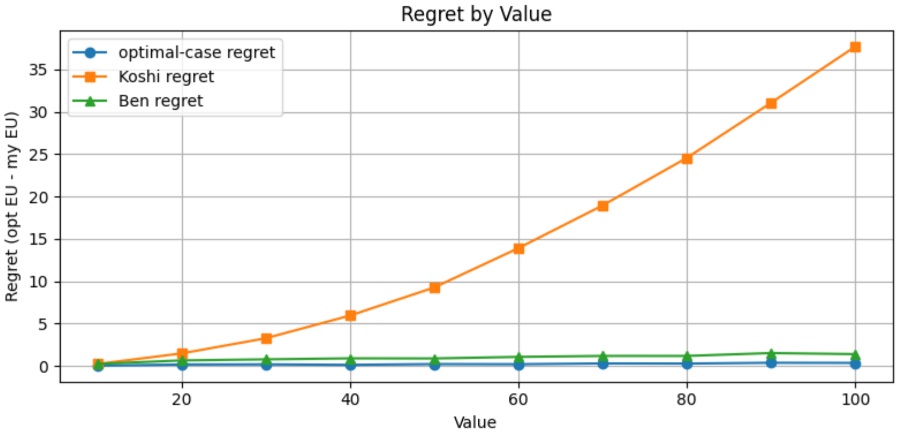


Figure: Regret

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Part 2

In Part 2, we consider two things;

- optimal data-driven bid strategy,
- the optimal bid in auction with two people in theory.

Goal & Method: Pick $b(v)$ from data

Goal. Choose a bid function $b(v)$ to maximize expected profit:

$$b^*(v) = \arg\max_{0 \le b \le v} (v - b)\,\Pr(B_{\text{opp}} \le b \mid v).$$

What we estimate. The win-probability CDF at value v:

$$F_v(b) = \Pr(B_{\text{opp}} \le b \mid v).$$

How we estimate (from `bid_data.csv`). Empirical CDF (no smoothing):

$$\hat{F}_v(b) = \frac{1}{n_v} \sum_{i=1}^{n_v} 1\{b_i \le b\}.$$

Plug-in rule (ties count as $1/2$).

$$\hat{b}^*(v) = \arg\max_{0 \le b \le v} (v - b)\,\hat{F}_v(b).$$

Benchmark (2 bidders, $U[0,1]$): $b(v) = \frac{1}{2}v$.

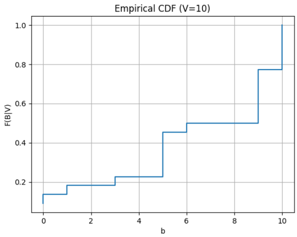

Figure: V=10

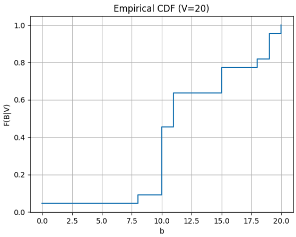

Figure: V=20

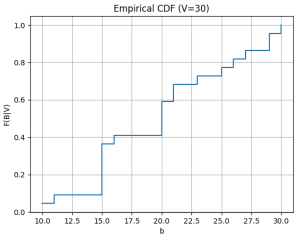

Figure: V=30

Appendix: Distribution

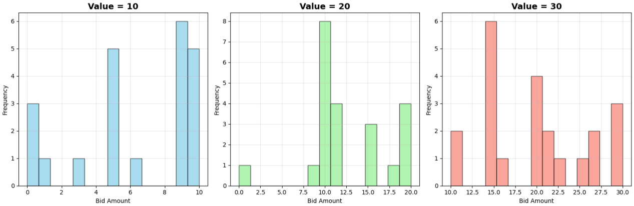

Figure: Discrete Distribution

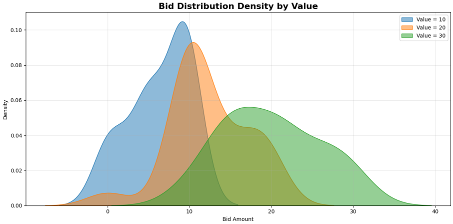

Figure: Continuous Distribution

Conclusion

Message. As n increases, the empirical win CDF $\hat{F}_v$ approaches the true F_v, so the plug-in bid converges to the optimum:

$$\hat{b}^*(v) = \arg \max_{0 \le b \le v} (v - b)\, \hat{F}_v(b) \;\longrightarrow\; b^*(v).$$

Estimator (simple, no smoothing).

$$\hat{F}_v(b) = \frac{1}{n_v} \sum_{i=1}^{n_v} 1\{b_i \le b\}.$$

Evidence. Regret decreases with n; Monte Carlo estimates converge to the exact value as the number of draws T grows.

Observation

- **Second-price:** Truthful bidding (bid = value) is a dominant strategy (referred in class).
- **First-price:** : It seems like the optimal bid is linear in value, but its coefficient is half not 1.
- Optimal-bid in First-price auction
  Optimal bid problem is formulated by following formula;

$$b^* = \arg\max_b (v - b) \Pr(\text{win at } b).$$

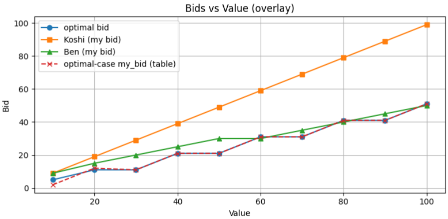

Figure: bid strategy

Bid Strategy in First-Price Auction

Setup: Two bidders, values i.i.d. $U[0,1]$ (ties have prob. 0).
considering Best respond to a opponent whose bid strategy is $b(x) = \alpha x$ for simplicity.

$$\Pr(\text{win at bid } b) = \Pr(\alpha X < b) = \tfrac{b}{\alpha}$$

so,

$$b^* = \arg\max_b (v - b)\tfrac{b}{\alpha}.$$

FOC in b: $\partial U/\partial b = \frac{1}{\alpha}(v - 2b) = 0 \;\Rightarrow\; b^*(v) = \frac{1}{2}v$.
By symmetry $b(v) = \alpha v = b^*(v)$, so $\alpha = \frac{1}{2}$.

Usage of AI