Dynamical Systems TIF155/FIM770

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Problem set 2

2.4 Homoclinic bifurcation

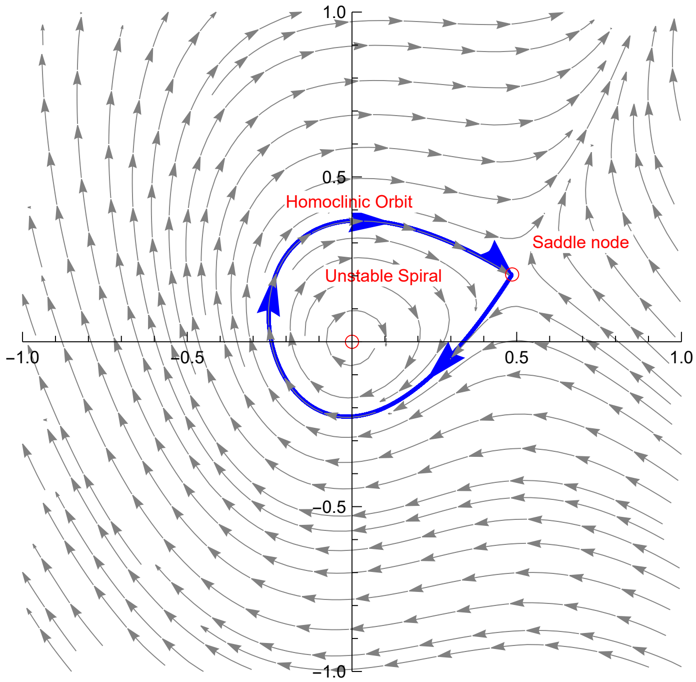
In[∘]:=
```
xDot[x_, y_, μ_] := μ x + y - x^2;
yDot[x_, y_, μ_] := -x + μ y + 2 x^2;
FPs = Solve[{xDot[x, y, μ] == 0, yDot[x, y, μ] == 0}, {x, y}]
```

Out[∘]= $\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{1+\mu^2}{2+\mu}, y \to \frac{1-2\mu+\mu^2-2\mu^3}{(2+\mu)^2}\right\}\right\}$

```
(*Using different values for mu we check the stability and
  the value of mu which gives a homoclinic bifurcation is μ=0.066*)
μ = 0.066;
t0 = 0;
tmax = 300;
x0 = (x /. FPs[[2]]) - 0.005;
y0 = (y /. FPs[[2]]);
sol = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
    y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, t0, tmax}];

p0 = ParametricPlot[{x[t], y[t]} /. sol, {t, t0, tmax}, PlotRange → {{-1, 1}, {-1, 1}},
    PlotLabel → StringForm["μ=``", μ] , PlotStyle → {Thick, Blue}] /.
   Line[x_] :→ {Arrowheads[{0.05, {0.05, 0.4}, {0.05, 0.2}, {0.05, 0.1}}], Arrow[x]};
p1 = StreamPlot[{xDot[x, y, μ], yDot[x, y, μ]}, {x, -1, 1}, {y, -1, 1},
   StreamStyle → Gray, StreamColorFunction → None];
p3 = Graphics[{Red, Circle[Evaluate[{x, y}] /. FPs[[1]], 0.02]}];
p4 = Graphics[{Red, Circle[Evaluate[{x, y}] /. FPs[[2]], 0.02]}];
Show[p0, p1, p3, p4]
```

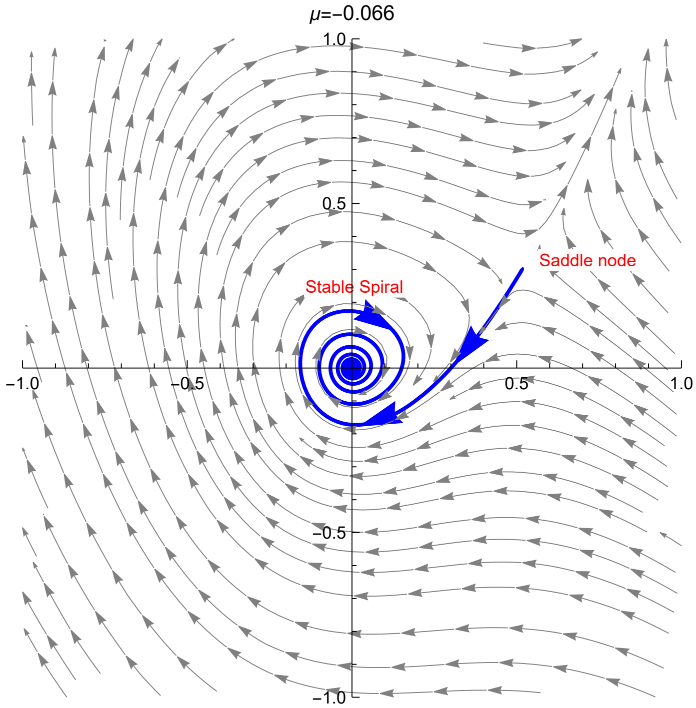
μ = 0.066



b)

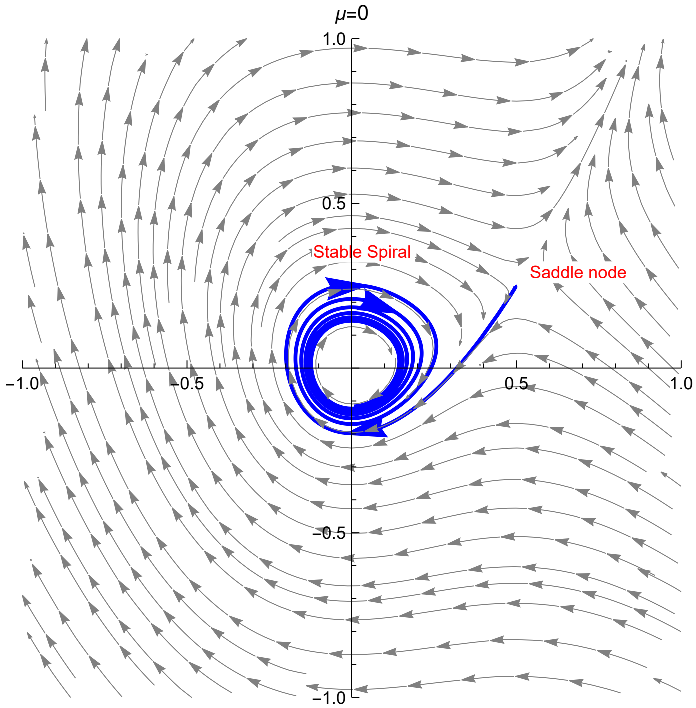
```
In[•]:= μ = -0.066;
t0 = 0;
tmax = 300;
x0 = (x /. FPs[[2]]) - 0.005;
y0 = (y /. FPs[[2]]);
sol = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
    y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, t0, tmax}];

p0 = ParametricPlot[{x[t], y[t]} /. sol, {t, t0, tmax}, PlotRange → {{-1, 1}, {-1, 1}},
    PlotLabel → StringForm["μ=``", μ], PlotStyle → {Thick, Blue}] /.
   Line[x_] :→ {Arrowheads[{{0.05, 0.4}, {0.05, 0.2}, {0.05, 0.1}}], Arrow[x]};
p1 = StreamPlot[{xDot[x, y, μ], yDot[x, y, μ]}, {x, -1, 1}, {y, -1, 1},
  StreamStyle → Gray, StreamColorFunction → None];
Show[p0, p1]
```



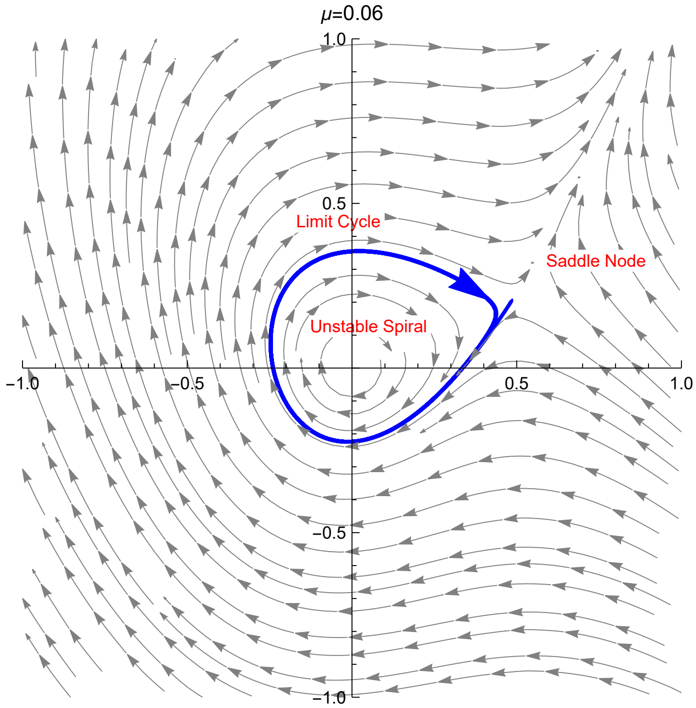
In[ ]:=
```
μ = 0;
t0 = 0;
tmax = 50;
x0 = (x /. FPs[[2]]) - 0.005;
y0 = (y /. FPs[[2]]);
sol = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
    y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, t0, tmax}];

p0 = ParametricPlot[{x[t], y[t]} /. sol, {t, t0, tmax}, PlotRange → {{-1, 1}, {-1, 1}},
    PlotLabel → StringForm["μ=``", μ], PlotStyle → {Thick, Blue}] /.
   Line[x_] :→ {Arrowheads[{{0.05, 0.4}, {0.05, 0.2}, {0.05, 0.1}}], Arrow[x]};
p1 = StreamPlot[{xDot[x, y, μ], yDot[x, y, μ]}, {x, -1, 1}, {y, -1, 1},
  StreamStyle → Gray, StreamColorFunction → None];
Show[p0, p1]
```



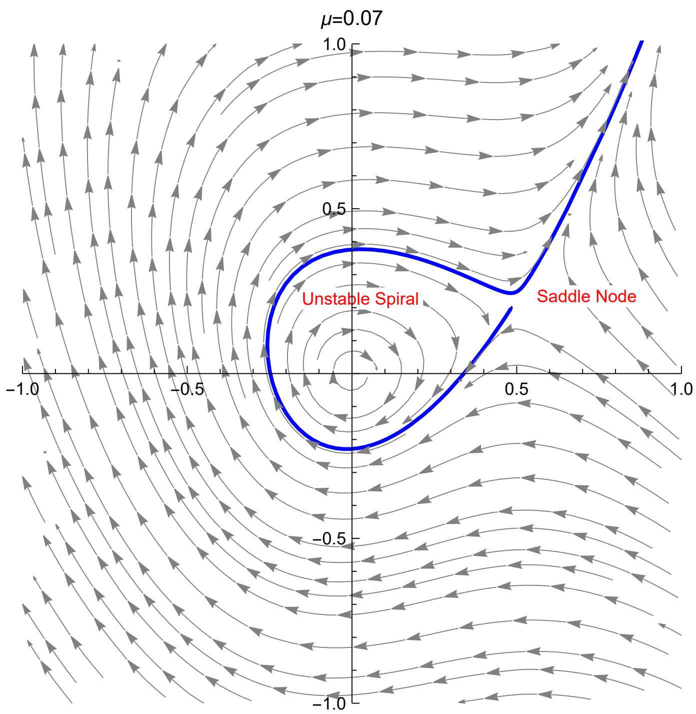
In[ ]:=
```
μ = 0.06;
t0 = 0;
tmax = 300;
x0 = (x /. FPs[[2]]) - 0.005;
y0 = (y /. FPs[[2]]);
sol = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
    y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, t0, tmax}];

p0 = ParametricPlot[{x[t], y[t]} /. sol, {t, t0, tmax}, PlotRange → {{-1, 1}, {-1, 1}},
    PlotLabel → StringForm["μ=``", μ], PlotStyle → {Thick, Blue}] /.
   Line[x_] :→ {Arrowheads[{{0.05, 0.3}, {0.05, 0.2}, {0.05, 0.1}}], Arrow[x]};
p1 = StreamPlot[{xDot[x, y, μ], yDot[x, y, μ]}, {x, -1, 1}, {y, -1, 1},
  StreamStyle → Gray, StreamColorFunction → None];
Show[p0, p1]
```



In[ ]:=
```
μ = 0.07;
t0 = 0;
tmax = 300;
x0 = (x /. FPs[[2]]) - 0.005;
y0 = (y /. FPs[[2]]);
sol = NDSolve[{x'[t] == xDot[x[t], y[t], μ],
    y'[t] == yDot[x[t], y[t], μ], x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, t0, tmax}];

p0 = ParametricPlot[{x[t], y[t]} /. sol, {t, t0, tmax}, PlotRange → {{-1, 1}, {-1, 1}},
    PlotLabel → StringForm["μ=``", μ], PlotStyle → {Thick, Blue}] /.
   Line[x_] :→ {Arrowheads[{{0.05}}], Arrow[x]};
p1 = StreamPlot[{xDot[x, y, μ], yDot[x, y, μ]}, {x, -1, 1}, {y, -1, 1},
  StreamStyle → Gray, StreamColorFunction → None];
Show[p0, p1]
```



c)

In[∘]:=
```
sols = DSolve[{x'[t] == u x[t], y'[t] == s y[t], x[0] == γ, y[0] == 1}, {x[t], y[t]}, t]
Solve[(x[t] /. Part[sols, 1, 1]) == 1, t]
```

Out[∘]= $\{\{x[t] \to e^{t u} \gamma, y[t] \to e^{s t}\}\}$

Out[∘]= $\left\{\left\{t \to \boxed{\frac{2 i \pi c_1 + \mathrm{Log}\left[\frac{1}{\gamma}\right]}{u} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$

d)

In[∘]:=
```
μ =.
saddlex = x /. Part[FPs, 2, 1]
```

Out[∘]= $\frac{1 + \mu^2}{2 + \mu}$

In[∘]:=
```
jacobianMatrix = {{μ - 2 saddlex, 1}, {-1 + 4 saddlex, μ}};
eigenValues = Eigenvalues[jacobianMatrix]
```

Out[∘]= $\left\{\frac{-1 + 2\mu - \sqrt{5 + 9\mu^2 + 4\mu^3 + \mu^4}}{2 + \mu}, \frac{-1 + 2\mu + \sqrt{5 + 9\mu^2 + 4\mu^3 + \mu^4}}{2 + \mu}\right\}$

In[∘]:=
```
Part[eigenValues, 2] // Simplify
```

Out[∘]= $\frac{-1 + 2\mu + \sqrt{5 + 9\mu^2 + 4\mu^3 + \mu^4}}{2 + \mu}$