Assignment 4- Probability and Random Variables

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Download all python codes from

```
https://github.com/KoteshSatvik/AI1103-
    Probability_and_Random_Variables/tree/
    main/Assignment-4/codes
```

and latex-tikz codes from

```
https://github.com/KoteshSatvik/AI1103-
    Probability_and_Random_Variables/blob/
    main/Assignment-4/Assignment4.tex
```

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Consider the following experiment.
Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places)

2 Solution

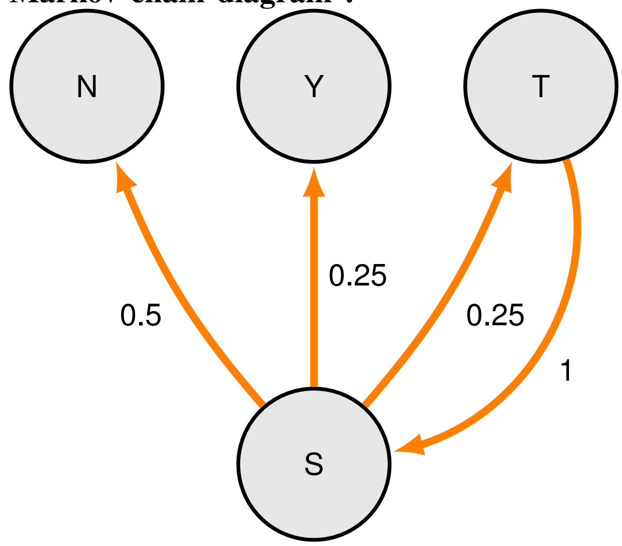
Given a fair coin is flipped twice. Let us represent the outcome as (x,y) where x represents the outcome in first throw and y represents the outcome in the second throw.

Variable	Event
S	Event of tossing a fair coin twice
N	Event of obtaining 'N' as the output
Y	Event of obtaining 'Y' as the output
T	Event of obtaining (TAIL,TAIL) as the output

TABLE 0: Variables representing different events

When a fair coin is tossed twice the output is,
'N' when the outcomes are (H,H), (H,T) and
'Y' when the outcome is (T,H).
Then,

Markov chain diagram :



Let : X_0 be defined as the initial state (at time 0).
Given : Initial state is 'S'(X_0)
To find : Probability of outcome being Y.
Therefore we have to find the probability of final state being state Y.
Let us define,

$$a_i = \Pr(\text{final state is Y}|X_0 = \text{i}) \tag{2.0.1}$$

Therefore by definition,

$$a_Y = 1 \tag{2.0.2}$$
$$a_N = 0 \tag{2.0.3}$$
$$a_T = a_S \tag{2.0.4}$$
$$a_S = \frac{1}{4}[a_Y + a_T] + \frac{1}{2}(a_N) \tag{2.0.5}$$
$$\implies a_S = \frac{1}{4}a_Y + \frac{1}{4}a_T \tag{2.0.6}$$
$$\implies a_S = \frac{1}{4}a_Y + \frac{1}{4}a_S \tag{2.0.7}$$
$$\implies \frac{3}{4}a_S = \frac{1}{4}a_Y \tag{2.0.8}$$
$$\therefore a_S = \frac{1}{3} \tag{2.0.9}$$

Therefore the probability of outcome being Y is 0.33 (rounded to two decimal places).