Dynamical Systems TIF155/FIM770
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Problem set 1

1.2 Subcritical pitchfork

a)

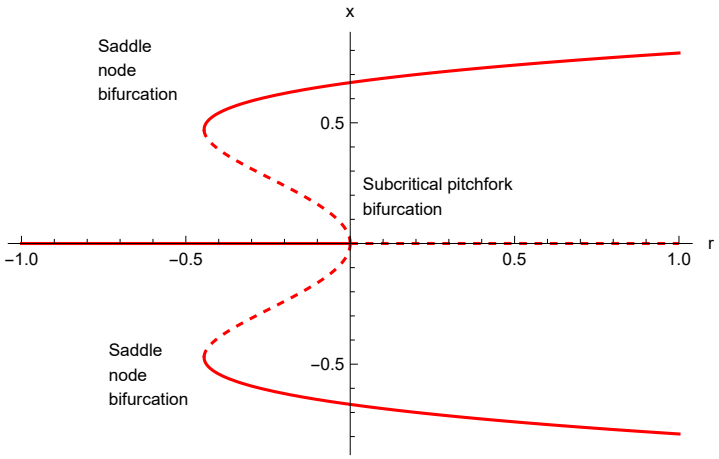
```
In[•]:= xDot[x_, r_] := r x + 4 x^3 - 9 x^5

In[•]:= sol = Solve[xDot[x, r] == 0, x];
        X = x /. sol;
        roots[r_] = X
```

$$\text{Out[•]}= \left\{0,\ -\frac{1}{3}\sqrt{2-\sqrt{4+9\,r}},\ \frac{1}{3}\sqrt{2-\sqrt{4+9\,r}},\ -\frac{1}{3}\sqrt{2+\sqrt{4+9\,r}},\ \frac{1}{3}\sqrt{2+\sqrt{4+9\,r}}\right\}$$

```
In[•]:= plot1 = Plot[{roots[r][[2]], roots[r][[3]], roots[r][[4]], roots[r][[5]]},
           {r, -1, 1}, PlotStyle → {{Red, Dashed}, {Red, Dashed}, {Red}, {Red}}];
        plot2 = Plot[roots[r][[1]], {r, -1, 0}, PlotStyle → {Red}];
        plot3 = Plot[roots[r][[1]], {r, 0, 1}, PlotStyle → {Red, Dashed}];

        plot = {plot1, plot2, plot3};
        Show[plot, AxesLabel → {"r", "x"}]
```



b)

In[]:=
```
sols = r /. Solve[xDot[x, r] == 0, r];
rF =.;
rF[x_] = sols
```

Out[]= $\left\{-4\,x^2+9\,x^4\right\}$

In[]:=
```
sol2 = Solve[D[rF[x], x] == 0]
```

Out[]= $\left\{\{x\to 0\},\left\{x\to -\frac{\sqrt{2}}{3}\right\},\left\{x\to \frac{\sqrt{2}}{3}\right\}\right\}$

In[]:=
```
rF[x /. sol2[[3]]]
```

Out[]= $\left\{-\frac{4}{9}\right\}$