Assignment 1- Probability and Random Variables

Songa Kotesh Satvik

Download all python codes from

```
https://github.com/KoteshSatvik/AI1103−
    Probability_and_Random_Variables/blob/
    main/Assignment−1/Assignment1.py
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and latex-tikz codes from

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https://github.com/KoteshSatvik/AI1103−
    Probability_and_Random_Variables/blob/
    main/Assignment−1/Assignment1.tex
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1 Problem 4.9

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

2 Solution

When two fare dice are rolled. The sum of the numbers obtained can have the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Pr(X) = probability of obtaining X as the sum and let us represent the case when first dice shows the number x_1 and the second dice shows the number x_2 as (x_1, x_2).
Then,

Pr(X=2) = 1/36 : [(1,1)]
Pr(X=3) = 2/36 : [(1,2),(2,1)]
Pr(X=4) = 3/36 : [(1,3),(2,2),(3,1)]
Pr(X=5) = 4/36 : [(1,4),(2,3),(3,2),(4,1)]
Pr(X=6) = 5/36 : [(1,5),(2,4),(3,3),(4,2),(5,1)]
Pr(X=7) = 6/36 : [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]
Pr(X=8) = 5/36 : [(2,6),(3,5),(4,4),(5,3),(6,2)]
Pr(X=9) = 4/36 : [(3,6),(4,5),(5,4),(6,3)]
Pr(X=10) = 3/36 : [(4,6),(5,5),(6,4)]
Pr(X=11) = 2/36 : [(5,6),(6,5)]
Pr(X=12) = 1/36 : [(6,6)]

The probability distribution table is

x	2	3	4	5	6	7	8	9	10	11	12
Pr(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

For the above problem,we know that

$$p_x(n) = \begin{cases} 0 & \text{if } n < 1 \\ \frac{n-1}{36} & \text{if } 2 \leq n \leq 7 \\ \frac{13-n}{36} & \text{if } 7 < n \leq 12 \\ 0 & \text{if } n > 12 \end{cases}$$

$$\begin{aligned} Mean, E(X) &= \sum XPr(X) \\ &= \frac{1}{36}[(2\times 1) + (3\times 2) + (4\times 3) + (5\times 4) \\ &+ (6\times 5) + (7\times 6) + (8\times 5) + (9\times 4) \\ &+ (10\times 3) + (11\times 2) + (12\times 1)] \\ &= \frac{252}{36} = 7 \end{aligned}$$

$$\begin{aligned} Variance, \sigma^2 &= E(X - E(X))^2 \\ &= E(X^2) - (E(X))^2 \end{aligned}$$

Let us consider $E(X^2)$,

$$\begin{aligned} &E(X^2) \\ &= \left(\sum_{k=1}^{12} k^2 p_x(k)\right) \\ &= \sum_{k=1}^{6} k^2 \times \frac{1}{36}[k-1] + \sum_{k=7}^{12} k^2 \times \frac{1}{36}[13-k] \end{aligned}$$

by rearrangement we get

$$\begin{aligned} &= \frac{1}{36}\left[\sum_{k=1}^{6} k^2(k-1) + \sum_{k=7-6}^{12-6} (k+6)^2 \times [13-(k+6)]\right] \\ &= \frac{1}{36}\left[\sum_{k=1}^{6} k^2(k-1) + \sum_{k=1}^{6}(k+6)^2(7-k)\right] \\ &= \frac{1}{36}\sum_{k=1}^{6}\left(k^2(k-1) + (k^2+36+12k)(7-k)\right) \end{aligned}$$

$$= \frac{1}{36}\sum_{k=1}^{6}\Big((k^3 - k^2) + (-k^3 - 5k^2 + 48k + 252)\Big)$$
$$= \frac{1}{36}\sum_{k=1}^{6}(-6k^2 + 48k + 252)$$
$$= \frac{1}{6}\sum_{k=1}^{6}(-k^2 + 8k + 42)$$
$$= \frac{1}{6}\left[-\frac{(6)(7)(13)}{6} + 8 \times \frac{(6)(7)}{2} + (42)(6)\right]$$
$$= \frac{1}{6}[-91 + 168 + 252]$$
$$= \frac{329}{6}$$

$$\begin{aligned}\text{Variance, } \sigma^2 &= E(X^2) - (E(X))^2 \\ &= \frac{329}{6} - ((7)^2) \\ &= \frac{329}{6} - 49 \\ &= \frac{35}{6}\end{aligned}$$

Therefore,

Standard deviation, $\sigma = \sqrt{\frac{35}{6}}$