

Dynamical Systems TIF155/FIM770

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Problem Set 3

3.1 Introduction to the Lorenz model

c)

```
In[ ]:= xDot[σ_, x_, y_, z_] = σ * (y - x);
yDot[r_, x_, y_, z_] = r * x - y - x * z;
zDot[b_, x_, y_, z_] = x * y - b * z;

jacobian[x_, y_, z_, σ_, b_, r_] =
  {{D[xDot[σ, x, y, z], x], D[xDot[σ, x, y, z], y], D[xDot[σ, x, y, z], z]},
   {D[yDot[r, x, y, z], x], D[yDot[r, x, y, z], y], D[yDot[r, x, y, z], z]},
   {D[zDot[b, x, y, z], x], D[zDot[b, x, y, z], y], D[zDot[b, x, y, z], z]}}

Out[ ]:= {{-σ, σ, 0}, {r - z, -1, -x}, {y, x, -b}}
```

a)

```
In[ ]:= σVal = 10;
bVal = 8 / 3;
rVal = 28;
sols = jacobian[0, 0, 0, σVal, bVal, rVal]

Out[ ]:= {{-10, 10, 0}, {28, -1, 0}, {0, 0, -8/3}}
```

```
In[ ]:= eigenValues = Eigenvalues[sols]

Out[ ]:= {1/2 * (-11 - sqrt(1201)), 1/2 * (-11 + sqrt(1201)), -8/3}
```

```
In[ ]:= eigenValues // N

Out[ ]:= {-22.8277, 11.8277, -2.66667}
```

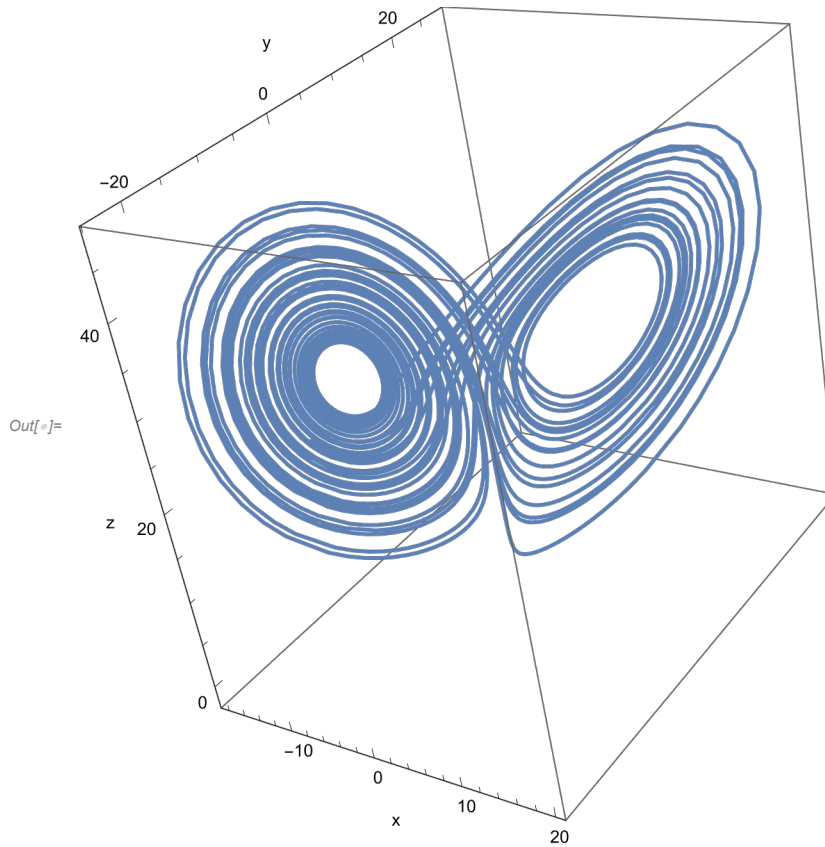
(*We get one positive eigenvalue and two negative so the origin is a saddle point and we have no stable fixed points*)

b)

```
In[ ]:= initialVal = 0.1;
tMax = 1000;
sols2 =
  NDSolve[{x'[t] == xDot[σVal, x[t], y[t], z[t]], y'[t] == yDot[rVal, x[t], y[t], z[t]],
    z'[t] == zDot[bVal, x[t], y[t], z[t]], x[0] == initialVal,
    y[0] == initialVal, z[0] == initialVal}, {x[t], y[t], z[t]}, {t, 0, tMax}];
```

```
In[ ]:= x[t_] = x[t] /. sols2[[1]];
        y[t_] = y[t] /. sols2[[1]];
        z[t_] = z[t] /. sols2[[1]];
```

```
In[ ]:= ParametricPlot3D[{x[t], y[t], z[t]}, {t, 100, 140}, AxesLabel -> {"x", "y", "z"}]
```



d)

```
In[ ]:= Tr[jacobian[x, y, z, σ, b, r]]
```

```
Out[ ]:= -1 - b - σ
```