1) Obter $y_2(x)$ nos exercícios abaixo.

a) $x^2y''-5xy'+9y=0$, com $y_1(x)=x^2$

2) $y'' - 5y' + 6y = 0$

$y=e^{\lambda x} \Rightarrow \lambda^2-5\lambda+6=0 \Rightarrow \lambda=2$ ou $\lambda=3$

$y=C_1e^{2x}+C_2e^{3x}$

3) $y''' + 3y'' - 4y' - 12y = 0$

$y=e^{\lambda x} \Rightarrow \lambda^3+3\lambda^2-4\lambda-12=0$

$\lambda=2 \Rightarrow \lambda^2+5\lambda+6=0$

$\Rightarrow \lambda=-3$ e $\lambda=-2$

$\Rightarrow y=C_1e^{-2x}+C_2e^{2x}+C_3e^{3x}$

$\lambda^3+3\lambda^2-4\lambda-12 \;\big|\, \lambda-2$

$-(\lambda^3-2\lambda^2)$ $\qquad \lambda^2+5\lambda+6$

$5\lambda^2-4\lambda-12$

$-(5\lambda^2-10\lambda)$

$6\lambda-12$

4) $y'' + 2y' + 2y = 0$, com $y(0) = 1$ e $y(\pi/2) = 0$

$y=e^{\lambda x} \Rightarrow \lambda^2+2\lambda+2=0 \Rightarrow \lambda=\frac{-2\pm\sqrt{4-4\cdot2}}{2}=-1\pm\sqrt{-1}=-1\pm i$

$y=e^{-x\pm ix}=e^{-x}e^{\pm ix}=e^{-x}(\cos(\pm x)+i\,\mathrm{sen}(\pm x))$

$y=e^{-x}[C_1(\cos x+i\,\mathrm{sen}\,x)+C_2(\cos(-x)+i\,\mathrm{sen}(-x))]$

$y(0)=1 \Rightarrow C_1+C_2=1$

$y(\pi/2)=0 \Rightarrow C_1i-C_2i=0 \Rightarrow C_1=C_2$ $\Big\}\ C_1=C_2=1/2$

$\therefore\ y=\frac{e^{-x}}{2}(\cos x+i\,\mathrm{sen}\,x+\cos(-x)+i\,\mathrm{sen}(-x))=e^{-x}\cos x$

(cancelam; $\cos x=\cos(-x)$)

5) $y'' - 25y = 0$ com $y(0) = 0$ e $y'(0) = 20$

$y=e^{\lambda x} \Rightarrow \lambda^2-25\lambda=0 \Rightarrow \lambda=0$ ou $\lambda=25$

$y=C_1+C_2e^{25x} \Rightarrow 0=C_1+C_2 \Rightarrow C_1=-C_2$

$y'(0)=20 \Rightarrow 25C_2=20 \Rightarrow C_2=\frac{20}{25}=\frac{4}{5}$ $\Big\}\ y=-\frac{4}{5}+\frac{4}{5}e^{25x}$

6) $y'' - y' - 2y = 0$ com $y(0) = -4$ e $y'(0) = -17$

$y=e^{\lambda x} \Rightarrow \lambda^2-\lambda-2=0 \Rightarrow \lambda=\frac{1\pm\sqrt{1-4\cdot(-2)}}{2}=\frac{1\pm3}{2} \Rightarrow \lambda=2$ ou $\lambda=-1$

$y=C_1e^{2x}+C_2e^{-x}$

$-4 = C_1 + C_2$ $\left.\begin{matrix} \\ \end{matrix}\right\} -21 = 3C_1 \Rightarrow C_1 = -7$
$-17 = 2C_1 - C_2$

$C_2 = -4 - C_1 = -4 + 7 = 3$

$y = -7e^{2x} + 3e^{-x}$

7) $y'' - 9\pi^2 y = 0$

$y = e^{\lambda x} \Rightarrow \lambda^2 - 9\lambda\pi^2 = 0 \Rightarrow \lambda = 0$ ou $\lambda = 9\pi^2$

$y = C_1 + C_2 e^{9\pi^2 x}$

8) $9y'' + 6y' + y = 0$ com $y(0) = 4$ e $y'(0) = -\frac{13}{3}$

$y = e^{\lambda x}$

$9\lambda^2 + 6\lambda + 1 = 0 \Rightarrow \lambda = \frac{-6 \pm \sqrt{36 - 36}}{2 \cdot 9} = \frac{-6}{2 \cdot 9} = \frac{-3}{9} = -\frac{1}{3}$

$y = C_1 e^{-\frac{1}{3}x} + x C_2 e^{-\frac{1}{3}x}$

$y(0) = C_1 = 4$ $\quad \therefore y = e^{-1/3 x}(4 - 3x)$

$y' = -\frac{1}{3} e^{-\frac{1}{3}x}(C_1 + x C_2) + C_2 e^{-1/3 x}$

$y'(0) = -\frac{13}{3} = -\frac{1}{3} C_1 + C_2 \Rightarrow 13 = C_1 - 3C_2 \Rightarrow 13 = 4 - 3C_2 \Rightarrow C_2 = -3$

9) $y'' + 2ky' + k^2 y = 0$

$y = e^{\lambda x} \Rightarrow \lambda^2 + 2K\lambda + K^2 = 0 \Rightarrow \lambda = -K$

$y = C_1 e^{-Kx} + x C_2 e^{-Kx}$

10) $8y'' - 2y' - y = 0$ com $y(0) = 0{,}2$ e $y'(0) = 0{,}325$

$8\lambda^2 - 2\lambda - 1 = 0 \Rightarrow \lambda = \frac{2 \pm \sqrt{4 + 32}}{16} = \frac{2 \pm 6}{16} \Rightarrow \lambda = \frac{1}{2}$ ou $\lambda = -1/4$

$y = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{4}x} \Rightarrow C_1 + C_2 = 0{,}2$

$y' = \frac{1}{2} C_1 e^{\frac{1}{2}x} - \frac{1}{4} C_2 e^{-\frac{1}{4}x} \Rightarrow \frac{1}{2} C_1 - \frac{1}{4} C_2 = 0{,}325 \Rightarrow 2C_1 - C_2 = 1{,}3$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\} 3C_1 = 1{,}5 \Rightarrow C_1 = \frac{1}{2}$

$\Rightarrow C_2 = 0{,}2 - 0{,}5 = -0{,}3$

$y = \frac{1}{2} e^{\frac{1}{2}x} - \frac{3}{10} e^{-\frac{1}{4}x}$

12) $y'' - 7y' + 12y = 0$

$\lambda^2 - 7\lambda + 12 = 0 \Rightarrow \lambda = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm 1}{2} \Rightarrow \lambda = 3 \text{ en } \lambda = 4$

$y = C_1 e^{3x} + C_2 e^{4x}$

13) $y''' - 4y'' + 5y' = 0$

$\lambda^3 - 4\lambda^2 + 5\lambda = 0 \Rightarrow \lambda = 0$

$\lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda = \frac{4 \pm \sqrt{16-4\cdot 5}}{2} = 2 \pm \sqrt{4-5} = 2 \pm i$

$y = C_1 + C_2 \cdot e^{2+i} + C_3 \cdot e^{2-i}$

$y = C_1 + e^2 \left[C_2 \left(\cos(1) + i \operatorname{sen}(1) \right) + C_3 \left(\cos(-1) + i \operatorname{sen}(-1) \right) \right]$