2019-09-12

Док-во

$$P = [t_0, t + h]$$

$$d_k : t_0 = t_0^k < t_1^k < ... < t_j^k < ... < t_{nk}^k = t_0 + h$$

$$rank\ d_k = \lambda_k = \max_{0 \le j \le n_k - 1} (t_{j+1}^k - t_j^k)$$

$$(3) \quad \lambda \underset{k \to +\infty}{\to} 0$$

$$(4) \quad \begin{cases} \phi_k(t_0) = x_0 \\ \phi_k(t) = \phi_k(t_j^k) + X(t_j^k, \phi_k(t_j^k))(t - t_j^k) \end{cases} \text{ - ломанные Эйлера}$$

$$t_j^k \le t \le t_{j+1}^k$$

рисунок 1

Лемма (1)

опред $\phi_k(t)$ *и*

$$|\phi_k(t) - x_0| \le M(t - t_0) \quad \forall t \in P \quad (5)$$

Замечание

$$(5) \Rightarrow$$

$$t \in P \Rightarrow 0 \le t - t_0 \le h \Rightarrow$$

$$\Rightarrow |\phi_k(t) - x_0| \le M \cdot h \le M \cdot \frac{b}{M} = b \quad (6)$$

Док-во (лемма 1)

$$j = 0 \quad t \in [t_0^k, t_1^k]$$

$$\phi_k(t) = x_0 + X(t_0, x_0) \cdot (t - t_0)$$

$$\Rightarrow |\phi_k(t) - x_0| = |\underbrace{X(t_0, x_0)}_{\le M}|(t - t_0) \le M(t - t_0)$$

Пусть (5) *Вып* $\forall t \in [t_0^k, t_j^k]$

$$\Rightarrow |\phi_k(t_j^k) - x_0| \le M(t_j^k - t_0) \le b \Rightarrow (t_j^k, \phi_k(t_j^k)) \in D$$

$$t_j^k \le t < t_{j+1}^k$$

$$|\phi_k(t) - x_0| = \underbrace{|\phi_k(t_j^k) - x_0|}_{\text{инд. предп}} + |X(t_j^k, \phi_k(t_j^k))|(t - t_j^k) \le$$

$$\le M(t_j^k - t_0) + M(t - t_j^k) = M(t - t_0)$$

Определение

$$(7)\quad \begin{cases}\psi_k(t) = X(t_j^K, \phi_k(t^k)), \quad t_j^k \le t \le t_{j+1}^k \qquad j = 0, n_k - 1\\ \phi_k(t_{nk}^k) = X(t_{nk}^k, \phi_k(t_{nk}^k))\end{cases}$$

Лемма (2)

$$\phi_k(t) = x_0 + \int_{t_0}^{t} \psi_k(\tau) d\tau \qquad (8)$$

Док-во

$$j = 0 \quad t \in [t_0^k, t_1^k]$$

$$\phi_k(t) = x_0 + X(t_0, x_0)(t - t_0)$$

$$= \int_{t_0}^{t} \underbrace{X(t_0, x_0)}_{=\psi_k(t)} d\tau$$

$$t \in [t_0^k, t_j^k] \Rightarrow \phi_k(t_j^k) = x_0 + \int_{t_0}^{t_j^k} \psi_k(\tau) d\tau$$

$$t \in [t_j^k, t_{j+1}^k]$$

$$\Rightarrow \phi_k(t) = \phi(t_j^k) + X(t_j^k, \phi_k(t_j^k))(t - t_j^k) =$$

$$= x_0 + \int_{t_0}^{t_j^k} \psi_k(\tau) d\tau + \int_{t_j^k}^{t} X(t_j^k, \phi_k(t_j^k)) d\tau =$$

$$= x_0 + \int_{t_0}^{t} \psi_k(\tau) d\tau$$

Лемма (3)

$\{\phi_k(t)\}_{k=1}^{\infty}$ - *равномерно огр, равностеп. непр.*

$t \in P$

Док-во

$$|\phi_k(t)| \le |\phi_k(t) - x_0| + |x_0| \le b + |x_0| \quad \forall k \in \mathbb{N}$$

$$\mathcal{E} > 0 \quad \delta$$

$$|\bar{t} - \bar{\bar{t}}| < \delta \quad (\bar{t}, \bar{\bar{t}} \in P)$$

$$|\phi_k(\bar{t}) - \phi_k(\bar{\bar{t}})| = |\int_{\bar{\bar{t}}}^{\bar{t}} \psi_k(\tau) d\tau| \le |\int_{\bar{\bar{t}}}^{\bar{t}} |\psi_k(t)| d\tau| \le$$

$$\le M\sigma = \mathcal{E}$$

$\exists$ подпослед $\{\phi_k(t)\}_1^\infty \quad t \in P$

$(9) \quad \phi_k(t) \underset{k\to+\infty}{\overset{P}{\rightrightarrows}} \phi(t)$

$\phi(t)$ - непр и $|\phi(t) - x_0| \leq b$

<u>Лемма</u> (4)

$(10) \quad \psi_k(t) \underset{k\to+\infty}{\overset{P}{\rightrightarrows}} X(t, \phi(t))$

<u>Док-во</u> (лемма 4)

$X(t,x) \in C(D) \Rightarrow X(t,x)$ *- равном непр. на* D

$\Rightarrow \forall \mathcal{E} > 0 \exists \delta > 0 : \forall (\bar{t}, \overline{x}), (\bar{\bar{t}}, \overline{\overline{x}}) \in D$

$|\bar{t} - \bar{\bar{t}}| < \delta, \quad |\overline{x} - \overline{\overline{x}}| < \delta \Rightarrow$

$\Rightarrow |X(\bar{t}, \overline{x}) - X(\bar{\bar{t}}, \overline{\overline{x}})| < \dfrac{\mathcal{E}}{2}$

фикс $\mathcal{E} > 0 \Rightarrow \exists \delta > 0$

$(12) \quad |X(t, \phi(t)) - \psi_k(t)| \leq |X(t, \phi(t)) - X(t, \phi_k(t))| + |X(t, \phi_k(t) - \phi_k(t)|$

из (9) $\quad \Rightarrow \exists k_1 : \forall k > k_1 \quad |\phi_k(t) - \phi(t)| < \delta \quad \forall t \in P$

$\Rightarrow \underbrace{|...|}_{(1)} < \dfrac{\mathcal{E}}{2}$

$t = t_{nk}^k \Rightarrow \underbrace{|...|}_{(2)} = 0$

$t \neq t_{nk}^k \to \exists j \in \{0, 1, ..., n_k - 1\} : t \in [t_j^k, t_{j+1}^k)$

$\underbrace{|...|}_{2} = |X(t, \phi_k(t)) - X(t_j^k, \phi_k(t_j^k))|$

$\exists k_2 : \forall k > k_2 \quad \lambda_k < \min(\delta, \dfrac{\delta}{M}) \quad$ (*из* (3))

$\Rightarrow (t - t_j^k) < (t_{j+1}^k - t_j^k) \leq \lambda_k < \delta$

$|\phi_k(t) - \phi_k(t_j^k)| \leq |\int_{t_j^k}^t \underbrace{|\psi_k(t)|}_{\leq M} \leq M \underbrace{(t - t_j^k)}_{\leq \lambda_k} < M \dfrac{\delta}{M} = \delta$

$$\Rightarrow |\underbrace{\dots}_{(2)}| < \frac{\mathcal{E}}{2}$$

$$\Rightarrow \forall k > \max(K_1, k_2) \qquad |X(t, \phi(t)) - \psi_k(t)| < \frac{\mathcal{E}}{2} + \frac{\mathcal{E}}{2} = \mathcal{E}$$

$$\phi(t) = x_0 + \int_{t_0}^{t} X(\tau, \phi(\tau)) d\tau \qquad (12)$$

$$t = t_0 : \phi(t_0) = x_0$$

Дифф $(13): \quad \dot{\phi}(t) = X(t, \phi(t))$

$\Rightarrow \phi(t)$ - *реш. задачи Коши* $(1), (2) \quad t \in P$