Dynamical Systems TIF155/FIM770

Konstantinos Zakkas

Problem set 2

2.2 Damped pendulum

e)

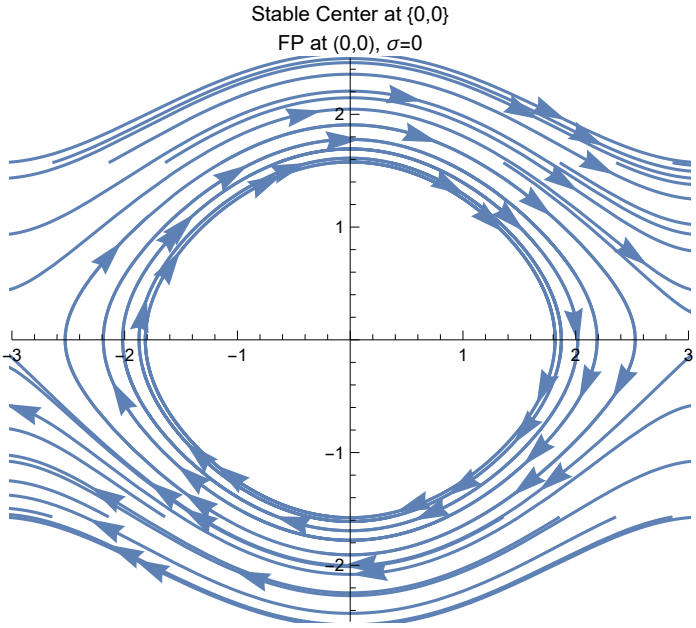
```
In[•]:= xDot[x_, y_, σ_] := y
        yDot[x_, y_, σ_] := -Sin[x] - σ y

In[•]:= Solve[{xDot[x, y, σ] == 0, yDot[x, y, σ] == 0}]

Out[•]= {{y → 0, x → 2 π C[1] if C[1] ∈ ℤ}, {y → 0, x → π + 2 π C[1] if C[1] ∈ ℤ}}
```

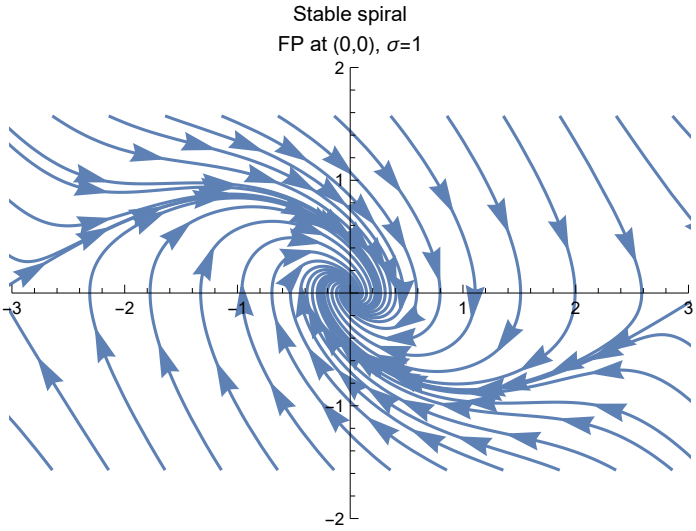
In[ ]:=
```
minx = -π;
miny = -π / 2;
maxx = π;
maxy = π / 2;
sol[x0_, y0_] :=
  Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t], x[0] == x0,
     y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {0}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.5}],
   Table[{maxx, y}, {y, miny, maxy, 0.5}], Table[{x, miny}, {x, minx, maxx, 0.5}],
   Table[{x, maxy}, {x, minx, maxx, 0.5}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions[[i, 1]],
       initialConditions[[i, 2]]]], {t, 0, 10}, PlotRange → {{-3, 3}, {-2.5, 2.5}},
    PlotLabel → "Stable Center at {0,0}\nFP at (0,0), σ=0"] /. Line[x_] :→
    {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



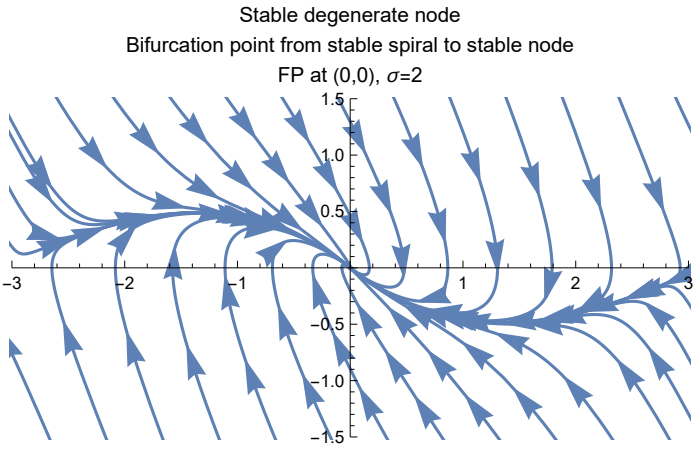
In[ ]:=
```
sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {1}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.5}],
   Table[{maxx, y}, {y, miny, maxy, 0.5}], Table[{x, miny}, {x, minx, maxx, 0.5}],
   Table[{x, maxy}, {x, minx, maxx, 0.5}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
        initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{-3, 3}, {-2, 2}},
      PlotLabel → "Stable spiral\nFP at (0,0), σ=1"] /. Line[x_] :↦
      {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



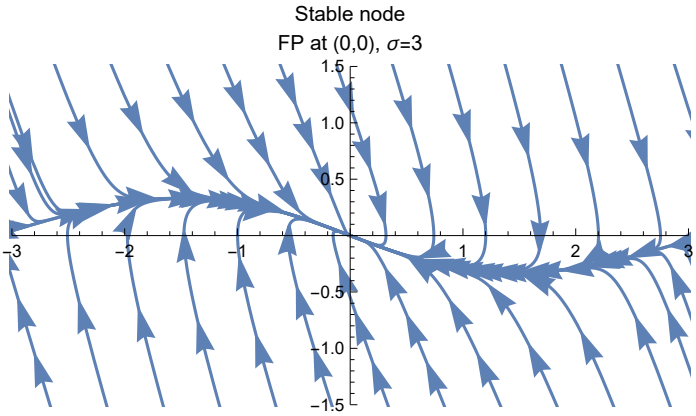
In[ ]:=
```
sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {2}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.5}],
   Table[{maxx, y}, {y, miny, maxy, 0.5}], Table[{x, miny}, {x, minx, maxx, 0.5}],
   Table[{x, maxy}, {x, minx, maxx, 0.5}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
       initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{-3, 3}, {-1.5, 1.5}},
    PlotLabel → "Stable degenerate node\nBifurcation point from
       stable spiral to stable node\nFP at (0,0), σ=2"] /.
   Line[x_] :↦ {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i,
   Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



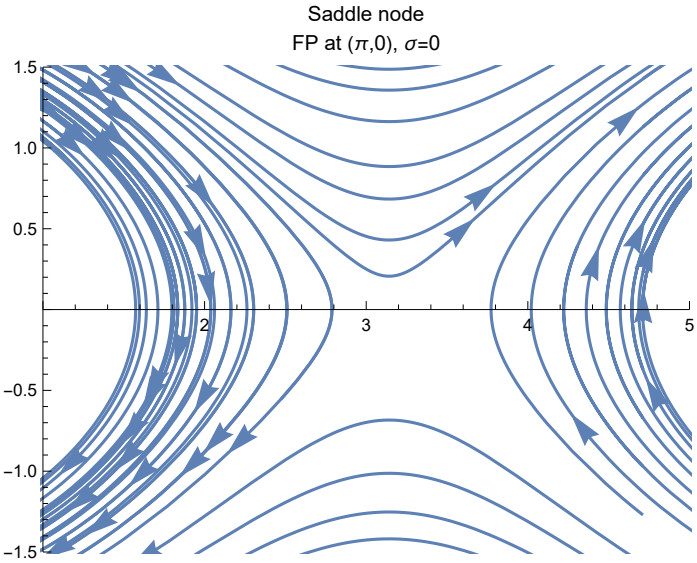
In[ ]:=
```
sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {3}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.5}],
   Table[{maxx, y}, {y, miny, maxy, 0.5}], Table[{x, miny}, {x, minx, maxx, 0.5}],
   Table[{x, maxy}, {x, minx, maxx, 0.5}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions[[i, 1]],
        initialConditions[[i, 2]]]], {t, 0, 10}, PlotRange → {{-3, 3}, {-1.5, 1.5}},
      PlotLabel → "Stable node\nFP at (0,0), σ=3"] /. Line[x_] :→
      {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



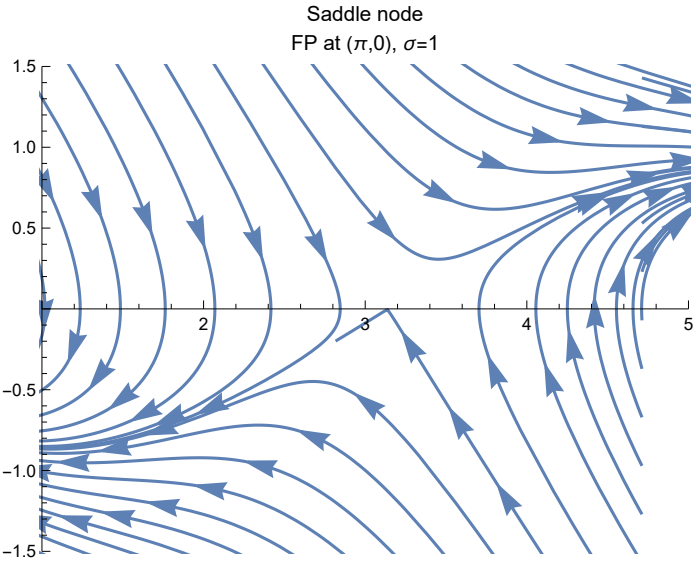
In[ ]:=
```
minx = -π / 2;
miny = -π / 2;
maxx = 3 π / 2;
maxy = π / 2;
sol[x0_, y0_] :=
  Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t], x[0] == x0,
     y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {0}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.3}],
   Table[{maxx, y}, {y, miny, maxy, 0.3}], Table[{x, miny}, {x, minx, maxx, 0.3}],
   Table[{x, maxy}, {x, minx, maxx, 0.3}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
       initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{1, 5}, {-1.5, 1.5}},
    PlotLabel → "Saddle node\nFP at (π,0), σ=0"] /. Line[x_] :↦
    {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



In[ ]:=
```
sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {1}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.3}],
   Table[{maxx, y}, {y, miny, maxy, 0.3}], Table[{x, miny}, {x, minx, maxx, 0.3}],
   Table[{x, maxy}, {x, minx, maxx, 0.3}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
        initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{1, 5}, {-1.5, 1.5}},
      PlotLabel → "Saddle node\nFP at (π,0), σ=1"] /. Line[x_] :↦
      {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



In[ ]:=
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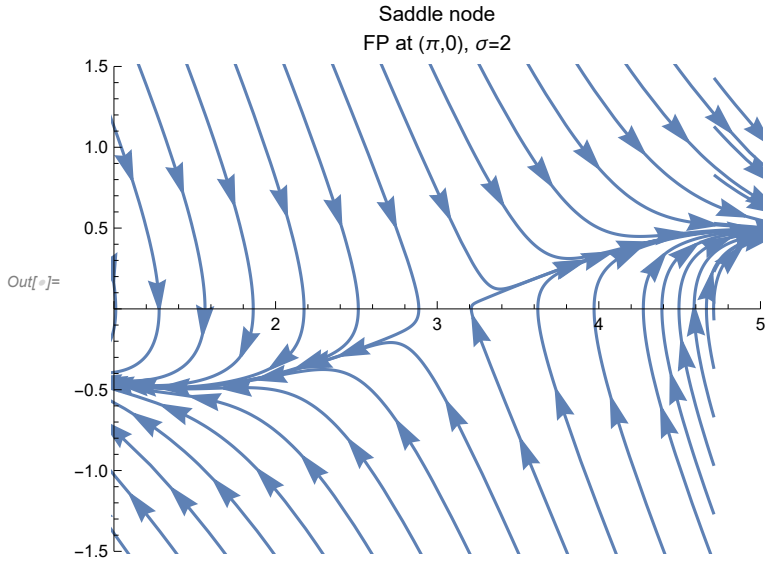
sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {2}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.3}],
   Table[{maxx, y}, {y, miny, maxy, 0.3}], Table[{x, miny}, {x, minx, maxx, 0.3}],
   Table[{x, maxy}, {x, minx, maxx, 0.3}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
       initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{1, 5}, {-1.5, 1.5}},
     PlotLabel → "Saddle node\nFP at (π,0), σ=2"] /. Line[x_] :↦
     {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=



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sol[x0_, y0_] := Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
      x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, 0, 10}], {σ, {3}}];
initialConditions = Join[Table[{minx, y}, {y, miny, maxy, 0.3}],
   Table[{maxx, y}, {y, miny, maxy, 0.3}], Table[{x, miny}, {x, minx, maxx, 0.3}],
   Table[{x, maxy}, {x, minx, maxx, 0.3}]];
p = Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialConditions⟦i, 1⟧],
        initialConditions⟦i, 2⟧]], {t, 0, 10}, PlotRange → {{1, 5}, {-1.5, 1.5}},
      PlotLabel → "Saddle node\nFP at (π,0), σ=3"] /. Line[x_] :↦
      {Arrowheads[{0, 0.04, 0.04, 0.04, 0}], Arrow[x]}, {i, Length[initialConditions]}];
Show[
 {p}]
```

Out[ ]=