Junio 2017

R \ H	[0,1]	(1,2]	(2,4]	Mas de 4 Hm.
[0,2]	0	0	2	2
(2,6]	0	2	2	0
(6,10]	2	2	0	0
(10,∞)	3	1	0	0

a) Ajustar recta $R=a+bH$ de mín. cuadrados y estimar bondad del ajuste

b) Ajustar: $R=\frac{c}{d+H^2}$ y analizar la bondad del ajuste.

c) Media y varianza para $H>2$

a) $R=a+bH \rightarrow \begin{pmatrix} N & \Sigma H_i \\ \Sigma H_i & \Sigma H_i^2 \end{pmatrix}\begin{pmatrix} b \\ a \end{pmatrix}=\begin{pmatrix} \Sigma R_i \\ \Sigma R_i H_i \end{pmatrix}$ $N=16$

$\Sigma H_i = 100$

$\Sigma H_i^2 = 900$

$\Sigma R_i = 32$

$\Sigma R_i H_i = 120$

$\begin{pmatrix} 16 & 100 \\ 100 & 900 \end{pmatrix}\begin{pmatrix} b \\ a \end{pmatrix}=\begin{pmatrix} 32 \\ 120 \end{pmatrix}$ $b=-2,318$ $a=10,887$

$\boxed{R=10,887-2,3188H}$

$r=\frac{Cov(x,y)}{\sigma_x\sigma_y}=\frac{\mu_{11}-\bar{x}\bar{y}}{\sigma_x\sigma_y}$ $\left\{ \begin{array}{l} \mu_{11}=\frac{\Sigma R_i H_i}{N}=\frac{120}{16}= \\ \bar{x}=6,25 \ ; \sigma_x=4,1457 \\ \bar{y}=2 \ ; \sigma_y=1,4684 \end{array} \right.$ $=\frac{7,5-6,25\cdot 2}{4,1457\cdot 1,4684}\Rightarrow \boxed{r=-0,8213}$

b)

$$R = \frac{c}{d+H^2} \Rightarrow \frac{1}{R} = \frac{d+H^2}{c} \Rightarrow \frac{1}{R} = \frac{d}{c} + \frac{H^2}{c} \quad \begin{cases} \hat{y} = \frac{1}{R} \\ \hat{a} = \frac{d}{c} \\ \hat{b} = \frac{1}{c} \\ \hat{x} = H^2 \end{cases}$$

$$\hat{y} = \hat{a} + \hat{b}\hat{x}$$

$\hat{y}$ \ $\hat{x}$	$0{,}5^2$	$1{,}5^2$	3^2	5^2
1	0	0	2	2
1/4	0	2	2	0
1/8	2	2	0	0
1/12	3	1	0	0

$$\begin{pmatrix} N & \sum \hat{x}_i \\ \sum \hat{x}_i & \sum \hat{x}_i^2 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} \sum \hat{y}_i \\ \sum \hat{x}\hat{y} \end{pmatrix}$$

$N = 16$

$\sum \hat{x}_i = 98{,}5$ $\qquad \sum \hat{y}_i = 5{,}8333$

$\sum \hat{x}_i^2 = 1599{,}625$ $\qquad \sum \hat{x}\hat{y} = 74{,}5$

$$\begin{pmatrix} 16 & 98{,}5 \\ 98{,}5 & 1599{,}625 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} 5{,}8333 \\ 74{,}5 \end{pmatrix} \quad \hat{a} = 0{,}1254$$

$$\hat{b} = 0{,}0385 \Rightarrow \hat{b} = \frac{1}{c} \Rightarrow \hat{c} = \frac{1}{\hat{b}} = \frac{1}{0{,}0385} = 25{,}974$$

$$d = \hat{a} \cdot c = 0{,}1254 \cdot 25{,}974 = 3{,}2577$$

$$\boxed{R = \frac{25{,}974}{3{,}2577 + H^2}}$$

R^2 Porque no es lineal

$$R^2 = 1 - \frac{MSE}{\sigma_y^2 = \sigma_R^2} = 1 - \frac{\sum e_i^2 / N}{\sigma_R^2} = 1 - \frac{\frac{\sum (y_i - y^*)^2}{N}}{\sigma_R^2}$$

R \ H	0,5	1,5	3	5
1	0	0	-1,1189	0,0808
4	0	-0,7159	1,881	0
8	0,5951	3,284	0	0
12	4,5951	7,284	0	0

$\to 1 - \frac{25{,}974}{3{,}2577 + 5^2}$

0	0	-2,2378	0,1616
0	-1,4318	3,762	0
1,1902	6,568	0	0
13,7871	7,284	0	0

$\sum e_i^2 =$ [redacted]

(2)

$$F(x) = P(Z \le x) = \begin{cases} 0 & x < 1 \\ K(x-1)^2 & 1 \le x \le 12 \\ 1 & x > 12 \end{cases}$$

a) Cuartil 1 (Q_1) y media

b) Prob. de que tarde menos de 10 años, habiendo durado más de 5 años

a) Hallar K $\Rightarrow F(12) = 1 \Rightarrow K((12-1))^2 = 1 \Rightarrow K = \frac{1}{121}$

$F(Q_1) = 0,25 \Rightarrow \frac{1}{121}(Q_1 - 1)^2 = 0,25 \Rightarrow Q_1^2 - 1 = 30,25 \Rightarrow Q_1^2 = 29,25 \Rightarrow$

$\Rightarrow Q_1 = \sqrt{29,25} \Rightarrow \underline{Q_1 = 5,4083}$

$f(t) = F'(t)$

$\frac{1}{121}(x-1)^2 = \frac{2}{121}(x-1)$

$$f(x) = \begin{cases} \frac{2}{121}(x-1) & 1 \le x \le 12 \\ 0 & \text{en otro caso} \end{cases}$$

$$E(x) = \int_1^{12} x \cdot \frac{2}{121}(x-1)\,dx = \frac{2}{121}\int_1^{12} x^2 - x\,dx = \frac{2}{121}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^{12} = \frac{2}{121}\left(\left(\frac{12^3}{3} - \frac{12^2}{2}\right) - \left(\frac{1^3}{3} - \frac{1^2}{2}\right)\right) =$$

$$\cancel{= \frac{2}{121}\left(-288 + \frac{1}{6}\right) = \frac{2}{121}\cdot\left(\frac{-1727}{6}\right) =}$$

$$= \frac{25}{3} = \boxed{8,3333}$$

b) $P(Z < 10 / Z > 5) = \frac{P(5 < Z < 10)}{P(Z > 5)} = \frac{F(10) - F(5)}{1 - F(5)} = \frac{\frac{1}{121}(10-1)^2 - \frac{1}{121}(5-1)^2}{1 - \left(\frac{1}{121}(5-1)^2\right)} =$

$$= \frac{13}{21} = \boxed{0,619}$$

(3)

500 productos

$X \sim P(\lambda)$

Algún Defectuoso $\rightarrow P(Z \geq 1) = 0{,}613259$

a) Media y varianza de Z

b) Prob. de encontrar más de 1 elemento defectuoso

c) Si compramos 50 de 500. Prob de encontrar + de 5 conteniendo más de un element defectuoso.

a) Si $P(Z \geq 1) = 0{,}613259$

$\downarrow$

$P(Z = 0) = 1 - 0{,}613259 = 0{,}386741$

$X \sim P(\lambda) \rightarrow P(0) = e^{-\lambda} \cdot \frac{\lambda^0}{0!} = 0{,}386741 \Rightarrow e^{-\lambda} = 0{,}386741 \Rightarrow$

$\Rightarrow \ln(e^{-\lambda}) = \ln(0{,}386741) \Rightarrow -\lambda = -0{,}95 \rightarrow \underline{\lambda = 0{,}95}$

Media
Varianza

Distrib. Poisson, media = varianza

b) $\underset{\overset{\downarrow}{P(>1)}}{P(Z > 1)} = 1 - \underset{P(0)}{P(Z=0)} - \underset{P(1)}{P(Z=1)} = 1 - e^{-0{,}95} \cdot \frac{0{,}95^0}{0!} - e^{-0{,}95} \cdot \frac{0{,}95}{1!} = \boxed{0{,}24585}$

c) $n = 50$
$p = 0{,}24585$

$X \sim B(np, \sqrt{npq}) \rightarrow$ grande

$X \sim B(12.2925, 3.04471)$

$\underset{\overset{\downarrow}{P(Z > 5+0{,}5)}}{P(Z > 5)} = P\left(\frac{Z-\mu}{\sigma} > \frac{5+0{,}5 - 12{,}2925}{3.04471}\right) = P(Z' > -2{,}23) =$

$= 1 - P(Z' > 2{,}23) = 1 - 0{,}0129 = \boxed{0{,}9871}$

$\downarrow$

Buscar
en la izquierda (negativ)

(4)

A: 10 elementos, $\mu = 0,5g$, $s^2 = 0,01g$. por cada 100g.

B: 16 elementos, $\mu = 0,45g$, $s^2 = 0,0064g$. " " "

Q: μ no debe pasar 0,38gr. por cada 100gr.

$\alpha = 0,1$

a) Contrastar contenido medio en A es mayor de 0,38

b) " " " " " " " que en B

a) Contraste de hipotesis para la media

$H_0: \mu_A \leq \mu_0$

$H_a: \mu_A > \mu_0$ $\quad \mu_0 = 0,38$

· Estadístico: $\frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{\frac{\alpha}{2}, n-1}$

Varianza desc.

Pequeñas

· Valor experimental:

$$\frac{0,5 - 0,38}{\frac{\sqrt{0,01}}{\sqrt{10}}} = 3,7947$$

- R.R.

$t_{\frac{0,1}{2}, n-1} = t_{0,05, 9} = 1,833$

· Conclusión:

$3,7947 \in RR \Rightarrow$ Rechazamos H_0, A es mayor de 0,38

b) Contraste hipotesis de igualdad de medias, varianzas?, muestras pequeñas

$H_0: \mu_A \leq \mu_B$

$H_a: \mu_A > \mu_B$

· Estadístico: $\frac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} > t_{\alpha, n_1 + n_2 - 2}$

Contraste previo:

OJO! Poner ∉ en las tablas

$H_0: \sigma_A^2 = \sigma_B^2$

$H_a: \sigma_A^2 \neq \sigma_B^2$

· $\frac{S_1^2}{S_2^2} = \frac{0,01}{0,0064} = 1,5625$

· Valor experimental: $S_p = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{9 \cdot 0,01 + 15 \cdot 0,0064}{24} = 0,088$

$$\frac{0,5 - 0,45}{0,088 \sqrt{\frac{1}{10} + \frac{1}{16}}} = 1,4089$$

· RR: $t_{0,1, 24} = 1,318$

· Conclusión:

$1,4089 \in RR \Rightarrow$ A es mayor que B

· RR

$F_{1 - \frac{0,1}{2}, 9, 15} = F_{0,95, 9, 15} = \frac{1}{F_{0,05, 15, 9}} = \frac{1}{3,006} = 0,3327$

$F_{\frac{0,1}{2}, 9, 15} = F_{0,05, 9, 15} = 2,588$

· Conclus.

$1,5625 \notin RR \Rightarrow$ varianzas iguales

Scanned by TapScanner