8
Electrical evaluation of piezoelectric transducers

K. NAKAMURA, Tokyo Institute of Technology, Japan

Abstract: The characteristics of a piezoelectric transducer can be found by measuring the electrical impedance/admittance. The impedance of a transducer changes steeply around the mechanical resonance frequencies. The constants in an electrical equivalent model are estimated from electrical measurements. An electrical evaluation of a piezoelectric transducer is described. Constants in an equivalent circuit vary as the vibration amplitude and there is non-linearity due to high stress and high temperatures. It is difficult to measure the constants under high-vibration amplitudes. Transducer constants at a high-vibration amplitude using a transient state are evaluated. High-power characteristics of piezoelectric ceramics are demonstrated. A load test for high-power transducers is described.

Key words: piezoelectric transducer, admittance characteristics, electrical equivalent circuit, admittance loop, high-power characteristics, load test.

8.1 Introduction

The first thing we would want to find out after fabricating a transducer is its resonance frequency. The second would be the electro-mechanical coupling factor or the quality factor. These can be found through electrical measurements at the electrical port of the piezoelectric transducer. It is possible to determine the equivalent electrical circuit constants such as the equivalent mass, equivalent compliance and loss factor from the resonance curves of the impedance/admittance of the transducer. In order to understand a transducer in its entirety, of course, we need to measure the shape of the vibration mode as well as the absolute value of the vibration displacement amplitude. Measurements of vibration have been widely conducted using laser Doppler velocimetry in recent years, which is described in the next chapter in this book. The radiation pattern and waveform of the emitted sound field are also important for characterizing transducers for medical use and non-destructive testing. These are measured with a visualization technique such as the schlieren method or hydrophones. Optical methods for the evaluation of the radiated sound field are explained in Chapter 10. Hydrophones are described in detail in Chapter 19.

This chapter discusses the electrical evaluation of piezoelectric transducers. The relation between the electrical resonance curves and the equivalent electrical circuit is briefly explained. The circuit constants can be found from the electrical impedance/admittance characteristics near resonance. This electrical evaluation is

usually carried out under low-voltage conditions. However, measurements at higher voltage are sometimes required, since the circuit constants, especially the loss factor, vary with the vibration amplitude due to high strain or temperature changes. We often encounter difficulties in the measurements in such a high-voltage region, because the resonance curve becomes asymmetric due to non-linearity. To overcome this problem, a method utilizing transient responses is introduced and typical high-power characteristics of actual piezoelectric ceramics are shown. For estimating output acoustic power and the efficiency of high-power transducers, a load test using an electrical power meter is demonstrated in the last section.

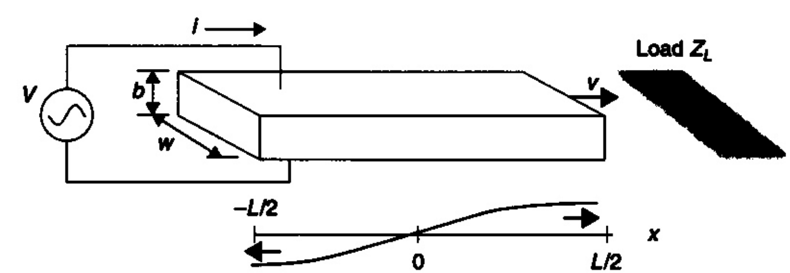
8.2 Equivalent electrical circuit

Figure 8.1 shows one of the simplest piezoelectric transducer configurations. A rectangular plate, with length L, width w and thickness b, vibrates in the fundamental longitudinal mode. The plate is polarized in the thickness direction, and both surfaces are metalized to act as electrodes. A sinusoidal voltage, with amplitude V, is applied between the electrodes, and the excitation electric field is in the thickness direction. The vibration direction is orthogonal to the electric field and the polarization, and its strength is given by the piezoelectric constant d_{31}. The angular frequency ω of the voltage source is chosen so that the length of the plate L is equal to the half the wavelength λ of the longitudinal wave. The longitudinal vibration velocity $v(x)$ is expressed as

$$v(x) = v \sin kx, \qquad [8.1]$$

using the vibration amplitude v at the end of the plate and the wave number k.

$$k = \omega_0 \sqrt{\frac{\rho}{E}}. \qquad [8.2]$$



8.1 Longitudinal transducer composed of a piezoelectric ceramic plate.

Here, ω_0, ρ and E are the resonance angular frequency, the density and Young's modulus of the plate, respectively. The resonance condition is

$$L=\frac{\pi}{\omega_0}\sqrt{\frac{E}{\rho}}. \qquad [8.3]$$

A mechanical load with an impedance z_L is assumed at the end of the plate.

The transducer working at or in the vicinity of the resonance can be modeled by an equivalent electrical circuit as shown in Fig. 8.2, where the mechanical resonance is denoted by a series connection of the equivalent mass l_m and the equivalent compliance c_m. The mechanical loss is represented by r_m. The mechanical resonance frequency is then determined as

$$\omega_0=\frac{1}{\sqrt{l_m c_m}}. \qquad [8.4]$$

The mechanical load z_L is connected in series to the resonance branch, and the current in the branch represents the vibration velocity v at the output end of the transducer. The kinetic energy in this circuit, $l_m v^2/2$, should be equal to the kinetic energy of the actual plate transducer. Then,

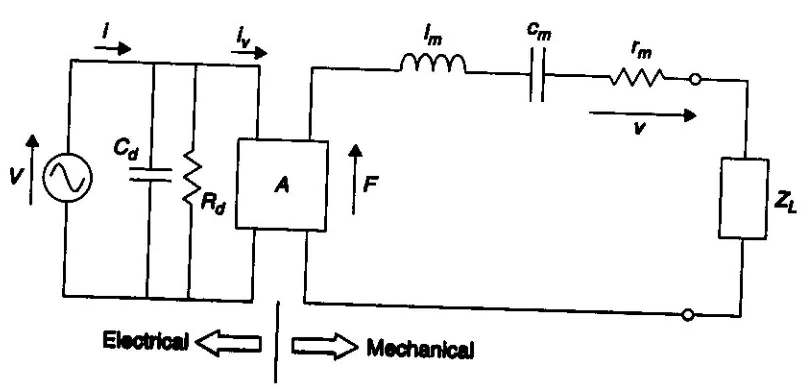
$$\frac{1}{2}l_m v^2=\int_{-L/2}^{L/2}\frac{1}{2}\rho b w v^2(x)\,dx. \qquad [8.5]$$

Evaluating the integral, the equivalent mass can be calculated as

$$l_m=\frac{\rho b w L}{2} \qquad [8.6]$$

Equation 8.6 shows that the equivalent mass is equal to half the total mass of the transducer. The equivalent compliance can be derived from Eqs 8.3, 8.4 and 8.6, or by comparing the reactive energy stored in c_m with the total elastic energy in the transducer.

The mechanical branch is connected to the electrical one through the force factor A. The capacitance of the plate C_d, which is called 'damped capacitance' or 'blocked capacitance', is connected to the electrical terminal in parallel with the



8.2 **Equivalent electrical circuit with lumped constants.**

dielectric loss R_d. For the vibration mode considered, the damped capacitance can be written simply as:

$$C_d=\frac{\varepsilon_{33}\varepsilon_0 L w}{b}, \qquad [8.7]$$

where ε_{33} and ε_0 are the permittivity of the material and the vacuum. The force factor is introduced as an analogy of an electrical transformer, relating the force F and the velocity v with the voltage V and the current i_v, respectively.

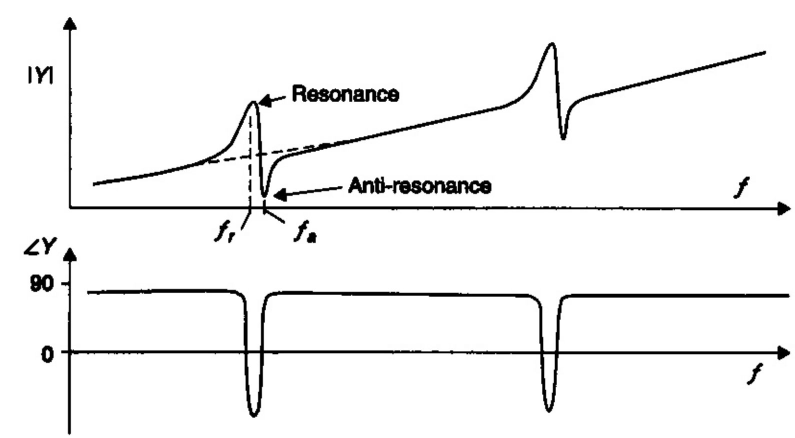
$$\begin{cases} Av=i_v \\ AV=F \end{cases} \qquad [8.8]$$

The motional current i_v is a virtual current proportional to the vibration velocity. The total current i, measured at the electrical terminal of the transducer, is assumed to be the sum of the motional current and the reactive current flowing to the damped capacitance C_d.

Different equivalent circuits should be defined for different resonance modes, since the circuit is valid only around the specific resonance frequency.

8.3 Electrical measurements

Figure 8.3 illustrates the typical frequency responses of the admittance $Y = i/V$ measured across the electrodes of the transducer. The amplitude $|Y|$ and phase angle $\angle Y$ of the admittance are separately displayed on the same frequency axis f ($=\omega/2\pi$). The amplitude is usually shown in log scale. Two resonances appear in this figure: the first is the fundamental mode while the second is the third mode. Even-order modes cannot be excited using the uniform electrodes shown in Fig. 8.1, and only the responses for odd-order modes are observed. We will focus on the fundamental resonance.



8.3 **Typical frequency responses of the electrical admittance of a piezoelectric transducer.**

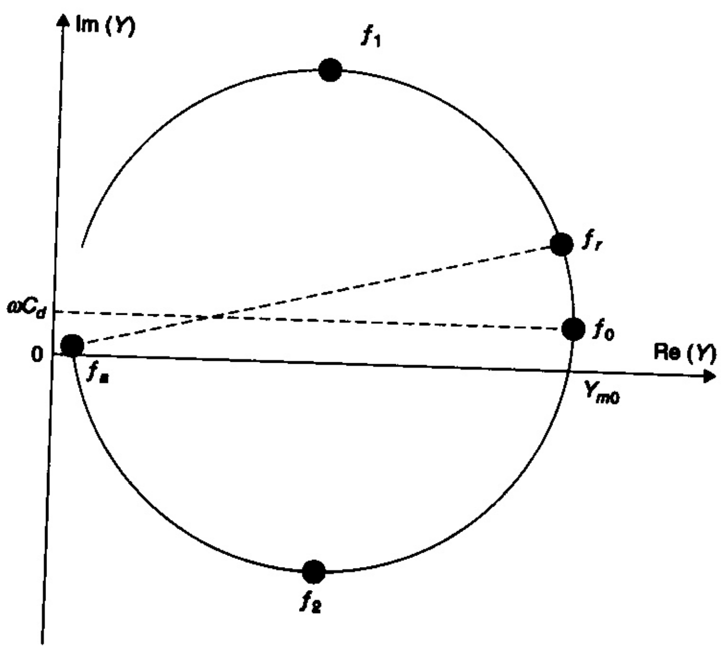
The admittance shows a peak at the resonance frequency, f_r, then it drops rapidly and has a minimum value at the so-called 'anti-resonance frequency', f_a. The resonance frequency observed here, f_r, is slightly lower than the mechanical resonance frequency, f_0 (= $\omega_0/2\pi$), given by Eq. 8.4. If the admittance near resonance is re-plotted in the complex plane, the trajectory by frequency has a circular shape, as shown in Fig. 8.4. This is sometimes called an 'admittance loop'. We will analyze the loop by comparing it with the simplified equivalent circuit illustrated in Fig. 8.5.

The mechanical components l_m, c_m and r_m are moved to the electrical branch by multiplying or dividing by the factor of A^2:

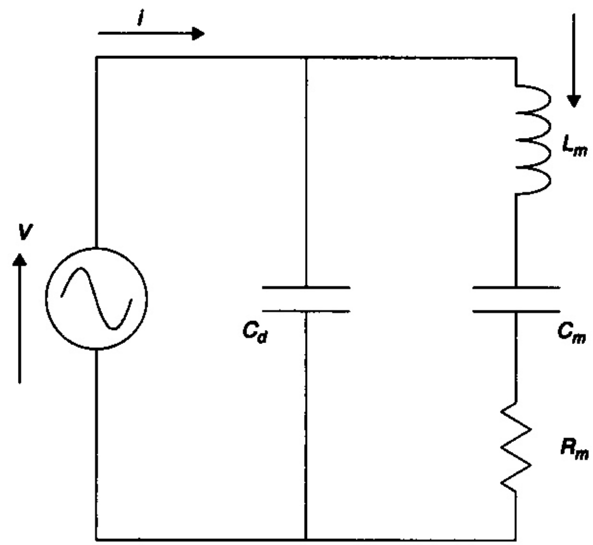
$$\begin{cases} R_m = r_m / A^2 \\ L_m = l_m / A^2 \\ C_m = A^2 c_m \end{cases} \qquad [8.9]$$

The dielectric loss is omitted for simplicity, while the mechanical load is removed ($z_L = 0$), assuming a free condition at the end of the transducer. The mechanical resonance frequency f_0 can be written as

$$f_0 = \frac{1}{2\pi\sqrt{L_m C_m}}. \qquad [8.10]$$



8.4 Admittance loop around the resonance frequency.



8.5 Simplified equivalent circuit model.

At this frequency, the real part of the admittance has a maximum as shown in Fig. 8.4. The diameter of the loop Y_{m0} is referred to as 'free motional admittance', and is equal to the inverse of the mechanical loss R_m. This means that we can find the value of R_m in the circuit from Y_{m0}:

$$R_m = 1/Y_{m0} \qquad [8.11]$$

Then, the quality factor, Q, of the resonance can be estimated from f_0, f_1 and f_2 as

$$Q = \frac{f_0}{f_2 - f_1}. \qquad [8.12]$$

Here, f_1 and f_2 are the frequencies where the imaginary part of the admittance is at the maximum and minimum values, respectively, and the amplitude of the admittance in the mechanical branch is $1/\sqrt{2}$ of that at the mechanical resonance frequency. Equation 8.12 is an approximate expression valid for large Q (>10). Most piezoelectric transducers have a high Q value ranging from 100 to over 1000, and Eq. 8.12 can be used in most cases. Using Q, the equivalent mass L_m is calculated as

$$L_m = \frac{QR_m}{2\pi f_0}. \qquad [8.13]$$

The compliance C_m is found from Eq. 8.10. When a mechanical load is applied to the mechanical output of the transducer, the diameter of the admittance loop becomes smaller.

If the admittance is measured at frequencies far from the resonance, the damped capacitance, C_d, and its loss can be evaluated. The electro-mechanical coupling factor, k_v^2, is expressed as the ratio of the compliance C_m to the electrical capacitance C_d, and is usually measured from the resonance and anti-resonance frequencies using the approximation

$$k_v^2 \approx \frac{2(f_a - f_r)}{f_a}. \qquad [8.14]$$

To find the mechanical values for the resonant components l_m, c_m and r_m, we need to find the force factor A through vibration velocity measurements using laser Doppler velocimetry and the simple relations shown in Eqs 8.8 and 8.9.

It is easy to calculate the force factor for a simple transducer, such as the plate transducer described here. The motional current is an integral of the flux density, D, over the entire electrode

$$i_v = j\,\omega \int_s D dS, \qquad [8.15]$$

where $j^2 = -1$. D is the product of the piezoelectric constant k_{31} and the longitudinal stress T. The stress is calculated from the gradient of the longitudinal displacement u:

$$T(x) = E\frac{\partial u}{\partial x} = \frac{\pi E v}{j\omega L}\cos\pi\frac{x}{L}. \qquad [8.16]$$

From Eqs 8.15 and 8.16, the force factor A is calculated as

$$A = 2wEd_{31} = 2wd_{31}/s_{11}, \qquad [8.17]$$

where s_{11} is the compliance of the material. The electro-mechanical coupling factor of the transducer, k_v^2, which can be estimated from C_m and C_d, is expressed using the coupling factor of the material k_{31}^2 as

$$k_v^2 = \frac{8}{\pi^2}k_{31}^2 \qquad [8.18]$$

This relation is generally used to evaluate the coupling factor of materials from the resonance measurement of a plate-shaped specimen.

Constants in the equivalent electrical circuit are evaluated through measurements of electrical admittance as a function of frequency around resonance as described in this section. Commercial impedance analyzers are widely used to carry out admittance measurements of transducers. A combination of a PC-controlled function generator and an A/D converter board is also a practical set-up for the measurements. There are two ways to measure the admittance: under constant voltage or under constant current. The former means that the vibration velocity changes with frequency, while the latter is a measurement under constant vibration velocity. The two give the same results when the vibration strain is limited to be under a small value. But, for measurements in the high-vibration strain region, the

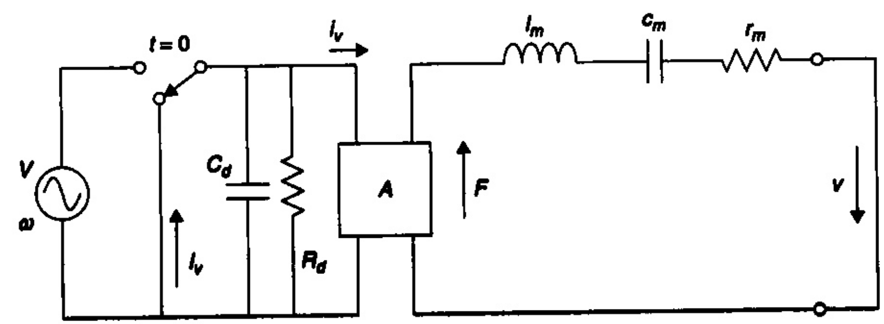
two methods exhibit different results since the circuit constants vary with the vibration amplitude due to non-linear effects.

8.4 Characterization of piezoelectric transducers under high-power operation

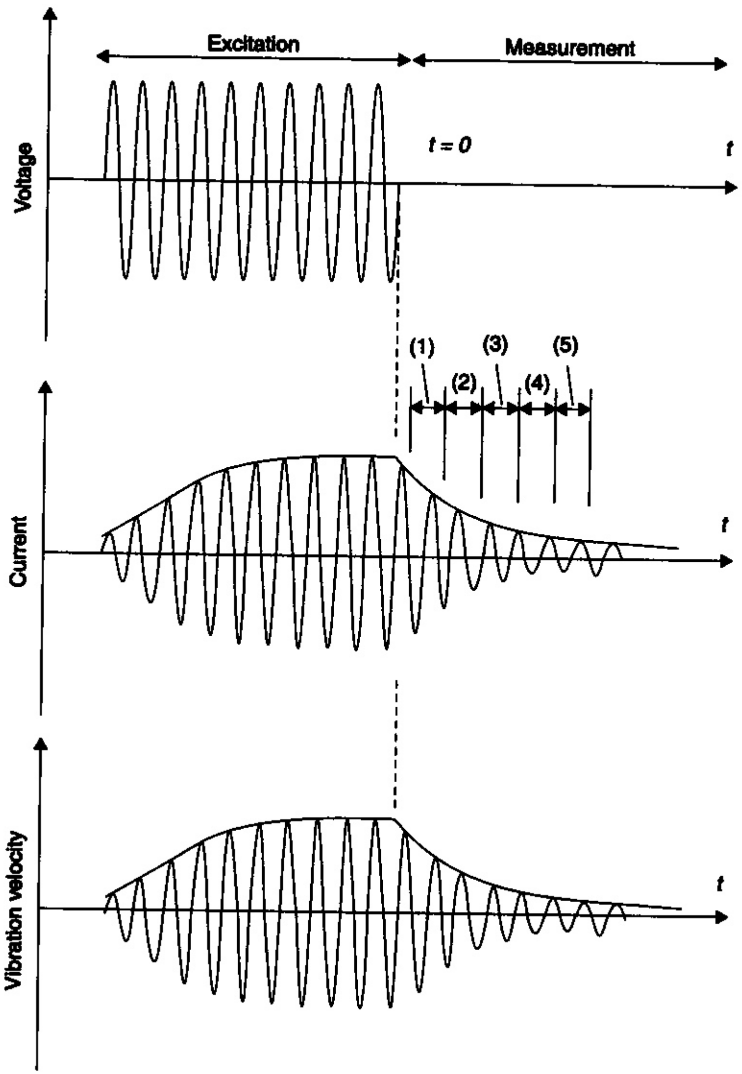
In general, the loss factor increases in the high-vibration amplitude region, and the increase in the loss causes a temperature rise. The temperature rise reduces the resonance frequency. In addition, the material is softened due to a high level of vibration strain. This also has the effect of lowering the resonance frequency. We sometimes encounter difficulties in making stable measurements of a resonance curve as described in the previous section. A computerized system is a powerful tool for measuring the admittance characteristics in the high-amplitude region. However, it is difficult to avoid the effects of the temperature rise. A piezoelectric ceramic transducer can be broken, in some cases, because of heating beyond the Curie temperature of the material.

In order to eliminate the effect of the temperature rise, the measurements should be completed in a short time. In this section, a transient response method (Umeda *et al.*, 1998) for making rapid measurements will be introduced.

The set-up for the measurement is shown in Fig. 8.6. A specimen transducer is excited by a continuous wave voltage near the resonance frequency, ω. After the vibration velocity has reached the required level, the electrical terminal is short circuited at $t = 0$. The short-circuited current, which is identical to the motional current, i_v, is measured and recorded for $t > 0$. The current shows a decay oscillation at the resonance frequency, ω_0, as illustrated in Fig. 8.7, and is proportional to the vibration velocity. The vibration velocity should be measured simultaneously. The duration for applying the voltage should be as short as possible to avoid temperature changes, and is less than 0.1 s for ordinary transducers. However, we should try to raise the velocity amplitude as much as possible.



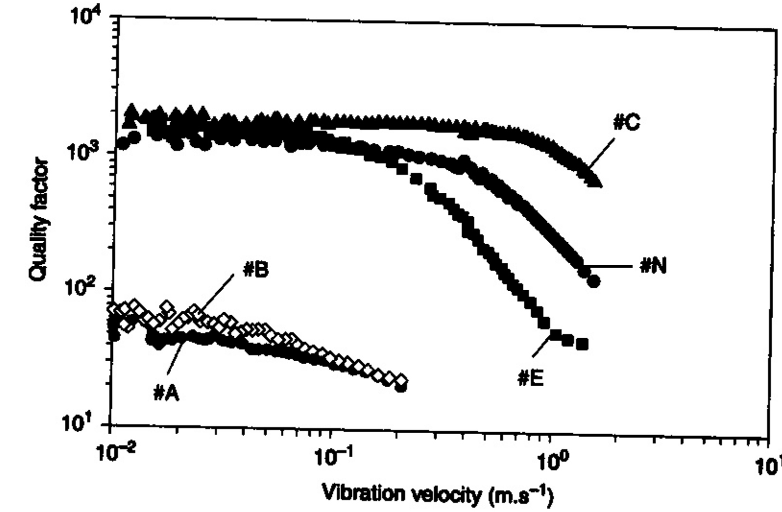
8.6 Transient method for measuring high-amplitude characteristics.



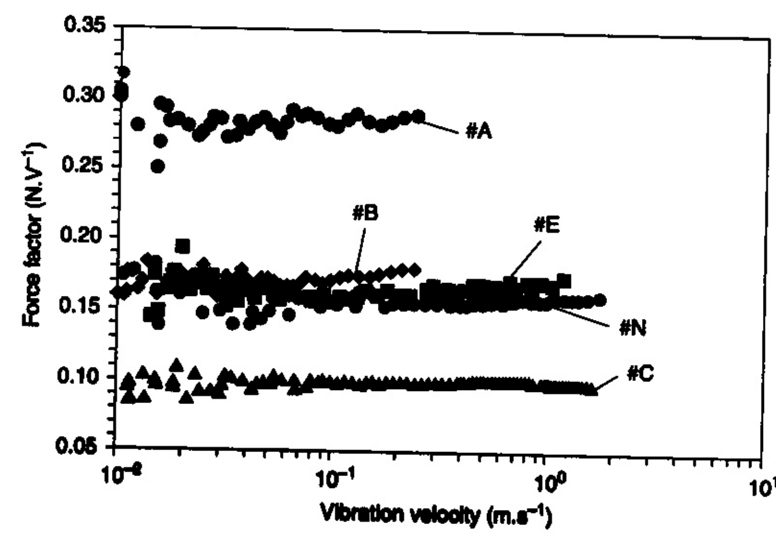
8.7 Waveforms observed in the transient method.

Carefully observe the decay oscillation of the current or the velocity by dividing it into many sections (1), (2), (3), etc., as indicated in Fig. 8.7, because the circuit constants vary with the vibration amplitude during the decay. For every section, the oscillation waveform is fitted by a curve $I_n \exp(-\alpha_n t)\sin(\omega_{0n} t + \phi_n)$. α_n and ω_{0n} are the attenuation constant and the angular resonance frequency for the nth section. The quality factor, Q_n, for the nth section can be determined as $Q_n = \omega_{0n}/(2\alpha_n)$. If we compare the current with the vibration velocity for every section, we can investigate the vibration amplitude dependence of the force factor, i.e. the

piezoelectric constant. Figures 8.8 and 8.9 demonstrate the difference between various kinds of PZT materials measured through the transient method (Umeda *et al.*, 1999). The quality factor and the force factor are plotted as functions of the vibration velocity for different PZTs: #A, #B, #E, #N and #C. #A and #B are soft PZTs, while #E, #N and #C are hard ones. Though the soft PZTs have a higher force factor (higher piezoelectric constant), the quality factor is low and decreases



8.8 Quality factor vs. vibration velocity.



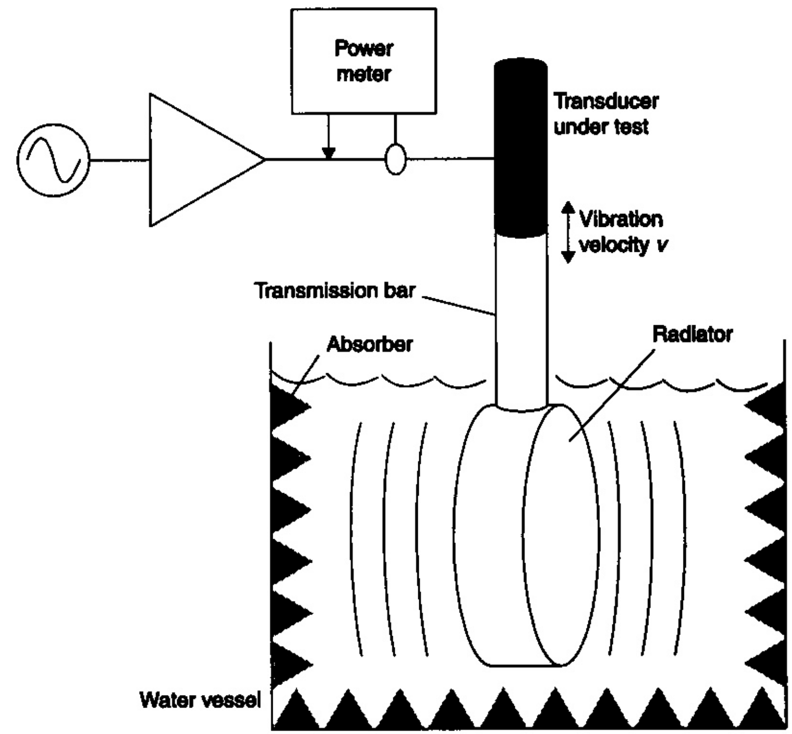
8.9 Force factor vs. vibration velocity.

rapidly with the vibration amplitude. On the other hand, the quality factor for the hard PZTs maintains a value of around 1000 even at $0.1\,m.s^{-1}$ or more. The hard PZT #C has a high quality factor in the high-vibration amplitude region up to $0.6\,m.s^{-1}$. This means that #C generates less heat in high-amplitude operation, and is suitable for high-power devices such as piezoelectric transformers, although its piezoelectric constant is rather low. An interesting point is the different behavior of the quality factors of #E and #N. The quality factor of #E has a higher value than #N at a low-vibration amplitude. However, #E drops more rapidly than #N, and #E shows worse characteristics at higher vibration velocity, over $0.2\,m.s^{-1}$. In general, the quality factor measured at low vibration velocities is indicated in the data sheets supplied by manufacturers. We should be careful when we choose PZT materials for high-power applications.

In the final part of this section, we will consider the maximum vibration velocity amplitude obtained at the end of a longitudinal transducer. The transducer will break mechanically at the center, if the vibration stress exceeds the fatigue limit. The maximum safety stress for extensional deformation ranges from 40 to 70 MPa for PZT ceramics. The vibration velocity at the end of the plate transducer is $1.7\,m.s^{-1}$ when the stress at the nodal point is 40 MPa. In practical continuous operation, the temperature increases quickly, even when the vibration amplitude is lower than this value, and thermal breakdown will occur faster than the mechanical breakdown. But we can conclude that the mechanical limit for the vibration velocity of a piezo-ceramic transducer is $1–2\,m.s^{-1}$. The kinetic energy stored in the transducer at these high amplitudes will reach a very high value. This high energy density of piezoelectric transducers attracted engineers, and was one of the motivating forces behind the development of high-power piezoelectric devices, such as ultrasonic motors and piezoelectric transformers.

8.5 Load test

The output acoustic power and efficiency of high-power transducers is measured by applying a mechanical load using an electrical power meter. Figure 8.10 illustrates a set-up for a load test. A large disk radiator is connected to the transducer under test via a transmission bar, and is immersed in water. Reflections from the water vessel's wall and bottom and the water surfaces should be minimized using an appropriate absorber. The radiator vibrates in an extensional mode, and both surfaces are loaded by the water. The input electrical power to the transducer, P_L, is measured and compared with the input power, P_0, under the no-load condition. The measurements should be carried out with the same vibration velocity under both the loaded and no-load conditions. The output acoustic power can be estimated from the difference $P_L - P_0$, since P_0 is wholly dissipated in the transducer under test. In general, a higher voltage is needed under the loaded condition to obtain the same vibration velocity as for the no-load case. The difference in the dielectric loss caused by the different voltages should



8.10 Transducer load test.

be compensated appropriately. The efficiency of the transducer is estimated from the ratio of the output acoustic power to the input electrical power, and reaches over 95% for the optimum load condition for Langevin transducers using PZT ceramics (Mori *et al.*, 1984).

This method can be used for different kinds of loads such as polymer specimens and other work pieces. Another transducer of the same type can be used as a dummy load, where the electrical port of the dummy transducer is terminated with an electrical resistor of the appropriate value. An inductor is often inserted in addition to the resistor in order to compensate the damped capacitance of the dummy load transducer.

8.6 Summary

The basic theory of the electrical evaluation of piezoelectric transducers has been explained in this chapter. The impedance/admittance measurements as functions of frequencies are easily carried out in the lower voltage region using a commercial impedance analyzer. The same measurements are possible with a combination of a computer-controlled signal generator and an A/D converter board. In the higher vibration amplitude region, both high strain and high temperature cause changes in the transducer characteristics; the loss factor especially increases rapidly as the vibration stress increases in ceramic transducers. The transient method is a powerful tool for high-power characterization of ultrasonic transducers. It should be noted that the quality factor given in the data sheets provided by manufacturers of piezo-ceramics is the value measured under low-vibration stress conditions. The loss factor increases dramatically in some piezoelectric ceramic materials at higher vibration stress. One should be careful when choosing piezoelectric materials for high-power applications.

A high-power load test is sometimes carried out using a water vessel or a dummy load transducer. The output acoustic power and the efficiency can be estimated by measuring the input electrical power.

8.7 References

Mori E, Ueha S, Tsuda Y, Kaneko S, Okada K, Masuda M and Ohoya K (1984) Load test of bolt-clamped Langevin type vibrator, *Jpn J Appl Phys*, 23, 209–11.

Umeda M, Nakamura K and Ueha S (1998) The measurement of high-power characteristics for a piezoelectric transducer based on the electrical transient response, *Jpn J Appl Phys*, 37, 5322–5.

Umeda M, Nakamura K and Ueha S (1999) Effects of vibration stress and temperature on the characteristics of piezoelectric ceramics under high vibration amplitude levels measured by electrical transient responses, *Jpn J Appl Phys*, 38, 5581–5.

9

Laser Doppler vibrometry for measuring vibration in ultrasonic transducers

M. JOHANSMANN and G. WIRTH, Polytec GmbH, Germany

Abstract: An experimental verification of an ultrasonic transducer or ultrasonic tool is an important design step. In this chapter we discuss laser Doppler vibrometry, which is an established experimental method for non-contact measurement of the transducer vibration amplitude and other properties. Basic measurements on a single test point are discussed as well as scanning entire surfaces and special technologies for ultra-high frequencies and for micro-structure devices.

Key words: laser Doppler vibrometry, scanning laser Doppler vibrometry, Mach–Zehnder interferometer, transducer characterization.

9.1 Introduction

To a certain extent numerical methods, such as finite element modelling (FEM), provide information about ultrasonic transducer performance and properties. Experimental verification and model updates of FEM data are frequently needed to achieve an accurate and reliable characterization of a transducer. Furthermore, an experiment allows a quick comparison of different transducer designs or materials.

Optical measurement methods are important because the testing of transducers and materials should be non-destructive and non-mass loading and should have a high spatial resolution. Laser Doppler vibrometry (LDV) is probably the most established optical method meeting these requirements.

Ultrasonic transducers and materials have an extremely wide range of designs and applications, ranging from conventional power transducers and ultrasonic wire-bonding machines to transducer designs for medical applications and surface wave transducers used in surface acoustic wave (SAW) devices. Because ultrasonic transducers cover such a wide field, some fundamental questions should be discussed before selecting the most appropriate measurement tool.

What is the frequency range of the transducer? Although many transducers start in the kilohertz range, medical applications require up to 100 MHz and the range of SAW filters may reach gigahertz.

Which velocity amplitudes are generated by the transducer? We will show later that any LDV has an upper velocity limit given by its optical design. For powerful transducers, especially at higher frequencies, these limits must be taken into account.

How is the vibration of the transducer composed? A Langevin transducer primarily generates a longitudinal out-of-plane (OOP) movement (displacement s_z, velocity v_z) but transversal in-plane (IP) components (s_x, s_y, v_x, v_y) may be present