Assignment 4- Probability and Random Variables

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Download all python codes from

https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/tree/main/Assignment−4/codes

and latex-tikz codes from

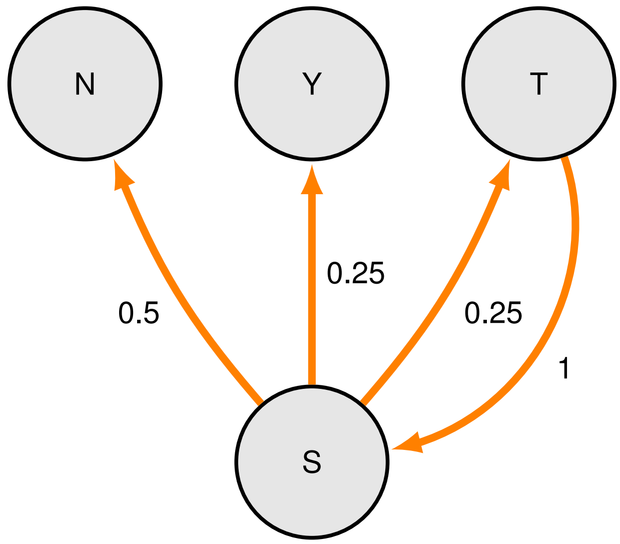
https://github.com/KoteshSatvik/AI1103−Probability_and_Random_Variables/blob/main/Assignment−4/Assignment4.tex

1 Gate 2016 (cs-set 1) Q:29

Consider the following experiment.
Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (upto two decimal places)

2 Solution

Given a fair coin is flipped twice. Let us represent the outcome as (x,y) where x represents the outcome in first throw and y represents the outcome in the second throw.
Let ,
S : Event of tossing a fair coin twice.
N : Event of obtaining N as the output.
Y : Event of obtaining Y as the output.
T : Event of obtaining (TAIL, TAIL) as the output.
Let us define the state of a Markov chain at time t is the value of X_t.
Then,
$\Pr(X_1 = N|X_0 = S) = 2/4$: [(H,T), (H,H)]
$\Pr(X_1 = Y|X_0 = S) = 1/4$: [(T,H)]
$\Pr(X_1 = T|X_0 = S) = 1/4$: [(T,T)]



Given : Initial state is 'S'(X_0)
To find : Probability of outcome being Y.
Therefore we have to find the probability of absorption in state Y.
Let us define,

$$a_i = \Pr(\text{absorption in Y}|X_0 = \text{i}) \tag{2.0.1}$$

Therefore by definition,

$$a_Y = 1 \tag{2.0.2}$$
$$a_N = 0 \tag{2.0.3}$$
$$a_T = a_S \tag{2.0.4}$$
$$a_S = \frac{1}{4}[a_Y + a_T] + \frac{1}{2}(a_N) \tag{2.0.5}$$
$$\implies a_S = \frac{1}{4}a_Y + \frac{1}{4}a_T \tag{2.0.6}$$
$$\implies a_S = \frac{1}{4}a_Y + \frac{1}{4}a_S \tag{2.0.7}$$
$$\implies \frac{3}{4}a_S = \frac{1}{4}a_Y \tag{2.0.8}$$
$$\implies a_S = \frac{1}{3} \tag{2.0.9}$$

$$\therefore \boxed{\Pr(\text{absorption in Y}|X_0 = \text{S}) = \frac{1}{3}}$$

Therefore the probability of outcome being Y is 0.33 (rounded to two decimal places).