Assignment 4- Probability and Random Variables

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Download all python codes from

https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/tree/main/Assignment-4/codes

and latex-tikz codes from

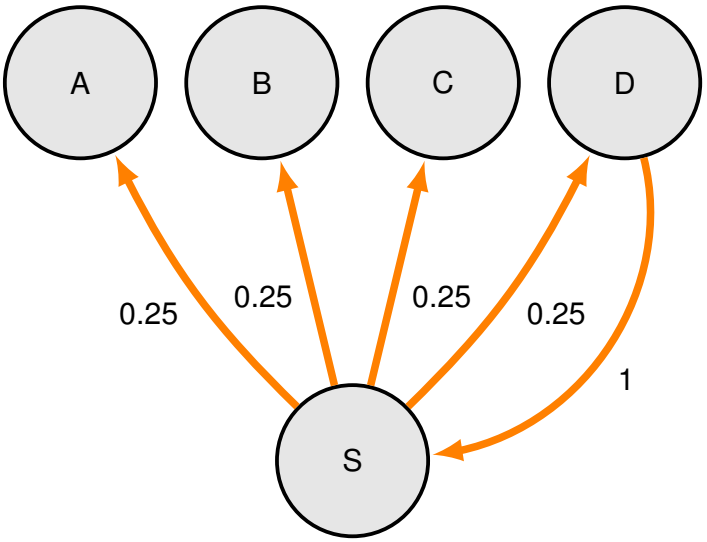
https://github.com/KoteshSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-4/Assignment4.tex

1 GATE 2016 (CS-SET 1) Q:29

Consider the following experiment.
Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places)

2 SOLUTION

Given a fair coin is flipped twice. Let us represent the outcome as (x,y) where x represents the outcome in first throw and y represents the outcome in the second throw.
Let ,
S : Event of tossing a fair coin twice.
A : Event of obtaining (Head, Head) as outcome.
B : Event of obtaining (Head, Tail) as outcome.
C : Event of obtaining (Tail, Head) as outcome.
D : Event of obtaining (Tail, Tail) as outcome.
Then the Markov chain diagram is,



Given : Initial state is 'S'(X_0)
To find : Probability of outcome being Y.
Therefore we have to find the probability of absorption in state C.
Let us define,

$$a_i = \Pr(\text{absorption in C}|X_0 = \text{i}) \tag{2.0.1}$$

Therefore by definition,

$$a_C = 1 \tag{2.0.2}$$
$$a_A = 0 \tag{2.0.3}$$
$$a_B = 0 \tag{2.0.4}$$
$$a_D = a_S \tag{2.0.5}$$
$$a_S = \frac{1}{4}[a_A + a_B + a_C + a_D] \tag{2.0.6}$$
$$\implies a_S = \frac{1}{4}a_C + \frac{1}{4}a_D \tag{2.0.7}$$
$$\implies a_S = \frac{1}{4}a_C + \frac{1}{4}a_S \tag{2.0.8}$$
$$\implies \frac{3}{4}a_S = \frac{1}{4}a_C \tag{2.0.9}$$
$$\implies a_S = \frac{1}{3} \tag{2.0.10}$$

$$\therefore \boxed{\Pr(\text{absorption in C}|X_0 = \text{S}) = \frac{1}{3}}$$

Therefore the probability of outcome being Y is 0.33 (rounded to two decimal places).