Assignment 2- Probability and Random Variables

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Download all python codes from

```
https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-2/codes
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and latex-tikz codes from

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https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-2/Assignment2.tex
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1 Gate Problem : 76

Let X and Y be two continuous random variables with joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.0.1}$$

Then $E(X|Y = \frac{1}{2})$ is

2 Solution

Given X and Y are two continuous random variables with joint probability density function,

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.1}$$

Then we know that

$$f_X(x) = \int_0^{1-x} 2dy \tag{2.0.2}$$

$$= 2(1 - x) \tag{2.0.3}$$

$$f_X(x) = \begin{cases} 2(1 - x) & 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.4}$$

Similarly ,

$$f_Y(y) = \begin{cases} 2(1 - y) & 0 \le y < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.5}$$

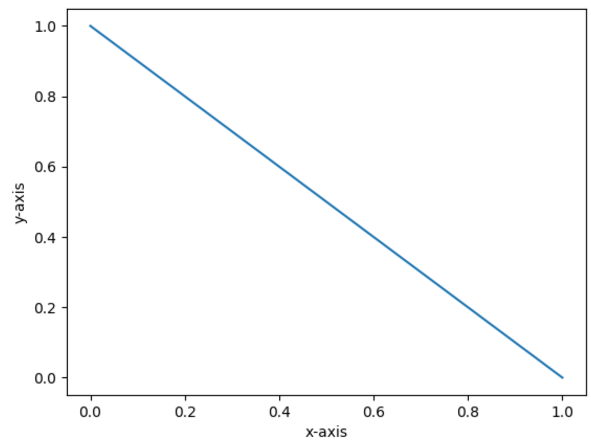

Fig. 0: Graph of x+y=1

Therefore ,

$$f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)} \tag{2.0.6}$$

$$= \begin{cases} \frac{2}{2(1-y)} & \text{if } 0 \le x + y < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.7}$$

Then,

$$E(X|Y = \frac{1}{2}) = \int_{-\infty}^{\infty} (x)(\frac{1}{1 - y})dx \tag{2.0.8}$$

$$= \frac{1}{1 - y} \int_0^{1-y} (x)dx \tag{2.0.9}$$

$$= \frac{1}{1 - y} \left[\frac{x^2}{2}\right]_0^{1-y} \tag{2.0.10}$$

$$= \frac{1 - y}{2} \tag{2.0.11}$$

$$= \frac{1 - \frac{1}{2}}{2} \tag{2.0.12}$$

Therefore (2.0.13)

$$E(X|Y = \frac{1}{2}) = \frac{1}{4} \tag{2.0.14}$$