

Assignment 4- Probability and Random Variables

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Download all python codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/tree/main/Assignment-4/codes

and latex-tikz codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-4/Assignment4.tex

1 GATE 2016 (CS-SET 1) Q:29

Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (upto two decimal places)

2 SOLUTION

Given a fair coin is flipped twice. Let us represent the outcome as (x,y) where x represents the outcome in first throw and y represents the outcome in the second throw.

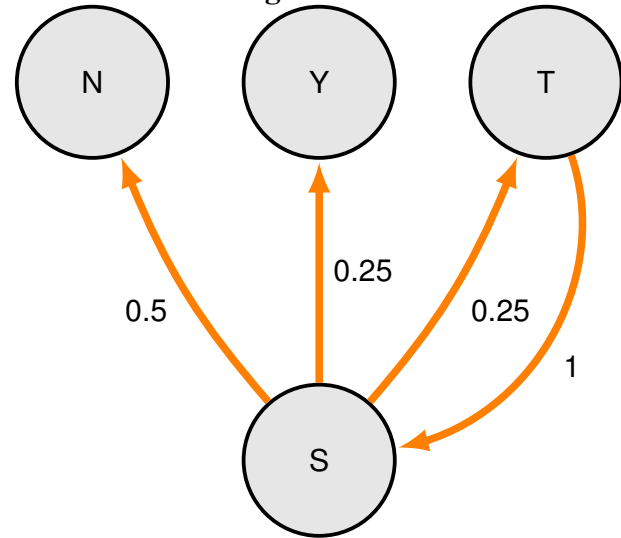
Variable	Event
S	Event of tossing a fair coin twice
N	Event of obtaining 'N' as the output
Y	Event of obtaining 'Y' as the output
T	Event of obtaining (TAIL,TAIL) as the output

TABLE 0: Variables representing different events

When a fair coin is tossed twice the output is, 'N' when the outcomes are (H,H), (H,T) and 'Y' when the outcome is (T,H).

Then,

Markov chain diagram :



Let : X_0 be defined as the initial state (at time 0).

Given : Initial state is 'S'(X_0)

To find : Probability of outcome being Y.

Therefore we have to find the probability of absorption in state Y.

Let us define,

$$a_i = \Pr(\text{final state is Y} | X_0 = i) \quad (2.0.1)$$

Therefore by definition,

$$a_Y = 1 \quad (2.0.2)$$

$$a_N = 0 \quad (2.0.3)$$

$$a_T = a_S \quad (2.0.4)$$

$$a_S = \frac{1}{4}[a_Y + a_T] + \frac{1}{2}(a_N) \quad (2.0.5)$$

$$\Rightarrow a_S = \frac{1}{4}a_Y + \frac{1}{4}a_T \quad (2.0.6)$$

$$\Rightarrow a_S = \frac{1}{4}a_Y + \frac{1}{4}a_S \quad (2.0.7)$$

$$\Rightarrow \frac{3}{4}a_S = \frac{1}{4}a_Y \quad (2.0.8)$$

$$\therefore a_S = \frac{1}{3} \quad (2.0.9)$$

Therefore the probability of outcome being Y is 0.33 (rounded to two decimal places).