Assignment 1- Probability and Random Variables

Songa Kotesh Satvik

Download all python codes from

```
https://github.com/KoteshSatvik/AI1103−
    Probability_and_Random_Variables/blob/
    main/Assignment−1/Assignment1.py
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and latex-tikz codes from

```
https://github.com/KoteshSatvik/AI1103−
    Probability_and_Random_Variables/blob/
    main/Assignment−1/Assignment1.tex
```

1 Problem 4.9

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

2 Solution

When two fare dice are rolled. The sum of the numbers obtained can have the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
$\Pr(X)$ = probability of obtaining X as the sum and let us represent the case when first dice shows the number x_1 and the second dice shows the number x_2 as (x_1, x_2).
Then,
$\Pr(X = 2)$= 1/36 : [(1,1)]
$\Pr(X = 3)$= 2/36 : [(1,2),(2,1)]
$\Pr(X = 4)$= 3/36 : [(1,3),(2,2),(3,1)]
$\Pr(X = 5)$= 4/36 : [(1,4),(2,3),(3,2),(4,1)]
$\Pr(X = 6)$= 5/36 : [(1,5),(2,4),(3,3),(4,2),(5,1)]
$\Pr(X = 7)$= 6/36 : [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
$\Pr(X = 8)$= 5/36 : [(2,6),(3,5),(4,4),(5,3),(6,2)]
$\Pr(X = 9)$= 4/36 : [(3,6),(4,5),(5,4),(6,3)]
$\Pr(X = 10)$= 3/36 : [(4,6),(5,5),(6,4)]
$\Pr(X = 11)$= 2/36 : [(5,6),(6,5)]
$\Pr(X = 12)$= 1/36 : [(6,6)]

x	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

TABLE 0: Probability Distribution Table of X

For the above problem,we know that.

$$p_x(n) = \begin{cases} 0 & \text{if } n \le 1, \\ \frac{n-1}{36} & \text{if } 2 \le n \le 7, \\ \frac{13-n}{36} & \text{if } 7 < n \le 12, \\ 0 & \text{if } n > 12. \end{cases} \tag{2.0.1}$$

$$Mean, E(X) \tag{2.0.2}$$

$$= \sum_{k=1}^{12} k p_x(k) \tag{2.0.3}$$

$$= \sum_{k=1}^{6} k \times \frac{1}{36}[k-1] + \sum_{k=7}^{12} k \times \frac{1}{36}[13-k] \tag{2.0.4}$$

$$= \frac{1}{36}\left[\sum_{k=1}^{6} k(k-1) + \sum_{k=7-6}^{12-6} (k+6) \times [13-(k+6)]\right] \tag{2.0.5}$$

$$= \frac{1}{36}\left[\sum_{k=1}^{6} k(k-1) + \sum_{k=1}^{6} (k+6) \times [13-(k+6)]\right] \tag{2.0.6}$$

$$= \frac{1}{36}\sum_{k=1}^{6} (k(k-1) + (k+6)(7-k)) \tag{2.0.7}$$

$$= \frac{1}{36}\sum_{k=1}^{6} \left((k^2-k) + (7k-k^2+42-6k)\right) \tag{2.0.8}$$

$$= \frac{1}{36}\sum_{k=1}^{6} (42) \tag{2.0.9}$$

$$= \frac{1}{36}[42 \times 6] \tag{2.0.10}$$

Therefore, Mean, E(X) = 7

$$Variance, \sigma^2 = E(X - E(X))^2 \tag{2.0.11}$$

$$= E(X^2) - (E(X))^2 \tag{2.0.12}$$

Let us consider $E(X^2)$,

$$E(X^2) \tag{2.0.13}$$
$$= \left(\sum_{k=1}^{12} k^2 p_x(k)\right) \tag{2.0.14}$$
$$= \sum_{k=1}^{6} k^2 \times \frac{1}{36}[k-1] + \sum_{k=7}^{12}(k)^2 \times [13-k] \tag{2.0.15}$$
$$\text{by rearrangement we get} \tag{2.0.16}$$
$$= \frac{1}{36}\left[\sum_{k=1}^{6} k^2(k-1) + \sum_{k=7-6}^{12-6}(k+6)^2 \times [13-(k+6)]\right] \tag{2.0.17}$$
$$= \frac{1}{36}\left[\sum_{k=1}^{6} k^2(k-1) + \sum_{k=1}^{6}(k+6)^2(7-k)\right] \tag{2.0.18}$$
$$= \frac{1}{36}\sum_{k=1}^{6}\left(k^2(k-1) + (k^2+36+12k)(7-k)\right) \tag{2.0.19}$$
$$= \frac{1}{36}\sum_{k=1}^{6}\left((k^3-k^2) + (-k^3-5k^2+48k+252)\right) \tag{2.0.20}$$
$$= \frac{1}{36}\sum_{k=1}^{6}(-6k^2+48k+252) \tag{2.0.21}$$
$$= \frac{1}{6}\sum_{k=1}^{6}(-k^2+8k+42) \tag{2.0.22}$$
$$= \frac{1}{6}\left[-\frac{(6)(7)(13)}{6} + 8 \times \frac{(6)(7)}{2} + (42)(6)\right] \tag{2.0.23}$$
$$= \frac{1}{6}[-91+168+252] \tag{2.0.24}$$
$$= \frac{329}{6} \tag{2.0.25}$$

$$\text{Variance, } \sigma^2 = E(X^2) - (E(X))^2 \tag{2.0.26}$$
$$= \frac{329}{6} - ((7)^2) \tag{2.0.27}$$
$$= \frac{329}{6} - 49 \tag{2.0.28}$$
$$\sigma^2 = \frac{35}{6} \tag{2.0.29}$$

Therefore,
Standard deviation, $\sigma = \sqrt{\frac{35}{6}}$