Assignment 2- Probability and Random Variables

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Download all python codes from

https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/tree/main/Assignment−2/codes

and latex-tikz codes from

https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/blob/main/Assignment−2/Assignment2.tex

1 Gate Problem : 76

Let X and Y be two continuous random variables with the joint probability density function

$f(x,y) = \begin{cases} 2 & 0 < x+y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$

Then $E(X|Y = \frac{1}{2})$ is

2 Solution

Given X and Y are two continuous random variables with joint probability density function,

$f(x,y) = \begin{cases} 2 & 0 < x+y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$

We know that,

$0 < x+y < 1 \implies 0 < y < 1-x$ for $0 < x < 1$.

Then,

$$f_X(x) = \int f_{XY}(x,y)dy \tag{2.0.1}$$

$$= \int_0^{1-x} (2)dy \tag{2.0.2}$$

$$= 2(1-x) \tag{2.0.3}$$

$$\implies f_X(x) = \begin{cases} 2(1-x) & 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.4}$$

Similarly,

$0 < x+y < 1 \implies 0 < x < 1-y$ for $0 < y < 1$

Then,

$$f_y(y) = \int f_{XY}(x,y)dx \tag{2.0.5}$$

$$= \int_0^{1-y} (2)dx \tag{2.0.6}$$

$$= 2(1-y) \tag{2.0.7}$$

$$\implies f_Y(y) = \begin{cases} 2(1-y) & 0 \le y < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2.0.8}$$

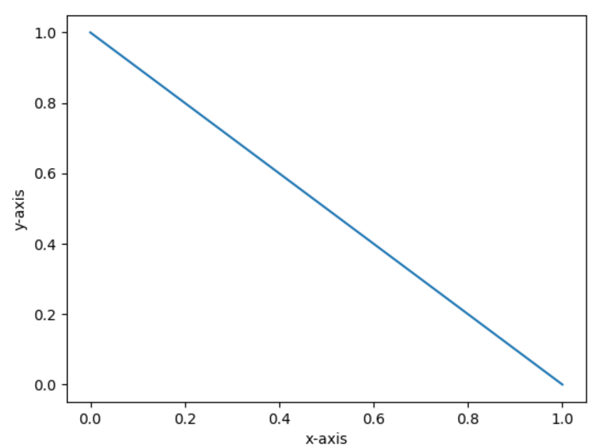

Fig. 0: Graph of x+y=1

Therefore ,

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} \tag{2.0.9}$$

$$= \begin{cases} \frac{2}{2(1-y)} & \text{if } 0 \le x+y < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2.0.10}$$

Then,

$$\begin{aligned}
E(X|Y = \frac{1}{2}) &= \int_{-\infty}^{\infty} (x)(\frac{1}{1-y})dx && (2.0.11)\\
&= \frac{1}{1-y}\int_{0}^{1-y} (x)dx && (2.0.12)\\
&= \frac{1}{1-y}\left[\frac{x^2}{2}\right]_0^{1-y} && (2.0.13)\\
&= \frac{1-y}{2} && (2.0.14)\\
&= \frac{1-\frac{1}{2}}{2} && (2.0.15)\\
&= \frac{1}{4} && (2.0.16)
\end{aligned}$$

Therefore,

$$E(X|Y = \frac{1}{2}) = \frac{1}{4}$$