

2019-09-12

Док-во

$$P = [t_0, t + h]$$

$$d_k : t_0 = t_0^k < t_1^k < \dots < t_j^k < \dots < t_{n_k}^k = t_0 + h$$

$$\text{rank } d_k = \lambda_k = \max_{0 \leq j \leq n_k - 1} (t_{j+1}^k - t_j^k)$$

$$(3) \quad \lambda \xrightarrow{k \rightarrow +\infty} 0$$

$$(4) \quad \begin{cases} \phi_k(t_0) = x_0 \\ \phi_k(t) = \phi_k(t_j^k) + X(t_j^k, \phi_k(t_j^k))(t - t_j^k) \end{cases} \text{ - ломанные Эйлера}$$

$$t_j^k \leq t \leq t_{j+1}^k$$

рисунок 1

Лемма (1)

опред $\phi_k(t)$ и

$$|\phi_k(t) - x_0| \leq M(t - t_0) \quad \forall t \in P \quad (5)$$

Замечание

$$(5) \Rightarrow$$

$$t \in P \Rightarrow 0 \leq t - t_0 \leq h \Rightarrow$$

$$\Rightarrow |\phi_k(t) - x_0| \leq M \cdot h \leq M \cdot \frac{b}{M} = b \quad (6)$$

Док-во (лемма 1)

$$j = 0 \quad t \in [t_0^k, t_1^k]$$

$$\phi_k(t) = x_0 + X(t_0, x_0) \cdot (t - t_0)$$

$$\Rightarrow |\phi_k(t) - x_0| = \underbrace{|X(t_0, x_0)|}_{\leq M} (t - t_0) \leq M(t - t_0)$$

$$\text{Пусть (5)} \quad \text{Вын } \forall t \in [t_0^k, t_j^k]$$

$$\Rightarrow |\phi_k(t_j^k) - x_0| \leq M(t_j^k - t_0) \leq b \Rightarrow (t_j^k, \phi_k(t_j^k)) \in D$$

$$t_j^k \leq t < t_{j+1}^k$$

$$|\phi_k(t) - x_0| = \underbrace{|\phi_k(t_j^k) - x_0|}_{\text{инд. предн}} + |X(t_j^k, \phi_k(t_j^k))|(t - t_j^k) \leq$$

$$\leq M(t_j^k - t_0) + M(t - t_j^k) = M(t - t_0)$$

Определение

$$(7) \quad \begin{cases} \psi_k(t) = X(t_j^K, \phi_k(t^k)), & t_j^k \leq t \leq t_{j+1}^k \\ \phi_k(t_{nk}^k) = X(t_{nk}^k, \phi_k(t_{nk}^k)) \end{cases} \quad j = 0, n_k - 1$$

Лемма (2)

$$\phi_k(t) = x_0 + \int_{t_0}^t \psi_k(\tau) d\tau \quad (8)$$

Док-во

$$j = 0 \quad t \in [t_0^k, t_1^k]$$

$$\begin{aligned} \phi_k(t) &= x_0 + X(t_0, x_0)(t - t_0) \\ &= \int_{t_0}^t X(t_0, x_0) d\tau \\ &= \psi_k(t) \end{aligned}$$

$$t \in [t_0^k, t_j^k] \Rightarrow \phi_k(t_j^k) = x_0 + \int_{t_0}^{t_j^k} \psi_k(\tau) d\tau$$

$$t \in [t_j^k, t_{j+1}^k]$$

$$\begin{aligned} \Rightarrow \phi_k(t) &= \phi(t_j^k) + X(t_j^k, \phi_k(t_j^k))(t - t_j^k) = \\ &= x_0 + \int_{t_0}^{t_j^k} \psi_k(\tau) d\tau + \int_{t_j^k}^t X(t_j^k, \phi_k(t_j^k)) d\tau = \\ &= x_0 + \int_{t_0}^t \psi_k(\tau) d\tau \end{aligned}$$

Лемма (3)

$\{\phi_k(t)\}_{k=1}^\infty$ - равномерно огр, равностеп. непр.

$$t \in P$$

Док-во

$$|\phi_k(t)| \leq |\phi_k(t) - x_0| + |x_0| \leq b + |x_0| \quad \forall k \in \mathbb{N}$$

$$\mathcal{E} > 0 \quad \delta$$

$$|\bar{t} - \bar{\bar{t}}| < \delta \quad (\bar{t}, \bar{\bar{t}} \in P)$$

$$\begin{aligned} |\phi_k(\bar{t}) - \phi_k(\bar{\bar{t}})| &= \left| \int_{\bar{\bar{t}}}^{\bar{t}} \psi_k(\tau) d\tau \right| \leq \left| \int_{\bar{\bar{t}}}^{\bar{t}} |\psi_k(t)| d\tau \right| \leq \\ &\leq M\sigma = \mathcal{E} \end{aligned}$$

\exists подпослед $\{\phi_k(t)\}_1^\infty \quad t \in P$

$$(9) \quad \phi_k(t) \xrightarrow[k \rightarrow +\infty]{P} \phi(t)$$

$\phi(t)$ - непр и $|\phi(t) - x_0| \leq b$

Лемма (4)

$$(10) \quad \psi_k(t) \xrightarrow[k \rightarrow +\infty]{P} X(t, \phi(t))$$

Док-во (лемма 4)

$X(t, x) \in C(D) \Rightarrow X(t, x)$ - равном непр. на D

$\Rightarrow \forall \mathcal{E} > 0 \exists \delta > 0 : \forall (\bar{t}, \bar{x}), (\bar{\bar{t}}, \bar{\bar{x}}) \in D$

$|\bar{t} - \bar{\bar{t}}| < \delta, \quad |\bar{x} - \bar{\bar{x}}| < \delta \Rightarrow$

$\Rightarrow |X(\bar{t}, \bar{x}) - X(\bar{\bar{t}}, \bar{\bar{x}})| < \frac{\mathcal{E}}{2}$

фикси $\mathcal{E} > 0 \Rightarrow \exists \delta > 0$

$$(12) \quad |X(t, \phi(t)) - \psi_k(t)| \leq |X(t, \phi(t)) - X(t, \phi_k(t))| + |X(t, \phi_k(t)) - \phi_k(t)|$$

из (9) $\Rightarrow \exists k_1 : \forall k > k_1 \quad |\phi_k(t) - \phi(t)| < \delta \quad \forall t \in P$

$$\Rightarrow \underbrace{|\dots|}_{(1)} < \frac{\mathcal{E}}{2}$$

$$t = t_{nk}^k \Rightarrow \underbrace{|\dots|}_{(2)} = 0$$

$t \neq t_{nk}^k \rightarrow \exists j \in \{0, 1, \dots, n_k - 1\} : t \in [t_j^k, t_{j+1}^k)$

$$\underbrace{|\dots|}_2 = |X(t, \phi_k(t)) - X(t_j^k, \phi_k(t_j^k))|$$

$\exists k_2 : \forall k > k_2 \quad \lambda_k < \min(\delta, \frac{\delta}{M}) \quad (из (3))$

$\Rightarrow (t - t_j^k) < (t_{j+1}^k - t_j^k) \leq \lambda_k < \delta$

$$|\phi_k(t) - \phi_k(t_j^k)| \leq \underbrace{\left| \int_{t_j^k}^t \right|}_{\leq M} |\psi_k(t)| \leq \underbrace{M(t - t_j^k)}_{\leq \lambda_k} < M \frac{\delta}{M} = \delta$$

$$\Rightarrow |\underbrace{\dots}_{(2)}| < \frac{\mathcal{E}}{2}$$

$$\Rightarrow \forall k > \max(K_1, k_2) \quad |X(t, \phi(t)) - \psi_k(t)| < \frac{\mathcal{E}}{2} + \frac{\mathcal{E}}{2} = \mathcal{E}$$

$$\phi(t) = x_0 + \int_{t_0}^t X(\tau, \phi(\tau)) d\tau \quad (12)$$

$$t = t_0 : \phi(t_0) = x_0$$

$$\text{Дифф (13) : } \dot{\phi}(t) = X(t, \phi(t))$$

$$\Rightarrow \phi(t) - \text{реш. задачи Коши (1), (2)} \quad t \in P$$