

0.1 03.10.2019

0.1.1 Ф-ла Тейлора для неявной функции

Пример

$$F(x, y; u) = u^3 + 3yu - 4x = 0, \quad u(x, y) \text{ в окр. } (1, 1)$$

Задача. Написать ф. Тейлора для $u(x, y)$ с точность. до $\underbrace{o(\sqrt{(x-1)^2 + (y-1)^2})}_\varphi^n$

$$(x, y) = (1, 1) \quad u^3 + 3u - 4 = 0 \Rightarrow (u^2 + u + 4)(u - 1) = 0 \Rightarrow u(1, 1) = 1$$

Проверим, что $F'_u(1, 1; 1) \neq 0$, $3u^2 + 3y \neq 0$

$$u'_x = -\frac{F'_x}{F'_u} = \frac{2}{3} \quad u'_y = -\frac{F'_y}{F'_u} = -\frac{1}{2}$$

$$u(x, y) = 1 - \frac{2}{3}(x - 1) - \frac{1}{2}(y - 1) + \bar{o}(\varphi) \quad n = 1$$

Способ 1 ($n = 2, 3, \dots$)

$$u'_x = -\frac{F'_x}{F'_u} = -\frac{4}{3u^2 + 3y} \quad u''_{xx} = \frac{4 * 6uu'_x}{(3u^2 + 3y^2)^2} = -\frac{16}{36} = -\frac{4}{9}$$

$$u''_{xy} = \frac{4(6uu'_y + 3)}{(3u^2 + 3y^2)^2} = 0 \quad u''_{yy} = \left(-\frac{3u}{3u^2 + 3y}\right)'_y = -\frac{u'_y(u^2 + y) - (2uu' + 1)u}{(u^2 + y)^2} = \frac{1}{4}$$

$$u(x, y) = 1 - \frac{2}{3}(x - 1) - \frac{1}{2}(y - 1) + \frac{1}{2}\left(-\frac{4}{9}(x - 1)^2 + \frac{1}{4}(y - 1)^2\right) + \bar{o}(\varphi^2)$$

Способ 2 (более высокие степени, метод неопр. коэф.)

$$u^3(x, y) = \left(1 + \frac{2}{3}(x - 1) - \frac{1}{2}(y - 1) + a(x - 1)^2 + b(x - 1)(y - 1) + c(y - 1)^2 + \bar{o}(\varphi^2)\right)^3$$

$$t = x - 1 \quad s = y - 1$$

$$0 = u^3 + 3yu - 4x = \bar{o}(\varphi^2) + 1 + 3 * 1^2 \left(\frac{2}{3}t - \frac{1}{2}s + at^2 + bts + cs^2\right) +$$

$$+ 3 \left(\left(\frac{2}{3}t\right)^2 + \frac{s^2}{4} - \frac{2}{3}ts\right) + 3(s + 1)u - 4(t + 1) =$$

$$\left((s + 1)u = s + \frac{2}{3}t - \frac{1}{2}s + s \left(\frac{2}{3}t - \frac{1}{2}s\right) + at^2 + bts + cs^2 + \bar{o}(\varphi^2)\right)$$

$$\begin{aligned}
&= \overline{o}(\varphi^2) + \underbrace{(1 + 3 - 4)}_{=0} + t \left(\underbrace{3\frac{2}{3} + 3\frac{2}{3} - 4}_{=0} \right) + s \left(\underbrace{-\frac{3}{2} + \frac{3}{2}}_{=0} \right) + t^2 \left(\underbrace{3a + 3\frac{4}{9} + 3a}_{=0} \right) + \\
&\quad + ts \left(\underbrace{3b - 2 + 3 \left(\frac{2}{3} + b \right)}_{=0} \right) + s^2 \left(\underbrace{3c + \frac{3}{4} - \frac{3}{2} + 3c}_{=0} \right)
\end{aligned}$$

Приравняли к 0, т.к. у найденного выше $u(x, y)$ эти коэф. = 0

$$\Rightarrow a = -\frac{2}{9} \quad b = 0 \quad c = \frac{1}{8}$$

ДЗ: 3127-3186 (10 задач)