

Assignment 1- Probability and Random Variables

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Download all python codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-1/Assignment1.py

and latex-tikz codes from

https://github.com/KotesSatvik/AI1103-Probability_and_Random_Variables/blob/main/Assignment-1/Assignment1.tex

1 PROBLEM 4.9

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X .

2 SOLUTION

When two fare dice are rolled. The sum of the numbers obtained can have the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

$\Pr(X)$ = probability of obtaining X as the sum and let us represent the case when first dice shows the number x_1 and the second dice shows the number x_2 as (x_1, x_2) .

Then,

$$\Pr(X = 2) = 1/36 : [(1,1)]$$

$$\Pr(X = 3) = 2/36 : [(1,2),(2,1)]$$

$$\Pr(X = 4) = 3/36 : [(1,3),(2,2),(3,1)]$$

$$\Pr(X = 5) = 4/36 : [(1,4),(2,3),(3,2),(4,1)]$$

$$\Pr(X = 6) = 5/36 : [(1,5),(2,4),(3,3),(4,2),(5,1)]$$

$$\Pr(X = 7) = 6/36 : [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]$$

$$\Pr(X = 8) = 5/36 : [(2,6),(3,5),(4,4),(5,3),(6,2)]$$

$$\Pr(X = 9) = 4/36 : [(3,6),(4,5),(5,4),(6,3)]$$

$$\Pr(X = 10) = 3/36 : [(4,6),(5,5),(6,4)]$$

$$\Pr(X = 11) = 2/36 : [(5,6),(6,5)]$$

$$\Pr(X = 12) = 1/36 : [(6,6)]$$

x	2	3	4	5	6	7	8	9	10	11	12
Pr(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

TABLE 0: Probability Distribution Table of X

For the above problem, we know that.

$$p_x(n) = \begin{cases} 0 & \text{if } n \leq 1, \\ \frac{n-1}{36} & \text{if } 2 \leq n \leq 7, \\ \frac{13-n}{36} & \text{if } 7 < n \leq 12, \\ 0 & \text{if } n > 12. \end{cases} \quad (2.0.1)$$

Mean, $E(X)$

$$= \sum_{k=1}^{12} (k \times p_x(k)) \quad (2.0.2)$$

$$= \sum_{k=1}^6 k \times \frac{1}{36} [k-1] + \sum_{k=7}^{12} k \times \frac{1}{36} [13-k] \quad (2.0.3)$$

$$= \frac{1}{36} \left[\sum_{k=1}^6 k(k-1) + \sum_{k=7}^{12-6} (k+6) \times [13 - (k+6)] \right] \quad (2.0.4)$$

$$= \frac{1}{36} \left[\sum_{k=1}^6 k(k-1) + \sum_{k=1}^6 (k+6) \times [13 - (k+6)] \right] \quad (2.0.5)$$

$$= \frac{1}{36} \sum_{k=1}^6 (k(k-1) + (k+6)(7-k)) \quad (2.0.6)$$

$$= \frac{1}{36} \sum_{k=1}^6 ((k^2 - k) + (7k - k^2 + 42 - 6k)) \quad (2.0.7)$$

$$= \frac{1}{36} \sum_{k=1}^6 (42) \quad (2.0.8)$$

$$= \frac{1}{36} [42 \times 6] \quad (2.0.9)$$

Therefore, Mean, $E(X) = 7$

$$\text{Variance, } \sigma^2 = E(X - E(X))^2 \quad (2.0.10)$$

$$= E(X^2) - (E(X))^2 \quad (2.0.11)$$

Let us consider $E(X^2)$,

$$E(X^2) = \left(\sum_{k=1}^{12} (k^2 \times p_x(k)) \right) \quad (2.0.12)$$

$$= \sum_{k=1}^6 k^2 \times \frac{1}{36} [k-1] + \sum_{k=7}^{12} (k)^2 \times [13-k] \quad (2.0.13)$$

by rearrangement we get (2.0.14)

$$= \frac{1}{36} \left[\sum_{k=1}^6 k^2(k-1) + \sum_{k=7-6}^{12-6} (k+6)^2 \times [13-(k+6)] \right] \quad (2.0.15)$$

$$= \frac{1}{36} \left[\sum_{k=1}^6 k^2(k-1) + \sum_{k=1}^6 (k+6)^2(7-k) \right] \quad (2.0.16)$$

$$= \frac{1}{36} \sum_{k=1}^6 (k^2(k-1) + (k^2 + 36 + 12k)(7-k)) \quad (2.0.17)$$

$$= \frac{1}{36} \sum_{k=1}^6 ((k^3 - k^2) + (-k^3 - 5k^2 + 48k + 252)) \quad (2.0.18)$$

$$= \frac{1}{36} \sum_{k=1}^6 (-6k^2 + 48k + 252) \quad (2.0.19)$$

$$= \frac{1}{6} \sum_{k=1}^6 (-k^2 + 8k + 42) \quad (2.0.20)$$

$$= \frac{1}{6} \left[-\frac{(6)(7)(13)}{6} + 8 \times \frac{(6)(7)}{2} + (42)(6) \right] \quad (2.0.21)$$

$$= \frac{1}{6} [-91 + 168 + 252] \quad (2.0.22)$$

$$= \frac{329}{6} \quad (2.0.23)$$

$$\text{Variance, } \sigma^2 = E(X^2) - (E(X))^2 \quad (2.0.24)$$

$$= \frac{329}{6} - ((7)^2) \quad (2.0.25)$$

$$= \frac{329}{6} - 49 \quad (2.0.26)$$

$$\sigma^2 = \frac{35}{6} \quad (2.0.27)$$

Therefore,

Standard deviation, $\sigma = \sqrt{\frac{35}{6}}$