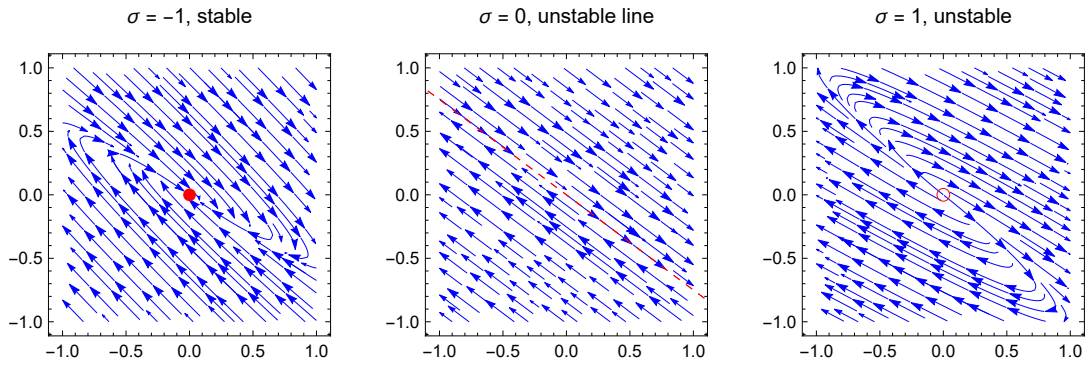
1.3 Degenerate linear system

a)

In[ ]:=
```
xDot[x_, y_, σ_] := (σ + 3) x + 4 y
yDot[x_, y_, σ_] := - (9 / 4) x + (σ - 3) y;
plot1 = Show[StreamPlot[{xDot[x, y, -1], yDot[x, y, -1]}, {x, -1, 1}, {y, -1, 1},
    StreamScale → Large, StreamStyle → Blue, StreamColorFunction → None,
    PlotLabel → "σ = -1, stable"], Graphics[{Red, Disk[{0, 0}, 0.05]}]];
plot2 = Show[StreamPlot[{xDot[x, y, 0], yDot[x, y, 0]}, {x, -1, 1},
    {y, -1, 1}, StreamScale → Large, StreamStyle → Blue,
    StreamColorFunction → None, PlotLabel → "σ = 0, unstable line"],
   Graphics[{Red, Dashed, Line[{{-4, 3}, {4, -3}}]}]];
plot3 = Show[StreamPlot[{xDot[x, y, 1], yDot[x, y, 1]}, {x, -1, 1}, {y, -1, 1},
    StreamScale → Large, StreamStyle → Blue, StreamColorFunction → None,
    PlotLabel → "σ = 1, unstable"], Graphics[{Red, Circle[{0, 0}, 0.05]}]];
GraphicsRow[{plot1, plot2, plot3}]
```

Out[ ]=



b)

```
In[•]:= M = {{(σ + 3), 4}, {- (9 / 4), σ - 3}}
```

$$\text{Out[•]= } \left\{\{3+\sigma,\ 4\},\ \left\{-\frac{9}{4},\ -3+\sigma\right\}\right\}$$

```
In[•]:= M // MatrixForm
```

$$\begin{pmatrix} 3+\sigma & 4 \\ -\frac{9}{4} & -3+\sigma \end{pmatrix}$$

```
In[•]:= Eigenvalues[M]
```

$$\text{Out[•]= } \{\sigma,\ \sigma\}$$

c)

```
In[•]:= Eigenvectors[M]
```

$$\text{Out[•]= } \left\{\left\{-\frac{4}{3},\ 1\right\},\ \{0,\ 0\}\right\}$$

```
In[•]:= Normalize[{-4/3, 1}]
```

$$\text{Out[•]= } \left\{-\frac{4}{5},\ \frac{3}{5}\right\}$$

d)

```
Inverse[M]
```

$$\left\{\left\{\frac{-3+\sigma}{\sigma^2},\ -\frac{4}{\sigma^2}\right\},\ \left\{\frac{9}{4\,\sigma^2},\ \frac{3+\sigma}{\sigma^2}\right\}\right\}$$

e)

```
detM = Det[M]
```

$$\text{Out[•]= } \sigma^2$$

```
Solve[detM == 0, σ]
```

$$\text{Out[•]= } \{\{\sigma \to 0\},\ \{\sigma \to 0\}\}$$

f)

```
Mg = {{σ - c d, d^2}, {-c^2, σ + c d}}
```

$$\text{Out[•]= } \left\{\left\{-c\,d+\sigma,\ d^2\right\},\ \left\{-c^2,\ c\,d+\sigma\right\}\right\}$$

```
Solve[Mg == M && c > 0, {c, d}]
```

Out[•]= $\left\{\left\{c \to \frac{3}{2}, d \to -2\right\}\right\}$

g)

```
Eigenvalues[Mg]
```

Out[•]= $\{\sigma, \sigma\}$

h)

```
eigenVec = Eigenvectors[Mg]
```

Out[•]= $\left\{\left\{\frac{d}{c}, 1\right\}, \{0, 0\}\right\}$

```
Normalize[eigenVec[[1]]]
```

Out[•]= $\left\{\frac{d}{c\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}, \frac{1}{\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}\right\}$