# Assignment 3- Probability and Random Variables

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Download all python codes from

```
https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/tree/main/Assignment-3/codes
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and latex-tikz codes from

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https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-3/Assignment3.tex
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## 1 Gate 2010 (ma) Q:49

Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} ae^{-2y} & 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases} \tag{1.0.1}$$

Then $E(X|Y = 2)$ is

## 2 Solution

Given X and Y are two continuous random variables with joint probability density function,

$$f(x, y) = \begin{cases} ae^{-2y} & 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.1}$$

We know that,
$0 < x < y < \infty \implies x < y < \infty$ for $0 < x < \infty$.
Then,

$$f_X(x) = \int f_{XY}(x, y)\, dy \tag{2.0.2}$$
$$= \int_x^{\infty} ae^{-2y} dy \tag{2.0.3}$$
$$= \left[\frac{ae^{-2y}}{(-2)}\right]_x^{\infty} \tag{2.0.4}$$
$$= \frac{-a}{2}\left[e^{-2y}\right]_x^{\infty} \tag{2.0.5}$$
$$= \frac{-a}{2}[0 - e^{-2x}] \tag{2.0.6}$$
$$\implies f_X(x) = \begin{cases} \frac{a}{2}e^{-2x} & 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.7}$$

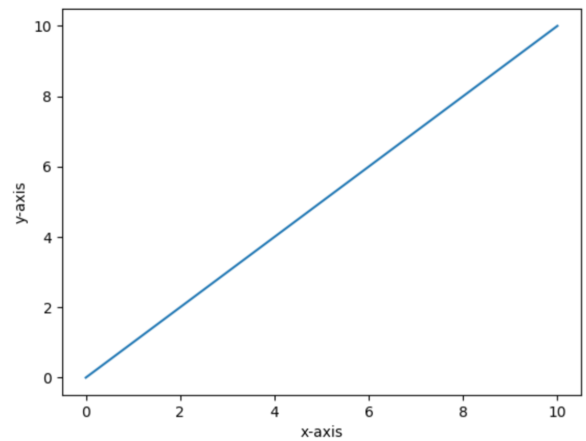


Fig. 0: Graph of x=y

Similarly,
$0 < x < y < \infty \implies 0 < x < y$ for $0 < y < \infty$
Then,

$$f_y(y) = \int f_{XY}(x, y)\, dx \tag{2.0.8}$$
$$= \int_0^{y} ae^{-2y} dx \tag{2.0.9}$$
$$= ae^{-2y}[x]_0^y \tag{2.0.10}$$
$$= aye^{-2y} \tag{2.0.11}$$
$$\implies f_Y(y) = \begin{cases} aye^{-2y} & 0 < y < \infty \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.12}$$

Therefore ,

$$f_{X|Y}(x|y) = \frac{f_{XY}(x, y)}{f_Y(y)} \tag{2.0.13}$$
$$= \frac{ae^{-2y}}{aye^{-2y}} \tag{2.0.14}$$
$$= \frac{1}{y} \tag{2.0.15}$$
$$\implies f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & \text{if } 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases} \tag{2.0.16}$$

Then,

$$
\begin{aligned}
E\left(X|Y=y\right) &= \int_{-\infty}^{\infty} (x) f_{X|Y}\left(x|y\right) dx && (2.0.17)\\
&= \int_{0}^{y} (x)\left(\frac{1}{y}\right) dx && (2.0.18)\\
&= \frac{1}{y}\int_{0}^{y} (x) dx && (2.0.19)\\
&= \frac{1}{y}\left[\frac{x^2}{2}\right]_0^y && (2.0.20)\\
&= \frac{1}{y}\left(\frac{y^2}{2}\right) && (2.0.21)\\
&= \frac{y}{2} && (2.0.22)\\
\implies E\left(X|Y=y\right) &= \frac{y}{2} && (2.0.23)\\
\therefore E\left(X|Y=2\right) &= 1 && (2.0.24)
\end{aligned}
$$