Engineering Tripos Part IIA

SA1 - Wing Analysis Interim Report 2

Paul Cumner - **Tom Kosteletos** - Pembroke College

1 Exercise 1

Zero pressure-gradient, laminar boundary layer.

1.1 ueintbit.m

```
function f = ueintbit(xa,ua,xb,ub)
%calculates integral of Thwaites solution
u_bar = (ua+ub)/2;
del_u = ub-ua;
del_x = xb-xa;

f = (u_bar^5 + 5/6 *u_bar^3*del_u^2 + 1/16*u_bar*del_u^4)*del_x;
end
```

1.2 Script

```
clear
close all

Re_L = 1;
x = linspace(0,1,101); % dimentionless x/L
ue = ones(1,101); % dimentionless ue/U
Int = 0;
theta = zeros(1,101); % theta/L
theta_b = zeros(1,101); %blasius theta/L

for i = (2:length(x))
   Int = Int + ueintbit(x(i-1),ue(i-1),x(i),ue(i));
   theta(i) = sqrt( 0.45/Re_L*(ue(i))^-6 * Int );

end

theta_b = 0.664/sqrt(Re_L)*sqrt(x);

plot(x,theta,'-' ,'color','r','linewidth',1.5)
hold on
plot(x,theta_b,'-' ,'color','b','linewidth',1.5)
hold off
legend('Thwaites','Blasius','location','northwest')
xlabel('x/L')
```

```
ylabel('theta/L')
set(gca,'Fontn','Times','FontSize',15,'linewidth',1)
title('zero pressure gradient laminar boundary layer momentum thickness')
print -deps2c ex1w2.eps
```

1.3 Plot

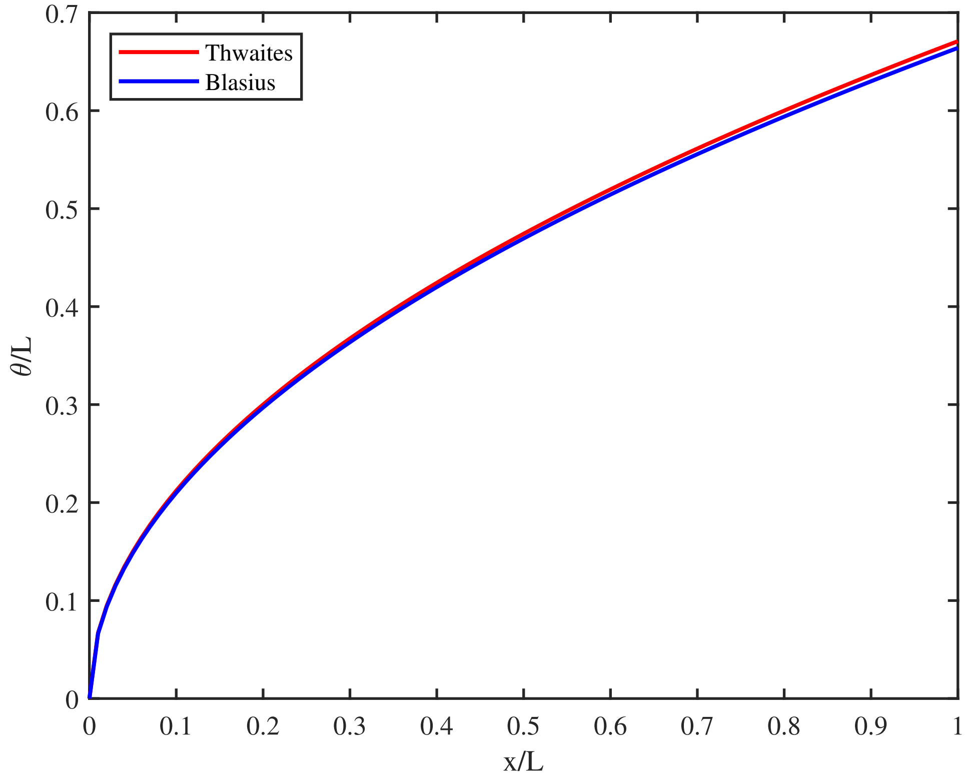


Figure 1: Zero pressure-gradient, laminar boundary layer momentum thickness.

2 Exercise 2

Constant-velocity-gradient, laminar boundary layer.

2.1 Script

```
clear
close all

% run for
%ue_grad = -0.1,0.1,0
%Re_L = 5*10^6,10*10^6,20*10^6

Re_L = 5*10^6;
Re_theta = 0;
x = linspace(0,1,101); % dimentionless x/L
ue_grad = -0.1;
ue = linspace(1,1+ue_grad,101); % dimentionless ue/U
Int = 0;
theta = zeros(1,101); % theta/L
theta_b = zeros(1,101); %blasius theta/L



 n = 101; % defines number of panels
 laminar = true; % initializes boundary layer state flag
 i = 1;
 while laminar && i < n
   i = i + 1;
   Int = Int + ueintbit(x(i-1),ue(i-1),x(i),ue(i));
   theta(i) = sqrt( 0.45/Re_L*(ue(i))^-6 * Int );
   Re_theta = Re_L * ue(i) * theta(i);

   m = -Re_L*theta(i)^2*ue_grad;
   H = thwaites_lookup(m);
   He = laminar_He(H);
   if log(Re_theta) >= 18.4*He - 21.74
        laminar = false;
        disp([x(i) Re_theta/1000])
   end
end
```

2.2 Transition Locations and Re_θ values

Re_L [$\times 10^6$]	$\frac{d(u_e/U)}{d(x/L)}$	Transition Location $\frac{x}{L}$	Re_θ [$\times 10^3$] at transition
5	-0.1	0.48	1.08
	0	0.74	1.29
	0.1	-	-
10	-0.1	0.29	1.17
	0	0.37	1.29
	0.1	0.55	1.51
20	-0.1	0.17	1.25
	0	0.19	1.31
	0.1	0.22	1.38

Table 1: Laminar to turbulent transition.

3 Exercise 3

Laminar boundary layer with transition or separation.

3.1 Script

```
clear
close all

% run for
%ue_grad = -0.25
%Re_L = 10^3,10^4,10^5

Re_L = 8.95*10^5;
Re_theta = 0;
x = linspace(0,1,101); % dimentionless x/L
ue_grad = -0.25;
ue = linspace(1,1+ue_grad,101); % dimentionless ue/U
Int = 0; %variable that stores integral value
theta = zeros(1,101); % theta/L
theta_b = zeros(1,101); %blasius theta/L

int = 0; %location of natural transition
ils = 0; %location of laminar separation

 n = 101; % defines number of panels
 laminar = true; % initializes boundary layer state flag
 i = 1;
 while laminar && i < n
   i = i + 1;
   Int = Int + ueintbit(x(i-1),ue(i-1),x(i),ue(i));
   theta(i) = sqrt( 0.45/Re_L*(ue(i))^-6 * Int );
   Re_theta = Re_L * ue(i) * theta(i);

   m = -Re_L*theta(i)^2*ue_grad;
   H = thwaites_lookup(m);
   He = laminar_He(H);
   if log(Re_theta) >= 18.4*He - 21.74
        laminar = false;
        disp([x(i) Re_theta/1000])
        int = i;
   elseif m >= 0.09
       laminar = false;
       ils = i;
   end

 end

if int ~= 0
```

```
    disp(['Natural transition at ' num2str(x(int)) ...
    ' with Re_theta ' num2str(Re_theta)])
end
if ils ~= 0
    disp(['Separation at ' num2str(x(ils)) ...
    ' with Re_theta ' num2str(Re_theta)])
end
```

3.2 Separation Locations

Re_L	$\frac{d(u_e/U)}{d(x/L)}$	Separation Location $\frac{x}{L}$
10^5	-0.25	0.50
10^4	-0.25	0.50
10^3	-0.25	0.50

Table 2: Laminar separation.

At $Re_L = 9.0 \times 10^5$ with $\frac{d(u_e/U)}{d(x/L)} = -0.25$, transition supplants separation.

4 Exercise 4

Zero-pressure-gradient, turbulent boundary layer.

4.1 thickdash.m

```
function dthickdx = thickdash(xmx0,thick)
global Re ue0 duedx;

ue = ue0+duedx*xmx0; % ue at first point solved at

Re_theta = Re*thick(1)*(ue); %for  L = 1

He = thick(2)/thick(1);
if He <1.46
    H = 2.803;
else
    H = (11*He+15)/(48*He-59);
end



cf = 0.091416*((H-1)*Re_theta)^-0.232*exp(-1.26*H);
cdiss = 0.010018*((H-1)*Re_theta)^(-1/6);


dthickdx = [cf/2-(H+2)/ue*duedx*thick(1); cdiss - 3/ue*duedx*thick(2)];
end
```

4.2 Script

```
clear
close all
global Re ue0 duedx;
Re = 10^7;
ue0 = 1;
duedx = 0;


x0 = 0.01;
thick0(1) = 0.023*x0*(Re*x0)^(-1/6);
thick0(2) = 1.83*thick0(1);


[delx thickhist] = ode45(@thickdash,[0 0.99],thick0);

theta = thickhist(:,1);
deltae = thickhist(:,2);
```

```
x = x0+delx;

theta7 = 0.037*x.*(Re.*x).^(-1/5);
theta9 = 0.023*x.*(Re.*x).^(-1/6);

plot(x,theta,'-' ,'color','b','linewidth',1.5);
xlabel('x/L')
ylabel('\theta/L')
set(gca,'Fontn','Times','FontSize',10,'linewidth',1)
title('Zero pressure-gradient, turbulent boundary layer momentum thickness')
hold on
plot(x,theta7,'-' ,'color','g','linewidth',1.5)
plot(x,theta9,'-' ,'color','r','linewidth',1.5)
hold off

legend('\theta','1/7th power law','1/9th power law','location','northwest')
print -deps2c ex4w2.eps
```

4.3 Graph

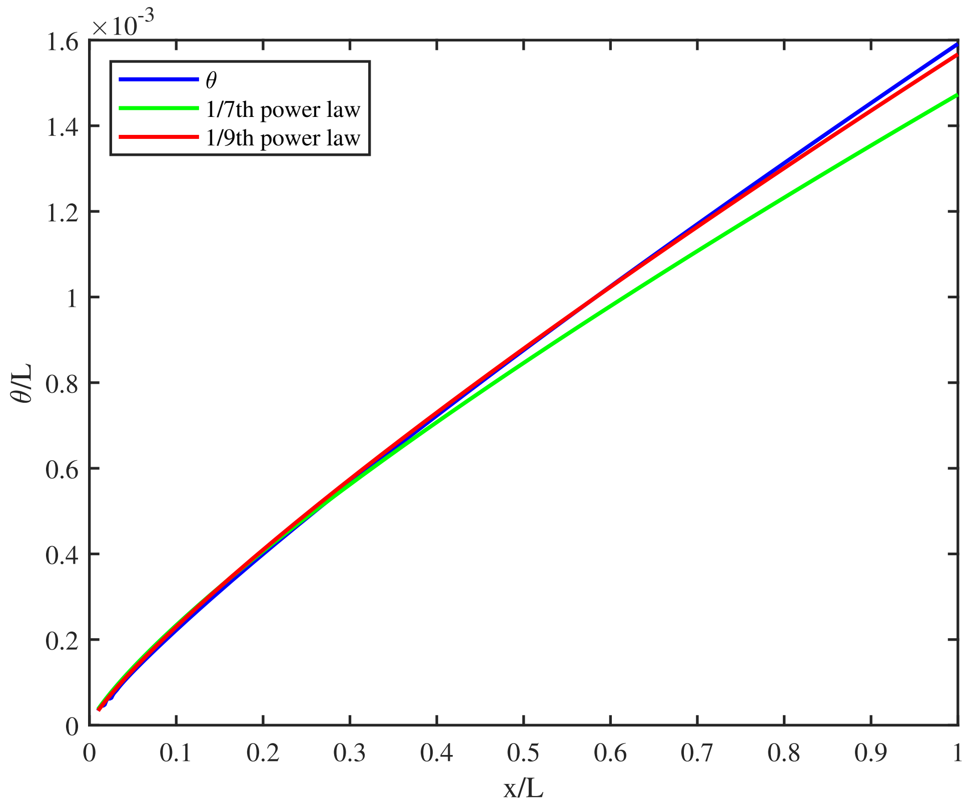


Figure 2: Zero pressure-gradient, turbulent boundary layer momentum thickness.

5 Exercise 5

Constant-velocity-gradient, turbulent boundary layer.

5.1 Script

```
clear
close all
global Re ue0 duedx;
Re = 10^7;
ue0 = 1;
duedx = -0.5;


x0 = 0.01;
thick0(1) = 0.023*x0*(Re*x0)^(-1/6);
thick0(2) = 1.83*thick0(1);


[delx thickhist] = ode45(@thickdash,[0 0.99],thick0);

theta = thickhist(:,1);
deltae = thickhist(:,2);
He = deltae./theta;


x = x0+delx;
He_sep = 1.46.*ones(1,length(x));

figure('Name', 'theta and delta e');
plot(x,theta,'-' ,'color','b','linewidth',1.5);
hold on
plot(x,deltae,'-','color','r','linewidth',1.5);

hold off
legend('\theta/L','\delta_e/L','location','northwest')
xlabel('x/L')
set(gca,'Fontn','Times','FontSize',10,'linewidth',1)
title('\theta/L ,\delta_e/L vs x/L  at duedx = -0.5, ReL = 10^7')

print -deps2c ex5w2.eps
%{
figure('Name', 'He against x');
plot(x,He,'-' ,'color','b','linewidth',1.5);
hold on
plot(x,He_sep,'-','color','r','linewidth',1.5);

hold off
%}
```

5.2 Separation Locations

Re_L	$\frac{d(u_e/U)}{d(x/L)}$	Separation Location $\frac{x}{L}$
10^6	-0.5	0.891
10^7	-0.5	0.985
10^8	-0.5	-
10^7	-0.25	-
10^7	-0.95	0.505

Table 3: Turbulent separation.

5.3 Plot

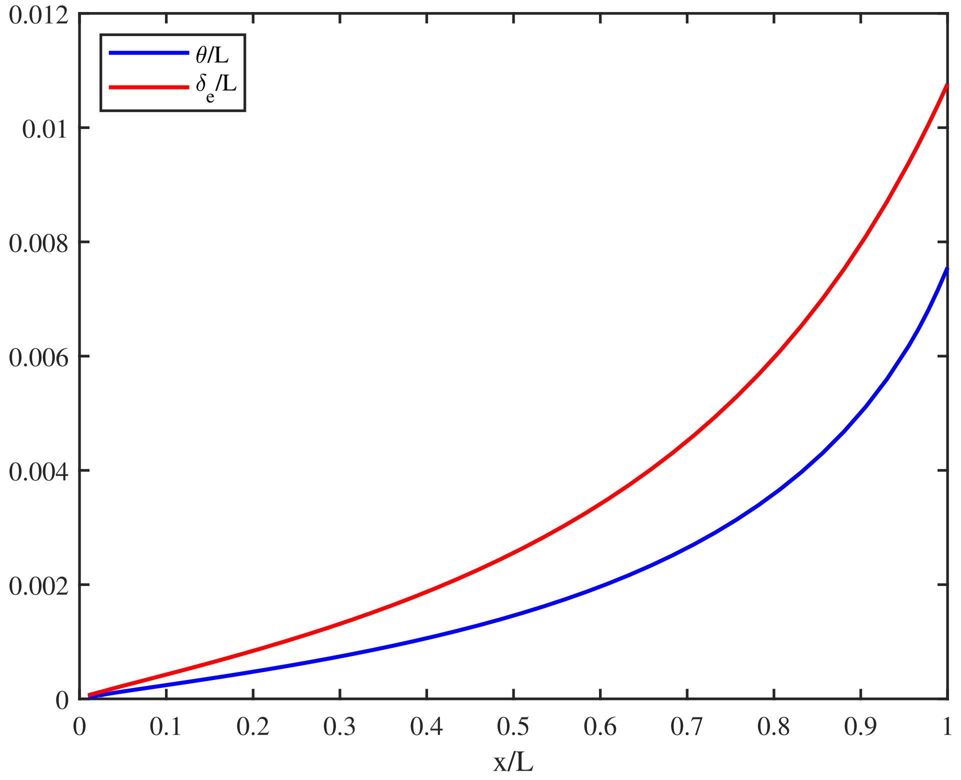
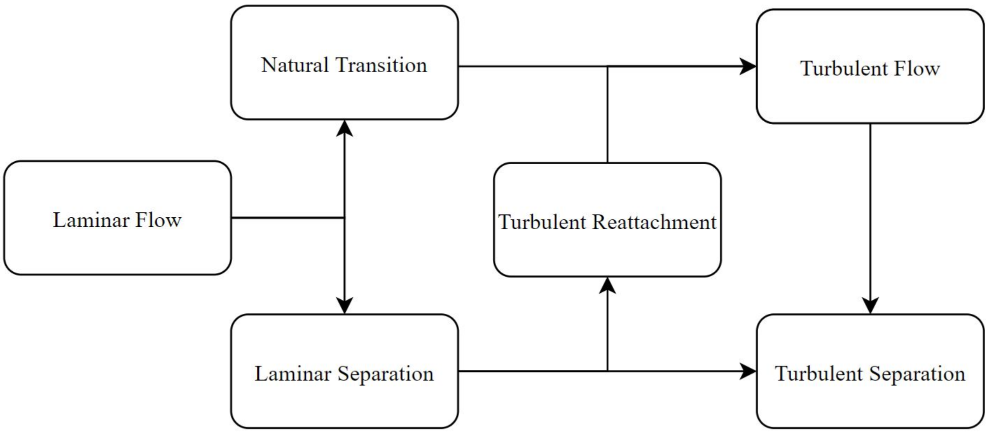


Figure 3: $\frac{\theta}{L}$ and $\frac{\delta_e}{L}$ against $\frac{x}{L}$ for a turbulent boundary layer at $Re_L = 10^7$ and $\frac{d(u_e/U)}{d(x/L)} = -0.50$.

6 Exercise 6

Combined Laminar and turbulent boundary layer evolution.

6.1 Flowchart



6.2 Script

```
clear
close all

% run for
%ue_grad = -0.25
%Re_L = 10^3,10^4,10^5
global Re ue0 duedx;
Re = 10^7;
ue0 = 1;
duedx = 0;

n = 101; % defines number of panels
Int = 0; %variable that stores integral value
x = linspace(0,1,n); % dimentionless x/L
ue = linspace(1,1+duedx,n); % dimentionless ue/U

m = zeros(1,n);
H = zeros(1,n);
He = zeros(1,n);
He(1) = 1.57258;
deltae = zeros(1,n);
Re_theta = zeros(1,n);
theta = zeros(1,n); % theta/L

int = 0; %natural transition i value
ils = 0; %laminar separation i value
```

```
itr = 0; %turbulent reattachment i value
its = 0; %turbulent separation i value

laminar = true; % initializes boundary layer state flag
i = 1;
 while laminar && i < n
   i = i + 1;
   Int = Int + ueintbit(x(i-1),ue(i-1),x(i),ue(i));
   theta(i) = sqrt( (0.45/Re)*(ue(i))^-6 * Int );
   Re_theta(i) = Re * ue(i) * theta(i);

   m(i) = -Re*theta(i)^2*duedx;
   H(i) = thwaites_lookup(m(i));
   He(i) = laminar_He(H(i));
   if log(Re_theta(i)) >= 18.4*He(i) - 21.74
       laminar = false;
       disp([x(i) Re_theta(i)/1000])
       int = i;
   elseif m(i) >= 0.09
       laminar = false;
       ils = i;
       He(i) = 1.51509;
   end

 end


deltae = He.*theta;

ue0 = ue0 + i/n*duedx;

 while its == 0 && i < n
   thick0 = [theta(i);deltae(i)];
   i = i + 1 ;
   [delx thickhist] = ode45(@thickdash,[0,x(i)-x(i-1)],thick0);
   ue0 = ue0 +1/n*duedx;
   theta(i) = thickhist(end,1);
   deltae(i) = thickhist(end,2);


   He(i) = deltae(i)/theta(i);
   if He(i)  < 1.46
        H(i) = 2.803;
   else
        H(i) = (11*He(i)+15)/(48*He(i)-59);
   end

   Re_theta(i) = Re * ue(i) * theta(i);
   if ils > 0 && itr ==0 && He(i)>1.58 %test for reattachment after laminar separation
```

```
      itr = i;
  elseif He(i)<1.46 % test for turbulent separation
      its = i;
  end
 end

 He (i:n) = He(i);
 H (i:n) = H(i);

 while i < n
 i = i+1;
 theta(i)= theta(i-1)*(ue(i-1)/ue(i))^(H(i)+2);
 end


 if int ~= 0
    disp(['Natural transition at ' num2str(x(int)) ...
    ' with Re_theta ' num2str(Re_theta(int))])
end
if ils ~= 0
    disp(['Laminar separation at ' num2str(x(ils)) ...
    ' with Re_theta ' num2str(Re_theta(ils))])
end

if itr ~= 0
    disp(['Turbulent reattachment at ' num2str(x(itr)) ...
    ' with Re_theta ' num2str(Re_theta(itr))])
end
if its ~= 0
    disp(['Turbulent separation at ' num2str(x(its)) ...
    ' with Re_theta ' num2str(Re_theta(its))])
end
```

6.3 Plots

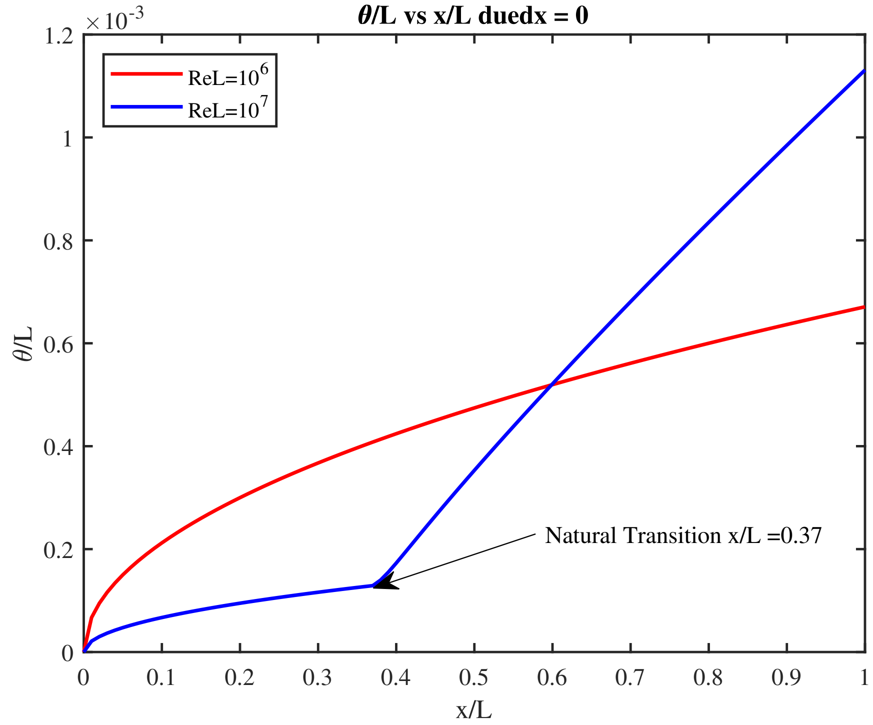


Figure 4: $\frac{\theta}{L}$ against $\frac{x}{L}$ for $\frac{d(u_e/U)}{d(x/L)} = 0$.

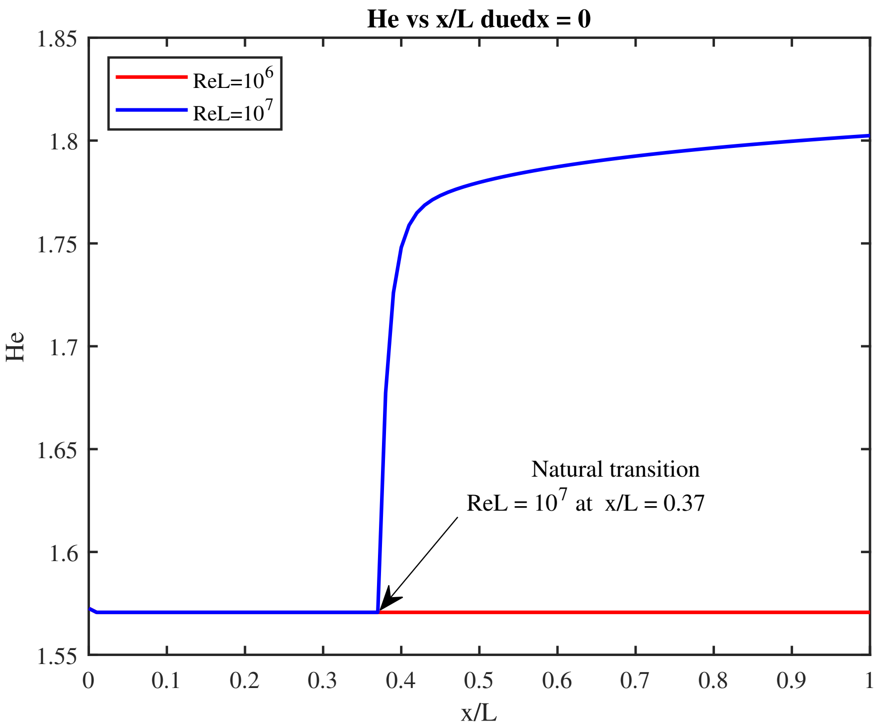


Figure 5: H_e against $\frac{x}{L}$ for $\frac{d(u_e/U)}{d(x/L)} = 0$.

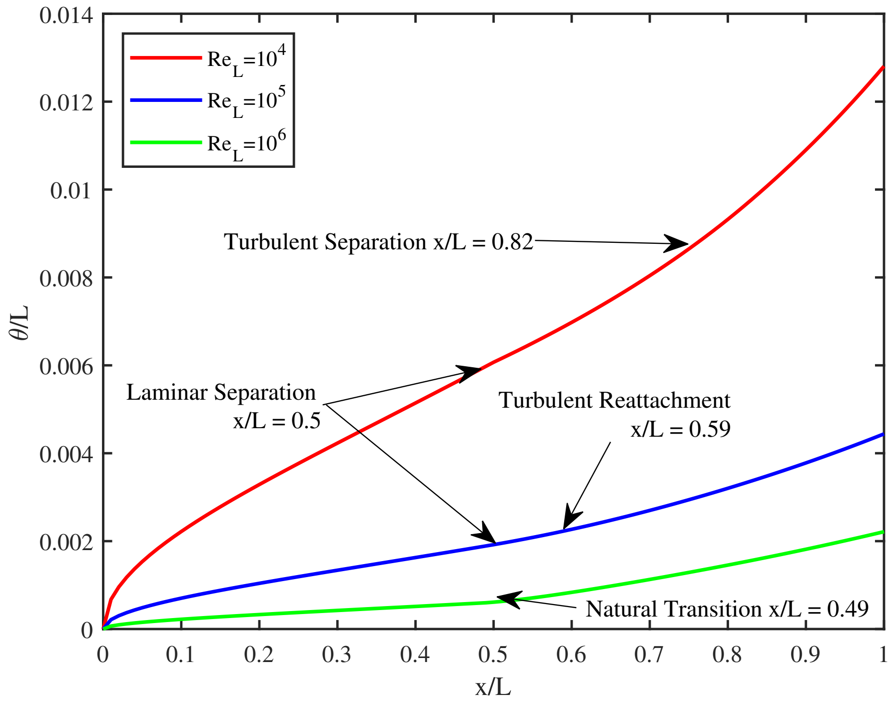


Figure 6: $\frac{\theta}{L}$ against $\frac{x}{L}$ for $\frac{d(u_e/U)}{d(x/L)} = -0.25$.

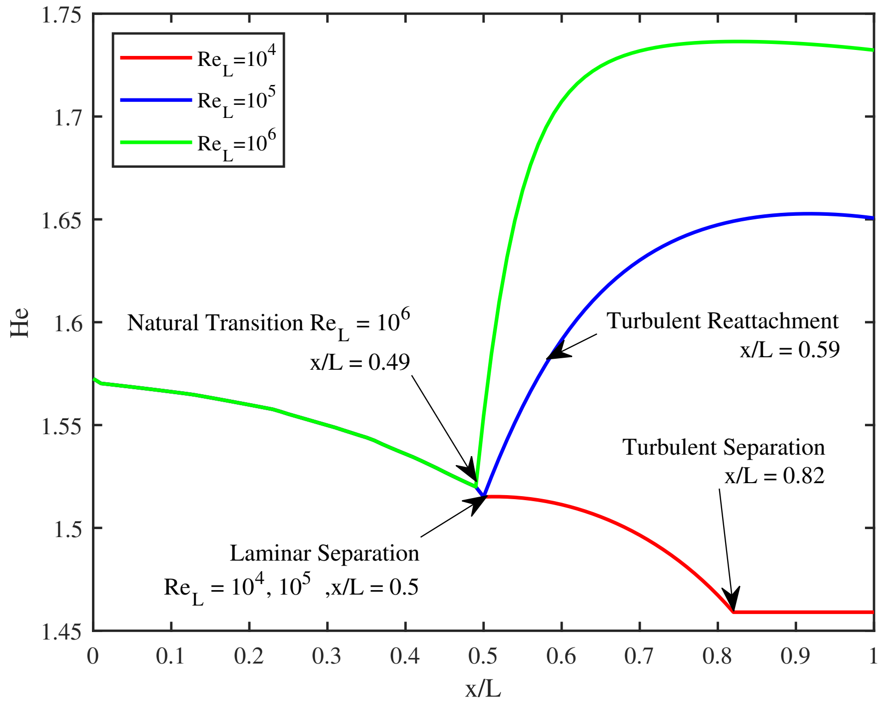


Figure 7: H_e against $\frac{x}{L}$ for $\frac{d(u_e/U)}{d(x/L)} = -0.25$.

6.4 Critical velocity gradient

At $Re_L = 10^5$, $\frac{d(u_e/U)}{d(x/L)} = -0.38$ gives separation at $\frac{x}{L} = 1$