Dynamical Systems TIF155/FIM770
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Problem Set 2

2.2 Damped pendulum

$$\ddot{\theta} = -\frac{g}{l}\sin(\theta) - \frac{\gamma}{m}\dot{\theta}$$
$$\omega = \dot{\theta}$$

Now the second order ODE will be written as two first order ODEs

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\frac{g}{l}\sin(\theta) - \frac{\gamma}{m}\omega$$

Next step is to de-dimensionalise the parameters

$$\theta = \theta_0 x$$
$$\omega = \omega_0 y$$
$$t = t_0 t'$$

where x, y, t' are the new dimensionless parameters. So if we substitute them we have

$$\frac{\theta_0}{t_0}\frac{dx}{dt} = \omega_0 y$$
$$\frac{\omega_0}{t_0}\frac{dy}{dt} = -\frac{g}{l}\sin(\theta_0 x) - \frac{\gamma}{m}\omega_0 y$$

thus if we choose $t_0 = \sqrt{(\frac{l}{g})}$ in order for t_0 to have s as its unit, since the unit for g is $\frac{m}{s^2}$ and for l is m we get

$$\frac{dx}{dt} = \sqrt{(\frac{l}{g})}\frac{\omega_0}{\theta_0} y$$
$$\frac{dy}{dt} = -\frac{g}{l}\sqrt{(\frac{l}{g})}\frac{\sin(\theta_0 x)}{\omega_0} - \frac{\gamma}{m}\sqrt{(\frac{l}{g})}\, y$$

and it was given that

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -\sin(x) - \sigma y$$

so comparing the our system with the one we want to have we get

$$\sqrt{(\frac{l}{g})}\frac{\omega_0}{\theta_0}=1$$
$$\sigma=\frac{\gamma}{m}\sqrt{(\frac{l}{g})}$$
$$\theta_0=1$$

and for $\theta_0=1$ we have

$$\omega_0=\sqrt{(\frac{g}{l})}$$