2019-09-23

Напоминание

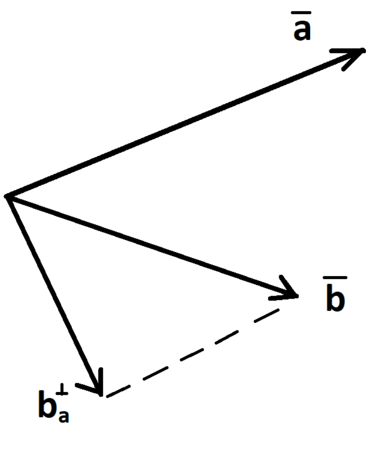
$$\left|\sum |f(t_i)-f(t_{i-1)}|\right| - \sum ||f(t_i)-f(t_{i-1})| - |f'(\tau_i)\Delta t_i|| \leqslant$$

$$\leqslant \sum \left| \int_{t_{i-1}}^{t_i} |f'(t)|\,dt - \int_{t_{i-1}}^{t_i} |f'(\tau_i)|\,dt \right| =$$

$$\sum \int_{t_{i-1}}^{t_i} |f'(t)-f'(t_i)|\,dt < \sum \mathcal{E}\Delta_i t = \mathcal{E}(b-a)$$

$$\forall \mathcal{E} > 0 \quad \exists \delta > 0 \text{ если } f_i - f_{i-1} < \delta$$

$$\Rightarrow |f'(t) - f'(\tau_i)| < \mathcal{E}$$

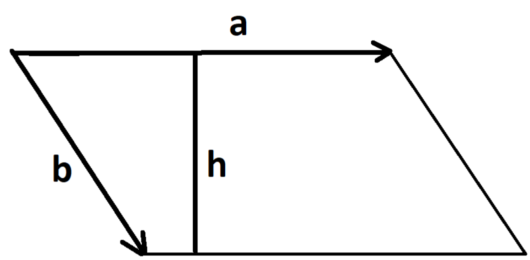


Лемма

$$\overrightarrow{b} = \text{Пр}_a b + b\frac{1}{a}$$

$$\overrightarrow{\text{Пр}_a b} = \frac{(a,b)}{|a|^2}\overrightarrow{a}$$

$$\left|b\frac{1}{a}\right| = \frac{\left|\overrightarrow{a} \times \overrightarrow{b}\right|}{|a|}$$



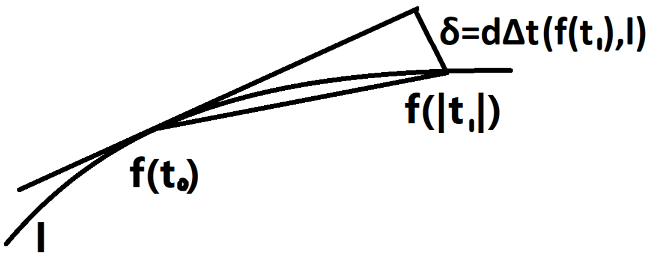
Док-во

$$h = \frac{S}{|a|}$$

$$\frac{(\overrightarrow{a} \times \overrightarrow{b}) \times \overrightarrow{a}}{|a|^2} = b\frac{1}{a}$$

$$(a, b, a \times b) \text{ - прав. тройка}$$

$$(a \times b, a, b) \text{ - прав. тройка}$$



Теорема

$$\lim_{t_1 \to t_0} \frac{\delta}{|f(t_1) - f(t_0)|} = 0$$

$$\overrightarrow{f'}(t_0) \Rightarrow \text{ по лемме}$$

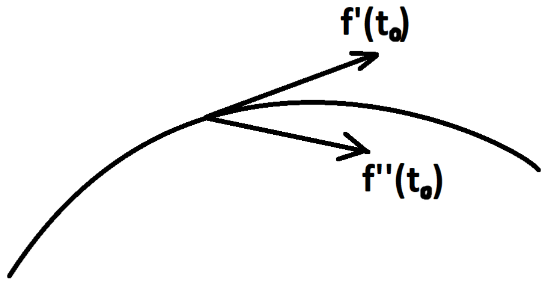
$$\delta = \frac{|f'(t_0) \times (f(t_1) - f(t_0))|}{|f'(t_0)|}$$

$$\lim_{t_1 \to t_0} \frac{\delta}{\underbrace{f(t_1) - f(t_0)}_{\overrightarrow{a}(t_0)}} = \lim_{t_1 \to t_0} \frac{|f'(t_0) \times \overrightarrow{a}(t_1)|}{|f'(t_0)| \cdot |a(t_1)|}$$

$$\lim_{t_1 \to t_0} \frac{\left| f'(t_0) \times \frac{f(t_1)-f(t_0)}{t_1-t_0} \right|}{\left| f'(t_0) \cdot \left| \frac{f(t_1)-f(t_0)}{t_1-t_0} \right| \right|} = \frac{f'(t_0) \times f'(t_0)}{|f'(t_0)|^2} = 0$$

$\Leftarrow$ очев

0.1 Вектор кривизны



Опр

$$g(\varphi(t)) = g(s) = f(t) \qquad s = \varphi(t)$$

$$\overrightarrow{f'}(t) = (g(\varphi_i t_i))' = \overrightarrow{g'} \cdot \varphi'(t)$$

$$\overrightarrow{v}(t_0) = \frac{f'(t_0)}{|f'(t_0)|} \qquad \overrightarrow{n} : |n| = 1; \quad \overrightarrow{n} \perp \overrightarrow{v}$$

$n \in < f', f'' >$ $\quad \overrightarrow{n}$ и $\overrightarrow{f}''$ в одной полуплоскости $f'(t)$

$$\overrightarrow{v}'(t) \perp \overrightarrow{v}(t)$$

$$\overrightarrow{v}'(t) = k \cdot \overrightarrow{n}$$

$|n| = 1 \quad k(t)$ - кривизна кривой $\quad k(t) \geqslant 0$ в точке t

$\overrightarrow{n}$ - вектор главной нормали

$\overrightarrow{v}$ - касат. вект

Утв

$f(t)$ - натуральная парам.

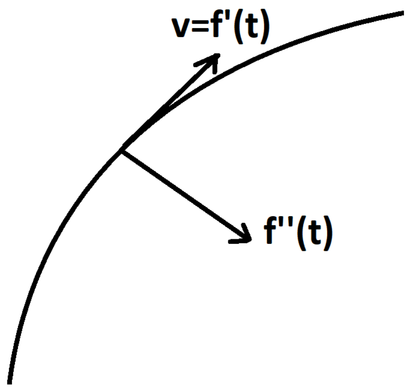
$$|f'(t)| = 1 \Rightarrow v = f'(t)$$

$$f''(t) = k\overrightarrow{n}$$

$$\overrightarrow{n} = \frac{f''(t)}{|f''(t)|}$$

$$k = |f''(t)|$$

рисунок 5 (центростр. ускорение)



$$f(t) \text{ - любая параметризация, } g(s) \text{ - натур. парам.}$$

$$f(t) = g(\varphi(t)) \qquad s = \varphi(t) \text{ - нат. парам}$$

$$s = \underbrace{\int_a^t (f'(\tau))d\tau}_{=\varphi(t)}$$

$$f'(t) = g'(s) \cdot \varphi'(t)$$

$$f''(t) = (g'(\varphi(t)))' \cdot \varphi'(t) + g'(s) \cdot \varphi''(t) =$$

$$= \underbrace{g''(s) \cdot \varphi'^2(t)}_{\perp \overrightarrow{v}} + \underbrace{g'(s)\varphi''(t)}_{\| g'(s)=v}$$

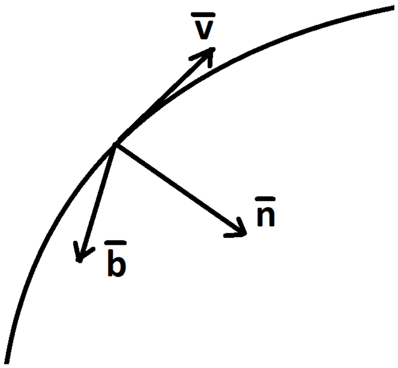
Теорема

Плоск. на вект $f'(t)$ и $f''(t)$ не зависит от параметризации

Опр

Эта плоскость (на вект. $\overrightarrow{v}$ и $\overrightarrow{n}$) наз. соприкасающейся плоск.

0.2 Формула Френе



Опр

$$\overrightarrow{b} = \overrightarrow{v} \times \overrightarrow{n} \text{ - вектор бинормали}$$

$$(\overrightarrow{v}, \overrightarrow{n}, \overrightarrow{b}) \text{ - базис Френе}$$

Трехвекторник Френе или ренер Френе

$$\overrightarrow{v}' = k \cdot \overrightarrow{n}$$

$$b' \perp b$$

$$b' = (\overrightarrow{v} \times \overrightarrow{n})' = \underbrace{\overrightarrow{v}' \times \overrightarrow{n}}_{=0} + \overrightarrow{v} \times n' \perp \overrightarrow{v}$$

$$\overrightarrow{v}' = k\overrightarrow{n}$$

$$\Rightarrow b' \parallel \overrightarrow{n} \Rightarrow b' = -æ \cdot \overrightarrow{n} \text{ - капа}$$

$$æ \text{ наз. кручением кривой}$$

Теорема

$$æ = 0 \Leftrightarrow \text{ Кривая плоская}$$

$$\text{Кривая плоская } \Leftrightarrow \text{ она лежит в плоск } < v, n > \Leftrightarrow$$

$$\Leftrightarrow \text{ нормаль к } < v, n > \text{ постоянна } \Leftrightarrow b = const \Leftrightarrow b' = 0 \Leftrightarrow æ = 0$$

$$n' = (\overrightarrow{b} \times v)' = b' \times v + b \times v' = -æ\, n \times v + k \cdot b \times n =$$

$$æ \cdot \overrightarrow{b} - k\overrightarrow{v}$$

$$v' = kn$$

$$n' = -kv + æb$$

$$b' = -æn$$

	v	n	b
v’	0	k	0
n’	-k	0	æ
b’	0	-æ	0

0.3 Вычисление кривизны кручения

Теорема

$$k = \frac{|f'(t) \times f''(t)|}{|t'(t)|^3}$$

Док-во

$$g(s) \text{ - нат. парам } f(t) = g(\varphi(t)) \quad s = \varphi(t) \quad \varphi(t) = \int_a^t |f'(\tau)| \, d\tau$$

$$g'(s) = \overrightarrow{v} \quad g''(s) = k\overrightarrow{n} \qquad \varphi'(t) = |f'(t)|$$

$$f''(t) = g''(s) \cdot \varphi^2(t) + g'(s) \cdot \varphi''(t) = k \cdot \overrightarrow{n} \cdot |f'(t)|^2 + v \cdot \varphi''(t)$$

$$f''(t) \times f'(t) = k \, |f'(t)|^2 \cdot \overrightarrow{n} \times f'(t) + 0 = \qquad v'(t) = |f'(t)| \, \overrightarrow{v}$$

$$k \cdot \overrightarrow{n} \times \overrightarrow{v} \, |f'(t)|^3$$

$$|f''(t) \times f'(t)| = k \, |f'(t)|^3$$

$$k = \frac{|f''(t) \times f'(t)|}{|f'(t)|^3}$$