Problem Set 1

Note: We will discuss the first problem in the problem-solving session. However, you still need to write your own solution to every problem.

Problem 1: BNE in an all-pay auction ..... (10 points)
Consider an all-pay single-item auction. The bidders can bid any real number and the bidder with the highest bid wins and everyone pays her bid.

Suppose there are two bidders with values distributed independently and uniformly over $[0, 2]$. The bidders know each other's value distributions, but not the others' realized values.

(a) (6 points) Verify the strategy $\boldsymbol{\sigma} = (\sigma, \sigma)$ where $\sigma : [0, 2] \to \mathbb{R}$ is a mapping from value to bid and $\sigma(v) = \frac{v^2}{4}$ is a BNE. Explain your answer.

(b) (4 points) What are the expected revenue and expected social welfare under the BNE in Part (a)? Explain your answer.

Problem 2: BNE in a first-price auction ..... (10 points)
Consider a first-price single-item auction. The bidders can bid any real number and the bidder with the highest bid wins and the winner pays her bid.

Suppose there are n bidders with values distributed independently and uniformly over $[0, 1]$. The bidders know each other's value distributions, but not the others' realized values.

(a) (6 points) Verify the strategy $\boldsymbol{\sigma} = (\sigma, \ldots, \sigma)$ where $\sigma : [0, 1] \to \mathbb{R}$ is a mapping from value to bid and $\sigma(v) = \frac{n-1}{n} v$ is a BNE. Explain your answer.

(b) (4 points) What are the expected revenue and expected social welfare under the BNE in Part (a)? Explain your answer.