Dynamical Systems TIF155/FIM770
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Problem Set 2

2.3 Hopf bifurcation

a) For μ = 0 the flows (1) and (2) become

$$\dot{x} = -3y - 1x^3$$
$$\dot{y} = 3x + 2y^3$$

$$\dot{x} = y - x^2$$
$$\dot{y} = -x + 2x^2$$

and we are given that the system at the bifurcation is of the form

$$\dot{x} = -\omega y + f(x, y)$$
$$\dot{y} = \omega x + g(x, y)$$

So comparing the coefficients for the first system we get

$\omega = 3$

and fot the second

$\omega = -1$

b) In the same way we determine the f and g as

$$f_{(1)} = -x^3$$
$$g_{(1)} = 2y^3$$

and for the second system

$$f_{(2)} = -x^2$$
$$g_{(2)} = 2x^2$$