ベクトル公式集

竹田航太

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1　ベクトルの公式

＊随時更新

Theorem 1.1. $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}^3$ とする．このとき以下が成り立つ．

(1) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$

(2) $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{b} \cdot \boldsymbol{b})\boldsymbol{a}$

(3) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) = 0$

(4) $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b}) = \det([\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}])$

Theorem 1.2. $\boldsymbol{f}, \boldsymbol{g} : \mathbb{R}^3 \to \mathbb{R}^3, \phi : \mathbb{R}^3 \to \mathbb{R}$ とする．このとき以下が成り立つ．

(1) $\nabla \cdot (\nabla \times \boldsymbol{f}) = 0$

(2) $\nabla \times (\nabla \phi) = \boldsymbol{o}$

(3) $\nabla(\boldsymbol{f} \cdot \boldsymbol{g}) = (\boldsymbol{f} \cdot \nabla)\boldsymbol{g} + (\boldsymbol{g} \cdot \nabla)\boldsymbol{f} + \boldsymbol{f} \times (\nabla \times \boldsymbol{g}) + \boldsymbol{g} \times (\nabla \times \boldsymbol{f})$

(4) $\nabla \times (\boldsymbol{f} \times \boldsymbol{g}) = \boldsymbol{f}(\nabla \cdot \boldsymbol{g}) - \boldsymbol{g}(\nabla \cdot \boldsymbol{f}) + (\boldsymbol{g} \cdot \nabla)\boldsymbol{f} - (\boldsymbol{f} \cdot \nabla)\boldsymbol{g}$

(5) $\frac{1}{2}\nabla(\boldsymbol{f} \cdot \boldsymbol{f}) = (u \cdot \nabla)\boldsymbol{f} + \boldsymbol{f} \times (\nabla \times \boldsymbol{f})$

(6) $\nabla \times (\nabla \times \boldsymbol{f}) = \nabla(\nabla \cdot \boldsymbol{f}) - (\nabla \cdot \nabla)\boldsymbol{f}$

参考文献

[1] J.E. Marsden Alexandre J. Chorin. *A mathematical introduction to fluid mechanics*, volume 3. Springer, 1993.