

# Uninformed Search

CS271P, Fall Quarter, 2018  
Introduction to Artificial Intelligence  
Prof. Richard Lathrop

Read Beforehand: R&N 3.4



# Uninformed search strategies

- Uninformed (blind):
  - You have no clue whether one non-goal state is better than any other. Your search is blind. You don't know if your current exploration is likely to be fruitful.
- Various blind strategies:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Iterative deepening search (generally preferred)
  - Bidirectional search (preferred if applicable)

# Basic graph/tree search scheme

- We have 3 kinds of states:
  - [only for graph search: explored (past states; = closed list) ]
  - frontier (current nodes; = open list, fringe, queue) [nodes now on the queue]
  - unexplored (future nodes) [implicitly given]
- Initially, frontier = MakeNode( start state)
- Loop until solution is found or state space is exhausted
  - pick/remove first node from queue/frontier/fringe/open using search strategy
  - if node is a goal then return node
  - [only for graph search: add node to explored/closed]
  - expand this node, add children to frontier only if not already in frontier
    - [only for graph search: add children only if their state is not in explored/closed list]
- Question:
  - what if a better path is found to a node already in frontier or on explored list?

# Search strategy evaluation

- A search **strategy** is defined by **the order of node expansion**
- Strategies are evaluated along the following dimensions:
  - **completeness**: does it always find a solution if one exists?
  - **time complexity**: number of nodes generated
  - **space complexity**: maximum number of nodes in memory
  - **optimality**: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - $b$ : maximum branching factor of the search tree (always finite)
  - $d$ : depth of the least-cost solution
  - $m$ : maximum depth of the state space (may be  $\infty$ )
  - (for UCS:  $C^*$ : true cost to optimal goal;  $\epsilon > 0$ : minimum step cost)

# Uninformed search design choices

- Queue for Frontier:
  - FIFO? LIFO? Priority? If Priority, what sort function?
- Goal-Test:
  - Do goal-test when node inserted into *Frontier*?
  - Do goal-test when node removed?
- Tree Search, or Graph Search:
  - Forget *Expanded* (or *Explored*, *Closed*, Fig. 3.7) nodes?
    - = Tree Search: Smaller memory cost, but larger search time
  - Or remember them?
    - = Graph Search: Smaller search time, but larger memory cost
  - Classic space/time computational tradeoff

# Queue for Frontier

- FIFO (First In, First Out)
  - Results in Breadth-First Search
- LIFO (Last In, First Out)
  - Results in Depth-First Search
- Priority Queue sorted by path cost so far
  - Results in Uniform Cost Search
- Iterative Deepening Search uses Depth-First
- Bidirectional Search can use either Breadth-First or Uniform Cost Search

# When to do goal test? (General)

- Do Goal-Test when node is popped from queue:  
IF you care about finding the optimal path  
AND your search space may have both short expensive and long cheap paths to a goal.
  - Guard against a short expensive goal.
  - E.g., Uniform Cost search with variable step costs.
- Otherwise, do Goal-Test when is node generated and inserted.
  - Usually, most of the search cost goes into creating the children (storage allocation, data structure creation, etc.), while the goal-test is usually fast and light-weight (am I in Bucharest? even the complicated 'check-mate?' goal-test in chess usually is fast because it does little or no storage allocation or data structure creation).
  - So most efficient search does goal-test as soon as nodes are generated.
- **REASON ABOUT your search space & problem.**
  - How could I possibly find a non-optimal goal?

# When to do Goal-Test? (Summary)

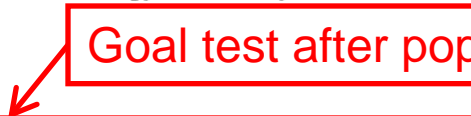
- For BFS, the goal test is done when the child node is generated.
  - Not an optimal search in the general case.
- For DLS, IDS, and DFS as in Fig. 3.17, goal test is done in the recursive call.
  - Result is that children are generated then iterated over. For each child DLS, is called recursively, goal-test is done first in the callee, and the process repeats.
  - More efficient search goal-tests children as generated. We follow your text.
- For DFS as in Fig. 3.7, goal test is done when node is popped.
  - Search behavior depends on how the LIFO queue is implemented.
- For UCS and A\*(next lecture), goal test when node removed from queue.
  - This avoids finding a short expensive path before a long cheap path.
- Bidirectional search can use either BFS or UCS.
  - Goal-test is search fringe intersection, see additional complications below
- For GBFS (next lecture) the behavior is the same either way
  - $h(\text{goal})=0$  so any goal will be at the front of the queue anyway.



# General tree search (R&N Fig. 3.7)

## Do not remember visited nodes

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```



---

```
function EXPAND(node, problem) returns a set of nodes
  successors ← the empty set
  for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
    s ← a new NODE
    PARENT-NODE[s] ← node; ACTION[s] ← action; STATE[s] ← result
    PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
    DEPTH[s] ← DEPTH[node] + 1
    add s to successors
  return successors
```

# General graph search (R&N Fig. 3.7)

## Do remember visited nodes

**function** GRAPH-SEARCH(*problem*, *fringe*) **returns** a solution, or failure

*closed* ← an empty set

*fringe* ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)

**loop do**

**if** *fringe* is empty **then return** failure

*node* ← REMOVE-FRONT(*fringe*)

**if** GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)

**if** STATE[*node*] is not in *closed* **then**

add STATE[*node*] to *closed*

*fringe* ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

Goal test after pop

These three statements change tree search to graph search.

# Tree-Search vs. Graph-Search

- Example : Assemble 5 objects {a, b, c, d, e}
- A state is a bit-vector (length 5), 1=object in assembly, 0= not in assembly
  - 11010 = a=1, b=1, c=0, d=1, e=0
  - $\Rightarrow$  a, b, d in assembly; c, e not in assembly
- State space:
  - Number of states =  $2^5 = 32$
  - Number of undirected edges =  $(2^5) \cdot 5 \cdot \frac{1}{2} = 80$
- Tree search space:
  - **Number of nodes** = number of paths =  $5! = 120$
  - States can be reached in multiple ways
    - 11010 can be reached by a+b+d or by a+d+b or by ... etc.
  - Often requires much more time, but much less space, than graph search
- Graph search space:
  - **Number of nodes** =  $\text{choose}(5,0) + \text{choose}(5,1) + \text{choose}(5,2) + \text{choose}(5,3) + \text{choose}(5,4) + \text{choose}(5,5) = 1 + 5 + 10 + 10 + 5 + 1 = 32$
  - States are reached in only one way, redundant paths are pruned
    - Question: What if a better path is found to a state that already has been explored?
  - Often requires much more space, but much less time, than tree search

# Checking for identical nodes (1)

Check if a node is already in fringe-frontier

- It is “easy” to check if a node is already in the fringe/frontier (recall fringe = frontier = open = queue)
  - Keep a hash table holding all fringe/frontier nodes
    - Hash size is same  $O(\cdot)$  as priority queue, so hash does not increase overall space  $O(\cdot)$
    - Hash time is  $O(1)$ , so hash does not increase overall time  $O(\cdot)$
  - When a node is expanded, remove it from hash table (it is no longer in the fringe/frontier)
  - For each resulting child of the expanded node:
    - If child is not in hash table, add it to queue (fringe) and hash table
    - Else if an old lower- or equal-cost node is in hash, discard the new higher- or equal-cost child
    - Else remove and discard the old higher-cost node from queue and hash, and add the new lower-cost child to queue and hash

Always do this for tree or graph search in BFS, UCS, GBFS, and A\*

# Checking for identical nodes (2)

Check if a node is in explored/expanded

- It is memory-intensive [  $O(b^d)$  or  $O(b^m)$  ] to check if a node is in explored/expanded (recall explored = expanded = closed)
  - Keep a hash table holding all explored/expanded nodes (hash table may be HUGE!!)
- When a node is expanded, add it to hash (explored)
- For each resulting child of the expanded node:
  - If child is not in hash table or in fringe/frontier, then add it to the queue (fringe/frontier) and process normally (BFS normal processing differs from UCS normal processing, but the ideas behind checking a node for being in explored/expanded are the same).
  - Else discard any redundant node.

Always do this for graph search

# Checking for identical nodes (3)

## Quick check for search being in a loop

- It is “moderately easy” to check for the search being in a loop
  - When a node is expanded, for each child:
    - Trace back through parent pointers from child to root
    - If an ancestor state is identical to the child, search is looping
      - Discard child and fail on that branch
    - Time complexity of child loop check is  $O(\text{depth}(\text{child}))$
    - Memory consumption is zero
      - Assuming good garbage collection
- Does NOT solve the general problem of repeated nodes — only the specific problem of looping
- For quizzes and exams, we will follow your textbook and NOT perform this loop check

# Breadth-first graph search (R&N Fig. 3.11)

**function** BREADTH-FIRST-SEARCH(**problem**) **returns** a solution, or failure

node  $\leftarrow$  a node with STATE = **problem**.INITIAL-STATE, PATH-COST = 0 **if**  
problem.GOAL-TEST(node.STATE) **then return** SOLUTION(node) **frontier**  $\leftarrow$   
a FIFO queue with node as the only element  
explored  $\leftarrow$  an empty set

**loop do**

**if** EMPTY?( **frontier**) **then return** failure

node  $\leftarrow$  POP(**frontier**) /\* chooses the shallowest node in **frontier** \*/

add node.STATE to explored

**for each action in** problem.ACTIONS(node.STATE) **do**

child  $\leftarrow$  CHILD-NODE(problem, node, action)

**if** child.STATE is not in explored or **frontier** **then**

**if** problem.GOAL-TEST(child.STATE) **then return** SOLUTION(child)

**frontier**  $\leftarrow$  INSERT(child, **frontier**)

Goal test before push

Avoid  
redundant  
frontier nodes

**Figure 3.11** Breadth-first search on a graph.

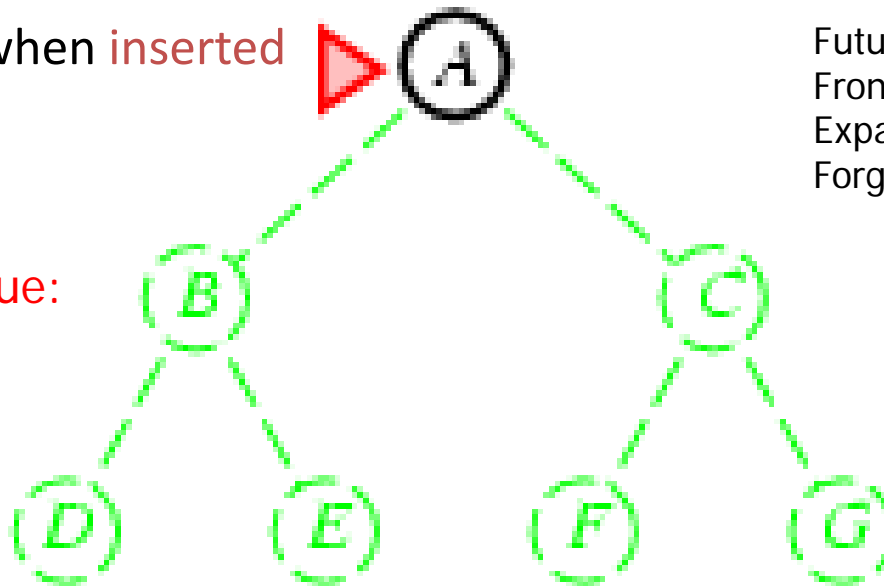
These three statements change tree search to graph search.

# Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or **OPEN**
- **Implementation:**
  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test when **inserted**

Initial state = A  
Is A a goal state?

Put A at end of queue:  
Frontier = [A]



Future= green dotted circles  
Frontier=white nodes  
Expanded/active=gray nodes  
Forgotten/reclaimed= black nodes

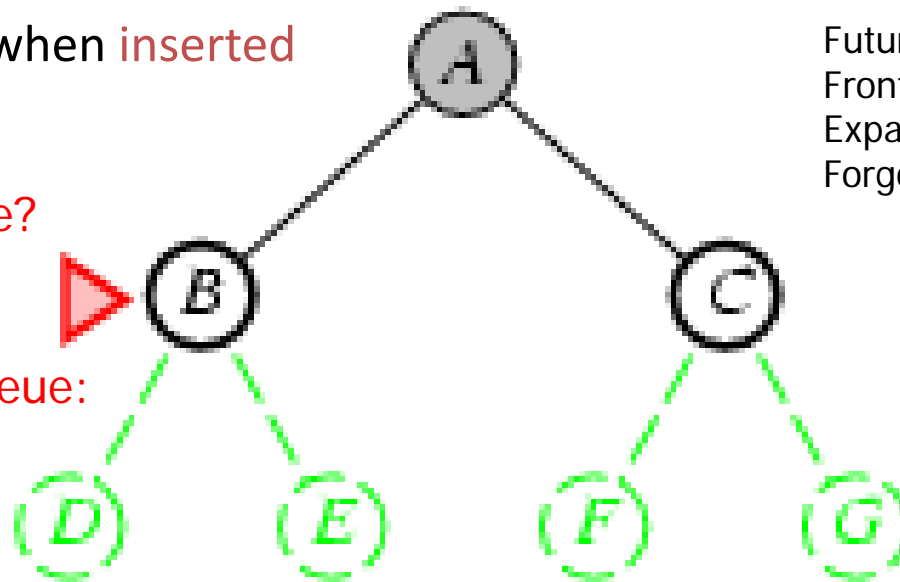


# Breadth-first search

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- **Implementation:**
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  - Goal test when **inserted**

Expand A to B,C  
Is B or C a goal state?

Put B,C at end of queue:  
Frontier = [B,C]



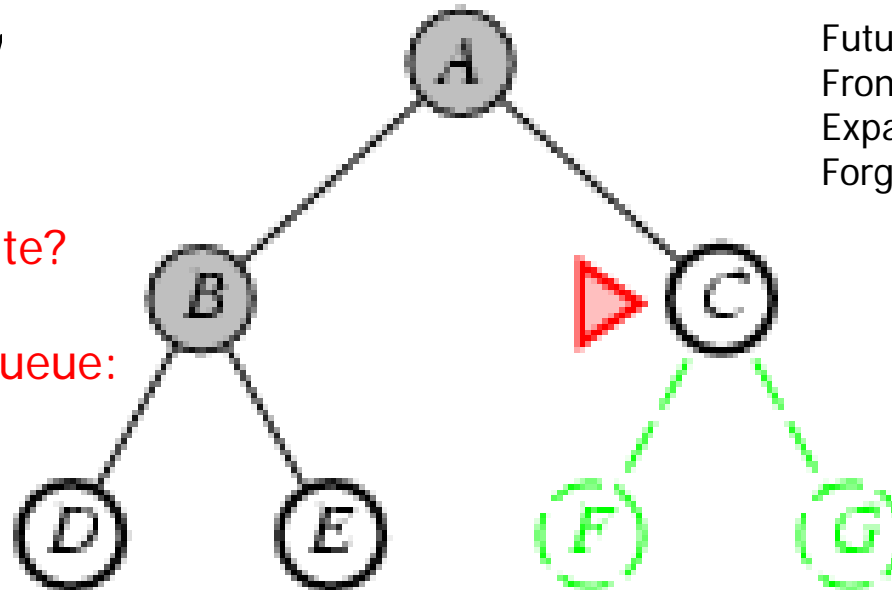
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# Breadth-first search

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  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test v

Expand B to D,E  
Is D or E a goal state?

Put D,E at end of queue:  
Frontier = [C,D,E]



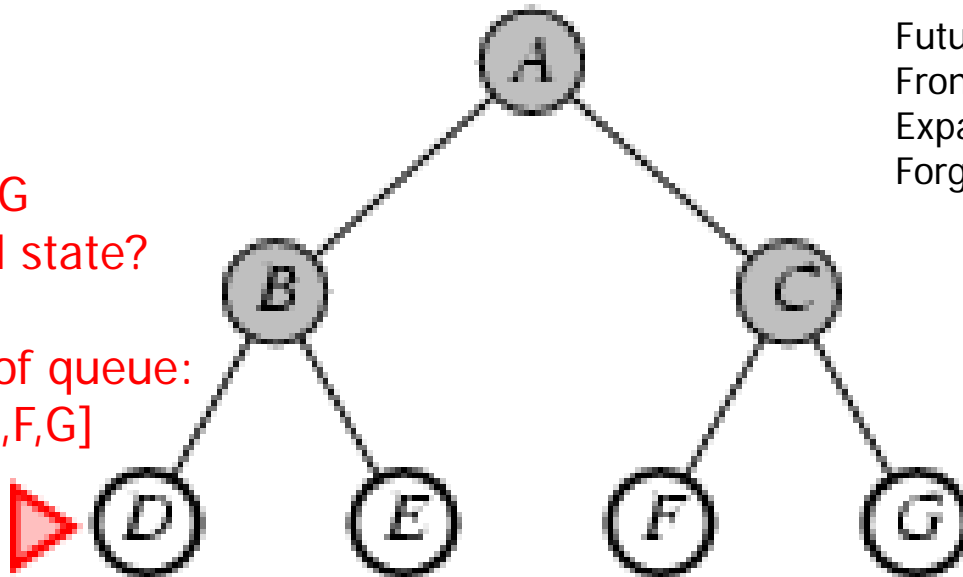
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# Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or **OPEN**
- **Implementation:**
  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test

Expand C to F, G  
Is F or G a goal state?

Put F,G at end of queue:  
Frontier = [D,E,F,G]

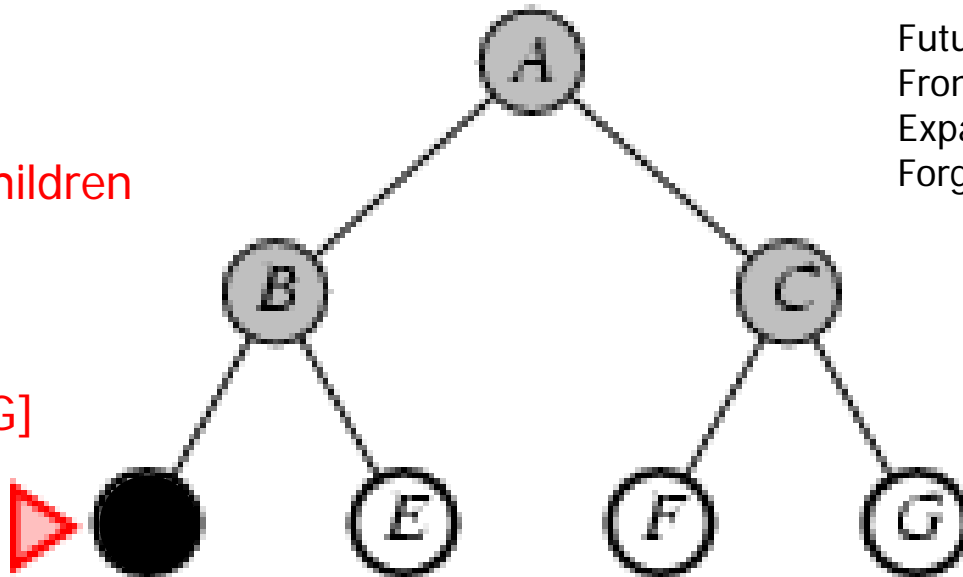


# Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or **OPEN**
- **Implementation:**
  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test

Expand D; no children  
Forget D

Frontier = [E,F,G]



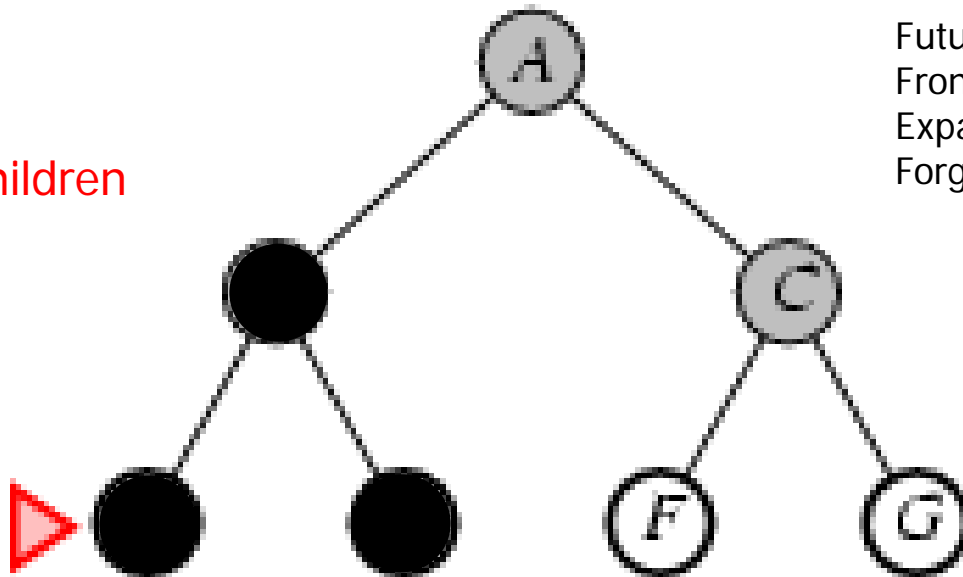
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# Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or **OPEN**
- **Implementation:**
  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test

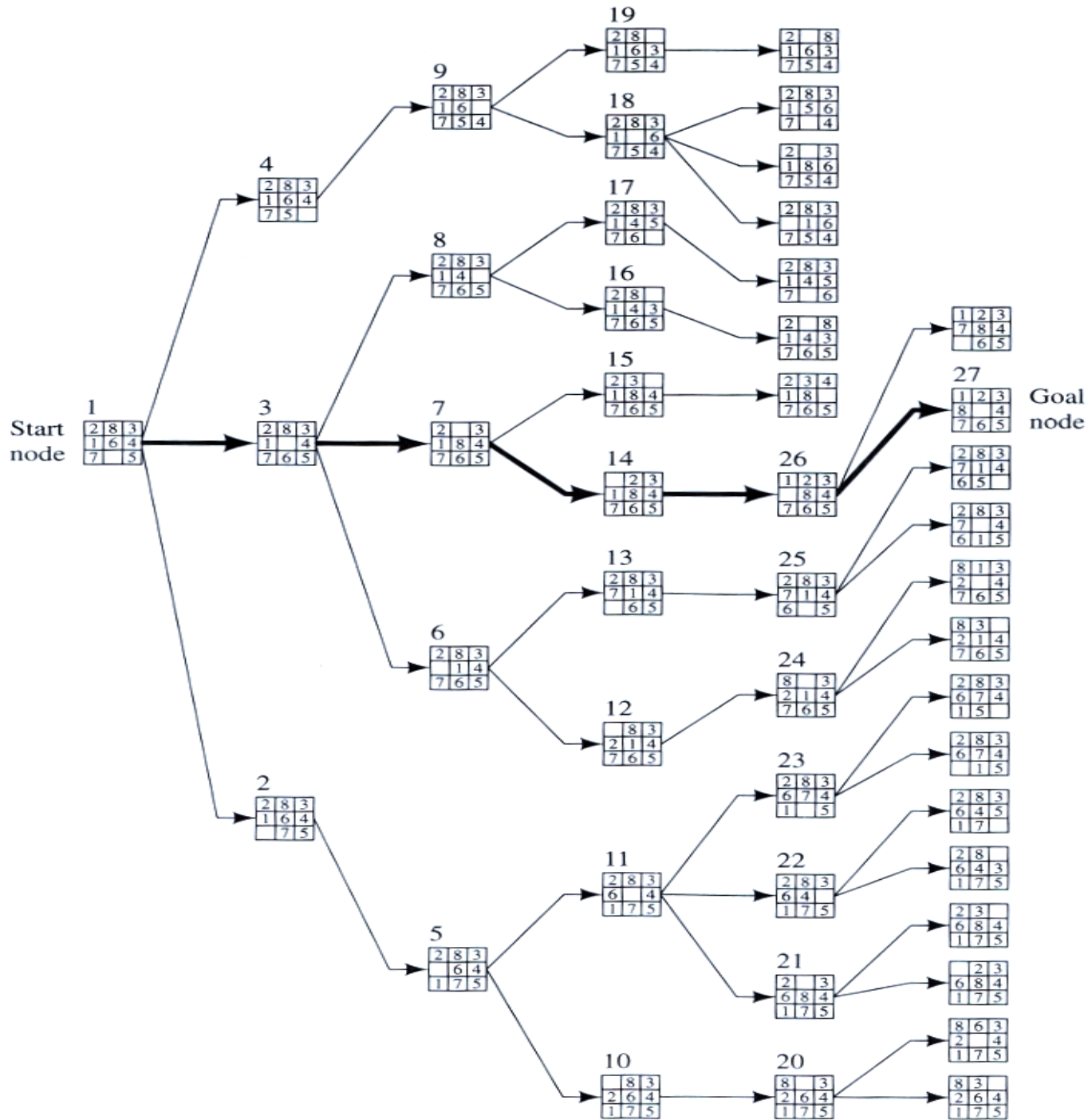
Expand E; no children  
Forget E; B

Frontier = [F,G]



Future= green dotted circles  
Frontier=white nodes  
Expanded/active=gray nodes  
Forgotten/reclaimed= black nodes

# Example BFS for 8-puzzle



# Properties of breadth-first search

- **Complete?** Yes, it always reaches a goal (if  $b$  is finite)
- **Time?**  $1 + b + b^2 + b^3 + \dots + b^d = O(b^d)$   
(this is the number of nodes we generate)
- **Space?**  $O(b^d)$   
(keeps every node in memory, either in frontier or on a path to frontier).
- **Optimal?** No, for general cost functions.  
Yes, if cost is a non-decreasing function only of depth.
  - With  $f(d) \geq f(d-1)$ , e.g., step-cost = constant:
    - All optimal goal nodes occur on the same level
    - Optimal goals are always shallower than non-optimal goals
    - An optimal goal will be found before any non-optimal goal
- Usually **Space** is the bigger problem (more than time)

# BFS: Time & Memory Costs

Depth of Solution	Nodes Expanded	Time	Memory
0	1	5 microseconds	100 bytes
2	111	0.5 milliseconds	11 kbytes
4	11,111	0.05 seconds	1 megabyte
8	$10^8$	9.25 minutes	11 gigabytes
12	$10^{12}$	64 days	111 terabytes

Assuming  $b=10$ ; 200k nodes/sec; 100 bytes/node



# Uniform-cost search

Breadth-first is only optimal if path cost is a non-decreasing function of depth, i.e.,  $g(d) \geq g(d-1)$ ; e.g., constant step cost, as in the 8-puzzle.

Can we guarantee optimality for variable positive step costs  $\geq \epsilon$ ?

(Why  $\geq \epsilon$ ? To avoid infinite paths w/ step costs 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ , ...)

## Uniform-cost Search:

Expand node with smallest path cost  $g(n)$ .

- *Frontier* is a priority queue, i.e., new successors are merged into the queue sorted by  $g(n)$ .
  - Can remove successors already on queue w/higher  $g(n)$ .
    - Saves memory, costs time; another space-time trade-off.
- *Goal-Test* when node is **popped off** queue.

# Uniform cost search (R&N Fig. 3.14)

## [A\* is identical except queue sort = $f(n)$ ]

**function** UNIFORM-COST-SEARCH(problem) **returns** a solution, or failure

node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0

frontier  $\leftarrow$  a priority queue ordered by PATH-COST, with node as the only element

explored  $\leftarrow$  an empty set

**loop do**

**if** EMPTY?(frontier) **then return** failure

  node  $\leftarrow$  POP(frontier) /\* chooses the lowest-cost node in frontier \*/

**if** problem.GOAL-TEST(node.STATE) **then return** SOLUTION(node)

  add node.STATE to explored

**for each** action **in** problem.ACTIONS(node.STATE) **do**

    child  $\leftarrow$  CHILD-NODE(problem, node, action)

**if** child.STATE is not in explored or frontier **then**

      frontier  $\leftarrow$  INSERT(child, frontier)

**else if** child.STATE is in frontier with higher PATH-COST **then**

      replace that frontier node with child

Goal test after pop

Avoid  
redundant  
frontier nodes

Avoid  
higher-cost  
frontier nodes

**Figure 3.14** Uniform-cost search on a graph. The algorithm is identical to the general graph search algorithm in Figure 3.7, except for the use of a priority queue and the addition of an extra check in case a shorter path to a frontier state is discovered. The data structure for **frontier** needs to support efficient membership testing, so it should combine the capabilities of a priority queue and a hash table.

These three statements change tree search to graph search.

# Uniform-cost search

## Proof of Completeness:

Assume (1) finite max branching factor =  $b$ ; (2) min step cost  $\geq \epsilon > 0$ ; (3) cost to optimal goal =  $C^*$ . Then a node at depth  $\lfloor 1 + C^*/\epsilon \rfloor$  must have a path cost  $> C^*$ . There are  $O(b^{\lfloor 1 + C^*/\epsilon \rfloor})$  such nodes, so a goal will be found.

## Proof of Optimality (given completeness):

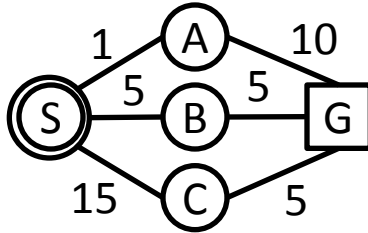
Suppose that UCS is not optimal. Then there must be an (optimal) goal state with path cost smaller than the found (suboptimal) goal state (invoking completeness).

However, this is impossible because UCS would have expanded that node first, by definition.

Contradiction.

(Search tree version)

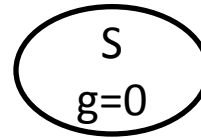
# Example: Uniform-cost search



Route finding problem.  
Steps labeled w/cost.

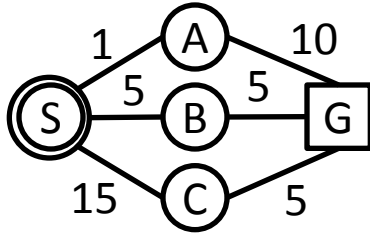
Order of node expansion: \_\_\_\_\_

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_



(Search tree version)

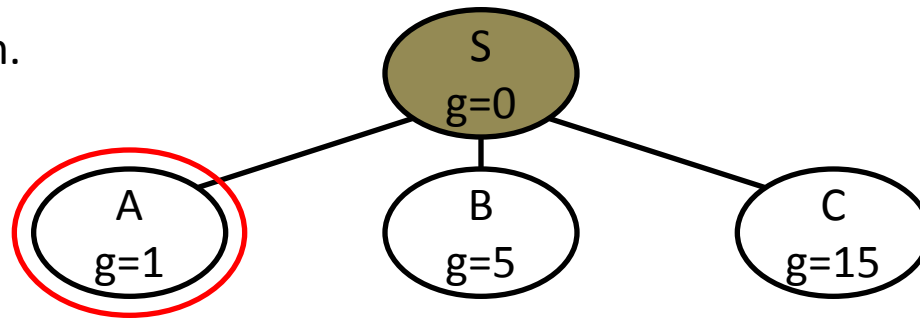
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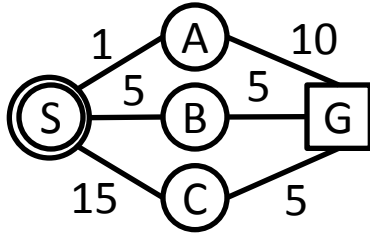
Order of node expansion: S

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.  
Steps labeled w/cost.



# Example: Uniform-cost search

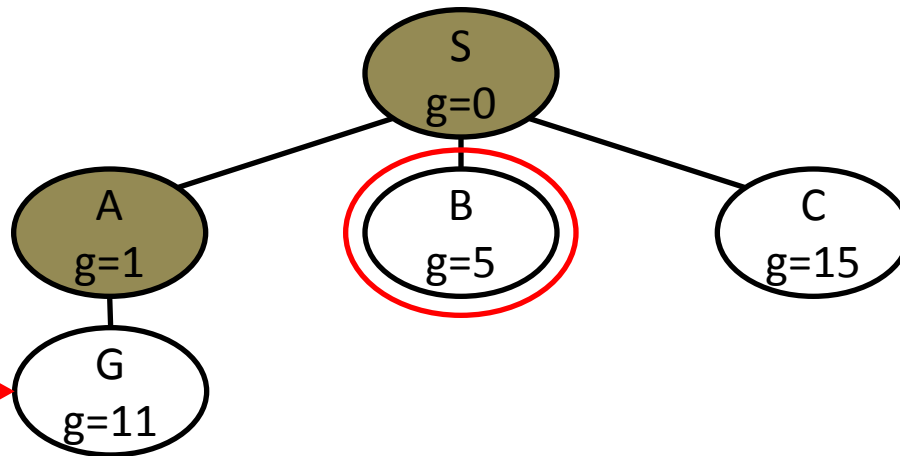


Order of node expansion: S A

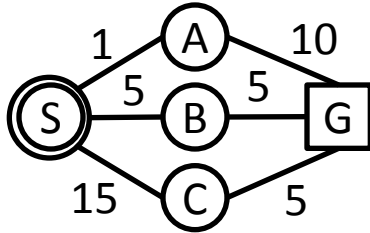
Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.  
Steps labeled w/cost.

This early  
expensive goal  
node will go  
back onto the  
queue until after  
the later  
cheaper goal is  
found.



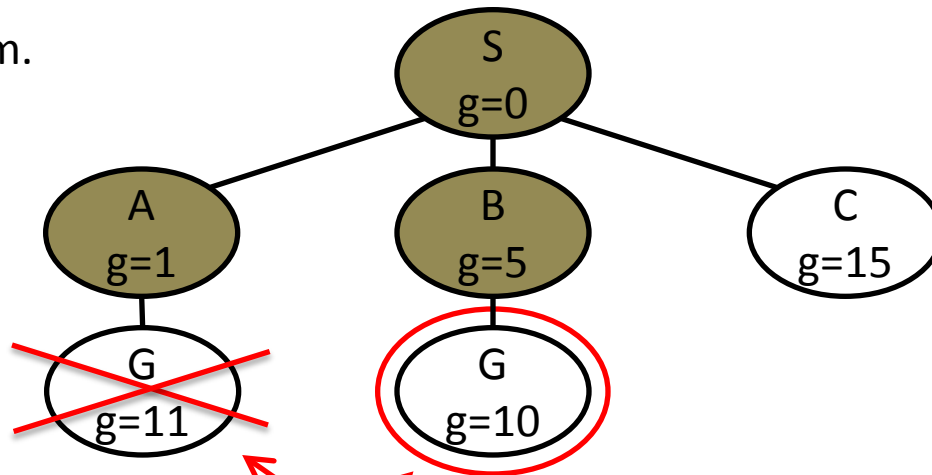
# Example: Uniform-cost search



Order of node expansion: S A B

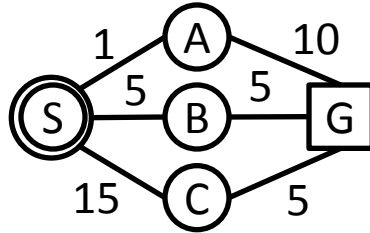
Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.  
Steps labeled w/cost.



Remove the higher-cost of identical nodes on the queue and save memory. However, UCS is optimal even if this is not done, since lower-cost nodes sort to the front.

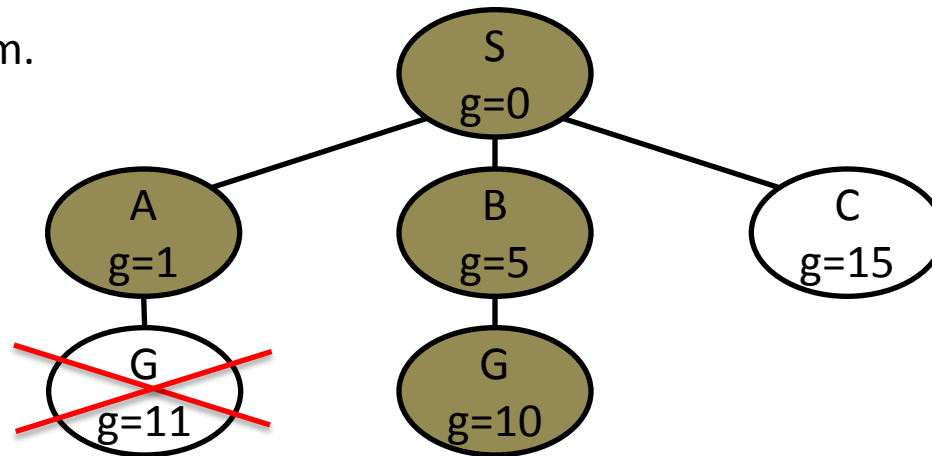
# Example: Uniform-cost search



Order of node expansion: S A B G

Path found: S B G Cost of path found: 10

Route finding problem.  
Steps labeled w/cost.

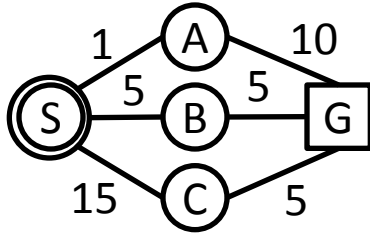


Technically, the goal node is not really expanded, because we do not generate the children of a goal node. It is listed in “Order of node expansion” only for your convenience, to see explicitly where it was found.



(Virtual queue version)

# Example: Uniform-cost search



Route finding problem.  
Steps labeled w/cost.

Order of node expansion: \_\_\_\_\_

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

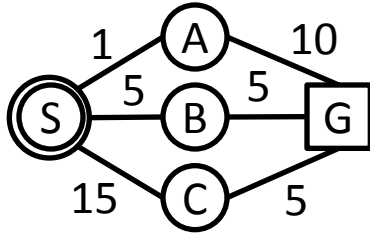
Expanded:

Next:

Children:

Queue: S/g=0

# Example: Uniform-cost search



Order of node expansion: S

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.

Steps labeled w/cost.

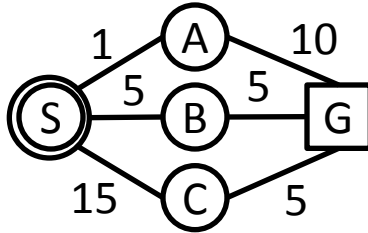
Expanded: S/g=0

Next: S/g=0

Children: A/g=1, B/g=5, C/g=15

Queue: S/g=0, A/g=1, B/g=5, C/g=15

# Example: Uniform-cost search



Order of node expansion: S A

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.  
Steps labeled w/cost.

Expanded: S/g=0, A/g=1

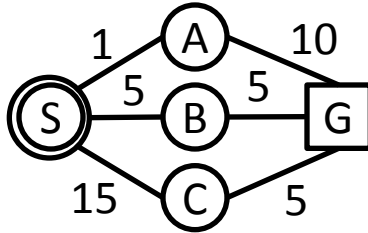
Next: A/g=1

Children: G/g=11

Queue: S/g=0, A/g=1, B/g=5, C/g=15, G/g=11

Note that in a proper priority queue in a computer system, this queue would be sorted by  $g(n)$ . For hand-simulated search it is more convenient to write children as they occur, and then scan the current queue to pick the highest-priority node on the queue.

# Example: Uniform-cost search



Order of node expansion: S A B

Path found: \_\_\_\_\_ Cost of path found: \_\_\_\_\_

Route finding problem.  
Steps labeled w/cost.

Expanded: S/g=0, A/g=1, B/g=5

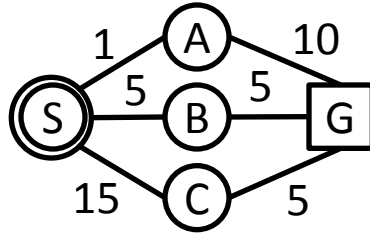
Next: B/g=5

Children: G/g=10

Queue: S/g=0, A/g=1, B/g=5, C/g=15, ~~G/g=11~~, G/g=10

Remove the higher-cost of identical nodes on the queue and save memory. However, UCS is optimal even if this is not done, since lower-cost nodes sort to the front.

# Example: Uniform-cost search



Route finding problem.  
Steps labeled w/cost.

Order of node expansion: S A B G  
Path found: S B G Cost of path found: 10

The same “Order of node expansion”, “Path found”, and “Cost of path found” is obtained by both methods. They are formally equivalent to each other in all ways.

Expanded: S/g=0, A/g=1, B/g=5, G/g=10

Next: G/g=10

Children: none

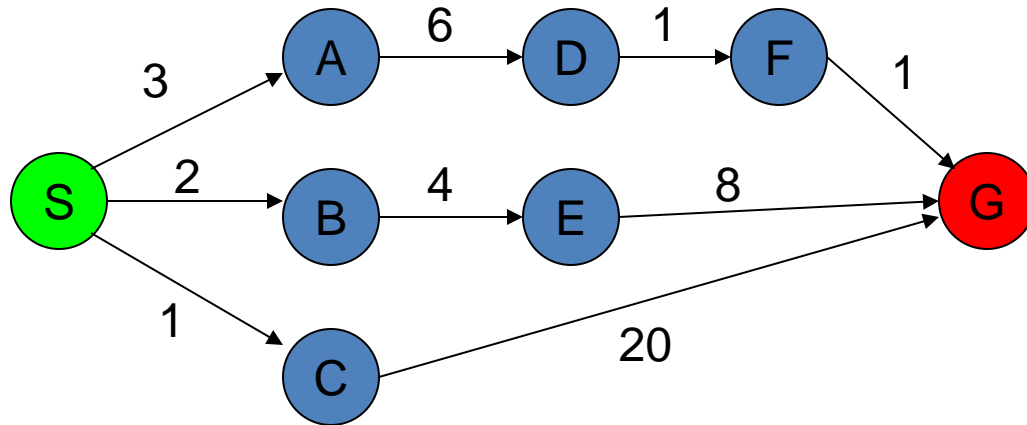
Queue: S/g=0, A/g=1, B/g=5, C/g=15, ~~G/g=11~~, G/g=10

Technically, the goal node is not really expanded, because we do not generate the children of a goal node. It is listed in “Order of node expansion” only for your convenience, to see explicitly where it was found.

# Uniform-cost search

**Implementation:** *Frontier* = queue ordered by path cost.  
Equivalent to breadth-first if all step costs all equal.

- **Complete?** Yes, if  $b$  is finite and step cost  $\geq \epsilon > 0$ .  
(otherwise it can get stuck in infinite regression)
- **Time?** # of nodes with path cost  $\leq$  cost of optimal solution.  
 $O(b^{\lfloor 1+C^*/\epsilon \rfloor}) \approx O(b^{d+1})$
- **Space?** # of nodes with path cost  $\leq$  cost of optimal solution.  
 $O(b^{\lfloor 1+C^*/\epsilon \rfloor}) \approx O(b^{d+1})$ .
- **Optimal?** Yes, for step cost  $\geq \epsilon > 0$ .



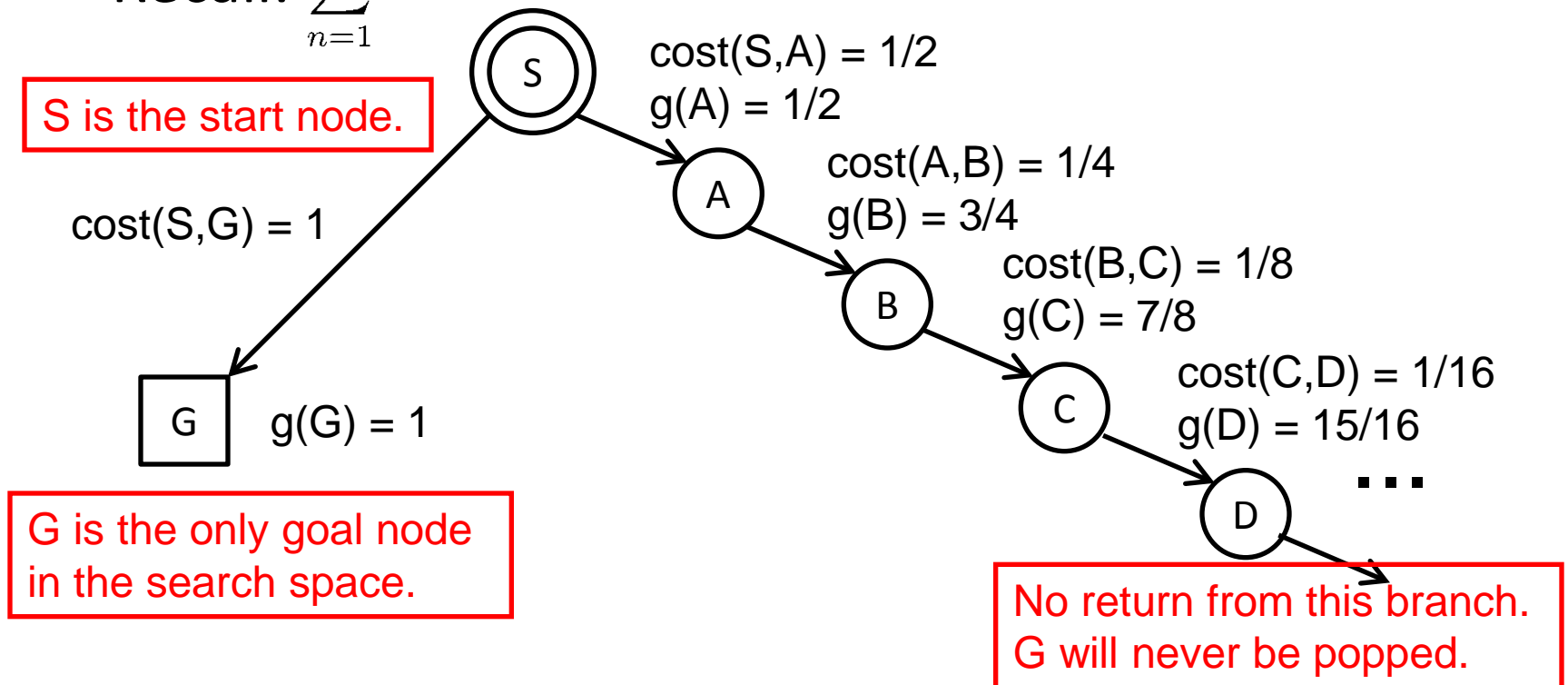
The graph above shows the step-costs for different paths going from the start (S) to the goal (G).

Use uniform cost search to find the optimal path to the goal.

Exercise for home

# Uniform cost search

- Why require step cost  $\geq \varepsilon > 0$ ?
  - Otherwise, an infinite regress is possible.
  - Recall:  $\sum_{n=1}^{\infty} 2^{-n} = 1$





# Iterative Deepening Search

- To avoid the infinite depth problem of DFS:
  - Only search until depth  $L$
  - i.e, don't expand nodes beyond depth  $L$
  - Depth-Limited Search
- What if solution is deeper than  $L$ ?
  - Increase depth iteratively
  - Iterative Deepening Search
- IDS — GENERALLY THE PREFERRED UNINFORMED SEARCH
  - Inherits the memory advantage of depth-first search
  - Has the completeness property of breadth-first search

# Depth-limited search & IDS (R&N Fig. 3.17-18)

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns soln/fail/cutoff
  RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
  cutoff-occurred?  $\leftarrow$  false
  if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
  else if DEPTH[node] = limit then return cutoff
  else for each successor in EXPAND(node, problem) do
    result  $\leftarrow$  RECURSIVE-DLS(successor, problem, limit)
    if result = cutoff then cutoff-occurred?  $\leftarrow$  true
    else if result  $\neq$  failure then return result
  if cutoff-occurred? then return cutoff else return failure
```

Goal test in  
recursive call,  
one-at-a-time

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or fail-
ure
  inputs: problem, a problem
  for depth  $\leftarrow$  0 to  $\infty$  do
    result  $\leftarrow$  DEPTH-LIMITED-SEARCH(problem, depth)
    if result  $\neq$  cutoff then return result
```

At *depth* = 0, IDS only goal-tests  
the start node. The start node is  
is not expanded at *depth* = 0.

# Iterative Deepening Search, $L=0$

Limit = 0



At  $L=0$ , the start node is goal-tested but no nodes are expanded. This is so that you can solve trick problems like, “Starting in Arad, go to Arad.”

# Iterative Deepening Search, $L=1$

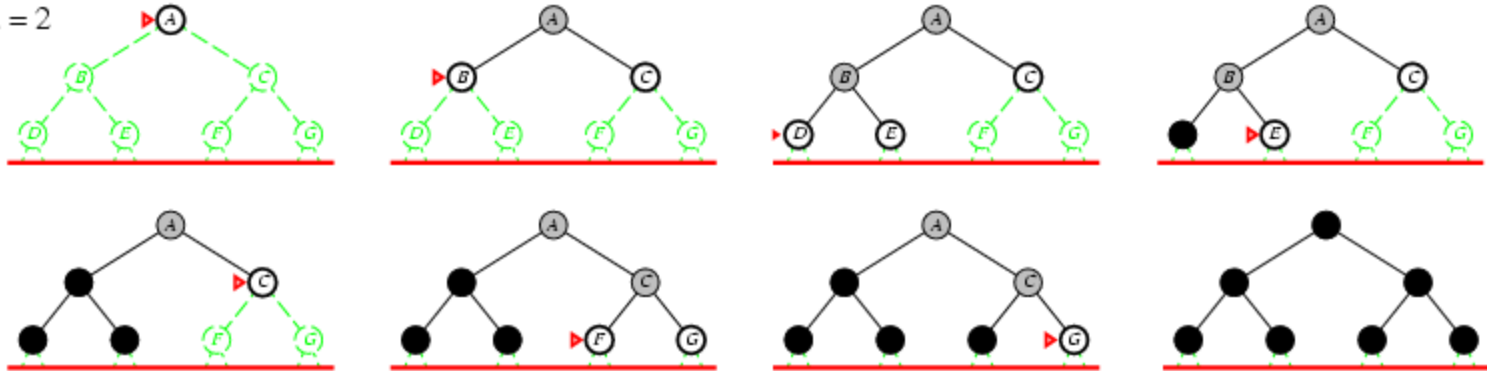
Limit = 1



At  $L=1$ , the start node is expanded. Its children are goal-tested, but not expanded. Recall that to expand a node means to generate its children.

# Iterative Deepening Search, L=2

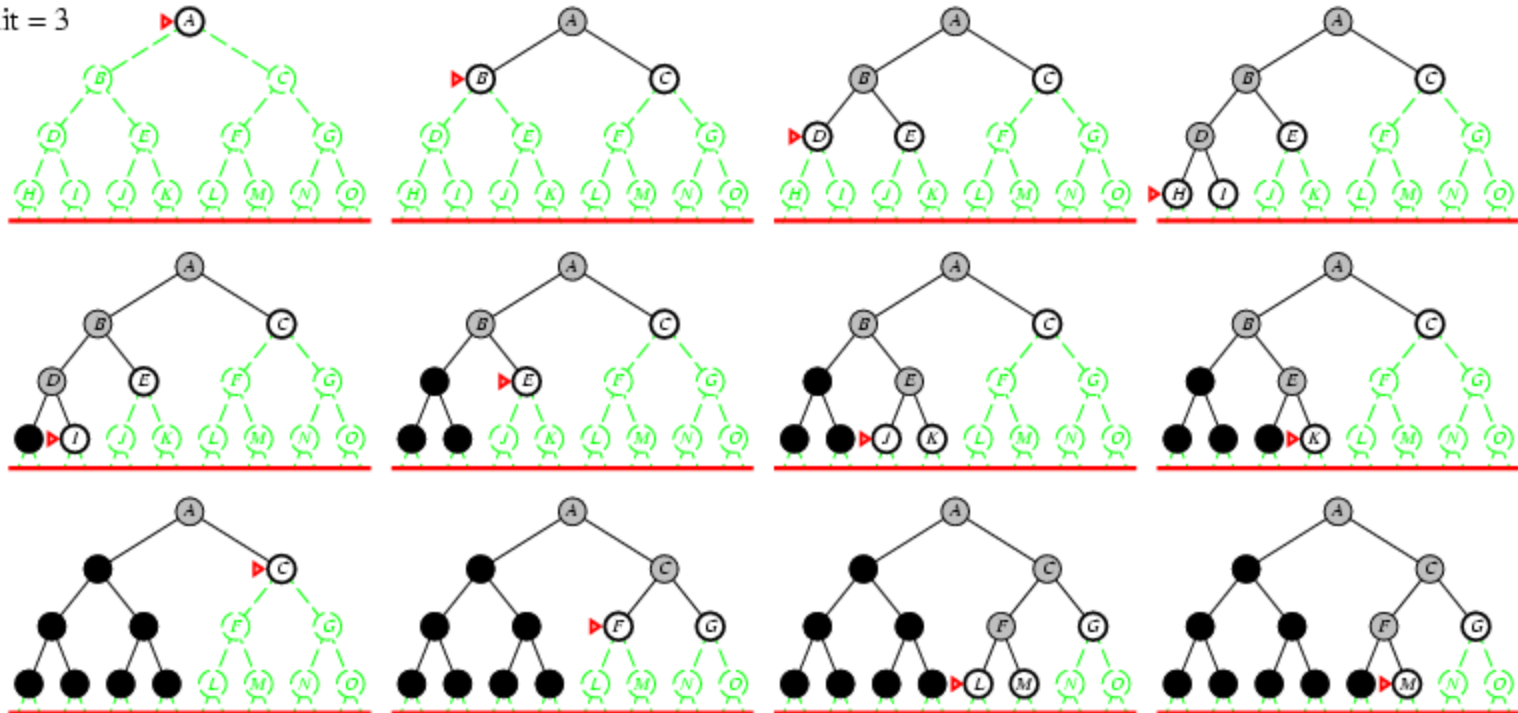
Limit = 2



At L=2, the start node and its children are expanded. Its grand-children are goal-tested, but not expanded.

# Iterative Deepening Search, L=3

Limit = 3



At L=3, the start node, its children, and its grand-children are expanded. Its great-grand-children are goal-tested, but not expanded.

# Iterative Deepening Search

- Number of nodes generated in a depth-limited search to depth  $d$  with branching factor  $b$ :

$$N_{DLS} = b^0 + b^1 + b^2 + \dots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth  $d$  with branching factor  $b$ :

$$\begin{aligned} N_{IDS} &= (d+1)b^0 + d b^1 + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d \\ &= O(b^d) \end{aligned}$$

- For  $b = 10$ ,  $d = 5$ ,

- $N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111$

- $N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$

[ Ratio:  $b/(b-1)$  ]

# Properties of iterative deepening search

- Complete? Yes
- Time?  $O(b^d)$
- Space?  $O(bd)$
- Optimal? No, for general cost functions.  
Yes, if cost is a non-decreasing function only of depth.

**Generally the preferred uninformed search strategy.**



# Depth-First Search (R&N Section 3.4.3)

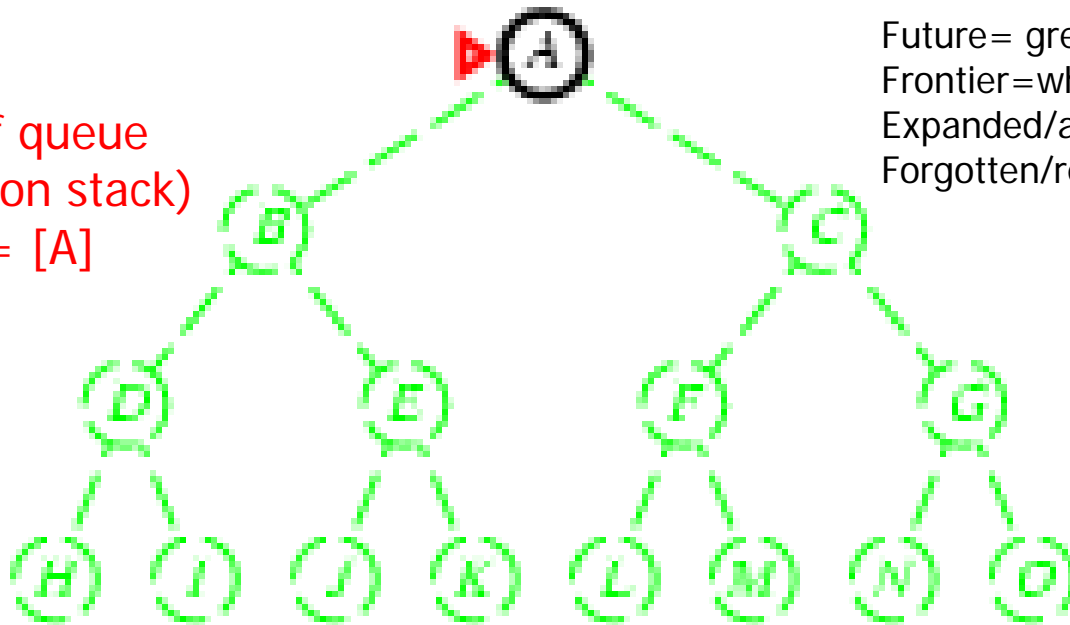
- Your textbook is ambiguous about DFS.
  - The second paragraph of R&N 3.4.3 states that DFS is an instance of Fig. 3.7 using a LIFO queue. Search behavior may differ depending on how the LIFO queue is implemented (as separate pushes, or one concatenation).
  - The third paragraph of R&N 3.4.3 says that an alternative implementation of DFS is a recursive algorithm that calls itself on each of its children, as in the Depth-Limited Search of Fig. 3.17 (above).
- **For quizzes and exams, we will follow Fig. 3.17.**

# Depth-first search

- Expand *deepest* unexpanded node
- *Frontier* = Last In First Out (LIFO) queue, i.e., new successors go at the front of the queue.
- *Goal-Test* first step of recursive call (R&N, Fig. 3.17).

Initial state = A

Put A at front of queue  
(note: queue is on stack)  
queue/frontier = [A]



Future= green dotted circles  
Frontier=white nodes  
Expanded/active=gray nodes  
Forgotten/reclaimed= black nodes

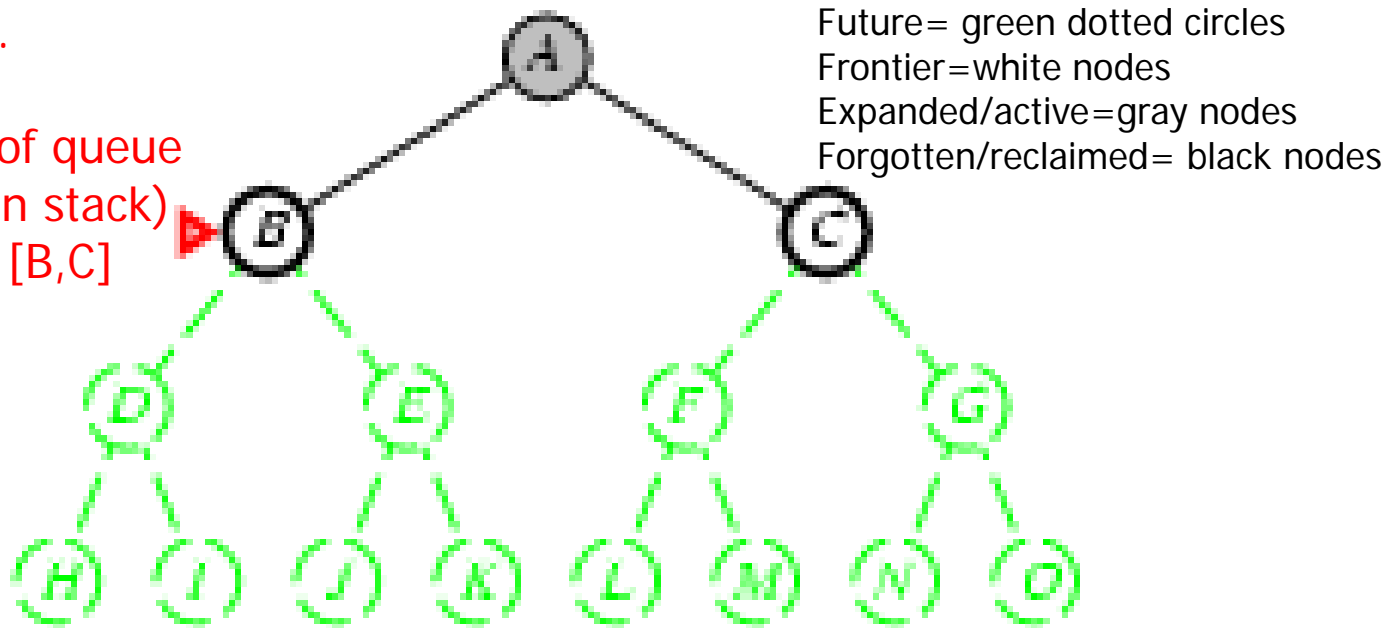
# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is A a goal state? No.

Expand A to B, C.

Put B, C at front of queue  
(note: queue is on stack)  
queue/frontier = [B,C]



Note: Can save a space factor of  $b$  by generating successors one at a time.  
See **backtracking search** in your book, p. 87 and Chapter 6.

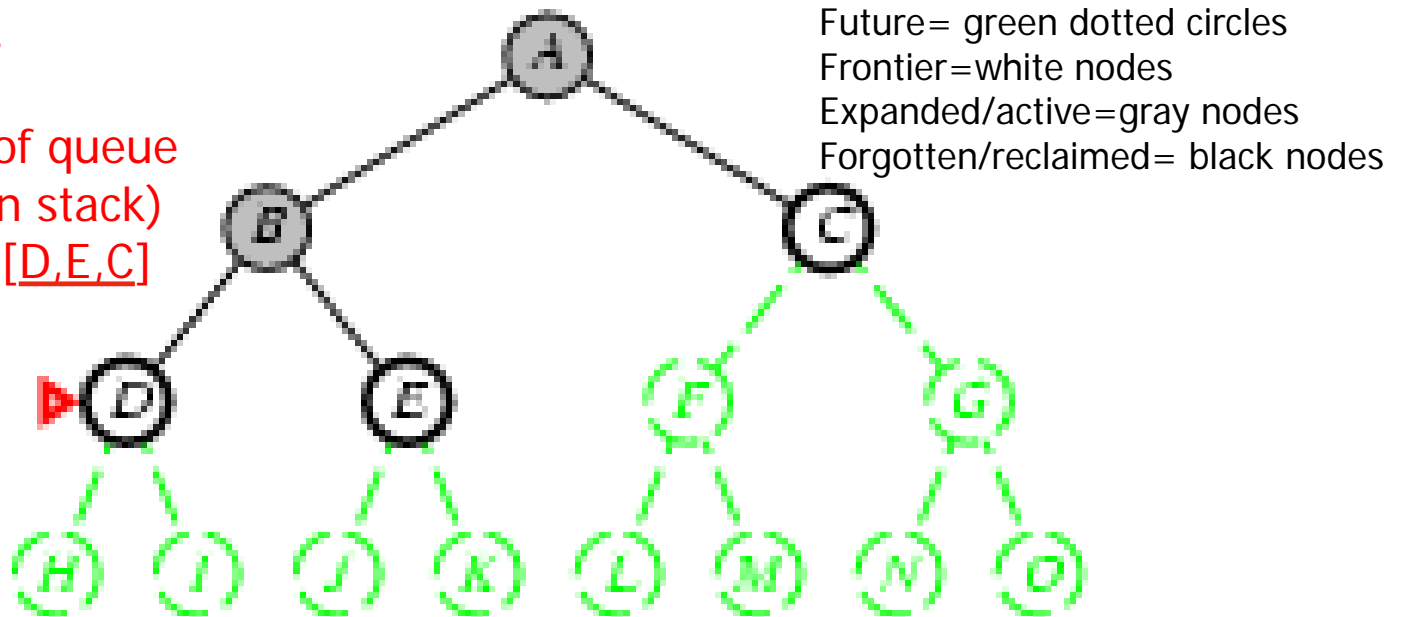
# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is B a goal state? No.

Expand B to D, E.

Put D, E at front of queue  
(note: queue is on stack)  
queue/frontier = [D, E, C]



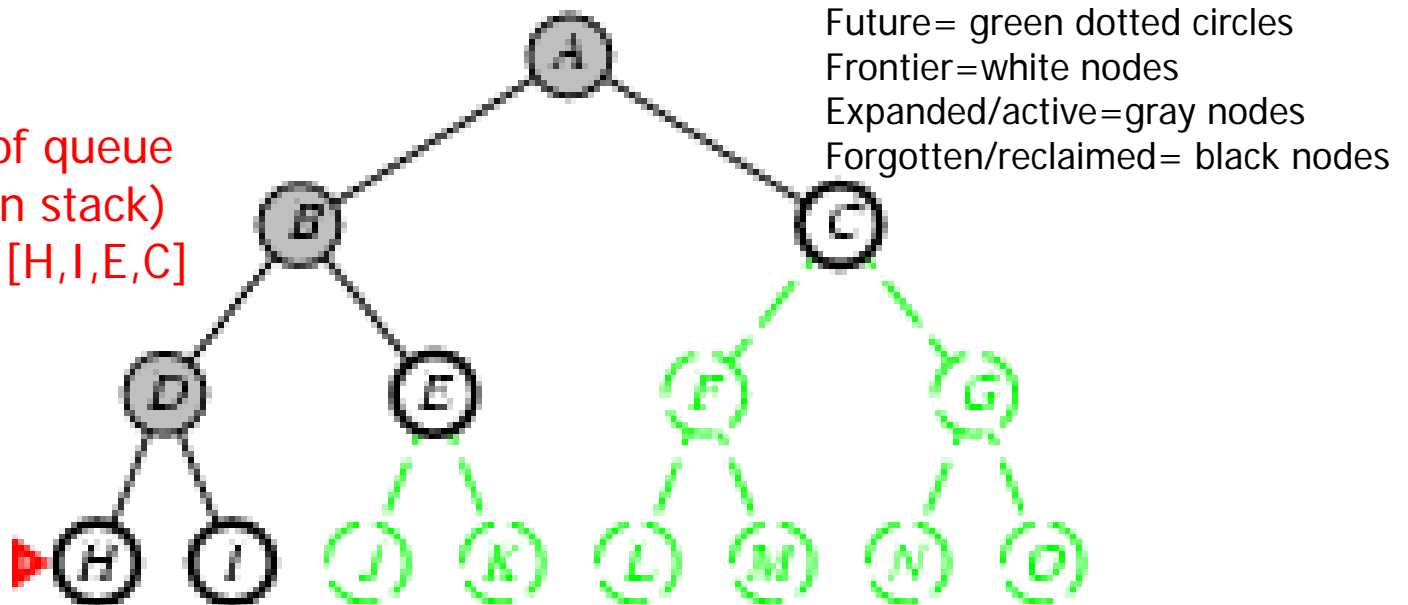
# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is D a goal state? No.

Expand D to H, I.

Put H, I at front of queue  
(note: queue is on stack)  
queue/frontier = [H,I,E,C]

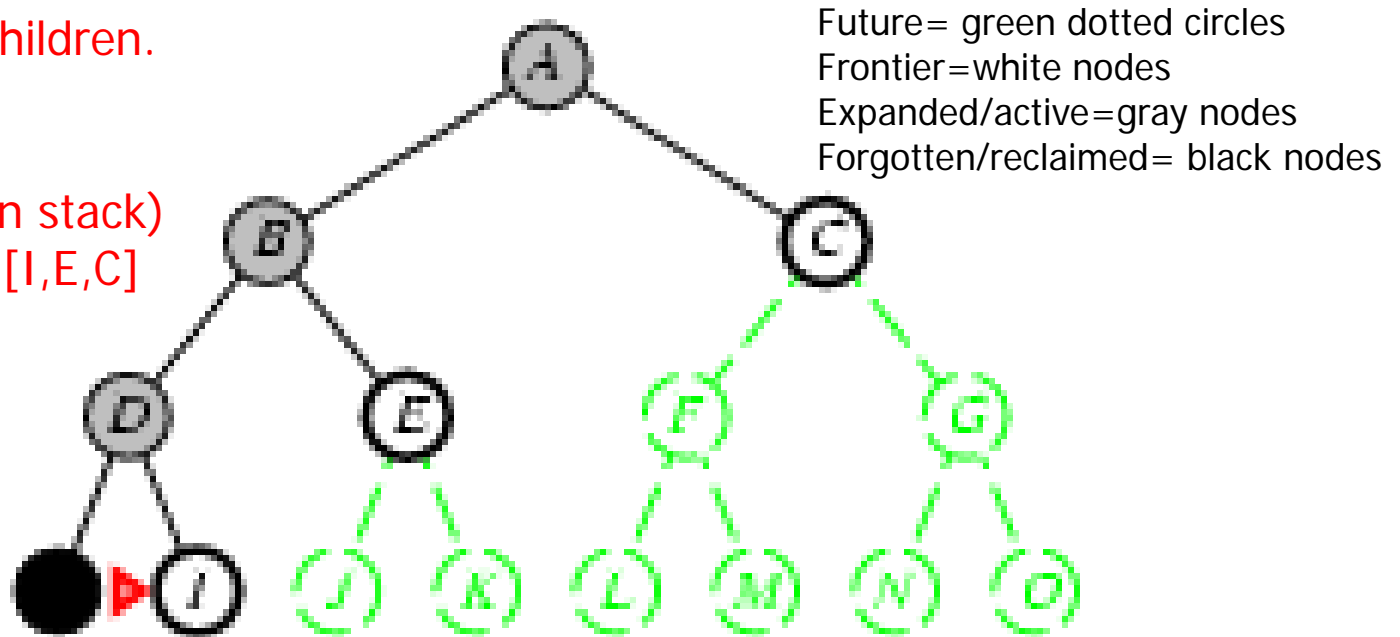


# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is H a goal state? No.  
Expand H to no children.  
Forget H.

(note: queue is on stack)  
queue/frontier = [I,E,C]

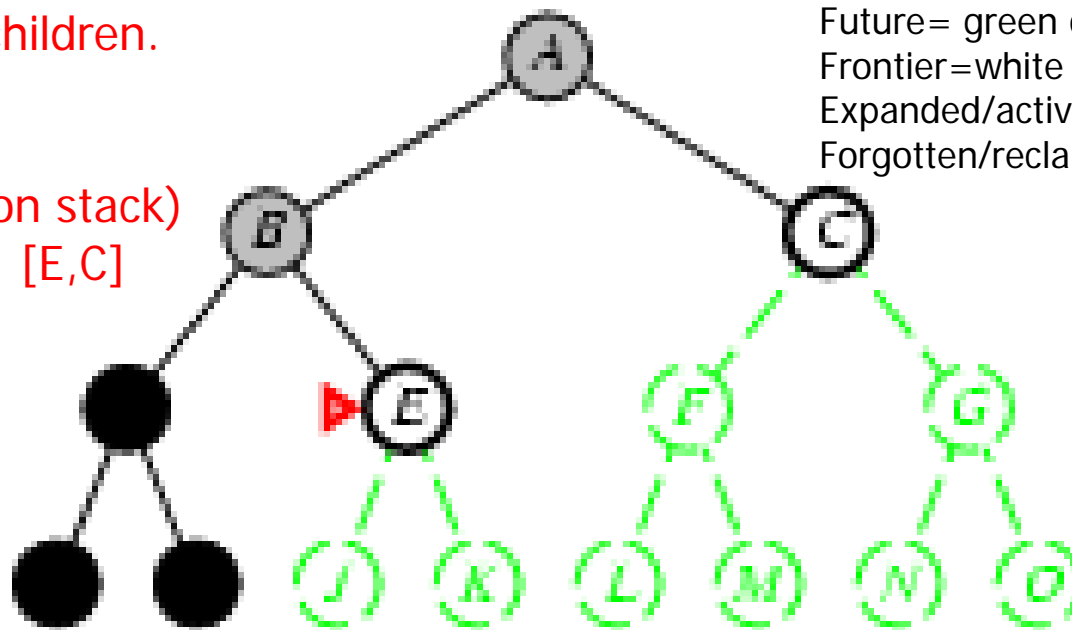


# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is I a goal state? No.  
Expand I to no children.  
Forget D, I.

(note: queue is on stack)  
queue/frontier = [E,C]



# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

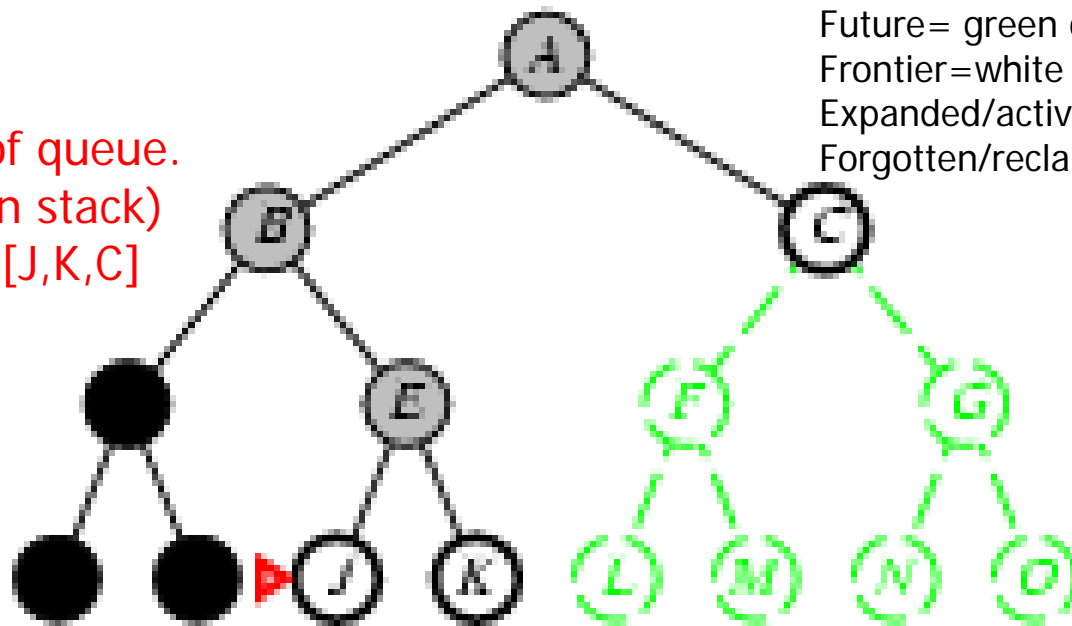
Is E a goal state? No.

Expand E to J, K.

Put J, K at front of queue.

(note: queue is on stack)

queue/frontier = [J,K,C]



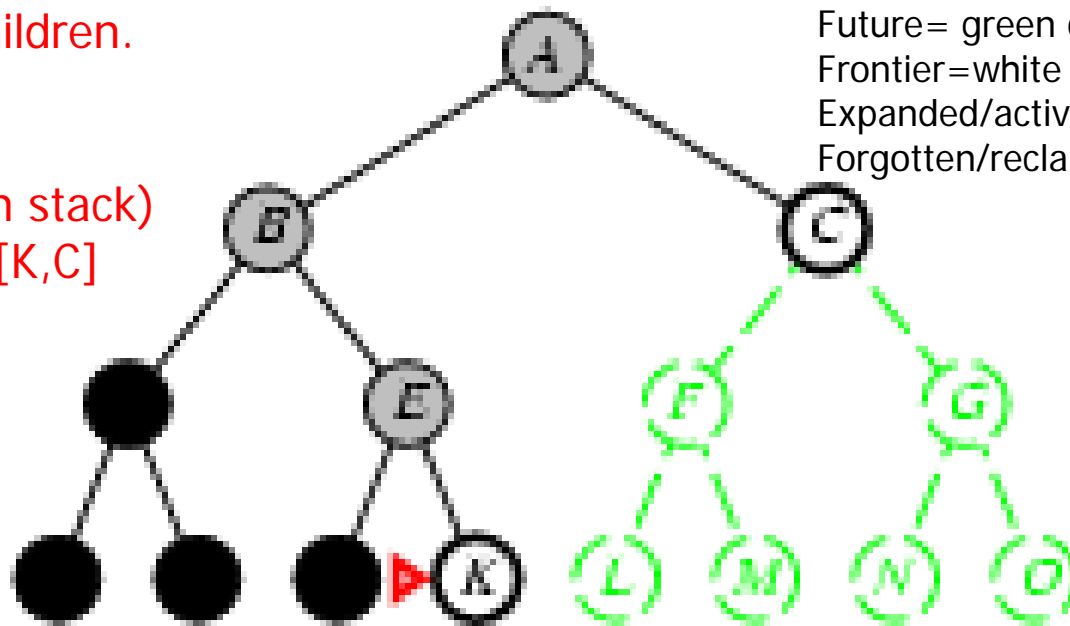


# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is J a goal state? No.  
Expand J to no children.  
Forget J.

(note: queue is on stack)  
queue/frontier = [K,C]

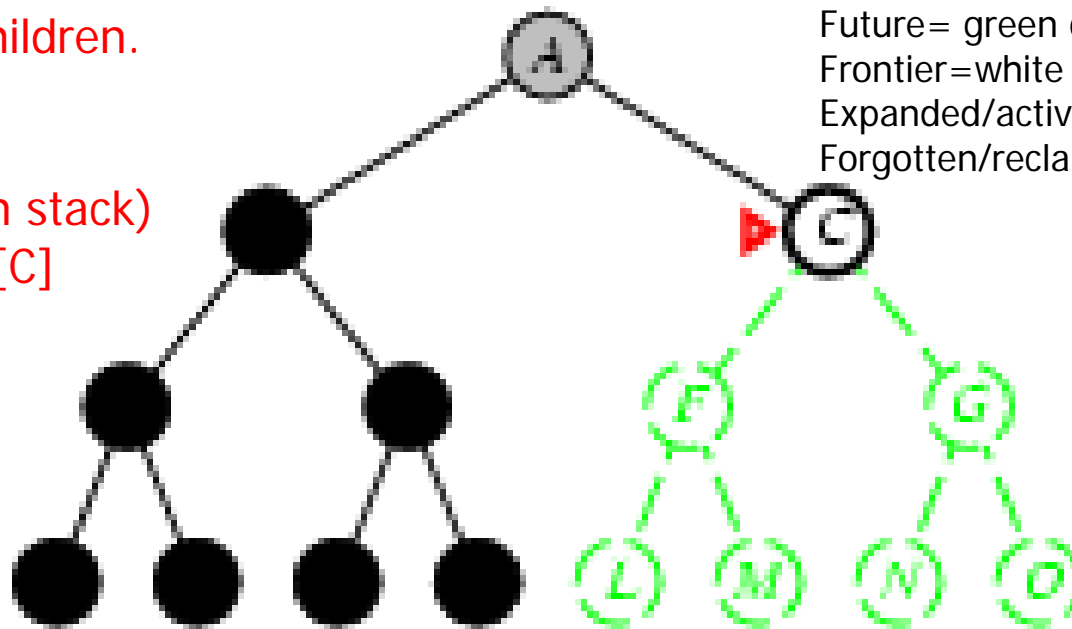


# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Is K a goal state? No.  
Expand K to no children.  
Forget B, E, K.

(note: queue is on stack)  
queue/frontier = [C]



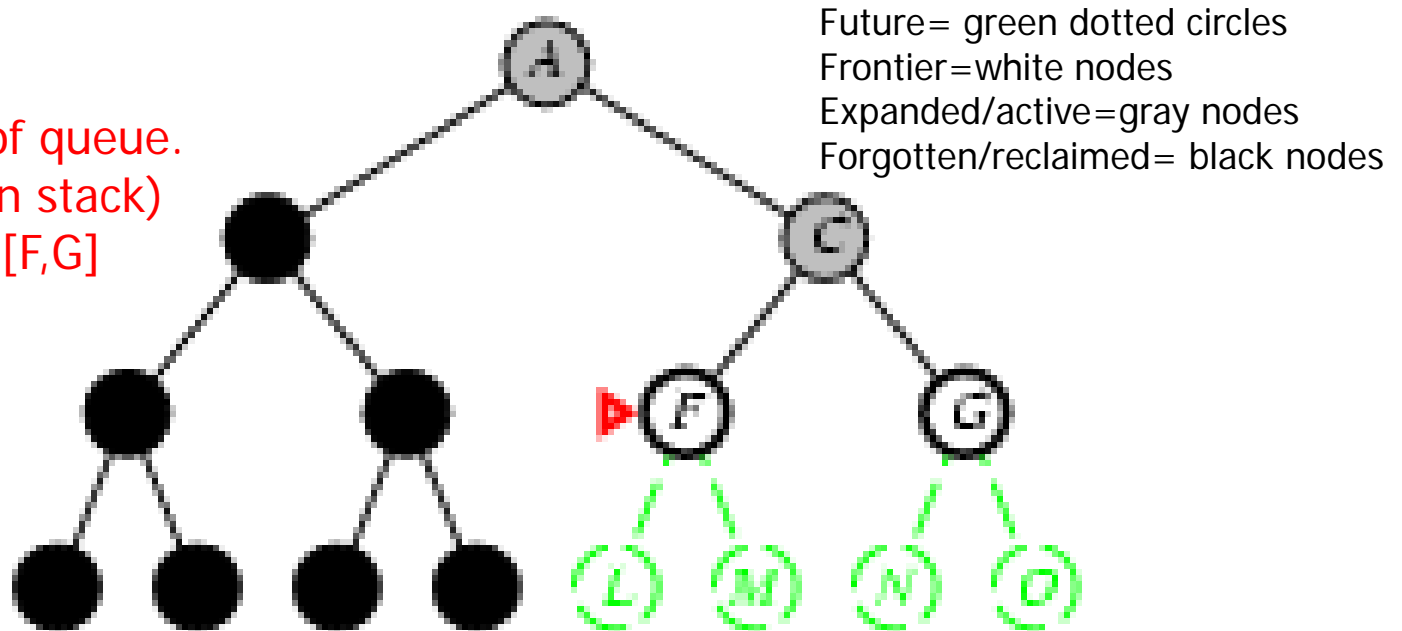
# Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

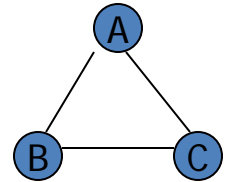
Is C a goal state? No.

Expand C to F, G.

Put F, G at front of queue.  
(note: queue is on stack)  
queue/frontier = [F,G]



# Properties of depth-first search



- **Complete?** No: fails in loops/infinite-depth spaces
  - Can modify to avoid loops/repeated states along path
    - check if current nodes occurred before on path to root
  - Can use graph search (remember all nodes ever seen)
    - problem with graph search: space is exponential, not linear
  - Still fails in infinite-depth spaces (may miss goal entirely)
- **Time?**  $O(b^m)$  with  $m$  = maximum depth of space
  - Terrible if  $m$  is much larger than  $d$
  - If solutions are dense, may be much faster than BFS
- **Space?**  $O(bm)$ , i.e., linear space!
  - Remember a single path + expanded unexplored nodes
- **Optimal?** No: It may find a non-optimal goal first

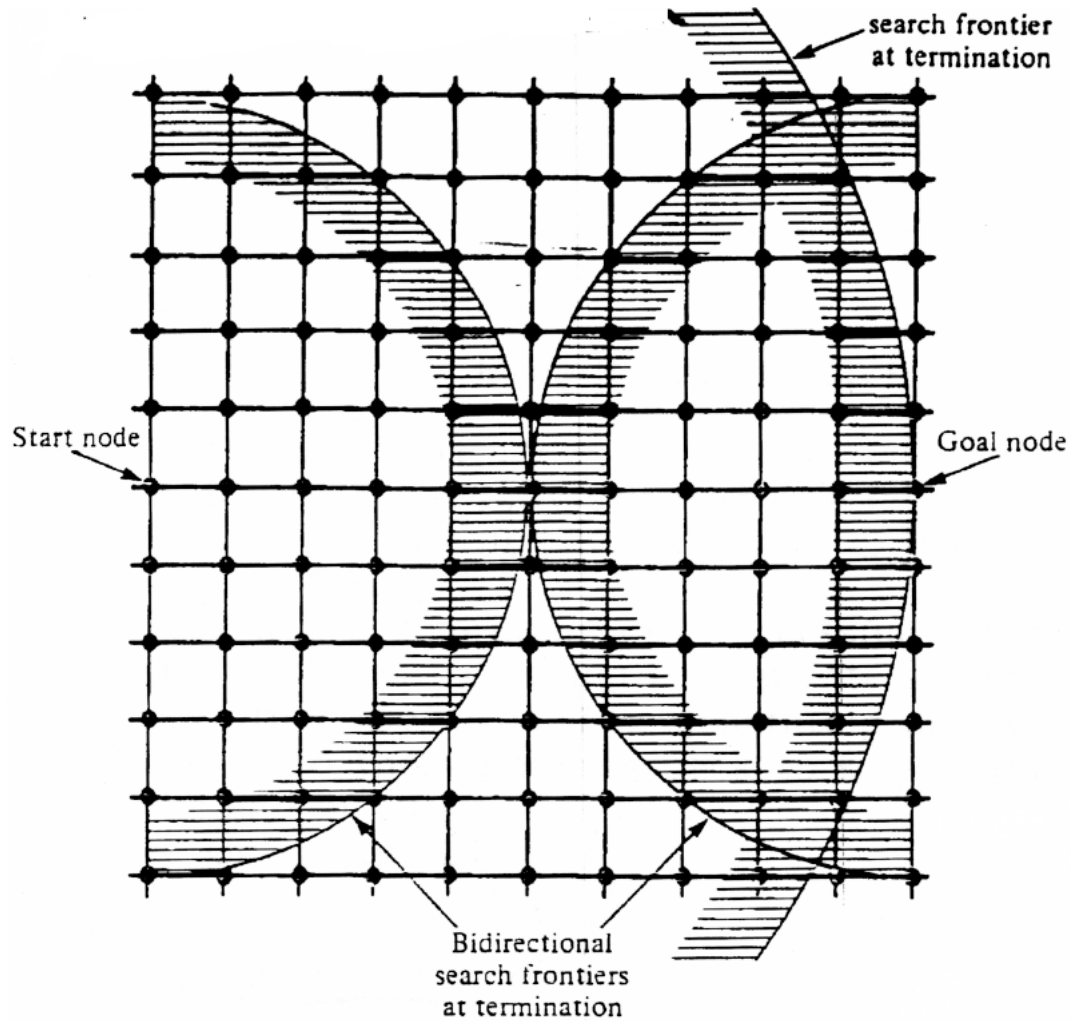
# Comparing DFS and BFS

- BFS is optimal if path cost is non-decreasing function of depth, DFS is not
- Worst-case Time Complexity: BFS =  $O(b^d)$ , DFS =  $O(b^m)$ ;  $m$  may be infinite
  - In the worst-case, BFS is always better than DFS
- Sometimes, on the average, DFS is better if:
  - Many goals, no loops, and no long or infinite paths
  - Thus, DFS may luckily blunder into an early goal
- BFS is much worse memory-wise
  - BFS may store the whole search space
- DFS can be linear space
  - Stores only the nodes on the path from the current leaf to the root
- In general:
  - BFS is better if shallow goals, many long paths, many loops, small search space
  - DFS is better if many goals, not many loops (easy to check), few long or infinite paths (hard to check), huge search space
  - DFS is always much better in terms of memory

# Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both “meet in the middle”
  - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - what if there are multiple goal states?
  - what if there is only a goal test, no explicit list?
- Complexity
  - time complexity is best:  $O(2 b^{(d/2)}) = O(b^{(d/2)})$
  - memory complexity is the same as time complexity

# Bi-Directional Search



*Fig. 2.10 Bidirectional and unidirectional breadth-first searches.*

# Bidirectional search termination

- R&N Sec. 3.4.6 discusses the BDS termination condition for BFS.
  - To clarify it, and to handle UCS:
- For BFS, the search terminates when one fringe expands a node and discovers that one of the new children is present in the other fringe. This is quick and easy because the other fringe already maintains a hash table holding its fringe, as discussed in the lecture slides about removing duplicate nodes from the fringe, so you just look up the new child in the other fringe's hash table. If present, then you join the path from the Start to that child to the reverse of the path from the Goal to that child, and you have your path from Start to Goal. The first such solution found may not be optimal; some additional search is required to make sure there isn't a short-cut across the gap.
- For UCS, the same applies, except that afterward you must continue searching until the sum of the costs of the nodes at the head of each queue is greater than or equal to the cost of the path you just found. This continuation guarantees that there is not a longer cheaper path somewhere in the queues. Of course, if you find a cheaper solution as the search winds down, it replaces the previous solution.



# Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening DLS	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{\lfloor 1+C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{\lfloor 1+C^*/\epsilon \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

There are a number of footnotes, caveats, and assumptions.

See Fig. 3.21, p. 91.

[a] complete if  $b$  is finite

[b] complete if step costs  $\geq \epsilon > 0$

[c] optimal if step costs are all identical

(also if path cost non-decreasing function of depth only)

[d] if both directions use breadth-first search

(also if both directions use uniform-cost search with step costs  $\geq \epsilon > 0$ )

Generally the preferred  
uninformed search strategy

Note that  $d \leq \lfloor 1+C^*/\epsilon \rfloor$

# You should know...

- Overview of uninformed search methods
- Search strategy evaluation
  - Complete? Time? Space? Optimal?
  - Max branching (b), Solution depth (d), Max depth (m)
  - (for UCS:  $C^*$ : true cost to optimal goal;  $\epsilon > 0$ : minimum step cost)
- Search Strategy Components and Considerations
  - Queue? Goal Test when? Tree search vs. Graph search?
- Various blind strategies:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Iterative deepening search (generally preferred)
  - Bidirectional search (preferred if applicable)

# Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

<http://www.cs.rmit.edu.au/AI-Search/Product/>  
<http://aima.cs.berkeley.edu/demos.html> (for more demos)