

MATH 390.4 Lecture 2

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Note: Half the class was spent on Lab work e.g. RStudio overview which is on class github.

1 Features

$$Y = t(Z1, Z2, Z3)$$

Y was the phenomenon of pay back mortgage $Y \in \{1, 0\}$

$y \in Y = \{0, 1\}$

t is the function, Z' are the true causal drivers

Z1: has the money (1) $Z1 \in \{1, 0\}$

$Z2 : \text{unforeseen emergency}(1) Z2 \in \{1, 0\}$

$Z3 : \text{criminal intent}(1) Z3 \in \{1, 0\}$

$$t = Z1(1 - Z2)(1 - Z3)$$

Next best thing: Find features that approximates the “information” in $Z1, Z2, Z3$.

Here are three that are directly related

X1: Salary at the time of application

$$X1 \in \mathbb{R}^+$$

X2: Payment history (Binary)

$$X2 \in R_+$$

X3: Criminal History/ do they have a record? (Binary) $X3 \in 0, 1$

X_j 's are called the following: features, characteristics, attributes, variables, independent variables, regressor, co-variance.

Let $P :=$ the number of features

Let $X_i := [X_{i1}, X_{i2}, \dots, X_{ip}]$ element of X input space e.g. R^P

X_i is called the i th: observation, setting, record, input, unit, subject

Can I measure $X3$ better?

$X3 \in \text{none, infraction, misdemeanor, felony}$

This is a categorical or factor variable with $L = 4$ levels Mathematical models require numerical values

Two options: Note: this is a factor variable with a monotonic order. Code this variable in $X3 = 0, 1, 2, 3$, but this has a downside, which is the coding is arbitrary e.g why not... $X = 0, 1, 5, 100$

Create multiple features: $X3a \in 0, 1$ is infraction? $X3b \in 0, 1$ is misdemeanor? $X3c \in 0, 1$ is felony? None is not required since it is captured by $X3a = X3b = X3c = 0$ $L - 1$ binary variable

Consider X_j element of Red, Green, Blue (un-ordered factor) $= i$ This must use option two

$$Y = t(Z1, Z2, Z3) = f(X1.....Xp)$$

BUT

$$Y = t(Z1, Z2, Z3) = f(X1.....Xp) + \delta$$

$\delta := t - f$ is the error due to ignorance $f :=$ is the "best" possible way of combining $X1, \dots, Xp$ to minimize δ .

How to get f ? Is there an analytical solution? No The approach we use is "learning from data". We use data to get an estimate of an f . This procedure is also called "supervised learning"

There are 3 ingredients: //

1. Training Data (Data) :=

$$D = \langle x1, y1 \rangle, \langle x2, y2 \rangle \dots \langle xn, yn \rangle$$

This is n *(n is sample size) historical examples (subjects with response) i.e it happened already // $x1$: Bob's features // $y1$: 1 (he paid it back) // $x2$: Bills features // $y2$: 0 (He did not pay back his loan) //

Fig 2:

Standard
Notation:

$$X = \begin{bmatrix} \leftarrow \vec{x}_1 \rightarrow \\ \leftarrow \vec{x}_2 \rightarrow \\ \vdots \\ \leftarrow \vec{x}_n \rightarrow \end{bmatrix}, \quad n \times p \text{ matrix}$$

$$\vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

col. vector length n

2. H = a set of candidate function for f , recall that $f: \mathbb{R}^P \rightarrow \mathbb{R}$ y e.g. $f: \mathbb{R}^P \rightarrow \mathbb{R}$
- R You need to simplify the set of all possible functions
3. A is an algorithm: s.t. $g = A(D, H)$, a way to select a model g which is element of H using D .