Assignment 1- Probability and Random Variables

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Download all python codes from

https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-1/Assignment1.py

and latex-tikz codes from

https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-1/Assignment1.tex

1 Problem 4.9

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

2 Solution

When two fare dice are rolled. The sum of the numbers obtained can have the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Pr(X) = probability of obtaining X as the sum and let us represent the case when first dice shows the number x_1 and the second dice shows the number x_2 as (x_1, x_2).
Then,

Pr(X=2) = 1/36 : [(1,1)]
Pr(X=3) = 2/36 : [(1,2),(2,1)]
Pr(X=4) = 3/36 : [(1,3),(2,2),(3,1)]
Pr(X=5) = 4/36 : [(1,4),(2,3),(3,2),(4,1)]
Pr(X=6) = 5/36 : [(1,5),(2,4),(3,3),(4,2),(5,1)]
Pr(X=7) = 6/36 : [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
Pr(X=8) = 5/36 : [(2,6),(3,5),(4,4),(5,3),(6,2)]
Pr(X=9) = 4/36 : [(3,6),(4,5),(5,4),(6,3)]
Pr(X=10) = 3/36 : [(4,6),(5,5),(6,4)]
Pr(X=11) = 2/36 : [(5,6),(6,5)]
Pr(X=12) = 1/36 : [(6,6)]

The probability distribution table is

x	2	3	4	5	6	7	8	9	10	11	12
Pr(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\begin{aligned} Mean, E(X) &= \sum XPr(X) \\ &= \frac{1}{36}[(2\times1) + (3\times2) + (4\times3) + (5\times4) \\ &+ (6\times5) + (7\times6) + (8\times5) + (9\times4) \\ &+ (10\times3) + (11\times2) + (12\times1)] \\ &= \frac{252}{36} = 7 \end{aligned}$$

$$\begin{aligned} Variance, \sigma^2 &= E(X - E(X))^2 \\ &= E(X^2) - (E(X))^2 \\ &= \left(\sum X^2Pr(X)\right) - (E(X))^2 \\ &= \frac{1}{36}[(2^2\times1) + (3^2\times2) + (4^2\times3) + (5^2\times4) \\ &+ (6^2\times5) + (7^2\times6) + (8^2\times5) + (9^2\times4) \\ &+ (10^2\times3) + (11^2\times2) + (12^2\times1)] - (7)^2 \\ &= \frac{1974}{36} - 49 \\ &= \frac{1974 - 1764}{36} \\ &= \frac{210}{36} = \frac{35}{6} \end{aligned}$$

Therefore,

Standard Deviation, $\sigma = \sqrt{\frac{35}{6}}$