Assignment 5- Probability and Random Variables

Songa Kotesh Satvik

Download all python codes from

https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/tree/main/Assignment−5/codes

and latex-tikz codes from

https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/blob/main/Assignment−5/Assignment5.tex

1 Gate 2015 (ee paper-1 new 2) Q.2(apt. section)

Given Set A = {2,3,4,5} and Set B = {11,12,13,14,15}, two numbers are randomly selected, one from each set. What is probability that the sum of the two numbers equals 16?

2 Solution

Let $X_1 \in \{2, 3, 4, 5\}$ and $X_2 \in \{11, 12, 13, 14, 15\}$ be the random variables such that X_1 represents the number chosen from set A and X_2 the number chosen from set B.
Then, the probability mass functions are

$$p_{X_1}(n) = \Pr(X_1 = n) = \begin{cases} \frac{1}{4} & 2 \le n \le 5 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

$$p_{X_2}(n) = \Pr(X_2 = n) = \begin{cases} \frac{1}{5} & 11 \le n \le 15 \\ 0 & otherwise \end{cases} \tag{2.0.2}$$

Let X be the random variable denoting the sum (X=X_1+X_2). Then, X can take the values $\{13, 14, 15, 16, 17, 18, 19, 20\}$.

$$\begin{aligned} p_X(n) &= \Pr(X_1 + X_2 = n) && (2.0.3) \\ &= \Pr(X_1 = n - X_2) && (2.0.4) \\ &= \Sigma_k \Pr(X_1 = n - k | X_2 = k)\, p_{X_2}(k) && (2.0.5) \end{aligned}$$

As X_1, X_2 are independent,

$$\Pr(X_1 = n - k | X_2 = k) = \Pr(X_1 = n - k) \tag{2.0.6}$$

from (2.0.5) and (2.0.6)

$$p_X(n) = \Sigma_k p_{X_1}(n-k) p_{X_2}(n) = p_{X_1}(n) * p_{X_2}(n) \tag{2.0.7}$$

where * denotes the convolution operator.
As,

$$\begin{aligned} p_X(n) &= \Sigma_k p_{X_1}(n-k) p_{X_2}(k) && (2.0.8) \\ &= \frac{1}{5} \sum_{k=11}^{15} p_{X_1}(n-k) && (2.0.9) \\ &= \frac{1}{5} \sum_{k=n-15}^{n-11} p_{X_1}(k) && (2.0.10) \end{aligned}$$

Since $p_{X_1}(k) = 0$ for $k < 2, k > 5$
Therefore, we get

$$p_x(n) = \begin{cases} 0 & n \le 12 \\ \frac{1}{5} \sum_{k=2}^{n-11} p_{X_1}(k) & 2 \le n - 11 \le 5 \\ \frac{1}{5} \sum_{k=n-15}^{5} p_{X_1}(k) & 2 \le n - 15 \le 5 \\ 0 & n > 20 \end{cases} \tag{2.0.11}$$

Therefore, from (2.0.1) we get

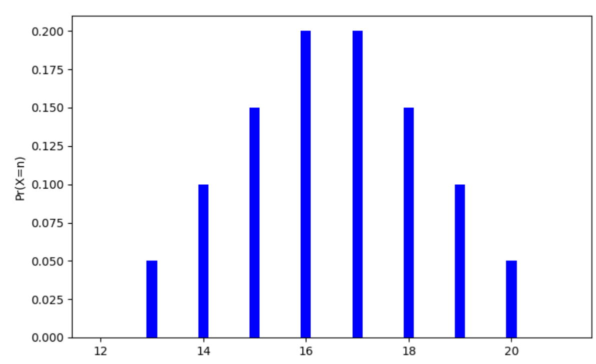

Fig. 0: Probability mass function of X

$$p_x(n) = \begin{cases} 0 & n \le 12 \\ \frac{n-12}{20} & 13 \le n \le 16 \\ \frac{21-n}{20} & 17 \le n \le 20 \\ 0 & n > 20 \end{cases} \tag{2.0.12}$$

Required probability is the probability of the sum of numbers selected from the sets, one from each set to be 16.
Therefore from (2.0.12),

$$p_X(16) = \left(\frac{16-12}{20}\right) \tag{2.0.13}$$

$$\implies p_X(16) = \frac{4}{20} \tag{2.0.14}$$

$$\implies \Pr\left(X_1 + X_2 = 16\right) = \frac{1}{5} \tag{2.0.15}$$

$$\therefore \Pr\left(X_1 + X_2 = 16\right) = 0.2 \tag{2.0.16}$$