

Assignment 2- Probability and Random Variables

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Download all python codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/tree/main/Assignment-2/codes

and latex-tikz codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-2/Assignment2.tex

1 GATE PROBLEM : 76

Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases} \quad (1.0.1)$$

Then $E(X|Y = \frac{1}{2})$ is

2 SOLUTION

Given X and Y are two continuous random variables with joint probability density function,

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases} \quad (2.0.1)$$

We know that,

$$0 < x + y < 1 \implies 0 < y < 1 - x \text{ for } 0 < x < 1.$$

Then,

$$f_X(x) = \int f_{XY}(x, y) dy \quad (2.0.2)$$

$$= \int_0^{1-x} (2) dy \quad (2.0.3)$$

$$= 2(1 - x) \quad (2.0.4)$$

$$\implies f_X(x) = \begin{cases} 2(1 - x) & 0 \leq x < 1 \\ 0 & \text{otherwise.} \end{cases} \quad (2.0.5)$$

Similarly,

$$0 < x + y < 1 \implies 0 < x < 1 - y \text{ for } 0 < y < 1$$

Then,

$$f_Y(y) = \int f_{XY}(x, y) dx \quad (2.0.6)$$

$$= \int_0^{1-y} (2) dx \quad (2.0.7)$$

$$= 2(1 - y) \quad (2.0.8)$$

$$\implies f_Y(y) = \begin{cases} 2(1 - y) & 0 \leq y < 1 \\ 0 & \text{otherwise.} \end{cases} \quad (2.0.9)$$

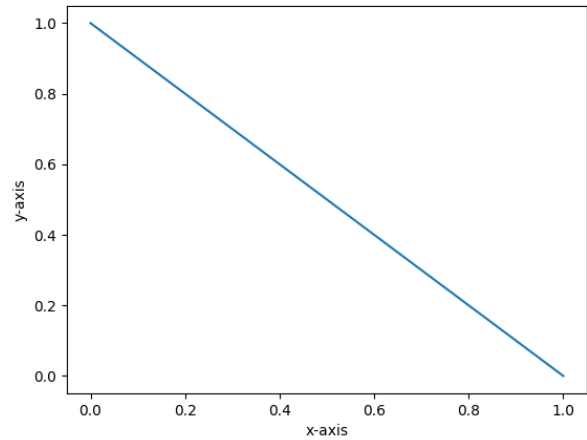


Fig. 0: Graph of $x+y=1$

Therefore ,

$$f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)} \quad (2.0.10)$$

$$= \begin{cases} \frac{2}{2(1-y)} & \text{if } 0 \leq x + y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.11)$$

Then,

$$E\left(X|Y = \frac{1}{2}\right) = \int_{-\infty}^{\infty} (x) \left(\frac{1}{1-y}\right) dx \quad (2.0.12)$$

$$= \frac{1}{1-y} \int_0^{1-y} (x) dx \quad (2.0.13)$$

$$= \frac{1}{1-y} \left[\frac{x^2}{2} \right]_0^{1-y} \quad (2.0.14)$$

$$= \frac{1-y}{2} \quad (2.0.15)$$

$$= \frac{1 - \frac{1}{2}}{2} \quad (2.0.16)$$

$$= \frac{1}{4} \quad (2.0.17)$$

$$\therefore E\left(X|Y = \frac{1}{2}\right) = \frac{1}{4} \quad (2.0.18)$$