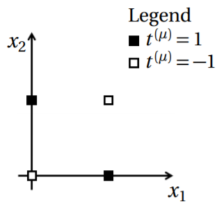
and

$$
\begin{aligned}
H' = & -\sum_{ij} w_{ij} h_i v'_j = H + 2\sum_{ij} w_{ij} h_i \delta_{j,m1} v_j + 2\sum_{ij} w_{ij} h_i \delta_{j,m2} v_j + \ldots && (11)\\
= & H + 2\sum_{i} w_{i,m1} h_i v_{m1} + 2\sum_{i} w_{i,m2} h_i v_{m2} + \ldots && (12)\\
= & H + 2 b^{(\nu)}_{m1} v_{m1} + 2 b^{(\nu)}_{m2} v_{m2} + \ldots && (13)\\
= & H - 2 b^{(\nu)}_{m1} \mathrm{sgn}\big[b^{(\nu)}_{m1}\big] - 2 b^{(\nu)}_{m2} \mathrm{sgn}\big[b^{(\nu)}_{m1}\big] - \ldots && (14)
\end{aligned}
$$

Chapter 5

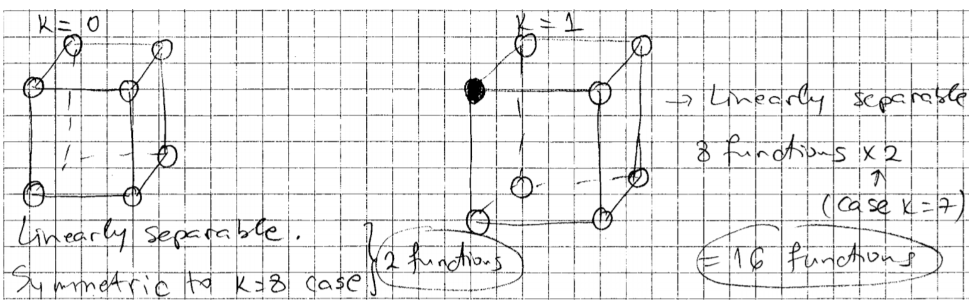
5.2 Boolean functions

(a)



In order for the problem to be solved by a simple perceptron with two input terminals and one output unit (no hidden layer), it must be linearly separable, i.e. there must exist a plane such that all inputs mapping to the target output +1 have to be on one side of the plane and all inputs mapping to the target output −1 have to be on the other side of the plane. By looking at the figure above, this is impossible for the Boolean XOR problem.

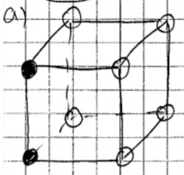
(b) 3D Boolean functions. Consider a function that has k input patterns that map to the target output +1 and $8-k$ input patterns that map to −1.



(2)

K=2 (total $\binom{8}{2} = \frac{8 \cdot 7}{2} = 28$)

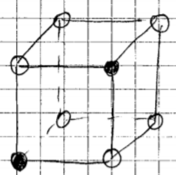
a)



Linearly separable

12 cases $\times$ 2 = 24 functions
(K=6)

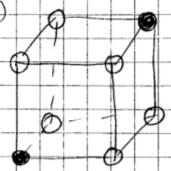
b)



Not lin. sep.

12 cases for K=2
(12 cases for K=6)

c)



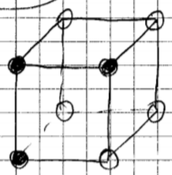
Not lin. separable

4 cases (for K=2)
4 cases (for K=6)

// Check : for K=2, there are 28 functions in total, 12 LS + 16 NLS //

K=3 total : $\binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$

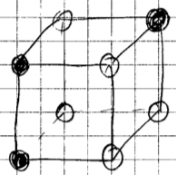
a)



Linearly separable

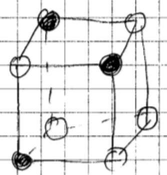
$\binom{4}{3} \cdot 6$ cases $= \frac{4 \cdot 3 \cdot 2}{3 \cdot 2} \cdot 6 = 24$ for K=3
+24 for K=5 } = 48

b)



NLS : $2 \cdot 12 = 24$
(and 24 for K=5)

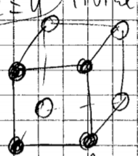
c)



NLS : $2 \cdot 4 = 8$
(and 8 for K=5)

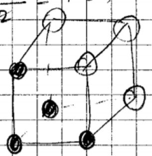
K=4 (Total $\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2} = 70$)

a)



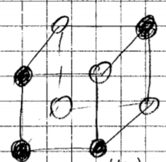
LS : 6 functions

b)



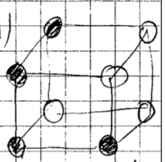
LS : 8 functions

c)



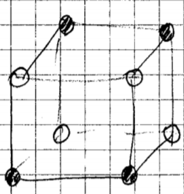
NLS : $6 \binom{4}{3} = 6 \cdot 4 = 24$

d)

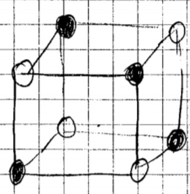


NLS : 24

e)



NLS : 4



NLS : 4

Total :

LS = 6 + 8 = 14
NLS = 24 + 24 + 8 = 56 } = 70