

# Assignment 4- Probability and Random Variables

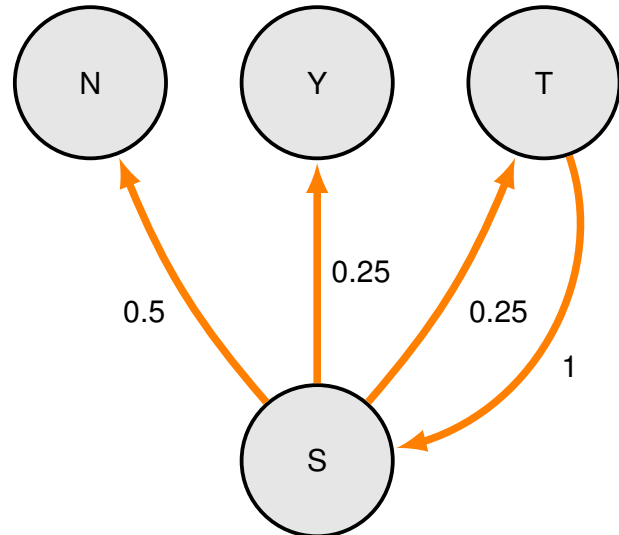
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Download all python codes from

[https://github.com/KotesSatvik/AI1103-Probability\\_and\\_Random\\_Variables/tree/main/Assignment-4/codes](https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/tree/main/Assignment-4/codes)

and latex-tikz codes from

[https://github.com/KotesSatvik/AI1103-Probability\\_and\\_Random\\_Variables/blob/main/Assignment-4/Assignment4.tex](https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-4/Assignment4.tex)



## 1 GATE 2016 (CS-SET 1) Q:29

Consider the following experiment.

**Step 1.** Flip a fair coin twice.

**Step 2.** If the outcomes are (TAILS, HEADS) then output Y and stop.

**Step 3.** If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

**Step 4.** If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (upto two decimal places)

## 2 SOLUTION

Given a fair coin is flipped twice. Let us represent the outcome as (x,y) where x represents the outcome in first throw and y represents the outcome in the second throw.

Let ,

S : Event of tossing a fair coin twice.

N : Event of obtaining N as the output.

Y : Event of obtaining Y as the output.

T : Event of obtaining (TAIL, TAIL) as the output.

Then,

$\Pr(N) = 2/4 : [(H,T), (H,H)]$

$\Pr(Y) = 1/4 : [(T,H)]$

$\Pr(T) = 1/4 : [(T,T)]$

**Given :** Initial state is 'S' ( $X_0$ )

**To find :** Probability of outcome being Y.

Therefore we have to find the probability of absorption in state Y.

Let us define,

$$a_i = \Pr(\text{absorption in Y} | X_0 = i) \quad (2.0.1)$$

Therefore by definition,

$$a_Y = 1 \quad (2.0.2)$$

$$a_N = 0 \quad (2.0.3)$$

$$a_T = a_S \quad (2.0.4)$$

$$a_S = \frac{1}{4}[a_Y + a_T] + \frac{1}{2}(a_N) \quad (2.0.5)$$

$$\Rightarrow a_S = \frac{1}{4}a_Y + \frac{1}{4}a_T \quad (2.0.6)$$

$$\Rightarrow a_S = \frac{1}{4}a_Y + \frac{1}{4}a_S \quad (2.0.7)$$

$$\Rightarrow \frac{3}{4}a_S = \frac{1}{4}a_Y \quad (2.0.8)$$

$$\therefore a_S = \frac{1}{3} \quad (2.0.9)$$

Therefore the probability of outcome being Y is 0.33 (rounded to two decimal places).