Optimization of Classroom Dimensions using Chemical Engineering

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2 Introduction

Classrooms are the foundational spaces where students begin their academic journey. Optimizing their design is crucial for enhancing comfort, concentration, and energy efficiency. This work examines how engineering principles can inform the design of more effective, student-centered learning environments.

In this study, we employ a numerical optimization approach using the **Gradient Descent algorithm** to minimize the total cost while satisfying environmental and comfort constraints.

3 Methodology

The optimization problem focuses on determining the ideal classroom dimensions (Length, Width, and Height) that minimize the total cost while ensuring compliance with indoor environmental comfort standards. The methodology includes the selection of relevant physical and environmental variables, defining appropriate assumptions, formulating an objective function, and applying a numerical optimization algorithm.

3.1 Selection of Variables

The primary decision variables in the model are:

- L – Length of the classroom (in meters)
- W – Width of the classroom (in meters)
- H – Height of the classroom (in meters)

Our main objective is to identify optimal values for these variables such that the total cost is minimized while maintaining acceptable levels of comfort, air quality, and acoustic performance.

The total cost is divided into two main components:

- **Capital Cost:** This includes one-time expenses for constructing the classroom and furnishing it. Specifically:
  - *Floor construction cost*, which depends on the floor area ($L \times W$)
  - *Wall construction cost*, which depends on the perimeter and height ($2H(L+W)$)
  - *Furniture cost*, calculated per student
- **Recurring Cost:** This includes annual costs required to maintain the desired environmental conditions in the classroom:
  - *Lighting energy cost*, based on required illuminance and area
  - *HVAC energy cost*, to maintain thermal comfort and compensate for heat loss

- *Sound system energy cost*, to maintain acceptable acoustic levels
- *Maintenance cost*, assumed constant per year

To ensure that the classroom provides a conducive learning environment, environmental performance metrics are also considered. These include:

- Indoor CO_2 concentration
- Illuminance (lighting intensity)
- Air temperature
- Reverberation time (acoustics)
- Sound transmission loss
- Heat loss and HVAC load

Any deviation from acceptable thresholds in the above parameters results in penalty costs being added to the objective function. These penalties act as soft constraints, steering the optimization toward feasible and student-friendly classroom designs.

3.2 Assumptions

To simplify the modeling process and ensure computational feasibility, the following assumptions have been made:

1. **Steady-State Conditions:** The model assumes steady-state behavior for both carbon dioxide (CO_2) concentration and thermal energy balance within the classroom environment.

2. **Uniform Illumination:** It is assumed that the lighting system provides uniform illumination throughout the classroom area, neglecting spatial variation in light distribution.

3. **Estimated Parameters:** Certain parameters, such as the cost of furniture per student, the base construction cost, and the annual maintenance, have been assumed based on reasonable estimates. Similarly, target thresholds for comfort variables such as CO_2 concentration, temperature, and illuminance are predefined for optimization purposes.

4. **Fixed Class Strength:** The number of students is fixed at 120, representing the actual classroom strength considered for this study.

3.3 Modeling

The main goal of this model is to determine the optimal classroom dimensions, minimizing the total cost per student. The total cost is divided into capital cost and recurring costs, which are impacted by various environmental factors.

3.3.1 Capital Cost

The capital cost includes the cost of flooring, walls, and furniture for students.
It is calculated as follows:

$$\text{Capital Cost} = A \times \text{base cost} + 2 \times H \times A \times \text{wall cost} + \text{students} \times \text{furniture cost per student}$$

Where: $A = L \times W$ is the floor area. Other values are constants.

3.3.2 Recurring Cost

Recurring costs are driven primarily by electricity for lighting, HVAC and sound systems. These are calculated as:

$$\text{Electricity Cost} = (\text{Lighting Energy} + \text{HVAC Energy} + \text{Sound Energy}) \times \text{electricity rate}$$

Where:

$$\text{Lighting Energy} = \text{light power density} \times A \times \text{daily usage} \times \text{days per year}$$
$$\text{HVAC Energy} = \text{hvac power density} \times A \times \text{daily usage} \times \text{days per year}$$
$$\text{Sound Energy} = \text{sound power density} \times V \times \text{daily usage} \times \text{days per year}$$

The total recurring cost is then:

$$\text{Recurring Cost} = \text{Electricity Cost} + \text{annual maintenance}$$

3.3.3 Total Cost

The total cost combines both capital and recurring costs:

$$\text{Total Cost} = \text{Capital Cost} + \text{Recurring Cost} \times \text{years}$$

The cost per student is given by:

$$\text{Cost per Student} = \frac{\text{Total Cost}}{\text{students}}$$

3.3.4 Penalties for Environmental Parameters

Penalties are applied when environmental conditions are not met. The penalties are calculated as:

$$P_{\mathrm{CO_2}} = (C_{\mathrm{CO_2}} - C_{\mathrm{limit}}), \quad P_E = (E_{\mathrm{target}} - E), \quad P_T = |T - T_{\mathrm{ref}}|, \quad P_{RT} = (RT - RT_{\max})$$

$$C_{\mathrm{CO_2}} = \frac{S + nVC_a}{nV}, \quad E = \frac{L}{A}, \quad T = 20 + \frac{5}{V}, \quad RT = \frac{0.161V}{A\alpha}$$

Where $P_{\mathrm{CO_2}}$, P_E, P_T, and P_{RT} are penalties for CO_2 concentration, illuminance, temperature, and reverberation time respectively. $C_{\mathrm{CO_2}}$ is indoor CO_2 concentration (ppm), C_{limit} is the permissible CO_2 limit, E is actual illuminance (lux), E_{target} is the target illuminance (lux), T is estimated temperature (°C), T_{ref} is the reference temperature (°C), RT is reverberation time (s), and $RT_{\max}$ is its upper limit. S is the CO_2 generation rate per person (kg/min), n is air exchange rate (1/hr), V is room volume (m^3), C_a is ambient CO_2 (ppm), L is total light output (lumens), A is surface area (m^2), and α is average absorption coefficient.

3.3.5 Objective Function

The objective function, which we aim to minimize, is:

$$\text{Objective Function} = \text{Cost per Student} + \frac{\text{Penalties}}{\text{Number of Students}}$$

3.4 Code Implementation

Gradient descent is very sensitive to the scale of the data, since the cost is 10^7 times more than the penalty term, so we divide the total cost by a factor of $10.^7$. Learning rate of 0.01 is used up to 100 iterations are performed.

Listing 1: Capital Cost Calculation

```
import numpy as np
import matplotlib.pyplot as plt

# Constants (Assumed)
CO2_target = 600  # Ideal CO2 concentration in ppm
Light_target = 400  # Ideal illuminance in lux
Temp_target = 25  # Ideal temperature in  C
Reverb_target = 1  # Ideal reverberation time in sec
students=120
number_years=5
# Learning rate for Adam
```

```
alpha = 0.001
beta1 = 0.9  # Momentum decay
beta2 = 0.999  # RMS decay
epsilon = 1e-8  # Small value to avoid division by zero

# Function to compute CO2 concentration (Simplified)
def co2_concentration(V):
    n = 5  # Air exchange rate (assumed)
    S = 0.005  # CO2 generation rate per person (assumed)
    Ca = 400  # Outdoor CO2 ppm
    return (S + n * V * Ca) / (n * V)

# Function to compute light intensity
def illuminance(A, lumens=3000):
    return lumens / A

# Function to compute temperature (simplified heat balance)
def temperature(V):
    return 20 + (5 / V)  # Assumption: Higher volume =
        better cooling

# Function to compute reverberation time (Sabine equation)
def reverberation_time(V, A):
    absorption_coeff = 0.2  # Assumed average absorption
        coefficient
    return 0.161 * V / (A * absorption_coeff)
# function for the capital cost
def capital_cost(L, W, H):
  base_cost=50000
  wall_cost=15000
  furniture_cost_per_student=2500
  wall_area=2*H*(L+W)
  capital_cost=base_cost+wall_area*wall_cost+students*
      furniture_cost_per_student
  return capital_cost/1e7
#function for the recurring cost
def recurring_cost(L, W):
  electricity_rate=8.5
  light_power_density=10
  hvac_power_density=15
  sound_power_density=0.1
  sound_efficiency=0.6
  daily_hr_usage=8
  days_per_year=250
  annual_maintenance=20000
  lighting_energy=light_power_density*L*W*daily_hr_usage*
      days_per_year
  hvac_energy=hvac_power_density*L*W*daily_hr_usage*
      days_per_year
  sound_energy=sound_power_density*L*W*daily_hr_usage*
```

```
      days_per_year
  electricity_cost=(lighting_energy+hvac_energy+sound_energy
      )*electricity_rate
  recurring_cost=electricity_cost+annual_maintenance
  return recurring_cost/1e7

# Objective Function (Deviation from optimal values)
def objective_function(L, W, H):
    V = L * W * H  # Classroom volume
    A = L * W  # Floor area
    cost_total=capital_cost(L, W, H)+recurring_cost(L, W)*
        number_years
    cost=cost_total/students
    # Deviation from target values
    co2_dev = abs(co2_concentration(V) - CO2_target)
    light_dev = abs(illuminance(A) - Light_target)
    temp_dev = abs(temperature(V) - Temp_target)
    reverb_dev = abs(reverberation_time(V, A) -
        Reverb_target)

    return (np.sqrt(co2_dev**2 + light_dev**2 + temp_dev**2
        + reverb_dev**2)/students+cost)  # Minimize this

# Optimization
def gradient_descent_optimization(lr=0.01, iterations=100):
    L, W, H = 15.0, 15.0, 5.0  # Initial classroom size
    obj_values = []

    for _ in range(iterations):
        # Compute gradients using numerical differentiation
        dL = (objective_function(L + 0.01, W, H) -
            objective_function(L, W, H)) / 0.01
        dW = (objective_function(L, W + 0.01, H) -
            objective_function(L, W, H)) / 0.01
        dH = (objective_function(L, W, H + 0.01) -
            objective_function(L, W, H)) / 0.01

        # Gradient descent updates
        L -= lr * dL
        W -= lr * dW
        H -= lr * dH

        obj_values.append(objective_function(L, W, H))

    return L, W, H, obj_values
```

4 Results and Discussion

4.1 Results

After running the above optimization, we get the final dimensions of the optimized classroom as: $L = 14.97\,\mathbf{m}$, $W = 14.97\,\mathbf{m}$, $H = 5.2\,\mathbf{m}$. We also get the optimized capital cost as **4839277 rupees** and the recurring cost as **9560092 rupees** per year . We get the convergence curve like below:

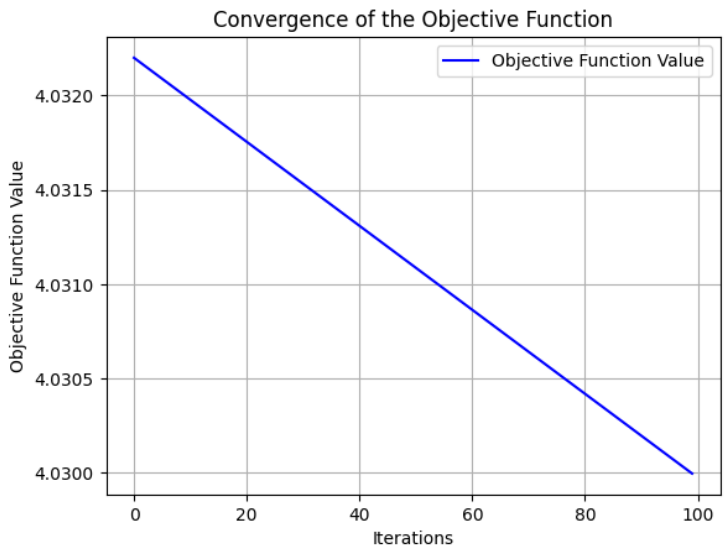

Figure 1: Convergence Curve

4.2 Improvements

While the current optimization provides a cost-effective classroom size, there is significant scope for future enhancements:

- **Integration of Real-time Sensor Data:** By incorporating real-time data from CO_2, temperature, humidity, and noise sensors, the model can dynamically adjust and re-optimize based on actual classroom conditions.
- **Machine Learning Integration:** Instead of manually defining the objective function, machine learning models like regression, decision trees, or even reinforcement learning can be used to learn optimal configurations from historical classroom usage and performance data.
- **Addition of More Variables:** Factors such as daylight exposure, ventilation patterns, student density, thermal comfort indices, and noise pollution can be added to make the model more holistic and realistic.